#### Fermilab DUS. DEPARTMENT OF Science



# Recent advances in the description of leptonnucleus scattering

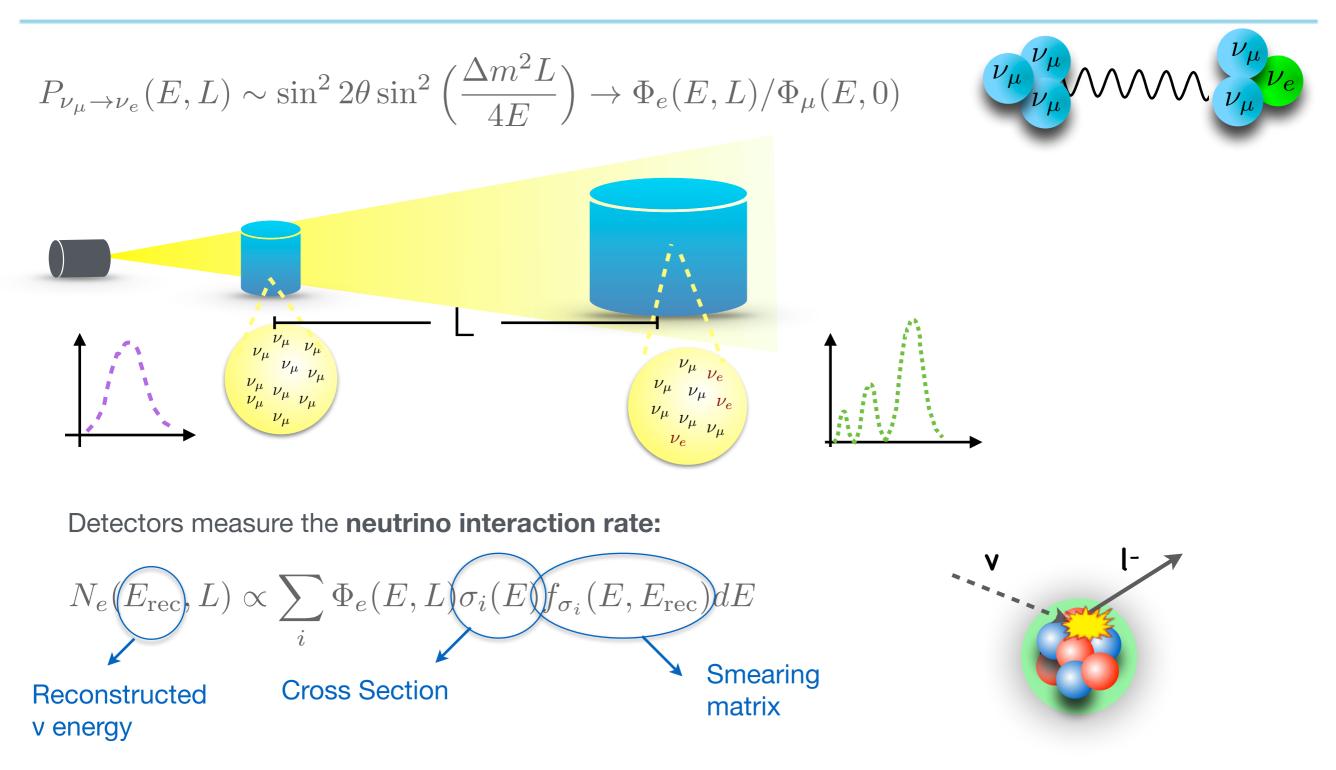
Noemi Rocco

Uncertainty Quantification in Nuclear Physics MITP – June 24 - 28, 2024

- Lepton nucleus interactions : GFMC
- Lepton nucleus interactions : Factorization Scheme
- Lepton nucleus interactions : BSM scenarios
  - Bayesian Artificial Neural network



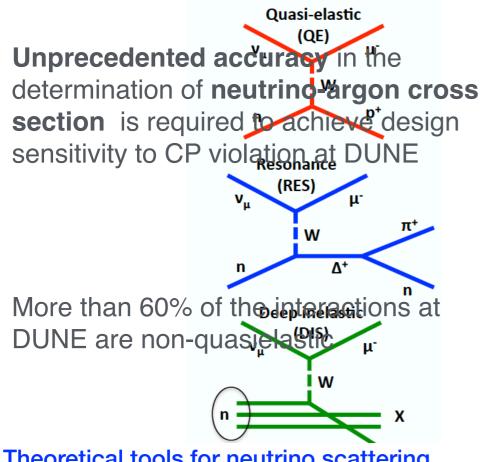
#### **Addressing Neutrino-Oscillation Physics**



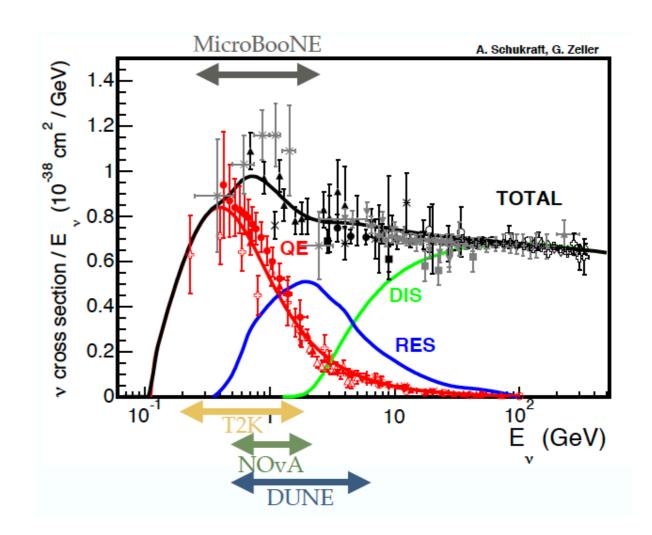
A precise determination of  $\sigma(E)$  is crucial to extract v oscillation parameters

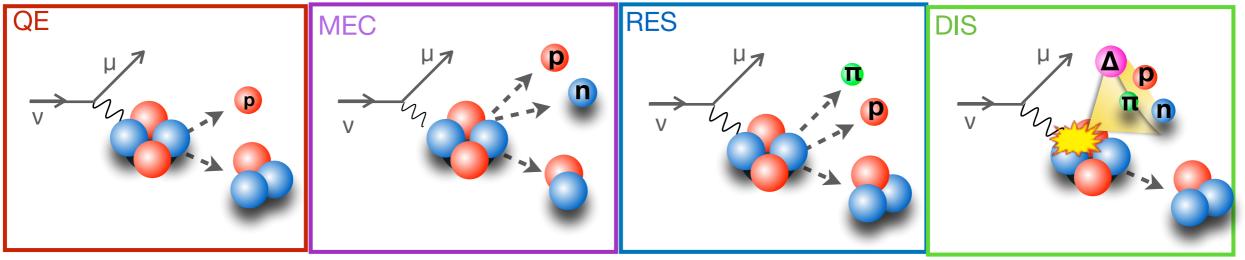


#### Inputs for the nuclear model



Theoretical tools for neutrino scattering, Contribution to: 2022 Snowmass Summer Study







# Why do we need more precision?

CLAS and e4v collaboration, Nature 599 (2021) 7886, 565-570

Used semi-exclusive electron scattering data to test models and event generators used in oscillation analyses

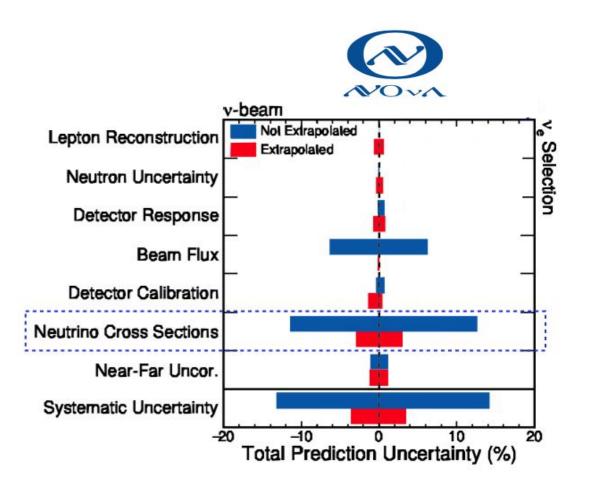
The results indicate	the <b>need</b>	for substantial , $\bar{v}_e$ , $v_e/\bar{v}_e$ ,	
Error source	FHC	RHC	FHC/RHC
Flux and (ND unconstrained)	15.1	12.2	1.2
cross section (ND constrained)	3.2	3.1	2.7
SK detector	2.8	3.8	1.5
SK FSI + SI + PN	3.0	2.3	1.6
Nucleon removal energy	7.1	3.7	3.6
$\sigma(\nu_e)/\sigma(\bar{\nu}_e)$	2.6	1.5	3.0
NC1γ	1.1	2.6	1.5
NC other	0.2	0.3	0.2
$\sin^2 \theta_{23}$ and $\Delta m_{21}^2$	0.5	0.3	2.0
$\sin^2 \theta_{13}$ PDG2018	2.6	2.4	1.1
All systematics	8.8	7.1	6.0

T2K Collaboration, Phys. Rev. D 103, 112008 (2021)

 $\mathbf{U}.\mathbf{U}$ 0.0

1.4 1.4  $C(e,e')_{0\pi} E_{QE} [GeV]$ 

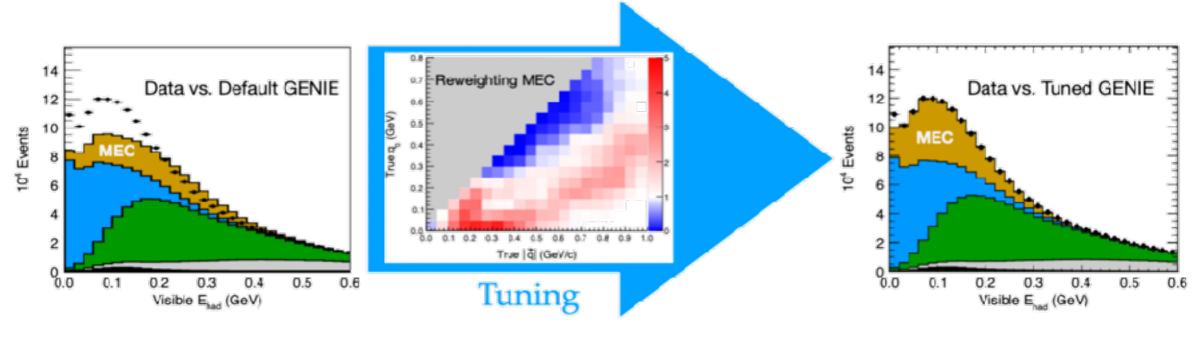
Current oscillation experiments report large systematic uncertainties associated with neutrino- nucleus interaction models.



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# Tuning

Discrepancies between generators and data often corrected by tuning an empirical model of the least well known mechanism: MEC ("meson exchange"/two-body currents)



Coyle, Li, and Machado, JHEP 12, 166 (2022)

Mis-modeling can distort signals of new physics, **biasing** measurement of **new physics parameters** 

Studies on the impact of different neutrino interactions and nuclear models on determining neutrino oscillation parameters are critical. These enable us to assess the level of precision we aim at.

Coloma, et al, Phys.Rev.D 89 (2014) 7, 073015



# Theory of lepton-nucleus scattering

• The cross section of the process in which a lepton scatters off a nucleus is given by

 $d\sigma \propto L^{lphaeta}R_{lphaeta}$ 

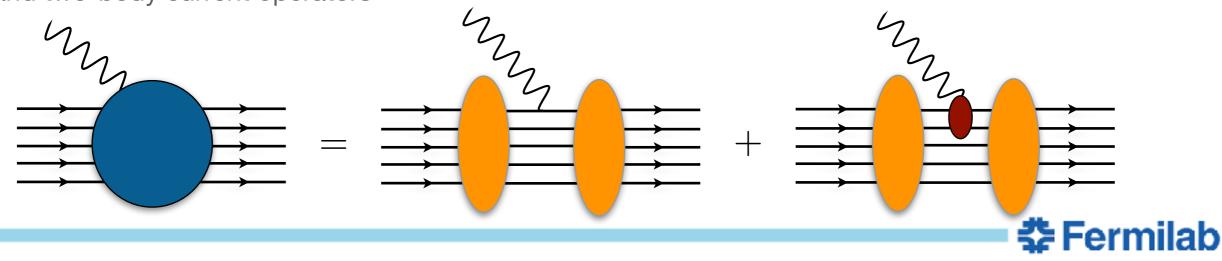
Leptonic Tensor: determined by lepton kinematics Hadronic Tensor: nuclear response function

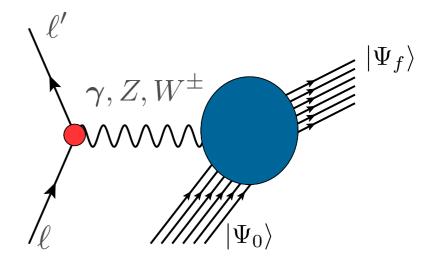
$$R_{\alpha\beta}(\omega,\mathbf{q}) = \sum_{f} \langle 0|J_{\alpha}^{\dagger}(\mathbf{q})|f\rangle \langle f|J_{\beta}(\mathbf{q})|0\rangle \delta(\omega - E_{f} + E_{0})$$

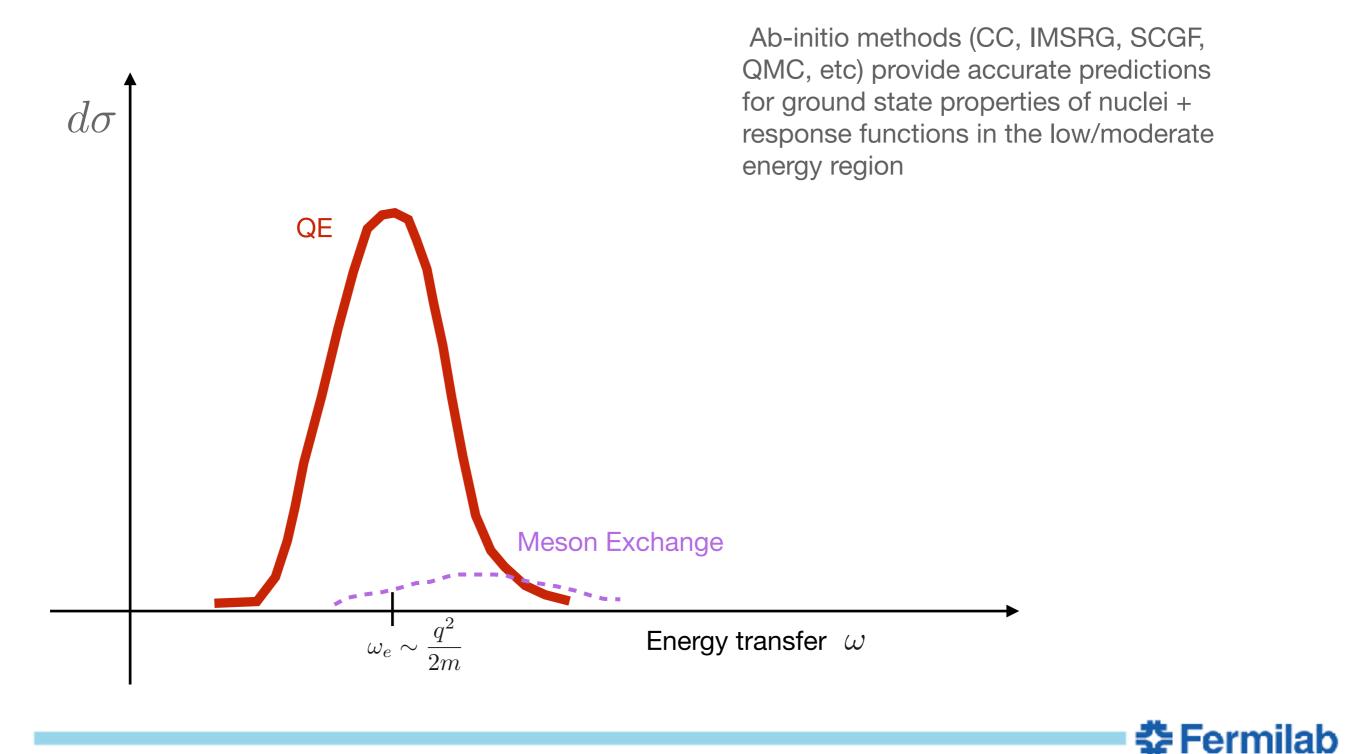
The initial and final wave functions describe many-body states:

$$|0\rangle = |\Psi_0^A\rangle , |f\rangle = |\Psi_f^A\rangle, |\psi_p^N, \Psi_f^{A-1}\rangle, |\psi_k^\pi, \psi_p^N, \Psi_f^{A-1}\rangle \dots$$

One and two-body current operators







## Many-Body method: GFMC

QMC techniques projects out the exact lowest-energy state:

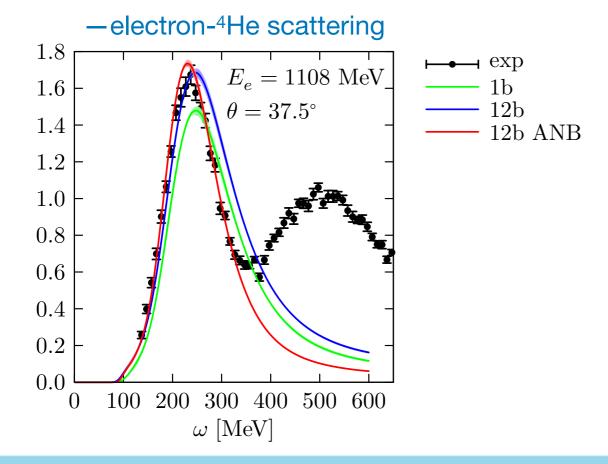
$$e^{-(H-E_0)\tau}|\Psi_T\rangle \to |\Psi_0\rangle$$

Nuclear response function involves evaluating a number of transition amplitudes. Valuable information can be obtained from the **integral transform of the response function** 

$$E_{\alpha\beta}(\sigma,\mathbf{q}) = \int d\omega K(\sigma,\omega) R_{\alpha\beta}(\omega,\mathbf{q}) = \langle \psi_0 | J_{\alpha}^{\dagger}(\mathbf{q}) K(\sigma,H-E_0) J_{\beta}(\mathbf{q}) | \psi_0 \rangle$$

Inverting the Laplace transform is a complicated problem

<u>A. Lovato et al, PRL117 (2016), 082501,</u> PRC97 (2018), 022502



Inclusive results which are virtually correct in the QE

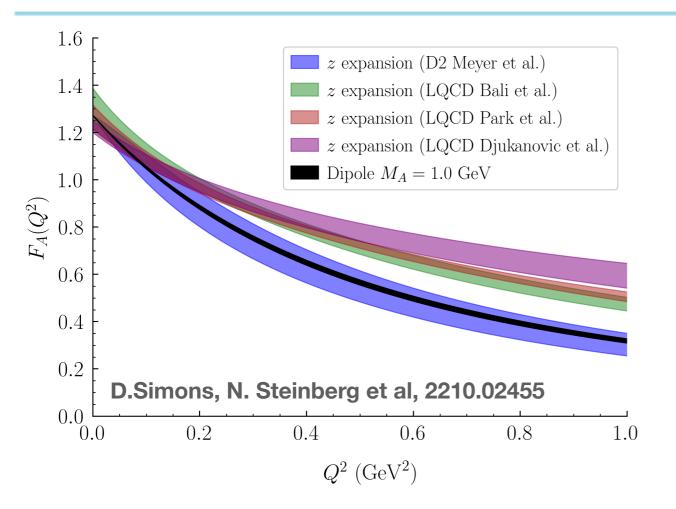
Different Hamiltonians can be used in the timeevolution operator

Relies on non-relativistic treatment of the kinematics

Can not handle explicit pion degrees of freedom



# Axial form factor determination



Comparison with recent MINERvA antineutrino-hydrogen charged-current measurements

1-2σ agreement with MINERvA data and LQCD prediction by PNDME Collaboration

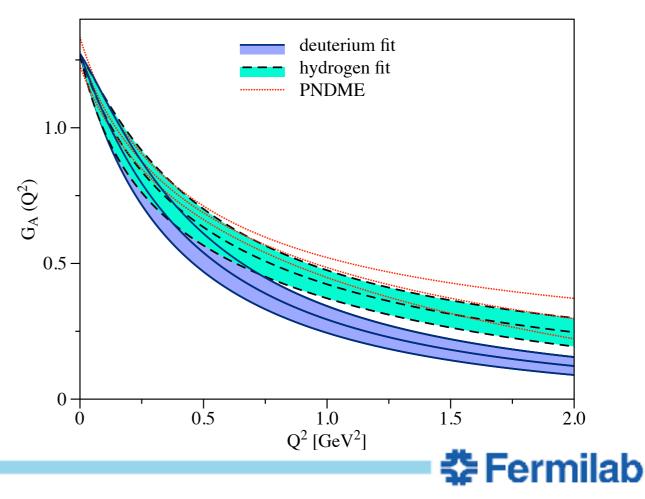
Novel methods are needed to remove excitedstate contributions and discretization errors A. Meyer, A. Walker-Loud, C. Wilkinson, 2201.01839

D2 Meyer et al: fits to neutrino-deuteron scattering data

LQCD result: general agreement between the different calculations

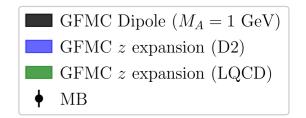
LQCD results are 2-3 $\sigma$  larger than D2 Meyer ones for Q<sup>2</sup> > 0.3 GeV<sup>2</sup>

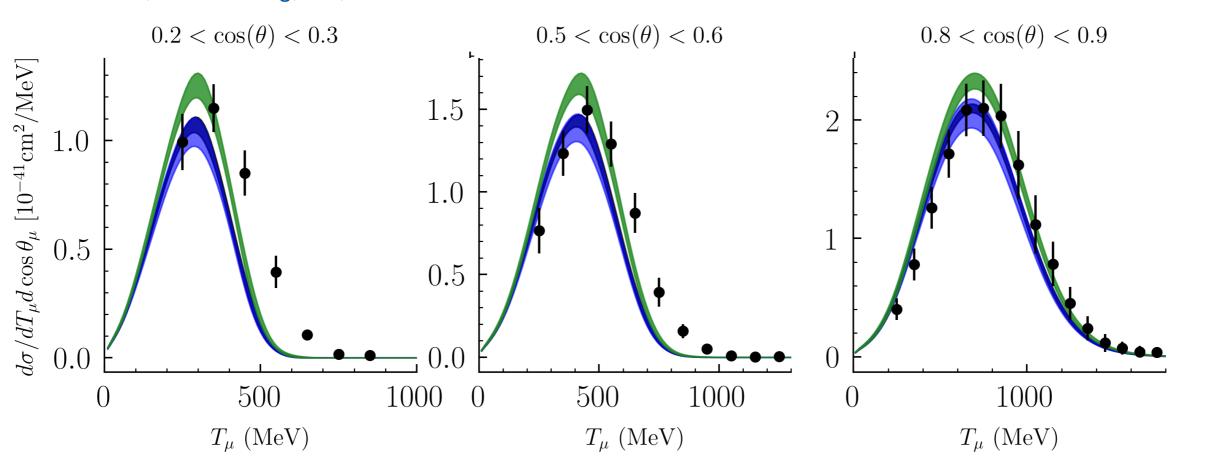
O. Tomalak, R. Gupta, T. Battacharaya, 2307.14920



# Study of model dependence in neutrino predictions

MiniBooNE results; study of the dependence on the axial form factor:





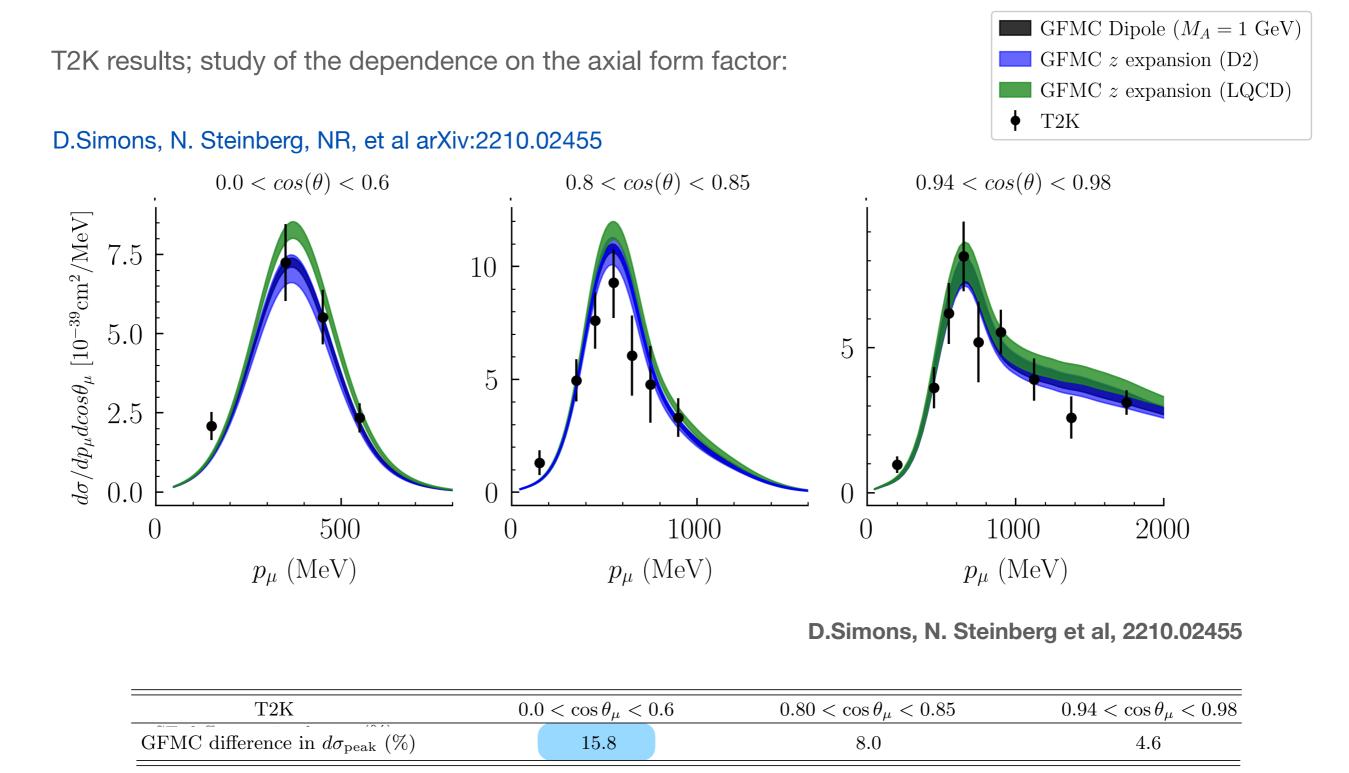
D.Simons, N. Steinberg, NR, et al arXiv:2210.02455

D.Simons, N. Steinberg et al, 2210.02455

MiniBooNE	$0.2 < \cos \theta_{\mu} < 0.3$	$0.5 < \cos \theta_{\mu} < 0.6$	$0.8 < \cos \theta_{\mu} < 0.9$
GFMC Difference in $d\sigma_{\text{peak}}$ (%)	18.6	17.1	12.2



# Study of model dependence in neutrino predictions



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#### Why relativity is important

$$R_{\alpha\beta}(\omega,\mathbf{q}) = \sum_{f} \langle 0|J_{\alpha}^{\dagger}(\mathbf{q})|f\rangle \langle f|J_{\beta}(\mathbf{q})|0\rangle \delta(\omega - E_{f} + E_{0}) \longrightarrow \text{Kinematics}$$
Currents

Covariant expression of the e.m. current:

$$j_{\gamma,S}^{\mu} = \bar{u}(\mathbf{p}') \Big[ \frac{G_E^S + \tau G_M^S}{2(1+\tau)} \gamma^{\mu} + i \frac{\sigma^{\mu\nu} q_{\nu}}{4m_N} \frac{G_M^S - G_E^S}{1+\tau} \Big] u(\mathbf{p})$$

Nonrelativistic expansion in powers of  $p/m_N$ 

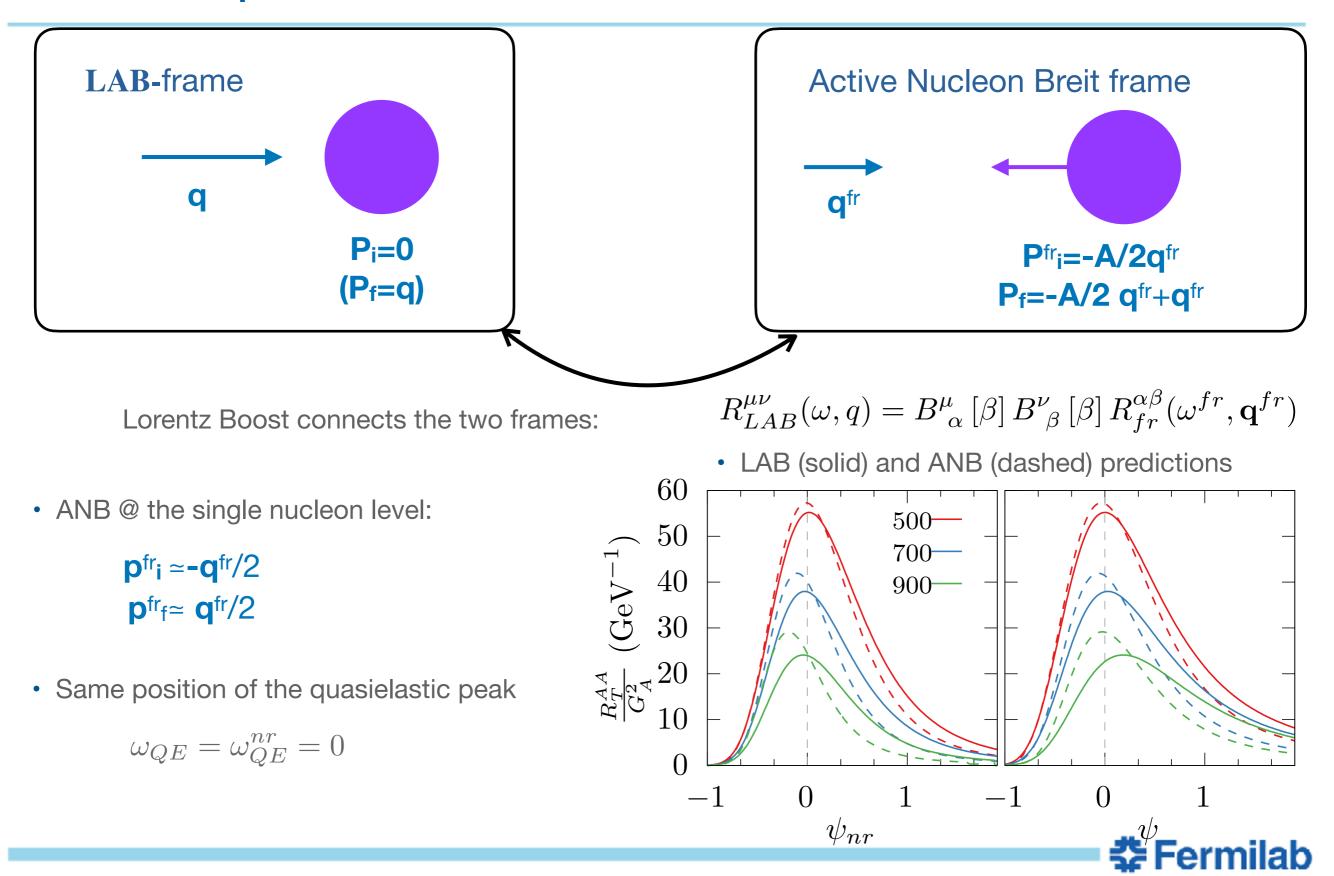
$$j^0_{\gamma,S} = \frac{G^S_E}{2\sqrt{1+Q^2/4m_N^2}} - i\frac{2G^S_M - G^S_E}{8m_N^2}\mathbf{q}\cdot(\pmb{\sigma}\times\mathbf{p})$$

Energy transfer at the quasi-elastic peak:

$$w_{QE} = \sqrt{\mathbf{q}^2 + m_N^2 - m_N}$$
  $w_{QE}^{nr} = \mathbf{q}^2/(2m_N)$ 



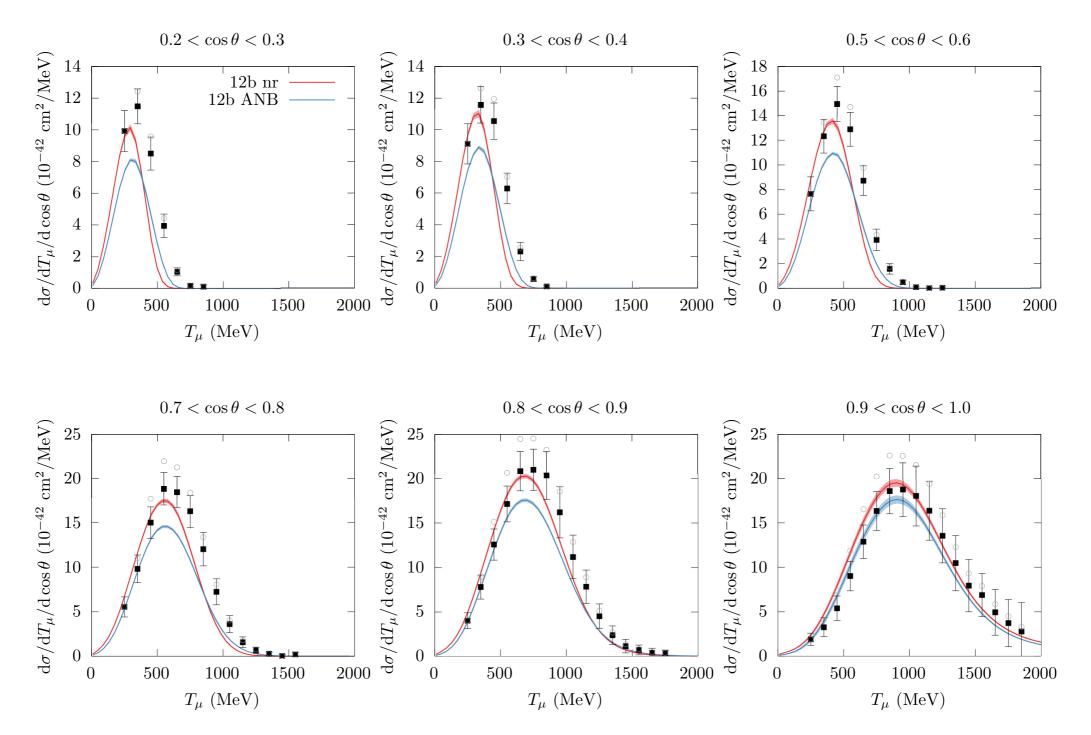
#### Frame dependence



#### Cross sections: Green's Function Monte Carlo

MiniBooNE results including relativistic corrections

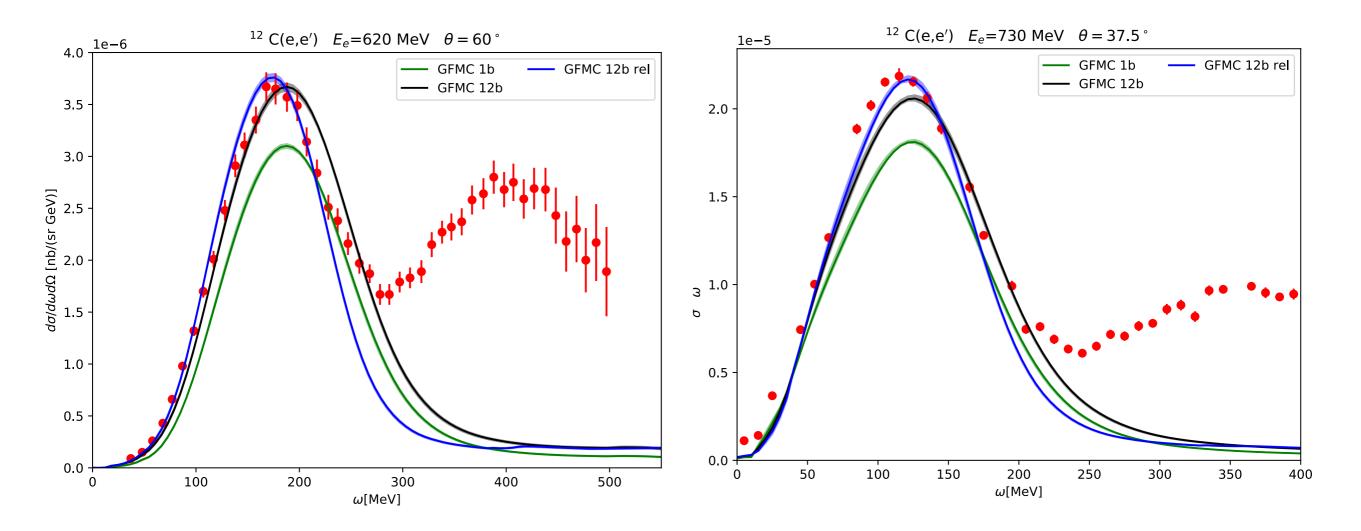
A.Nikolakopoulos, A.Lovato, NR, PRC 109 (2024) 1, 014623





#### Cross sections: Green's Function Monte Carlo

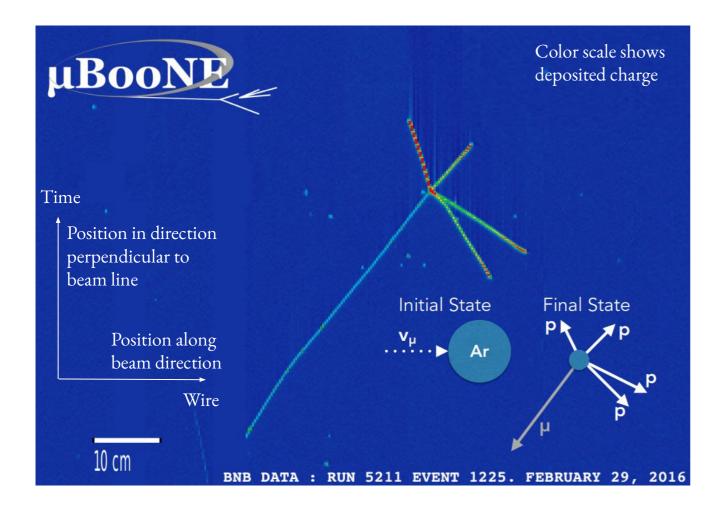
Electron scattering results including relativistic corrections for some kinematics covered by the calculated responses



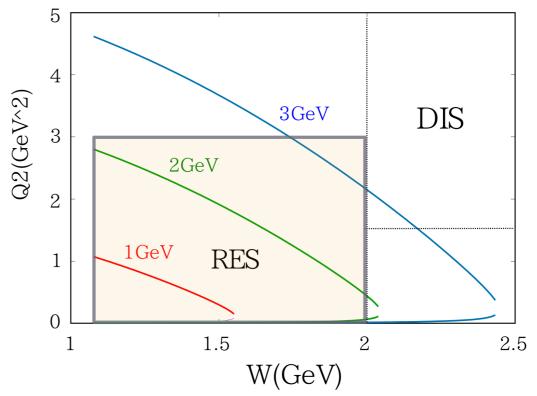
A.Lovato, A.Nikolakopoulos, NR, N. Steinberg, Universe 9 (2023) 8, 36



#### Address new experimental capabilities



T.Sato talks @ NuSTEC Workshop on Neutrino-Nucleus Pion Production in the Resonance Region

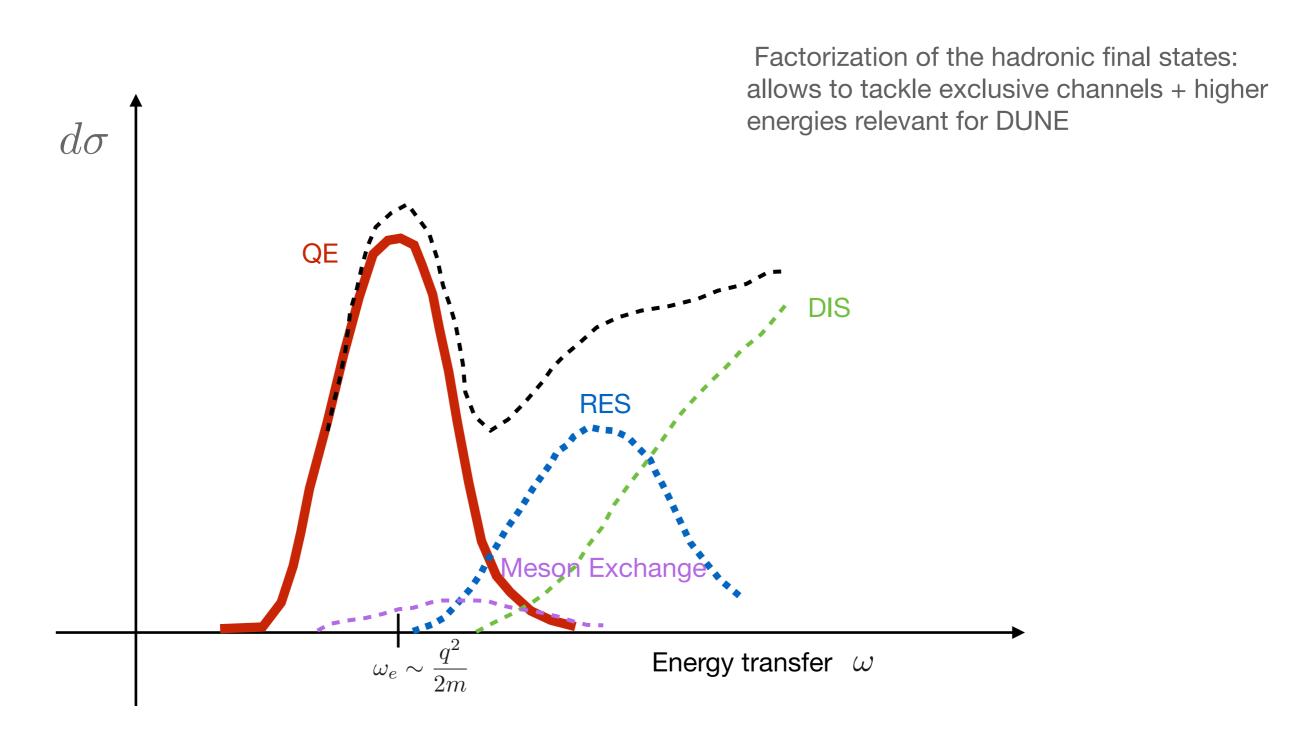


- Excellent spatial resolution
- Precise calorimetric information
- Powerful particle identification

$$W = \sqrt{(p+q)^2}, Q^2 = -q^2 = -(p_{\nu} - p_l)^2$$



#### **Factorization Based Approaches**





# Spectral function approach

At large momentum transfer, the scattering reduces to the sum of individual terms

$$J_{\alpha} = \sum j_{\alpha}^{i} \qquad |\Psi_{f}\rangle \to |p\rangle \otimes |\Psi_{f}\rangle_{A-1}$$
$$J^{\mu} \to \sum j_{i}^{\mu} \qquad |\psi_{f}^{A}\rangle \to |p\rangle \otimes |\psi_{f}^{A-1}\rangle \qquad E_{f} = E_{f}^{A-1} + e(\mathbf{p})$$

The incoherenticontribution of the one-body response reads

$$R_{\alpha\beta} \simeq \int \frac{d^{3}k}{(2\pi)^{3}} dEP_{h}(\mathbf{k}, E) \sum_{i} \langle k | j_{\alpha}^{i}{}^{\dagger} | k + q \rangle \langle k + q | j_{\beta}^{i} | k \rangle \delta(\omega + E - e(\mathbf{k} + \mathbf{q}))$$

$$= \Psi_{0} \rangle = \frac{1}{|\Psi_{f}\rangle_{A-1}} P_{A-1}$$

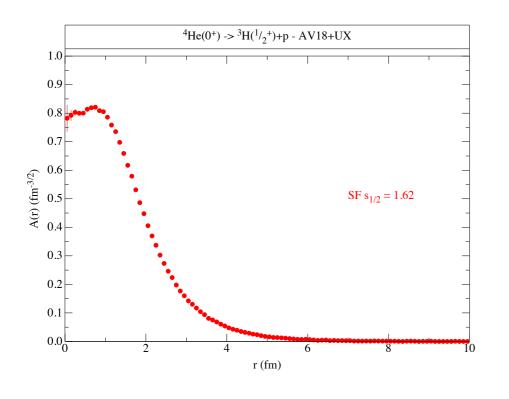
$$= \frac{1}{|\Psi_{f}\rangle_{A-1}} P_{A-1} P$$

lab

# QMC Spectral function of light nuclei

• Single-nucleon spectral function:

 $P_{p,n}(\mathbf{k}, E) = \sum_{n} \left| \langle \Psi_0^A | [|k\rangle \otimes |\Psi_n^{A-1}\rangle] \right|^2 \delta(E + E_0^A - E_n^{A-1}) = P^{MF}(\mathbf{k}, E) + P^{\text{corr}}(\mathbf{k}, E)$ 

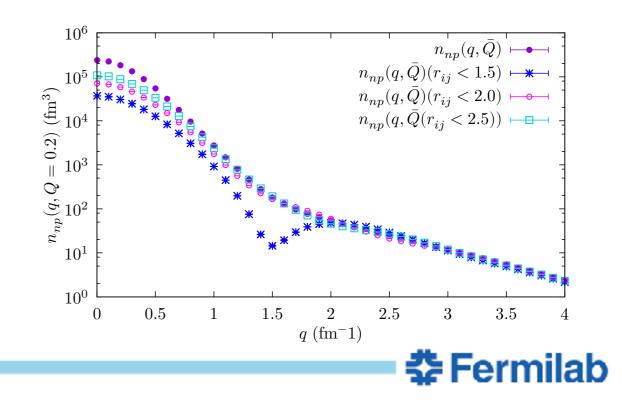


$$P^{\text{corr}}(\mathbf{k}, E) = \int d^3k' \Big| \langle \Psi_0^A | [|k\rangle | k'\rangle \otimes |\Psi_n^{A-2}\rangle] \Big|^2$$
$$\times \delta \Big( E - B_A - e(\mathbf{k}') + B_{A-2} - \frac{(\mathbf{k} + \mathbf{k}')^2}{2m_{A-2}} \Big)$$

• Written in terms of two-body momentum distribution

$$P^{MF}(\mathbf{k}, E) = \left| \langle \Psi_0^A | [|k\rangle \otimes |\Psi_n^{A-1}\rangle] \right|^2 \\ \times \delta \left( E - B_A + B_{A-1} - \frac{\mathbf{k}^2}{2m_{A-1}} \right)^2$$

• The single-nucleon overlap has been computed within VMC (center of mass motion fully accounted for)



# Spectral function approach

The hadronic tensor for two-body current factorizes as

$$R_{2b}^{\mu\nu}(\mathbf{q},\omega) \propto \int dE d^3k d^3k' P_{2b}(\mathbf{k},\mathbf{k}',E)$$
$$\times d^3p d^3p' |\langle kk' | j_{2b}^{\mu} | pp' \rangle|^2$$

$$|f\rangle \to |p_{\pi}p\rangle \otimes |f_{A-1}\rangle \to =$$

11

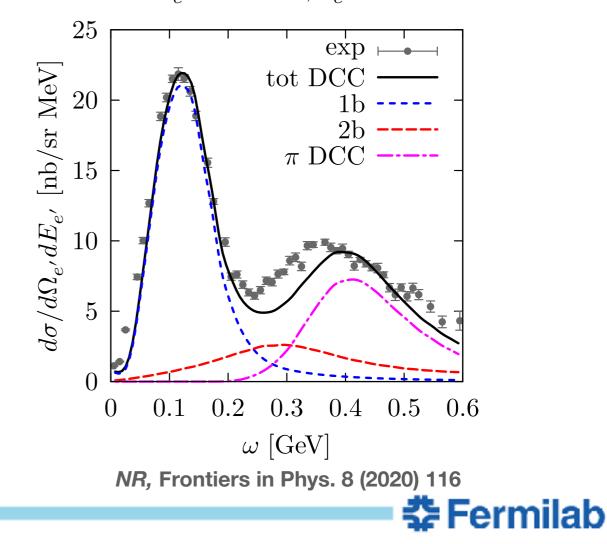
.

Production of real  $\pi$  in the final state

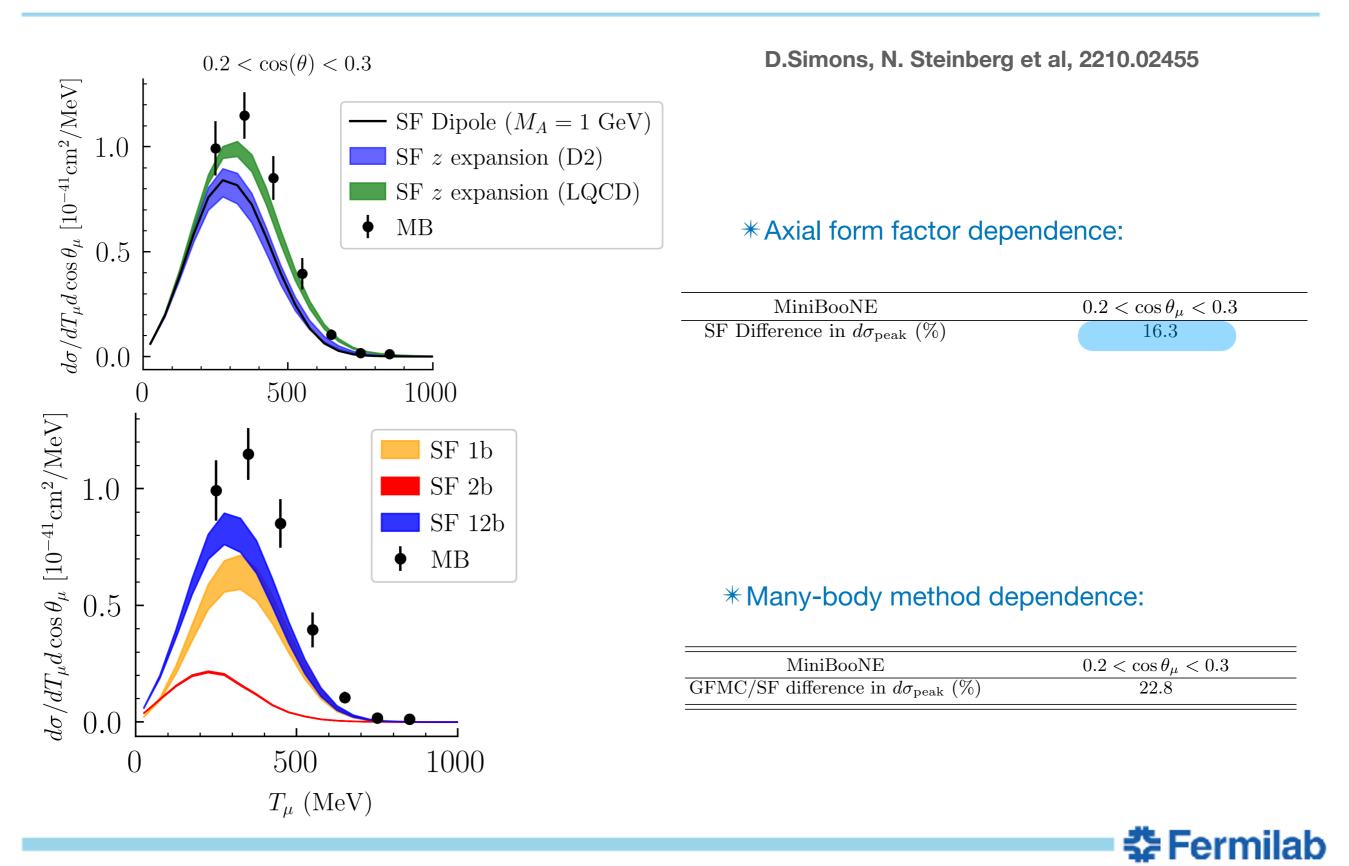
$$R_{1b\pi}^{\mu\nu}(\mathbf{q},\omega) \propto \int dE d^3 k P_{1b}(\mathbf{k},E) \times d^3 p d^3 k_{\pi} |\langle k|j^{\mu}|pk_{\pi}\rangle|^2$$

Pion production elementary amplitudes currently derived within the extremely sophisticated Dynamic Couple Chanel approach;

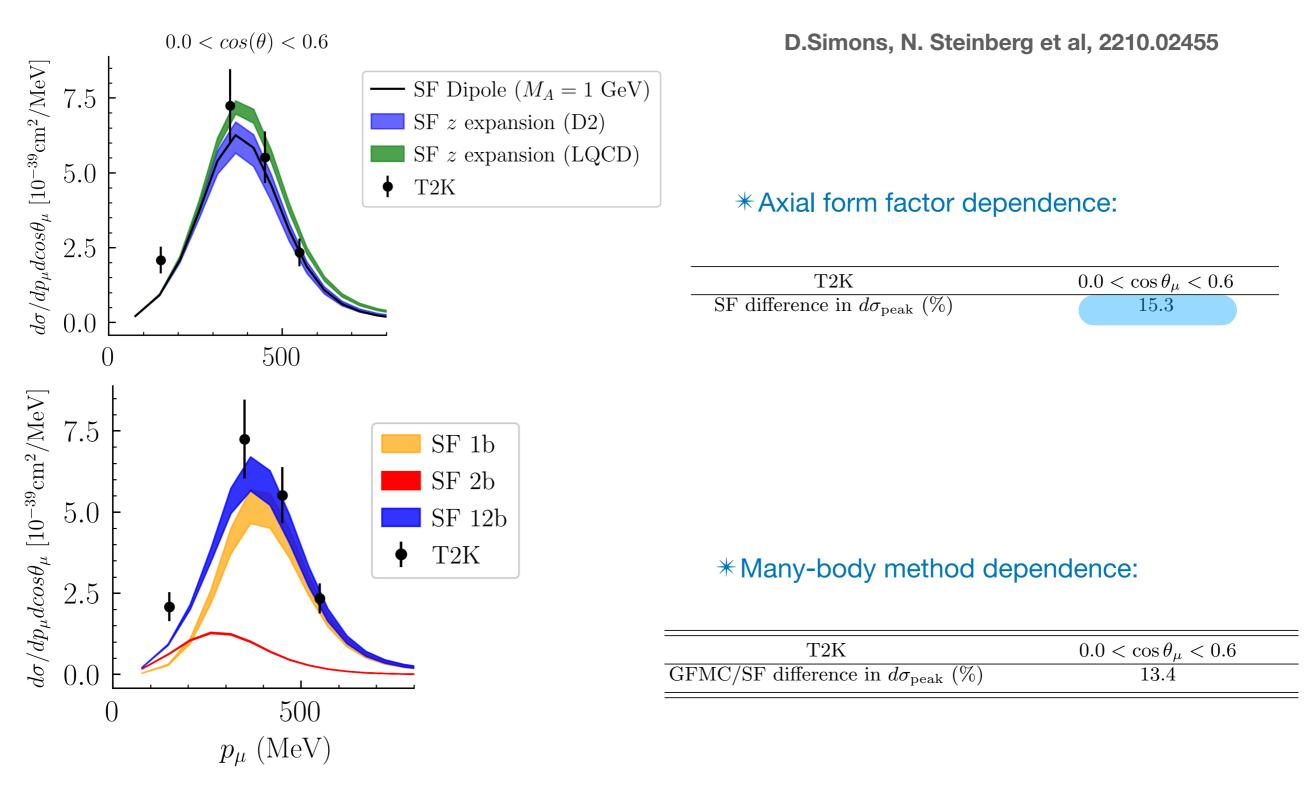
S.X.Nakamura, et al PRD92(2015) T. Sato, et al PRC67(2003)  $E_e = 730 \text{ MeV}, \theta_e = 37.0^{\circ}$ 



# **Axial Form Factors Uncertainty needs**

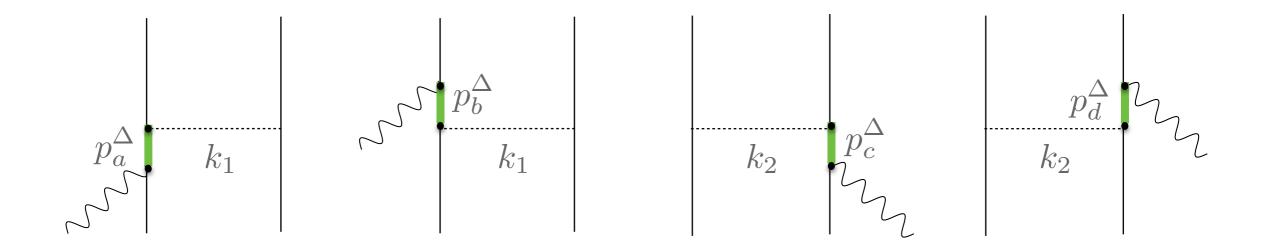


## **Axial Form Factors Uncertainty needs**





#### Two-body currents - Delta contribution



$$j_{\Delta}^{\mu} = \frac{3}{2} \frac{f_{\pi NN} f^*}{m_{\pi}^2} \left\{ \Pi(k_2)_{(2)} \left[ \left( -\frac{2}{3} \tau^{(2)} + \frac{I_V}{3} \right)_z F_{\pi NN}(k_2) F_{\pi N\Delta}(k_2) (J_a^{\mu})_{(1)} - \left( \frac{2}{3} \tau^{(2)} + \frac{I_V}{3} \right)_z F_{\pi NN}(k_2) F_{\pi N\Delta}(k_2) (J_b^{\mu})_{(1)} \right] + (1 \leftrightarrow 2) \right\}$$

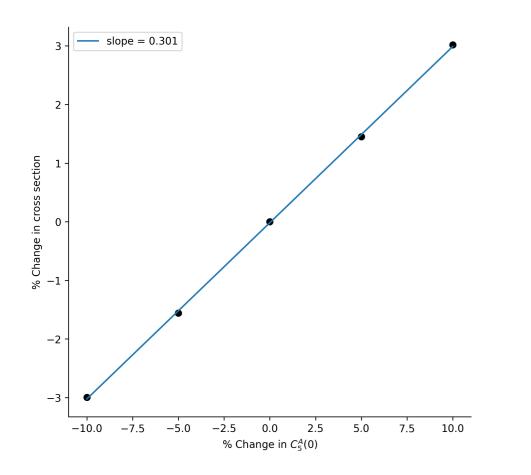
Rarita Schwinger propagator  $(j_{a}^{\mu})_{V} = (k_{\pi}')^{\alpha} G_{\alpha\beta}(p_{\Delta}) \begin{bmatrix} C_{3}^{V} \\ m_{N} \end{bmatrix} \qquad (j_{a}^{\mu})_{A} = (k_{\pi}')^{\alpha} G_{\alpha\beta}(p_{\Delta}) C_{5}^{A} g^{\beta\mu}$   $(j_{a}^{\mu})_{A} = (k_{\pi}')^{\alpha} G_{\alpha\beta}(p_{\Delta}) C_{5}^{A} g^{\beta\mu}$ 



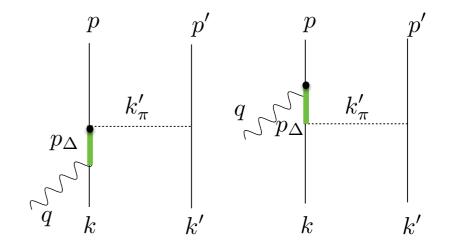
where

## **Resonance Uncertainty needs**

The largest contributions to two-body currents arise from resonant  $N\to \Delta$  transitions yielding pion production



D.Simons, N. Steinberg et al, 2210.02455



The normalization of the dominant  $N \to \Delta$  transition form factor needs be known to 3% precision to achieve 1% cross-section precision for MiniBooNE kinematics

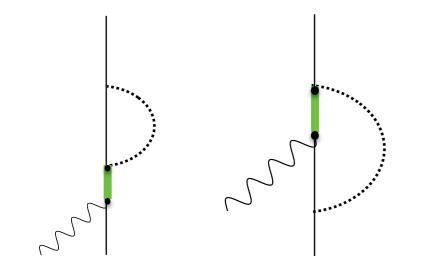
State-of-the-art determinations of this form factor from experimental data on pion electroproduction achieve 10-15% precision (under some assumptions)

#### Hernandez et al, PRD 81 (2010)

Further constraints on  $N \to \Delta$  transition relevant for two-body currents and  $\pi$  production will be necessary to achieve few-percent cross-section precision



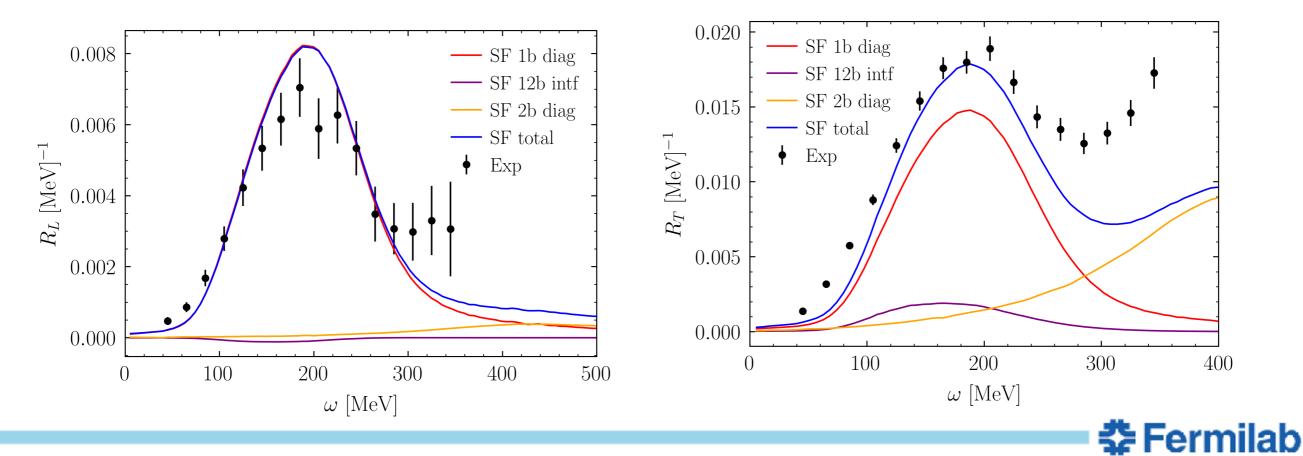
#### Including the one- and two-body interference



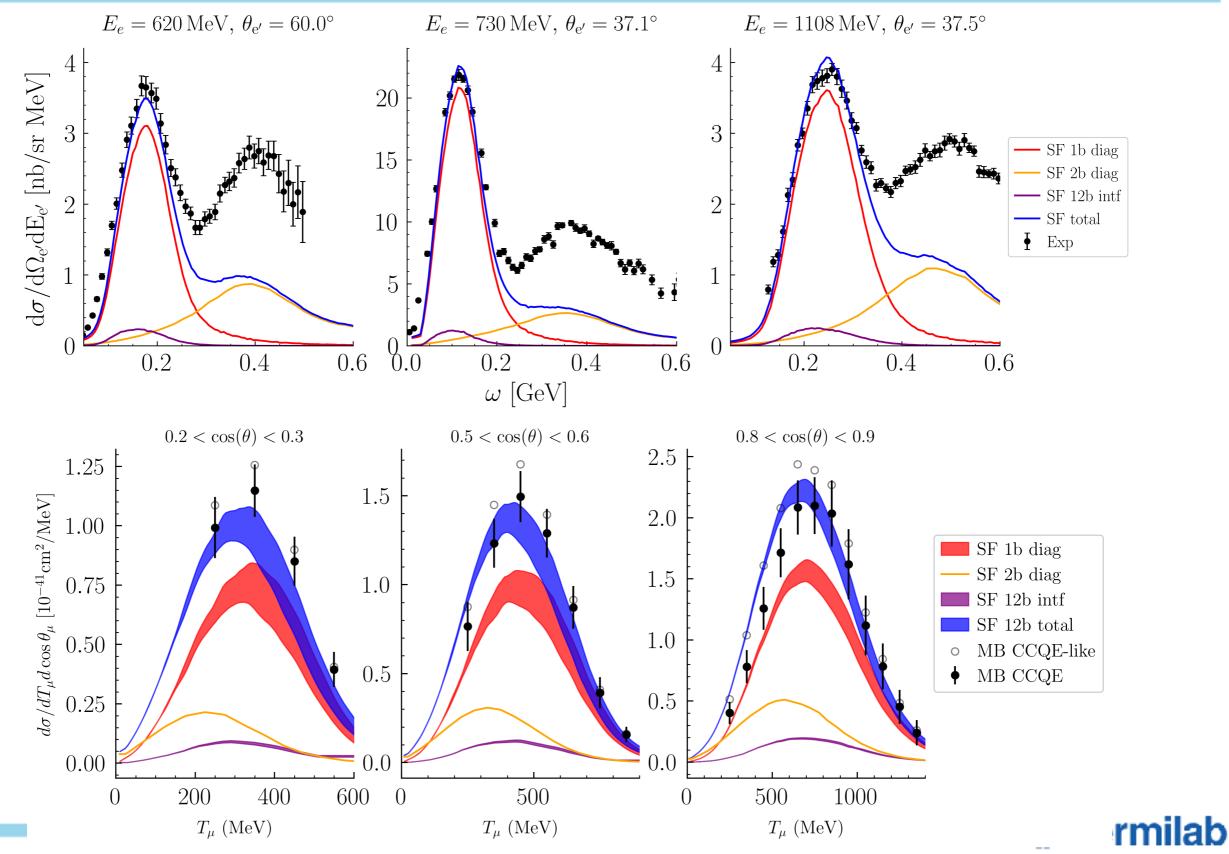
We recently included interference effects between oneand two-body currents yielding single nucleon knock-out

Observe a small quenching in the longitudinal channel and an enhancement in the q.e. peak in the transverse → agreement with the GFMC

#### N. Steinberg, NR, A. Lovato, arXiv: 2312.12545



#### Including the one- and two-body interference



N. Steinberg, NR, A. Lovato, arXiv: 2312.12545

#### Interplay with BSM scenarios

- Interested in Weak Effective Field Theory (WEFT), valid below the electroweak scale, with the electroweak gauge bosons, the Higgs boson, and the top quark integrated out
- CC: New left/right handed, (pseudo)scalar and tensor interactions

$$\mathcal{L}_{\text{WEFT}} \supset -\frac{2V_{ud}}{v^2} \{ [\mathbf{1} + \epsilon_L_{\alpha\beta} (\bar{u}\gamma^{\mu}P_L d)(\bar{\ell}_{\alpha}\gamma_{\mu}P_L\nu_{\beta}) \\ + \epsilon_R_{\alpha\beta} (\bar{u}\gamma^{\mu}P_R d)(\bar{\ell}_{\alpha}\gamma_{\mu}P_L\nu_{\beta}) \\ + \frac{1}{2} \epsilon_S_{\alpha\beta} (\bar{u}d)(\bar{\ell}_{\alpha}P_L\nu_{\beta}) - \frac{1}{2} \epsilon_P_{\alpha\beta} (\bar{u}\gamma_5 d)(\bar{\ell}_{\alpha}P_L\nu_{\beta}) \\ + \frac{1}{4} \epsilon_T_{\alpha\beta} (\bar{u}\sigma^{\mu\nu}P_L d)(\bar{\ell}_{\alpha}\sigma_{\mu\nu}P_L\nu_{\beta}) + \text{h.c.} \}$$

SM Interactions:

$$: \langle p(p_p) | \bar{q}_u \gamma_\mu q_d | n(p_n) \rangle = \bar{u}_p(p_p) \bigg[ G_V(Q^2) \gamma_\mu + i \frac{\tilde{G}_{T(V)}(Q^2)}{2M_N} \sigma_{\mu\nu} q^\nu - \frac{\tilde{G}_S(Q^2)}{2M_N} q_\mu \bigg] u_n(p_n)$$

$$: \langle p(p_p) | \bar{q}_u \gamma_\mu \gamma_5 q_d | n(p_n) \rangle = \bar{u}_p(p_p) \bigg[ G_A(Q^2) \gamma_\mu \gamma_5 + i \frac{\tilde{G}_{T(A)}(Q^2)}{2M_N} \sigma_{\mu\nu} q^\nu \gamma_5 - \frac{\tilde{G}_P(Q^2)}{2M_N} q_\mu \gamma_5 \bigg] u_n(p_n)$$



#### Form factors - new interactions

• Scalar: conservation of the vector current (CVC)

 $G_{S}(Q^{2}) = -\frac{\delta M_{N}^{QCD}}{\delta M_{N}^{QCD}} G_{V}(Q^{2}) + \frac{Q^{2}/2M_{N}\tilde{\sigma}}{\delta m_{q}} G_{V}(Q^{2}) + \frac{Q^{2}/2M_{N}\tilde{\sigma}}{\delta m_{q}} G_{V}(Q^{2})$ 

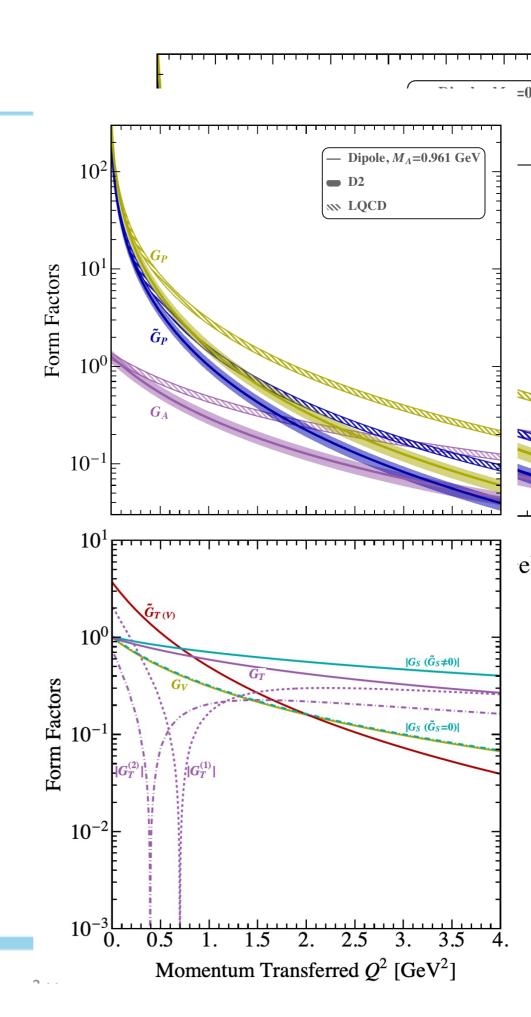
Partially-conserved axial current (PCAC)

$$G_P(Q^2 \ G_P(Q^2) = \frac{M_N}{m_q} G_A(Q^2) + \frac{Q^2/2M_N}{2m_q} \tilde{G}_P(Q^2)$$

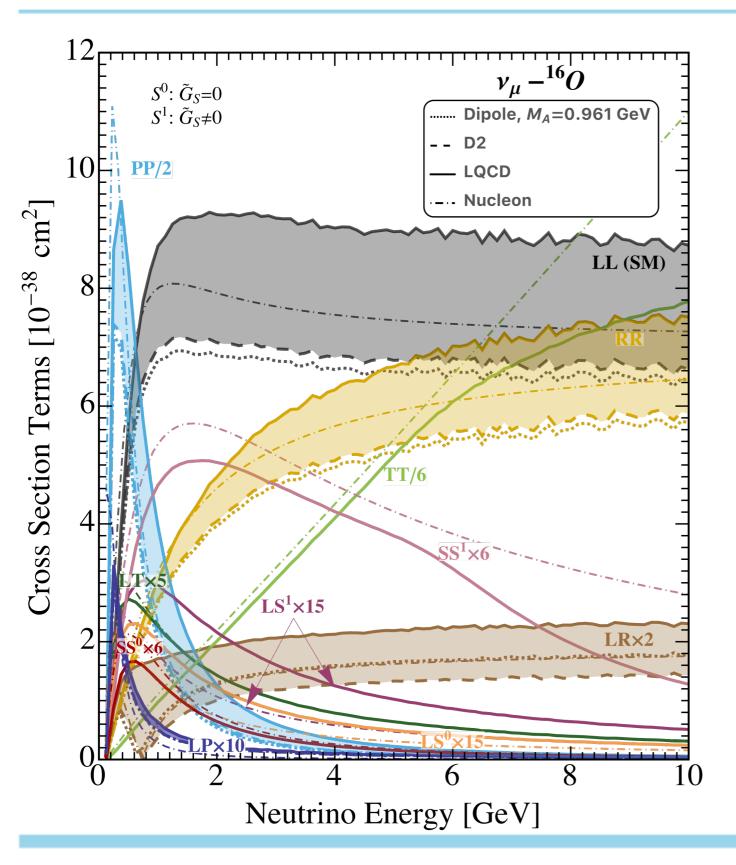
Tensor: I and theoretical considerations

We can not neglect  $\tilde{G}_{S}(Q^{2})$  anymore.

We analyze for the first time how the **axial form factor uncertainty affects the study of new interactions** beyond the SM and we find a sizable effect



# Interplay with BSM scenarios



 Specific Lorentz structures, especially pseudoscalar and tensor interactions, exhibit cross sections notably enhanced compared to those of the Standard Model. {there is a considerable margin of uncertainty}

 The axial form factor introduces significant systematic uncertainties, true for both SM and BSM interactions

 Nuclear effects are crucial even at multi-GeV energies, this is particularly apparent for tensor interactions at energies ≥ 6GeV



#### J. Sobczyk, NR, A. Lovato, arxiv:2406.06292

The inclusive electron-nucleus cross section can be written in terms of the longitudinal and transverse response function

$$\left(\frac{d^2\sigma}{dE'd\Omega'}\right)_e = \left(\frac{d\sigma}{d\Omega'}\right)_{\rm M} \left[\frac{q^4}{\mathbf{q}^4}R_L(\mathbf{q},\omega) + \left(\tan^2\frac{\theta}{2} - \frac{1}{2}\frac{q^2}{\mathbf{q}^2}\right)R_T(\mathbf{q},\omega)\right]$$

Traditionally, the **Rosenbluth separation** is adopted to obtain  $R_L(\mathbf{q}, \omega)$  and  $R_T(\mathbf{q}, \omega)$ 

$$\Sigma(\mathbf{q},\omega,\epsilon) = \epsilon \frac{\mathbf{q}^4}{Q^4} \left(\frac{d^2\sigma}{dE'd\Omega'}\right)_e \left/ \left(\frac{d\sigma}{d\Omega'}\right)_{\mathrm{M}} = \epsilon R_L(\mathbf{q},\omega) + \frac{1}{2} \frac{\mathbf{q}^2}{Q^2} R_T(\mathbf{q},\omega) \right|_{\mathrm{M}}$$
Photon polarization

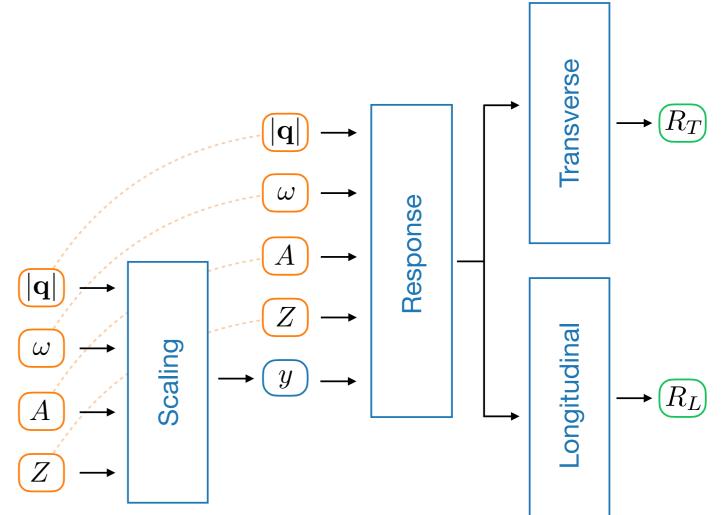
As  $\theta$  ranges between 180 to 0 degrees,  $\epsilon$  varies between 0 and 1. Within this approach,  $R_L$  is the **slope** while  $(\mathbf{q}^2/2Q^2)R_T$  is the **intercept** of the linear fit to data

This definition can only be applied if the Born approximation is valid and if the data have already been corrected to account for <u>Coulomb distortions of the electron wave function</u>.



We used ANN architecture to obtain the longitudinal and transverse responses

- We preprocess the input through a 'scaling' net whose output is  $y(\mathbf{q}, \omega, A, Z)$ .
- We concatenate *y* with the other inputs to compute the 'Response' net which gives a 32-dim output.
- This input is used to built two completely independent nets; each provides a single output corresponding to the longitudinal and transverse responses, respectively.



We train our ANN using the quasielastic electron nucleus scattering archive of <u>arXiv:nucl-ex/0603032</u> considering five different light and medium-mass nuclei, symmetric: <sup>4</sup>He, <sup>6</sup>Li, <sup>12</sup>C, <sup>16</sup>O and <sup>40</sup>Ca.



We used **Bayesian statistics** to quantify the uncertainty of the ANN. We treat the weights  $\mathcal{W}$  as a probability distribution.

The posterior probability of the parameters  ${\mathscr W}$  given the measured cross sections Y can be written as

$$P(\mathcal{W} \mid Y) = \frac{P(Y \mid \mathcal{W})P(\mathcal{W})}{P(Y)}$$

We assign a normal Gaussian prior for each neural network parameter and assume a **Gaussian distribution** for the likelihood based on a loss function obtained from a least-squares fit to the empirical data

$$P(Y|\mathcal{W}) = \exp\left(-\frac{\chi^2}{2}\right) \qquad \qquad \chi^2 = \sum_{i=1}^N \frac{\left[y_i - \hat{y}_i(\mathcal{W})\right]^2}{\sigma_i^2}$$

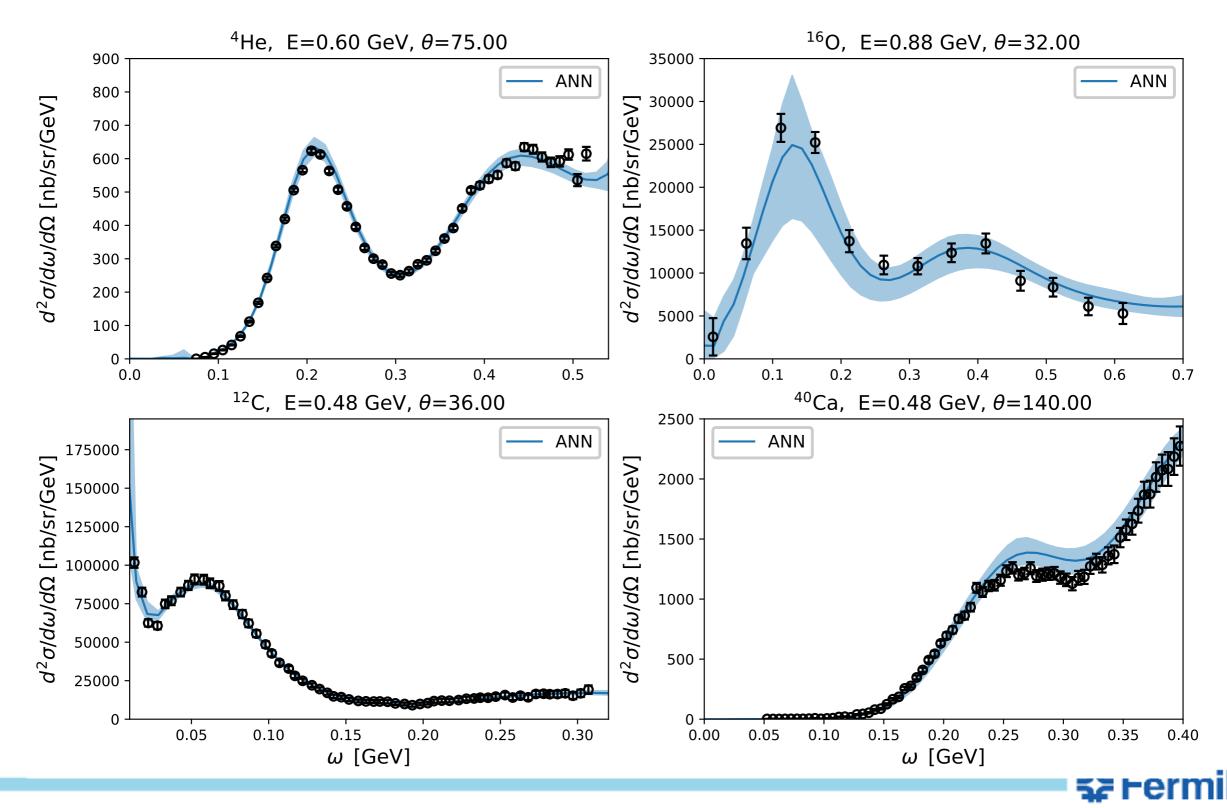
We increase the experimental errors  $\sigma_i$  listed in <u>arXiv:nucl-ex/0603032</u> including an additional term proportional to the experimental cross section value:  $\sigma_i \rightarrow \sigma_i + 0.05y_i$ .

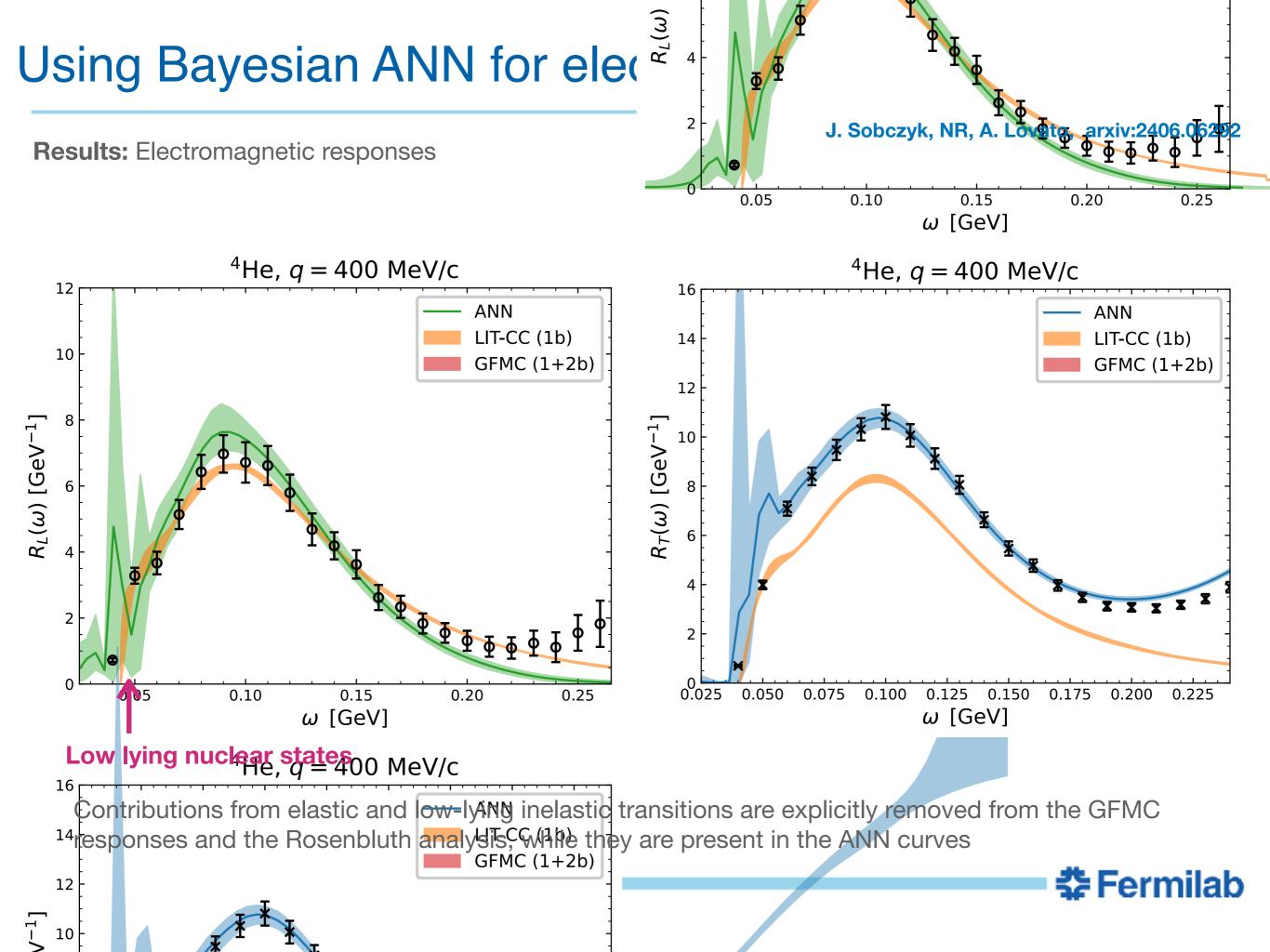
The posterior distribution is sampled using the **NumPyro No-U-Turn Sampler** extension of HMC. We also implemented the standard HMC algorithm and validated results.

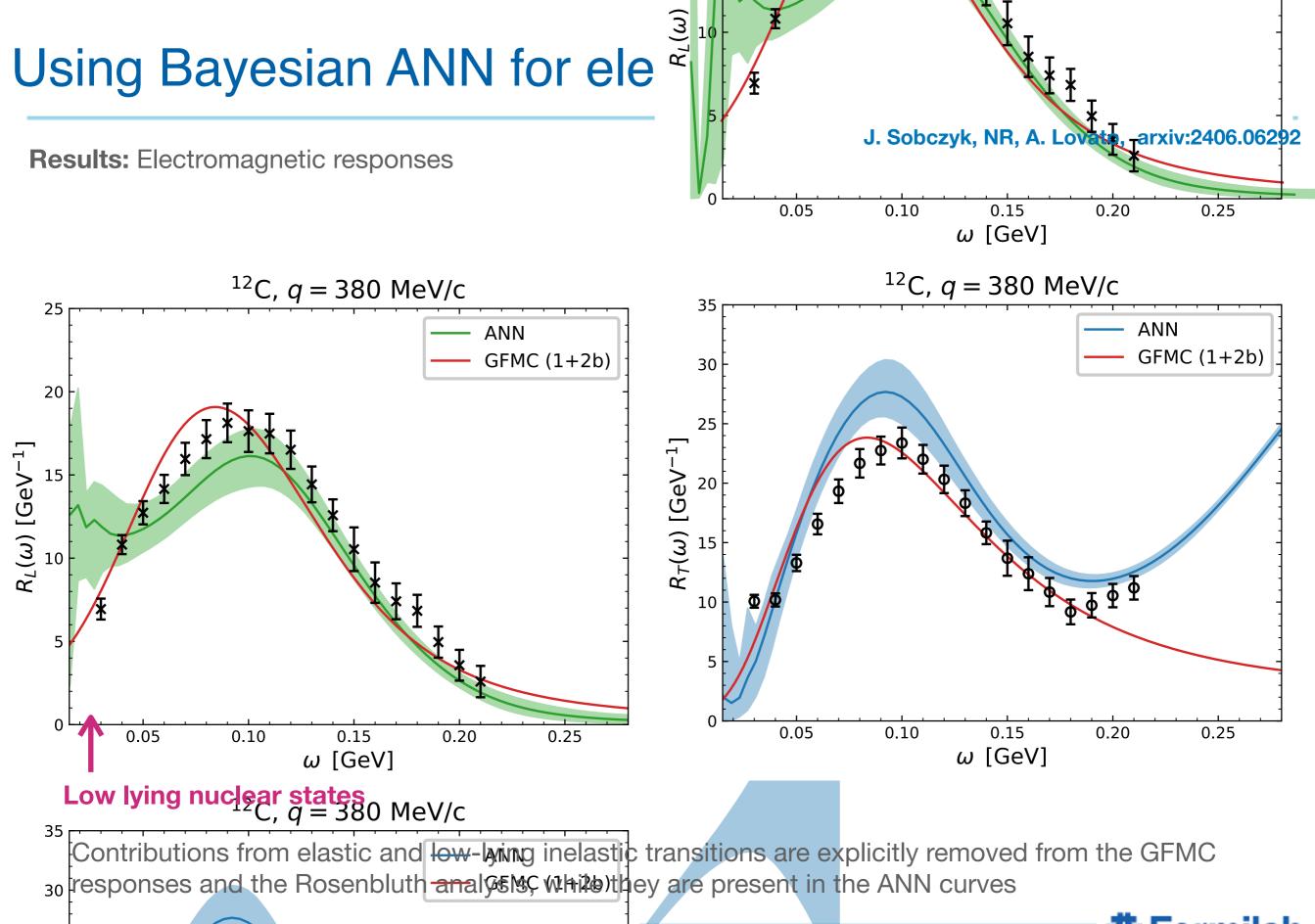


#### **Results:** Cross sections for different nuclei

J. Sobczyk, NR, A. Lovato, arxiv:2406.06292





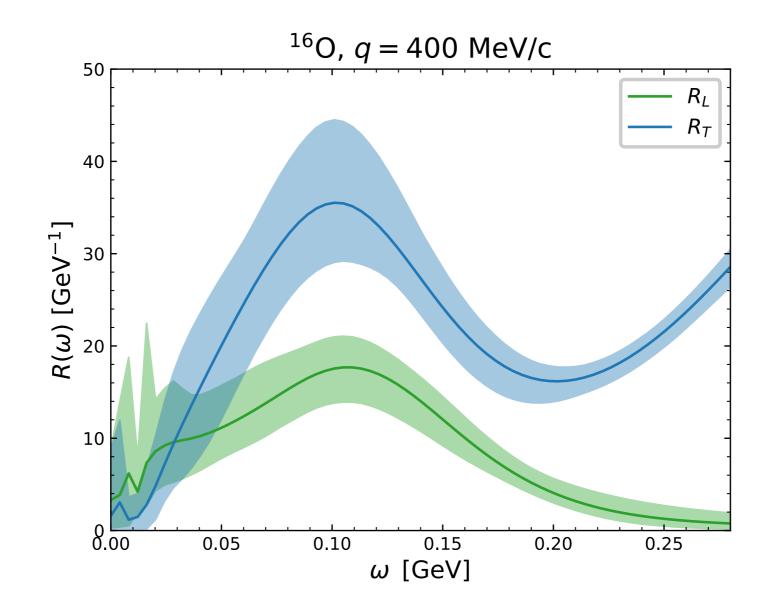


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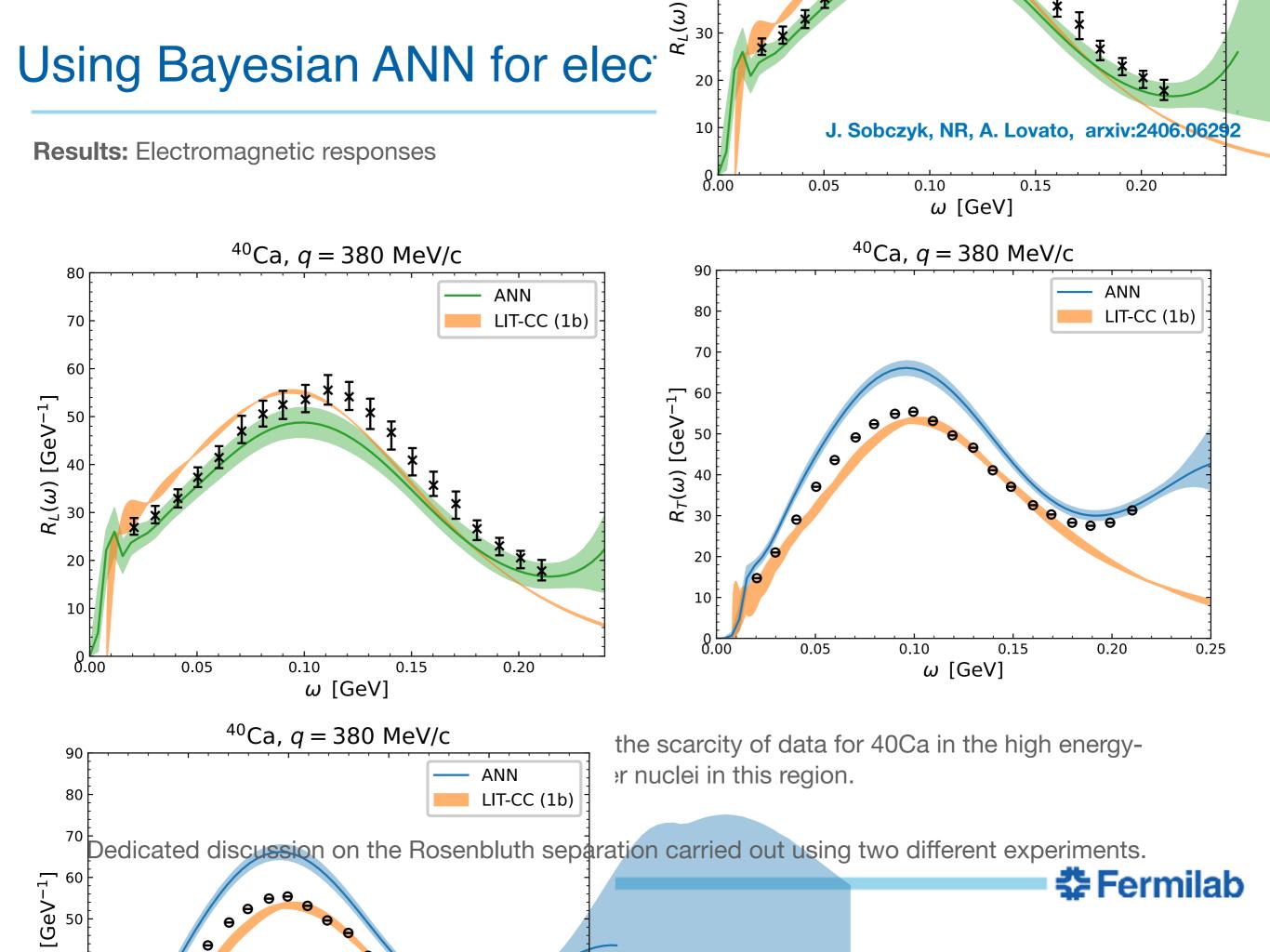
J. Sobczyk, NR, A. Lovato, arxiv:2406.06292

**Results:** Electromagnetic responses



First separation of the longitudinal and transverse responses of <sup>16</sup>O. Large uncertainty bands reflect the **scarcity of inclusive cross section data**.





#### Conclusions

\* Neutrino oscillation experiments are entering a new precision era

\* To match these precision goals accurate predictions of neutrino cross sections are crucial

Ab initio methods: almost exact results but limited in energy, fully inclusive

Approaches based on factorization schemes are being further developed

\* Uncertainty associated with the theory prediction of the hard interaction vertex needs to be assessed. Initial work has been carried out in this direction studying the dependence on:

Form factors: one- and two-body currents, resonance/ $\pi$  production

Error of factorizing the hard interaction vertex / using a non relativistic approach

\* Combine state-of-the art neutrino-nucleus calculations with BSM theories is gaining momentum; UQ is very interesting (and challenging) in this case as well



# Thank you for your attention!