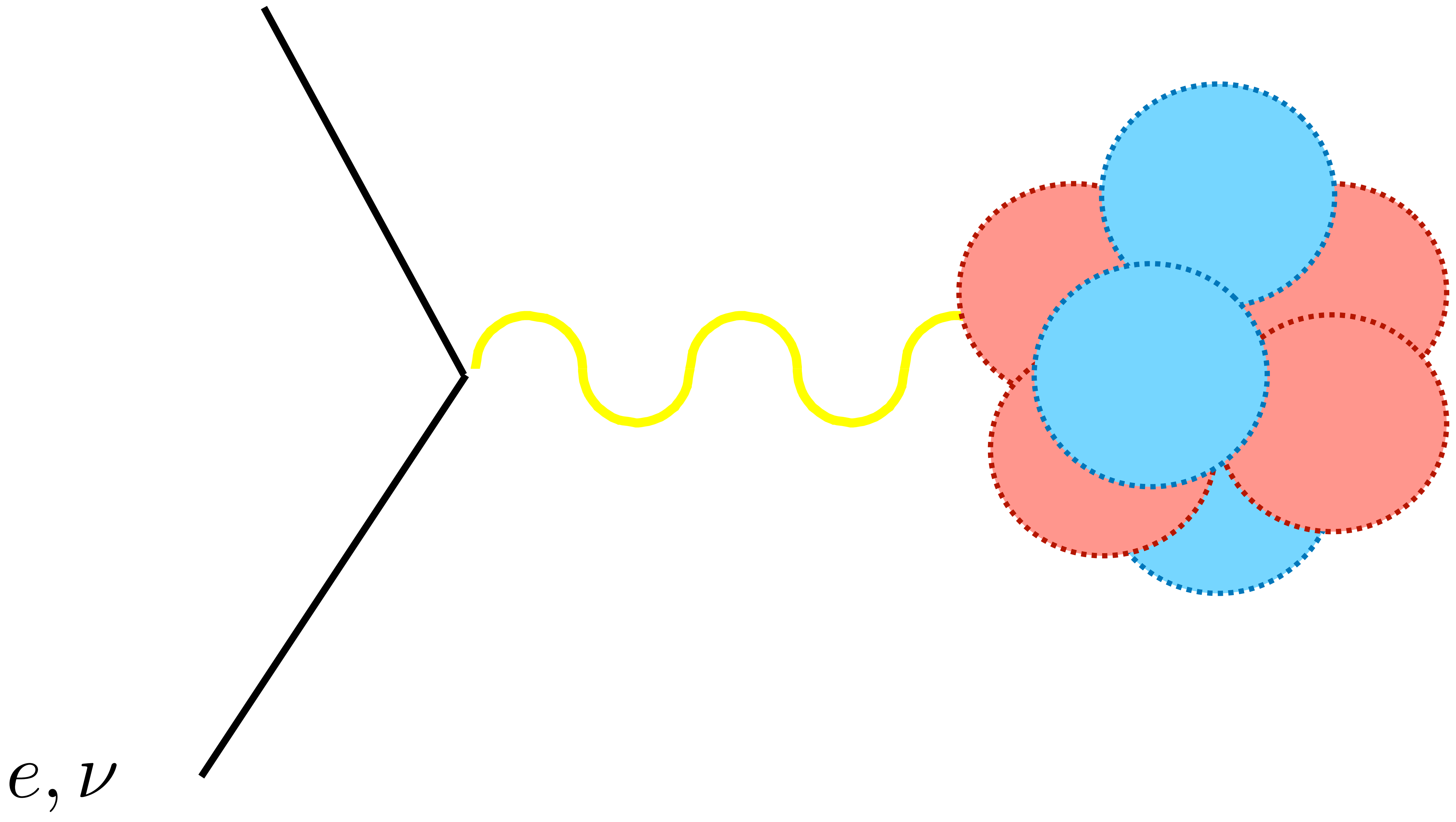
Uncertainty quantification in electroweak reactions

Sonia Bacca

Electroweak Reactions



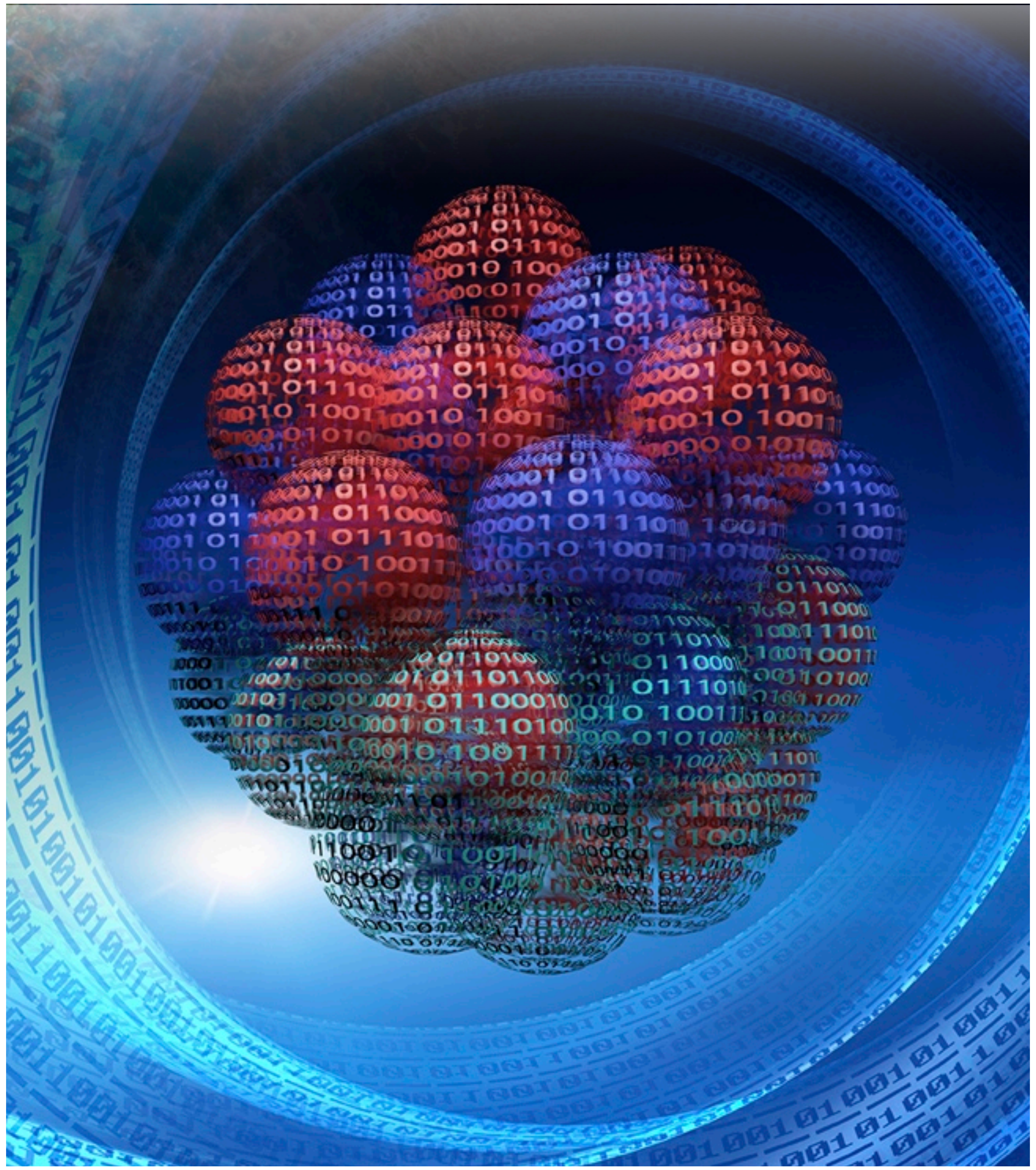
The ab-initio approach

- Start from protons and neutrons
- Solve the quantum mechanics of A=Z+N interacting nucleons

$$H|\Psi\rangle = E|\Psi\rangle$$

$$H = T + V$$

- Find numerically exact solutions or controlled approximations



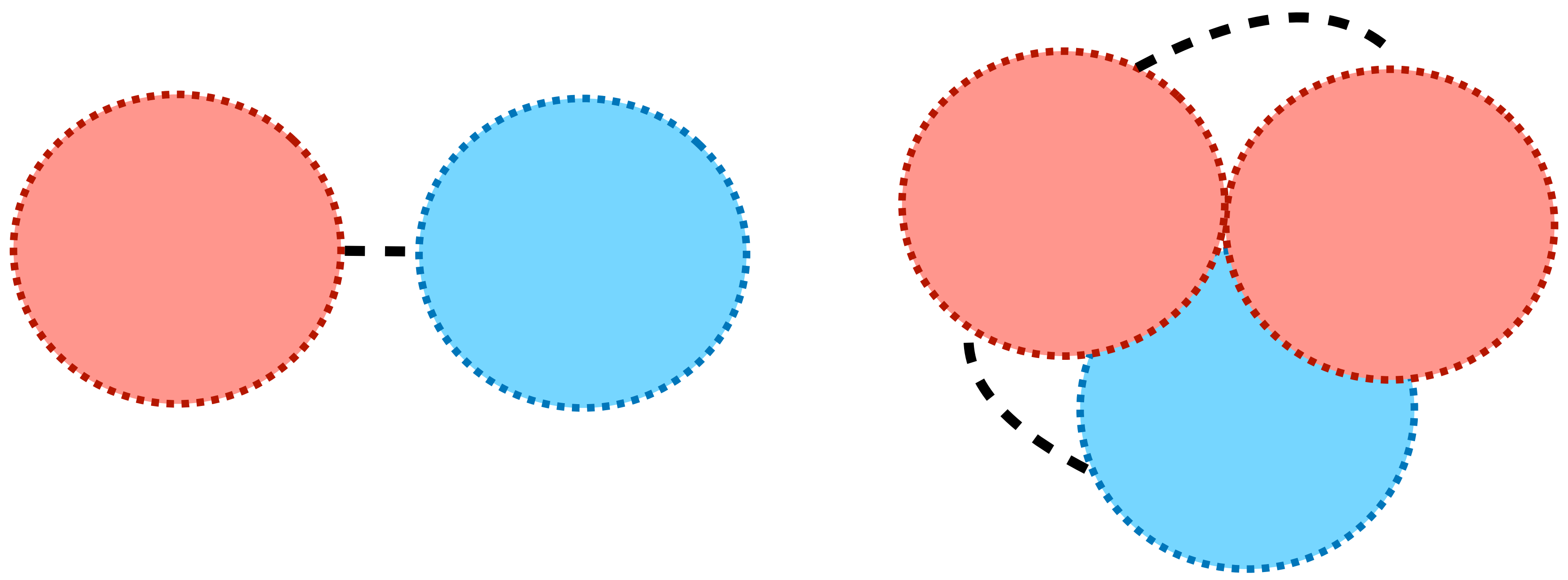
Credits: ORNL, LeJean Hardin and Andy Sproles

Chiral effective field theory

The Hamiltonian

Three-nucleon forces appear

$$V = V_{NN} + V_{3N} + \ldots$$

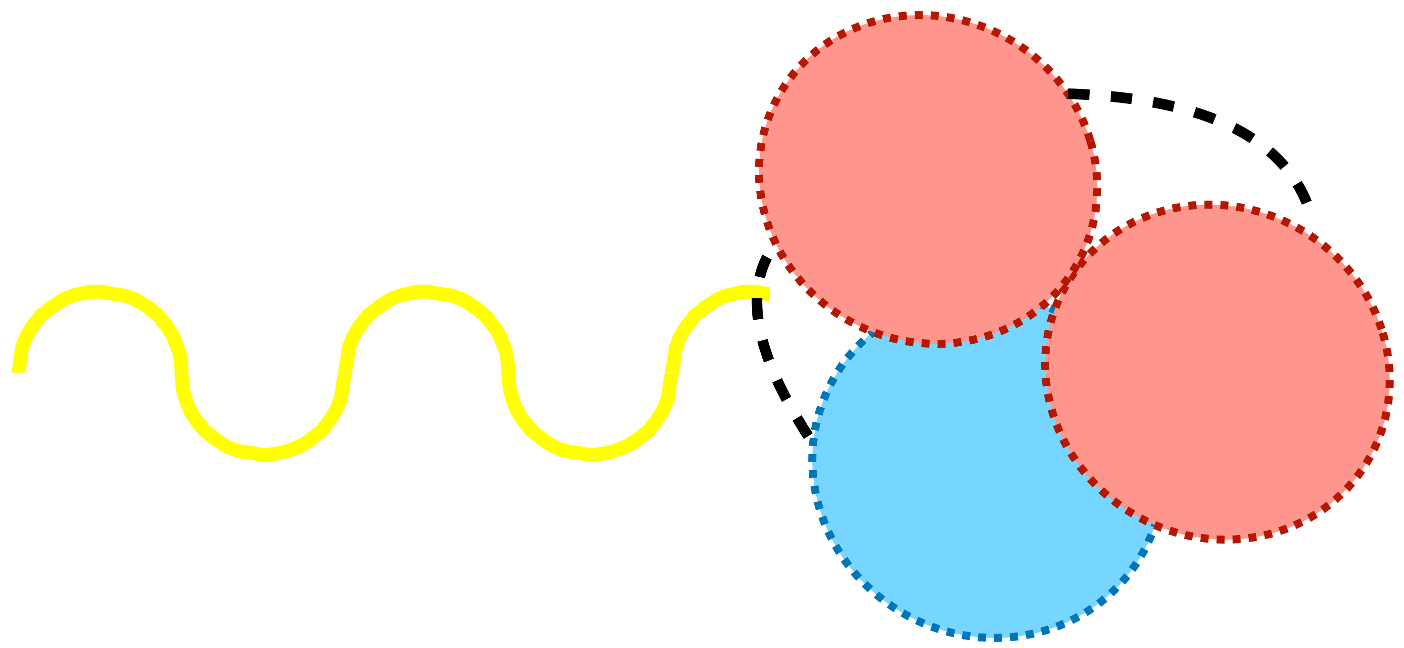


Systematic expansion

$$V = V_{\mathrm{LO}} + V_{\mathrm{NLO}} + V_{\mathrm{NNLO}} \ldots$$

Chiral effective field theory

Electroweak currents



Two-body currents appear in a systematic expansion

$$J^{\mu} = J^{\mu}_{\mathrm{1BC}} + J^{\mu}_{\mathrm{2BC}} + \dots$$

Sources of uncertainty

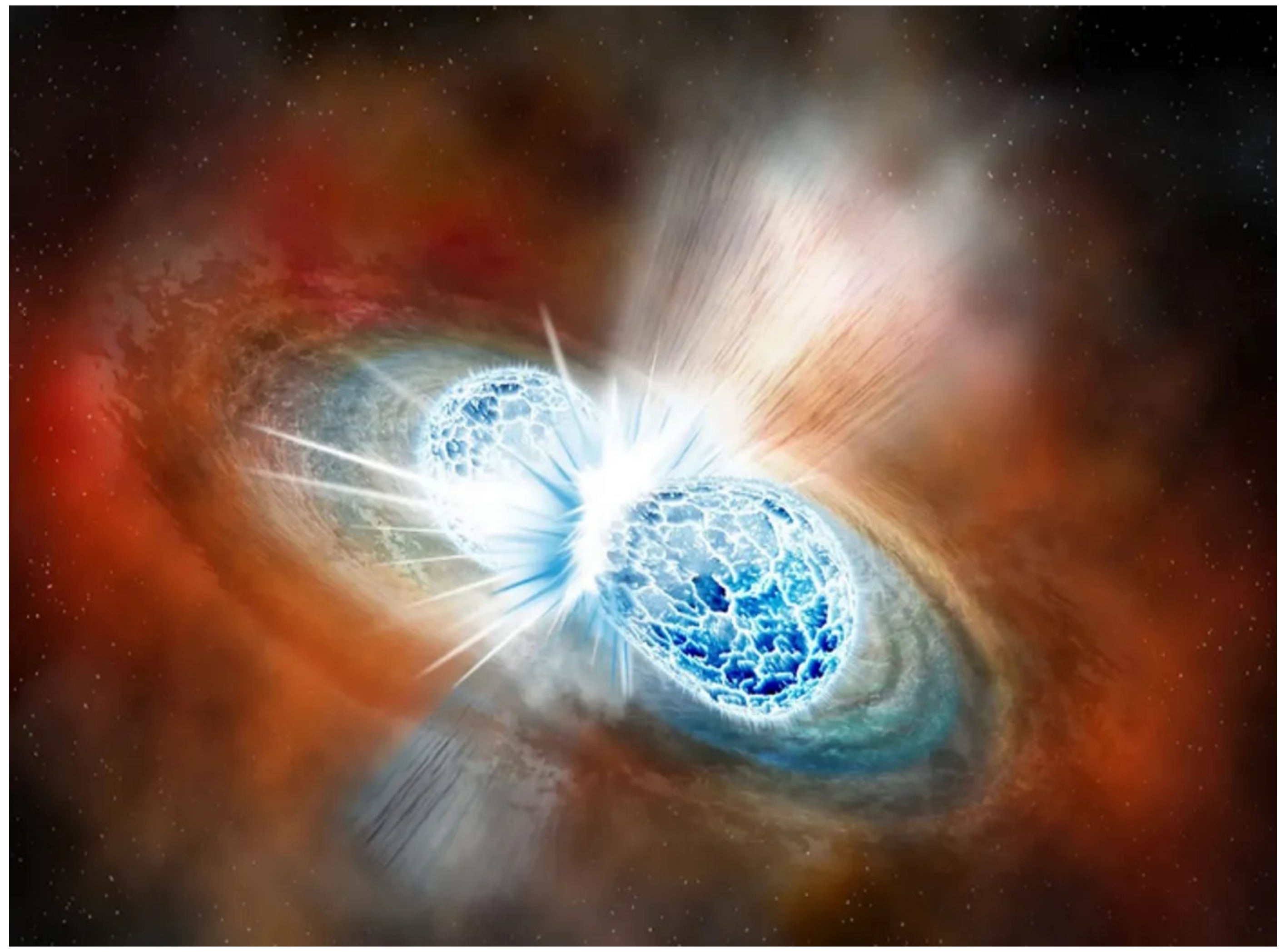
- Numerical errors
- Many-body truncations
- Model

We estimate these uncertainties with different levels of sophistications depending on the problem we are dealing with. More difficult with increasing A

GoComics.com

Relevance of EW reactions

nuclear astrophysics

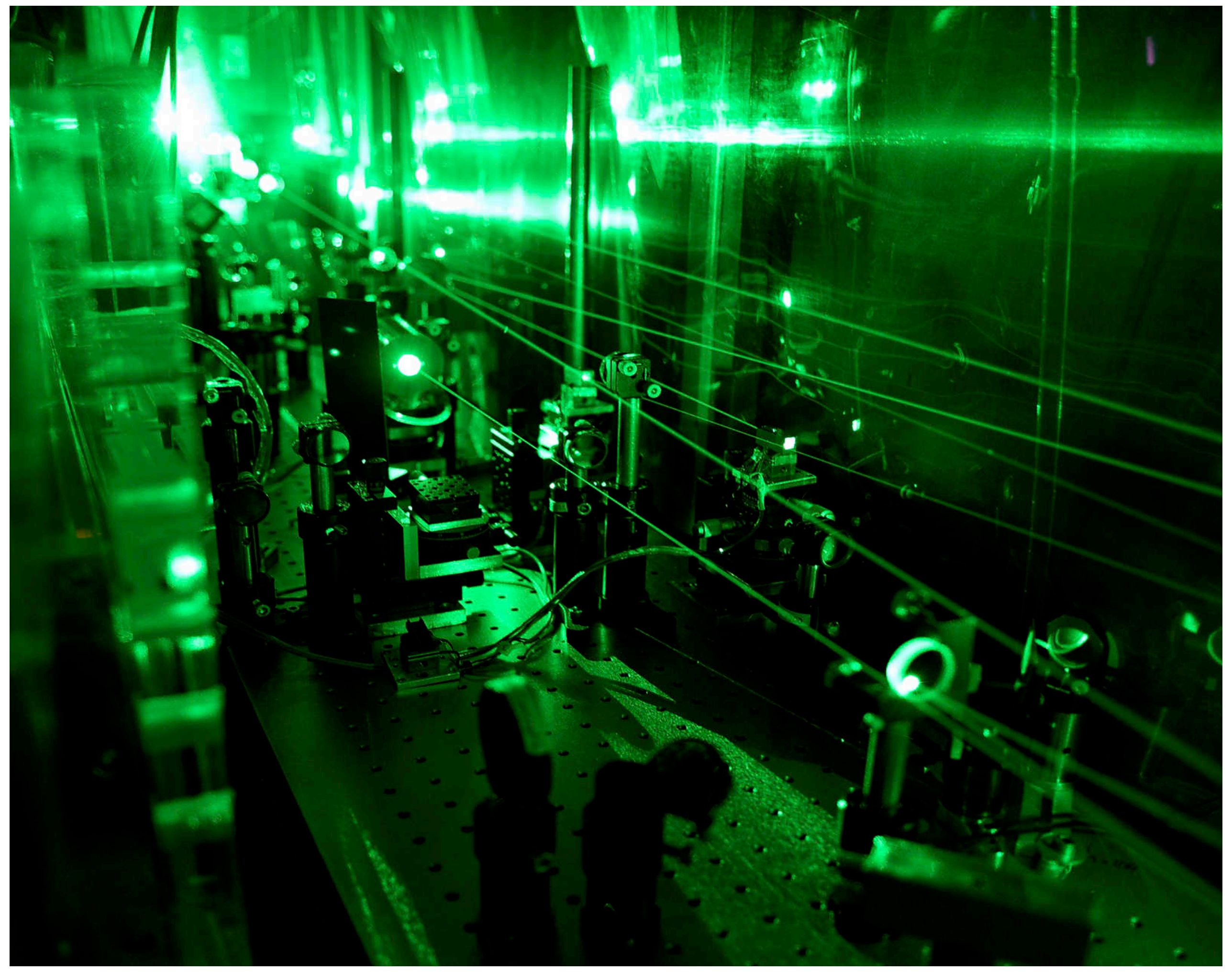


Credit: Robin Dienel/Carnegie Institution for Science

particle physics

Credit:nures.uta.edu

atomic physics



Credit: R.Pohl

One first example

Application to nuclear astrophysics

Big bang nucleosynthesis

Bayesian analysis for uncertainty quantification in $n\,p \to D\,\gamma$

Uncertainty quantifications with tools developed by the BUQEYE collaboration

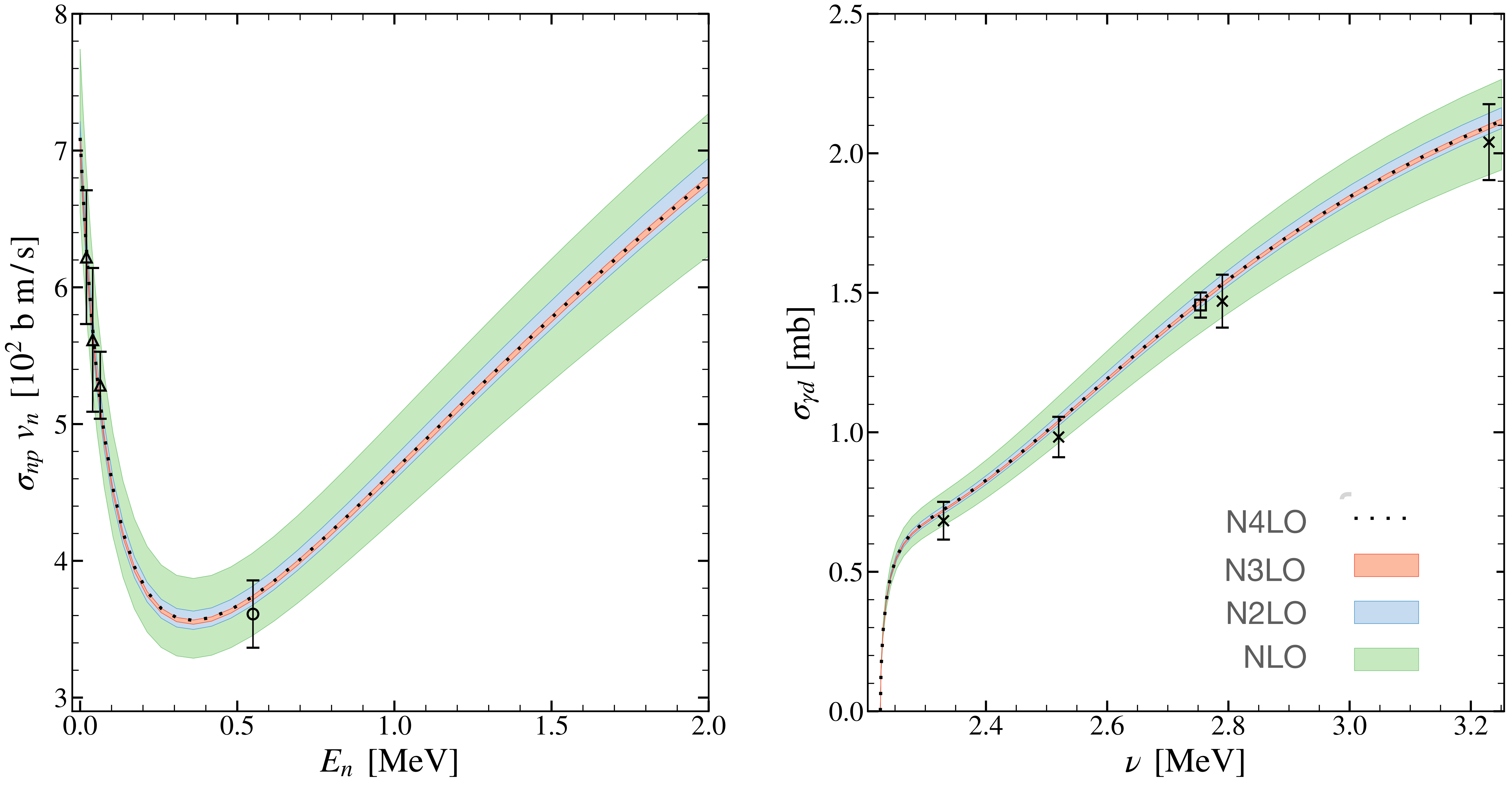
- Express observable as

$$y(\nu) = y_{ref}(\nu) \sum_{n=0}^{\infty} c_n(\nu)(Q/\Lambda)^n$$

$$\delta y_k(\nu) = y_{\rm ref}(\nu) \sum_{n=k+1}^{\infty} c_n(\nu)\,(Q/\Lambda)^n$$

- Calibrate a Gaussian process emulator using physics-based info on $c_n(\nu)$ as "prior"
- Calculate "Bayesian posterior" for $c_{n>k}(\nu)$ obtaining statistically interpretable truncation error, amounting to 0.2% at the highest order.
- Fix current at NLO, vary potential

B. Acharya and SB, Phys. Lett. B 827, 137011 (2022)



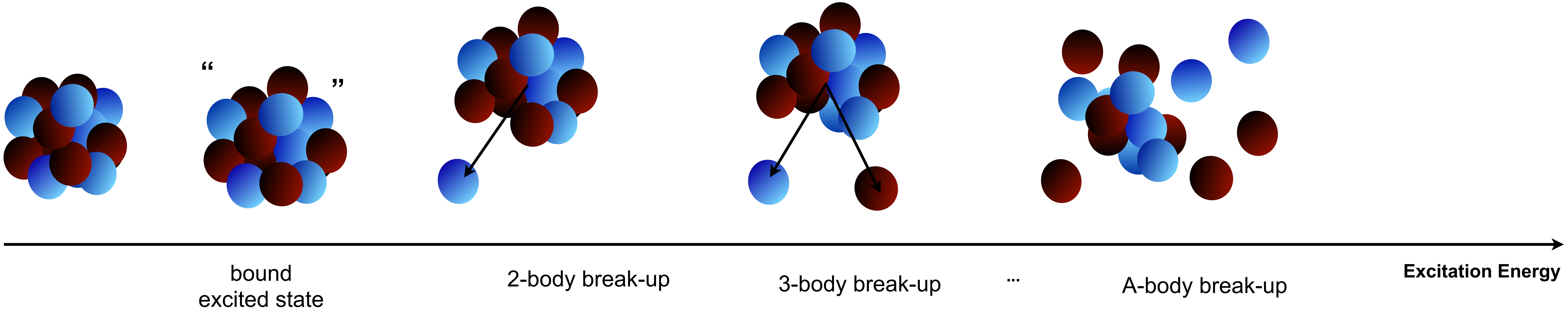
At zero energy N4LO gives 7.0932 and the experiment 7.3128 ⇒ 3% difference due to missing two-body currents

What about reactions with A>2?

The continuum problem

$$R(\omega) = \sum\!\!\!\!\!\!\int_f \left| \left\langle \psi_f \left| J^\mu \right| \psi_0 \right\rangle \right|^2 \delta(E_f - E_0 - \omega)$$

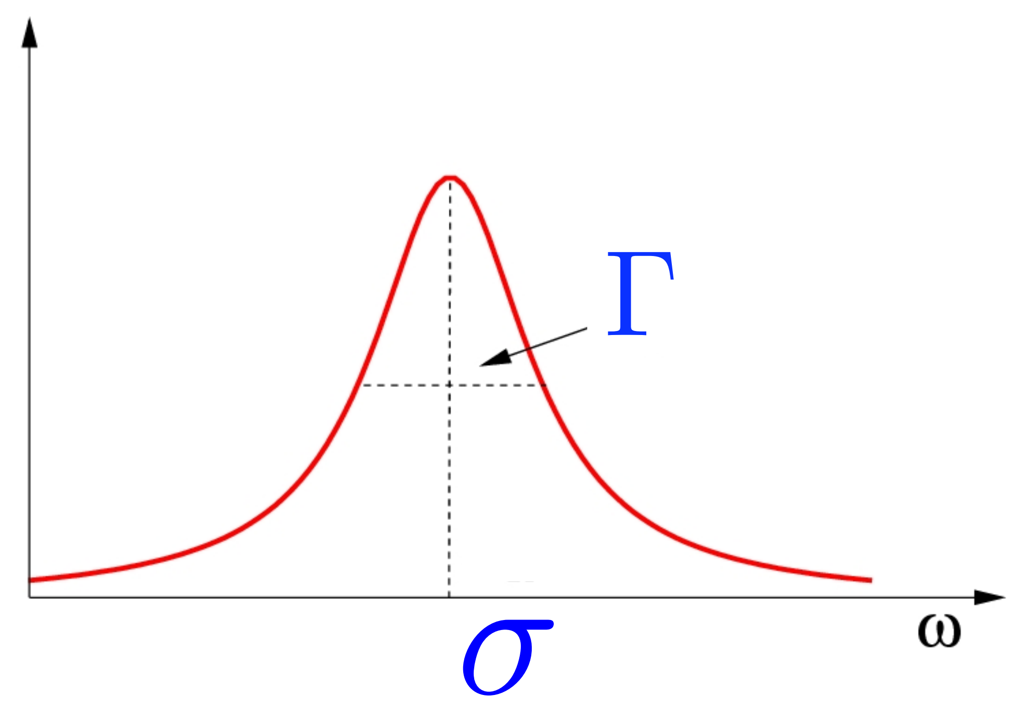
Depending on E_f, many channels may be involved



The Lorentz integral transform (LIT)

inversion

$$L(\sigma,\Gamma) = \frac{\Gamma}{\pi}\int d\omega \frac{R(\omega)}{(\omega-\sigma)^2+\Gamma^2} = \langle\tilde{\psi}|\tilde{\psi}\rangle$$



Efros, *et al.*, JPG.: Nucl.Part.Phys. **34** (2007) R459

$$(H - E_0 - \sigma + i\Gamma)\,|\,\tilde{\psi}\rangle = J^\mu|\,\psi_0\rangle$$

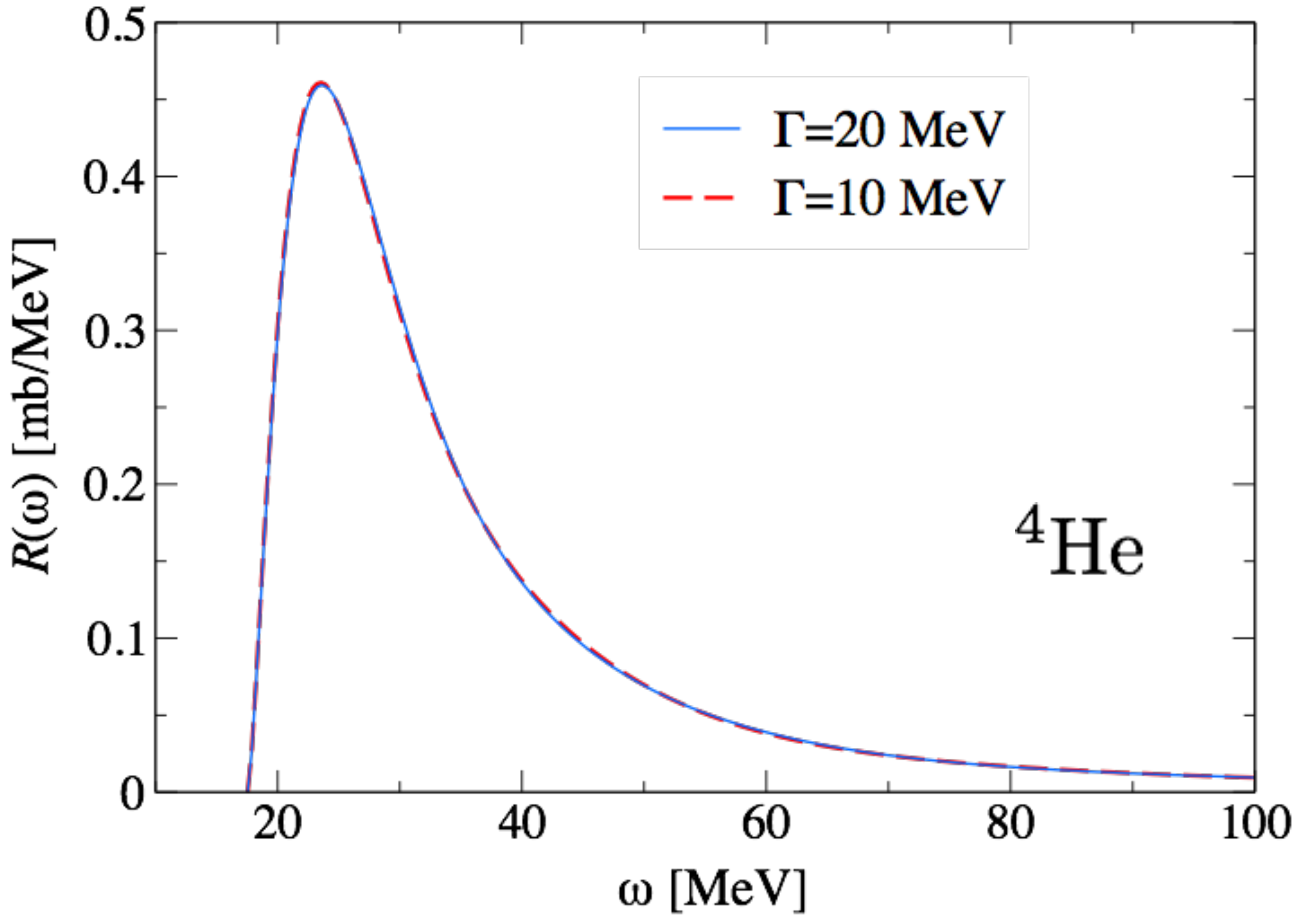
Reduce the continuum problem to a bound-state-like equation

The Lorentz integral transform (LIT)

The inversion is performed numerically with a regularization procedure (ill-posed problem)

Ansatz $$R(\omega) = \sum_i^{I_{\max}} c_i \chi_i(\omega, \alpha) \quad \Longrightarrow \quad L(\sigma, \Gamma) = \sum_i^{I_{\max}} c_i \mathcal{L}[\chi_i(\omega, \alpha)]$$

fit

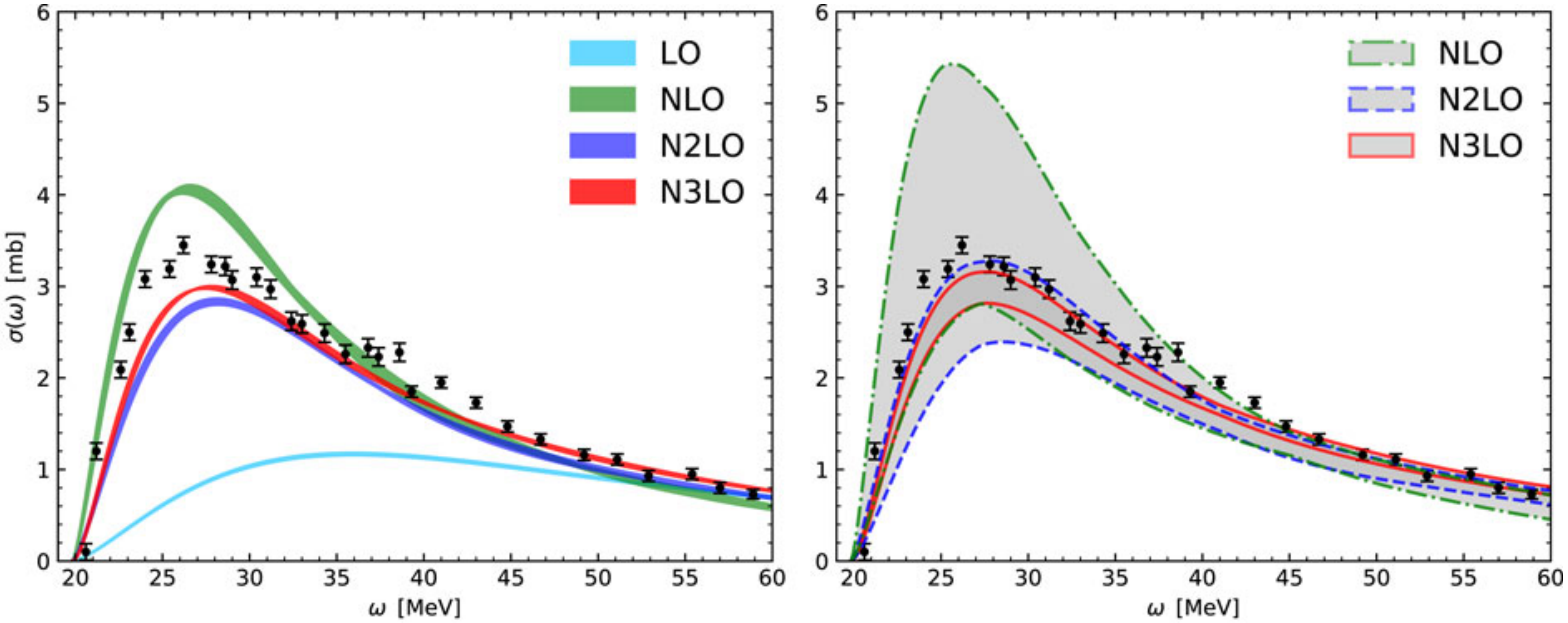


Message: Inversions are stable if the LIT is calculated precisely enough

^{4}He photoabsorption cross-section

Acharya, SB, Bonaiti, Li Muli, Sobczyk, Front. Phys.10:1066035 (2023)

With local chiral potentials from Phys. Rev. C **90**, 054323 up to N2LO, Method: LIT with hyper-spherical harmonics



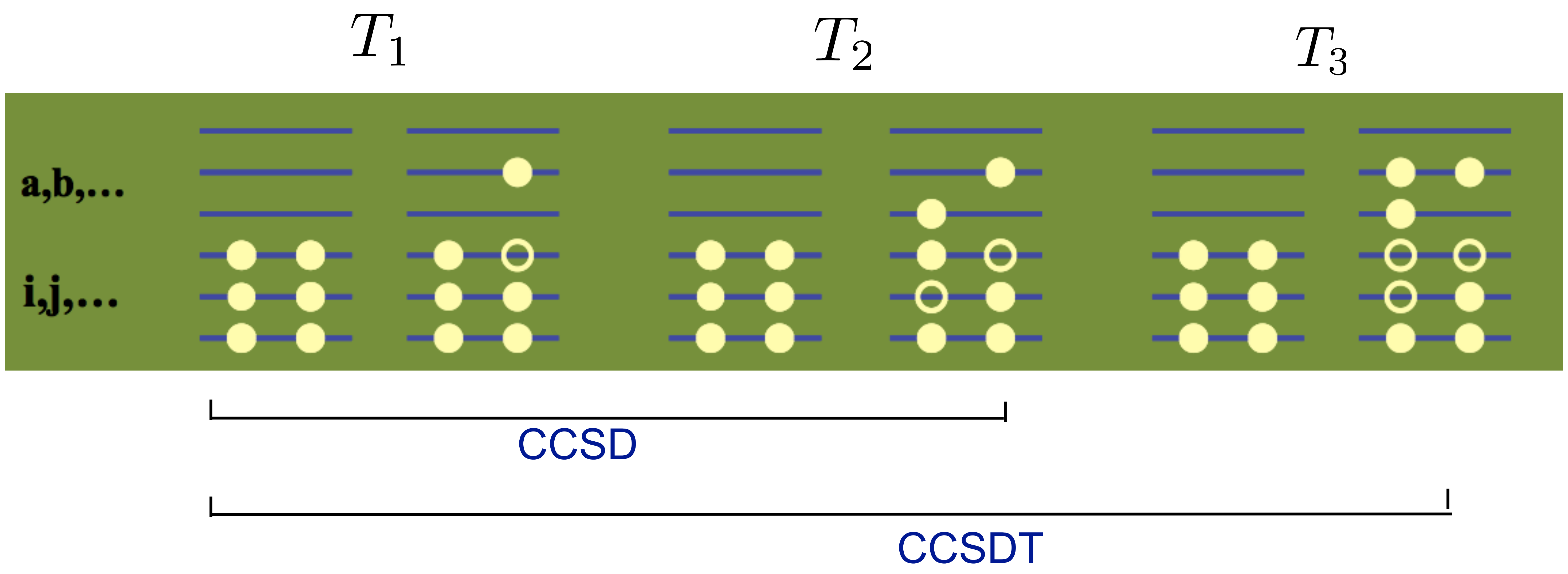
$$\delta_{\mathcal{O}}^{\chi\mathrm{EFT}} = \max\left\{\left(\frac{Q}{\Lambda}\right)^{k+1}|\mathcal{O}_{\nu_0}|, \left(\frac{Q}{\Lambda}\right)^{k}|\mathcal{O}_{\nu_0+1} - \mathcal{O}_{\nu_0}|, \ldots, \left(\frac{Q}{\Lambda}\right)|\mathcal{O}_{\nu_0+k} - \mathcal{O}_{\nu_0+k-1}|\right\}$$

What about heavier nuclei?

Coupled-cluster theory

$$|\psi_0(\vec{r}_1, \vec{r}_2, ..., \vec{r}_A)\rangle = e^T |\phi_0(\vec{r}_1, \vec{r}_2, ..., \vec{r}_A)\rangle \qquad T = \sum T_{(A)}$$

cluster expansion



CCSD algorithm scales as ~A^6

Coupled-cluster formulation of the LIT
LIT-CC

SB et al., Phys. Rev. Lett. 111, 122502 (2013)

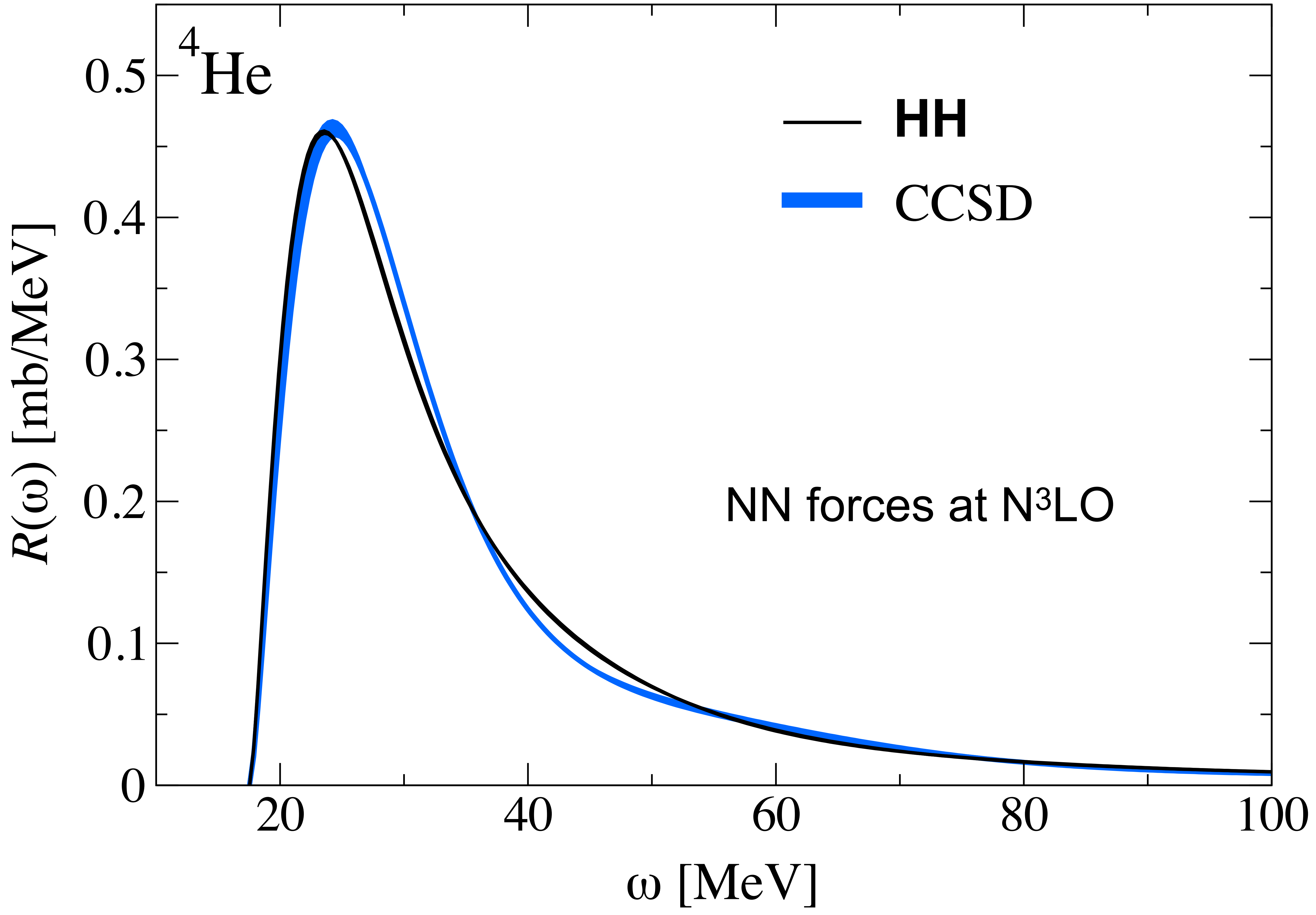
$$(\bar{H} - E_0 - \sigma + i\Gamma)|\tilde{\Psi}_R\rangle = \bar{\Theta}|\Phi_0\rangle$$

$$\begin{cases} \bar{H} = e^{-T} H e^{T} \\ \bar{\Theta} = e^{-T} \Theta e^{T} \\ |\tilde{\Psi}_R\rangle = \hat{R}|\Phi_0\rangle \end{cases}$$

$$\mathcal{R}(z) = r_0(z) + \sum_{ai} r_i^a(z) a_a^\dagger a_i + \frac{1}{4} \sum_{abij} r_{ij}^{ab}(z) a_a^\dagger a_b^\dagger a_j a_i + \ldots$$

Benchmark on ^{4}He

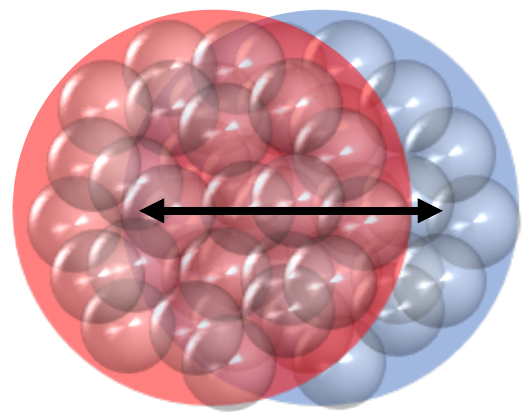
SB et al., Phys. Rev. Lett. 111, 122502 (2013)



Application to Astrophysics

Neutron stars

The nuclear EOS



Constraining the symmetry energy *S(ρ) through properties of finite nuclei*

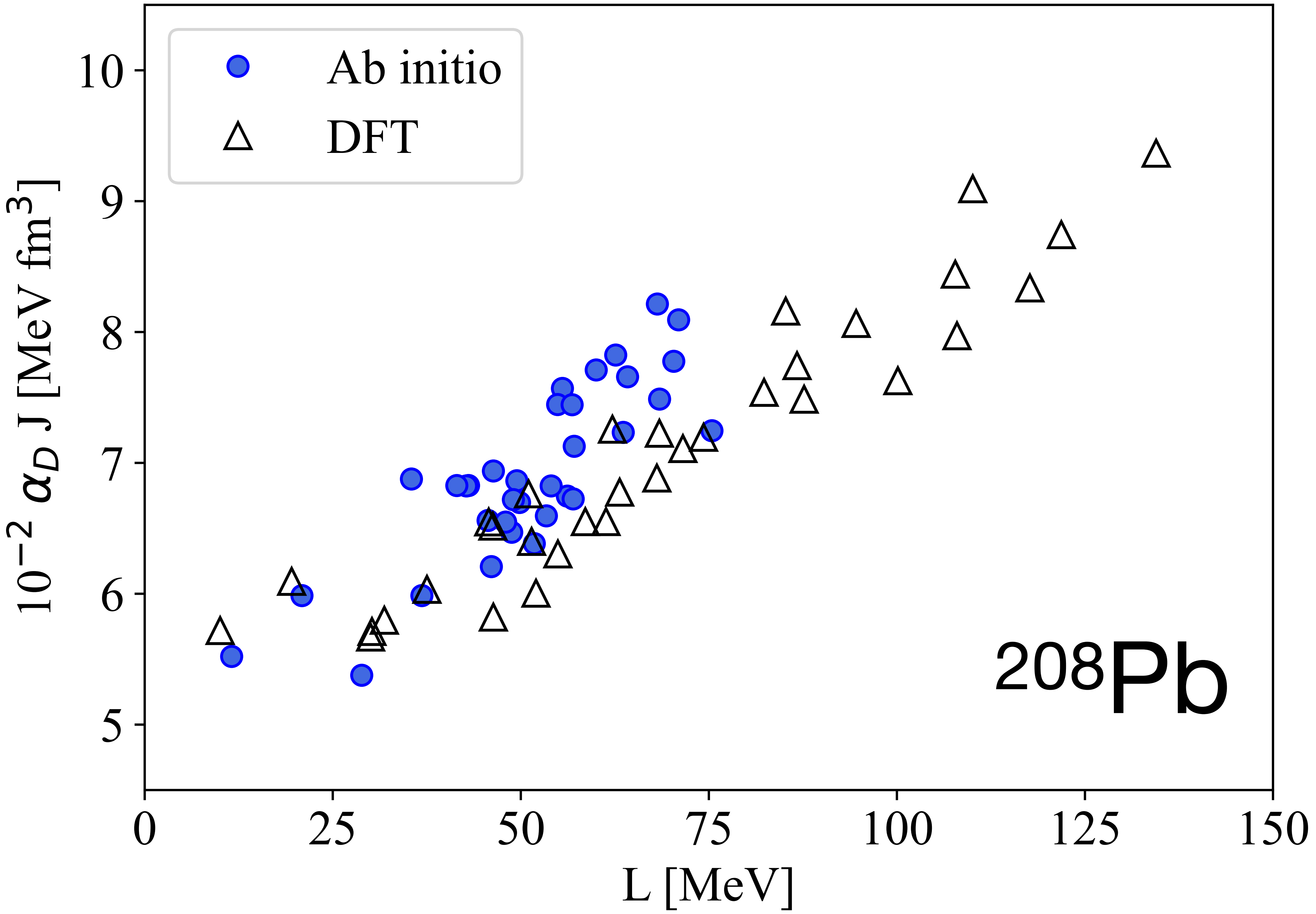
$$\mathcal{E}(\rho,\alpha) = \mathcal{E}_{\mathrm{SNM}}(\rho) + \alpha^2 \mathcal{S}(\rho) + \mathcal{O}(\alpha^4)$$

$$\rho = (\rho_n + \rho_p) \qquad \alpha = (\rho_n - \rho_p)/\rho$$

$$\mathcal{S}(\rho) = J + L\frac{(\rho-\rho_0)}{3\rho_0} + \ldots$$

symmetry energy

slope parameter

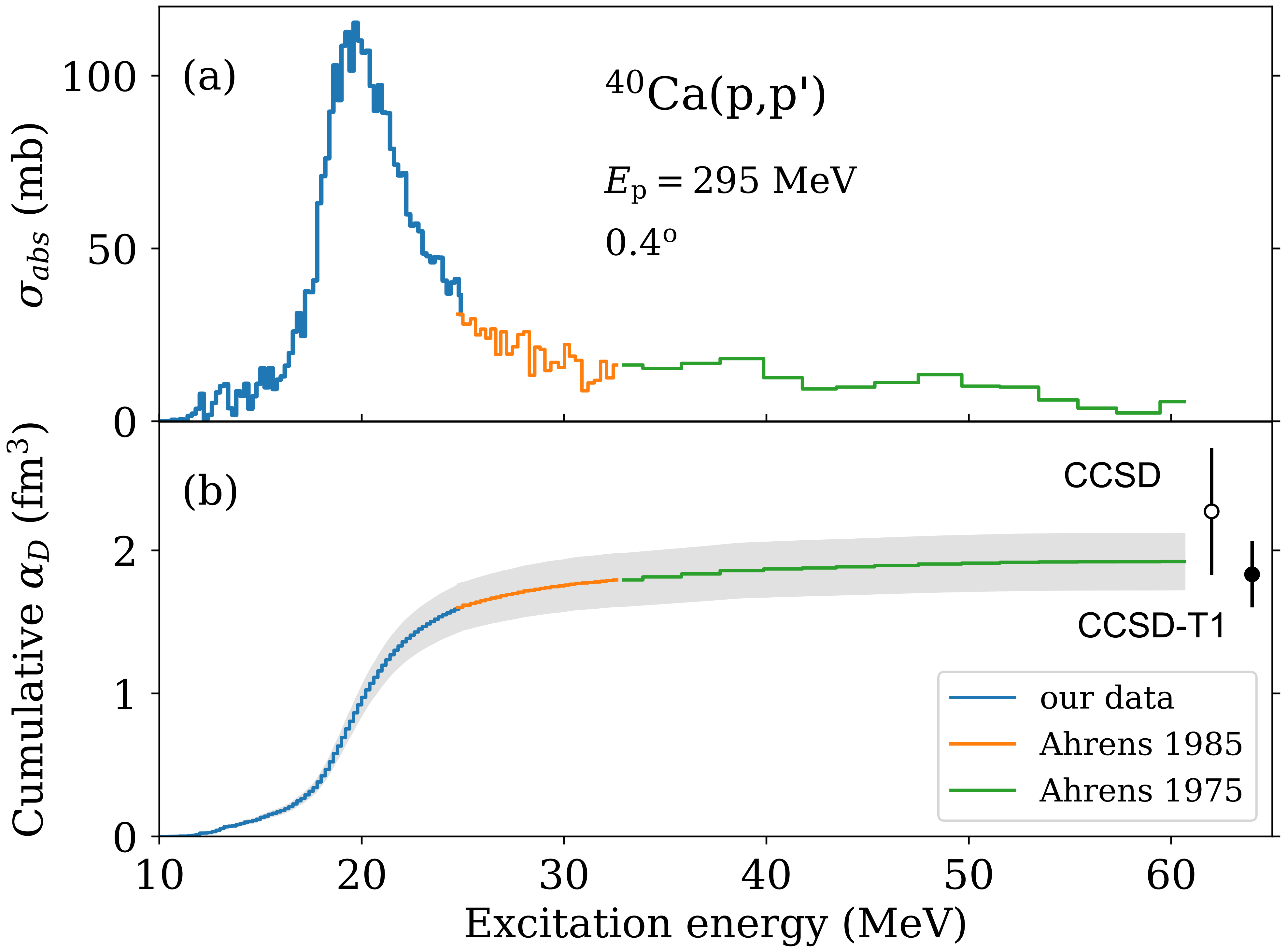


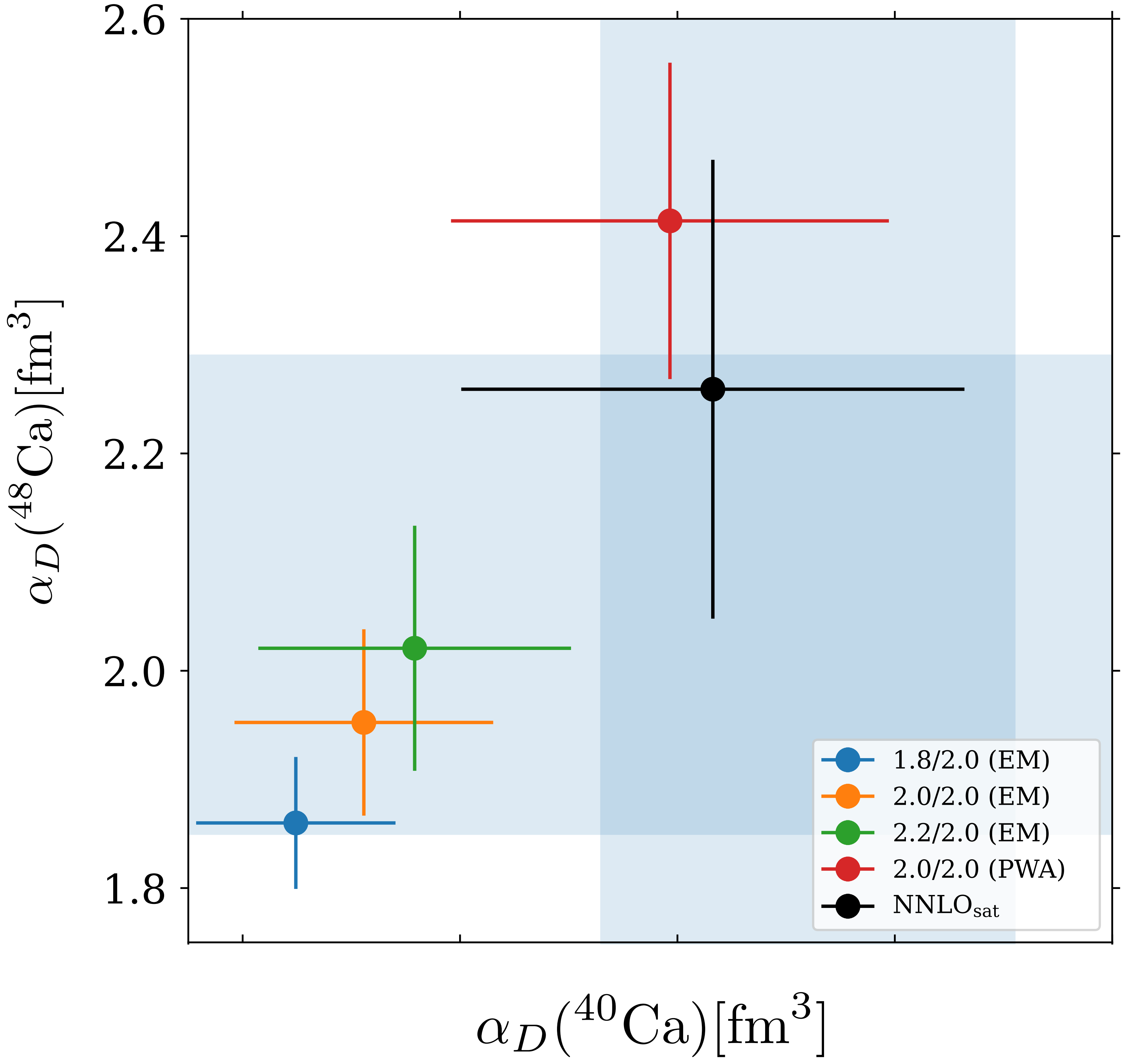
Curtesy from Francesca Bonaiti using data from Hu et al. Nature Phys. 18 (2022)

Electric Dipole Polarizability

Comparison of experiments to ab-initio

Fearick et al, PRRL (2023)

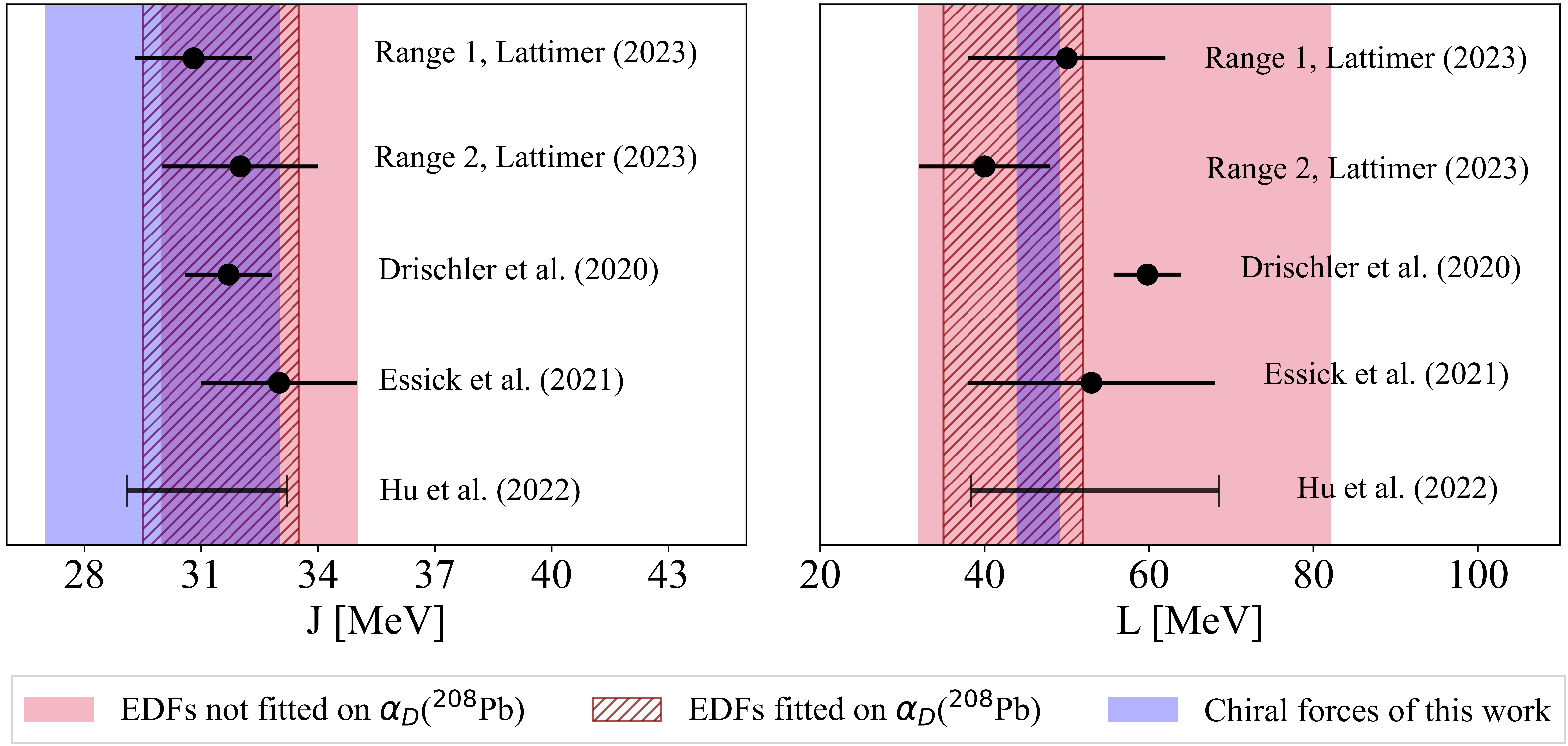




More on Ca-isotopes in F. Bonaiti's talk

Constraints on symmetry energy slope L = 41-49 MeV

Comparison to other analyses



Curtesy of Francesca Bonaiti

Applications to particle physics

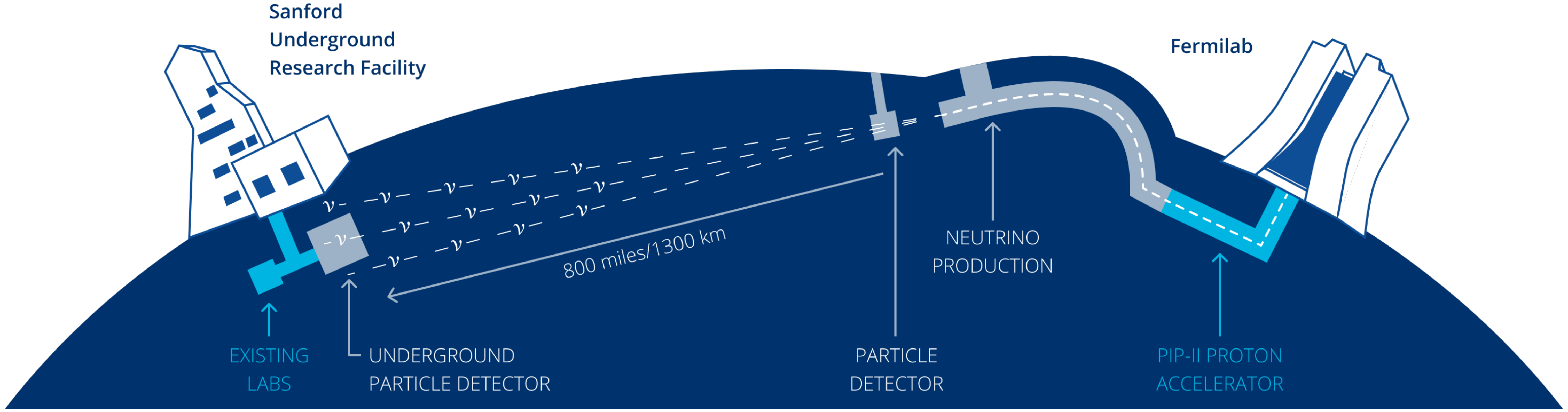
Neutrino oscillations

Next generation experiments

Hyper-K

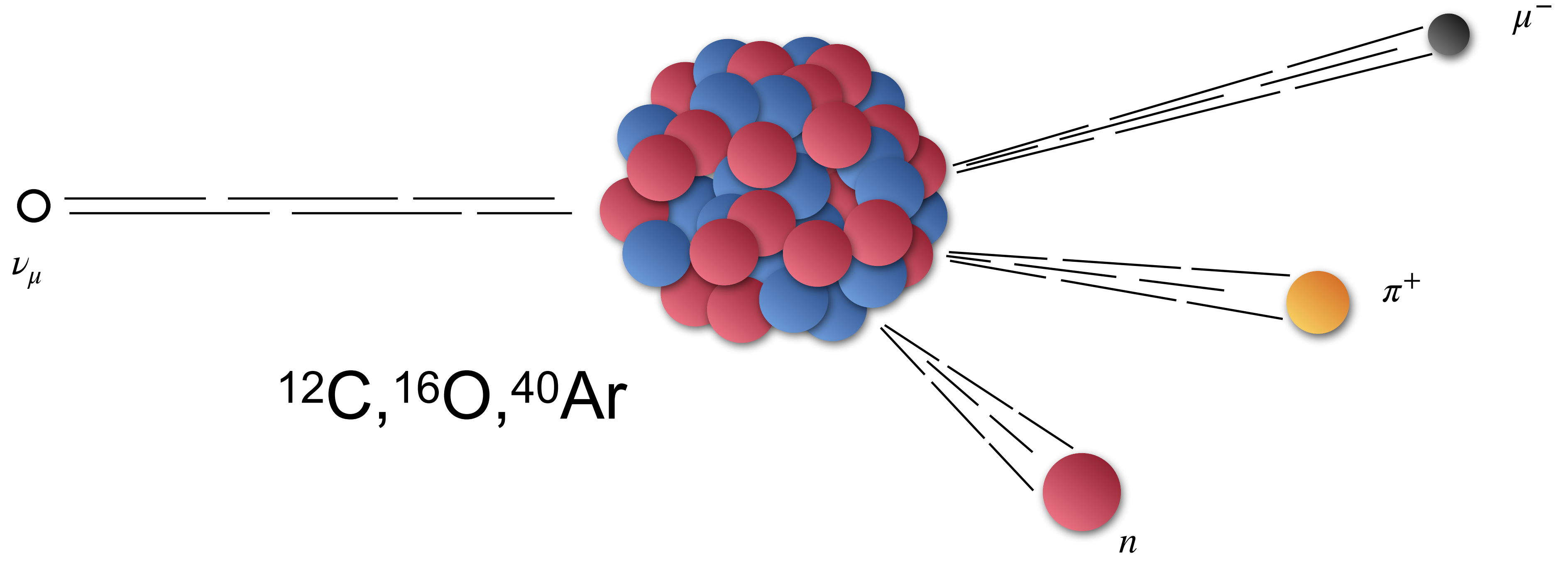
https://cerncourier.com/

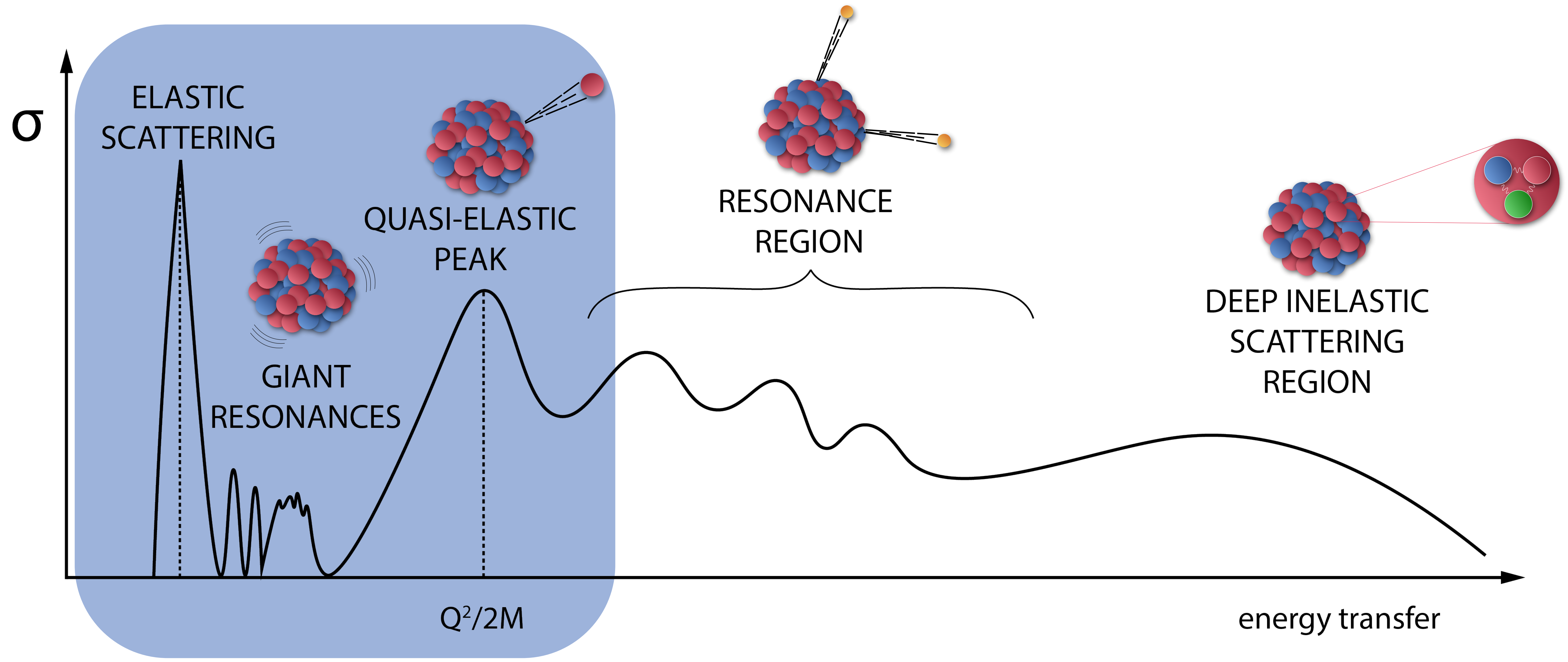
DUNE DEEP UNDERGROUND NEUTRINO EXPERIMENT



https://lbnf-dune.fnal.gov/

Challenges and opportunities





Electrons for neutrinos (e4ν)

ν-A scattering

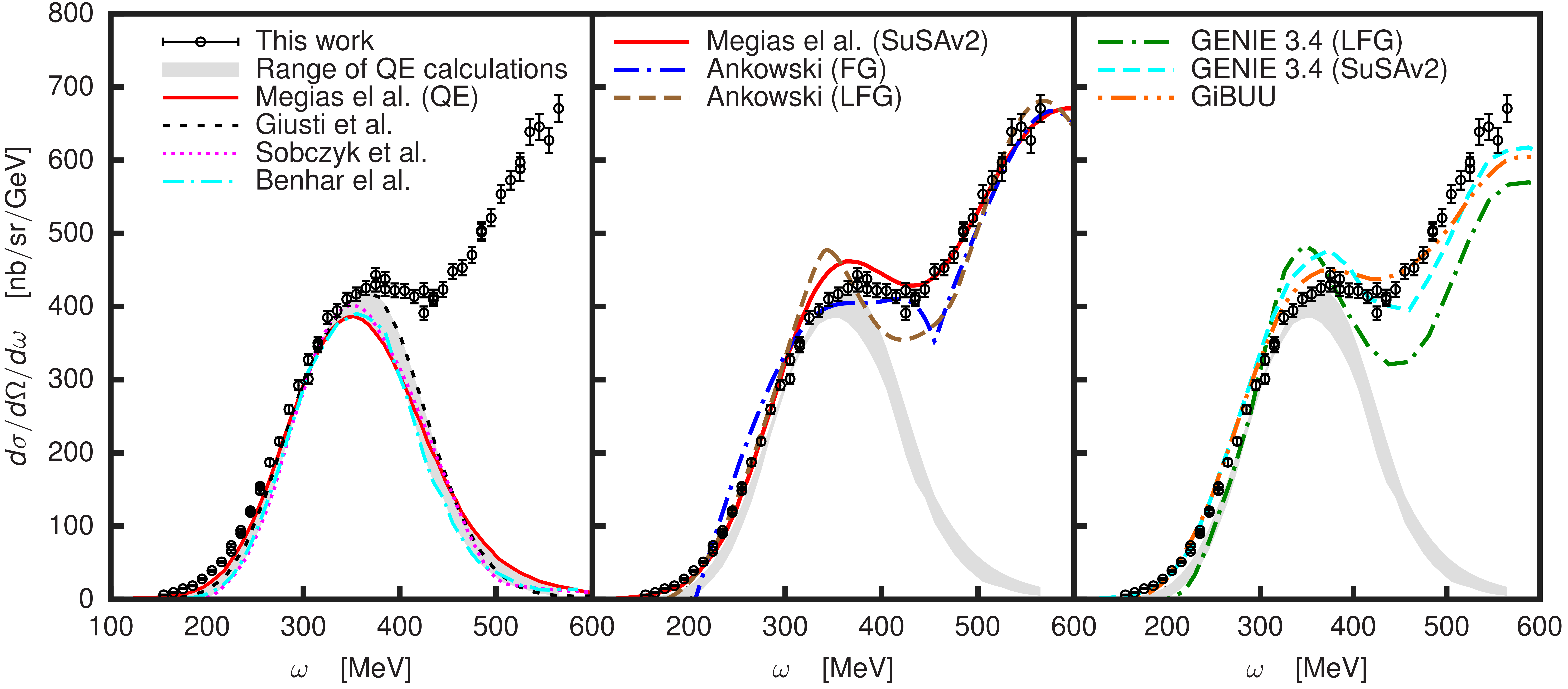
$$\left.\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega\,\mathrm{d}\omega}\right|_{\nu/\bar{\nu}} = \sigma_0\left[\ell_{CC}R_{CC} + \ell_{CL}R_{CL} + \ell_{LL}R_{LL} + \ell_T R_T \pm \ell_{T'}R_{T'}\right]$$

e-A scattering

$$\left.\frac{d^2\sigma}{d\Omega d\omega}\right|_e = \sigma_M\left[\frac{Q^4}{q^4}R_L + \left(\frac{Q^2}{2q^2} + \tan^2\frac{\theta_e}{2}\right)R_T\right]$$

e4ν in Mainz: ^{12}C exp

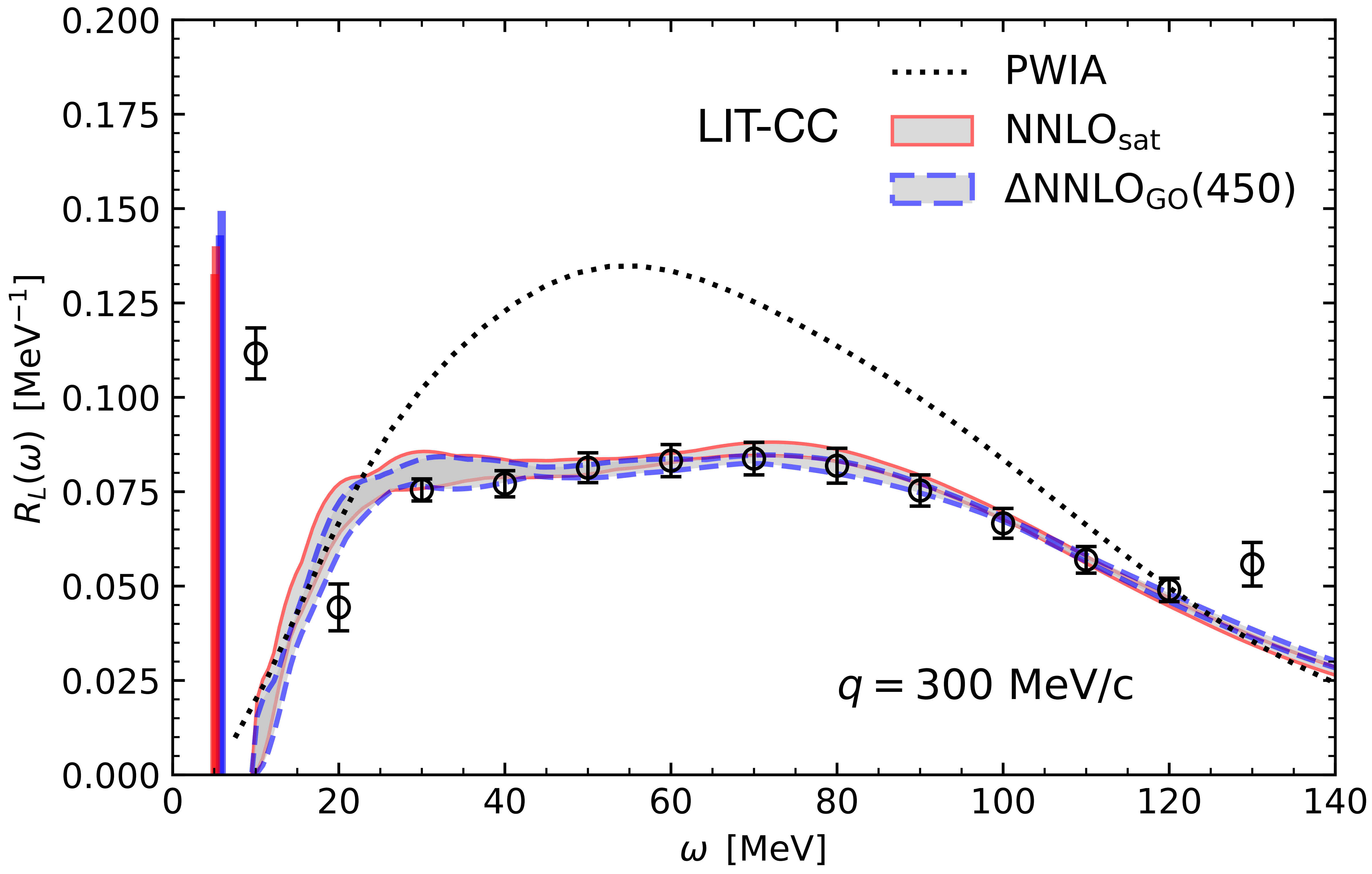
Mihovilovic, Doria et al, arXiv:2406.16059, E=855 MeV/c, θ=70°



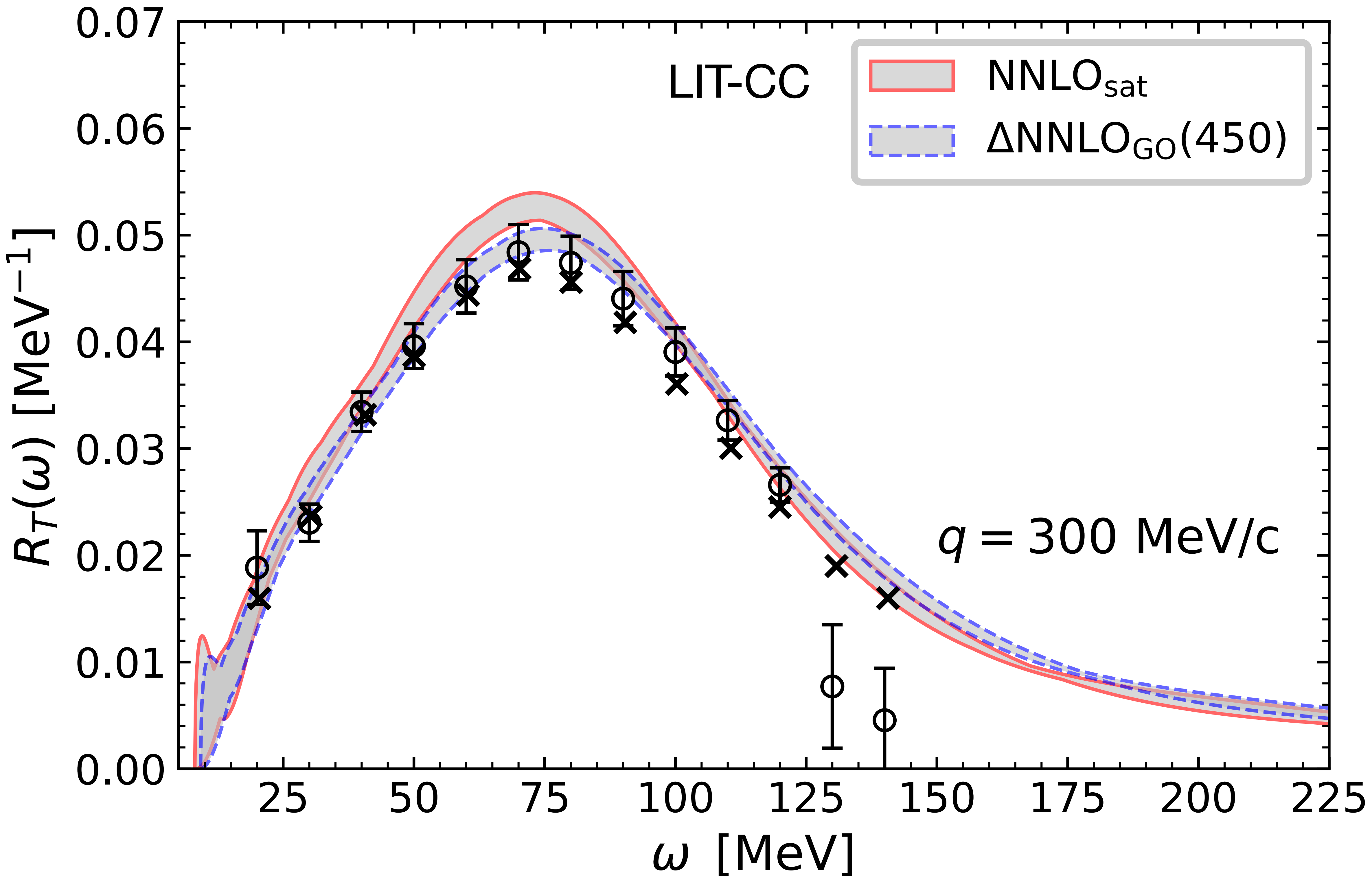
e4ν in Mainz: theory

$^{40}Ca(e,e')X$

Sobczyk, Acharya, SB, Hagen, PRL 127 (2021) 7, 072501



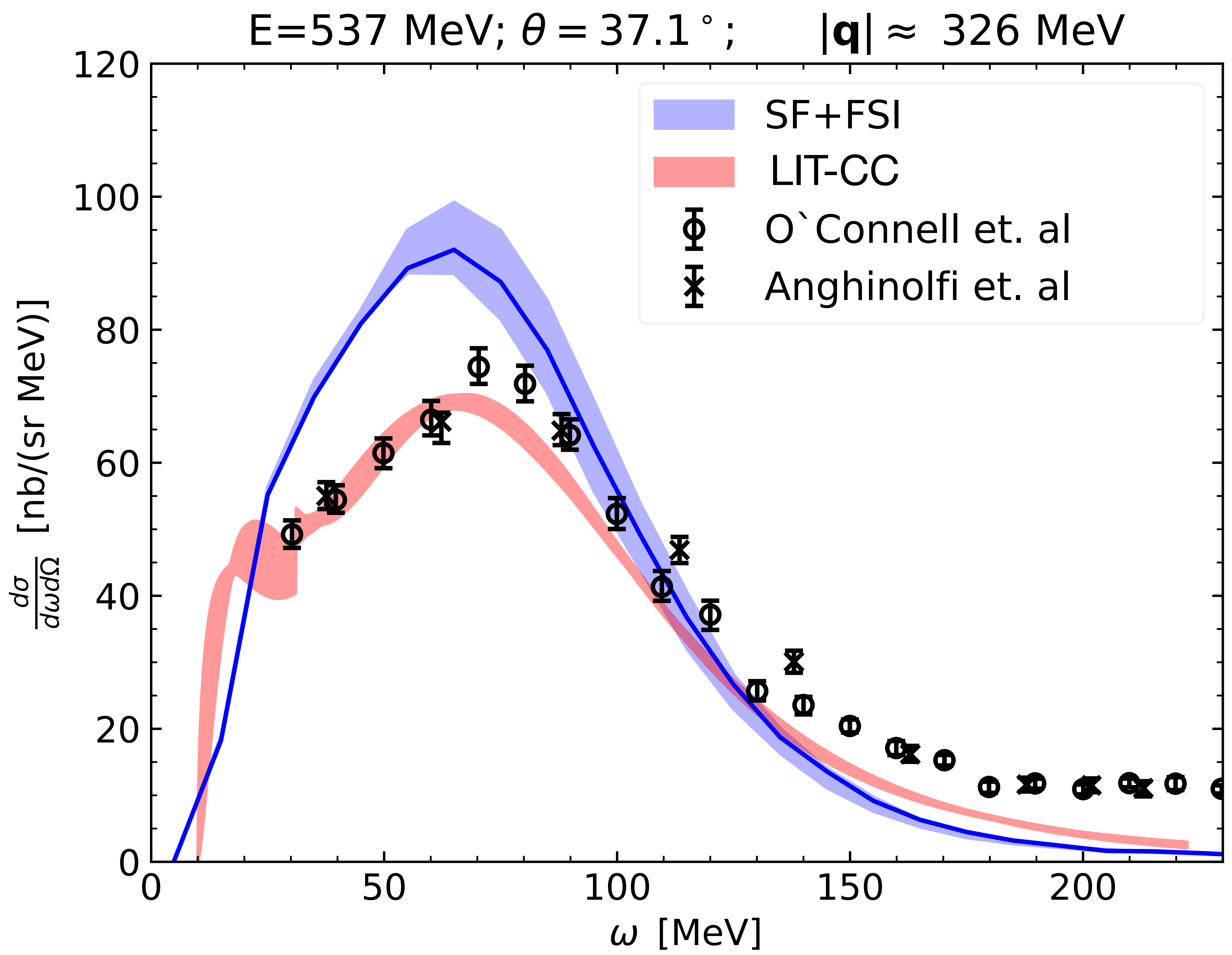
Sobczyk, Acharya, SB, Hagen, PRC 109 (2024) 2, 025502



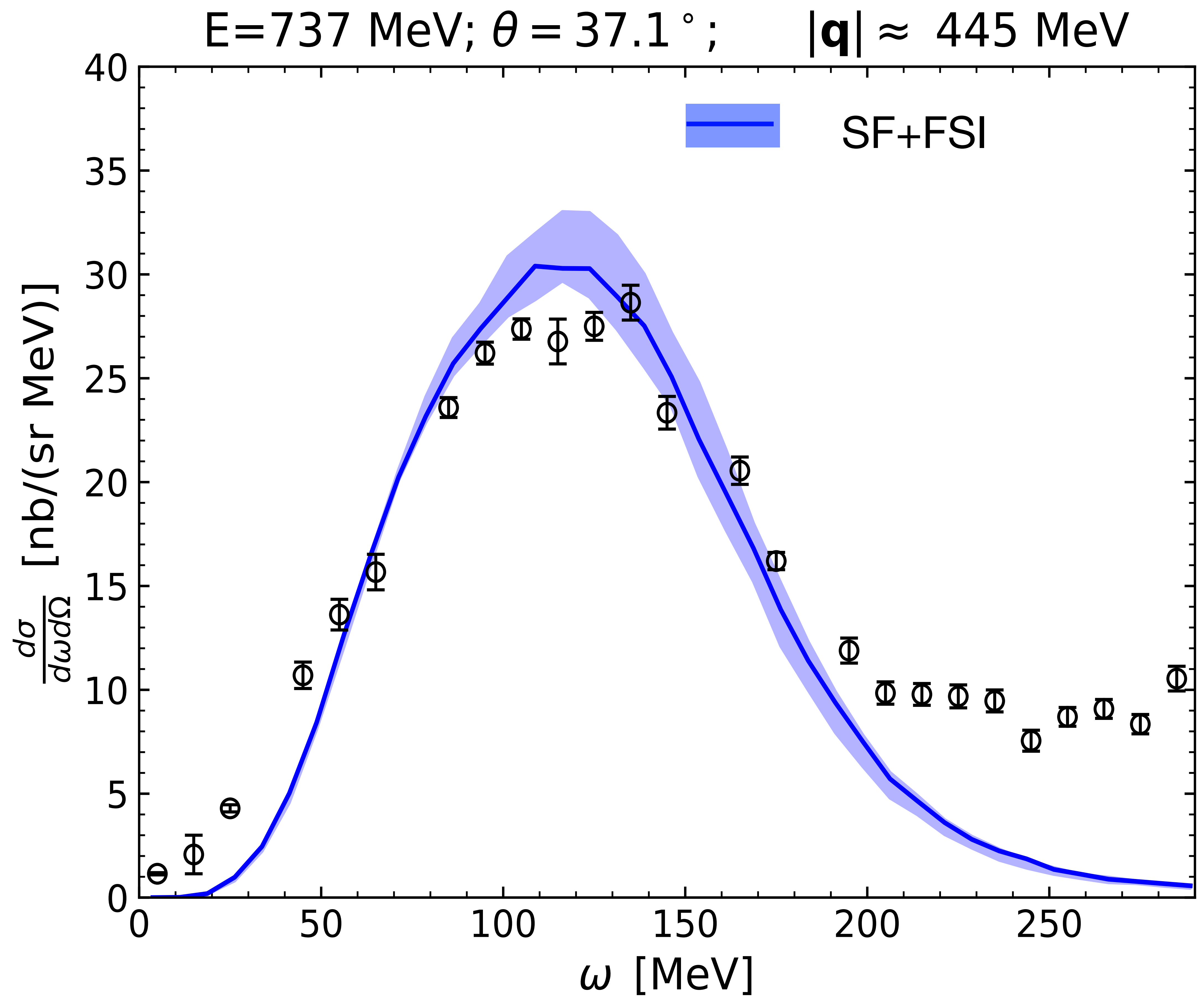
e4ν in Mainz: theory

^{16}O(e,e')X

Sobcyzk, Acharya, SB, Hagen, in preparation



Sobczyk, SB, PRC 109, 044314 (2024)

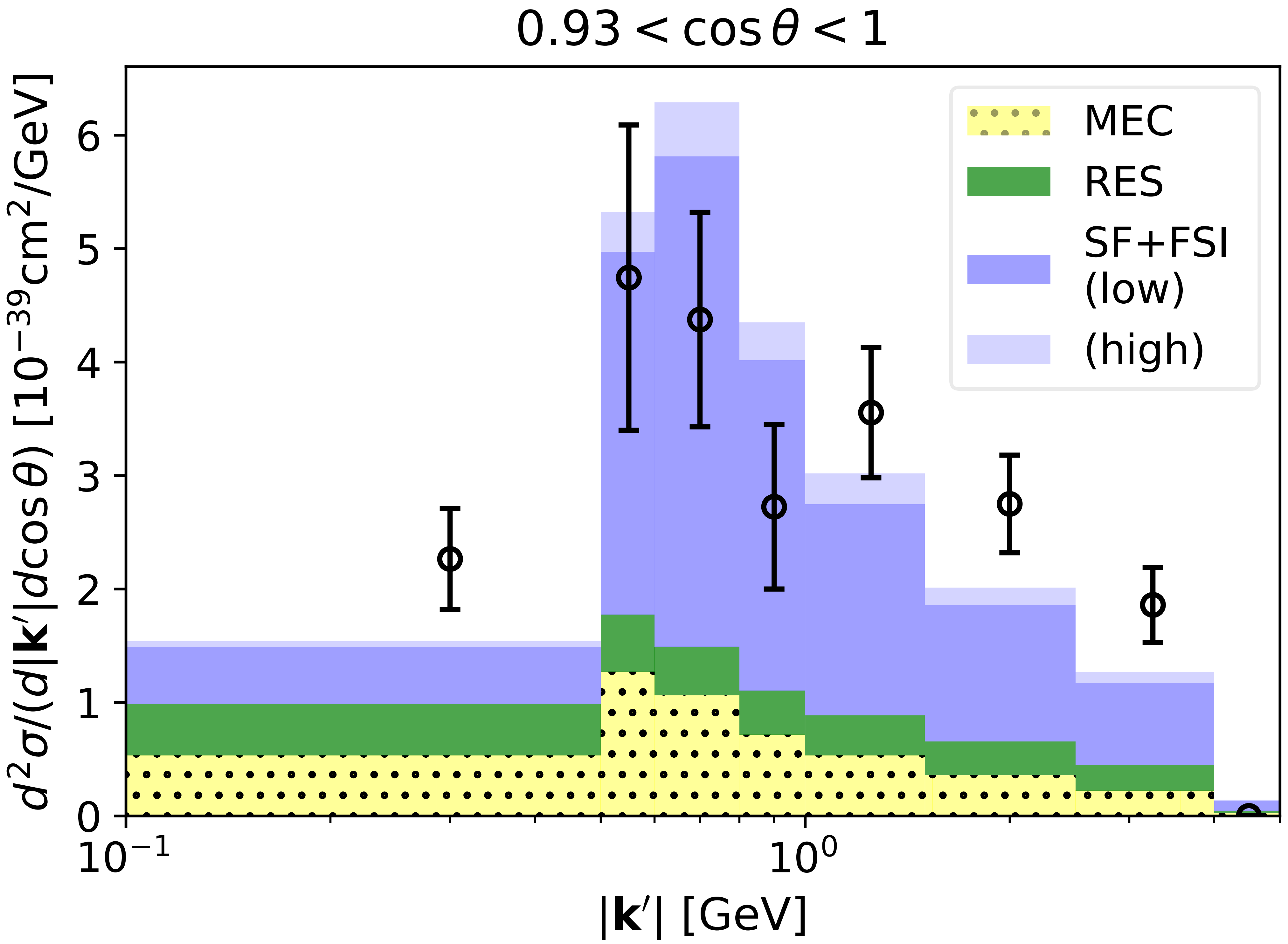


Towards neutrino scattering

Spectral function formalism

Sobczyk, SB, PRC 109, 044314 (2024)

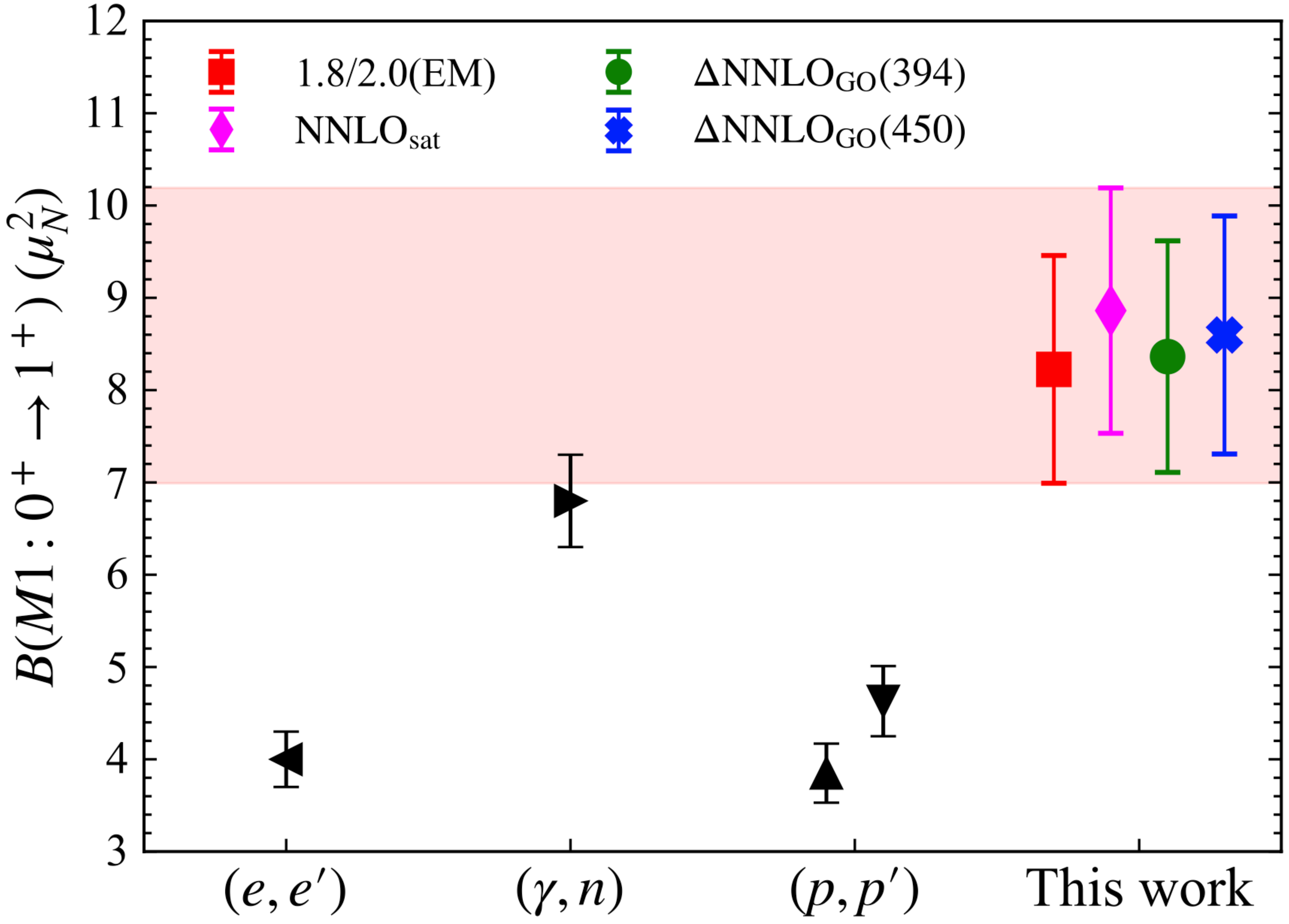
$$\nu_\mu + {}^{16}\mathrm{O} \to \mu^- + X$$

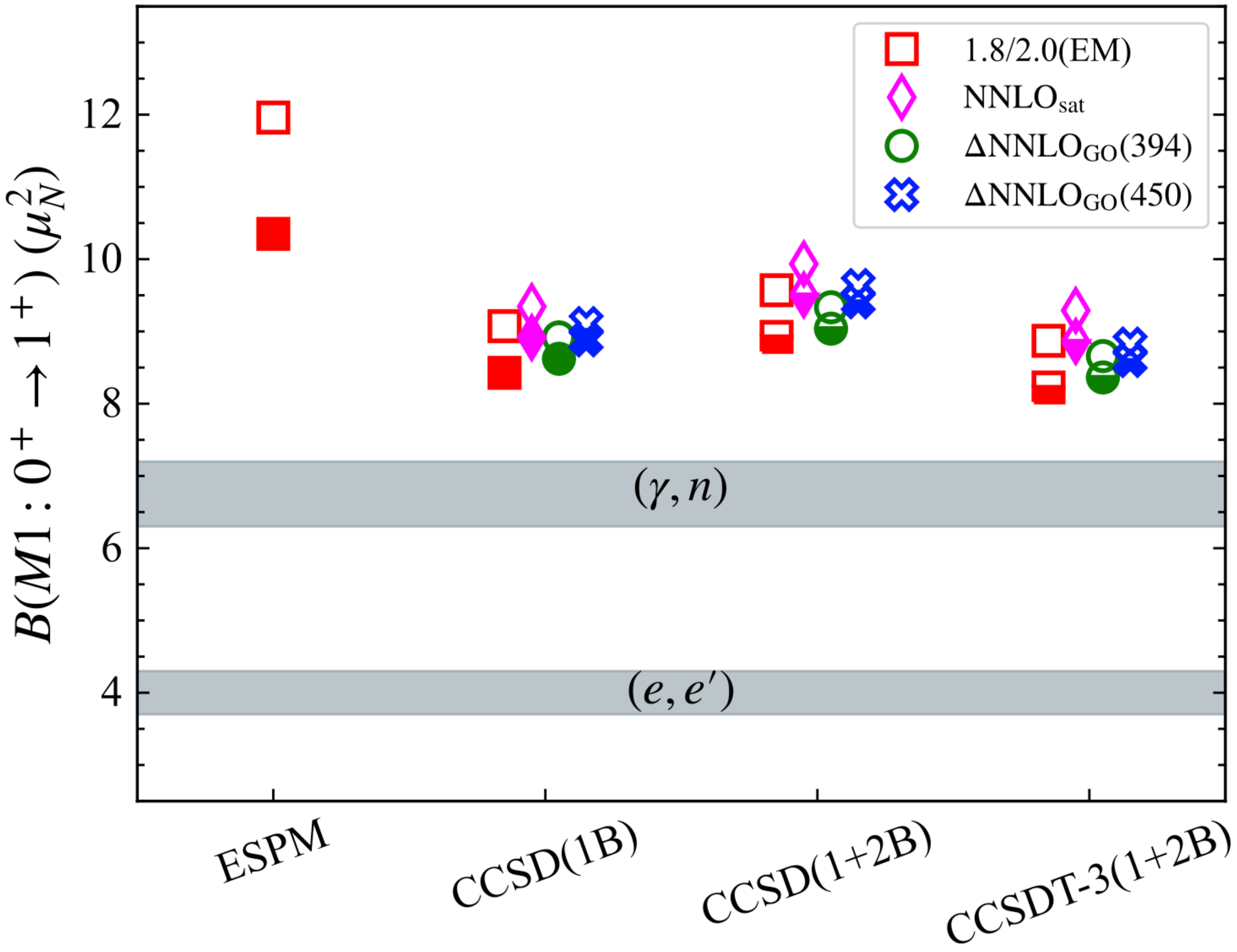


Back to nuclear structure

^{48}Ca M1 transition

Acharya et al., PRL 132 (2024) 23, 232504

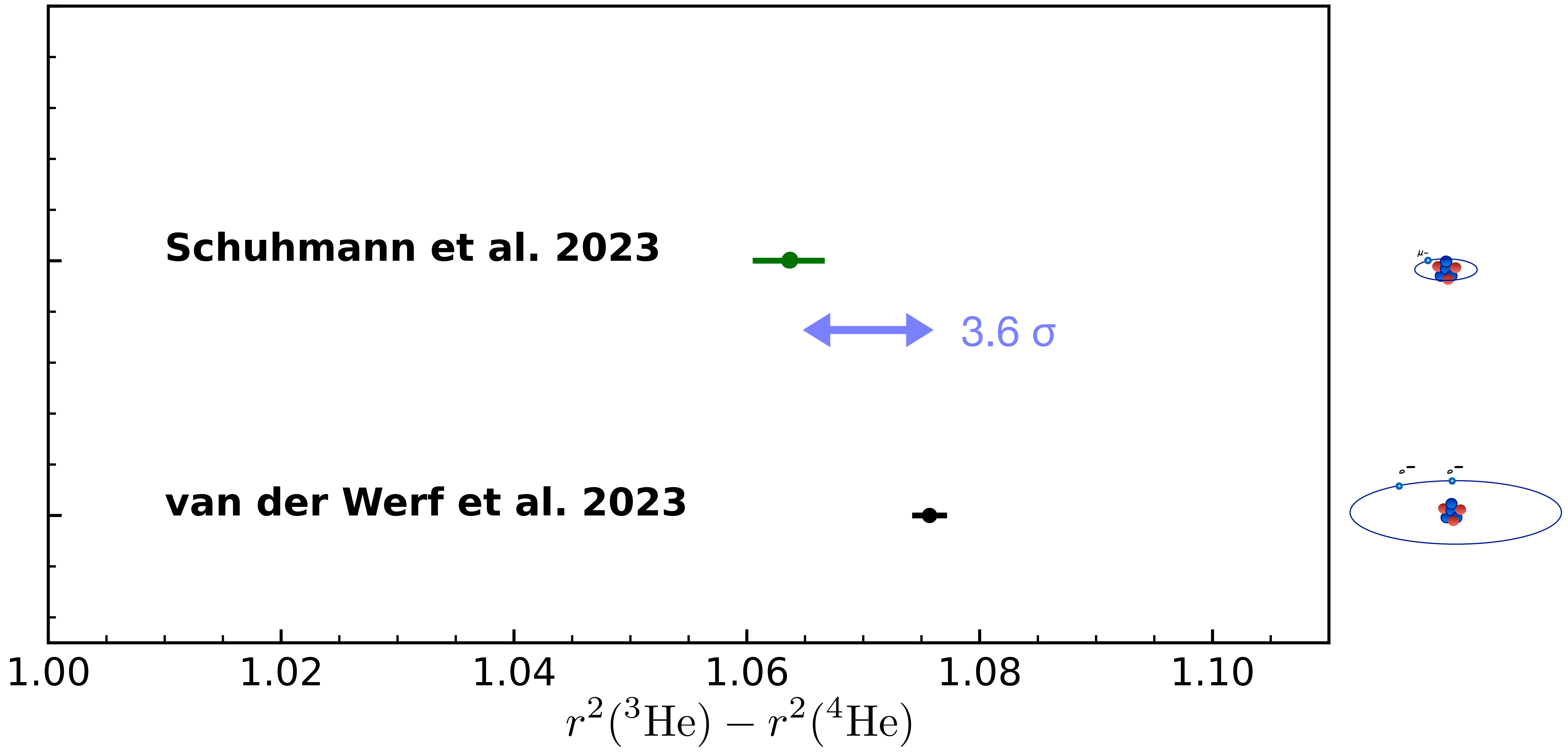




Applications to Atomic Physics

The helium isotope-shift puzzle

Li Muli, Richardson, SB, arXiv:2401.13424



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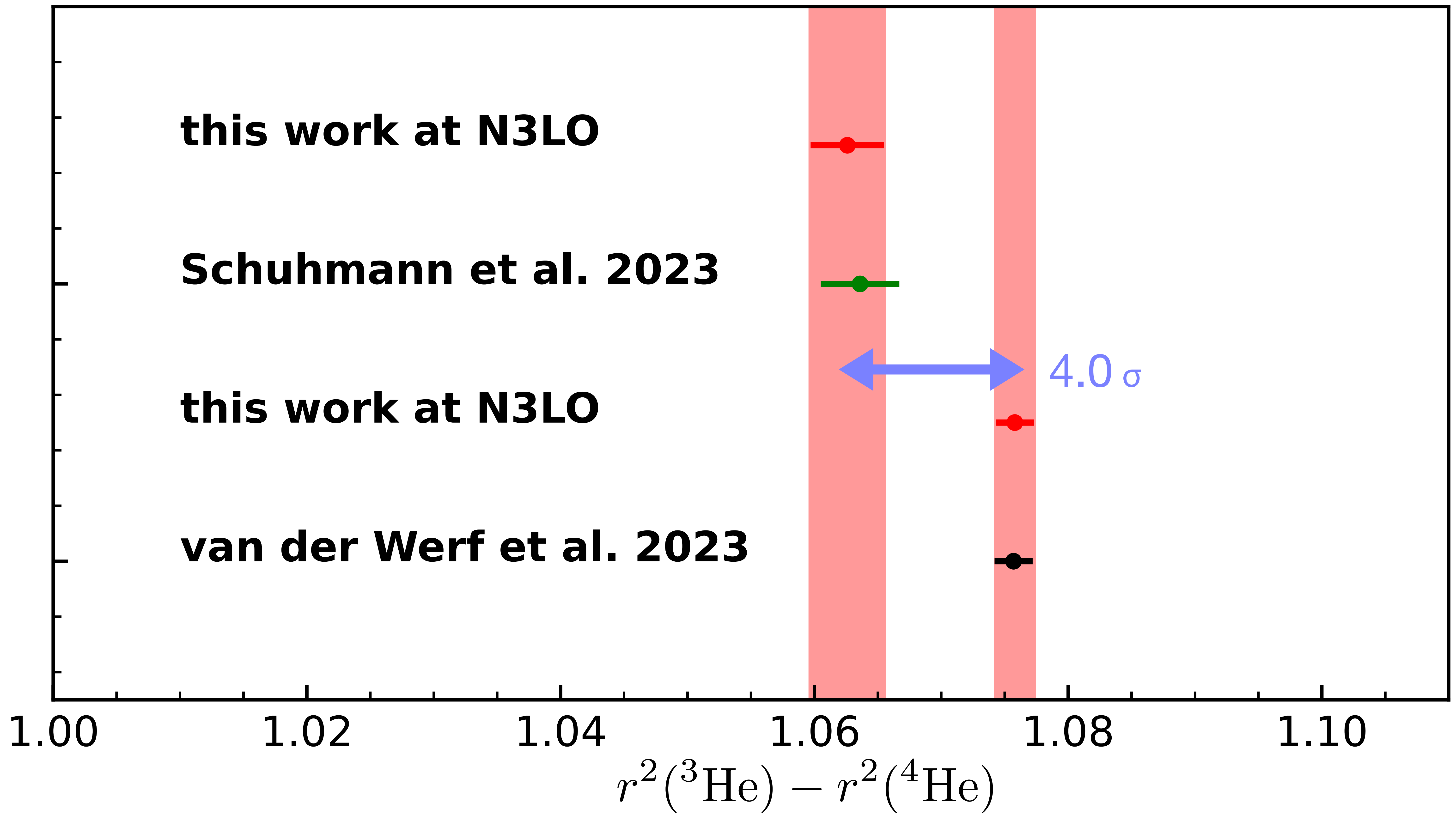

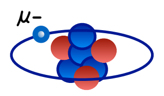

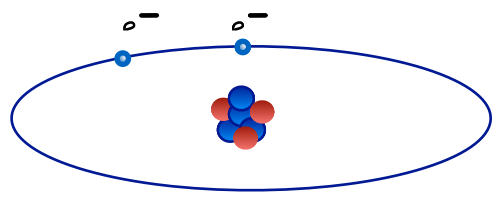

See Simone Li Muli's talk

Thanks to all my collaborators:

B. Acharya, F. Bonaiti, G. Hagen, G.R. Jansen, **W. Jiang,** W. Leidemann, **S.S. Li Muli**, G. Orlandini, T. Papenbrock, **I. Reis**, A. Schwenk, J. Simonis, **J.E. Sobczyk,** et al.

\+

experimental colleagues

THANK YOU

Challenges & Questions

- Should we do model mixing?
- Should we develop emulators?
- What can we do with the ^{4}He monopole puzzle?

Experimental extractions of 0^+_2

E_r (MeV)	Γ_r (MeV)	Probe
20.31 ± 0.01	0.29 ± 0.01	(α, α)
20.40 ± 0.04	0.33 ± 0.03	(α, α)
20.28 ± 0.05	0.41 ± 0.05	(α, α)
20.29 ± 0.02	0.89 ± 0.04	(α, α)
20.46 ± 0.14	0.34 ± 0.04	(p,p)
20.26 ± 0.16	0.39 ± 0.10	(e,e)
20.10 ± 0.05	0.27 ± 0.05	(e,e)
20.12 ± 0.16	0.24 ± 0.07	(e,e)
20.21	0.26 ± 0.05	(e,e)
20.21	0.29 ± 0.03	(e,e)

Year	Potential	Method	Continuum	E_r=Position of 0^+_2	Comments
2004	AV8+Central 3NF	GEM	No	20.25	Describes data
2013	AV18+UIX	LIT/HH	Yes*	21	Too high
2013	N3LO+N2LO	LIT/HH	Yes*	21	Too high
2023	AV8+Central 3NF	LIT/HH	No	20	Describes data
2023	Vlowk	NCGSM-CCC	Yes	Tuned	No 3NF, describes data
2023	SU4	NLEFT	No	20-20.30	Describes data, modified tail
2024	N3LO+N2LO*	HH	Yes	20.30	Describes data, a bit high
2024	Daejeon16	NCSM	No	?	Too high