Uncertainty Quantification in Nuclear Physics
MITP Workshop, Mainz June 2024

Bayesian uncertainty quantification to nuclear structure effects in atomic systems

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Short History of Atomic Physics

Early 20th century

- Relativistic corrections $\sim Z\alpha$
- Radiative corrections $\sim \alpha$
- Recoil corrections $\sim m_e/m_N$
- Nuclear structure $\sim (r_N/r_e)^3$

Birth of Quantum Mechanics

mid 20th century

Fine-structure
Relativistic Quantum Mechanics

Lamb-shift
QED

late 20th century

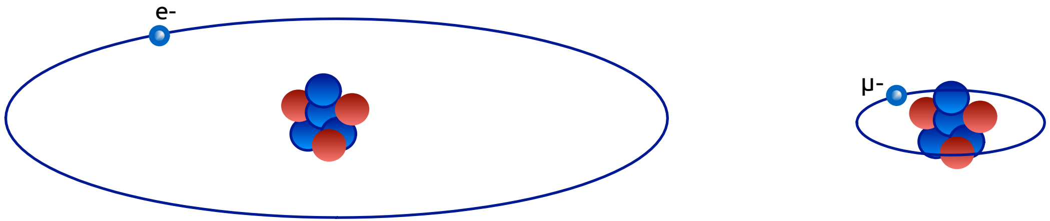
Systems with different expansion parameters

Muonic Atoms

Recoil corrections

Nuclear structure effects

Muonic Atoms



Nuclear structure effects are enhanced in muonic atoms

Experimental campaign at **PSI** by the **CREMA** collaboration

Muonic Hydrogen
- Pohl et al., Nature (2010)
- Antognini et al., Science (2013)

Muonic Deuterium
- Pohl et al., Science (2016)

Muonic Helium isotopes
- Krauth et al., Nature (2021)
- Schuhmann et al., Arxiv (2023)

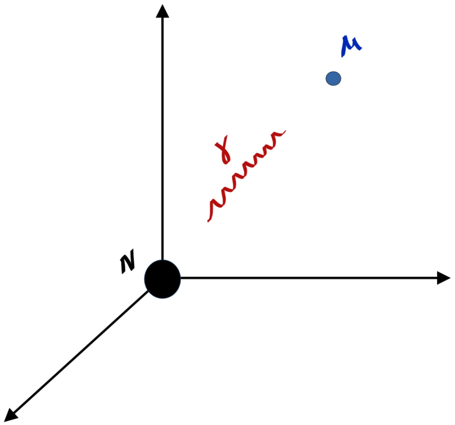
Lamb-shift and nuclear charge radii

$$\mathrm{E_{LS}} = \mathrm{E_{QED}} + \mathrm{C}r_c^2 + \mathrm{E_{TPE}} + ...$$

Lamb-shift and nuclear charge radii

$$E_{LS} = E_{QED} + Cr_c^2 + E_{TPE} + \ldots$$

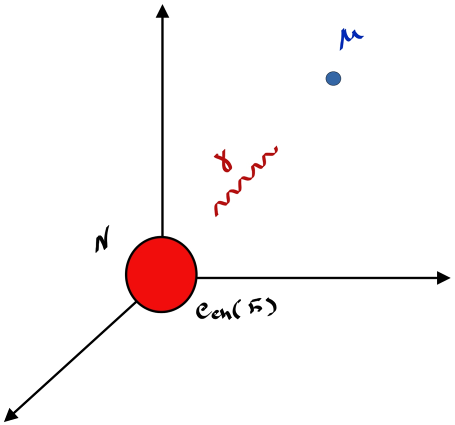
$$E_{QED} =$$



Lamb-shift and nuclear charge radii

$$\mathrm{E_{LS}} = \mathrm{E_{QED}} + \mathrm{C}r_c^2 + \mathrm{E_{TPE}} + \ldots$$

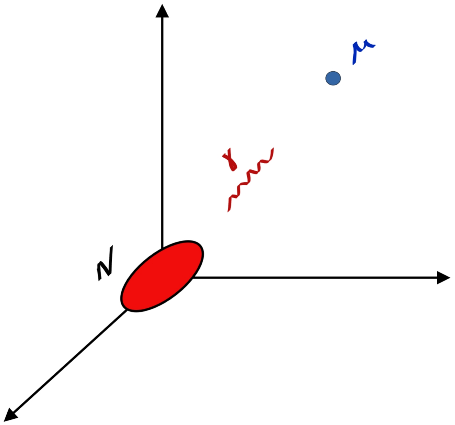
$$\mathrm{C}r_c^2 =$$



Lamb-shift and nuclear charge radii

$$E_{LS} = E_{QED} + C r_c^2 + E_{TPE} + \dots$$

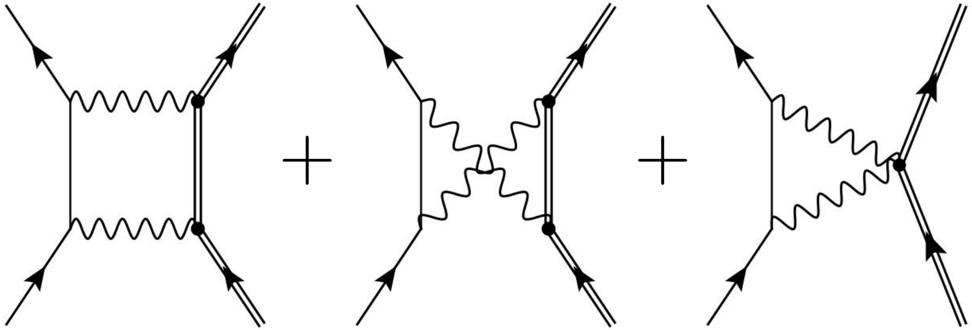
$$E_{TPE} =$$



Lamb-shift and nuclear charge radii

$$\mathrm{E_{LS}} = \mathrm{E_{QED}} + \mathrm{C}r_c^2 + \mathrm{E_{TPE}} + ...$$

$$\mathrm{E_{TPE}} =$$



A matter of precision

$$E_{LS} = E_{QED} + Cr_c^2 + E_{TPE} + E_{3PE} + ...$$

For muonic Helium-4 ion

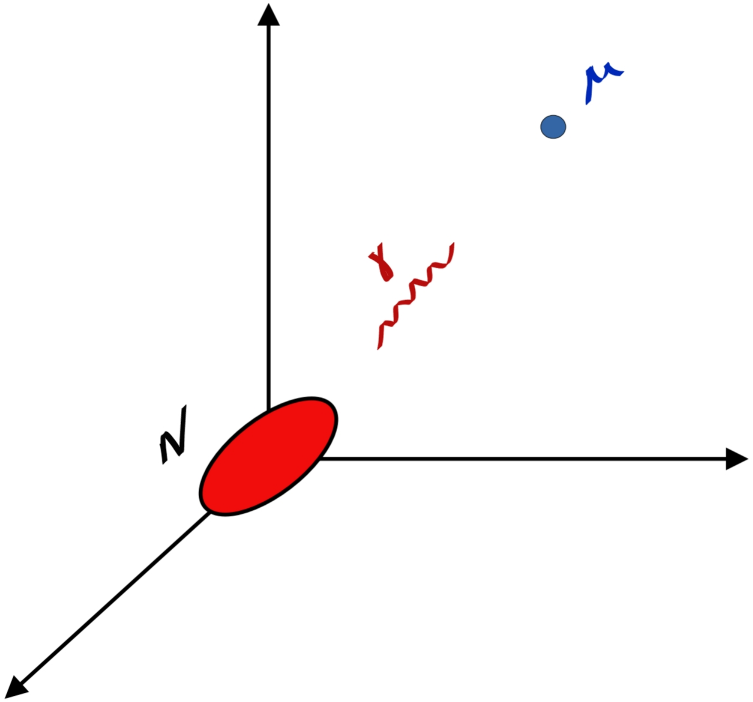
$$E_{QED} = +1,644.348(8)\ \text{meV}$$
$$C = -106.220(8)\ \text{meV fm}^{-2}$$
$$E_{TPE} = +9.340(250)\ \text{meV}$$
$$E_{3PE} = -0.150(150)\ \text{meV}$$

→

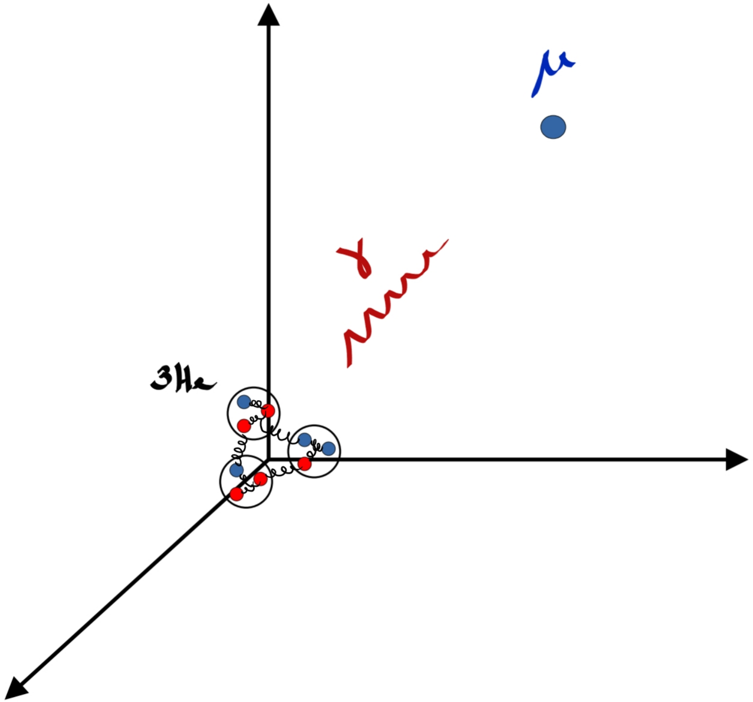
$$r_c = 1.67824(13)_{\text{ex}}(82)_{\text{th}}\ \text{fm}$$

J. J. Krauth et. al. Nature 589,527 (2021)

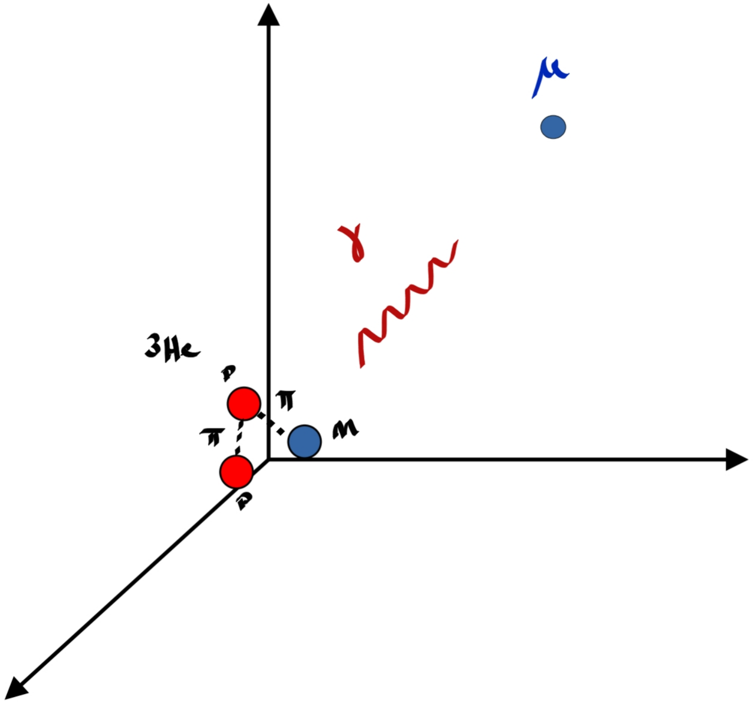
Ab-initio nuclear theory



Ab-initio nuclear theory



Ab-initio nuclear theory



Ab-initio nuclear theory

- **Our strategy is to build models for the operators from first principles**

$$\mathrm{H}$$

$$\{\rho_{\mathrm{ch}}, J_i, B_{ij}\}$$

from chiral effective field theory

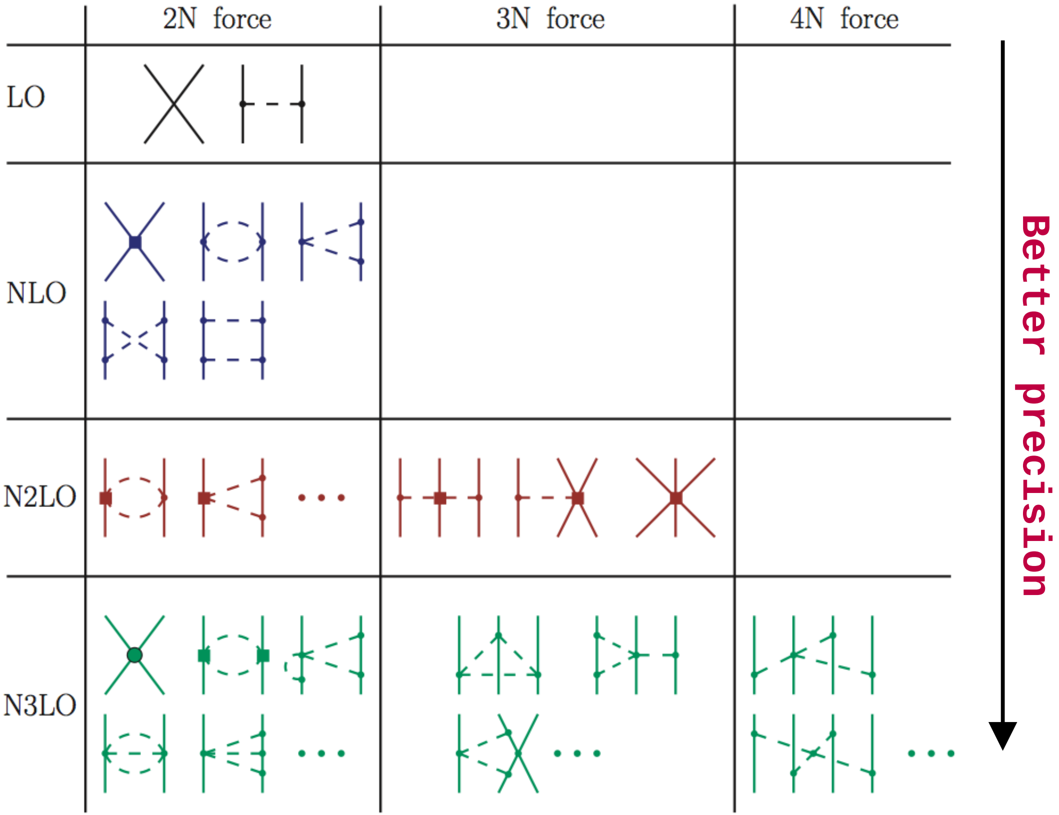
- **Solve the many-body Schrödinger equation of the nucleus**

$$\mathrm{H}\,|\mathrm{N}\rangle = \mathrm{E}_{\mathrm{N}}\,|\mathrm{N}\rangle$$

- **Calculate the relevant matrix elements with controlled approximations.**

$$\langle\mathrm{N}|\,\rho_{\mathrm{ch}}(\mathbf{x})\,|0\rangle$$

Nuclear Hamiltonians from ChEFT



- **Degrees of freedom**
- **Symmetries**
- **Power counting**

Bayesian uncertainty quantification of ChEFT truncation errors

ChEFT is an expansion in powers of $Q = \frac{m_\pi}{\Lambda_\chi} \sim 0.3$

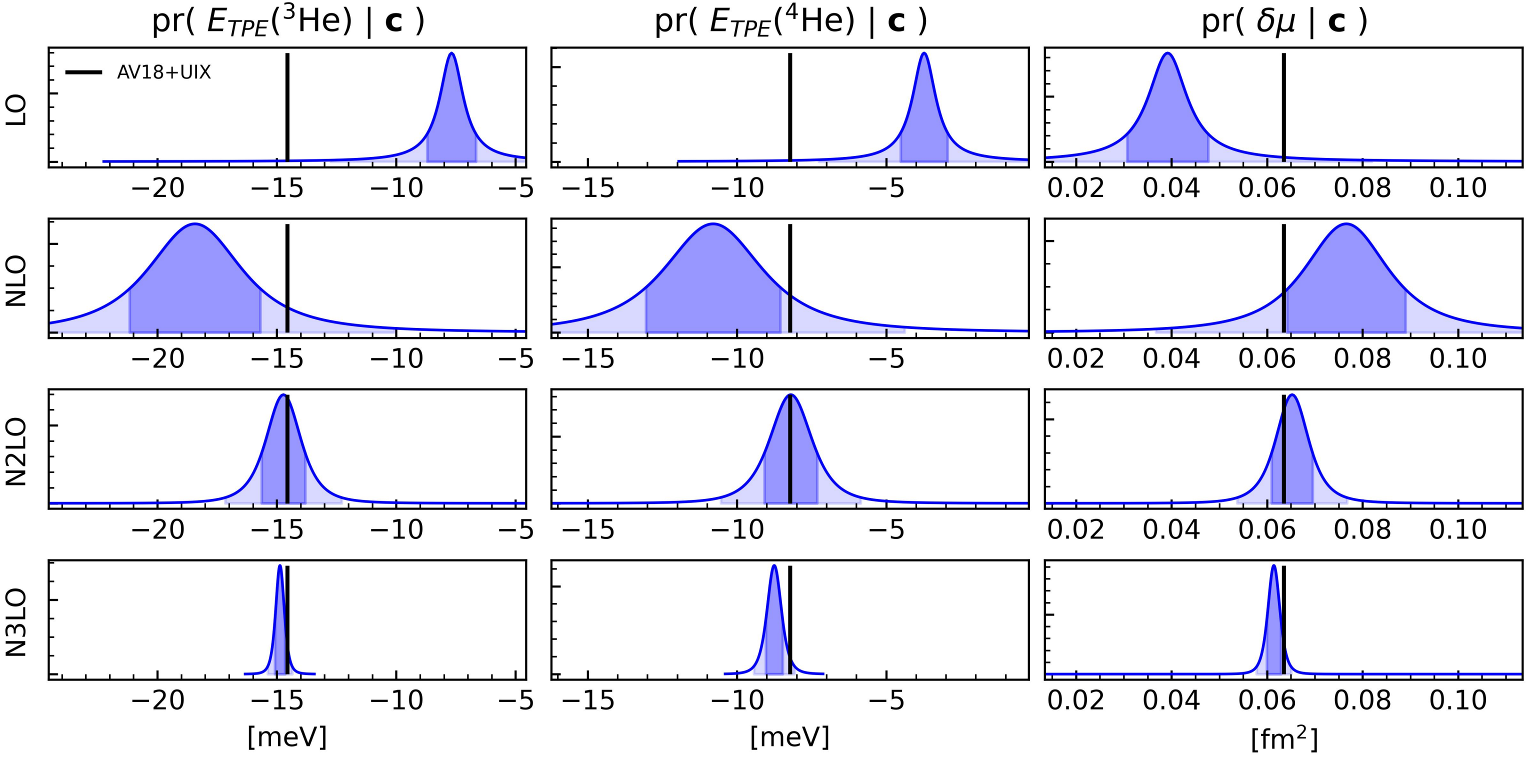
We assume that a similar expansion holds also for the calculated observables

$$X = \sum_{n=0}^{k} D_n + \sum_{n=k+1}^{\infty} D_n$$

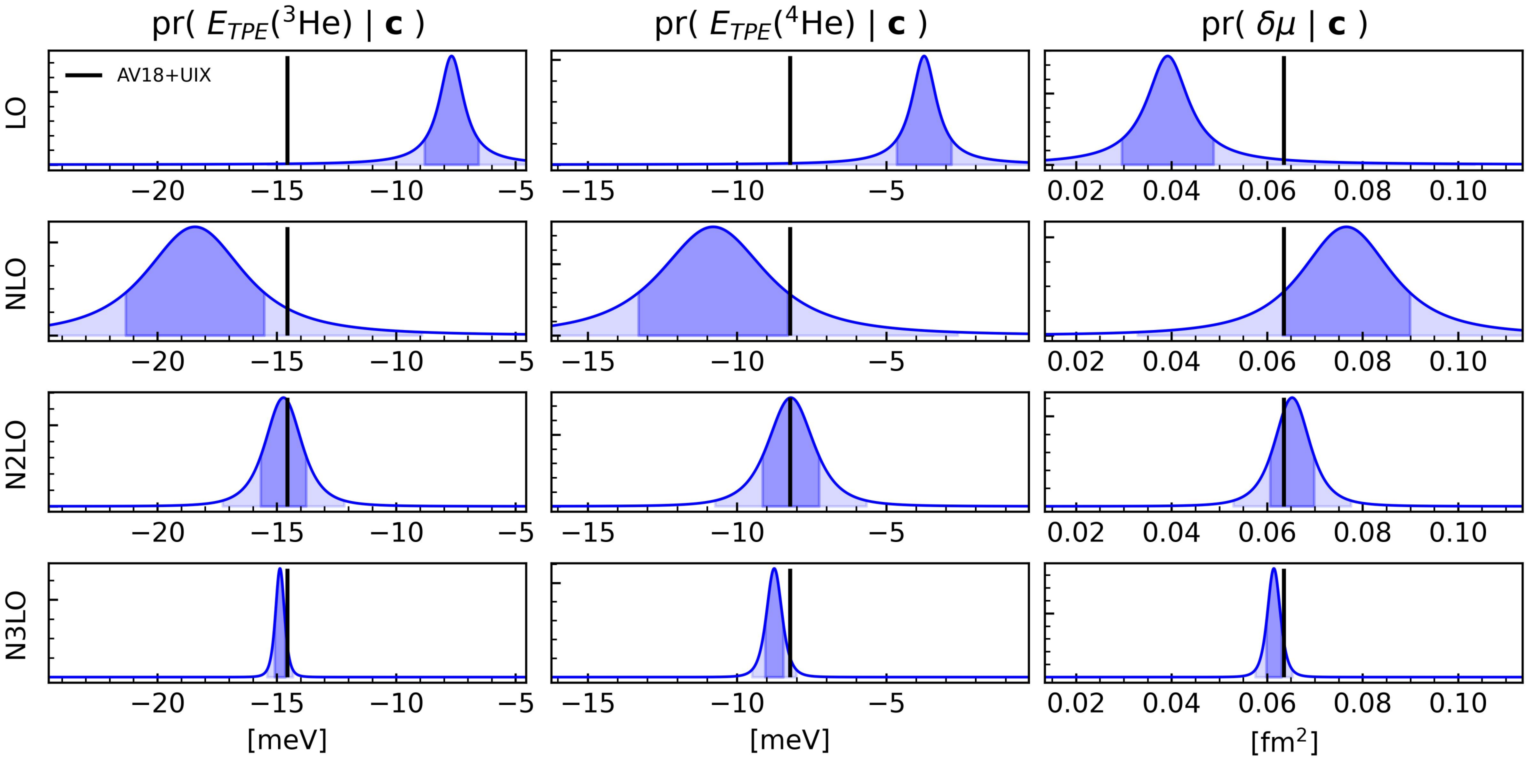
$$= X_{\text{ref}} \left[\sum_{n=0}^{k} c_n \, Q^n + \sum_{n=k+1}^{\infty} c_n \, Q^n \right]$$

We assume that the expansion coefficients follow the same underlying distribution and use the calculated coefficients to learn about the distribution.

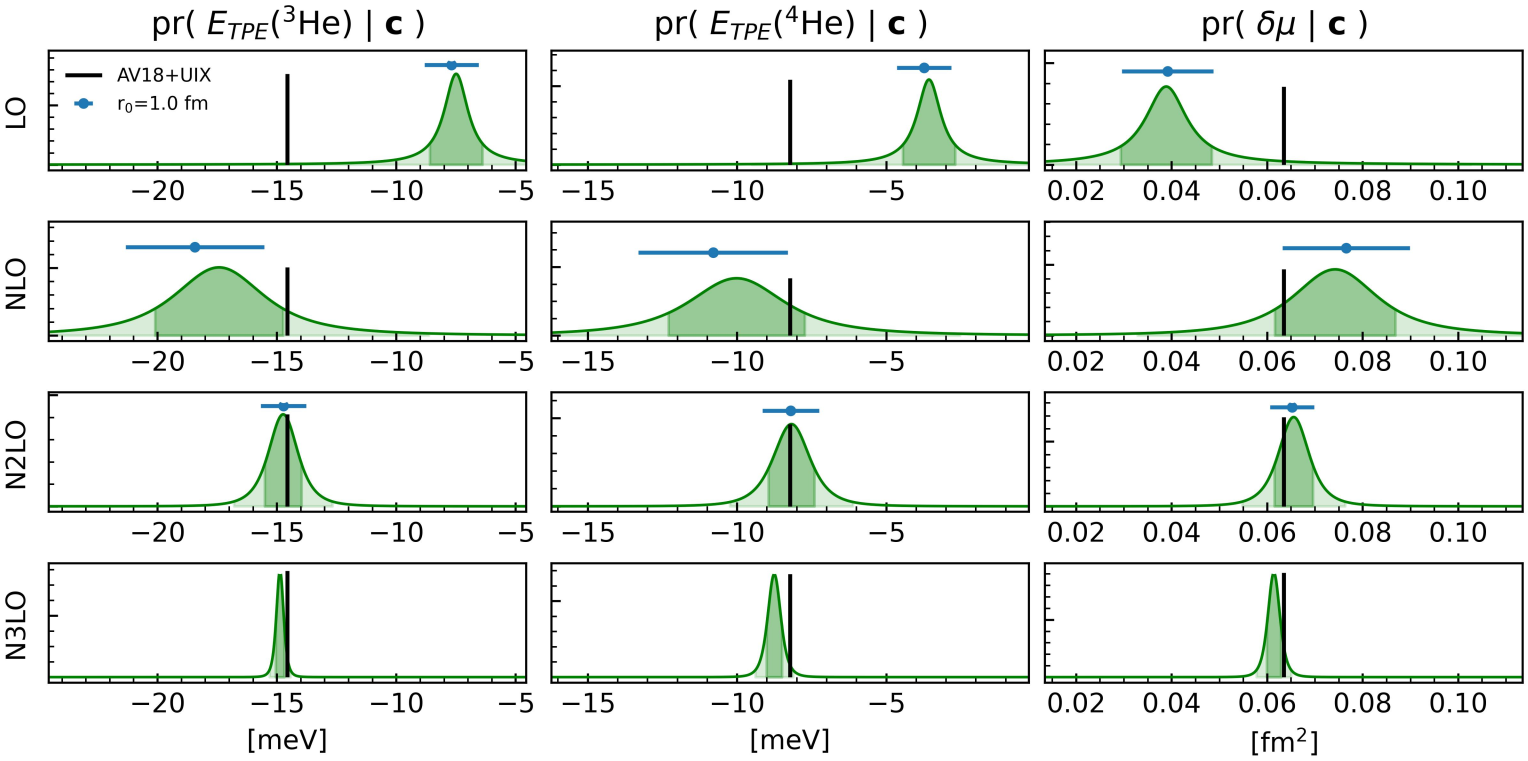
TPE corrections in muonic helium



TPE corrections in muonic helium: sub-leading truncation errors



TPE corrections in muonic helium: Regulator artifacts



TPE corrections in muonic helium: Total uncertainty

Total uncertainty
in the
Two-photon nuclear structure effects

	$\mu^3\text{He}^+$ [%]	$\mu^4\text{He}^+$ [%]
Numerical	0.1	0.2
Nuclear model	1.4	3.0
η-expansion	1.4	0.2
ISB	0.5	0.5
Nucleon-size	0.9	1.2
Relativistic	0.1	0.0
Coulomb	0.9	0.1
Total	2.4	3.5

– SSLM, Thomas R. Richardson, Sonia Bacca, Arxiv:2401.13424

– SSLM, et al. J. Phys. G: Nucl. Part. Phys. 49 105101 (2022)

– C. Ji, et al. J. Phys. G: Nucl. Part. Phys. 45 (2018)

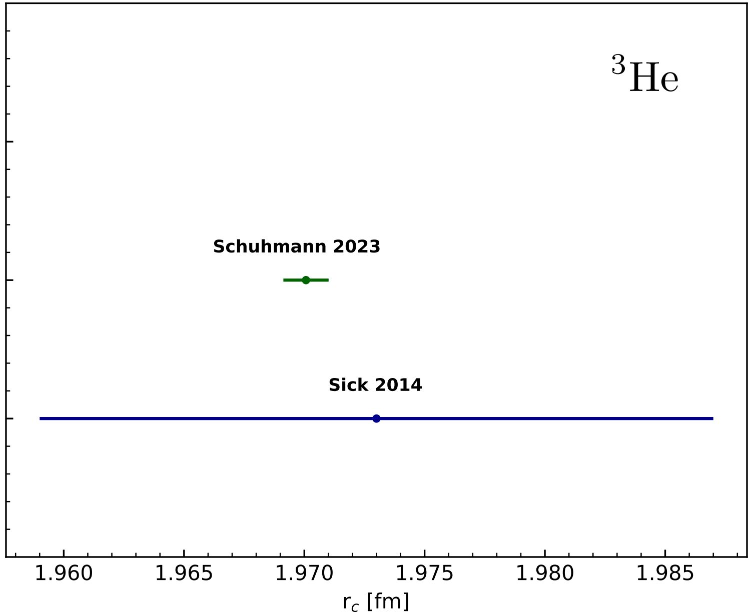
Nuclear structure in muonic helium: Total uncertainty

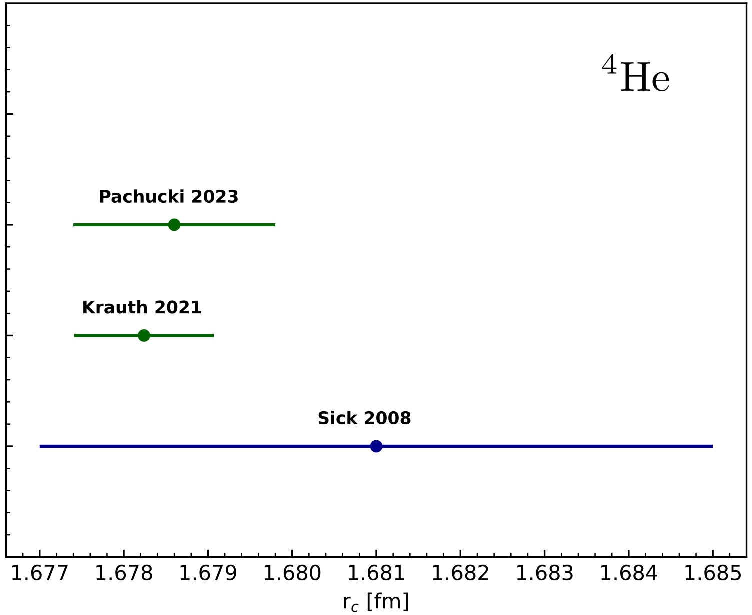
Section	Order	Correction	μH	μD	μ^3He$^+$	μ^4He$^+$
III.A	$\alpha(Z\alpha)^2$	eVP$^{(1)}$	205.007 38	227.634 70	1641.886 2	1665.773 1
III.A	$\alpha^2(Z\alpha)^2$	eVP$^{(2)}$	1.658 85	1.838 04	13.084 3	13.276 9
III.A	$\alpha^3(Z\alpha)^2$	eVP$^{(3)}$	0.007 52	0.008 42(7)	0.073 0(30)	0.074 0(30)
III.B	$(Z, Z^2, Z^3)\alpha^5$	Light-by-light eVP	−0.000 89(2)	−0.000 96(2)	−0.013 4(6)	−0.013 6(6)
III.C	$(Z\alpha)^4$	Recoil	0.057 47	0.067 22	0.126 5	0.295 2
III.D	$\alpha(Z\alpha)^4$	Relativistic with eVP$^{(1)}$	0.018 76	0.021 78	0.509 3	0.521 1
III.E	$\alpha^2(Z\alpha)^4$	Relativistic with eVP$^{(2)}$	0.000 17	0.000 20	0.005 6	0.005 7
III.F	$\alpha(Z\alpha)^4$	μSE$^{(1)}$ + μVP$^{(1)}$, LO	−0.663 45	−0.769 43	−10.652 5	−10.926 0
III.G	$\alpha(Z\alpha)^5$	μSE$^{(1)}$ + μVP$^{(1)}$, NLO	−0.004 43	−0.005 18	−0.174 9	−0.179 7
III.H	$\alpha^2(Z\alpha)^4$	μVP$^{(1)}$ with eVP$^{(1)}$	0.000 13	0.000 15	0.003 8	0.003 9
III.I	$\alpha^2(Z\alpha)^4$	μSE$^{(1)}$ with eVP$^{(1)}$	−0.002 54	−0.003 06	−0.062 7	−0.064 6
III.J	$(Z\alpha)^5$	Recoil	−0.044 97	−0.026 60	−0.558 1	−0.433 0
III.K	$\alpha(Z\alpha)^5$	Recoil with eVP$^{(1)}$	0.000 14(14)	0.000 09(9)	0.004 9(49)	0.003 9(39)
III.L	$Z^2\alpha(Z\alpha)^4$	nSE$^{(1)}$	−0.009 92	−0.003 10	−0.084 0	−0.050 5
III.M	$\alpha^2(Z\alpha)^4$	$\mu F_1^{(2)}$, $\mu F_2^{(2)}$, μVP$^{(2)}$	−0.001 58	−0.001 84	−0.031 1	−0.031 9
III.N	$(Z\alpha)^6$	Pure recoil	0.000 09	0.000 04	0.001 9	0.001 4
III.O	$\alpha(Z\alpha)^5$	Radiative recoil	0.000 22	0.000 13	0.002 9	0.002 3
III.P	$\alpha(Z\alpha)^4$	hVP	0.011 36(27)	0.013 28(32)	0.224 1(53)	0.230 3(54)
III.Q	$\alpha^2(Z\alpha)^4$	hVP with eVP$^{(1)}$	0.000 09	0.000 10	0.002 6(1)	0.002 7(1)
IV.A	$(Z\alpha)^4$	r_C^2	$-5.197\,5 r_p^2$	$-6.073\,2 r_d^2$	$-102.523 r_h^2$	$-105.322 r_\alpha^2$
IV.B	$\alpha(Z\alpha)^4$	eVP$^{(1)}$ with r_C^2	$-0.028\,2 r_p^2$	$-0.034\,0 r_d^2$	$-0.851 r_h^2$	$-0.878 r_\alpha^2$
IV.C	$\alpha^2(Z\alpha)^4$	eVP$^{(2)}$ with r_C^2	$-0.000\,2 r_p^2$	$-0.000\,2 r_d^2$	$-0.009(1) r_h^2$	$-0.009(1) r_\alpha^2$
V.A	$(Z\alpha)^5$	TPE	0.029 2(25)	1.979(20)	16.38(31)	9.76(40)
V.B	$\alpha^2(Z\alpha)^4$	Coulomb distortion	0.0	−0.261	−1.010	−0.536
V.C	$(Z\alpha)^6$	3PE	−0.001 3(3)	0.002 2(9)	−0.214(214)	−0.165(165)
V.D	$\alpha(Z\alpha)^5$	eVP$^{(1)}$ with TPE	0.000 6(1)	0.027 5(4)	0.266(24)	0.158(12)
V.E	$\alpha(Z\alpha)^5$	μSE$^{(1)}$ + μVP$^{(1)}$ with TPE	0.000 4	0.002 6(3)	0.077(8)	0.059(6)
III	E_{QED}	Point nucleus	206.034 4(3)	228.774 0(3)	1644.348(8)	1668.491(7)
IV	$\mathcal{C} r_C^2$	Finite size	$-5.225\,9 r_p^2$	$-6.107\,4 r_d^2$	$-103.383 r_h^2$	$-106.209 r_\alpha^2$
V	E_{NS}	Nuclear structure	0.028 9(25)	1.750 3(200)	15.499(378)	9.276(433)
	E_L (exp)	Experiment[a]	202.370 6(23)	202.878 5(34)	1258.598(48)	1378.521(48)
	r_C	This review	0.840 60(39)	2.127 58(78)	1.970 07(94)	1.678 6(12)
	r_C	Previous work[a]	0.840 87(39)	2.125 62(78)	1.970 07(94)	1.678 24(83)

K. Pachucki et al. Rev. Mod. Phys. 96.015001 (2024)

Helium isotopes charge radii

$$E_{LS} = E_{QED} + C r_c^2 + E_{TPE} + E_{3PE} + \dots$$



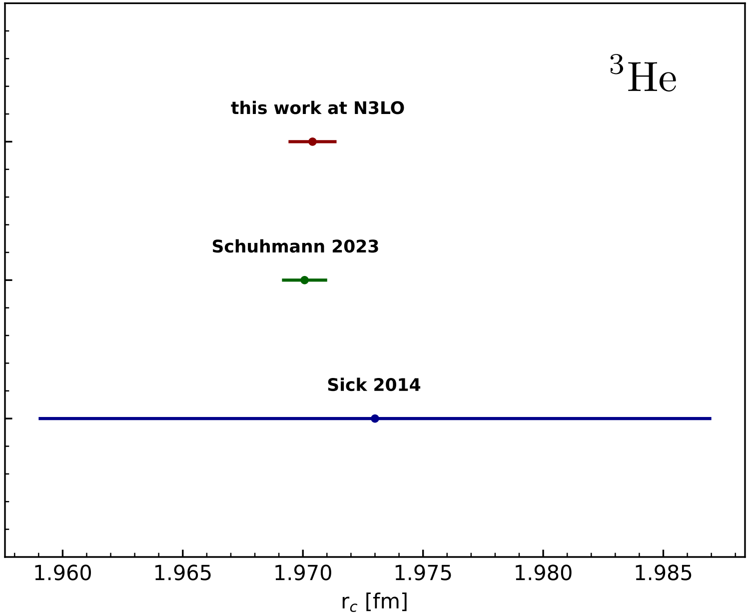


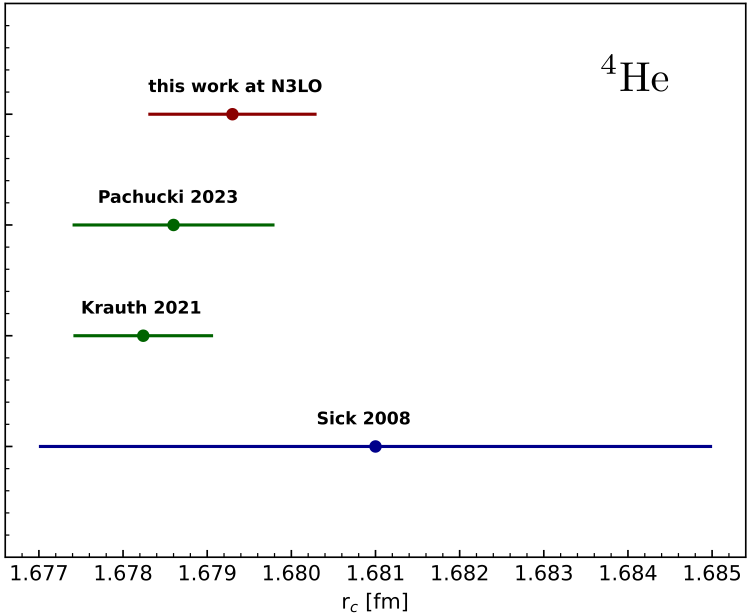
Helium isotopes charge radii

$$E_{LS} = E_{QED} + Cr_c^2 + E_{TPE} + E_{3PE} + \ldots$$

Updating this

SSLM, Thomas R. Richardson, Sonia Bacca, Arxiv:2401.13424





Isotope shift of muonic helium

$$\delta r_c^2 = \left[\frac{\mathrm{E}_{\mathrm{LS}(^3\mathrm{He})}}{\mathrm{C}(^3\mathrm{He})} - \frac{\mathrm{E}_{\mathrm{LS}(^4\mathrm{He})}}{\mathrm{C}(^4\mathrm{He})}\right] + \underbrace{\left[\frac{\mathrm{E}_{\mathrm{TPE}(^4\mathrm{He})}}{\mathrm{C}(^4\mathrm{He})} - \frac{\mathrm{E}_{\mathrm{TPE}(^3\mathrm{He})}}{\mathrm{C}(^3\mathrm{He})}\right]}_{\text{Define the nuclear structure part of this term as } \delta\mu}$$

$$+ \left[\frac{\mathrm{E}_{\mathrm{3PE}(^4\mathrm{He})}}{\mathrm{C}(^4\mathrm{He})} - \frac{\mathrm{E}_{\mathrm{3PE}(^3\mathrm{He})}}{\mathrm{C}(^3\mathrm{He})}\right] + \left[\frac{\mathrm{E}_{\mathrm{QED}(^4\mathrm{He})}}{\mathrm{C}(^4\mathrm{He})} - \frac{\mathrm{E}_{\mathrm{QED}(^3\mathrm{He})}}{\mathrm{C}(^3\mathrm{He})}\right] + \ldots$$

Exploiting correlations

$$\Delta\Big[X - Y\Big] = \sqrt{\Delta X^2 + \Delta Y^2 - 2\rho\underline{\Delta X}\,\underline{\Delta Y})}$$

Is obtained from the Bayesian analysis

Are obtained from the Bayesian analysis

with

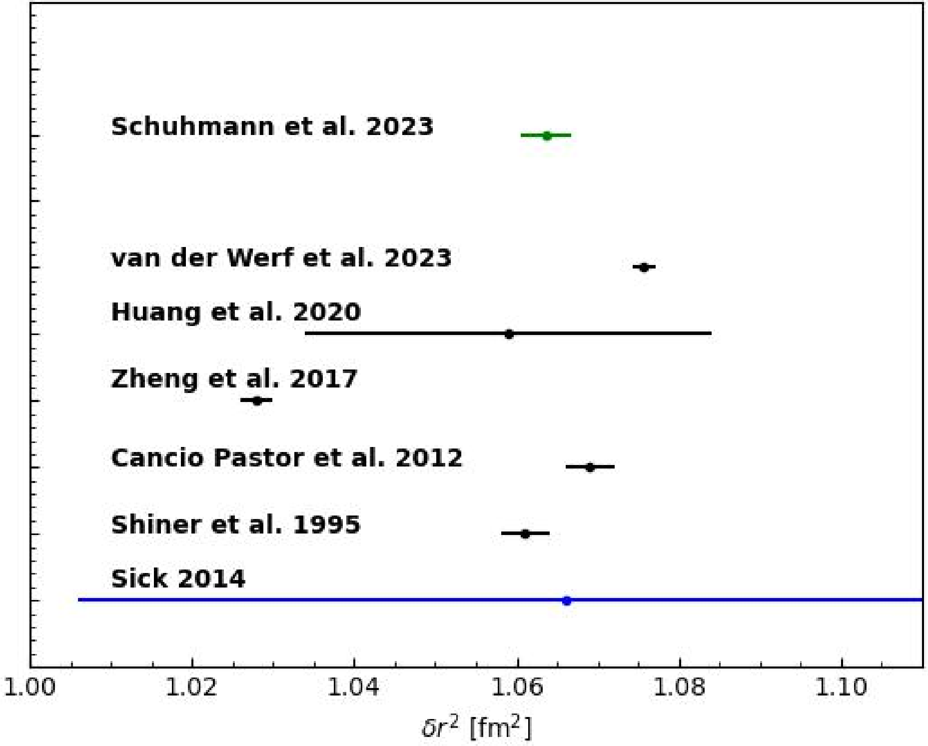
$$X = \mathrm{E}_{\mathrm{TPE}}(^4\mathrm{He})/\mathrm{C}(^4\mathrm{He})$$

and

$$Y = \mathrm{E}_{\mathrm{TPE}}(^3\mathrm{He})/\mathrm{C}(^3\mathrm{He})$$

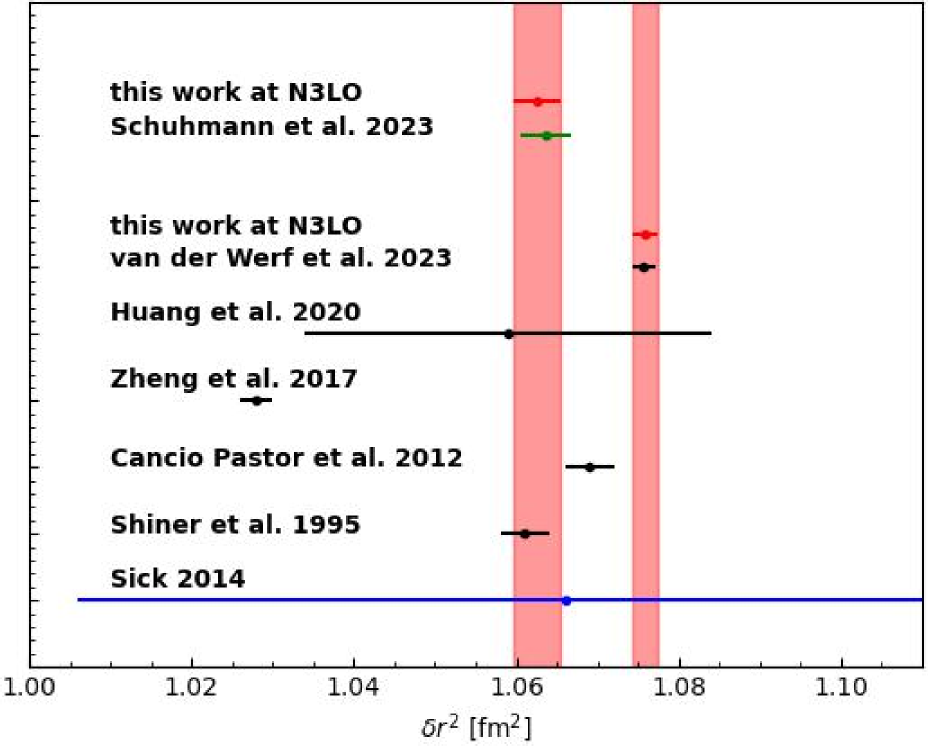
- **We can extract the correlation coefficient**
- $\rho \approx 0.8$
- **Assume that remaining nuclear structure effects (3-photon-exchange, etc…) are correlated with the same correlation coefficient.**

Isotope shift of muonic helium



Isotope shift of muonic helium

SSLM, Thomas R. Richardson, Sonia Bacca, Arxiv:2401.13424



Discussion points

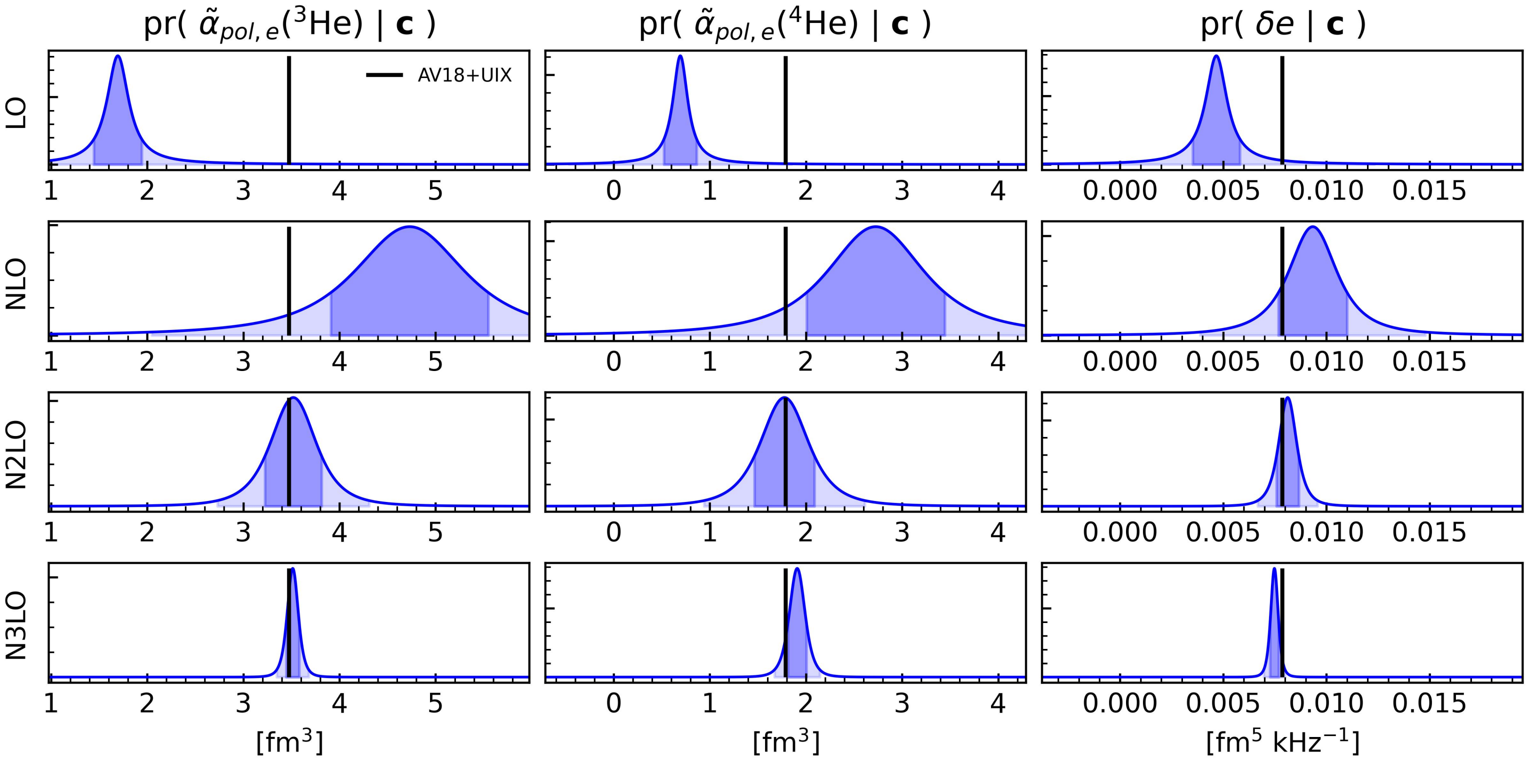
Uncertainty quantification is an excellent starting point to coordinate meaningful exchanges with experimental physics.

The LO ChEFT interaction is statistical worse than expected.

How to improve the predictive power of ChEFT by including information on correlations among observables in nuclei.

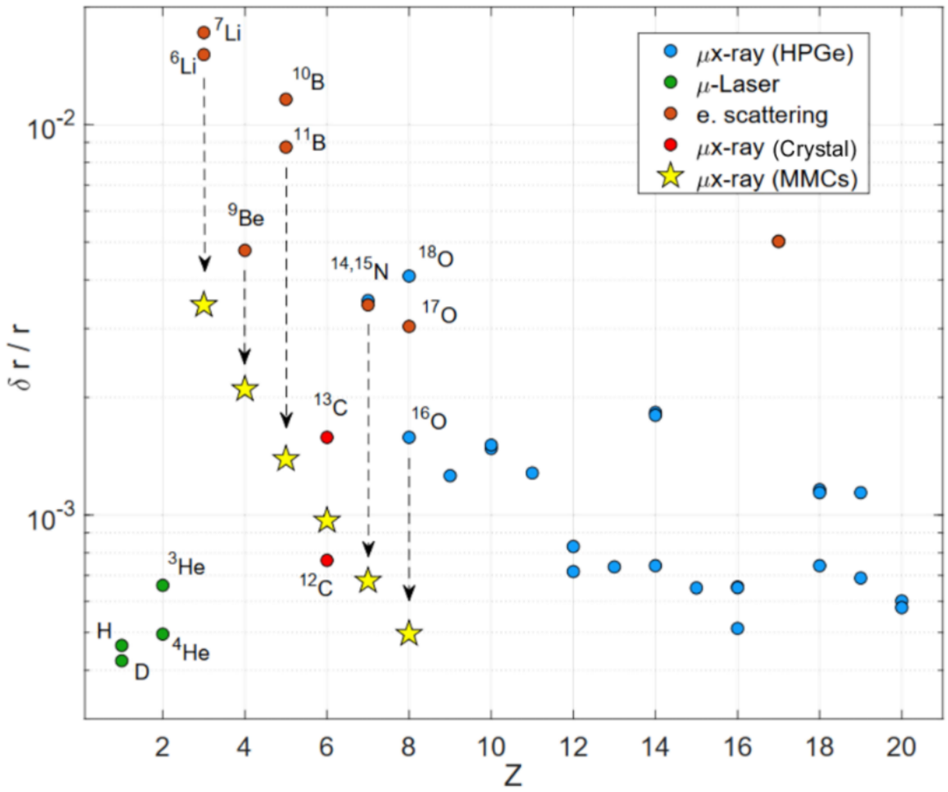
Backup

Two-photon nuclear structure effects in helium atoms

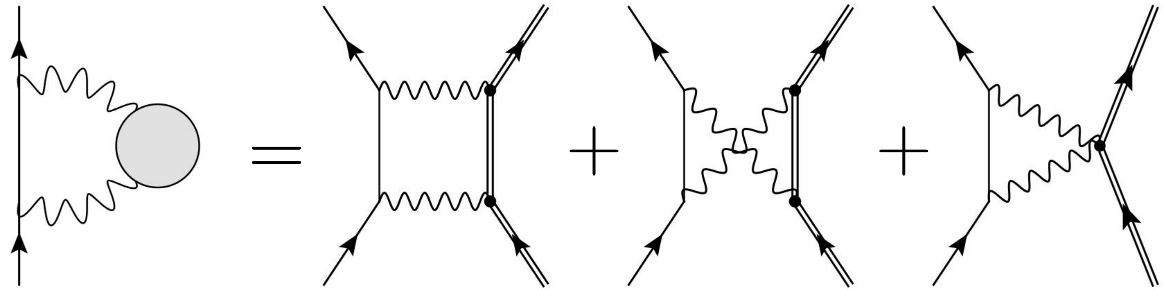


The future pipeline

A. Antognini et al, Arxiv:2210.16929



Evaluation of the TPE term



$= i\mathcal{M}$

The amplitudes define a complex potential that shift the 2S-energy of the bound muon by

$$\Delta \mathrm{E}_{2\mathrm{S}} = \langle \phi_{2\mathrm{S}} | \,\mathrm{Re}\left[i\mathcal{M} \right] | \phi_{2\mathrm{S}} \rangle$$

With $|\phi_{2\mathrm{S}}\rangle$ being the 2S-state of the muon. At our level of precision there are no corrections to 2P-states.

Evaluation of the TPE term

$$\Delta E_{2S} = -8\alpha^2 m\, \phi_{2S}^2 \Bigg\{ \sum_{N\neq 0} \int d^3x\, d^3y\ \langle 0|\, \rho_{\mathrm{ch}}^\dagger(\mathbf{y})\, |N\rangle \langle N|\, \rho_{\mathrm{ch}}(\mathbf{x})\, |0\rangle\ \mathrm{I}_{\mathrm{N}}(z)$$

$$+ \sum_{N\neq 0} \int d^3x\, d^3y\, \langle 0|\, J_i^\dagger(\mathbf{y})\, |N\rangle \langle N|\, J_j(\mathbf{x})\, |0\rangle \left[\delta_{ij} J_N(z) + z^i z^j \bar{J}_N(z)\right]$$

$$+ \int d^3x\, d^3y\ B^{ij}(\mathbf{x},\mathbf{y})\, \frac{1}{2}\left[\delta_{ij} K(z) + z_i z_j \bar{K}(z)\right] \Bigg\} \,.$$

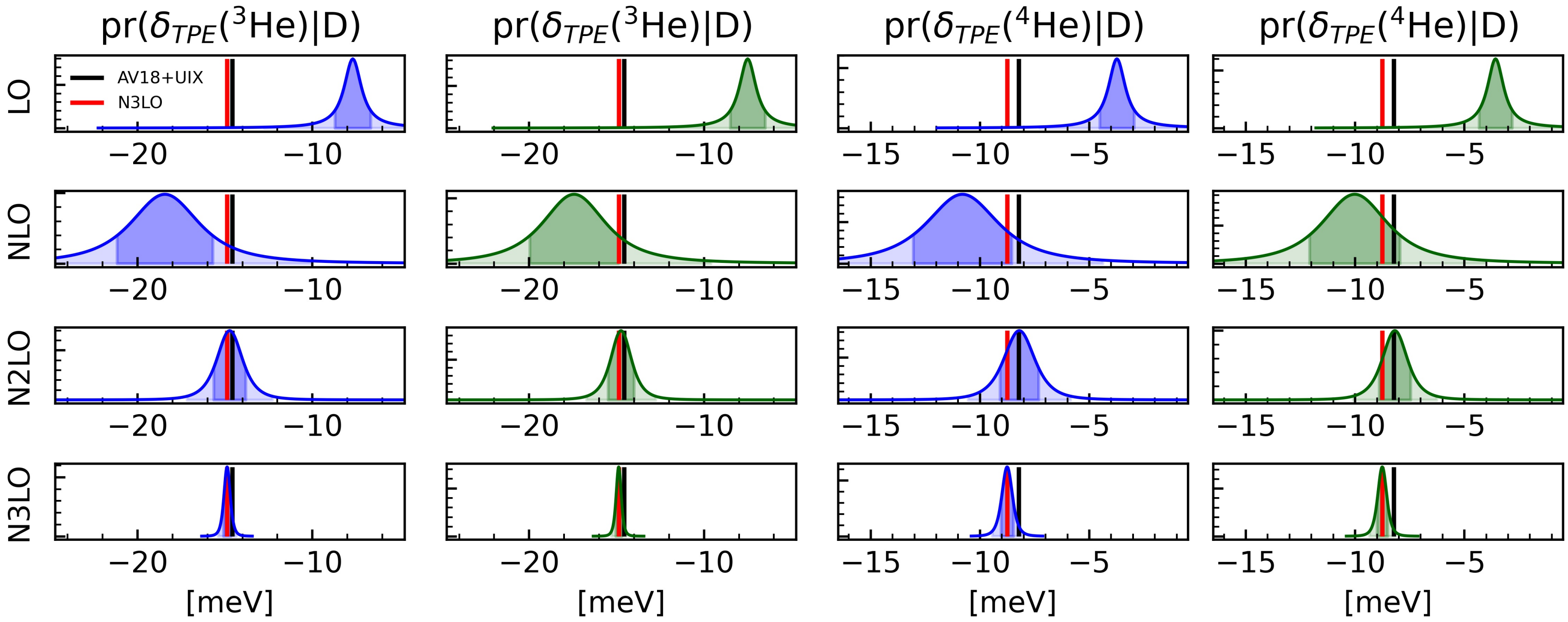
With $z = |\mathbf{x} - \mathbf{y}|$**.**

– The five structure functions are known by calculating the leptonic part of the Feynman diagrams.

– The nuclear matrix elements must be calculated numerically from Nuclear Theory.

TPE corrections in muonic Helium

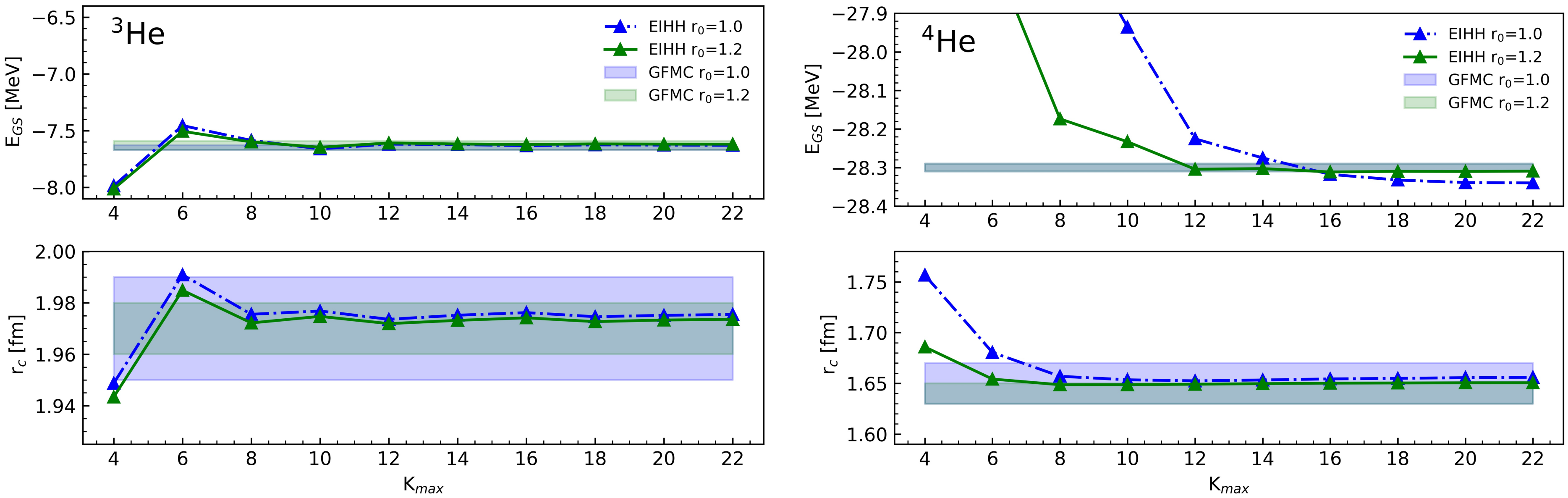
S.S.LM, et al. In preparation for 2023



C. Ji, et al. J. Phys. G: Nucl. Part. Phys. 45 (2018)

Benchmark tests

S.S. LM, S. Bacca , N. Barnea, Front. Phys. 9, 671869 (2021)



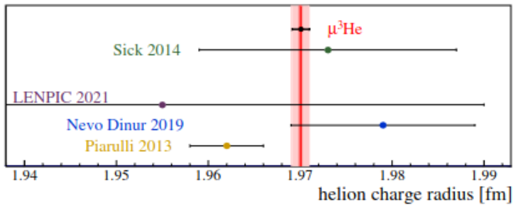
A matter of precision

$$\delta_{\mathrm{LS}} = \delta_{\mathrm{QED+NR}} + \delta_{\mathrm{FS}}^{(4)} \times r_c^2 + \delta_{\mathrm{TPE}}^{(5)} + \delta_{\mathrm{3PE}}^{(6)} + \ldots$$

For the muonic Helium-3 ion

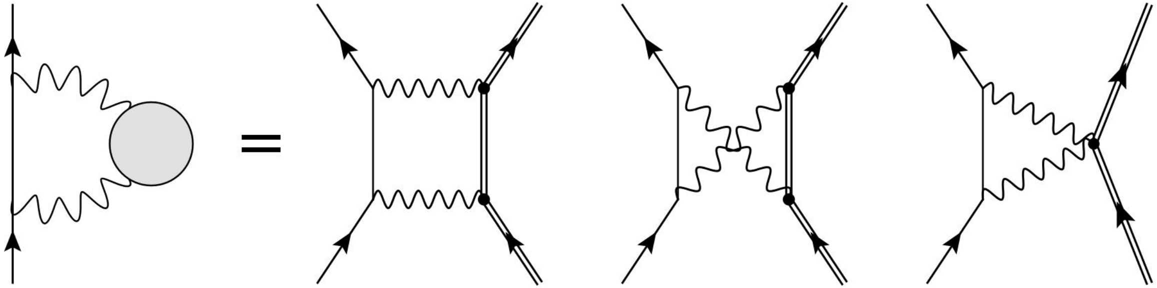
$$\delta_{\mathrm{QED+NR}} = +1,644.348(8)\ \mathrm{meV}$$
$$\delta_{\mathrm{FS}}^{(4)} = -103.383\ \mathrm{meV\ fm}^{-2}$$
$$\delta_{\mathrm{TPE}}^{(5)} = +15.499(378)\ \mathrm{meV}$$
$$\delta_{\mathrm{3PE}}^{(6)} = -0.214(214)\ \mathrm{meV}$$
$$\delta_{\mathrm{HO}}^{(5)} = -0.667(25)\ \mathrm{meV}$$

$$r_c = 1.97007(12)_{\mathrm{ex}}(93)_{\mathrm{th}}\ \mathrm{fm}$$



K. Schuhmann et. al. Arxiv 2305.11679 (2023)

Evaluation of the NS effects



$$\Delta E_{nl} = -8\alpha^2 m\ \phi_{nL}^2 \int \frac{d^3q}{4\pi} \left\{ \sum_{N\neq 0} |\langle N|\,\rho_{\text{ch}}(\mathbf{q})\,|0\rangle|^2 \frac{(2E_q + \omega_N)}{q^4 E_q\left[(E_q+\omega_N)^2 - m^2\right]} \right.$$

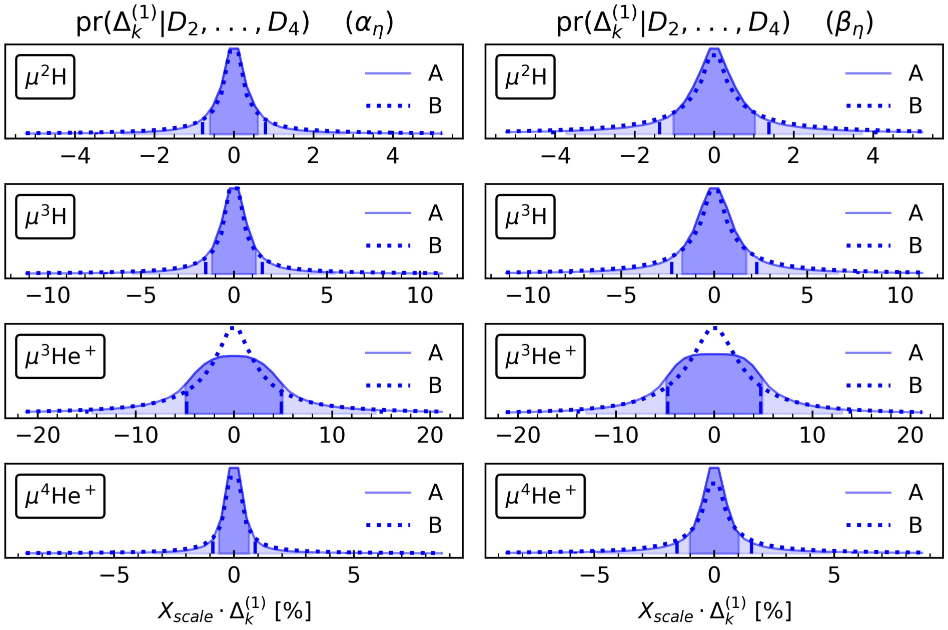
$$+ \sum_{N\neq 0} |\langle 0|\,\mathbf{J}_\perp(\mathbf{q})\,|N\rangle|^2 \left[\frac{q^2}{4m^2} \frac{(2E_q + \omega_N)}{q^4 E_q\left[(E_q+\omega_N)^2 - m^2\right]} - \frac{1}{4m^2 q^3} \frac{\omega_N + 2q}{(\omega_N + q)^2} \right]$$

$$\left. + B_\perp(\mathbf{q}) \frac{1}{8m^2 q^2} \left(\frac{1}{q} - \frac{1}{E_q} \right) \right\}$$

eta-expansion uncertainty

$$\Delta E_{nl}^{\mathrm{NR}} = -8\alpha^2\ \phi_{nl}^2 \sum_{N\neq 0} \int d^3x\ d^3y\ \langle 0|\,\rho_{\mathrm{ch}}^{\dagger}(\mathbf{y})\,|N\rangle\,\langle N|\,\rho_{\mathrm{ch}}(\mathbf{x})\,|0\rangle\ \ \mathrm{I}_{\mathrm{NR}}(z)$$

$$= -8\alpha^2\ \phi_{nl}^2 \sum_{N\neq 0} \int d^3x\ d^3y\ \langle 0|\,\rho_{\mathrm{ch}}^{\dagger}(\mathbf{y})\,|N\rangle\,\langle N|\,\rho_{\mathrm{ch}}(\mathbf{x})\,|0\rangle\ \left(\mathrm{I}_{\mathrm{NR}}^{(2)}(z) + \mathrm{I}_{\mathrm{NR}}^{(3)}(z) + \mathrm{I}_{\mathrm{NR}}^{(4)}(z) + ..\right)$$



S.S.LM, et al. 2022 J. Phys. G: Nucl. Part. Phys. 49 105101

	μ^2H	μ^3H	μ^3He$^+$	μ^4He$^+$
C. Ji, et al. (2018)	0.4%	1.3%	1.1%	0.8%
This work	0.8%	1.5%	4.8%	0.9%

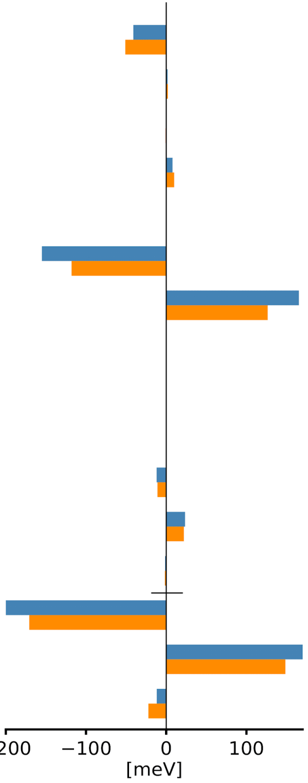
Preliminary works on Li atoms

$$\delta_{\rm TPE} = \delta^{(0)}_{\rm D1} + \delta^{(0)}_{\rm C} + \delta^{(1)}_{\rm Z1} + \delta^{(1)}_{\rm Z3} + \delta^{(2)}_{\rm NS} + \delta^{(2)}_{\rm Q} + \dots$$

δ_{TPE}	Ref. **(1) (meV)**	Ref. **(2) (meV)**
μ-$^6Li^{2+}$	-11.8(3)	-15(4)
μ-$^7Li^{2+}$	-22.2(4)	-21(4)

(1) S.L., A.Poggialini, S.Bacca, SciPost Phys. Proc. 3, 028 (2020)

(2) Drake et al, Phys. Rev. A **32**, 713 (1985)



	μ-$^6Li^{2+}$	μ-$^7Li^{2+}$
$\delta^{(0)}_{D1}$	-41.0	-51.0
$\delta^{(0)}_{L}$	1.66	2.04
$\delta^{(0)}_{T}$	-0.75	-0.92
$\delta^{(0)}_{C}$	7.85	9.89
$\delta^{(0)}_{M}$	-	-
$\delta^{(1)}_{R3}$	-155.0	-118.0
$\delta^{(1)}_{Z3}$	165.4	126.5
$\delta^{(2)}_{R2}$	-	-
$\delta^{(2)}_{Q}$	-	-
$\delta^{(2)}_{D1D3}$	-	-
$\delta^{(1)}_{R1}$	-12.0	-11.0
$\delta^{(1)}_{Z1}$	23.47	22.03
$\delta^{(2)}_{NS}$	-1.41	-1.75
δ^{A}_{pol}	-200.65	-170.74
δ^{A}_{Zem}	188.87	148.53
$\delta^{A}_{pol} + \delta^{A}_{Zem}$	-11.78	-22.21

Priors

$$X = \sum_{n=0}^{k} D_n + \sum_{n=k+1}^{\infty} D_n$$

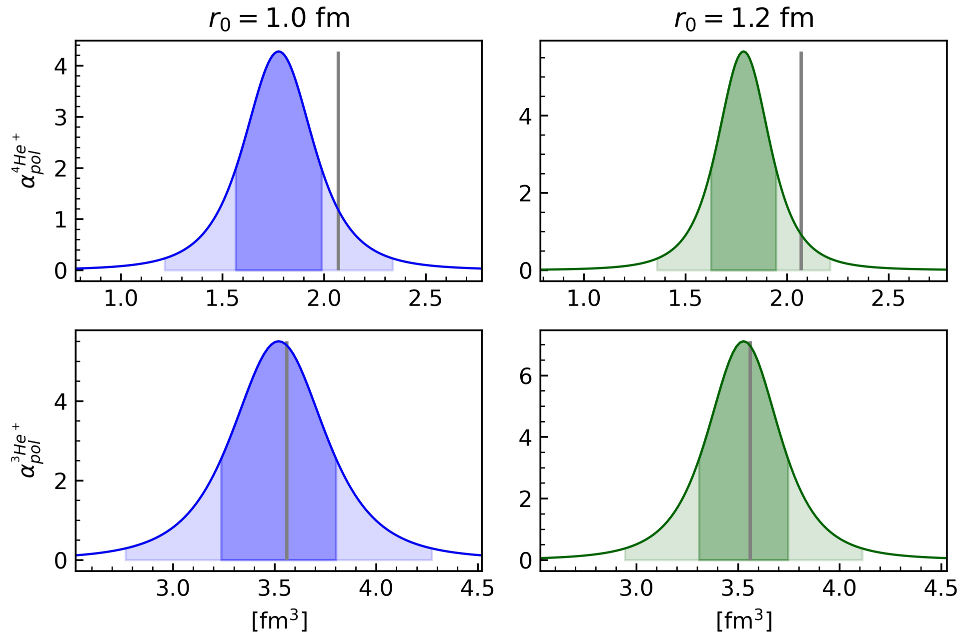
$$= X_{\text{ref}} \left[\sum_{n=0}^{k} c_n \; Q^n + \sum_{n=k+1}^{\infty} c_n \; Q^n \right]$$

Priors	$\mathrm{pr}(\eta)$
α_η	$\frac{1}{\lambda} \exp\left(\frac{\eta}{\lambda}\right)$
β_η	$\beta(a, b)$

Priors	$\mathrm{pr}(c_i \vert \bar{c})$	$\mathrm{pr}(\bar{c})$
A	$\frac{1}{2\bar{c}}\theta(\bar{c} - \vert c_i \vert)$	$\frac{1}{\ln(\bar{c}_>/\bar{c}_<)\bar{c}}\theta(\bar{c} - \bar{c}_<)\theta(\bar{c}_> - \bar{c})$
B	$\frac{1}{\sqrt{2\pi}\bar{c}}\exp\left(-\frac{c_i^2}{2\bar{c}^2}\right)$	$\frac{1}{\ln(\bar{c}_>/\bar{c}_<)\bar{c}}\theta(\bar{c} - \bar{c}_<)\theta(\bar{c}_> - \bar{c})$

NS effects in 3He+ and 4He+

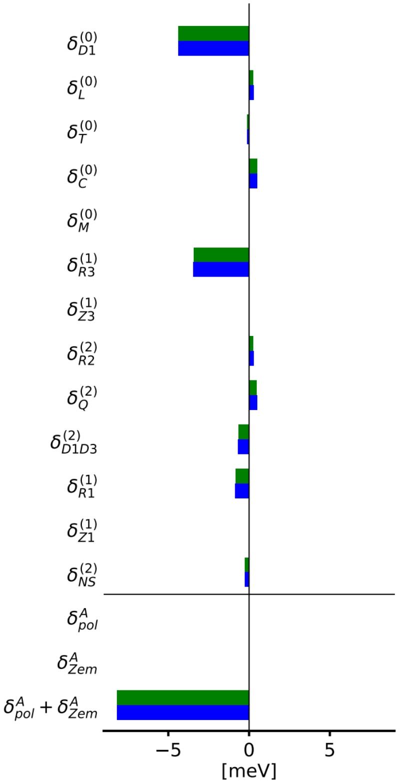
$$\delta^{(5)}_{\rm pol} = -m\alpha^4 \left\langle \sum_a \delta^3(r_a) \right\rangle (m^3 \alpha_{\rm pol})$$



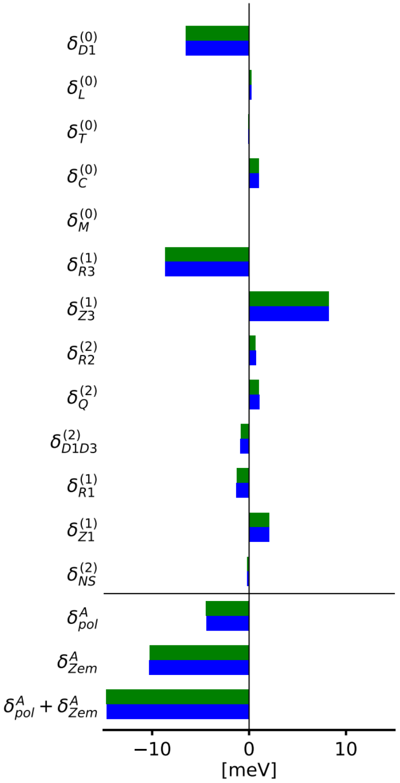
1S-2S	^{3}He$^+$	^{4}He$^+$
[1]	48(5)kHz	28(3)kHz
This work	48(6)kHz	24(4)kHz

— [1] K. Pachucki, A.M. Moro Phys.Rev.A 75,032521(2007)

NS corrections in μ4He+



	$r_0 = 1.2$	$r_0 = 1.0$
$\delta_{D1}^{(0)}$	-4.386	-4.373
$\delta_{L}^{(0)}$	0.272	0.287
$\delta_{T}^{(0)}$	-0.124	-0.125
$\delta_{C}^{(0)}$	0.517	0.514
$\delta_{M}^{(0)}$	0.011	0.011
$\delta_{R3}^{(1)}$	-3.422	-3.477
$\delta_{Z3}^{(1)}$	-	-
$\delta_{R2}^{(2)}$	0.267	0.285
$\delta_{Q}^{(2)}$	0.484	0.505
$\delta_{D1D3}^{(2)}$	-0.668	-0.69
$\delta_{R1}^{(1)}$	-0.846	-0.856
$\delta_{Z1}^{(1)}$	-	-
$\delta_{NS}^{(2)}$	-0.272	-0.277
δ_{pol}^{A}	-	-
δ_{Zem}^{A}	-	-
$\delta_{pol}^{A} + \delta_{Zem}^{A}$	-8.174	-8.196



	$r_0 = 1.2$	$r_0 = 1.0$
$\delta_{D1}^{(0)}$	-6.562	-6.552
$\delta_{L}^{(0)}$	0.232	0.232
$\delta_{T}^{(0)}$	-0.104	-0.104
$\delta_{C}^{(0)}$	1.018	1.015
$\delta_{M}^{(0)}$	0.008	0.008
$\delta_{R3}^{(1)}$	-8.674	-8.691
$\delta_{Z3}^{(1)}$	8.223	8.255
$\delta_{R2}^{(2)}$	0.67	0.696
$\delta_{Q}^{(2)}$	1.043	1.062
$\delta_{D1D3}^{(2)}$	-0.877	-0.899
$\delta_{R1}^{(1)}$	-1.297	-1.302
$\delta_{Z1}^{(1)}$	2.054	2.057
$\delta_{NS}^{(2)}$	-0.191	-0.193
δ_{pol}^{A}	-4.458	-4.411
δ_{Zem}^{A}	-10.277	-10.312
$\delta_{pol}^{A} + \delta_{Zem}^{A}$	-14.735	-14.723