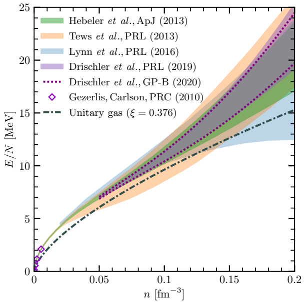
Uncertainty quantification for the nuclear equation of state

Hannah Göttling, Kai Hebeler, Achim Schwenk and Ingo Tews

From nuclei to neutron stars

- connect microscopic calculations of dense matter to astrophysical observations



Huth et al., PRC (2021)

The equation of state (EOS) is ...

... given by pressure as a function of density and temperature

... key to neutron stars and mergers

- compact objects under extreme conditions of density and temperature
- observations yield novel insights into nature of strongly interacting matter

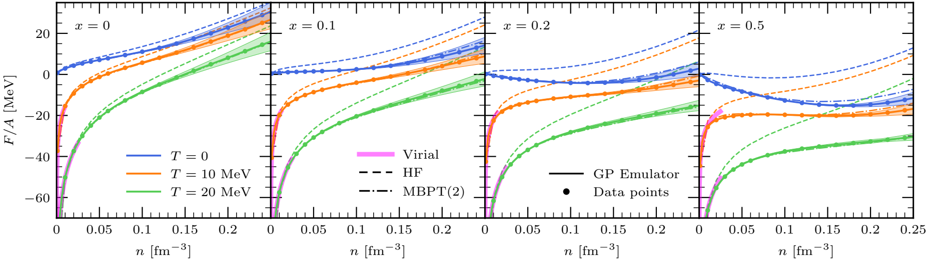
- provide microscopic information with quantified uncertainties

see also Drischler et al., PRC (2020)

Newest EOS developments

Keller et al., PRL (2023)

- nucleonic matter results for different proton fractions (x) and temperatures (T)



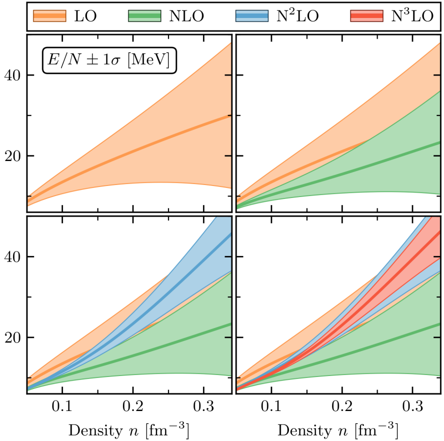
- Gaussian process (GP) emulator used to access arbitrary conditions
- effective field theory (EFT) uncertainties using pointwise predictions from **Epelbaum et al., (2015)**

State-of-the-art uncertainty quantification

Drischler et al., PRC (2020)

Correlated EFT uncertainties

- implemented using GP-Bayes approach
- explored for pure neutron matter (PNM) and symmetric nuclear matter (SNM)



Gaussian processes
following Rasmussen & Williams (2006)

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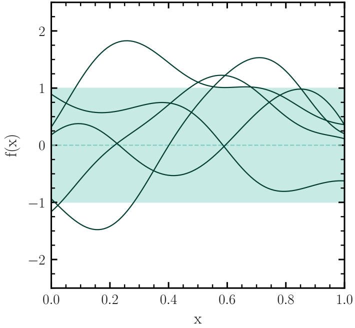
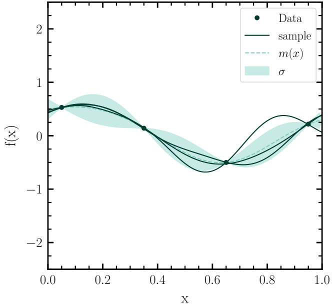
- collection of random variables: $f(x) \sim GP[m(x), \kappa(x, x'; \theta)]$
- kernel: $\kappa(x, x'; \theta) = \bar{c}^2 e^{-\frac{1}{2}(x-x')^T L (x-x')}$





- optimize to training data

GP-Bayes uncertainty quantification

Melendez et al. PRC (2019)

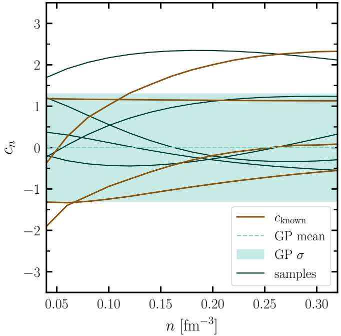
- consider EFT expansion for observable

$$E(x) = E_{\text{ref}}(x) \sum_{i=0}^{k} c_i(x) Q^i(x)$$

$\rightarrow$ reference energy E_{ref} and expansion parameter $Q = \frac{k_F}{\Lambda_b}$

- order-by-order convergence with natural size of coefficients
- coefficients are random variables

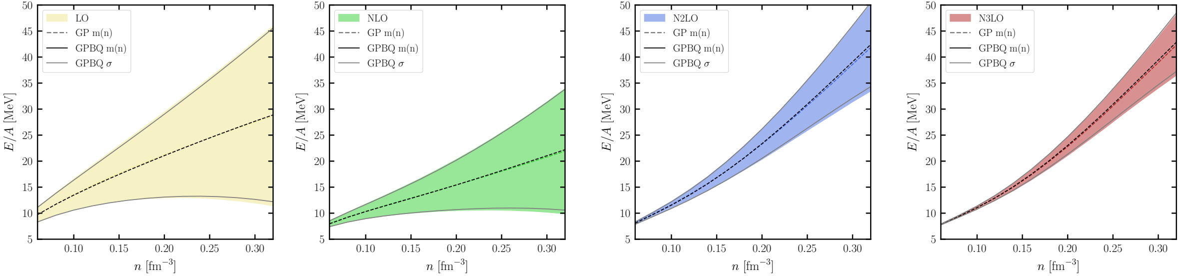
$$c_n(x)|\theta \sim GP[m(x), \kappa(x, x'; \theta)]$$



$\Rightarrow$ **hyperparameters are determined from expansion coefficients**

GP-B for pure neutron matter

- apply GP-B to results from **Keller et al. PRL (2023)** (colored)



- check our implementation against BUQEYE package **Drischler et al. (2020)** (GPBQ)

Goals

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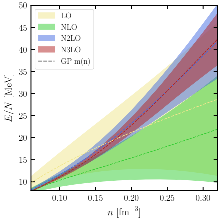
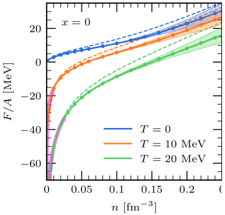
PNM

- investigate sensitivities of uncertainty bands to GP choices
- explore nonlocal and local chiral EFT and Δ-full results





multidimensional GP

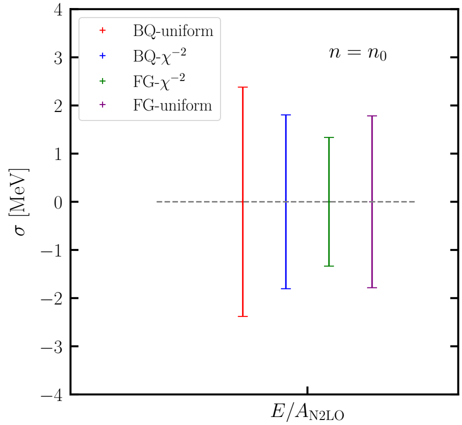
- decscribe the EOS (n,x,T) with one GP

Reference and prior choices

Melendez et al. PRC (2019)

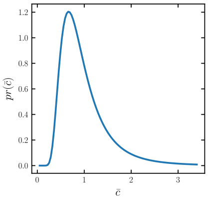
explore sensitivity to the reference energy

- using free gas reference energy E_{FG}
- applying the BUQEYE ansatz $E_{\mathrm{BQ}} = 16\ \mathrm{MeV} \left(\frac{n}{n_0}\right)^{2/3}$



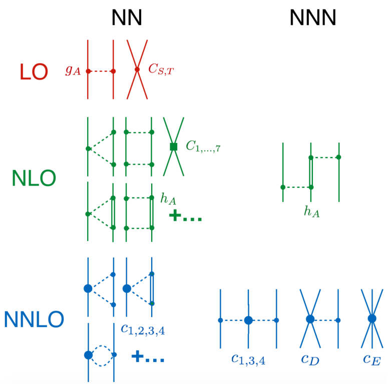
impact of priors for GP hyperparameters

- with normal inverse χ^2 prior
- uniform prior



Results for Δ-full chiral EFT interactions

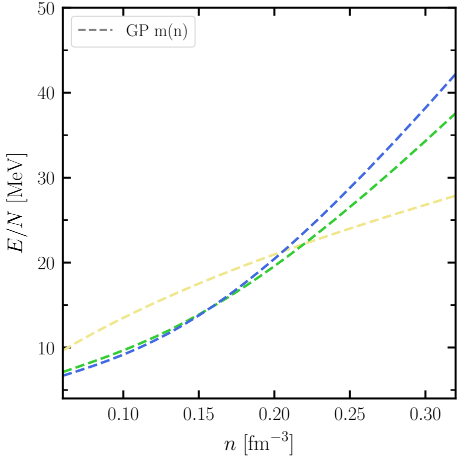
- 3N forces enter already at NLO
- 3N density dependence comes in earlier
  $\rightarrow$ expect better EFT convergence



Ekström et al., PRC (2018)

GP-B uncertainties for Δ-full interactions

- MBPT results from **Yannick Dietz** for $\Delta N2LO_{\mathrm{Go}}$ interaction



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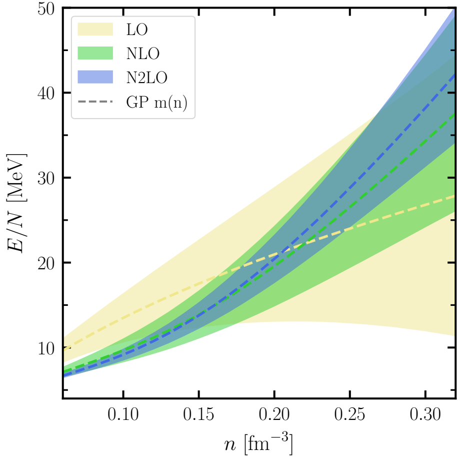
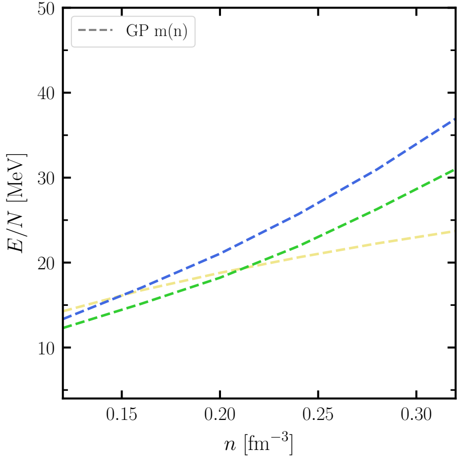
- GP-B calculated with $E_{\mathrm{ref}} = E_{BQ}$ and $pr(\bar{c}^2) \propto \chi^{-2}$



QMC results with new local interactions
from Ingo Tews and Rahul Somasundaram

- using quantum Monte Carlo (QMC) as many-body method
- using new local chiral N2LO interactions with large cutoff as input

  Somasundaram et al., PRC (2024)



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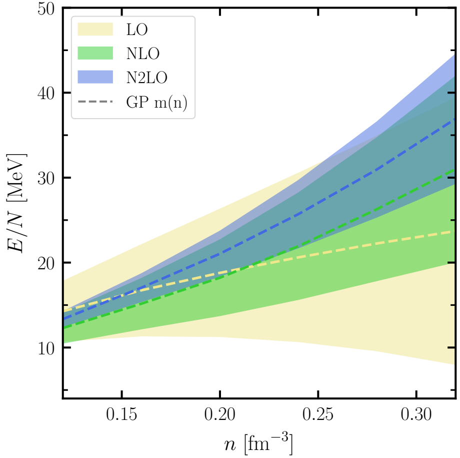
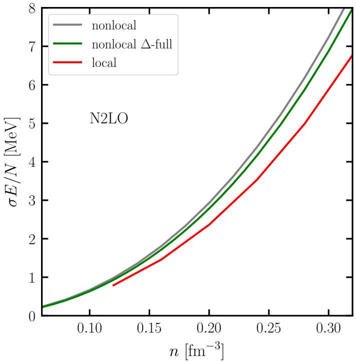
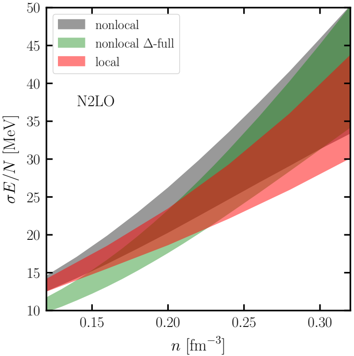


Comparison of GP-B uncertainties





- similar uncertainties for different interactions
- difference at subnuclear densities probably from different NN phaseshifts

Multidimensional GP
following Melendez et al., PRC (2019)

- extending the input space to multiple dimensions

1D

$x = n; k_\mathrm{F}$

$$\kappa(x, x'; \theta) = \bar{c}^2 e^{-\frac{1}{2l^2}(x-x')^2}$$

ND

$x = [n, x_p]^T \rightarrow [k_p, k_n]^T$

$$\kappa(x, x'; \theta) = \bar{c}^2 e^{-\frac{1}{2}(x-x')^T L (x-x')}$$

$$L = \begin{pmatrix} l_{k_p}^{-2} & 0 \\ 0 & l_{k_n}^{-2} \end{pmatrix}$$

$\Rightarrow$ $\bar{c}^2$ **is the same for the whole system**

Observable expansion for multiple variables

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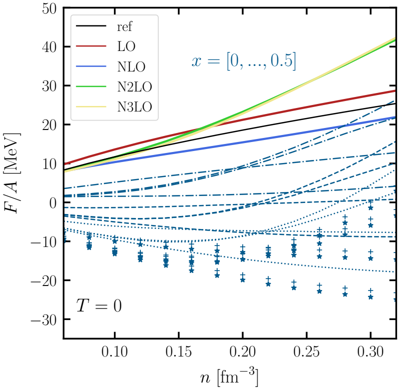
- use reference energy:

$$E_{\mathrm{ref}} = 16\ \mathrm{MeV} \left(\frac{n}{n_0}\right)^{2/3} \rightarrow \frac{16\ \mathrm{MeV}}{(3\pi^2 n_0)^{2/3}} \, (k_p^3 + k_n^3)^{2/3}$$

- consider relevant momentum scale:

$$Q = \frac{\max(k_p, k_n)}{\Lambda_{\mathrm{b}}}$$



nuclear matter calculations from Keller et al. PRL (2023)

Consistency check

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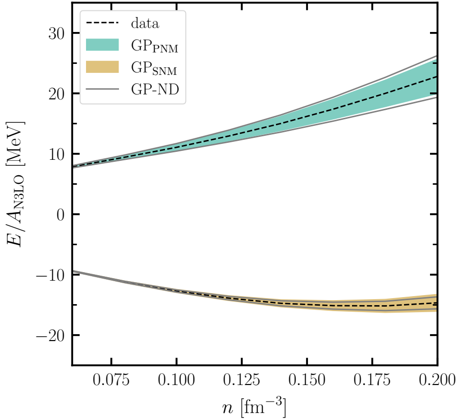
- include proton fractions $x = 0, 0.1, 0.2, 0.3, 0.4, 0.5$
- compare GP-ND with GP-1D PNM and SNM bands

$\rightarrow$ bands are similar



GP-bands for the EOS as function of proton fraction

Preliminary

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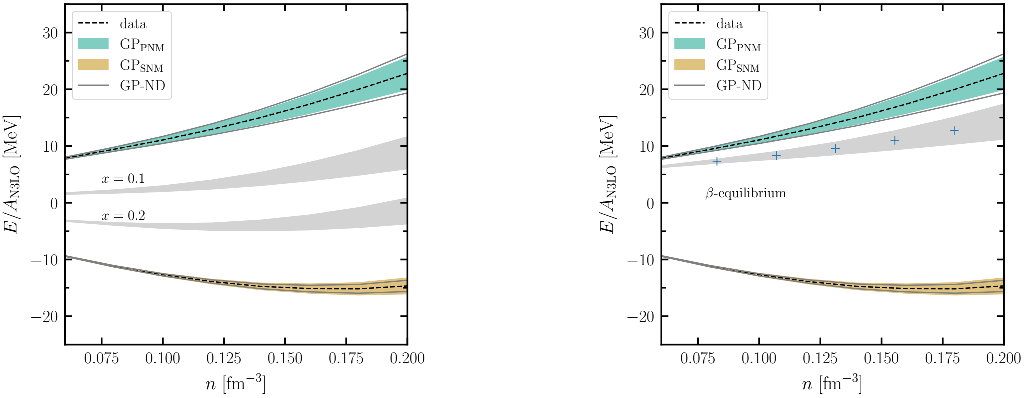

- provide uncertainties for finite x and β-equilibrium

Summary and outlook

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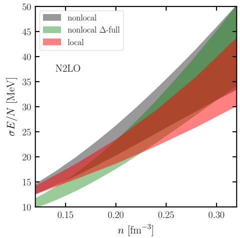
GPs ...

... provide an excellent **emulator**

... can access arbitrary points in **n,x,T**

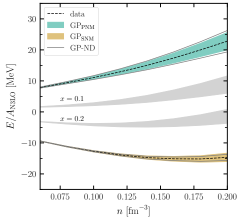
... quantify correlated **EFT uncertainties**

next: other thermodynamic properties



thoughts and questions

- Do we need a more physically informed reference scale? (e.g. at higher densities)
- Can the EOS (n,x,T) be described by a single GP?



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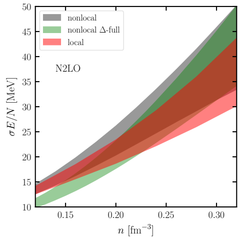

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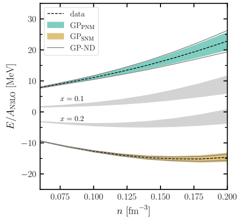

Thank you for your attention!