

# Uncertainty Estimation in Low-Energy **Fundamental Symmetry Tests:** The Case of Hadronic Parity Violation

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# Perspective

Non-zero  $\nu$  masses, a cosmic BAU, dark matter, dark energy are all established, but *the underlying dynamics — and any interconnections — are unclear*

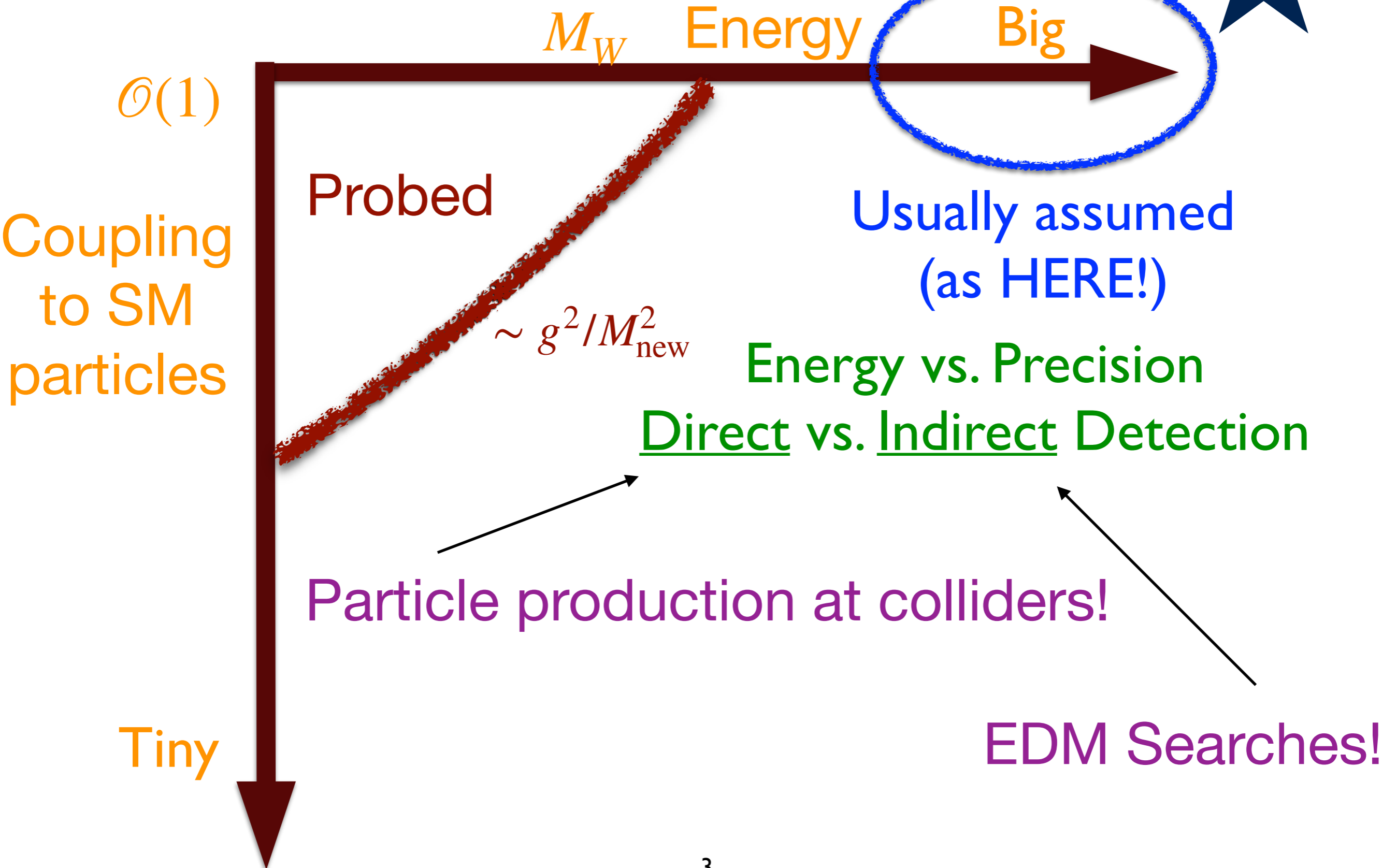
Discovering the mechanisms of this new physics rely, in part, on **fundamental symmetry** studies in hadrons, nuclei, atoms, and molecules

These are multi-scale problems, and QCD, and the ability to control it, flows through the interpretation of the experimental results

**TODAY: we consider “UQ” in these new physics searches — and use hadronic parity violation in the SM as a particular example**

# New Particle Discovery Space

## Dichotomies

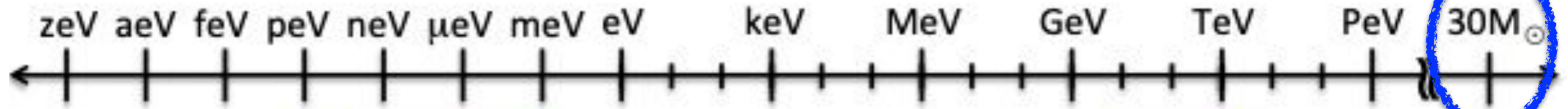


# A Vast Range of Dark Matter Candidates

## Particle Masses

Elementary Particle

Fits in Galaxy



“Fuzzy DM”

“WIMPs”

“Exotics”

Phenomenology controlled by deBroglie  $\lambda$   
at the Sun’s location:

$$n_{\text{DM}} \lambda_{\text{dB}}^3 \gg 1$$

Behaves like  
a classical field

*Wave-like!*

$$n_{\text{DM}} \lambda_{\text{dB}}^3 \ll 1$$

Rare collisions

*Particle-like!*

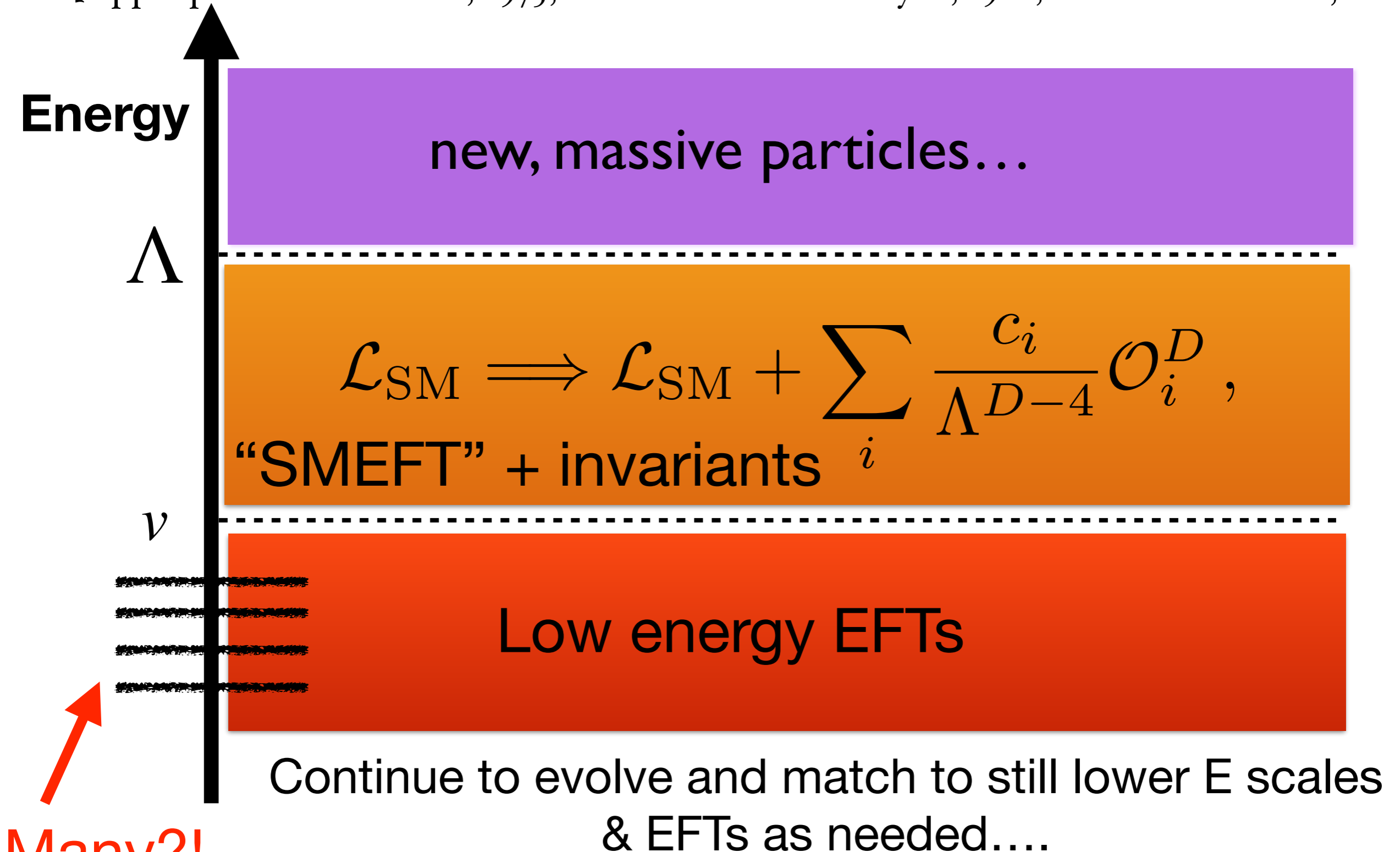
“Black Holes”!

Cosmic small-scale structure is washed out if DM is too light  
New opportunities!! New probes!! (Also theory driven!)

# Model Independent Analysis

Assuming new physics scale  $\Lambda$  heavy cf. to the weak scale  $v$

[Appelquist & Carazzone, 1975; note Buchmuller & Wyler, 1986; Grzadkowski et al., 2010]



Many?!

# “Fundamental Symmetries”

➔ Studies of B, C, CP, L, P, & T Violation  
in Nuclei —and Light Hadrons (& more!)

**Motivation: A cosmic baryon asymmetry...**

The particle physics of the early universe can explain this asymmetry if **B** (baryon number), **C** (particle-antiparticle), and **CP** (matter-antimatter) **violation** all exist in a non-equilibrium environment. [Sakharov, 1967]

**But what is the mechanism?**

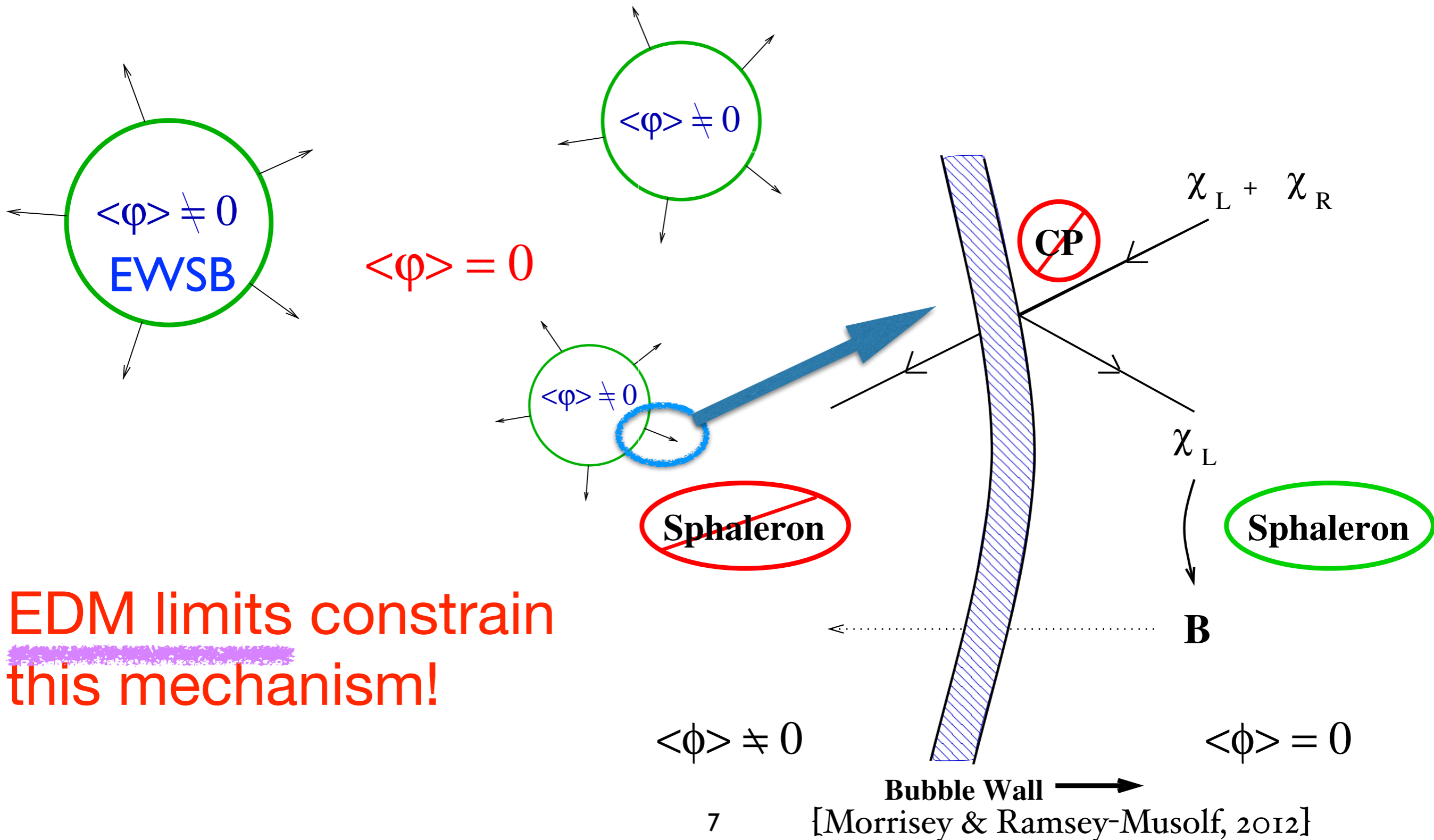
The SM **almost** has the right ingredients, but we need BSM physics to explain it. Probes?

permanent EDMs       $0\nu\beta\beta$  decay       $n\bar{n}$  oscillations

Also...  $\mu \rightarrow e$  conversion;  $\beta$ -decay correlations...

# A Cosmic Baryon Asymmetry

Via electroweak baryogenesis (& a SFO EWPT!)



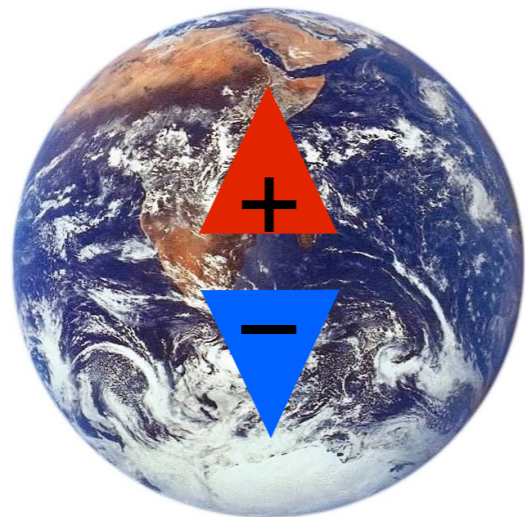
EDM limits constrain this mechanism!

# EDMs to Probe CPV for a BAU?

**Current limits for the electron and neutron strongly constrain models of EW baryogenesis**

Neutron:  $|d_n| < 1.8 \times 10^{-26}$  e-cm [90 % C.L.] [Abel et al., 2020]

For a sense of scale:



Scaling the  $n$  to Earth's size implies a charge separation of  $< 4\mu\text{m}$   
(cf. human hair width  $40\mu\text{m}$ )

**Expts under development reach for 10-100x sensitivity**

**Applied electric fields can be enormously enhanced**

**in atoms and molecules** [Purcell and Ramsey, 1950]

ACME II, 2018 (ThO):  $|d_e| < 1.1 \times 10^{-29}$  e-cm [90 % C.L.]

Roussy et al., 2023 (HfF<sup>+</sup>):  $|d_e| < 4.1 \times 10^{-30}$  e-cm [90 % C.L.]

**New CPV sources not yet observed....**



# EFT+QCD for New Physics Searches

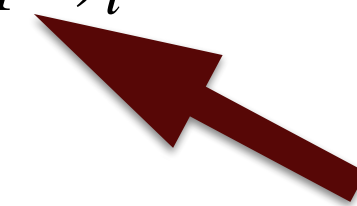
Assuming new physics **heavy** cf. to the weak scale

Hadronic Matrix Elements (indirect):

**Context:** Assert T &/or P, or B &/or B-L... broken at a high scale  $\Lambda_{\text{new}}$  to extend the SM: enter **SMEFT**

$$\mathcal{L}_{\text{SM}} \implies \mathcal{L}_{\text{SM}} + \sum_i \frac{c_i}{\Lambda^{d-4}} \mathcal{O}_i^d \quad \text{[Buchmuller & Wyler, 1986; Grzadkowski et al., 2010]}$$

e.g.:  $p \rightarrow e^+ \pi^0$

$$\mathcal{L}_{|\Delta B|=1}^{(d=6)} \supset \sum_i \frac{c_i}{\Lambda_{|\Delta B|=1}^2} (qqq\ell)_i + \text{h.c.}$$


For  $c_i$  work in an explicit BSM model or make  $\mathcal{O}(1)$  — with matrix element

experimental limit bounds  $\Lambda_{\text{new}} \dots$

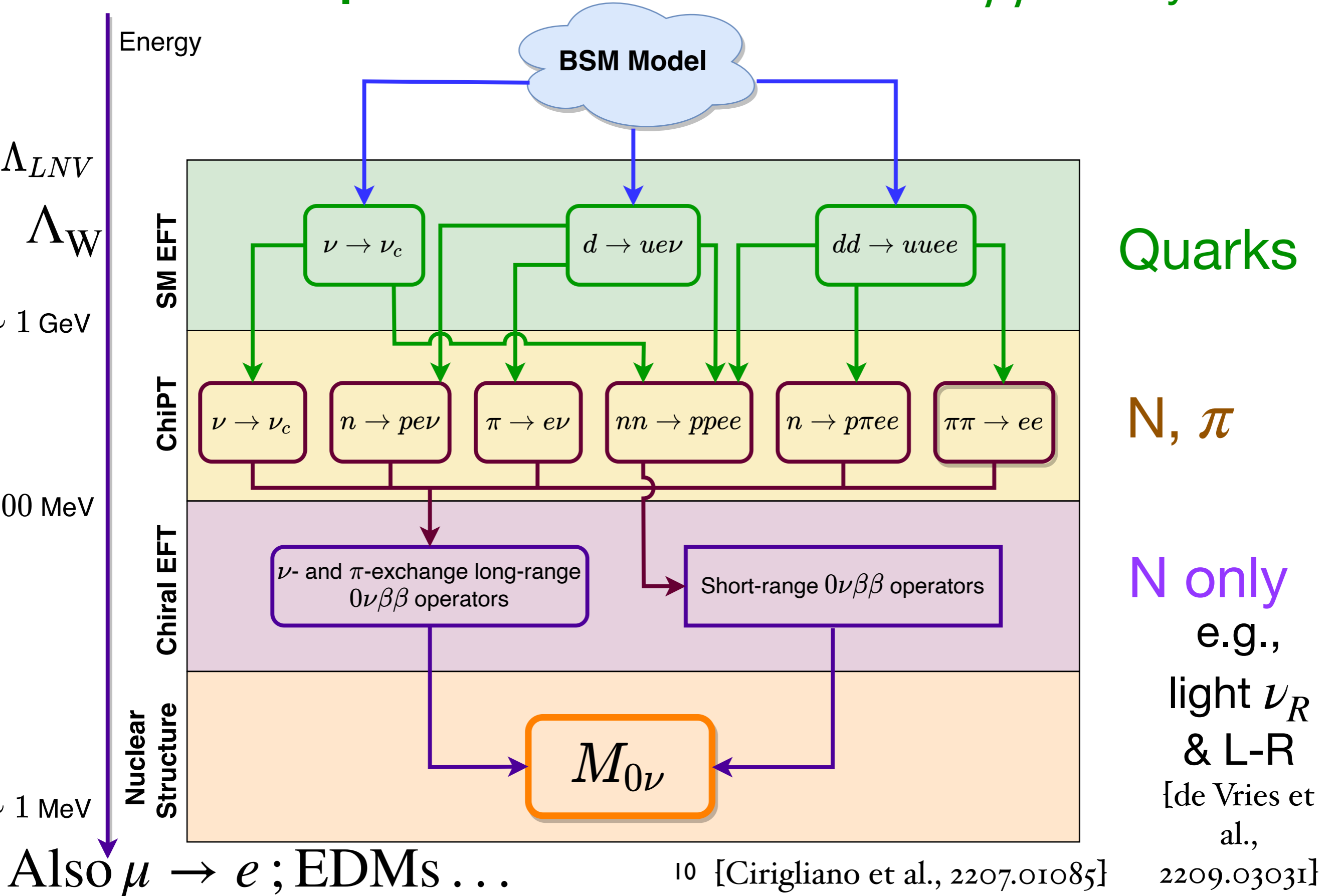
$$p \rightarrow \ell^+ \pi^0 \implies \Lambda_{\text{new}} > 10^{15} \text{ GeV!}$$

Local operator:  
LQCD to compute its  
hadronic matrix element

[e.g., Aoki et al., FLAG review, 2111.09849]

# Connecting LNV to Complex Systems

**Example:** “Tower of theories” for  $0\nu\beta\beta$  decay



# EFT+QCD for New Physics Searches

Assume new physics is heavy but much more accessible

Here SM / QCD RG effects must be addressed

Nonzero signals in

permanent EDMs	$0\nu\beta\beta$ decay	$n\bar{n}$ oscillations
BSM sources of CPV(*)	$ \Delta L  = 2$	$ \Delta B  = 2$

would speak to BSM physics

Here UQ is essential to understanding  
the BSM landscape

How can we test our assessments?

# SM + QCD at scale $M_W$



SM example: hadronic parity violation ( $\Delta F = 0$ )

$$\mathcal{H}_{\text{eff}} = \mathcal{H}_Z + \mathcal{H}_W$$

$$\mathcal{H}_Z(M_W) = \frac{G_F s_W^2}{3\sqrt{2}} \left( \Theta_1 - 3 \left( \frac{1}{2s_W^2} - 1 \right) \Theta_5 \right)$$

$$\Theta_1 = [(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V]^{\alpha\alpha} [(\bar{u}u)_A - (\bar{d}d)_A - (\bar{s}s)_A]^{\beta\beta}$$

$$\Theta_5 = [(\bar{u}u)_V - (\bar{d}d)_V - (\bar{s}s)_V]^{\alpha\alpha} [(\bar{u}u)_A - (\bar{d}d)_A - (\bar{s}s)_A]^{\beta\beta}$$

$$\mathcal{H}_W(M_W) = -\frac{G_F}{\sqrt{2}} (\cos^2(\theta_c)\Theta_9 + \sin^2(\theta_c)\Theta_{11})$$

$$\Theta_9 = (\bar{u}d)_V^{\alpha\alpha} (\bar{d}u)_A^{\beta\beta} + (\bar{d}u)_V^{\alpha\alpha} (\bar{u}d)_A^{\beta\beta}$$

$$\Theta_{11} = (\bar{u}s)_V^{\alpha\alpha} (\bar{s}u)_A^{\beta\beta} + (\bar{s}u)_V^{\alpha\alpha} (\bar{u}s)_A^{\beta\beta}$$

Gluon radiation modifies  $\mathcal{H}_{\text{eff}}$

Can we determine its outcomes at low(er) energies?  
Compare to experiments?

# Hadronic Parity Violation ( $\Delta F = 0$ ) in Nuclei

The experiments are very challenging

[SG, Haxton, & Holstein, 2017]

$$A_L(\vec{p} p) = \begin{cases} (-0.93 \pm 0.20 \pm 0.05) \times 10^{-7} \\ (-1.7 \pm 0.8) \times 10^{-7} \\ (-1.57 \pm 0.23) \times 10^{-7} \\ (0.84 \pm 0.34) \times 10^{-7} \end{cases}$$

13.6 MeV (40)

15 MeV (41)

45 MeV (42–44)

221 MeV (45, 46)

$$A_L(\vec{p} \alpha) \Big|_{46 \text{ MeV}} = -(3.3 \pm 0.9) \times 10^{-7}$$

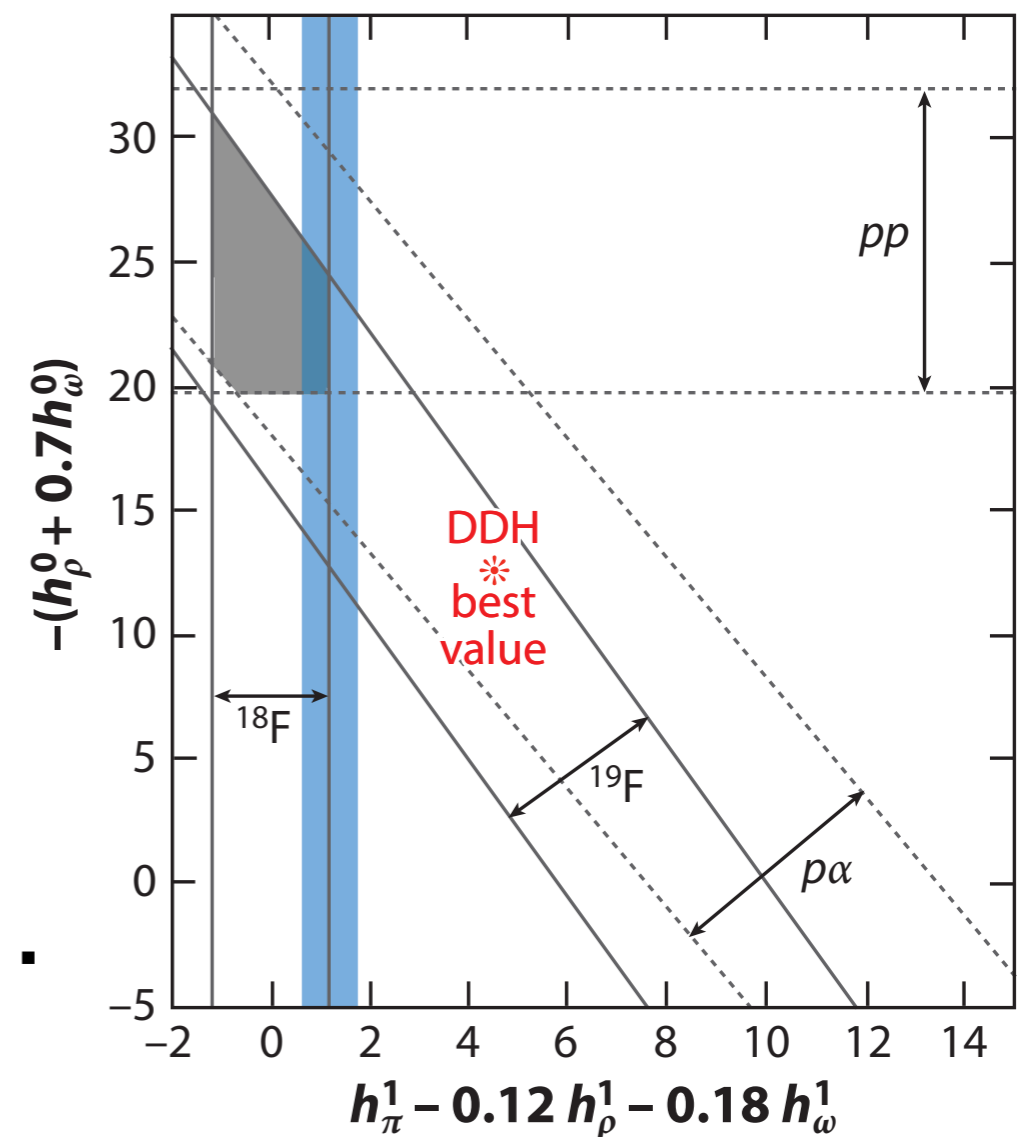
angular asymmetry in  $^{19}\text{F}$

$$A_\gamma = \begin{cases} (-8.5 \pm 2.6) \times 10^{-5} & \text{Seattle (16)} \\ (-6.8 \pm 1.8) \times 10^{-5} & \text{Mainz (49, 50)} \end{cases}$$

limits in  $^{18}\text{F}$ ...

And newer results in  $A = 2, 4, \dots$

[Desplanques, Donoghue, Holstein (DDH), 1980]



# Hadronic Parity Violation ( $\Delta F = 0$ ) in Nuclei

At very low energies, 5 PV NNNN contact interactions exist

[Danilov 1965, 1971, Zhu et al. 2005, Girlanda, 2008]

$$\begin{aligned}
 V_{\text{LO}}^{\text{PNC}}(\mathbf{r}) = & \Lambda_0^{1S_0-3P_0} \left( \frac{1}{i} \frac{\overleftrightarrow{\nabla}_A \delta^3(\mathbf{r})}{2m_N m_\rho^2} \cdot (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) - \frac{1}{i} \frac{\overleftrightarrow{\nabla}_S \delta^3(\mathbf{r})}{2m_N m_\rho^2} \cdot i(\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2) \right) \\
 & + \Lambda_0^{3S_1-1P_1} \left( \frac{1}{i} \frac{\overleftrightarrow{\nabla}_A \delta^3(\mathbf{r})}{2m_N m_\rho^2} \cdot (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) + \frac{1}{i} \frac{\overleftrightarrow{\nabla}_S \delta^3(\mathbf{r})}{2m_N m_\rho^2} \cdot i(\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2) \right) \\
 & + \Lambda_1^{1S_0-3P_0} \left( \frac{1}{i} \frac{\overleftrightarrow{\nabla}_A \delta^3(\mathbf{r})}{2m_N m_\rho^2} \cdot (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)(\tau_{1z} + \tau_{2z}) \right) \\
 & + \Lambda_1^{3S_1-3P_1} \left( \frac{1}{i} \frac{\overleftrightarrow{\nabla}_A \delta^3(\mathbf{r})}{2m_N m_\rho^2} \cdot (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)(\tau_{1z} - \tau_{2z}) \right) \\
 & + \Lambda_2^{1S_0-3P_0} \left( \frac{1}{i} \frac{\overleftrightarrow{\nabla}_A \delta^3(\mathbf{r})}{2m_N m_\rho^2} \cdot (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1 \otimes \boldsymbol{\tau}_2)_{20} \right),
 \end{aligned}$$

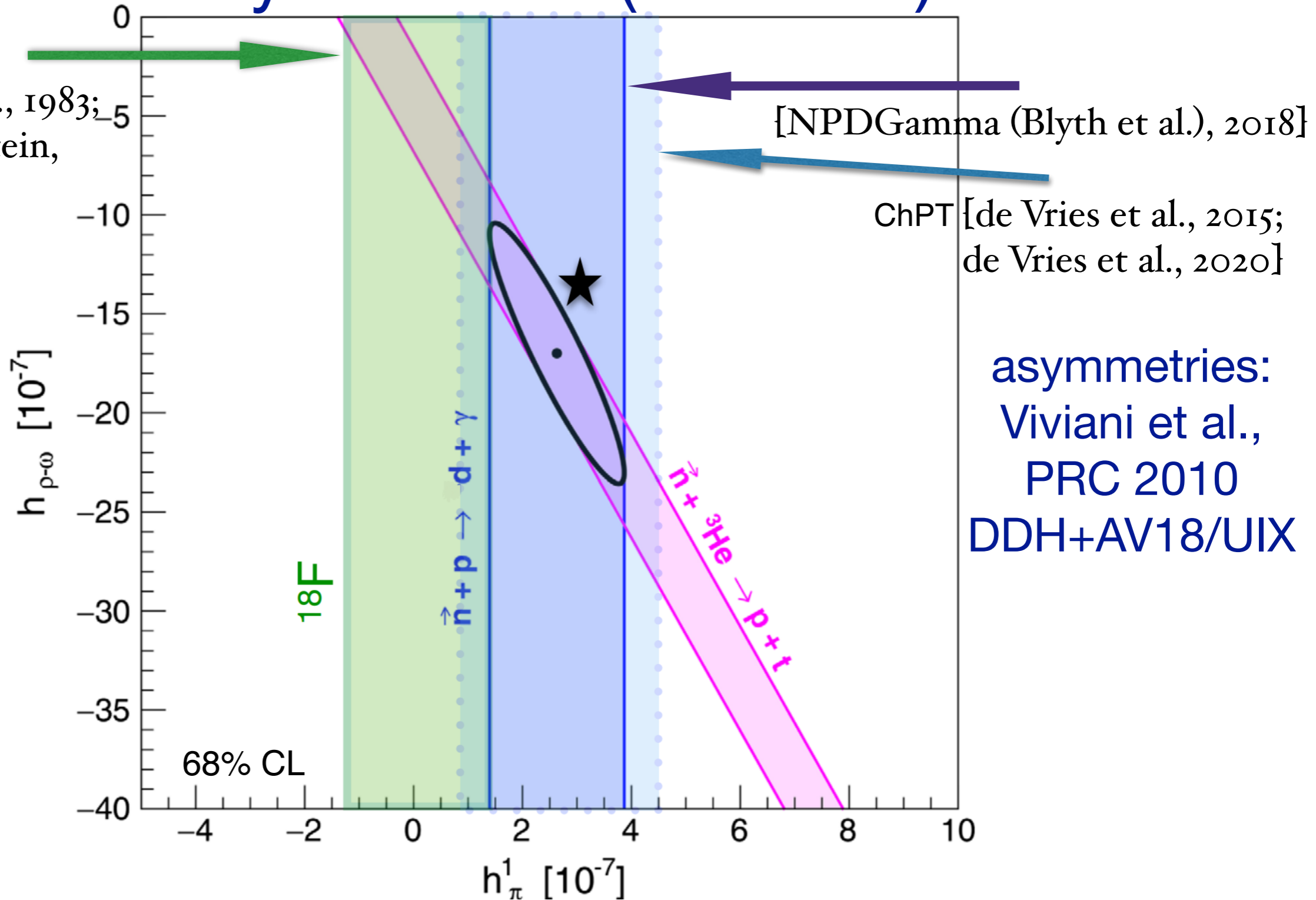
$2s+1 S_I - 2s'+1 P_I$   
 mixing

But there are **not enough experiments** to fix all the coefficients (& note map to DDH) [Haxton & Holstein, 2013]

And it may be that none are negligibly small

# Hadronic Parity Violation ( $\Delta F = 0$ ) in Nuclei

[Haxton, 1981;  
Adelberger et al., 1983;  
Haxton & Holstein,  
2013]



$${}_{n3\text{He}}: h_{\rho-\omega} \equiv h_{\rho}^0 + 0.605h_{\omega}^0 - 0.605h_{\rho}^1 - 1.316h_{\omega}^1 + 0.026h_{\rho}^2 = (-17.0 \pm 6.56) \times 10^{-7}$$

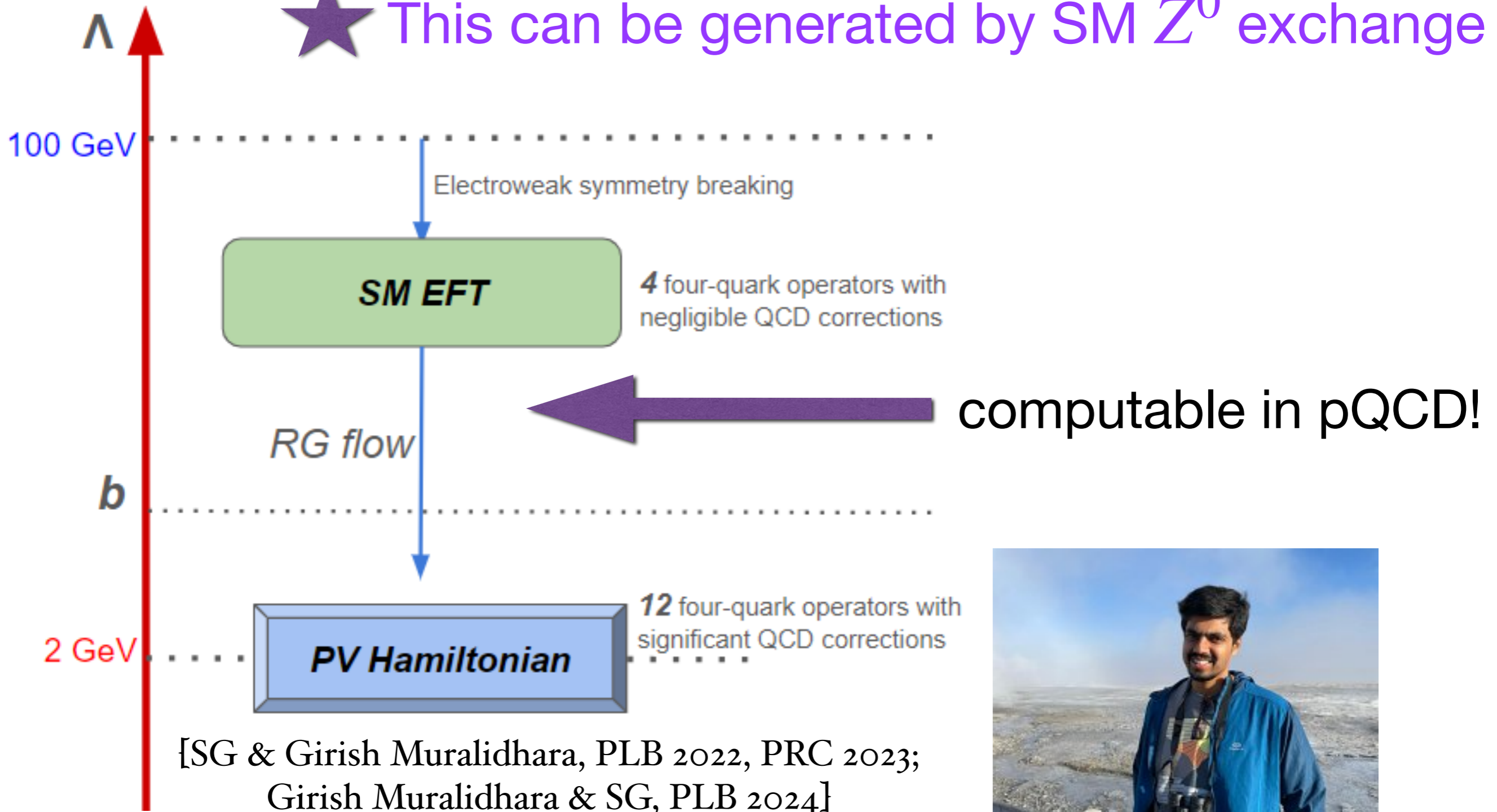
$$\text{LOQCD+LQCD}: h_{\rho-\omega} = -12.9 \pm 0.52 + \begin{pmatrix} 0.97 \\ -1.9 \end{pmatrix} + 0.62 + (-3.4) \times 10^{-7};$$

[SG & Muralidhara, 2023; 15 after  $n_3\text{He}$  (Gericke et al.), 2020]

# EFT+QCD for New Physics Searches

SM example: hadronic parity violation ( $\Delta F = 0$ )

★ This can be generated by SM  $Z^0$  exchange



Analysis through 2-loop order



# An Effective Hamiltonian for HPV

At  $\mu = 2 \text{ GeV}$  for all isosectors

[Dai et al., 1991; SG & Girish Muralidhara, 2022; Girish Muralidhara & SG, 2024]

$$\mathcal{H}_{\text{eff}}^{l=1}(\mu) = \frac{G_F s_W^2}{3\sqrt{2}} \sum_{i=1}^{10} C_i^{l=1}(\mu) \Theta_i^{l=1}$$

$$\mathcal{H}_{\text{eff}}^{l=0\oplus 2}(\mu) = \frac{G_F s_W^2}{3\sqrt{2}} \sum_{i=1}^{10} C_i^{l=0\oplus 2}(\mu) \Theta_i^{l=0\oplus 2}$$

$$\Theta_1^{l=1} = [(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V]^{\alpha\alpha} [(\bar{u}u)_A - (\bar{d}d)_A]^{\beta\beta}$$

$$\Theta_2^{l=1} = [(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V]^{\alpha\beta} [(\bar{u}u)_A - (\bar{d}d)_A]^{\beta\alpha}$$

$$\Theta_3^{l=1} = [(\bar{u}u)_A + (\bar{d}d)_A + (\bar{s}s)_A]^{\alpha\alpha} [(\bar{u}u)_V - (\bar{d}d)_V]^{\beta\beta}$$

$$\Theta_4^{l=1} = [(\bar{u}u)_A + (\bar{d}d)_A + (\bar{s}s)_A]^{\alpha\beta} [(\bar{u}u)_V - (\bar{d}d)_V]^{\beta\alpha}$$

$$\Theta_5^{l=1} = (\bar{s}s)_V^{\alpha\alpha} [(\bar{u}u)_A - (\bar{d}d)_A]^{\beta\beta}$$

$$\Theta_6^{l=1} = (\bar{s}s)_V^{\alpha\beta} [(\bar{u}u)_A - (\bar{d}d)_A]^{\beta\alpha}$$

$$\Theta_7^{l=1} = (\bar{s}s)_A^{\alpha\alpha} [(\bar{u}u)_V - (\bar{d}d)_V]^{\beta\beta}$$

$$\Theta_8^{l=1} = (\bar{s}s)_A^{\alpha\beta} [(\bar{u}u)_V - (\bar{d}d)_V]^{\beta\alpha}$$

$$\Theta_9^{l=1} = (\bar{u}s)_V^{\alpha\alpha} (\bar{s}u)_A^{\beta\beta} + (\bar{s}u)_V^{\alpha\alpha} (\bar{u}s)_A^{\beta\beta}$$

$$\Theta_{10}^{l=1} = (\bar{u}s)_V^{\alpha\beta} (\bar{s}u)_A^{\beta\alpha} + (\bar{s}u)_V^{\alpha\beta} (\bar{u}s)_A^{\beta\alpha}$$

$[\sin^2 \theta_C]$

$$\Theta_1^{l=0\oplus 2} = [(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V]^{\alpha\alpha} [(\bar{s}s)_A]^{\beta\beta}$$

$$\Theta_2^{l=0\oplus 2} = [(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V]^{\alpha\beta} [(\bar{s}s)_A]^{\beta\alpha}$$

$$\Theta_3^{l=0\oplus 2} = [(\bar{u}u)_A + (\bar{d}d)_A + (\bar{s}s)_A]^{\alpha\alpha} [(\bar{s}s)_V]^{\beta\beta}$$

$$\Theta_4^{l=0\oplus 2} = [(\bar{u}u)_A + (\bar{d}d)_A + (\bar{s}s)_A]^{\alpha\beta} [(\bar{s}s)_V]^{\beta\alpha}$$

$$\Theta_5^{l=0\oplus 2} = [(\bar{u}u)_V - (\bar{d}d)_V]^{\alpha\alpha} [(\bar{u}u)_A - (\bar{d}d)_A]^{\beta\beta} + (\bar{s}s)_V^{\alpha\alpha} (\bar{s}s)_A^{\beta\beta}$$

$$\Theta_6^{l=0\oplus 2} = [(\bar{u}u)_V - (\bar{d}d)_V]^{\alpha\beta} [(\bar{u}u)_A - (\bar{d}d)_A]^{\beta\alpha} + (\bar{s}s)_V^{\alpha\beta} (\bar{s}s)_A^{\beta\alpha}$$

$$\Theta_7^{l=0\oplus 2} = [(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V]^{\alpha\alpha} [(\bar{u}u)_A + (\bar{d}d)_A + (\bar{s}s)_A]^{\beta\beta}$$

$$\Theta_8^{l=0\oplus 2} = [(\bar{u}u)_A + (\bar{d}d)_A + (\bar{s}s)_A]^{\alpha\beta} [(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V]^{\beta\alpha}$$

$$\Theta_9^{l=0\oplus 2} = (\bar{u}d)_V^{\alpha\alpha} (\bar{d}u)_A^{\beta\beta} + (\bar{d}u)_V^{\alpha\alpha} (\bar{u}d)_A^{\beta\beta}$$

$$\Theta_{10}^{l=0\oplus 2} = (\bar{u}d)_V^{\alpha\beta} (\bar{d}u)_A^{\beta\alpha} + (\bar{d}u)_V^{\alpha\beta} (\bar{u}d)_A^{\beta\alpha}$$

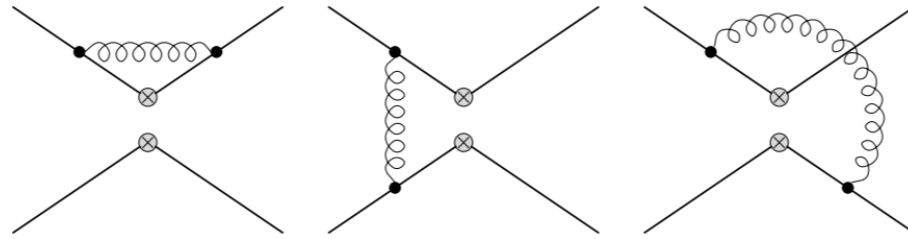
$[\cos^2 \theta_C]$

These operator sets close under renormalization  
& their mixing is characterized by an  
**anomalous dimension matrix**

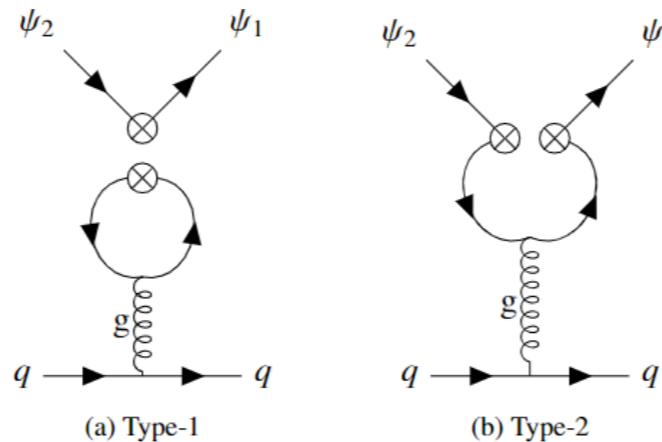
# Example

Operators renormalize and mix even under LO QCD corrections. Inserting

$$\Theta_1 = [(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V]^{\alpha\alpha} [(\bar{u}u)_A - (\bar{d}d)_A - (\bar{s}s)_A]^{\beta\beta}$$



LO current-current corrections



LO Penguin corrections

$$\Theta_1 \rightarrow \Theta_1 + \frac{g^2 \Gamma(\frac{\epsilon}{2})}{(4\pi)^2 \mu^\epsilon} \left( \frac{2}{9} \Theta_1 - \frac{2}{3} \Theta_2 + 1 \Theta_3 - 3 \Theta_4 \right)$$

# An Effective Hamiltonian for HPV

Since QCD is flavor blind up to quark mass effects...

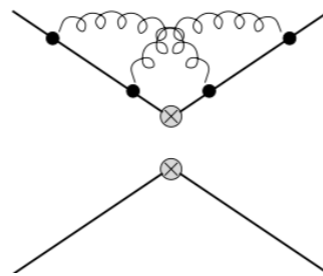
## Current-Current Basis

Renormalization of the  $\Delta S = 1$  physics operators:

$$\begin{aligned} \mathbf{O}_1 &= (\bar{s}u)_{V-A}^{\alpha\alpha} (\bar{u}d)_{V-A}^{\beta\beta} & \mathbf{O}_2 &= (\bar{s}u)_{V-A}^{\alpha\beta} (\bar{u}d)_{V-A}^{\beta\alpha} \\ \mathbf{O}_5 &= (\bar{s}d)_{V-A}^{\alpha\alpha} \sum_q^f (\bar{q}q)_{V+A}^{\beta\beta} & \mathbf{O}_6 &= (\bar{s}d)_{V-A}^{\alpha\beta} \sum_q^f (\bar{q}q)_{V+A}^{\beta\alpha} \end{aligned}$$

Buras et al.,  
Nucl. Phys. B (1993)

due to insertions into diagrams such as:



give the information of the mixing of prototype basis:

$$\begin{aligned} \Phi_1 &= (\psi_1 \psi_2)_V^{\alpha\alpha} (\psi_3 \psi_4)_A^{\beta\beta} \\ \Phi_2 &= (\psi_1 \psi_2)_V^{\alpha\beta} (\psi_3 \psi_4)_A^{\beta\alpha} \\ \Phi_3 &= (\psi_1 \psi_2)_A^{\alpha\alpha} (\psi_3 \psi_4)_V^{\beta\beta} \\ \Phi_4 &= (\psi_1 \psi_2)_A^{\alpha\beta} (\psi_3 \psi_4)_V^{\beta\alpha} \end{aligned}$$

$$C_{NLO} = \left( \frac{\alpha_s}{4\pi} \right)^2 \begin{pmatrix} \frac{1279}{12} - \frac{20f}{3} & \frac{17}{4} - \frac{4f}{3} & \frac{2f}{9} - \frac{173}{12} & \frac{173}{4} - \frac{2f}{3} \\ \frac{95}{2} - \frac{5f}{3} & \frac{149}{6} - \frac{17f}{3} & -\frac{f}{3} & \frac{202}{3} - \frac{7f}{9} \\ \frac{2f}{9} - \frac{173}{12} & \frac{173}{4} - \frac{2f}{3} & \frac{1279}{12} - \frac{20f}{3} & \frac{17}{4} - \frac{4f}{3} \\ -\frac{f}{3} & \frac{202}{3} - \frac{7f}{9} & \frac{95}{2} - \frac{5f}{3} & \frac{149}{6} - \frac{17f}{3} \end{pmatrix}$$

with penguin  
type-1,2  
contributions  
we get the  
full a.d. matrix

# An Effective Hamiltonian for HPV

- ▶ Use prototype basis  $\vec{\Phi}_{cc}$  and corresponding ADM  $C_{NLO}$  on operators of HPV:  $\gamma_{cc,NLO}^{HPV}$
- ▶ Use prototype basis  $\vec{\Phi}_p$  and mixing schemes  $\mathcal{P}_1$  and  $\mathcal{P}_2$  on operators of HPV:  $\gamma_{penguin,NLO}^{HPV}$
- ▶ The NLO mixing matrix for the Z-sector:  $\gamma_{cc,NLO}^{HPV} + \gamma_{penguin,NLO}^{HPV} = \gamma_{NLO}^{HPV} =$

$$\frac{\alpha_S^2}{(4\pi)^2} \begin{pmatrix} \frac{97087}{972} - \frac{64f}{9} & \frac{6737}{324} & \frac{2f}{9} - \frac{1205}{108} & \frac{1717}{36} - \frac{2f}{3} & 0 & 0 & -\frac{142q}{9} & -\frac{2q}{3} \\ \frac{281}{6} - \frac{1970f}{243} & \frac{818f}{81} + \frac{161}{6} & 16 - \frac{20f}{27} & \frac{202}{3} - \frac{4f}{3} & 0 & 0 & -\frac{92q}{27} & -\frac{52q}{9} \\ -\frac{20525}{972} & \frac{19373}{324} & \frac{28f}{3} + \frac{11863}{108} & \frac{313}{36} - \frac{4f}{3} & 0 & 0 & \frac{4q}{9} & -\frac{4q}{3} \\ -\frac{16f+18}{27} & \frac{208}{3} & 2f + \frac{127}{2} & \frac{149-4f}{6} & 0 & 0 & \frac{1664q}{243} & -\frac{1232q}{81} \\ \frac{2q}{3} & -2q & -16q & 0 & \frac{553}{6} - \frac{58f}{9} & \frac{95}{2} - 2f & -\frac{836}{243} & \frac{1700}{81} \\ \frac{1628q}{243} & -\frac{1340q}{81} & -\frac{88q}{27} & -\frac{40q}{9} & \frac{95}{2} - 2f & \frac{553}{6} - \frac{58f}{9} & \frac{46}{3} & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{80f}{9} + \frac{43121}{486} & \frac{11095}{162} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{377}{6} - \frac{1322f}{243} & \frac{1178f}{81} + \frac{565}{6} \end{pmatrix}$$

& compute RG flow  
to yield  $\vec{C}(2 \text{ GeV})$

[Girish Muralidhara & SG, PLB, 2024]

# Example: Meson-nucleon Couplings

## Use nucleon charges from lattice QCD

- The DDH's meson-exchange phenomenological HPV Hamiltonian is dictated by couplings  $h_M^I$  for meson  $M$  and isosector  $I$ :  
 $h_\pi^1, h_{\rho^0}^1, h_\rho^0, h_\rho^2, h_\omega^0$  and  $h_\omega^1$  [Desplanques, Donoghue, Holstein (DDH), 1980]
- To obtain them from our RG Hamiltonian, we make the following matching from the quark to hadron level:  $\langle MN' | \mathcal{H}_{\text{eff}}^I | N \rangle = \langle MN' | \mathcal{H}_{\text{DDH}} | N \rangle$
- For example, the pion contribution to hadronic PV:

$$\mathcal{H}_{\text{DDH}}^\pi = ih_\pi^1 (\pi^+ \bar{p}n - \pi^- \bar{n}p) \implies -ih_\pi^1 \bar{u}_n u_p = \langle n\pi^+ | \mathcal{H}_{\text{eff}}^{I=1} | p \rangle$$

$u_N$  is a Dirac spinor.

- Next, we make use of factorization approximation to evaluate these matrix elements. If we consider vector meson (V) emission, the factorization approximation for long-distance hadronic interaction matrix elements in terms of four-quark operators separate as

$$\langle VN' | (\bar{q}_1 q_2)_v \bar{q}_3 q_4)_A | N \rangle = \langle V | (\bar{q}_1 q_2)_v | 0 \rangle \langle N' | (\bar{q}_3 q_4)_A | N \rangle$$

# Example: $h_\pi^1$

As a pseudoscalar meson

$$\begin{aligned} \langle \pi^+ n | \mathcal{H}_{I=1} | p \rangle &= -i h_\pi^1 \bar{u}_n u_p = \\ &= \frac{G_F s_W^2}{3\sqrt{2}} \langle \pi^+ | (\bar{u} \gamma_5 d) | 0 \rangle \left( \frac{2c_1^{I=1}}{3} + 2c_2^{I=1} - \frac{2c_3^{I=1}}{3} + 2c_4^{I=1} \right) \langle n | \bar{d} u | p \rangle \end{aligned}$$

With  $f_\pi$  the charged pion decay constant

$$\langle \pi^+ | (\bar{u} \gamma_5 d) | 0 \rangle = \frac{m_\pi^2 f_\pi}{i(m_u + m_d)}$$

$$m_\pi = 135 \text{ MeV}; f_\pi = 130; (m_u + m_d)[\text{RGI}] = 2(4.736(60)_m(1.5)_\Lambda) \text{ MeV}$$

and isovector quark scalar charge of the nucleon<sup>1</sup>

$$\langle n | \bar{d} u | p \rangle = g_s^{u-d} \bar{u}_n u_p; \quad g_s^{u-d} = 1.06(10)(06)_{\text{sys}}$$

$$h_\pi^1 = (3.06 \pm 0.34 + \left( \begin{smallmatrix} +1.29 \\ -0.64 \end{smallmatrix} \right) + 0.42 + (1.00)) \times 10^{-7} \text{ (npdGamma}^2 : 2.6(1.2)(0.2) \times 10^{-7}$$

$$h_\pi^1 = 2.13 \pm 0.22 + \left( \begin{smallmatrix} +0.19 \\ -0.33 \end{smallmatrix} \right) \times 10^{-7} \text{ (NLO)}$$

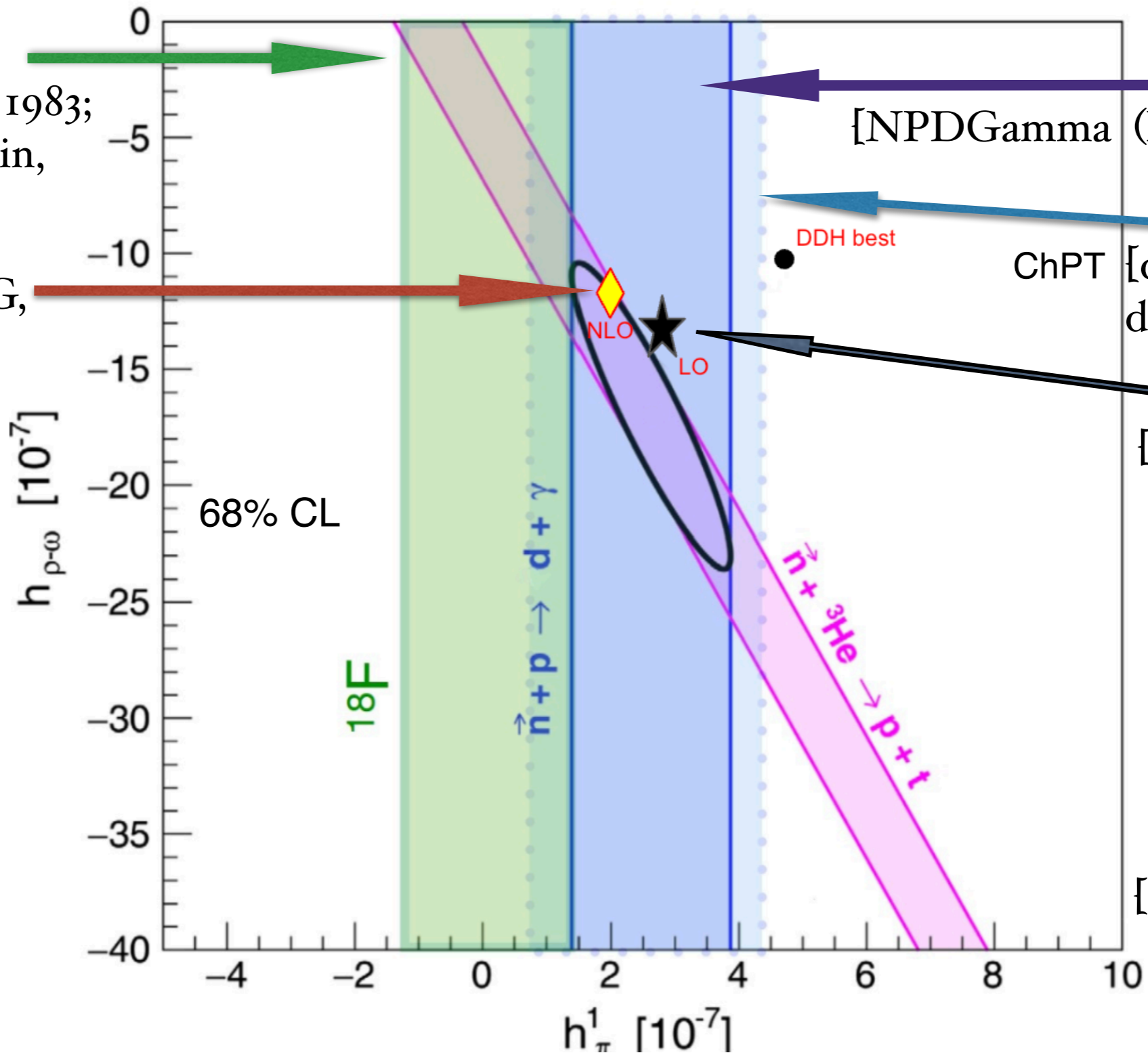
<sup>1</sup>FLAG review, 2021      Can also evaluate  $h_{\omega,\rho}^{0,1,2}$

<sup>2</sup>Blyth et al., 2018

# Implications

[Haxton, 1981;  
Adelberger et al., 1983;  
Haxton & Holstein,  
2013]

[Muralidhara & SG,  
PLB 2024 (NLO)  
+LQCD inputs]



[NPDGamma (Blyth et al.), 2018]

ChPT [de Vries et al., 2015;  
de Vries et al., 2020]

[SG & Muralidhara,  
2022 (LO)]

Scale  
dependence?  
cf. exact  
NN EFT  
study

[Epelbaum, Gegelia,  
Meissner, 2017]

$$n_3\text{He}: h_{p-\omega} \equiv h_{\rho}^0 + 0.605h_{\omega}^0 - 0.605h_{\rho}^1 - 1.316h_{\omega}^1 + 0.026h_{\rho}^2 = (-17.0 \pm 6.56) \times 10^{-7}$$

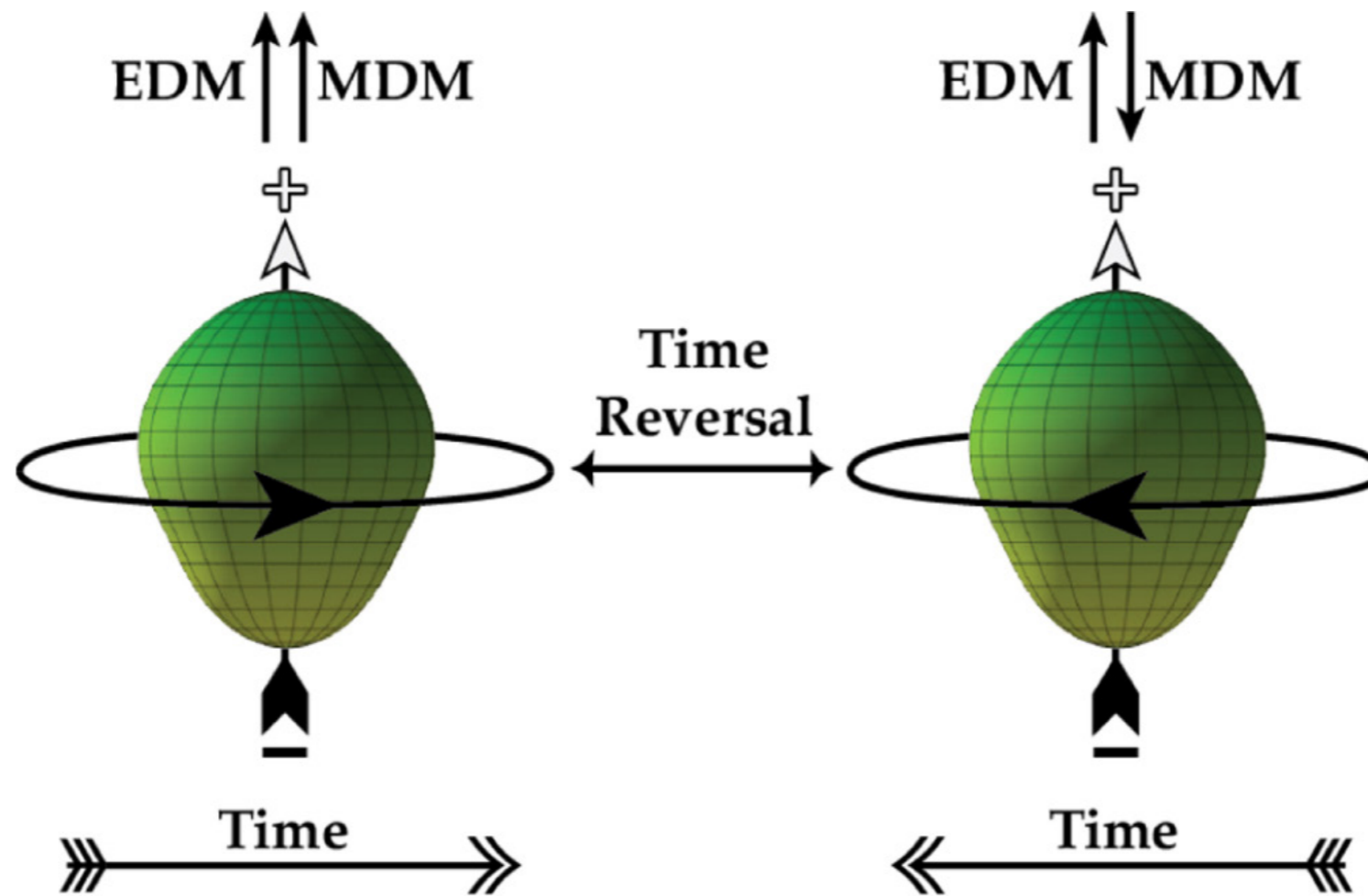
$$\text{LOQCD+LQCD}: h_{p-\omega} = (-12.9 \pm 0.52 + \binom{0.97}{-1.9}) + 0.62 + (-3.4) \times 10^{-7}; \quad h_{p-\omega} = -11.9 \pm 0.65 + \binom{0.09}{-0.77} \times 10^{-7}$$

[SG & Muralidhara, 2023 with 2024 updates; 23 after  $n_3\text{He}$  (Gericke et al.), 2020] (NLO)

# Some Future BSM Tests

## Radioisotope Harvesting at FRIB

Pear-shaped nuclei for permanent EDMs searches:



{2023 LRP for Nuclear Science}

Quantum sensing & BSM searches with  
(radioactive) molecules

{DeMille et al., Nature Physics, 2024}



# Summary

- We have considered BSM tests in complex nuclei & UQ re the “tower of EFTs” that can appear
- We have considered limited aspects of this task within the example of hadronic parity violation
- We have constructed the PV effective Hamiltonian  $\mathcal{H}_{\text{eff}}$  of the SM in LO & NLO QCD at  $\mu = 2 \text{ GeV}$
- Using our result, the factorization approximation, and lattice QCD charges, we have computed meson-nucleon coupling constants that compare favorably to few-body experiments
- Forthcoming AMO studies may yield information on PV in lighter systems

# For Discussion

Can we use our  $\mathcal{H}_{\text{eff}}$  to compute the PV LECs in chiral EFT?

Will computations of PV in complex nuclei (anapole moments?) prove possible?

Will we eventually be able to compute the Schiff moments of heavy, deformed nuclei (for future EDM searches) with defendable errors?

How well will we be able to realize UQ in “Tower of EFTs” BSM searches?

# Backup Slides

# Permanent Electric Dipole Moments

## Atomic Scale Effects & Enhancements

Limits on the electron EDM  $d_e$  come from paramagnetic and (to a limited extent) diamagnetic atoms — and from molecules

Schiff Theorem (1963):

In the non-relativistic limit a neutral, point-like atom will shield an applied electric field, so that there is no atomic EDM even if  $d_{\text{nucleus}}$  is not zero!

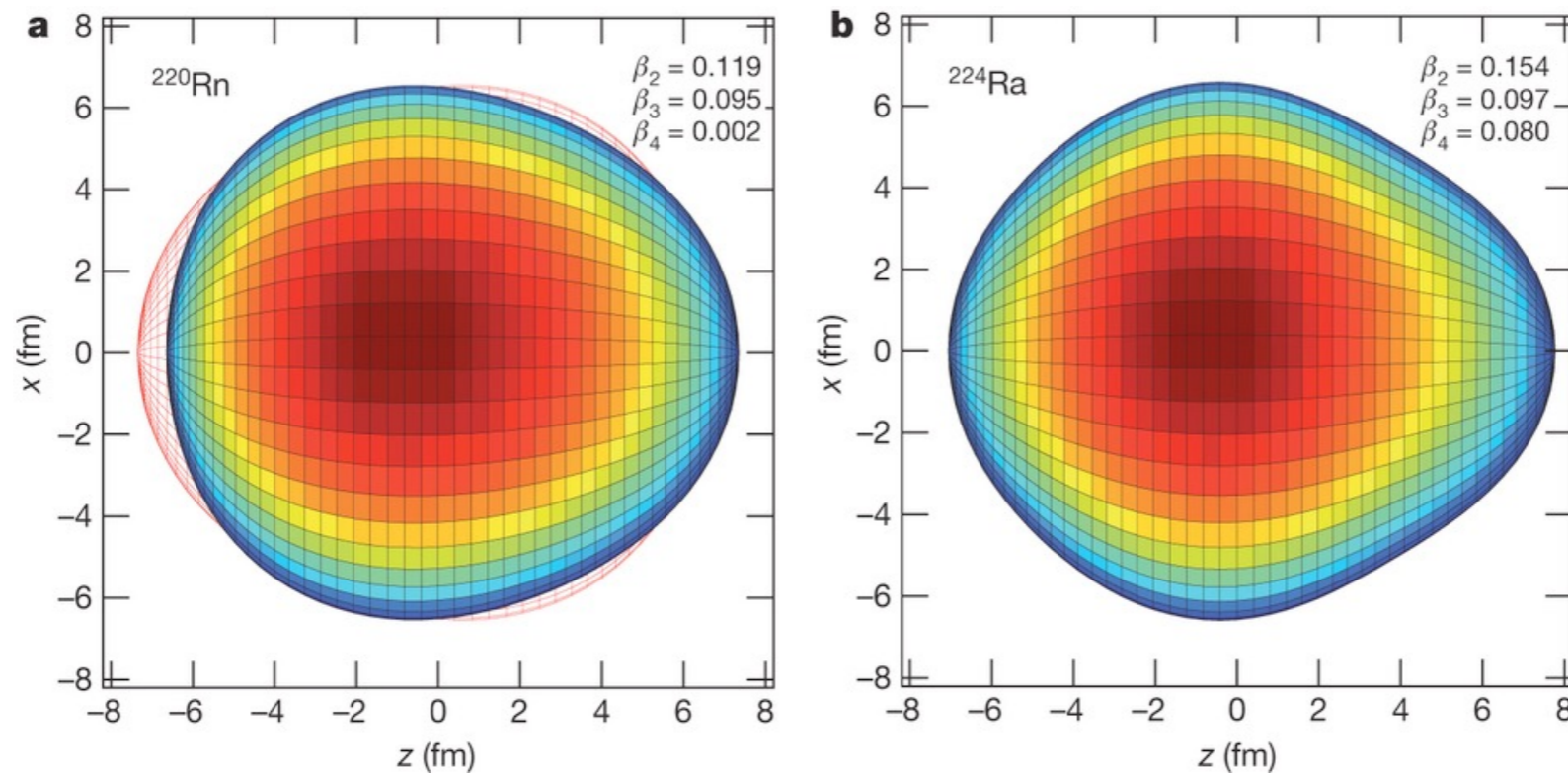
**Schiff's theorem can be strongly violated by relativistic and finite-size effects!**

In paramagnetic atoms & polar molecules relativistic effects dominate. Note in alkali atoms  $d_{\text{atom}} \sim Z^3 a^2 d_e$   
( $d_{\text{Tl}} \sim 585d_e + \dots$  !)

[Sandars, 1965]

# Heavy Atom EDMs

evade Schiff's theorem through large  $Z$ , finite nuclear size, and permanent (octupole) deformation



[Gaffney et al.,  
Nature (2013)]

Permanent deformation in Ra-225 makes the nucleus more “rigid” and the Schiff moment computation more robust and 1000x bigger than  $^{199}\text{Hg}$  (existing best atomic EDM limit)

**This is just one example...**

# Electric & Magnetic Dipole Moments

A permanent EDM breaks P & T

$$\mathcal{H} = -\mu \frac{\vec{S}}{S} \cdot \vec{B} - d \frac{\vec{S}}{S} \cdot \vec{E}$$

Maxwell Equations...

$$\vec{B} \xleftrightarrow{P} \vec{B} \quad \vec{E} \xleftrightarrow{P} -\vec{E} \quad \vec{S} \xleftrightarrow{P} \vec{S}$$

$$\vec{B} \xleftrightarrow{T} -\vec{B} \quad \vec{E} \xleftrightarrow{T} \vec{E} \quad \vec{S} \xleftrightarrow{T} -\vec{S}$$

**MDM: P even, T even**

**EDM: P odd, T odd**

→ under CPT, CP is also broken

# EDMs & Sensitivity to New Physics

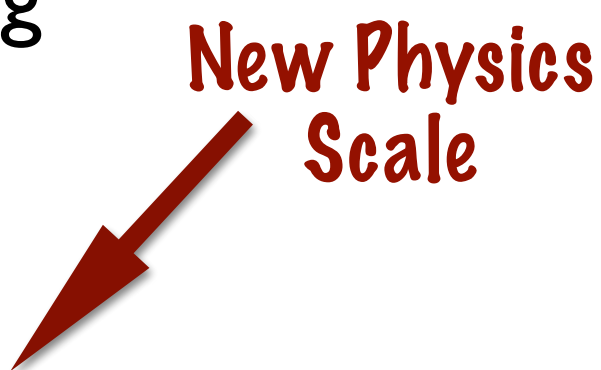
The electric and (anomalous) magnetic moments change chirality

$$\bar{\psi}\sigma^{\mu\nu}\psi = (\bar{\psi}_L\sigma^{\mu\nu}\psi_R + \bar{\psi}_R\sigma^{\mu\nu}\psi_L)$$

$$\bar{\psi}\sigma^{\mu\nu}\gamma_5\psi = (\bar{\psi}_L\sigma^{\mu\nu}\gamma_5\psi_R + \bar{\psi}_R\sigma^{\mu\nu}\gamma_5\psi_L)$$

By dimensional analysis we infer the scaling

$$d_f \sim e \frac{\alpha}{4\pi} \frac{m_f}{\Lambda^2} \sin \phi_{\text{CP}}$$

$$d_{d \text{ quark}} \sim 10^{-3} e \frac{m_d(\text{MeV})}{\Lambda(\text{TeV})^2} \sim 10^{-25} \frac{1}{\Lambda(\text{TeV})^2} e - \text{cm}$$


Note ILL limit on neutron EDM:

$$d_n < 3 \times 10^{-26} \text{ e-cm @ 90\%CL} \quad [\text{Pendlebury et al., 2015}]$$

EDM experiments have TeV scale sensitivity

# EDM Measurement Principle

**Much simplified!**

Consider the precession frequency

$$\nu = \frac{1}{2\pi} \frac{d\varphi}{dt} = \frac{2\vec{\mu} \cdot \vec{B} \pm 2\vec{d} \cdot \vec{E}}{h}$$

and its *change* under  $\vec{E}$  field reversal

**B must be very well determined!**

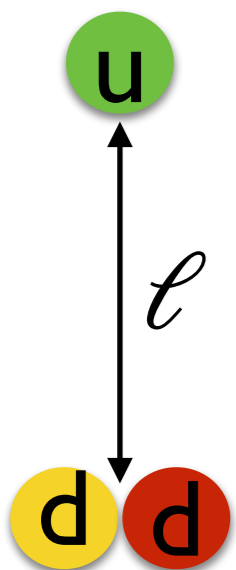
The experimental sensitivity to the energy  $\vec{d} \cdot \vec{E}$  is set by

$$\sigma_d \sim \frac{\hbar}{|\vec{E}| T_m \sqrt{N}} \quad \begin{array}{l} T_m \text{ measurement time} \\ N \text{ number of counts} \end{array}$$

Neutron:  $d_n < 1.8 \times 10^{-26}$  e-cm [90 % C.L.]

[Abel et al., 2020]

Estimate:  $d \sim \frac{2}{3} e\ell \sim 6 \times 10^{-15}$  e-cm if  $\ell \sim 0.1 r_p$  (!)





# Operator Analysis of EDMs

Multiple sources with  $d \leq 6$  exist

Even a single TeV scale source can give rise to multiple GeV scale sources through QCD effects

[Chien et al., 2016]

$$\mathcal{L}^{d \leq 6} \supset \bar{\theta} \alpha_s G \tilde{G} + \sum_{i \in u, d, s} i(d_i \bar{q}_i (F \sigma) \gamma_5 q + \tilde{d}_i \bar{q}_i (G \sigma) \gamma_5 q) + d_G G G \tilde{G}$$

[Pospelov & Ritz, 2005]

LQCD studies of apropos neutron matrix elements

exist (e.g., tensor charges) and is ongoing

[note FLAG review; Snowmass white paper 2203.08103]



Can all the low-energy CPV sources be determined?

Need to interpret EDM limits in nuclei, atoms, molecules

Note  $a G \tilde{G}$ ,  $\partial_\mu a \bar{N} \gamma^\mu \gamma_5 N$  can act as axion portals

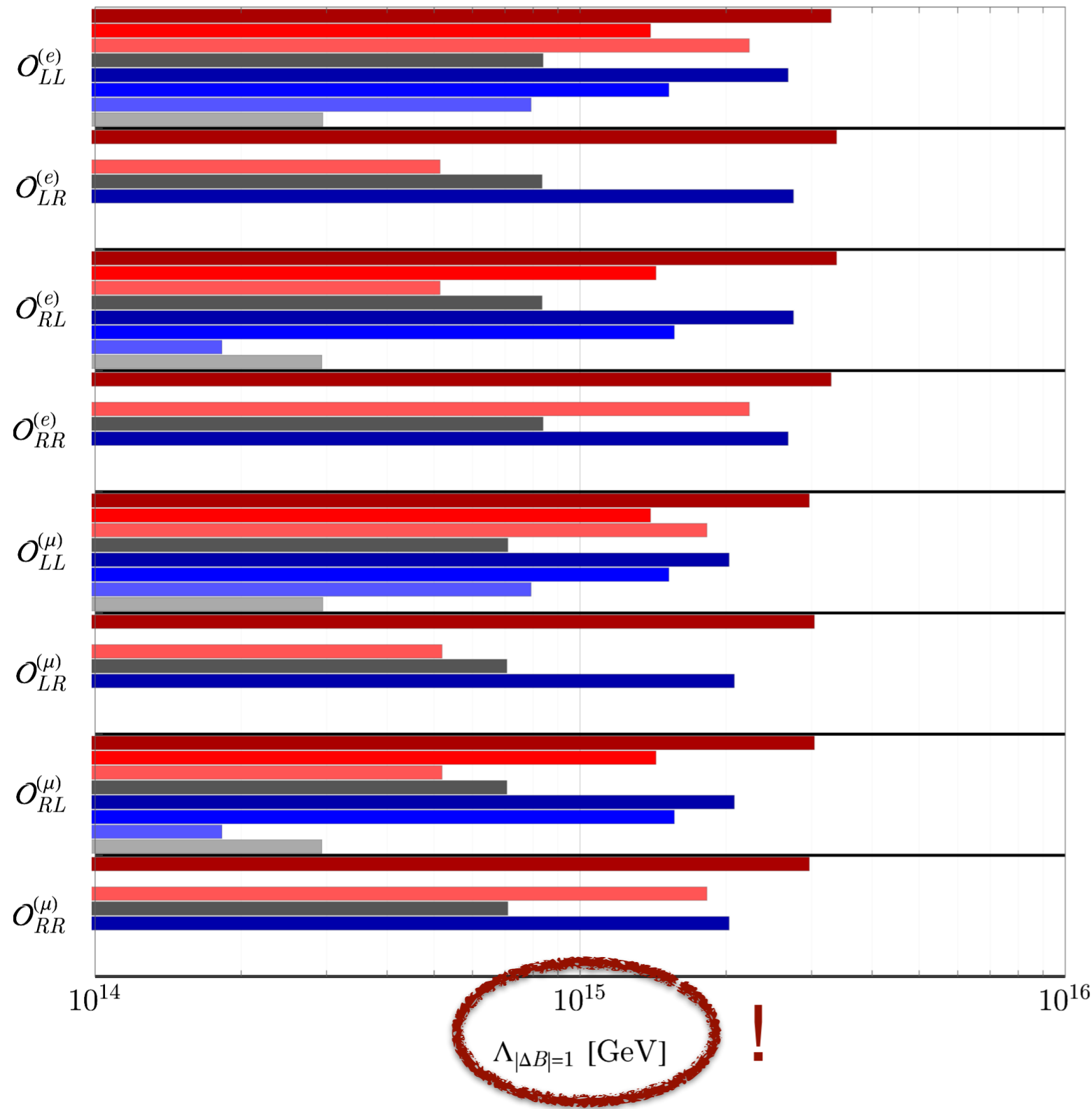
[Batell, Pospelov, & Ritz, 2009]

$$\begin{aligned}
\Lambda_0^{1S_0-3P_0} &= -g_\rho(2 + \chi_\rho)b_\rho^0 - g_\omega(2 + \chi_\omega)b_\omega^0 \\
\Lambda_0^{3S_1-1P_1} &= -3g_\rho\chi_\rho b_\rho^0 + g_\omega\chi_\omega b_\omega^0 \\
\Lambda_1^{1S_0-3P_0} &= -g_\rho(2 + \chi_\rho)b_\rho^1 - g_\omega(2 + \chi_\omega)b_\omega^1 \\
\Lambda_1^{3S_1-3P_1} &= \sqrt{\frac{1}{2}} g_{\pi NN} \left(\frac{m_\rho}{m_\pi}\right)^2 b_\pi^1 + g_\rho(b_\rho^1 - b_\rho^{1'}) - g_\omega b_\omega^1 \\
\Lambda_2^{1S_0-3P_0} &= -g_\rho(2 + \chi_\rho)b_\rho^2
\end{aligned}$$

# Limits on $|\Delta B| = 1$ Decays

Mediated by mass dimension 6 operators in SMEFT

[Berryman, SG, & Zakeri, 2022]



$$\mathcal{L}_{|\Delta B|=1}^{(d=6)} \supset \sum_i \frac{c_i}{\Lambda_{|\Delta B|=1}^2} (qqq\ell)_i + \text{h.c.}$$

But the origin of  $|\Delta B| = 2$  processes can be distinct!

- $p \rightarrow \ell^+ \pi^0$
- $p \rightarrow \bar{\nu}_\ell \pi^+$
- $p \rightarrow \ell^+ \eta$
- $p \rightarrow \ell^+ \gamma$
- $n \rightarrow \ell^+ \pi^-$
- $n \rightarrow \bar{\nu}_\ell \pi^0$
- $n \rightarrow \bar{\nu}_\ell \eta$
- $n \rightarrow \bar{\nu}_\ell \gamma$

[Marshak & Mohapatra, 1980; Babu & Mohapatra, 2001 & 2012; Arnold, Fornal, & Wise, 2013....]

$$\mathcal{L}_{|\Delta B|=2}^{(d=9)} \supset \sum_i \frac{c_i}{\Lambda_{|\Delta B|=2}^5} (qqqqqq)_i + \text{h.c.}$$

$n\bar{n}$  expt'l limit yields

$$\Gamma_{|\Delta B|=2} \gtrsim 10^{5.5} \text{ GeV}$$

# On Neutrinoless Double Beta ( $0\nu \beta\beta$ ) decay

If observed, the  $\nu$  has a Majorana mass

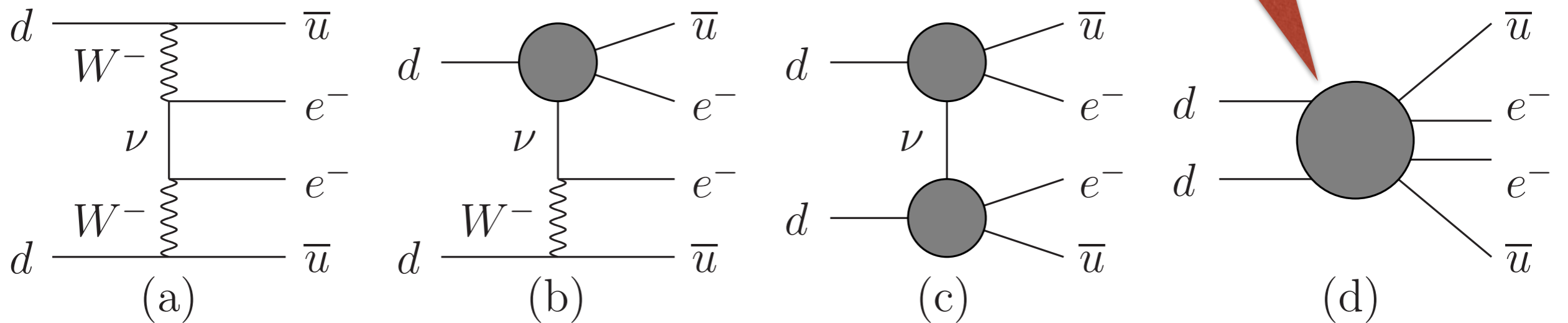
[Schechter & Valle, 1982]

$0\nu \beta\beta$  mediated by a dimension 9 operator:

$$\mathcal{O} \propto \bar{u}\bar{u}dd\bar{e}\bar{e}$$

(or  $\pi^- \pi^- \rightarrow e^- e^-$ )

“mass mechanism”



“long range”

★ “short range”

[Bonnet, Hirsch, Ota, & Winter, 2013]

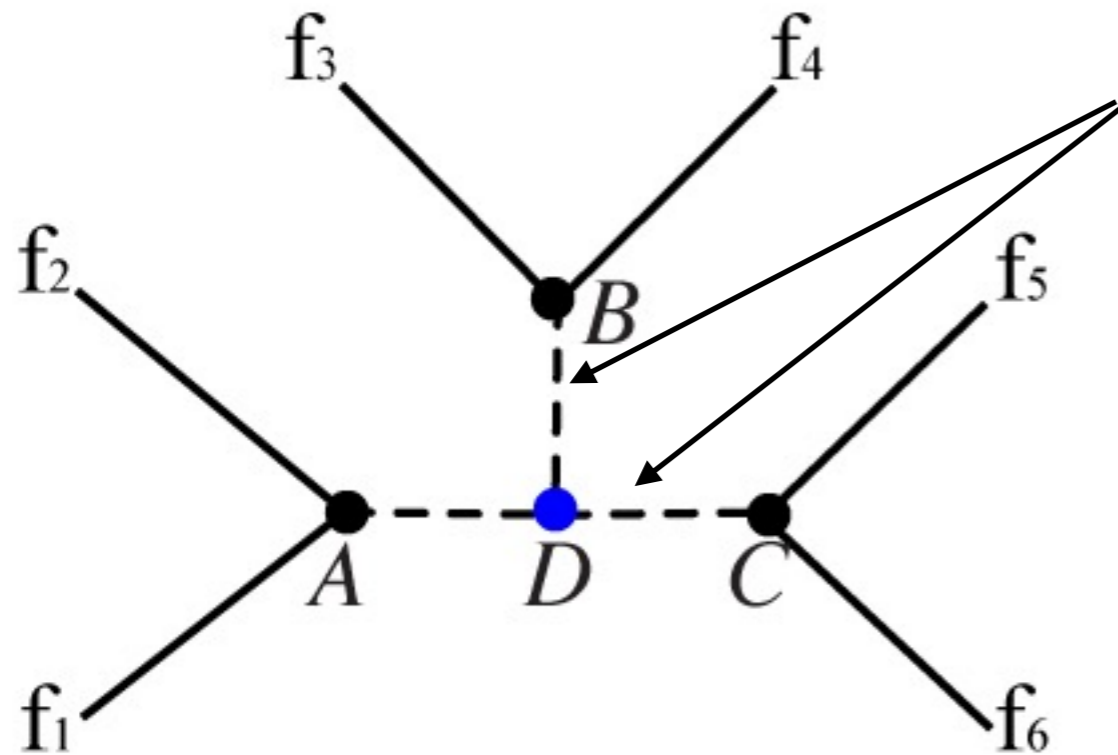
[Note Cirigliano et al., EFT analysis]

mediated by B-L breaking!

# $0\nu \beta\beta$ Decay in Nuclei

Can be mediated by “short-” or “long”-range mechanisms

The “short-range” mechanism involves new B-L violating dynamics; e.g.,



S or V that carries B or L

For choices of fermions  $f_i$  this decay topology can yield  $n-\bar{n}$  or  $0\nu \beta\beta$  decay

[Bonnet, Hirsch, Ota, & Winter, 2013]

Can we relate the possibilities in a data-driven way?

[Yes!] [S.G. & Xinshuai Yan, PLB 2019]

# Fundamental Majorana Dynamics

Can exist for electrically neutral massive fermions:  
either leptons ( $\nu$ 's) or combinations of quarks ( $n$ 's)

Lorentz invariance allows

$$\mathcal{L} = \bar{\psi}i\not{\partial}\psi - \frac{1}{2}m(\psi^T C\psi + \bar{\psi}C\bar{\psi}^T) \quad [\text{Majorana, 1937}]$$

where  $m$  is the Majorana mass.

A “Majorana neutron” is an entangled  $n$  and  $\bar{n}$  state,  
but a Majorana neutrino can be a two-component field

## Bibliography:

S.G. & Xinshuai Yan, Phys. Rev. D93, 096008 (2016) [arXiv:1602.00693];  
S.G. & Xinshuai Yan, Phys. Rev. D97, 056008 (2018) [arXiv:1710.09292];  
S.G. & Xinshuai Yan, Phys. Lett. B790 (2019) 421 [arXiv:1808.05288];  
and on ongoing work in collaboration with Xinshuai Yan

# A Dark-Dominated Universe

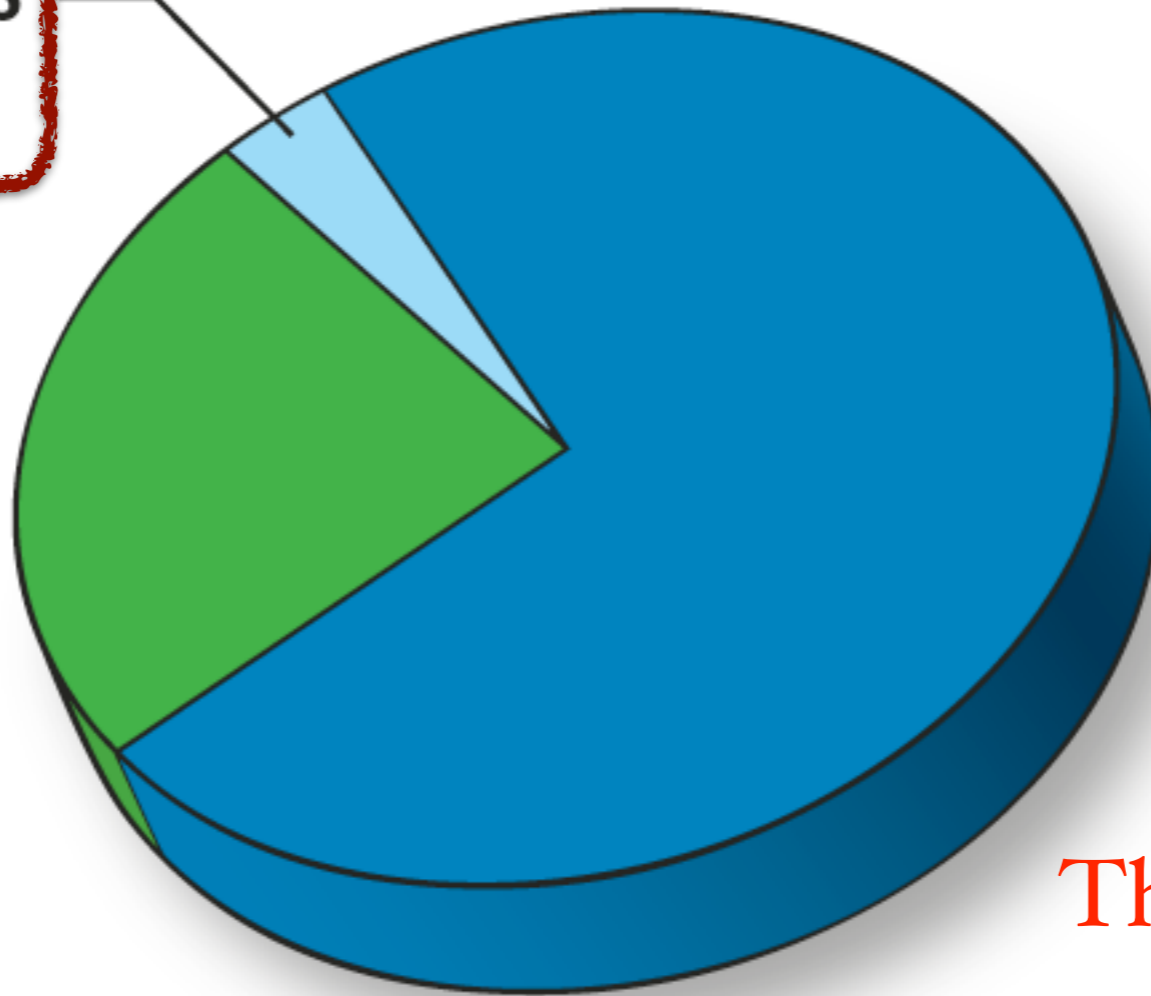
## Why should quarks (& QCD) matter?

[map.gsfc.nasa.gov/universe/uni\\_matter.html](https://map.gsfc.nasa.gov/universe/uni_matter.html)

Linked?

Atoms  
4.6%

Dark  
Matter  
24%



Dark  
Energy  
71.4%

The dark content  
is unknown

a baryon **excess**: TODAY

$$\eta = n_{\text{baryon}}/n_{\text{photon}} = (6.12 \pm 0.04) \times 10^{-10}$$

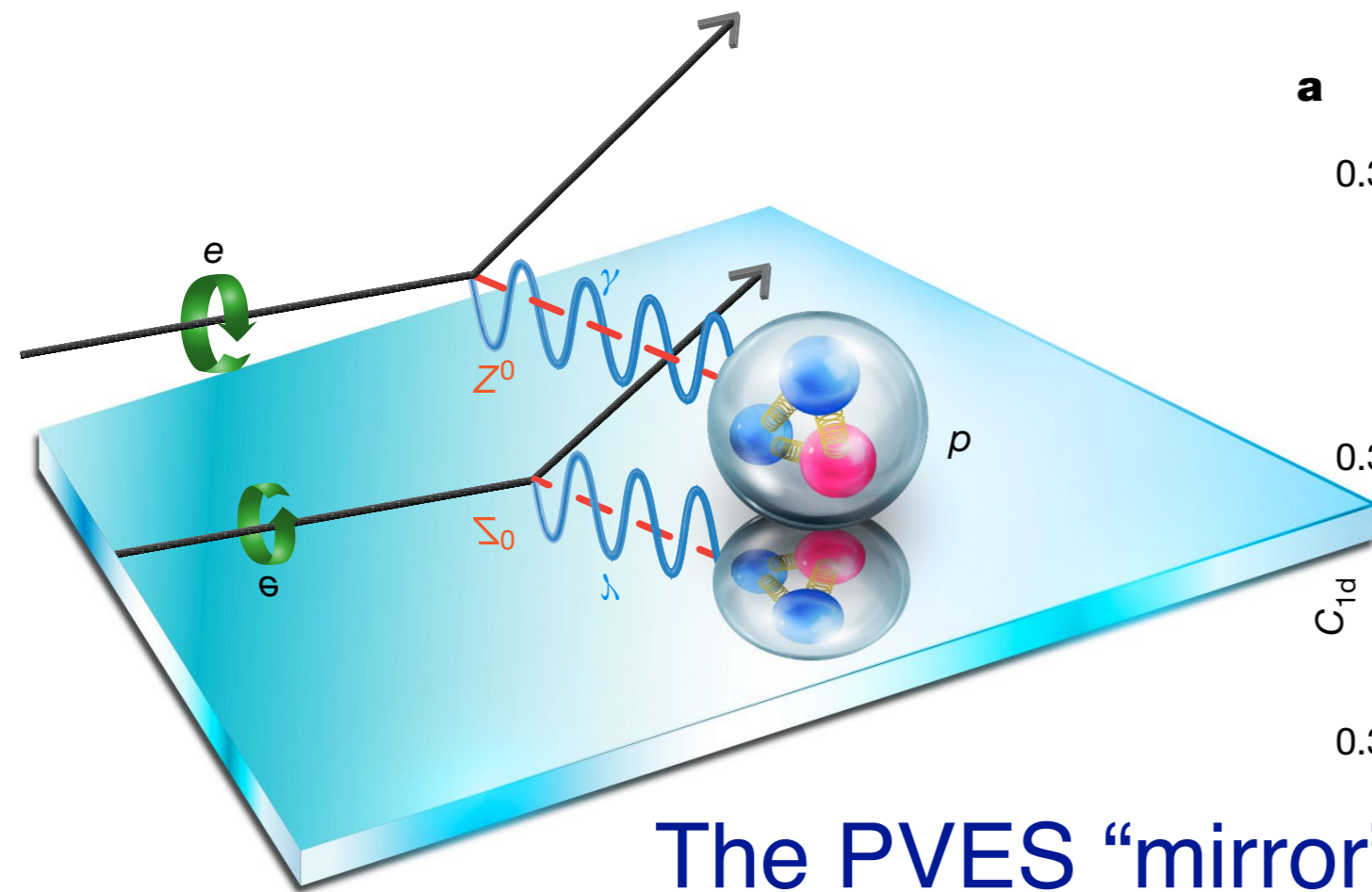
N.B. primordial D/H abundance...

[Planck, 2020; PDG, 2022]

# PVES probes SM fermion couplings

Limits BSM possibilities

$$\mathcal{L}_{\text{PV}}^{\text{SM}} = \bar{e} \gamma_{\mu} \gamma_5 e \sum_q \left( \frac{G_F}{\sqrt{2}} C_{1q} \right) \bar{q} \gamma^{\mu} q$$



[Qweak Collaboration, Nature, 2018]

★ HPV studies should help probe QCD framework

