

# Advances and challenges for ab initio calculations of medium-mass to heavy nuclei and nuclear matter

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TECHNISCHE  
UNIVERSITÄT  
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MITP  
TOPICAL  
WORKSHOP

Uncertainty Quantification in Nuclear Physics  
June 24 – 28, 2024

 <https://indico.mtp.uni-mainz.de/event/357>



MITP Workshop

June 25, 2024



European Research Council  
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Hessens Zukunft

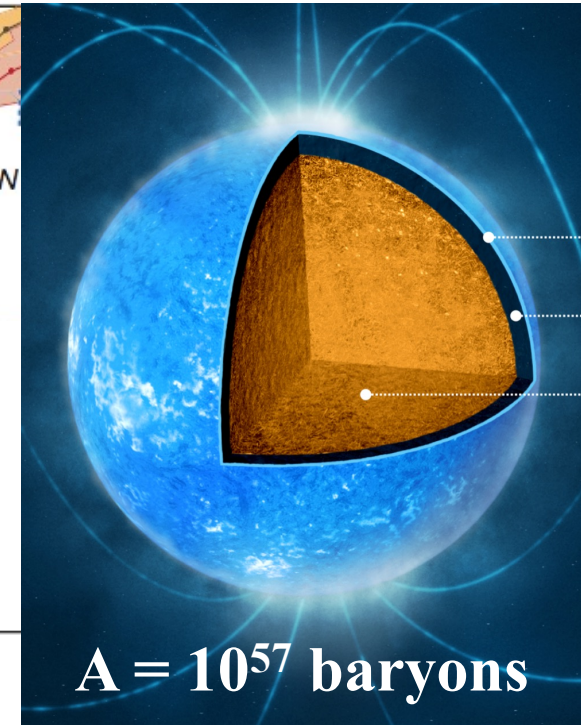
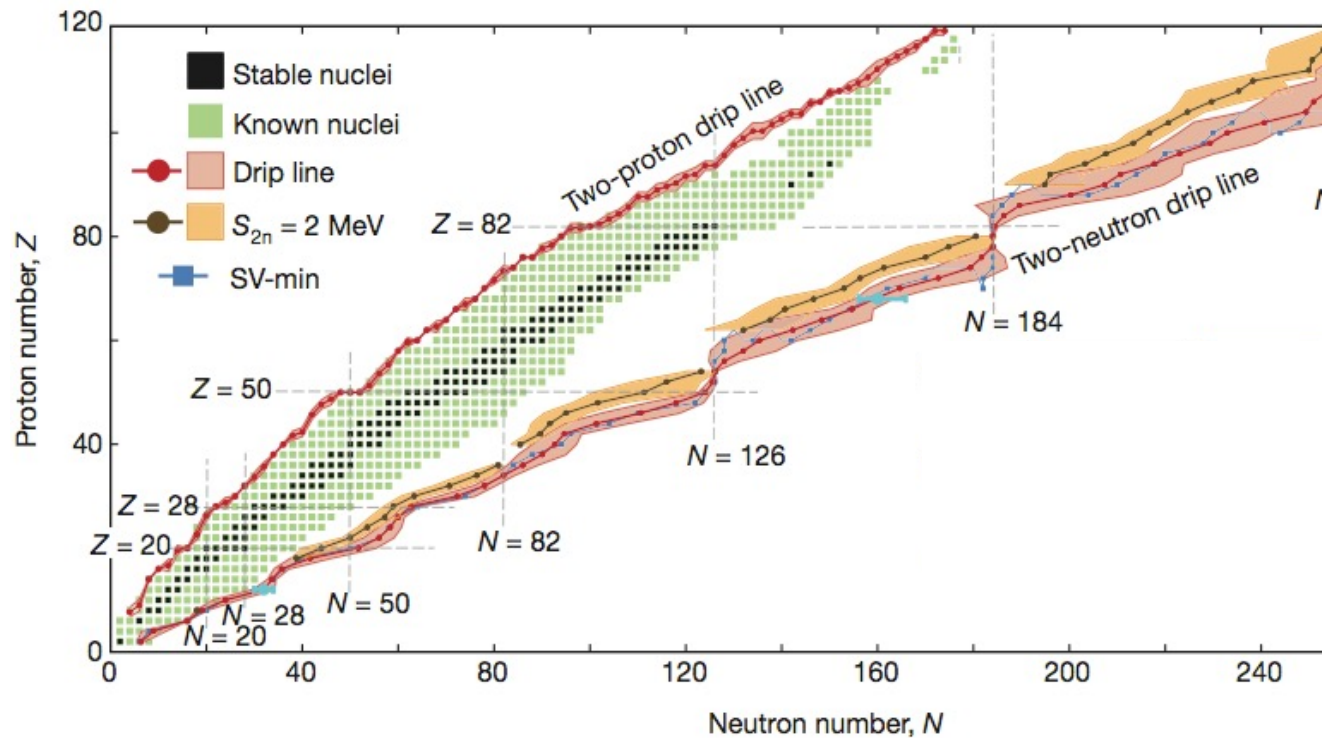
# Structure of nuclei and dense matter in neutron stars

doi:10.1038/nature11188

## The limits of the nuclear landscape

Jochen Erler<sup>1,2</sup>, Noah Birge<sup>1</sup>, Markus Kortelainen<sup>1,2,3</sup>, Witold Nazarewicz<sup>1,2,4</sup>, Erik Olsen<sup>1,2</sup>, Alexander M. Perhac<sup>1</sup> & Mario Stoitsov<sup>1,2,†</sup>

~ 4000 ± 500 nuclei unknown, extreme neutron-rich



Extreme neutron-rich matter in neutron stars

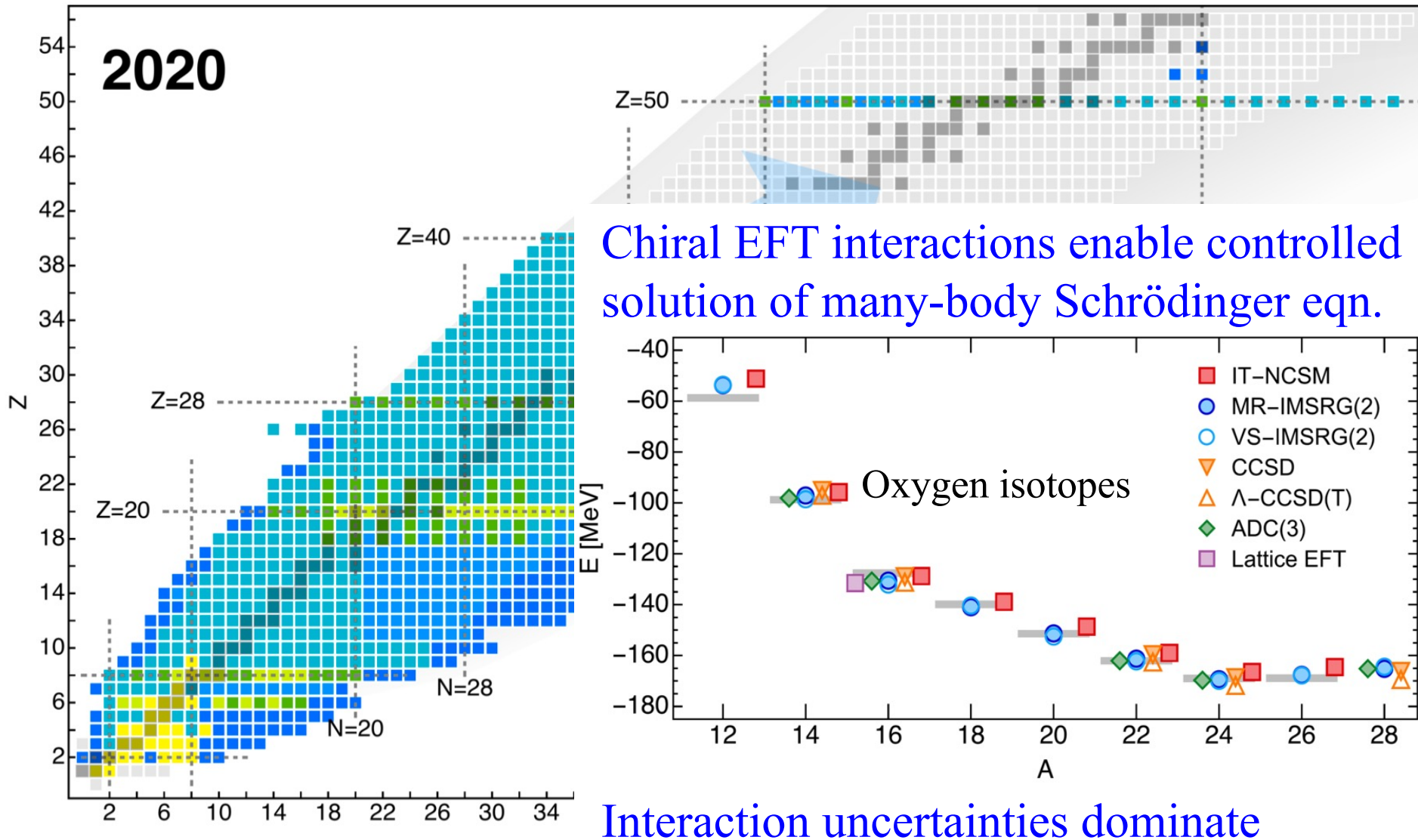
# Chiral effective field theory for nuclear forces

Systematic expansion (power counting) in low momenta  $(Q/\Lambda_b)^n$

	NN	3N	4N	
LO $\mathcal{O}\left(\frac{Q^0}{\Lambda^0}\right)$				based on symmetries of strong interaction (QCD)
NLO $\mathcal{O}\left(\frac{Q^2}{\Lambda^2}\right)$				long-range interactions governed by pion exchanges  powerful approach for many-body interactions
N <sup>2</sup> LO $\mathcal{O}\left(\frac{Q^3}{\Lambda^3}\right)$				all 3- and 4-neutron forces predicted to N <sup>3</sup> LO Tews et al., PRL (2013)
N <sup>3</sup> LO $\mathcal{O}\left(\frac{Q^4}{\Lambda^4}\right)$				
	+ ...	(2011) ...	(2006) ...	

Weinberg (1990,91), van Kolck, Kaplan, Savage, Wise, Bernard, Epelbaum, Kaiser, Meißner,...

# Great progress in ab initio calculations of nuclei

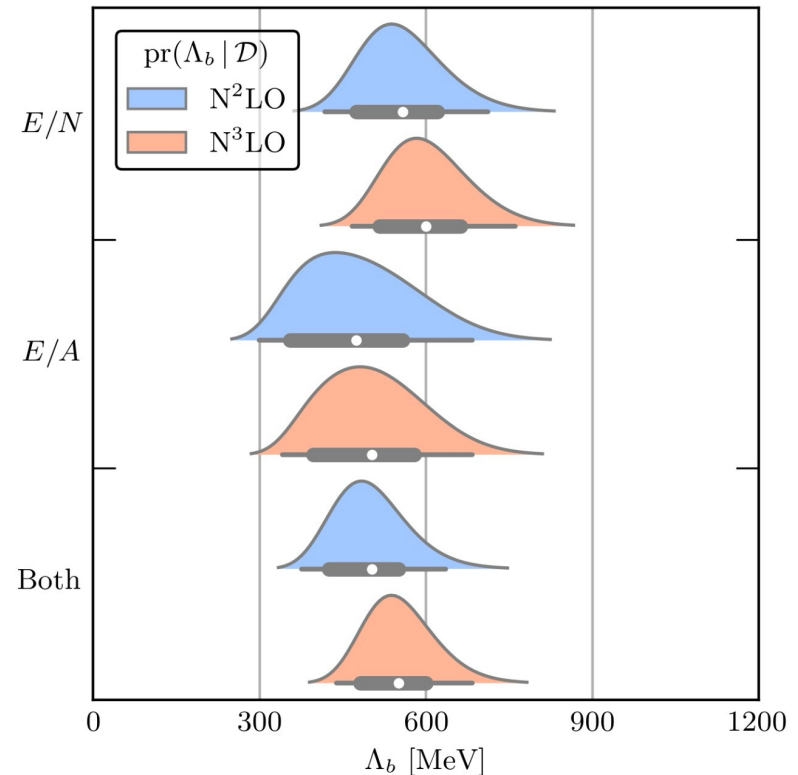
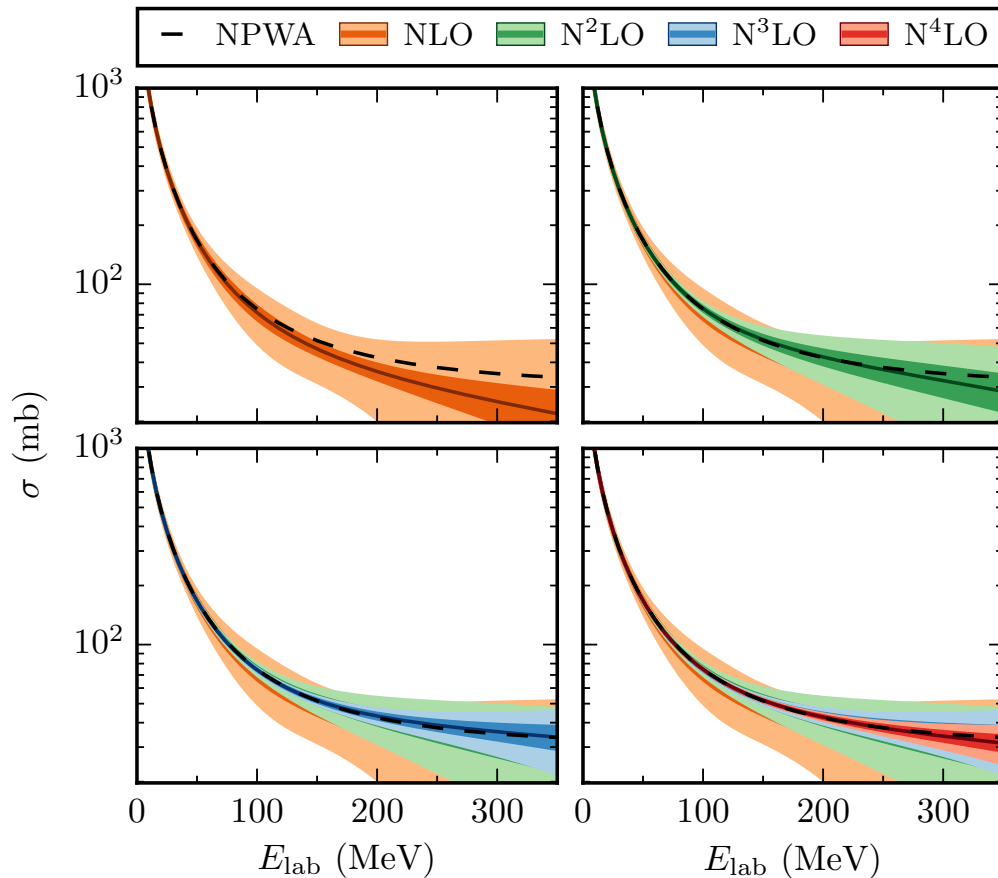


figures from Hergert (2020)

# Chiral effective field theory for nuclear forces

Systematic expansion (power counting) in low momenta  $(Q/\Lambda_b)^n$

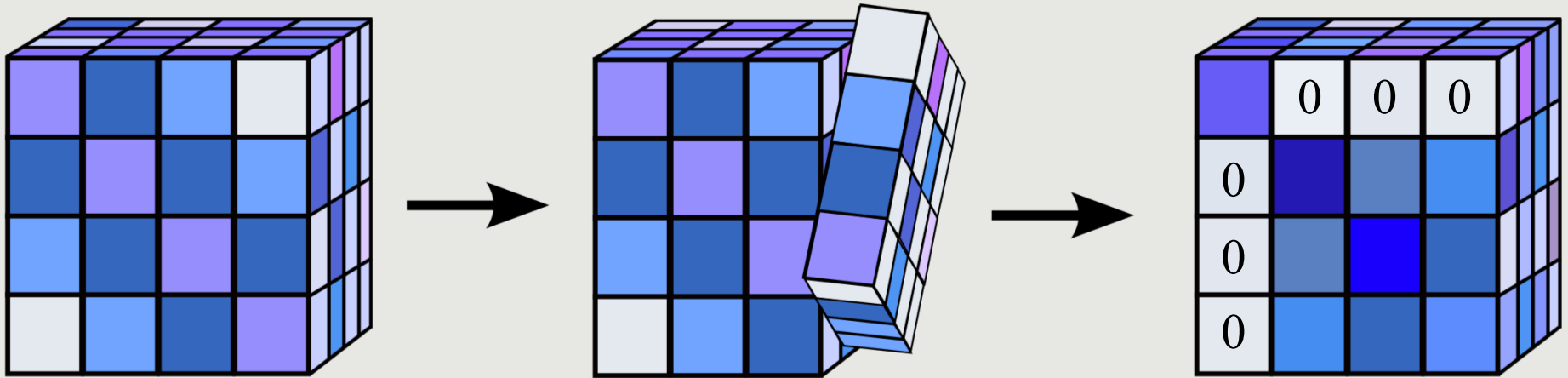
Bayesian order-by-order uncertainty estimates based on power series in  $(Q/\Lambda_b)^n$  with prior distribution for expansion coefficients



# In-medium similarity renormalization group (IMSRG)

Tsukiyama, Bogner, AS, PRL (2011), Hergert et al., Phys. Rep. (2016)

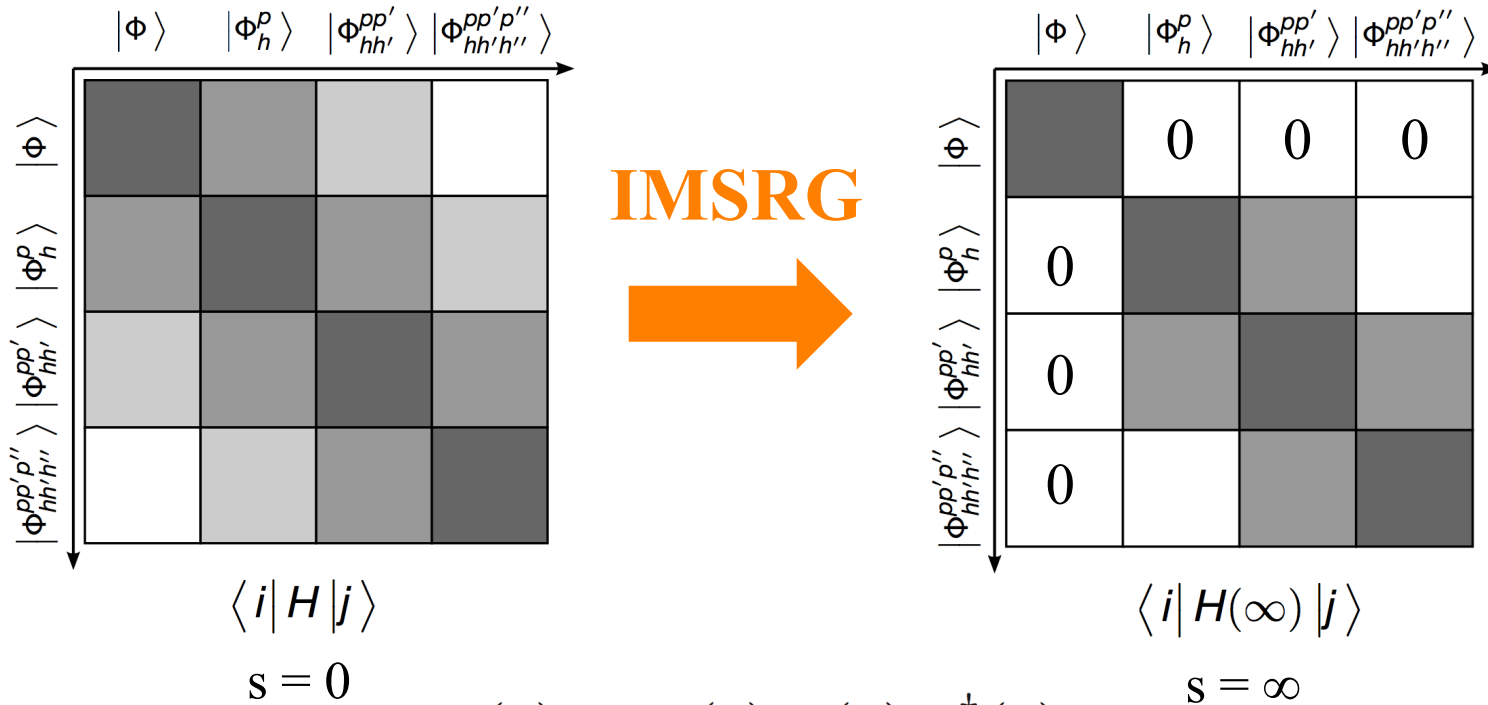
continuous transformation to block-diagonal form ( $\rightarrow$  decoupling)



# In-medium similarity renormalization group (IMSRG)

Tsukiyama, Bogner, AS, PRL (2011), Hergert et al., Phys. Rep. (2016)

RG flow equations to decouple higher-lying particle-hole states

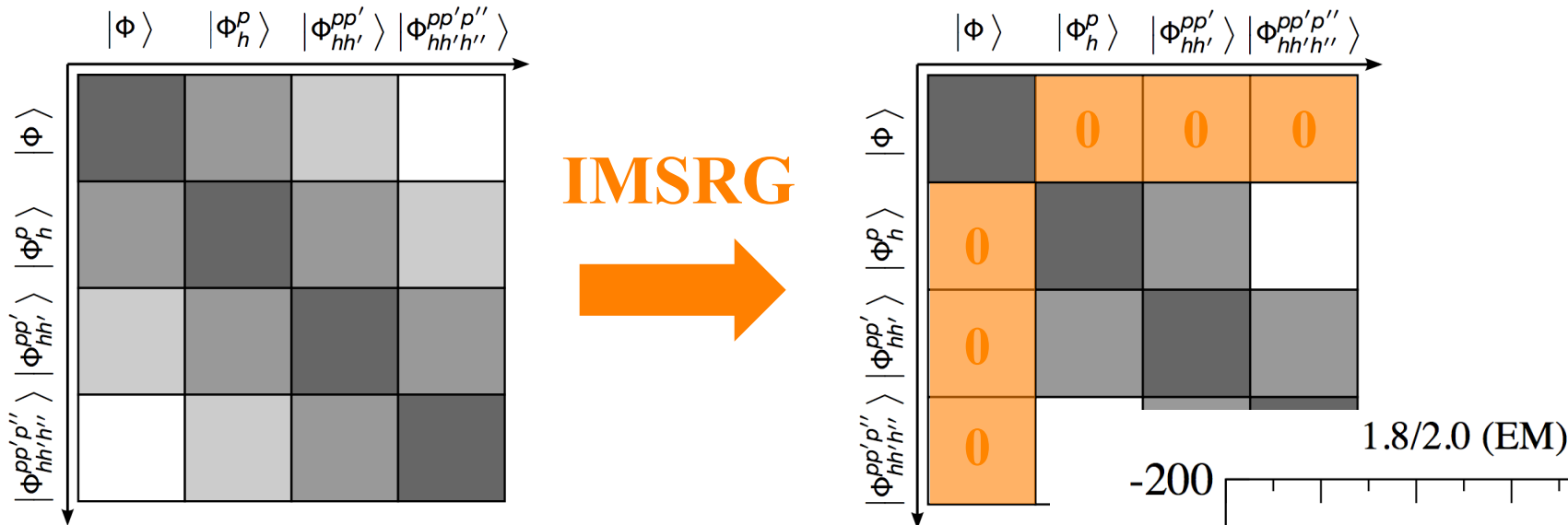


$$H(s) = U(s)H(0)U^\dagger(s)$$

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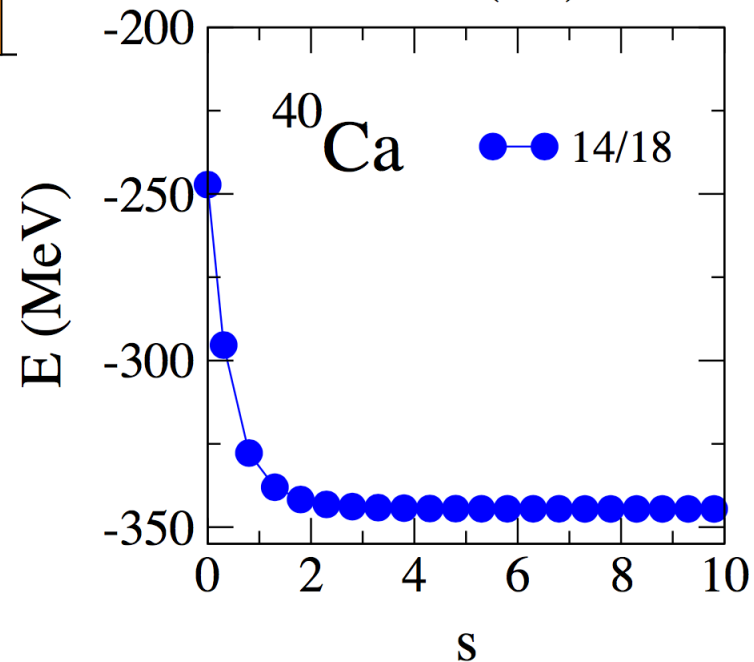
$$\langle i | H | j \rangle$$

$$s = 0$$

$$H(s) = U(s)H(0)U^\dagger(s)$$

$$\frac{d}{ds}H(s) = [\eta(s), H(s)]$$

$$\text{with generator } \eta(s) = [H_{\text{od}}(s), H_{\text{d}}(s)]$$

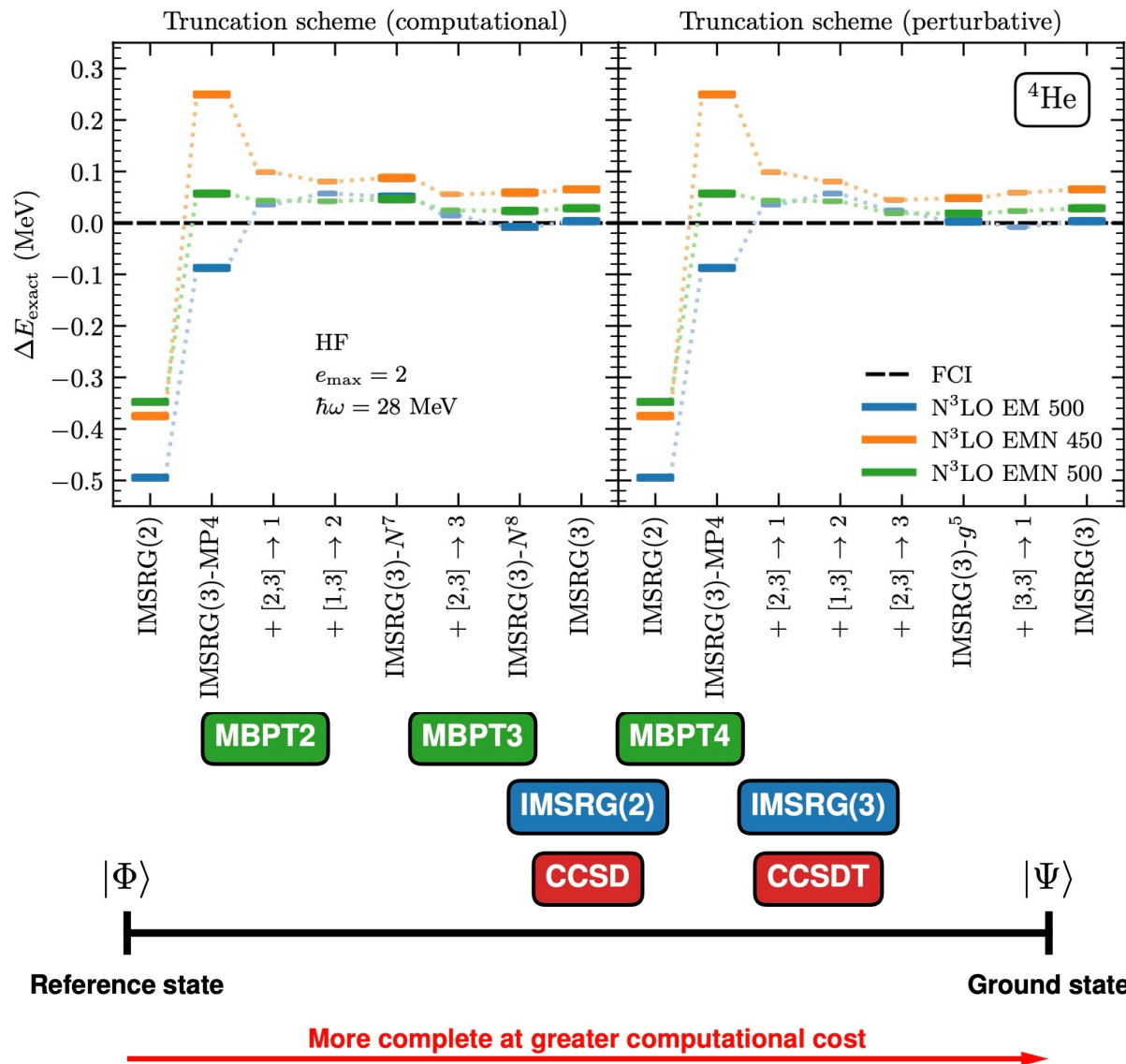
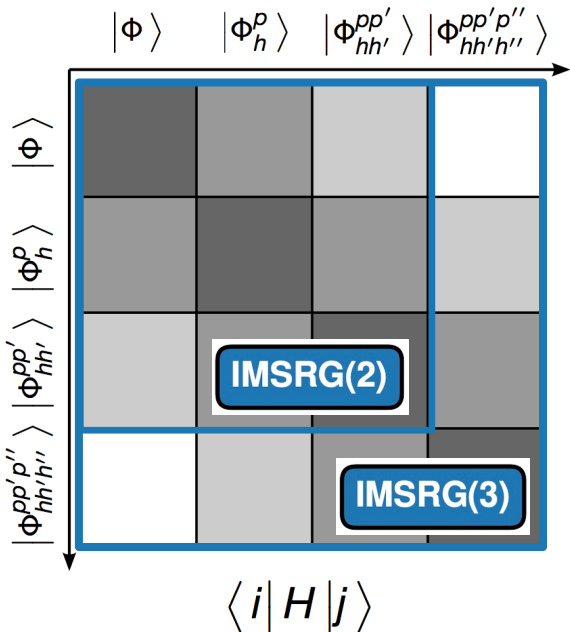




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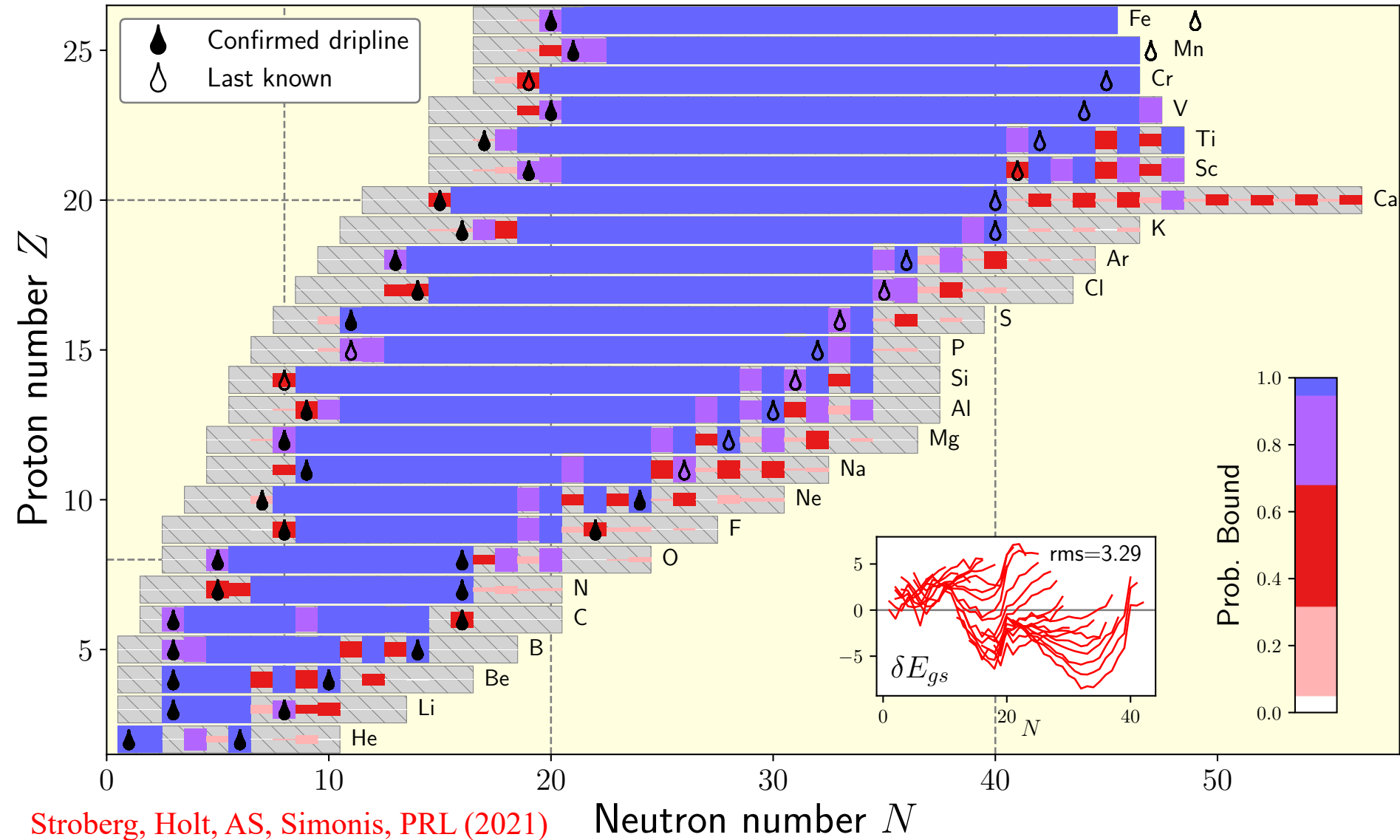


Standard truncation of IMSRG flow equations at normal-ordered 2-body level: IMSRG(2)

First IMSRG(3) results

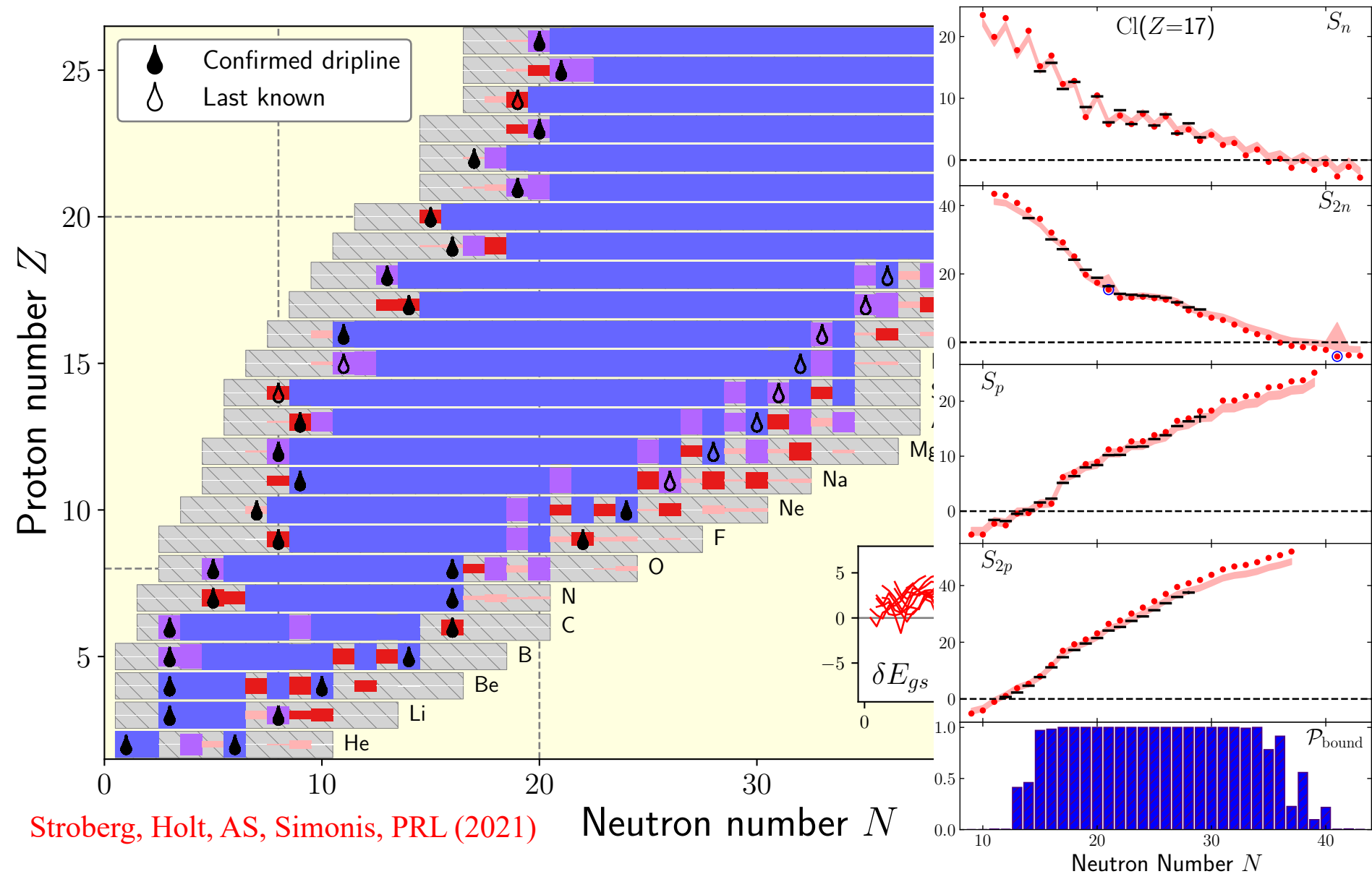
Heinz et al., PRC (2021)

# Nuclear landscape based on a chiral NN+3N interaction



ab initio is advancing to global theories, limitations due to input NN+3N

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ab initio is advancing to global theories, limitations due to input NN+3N

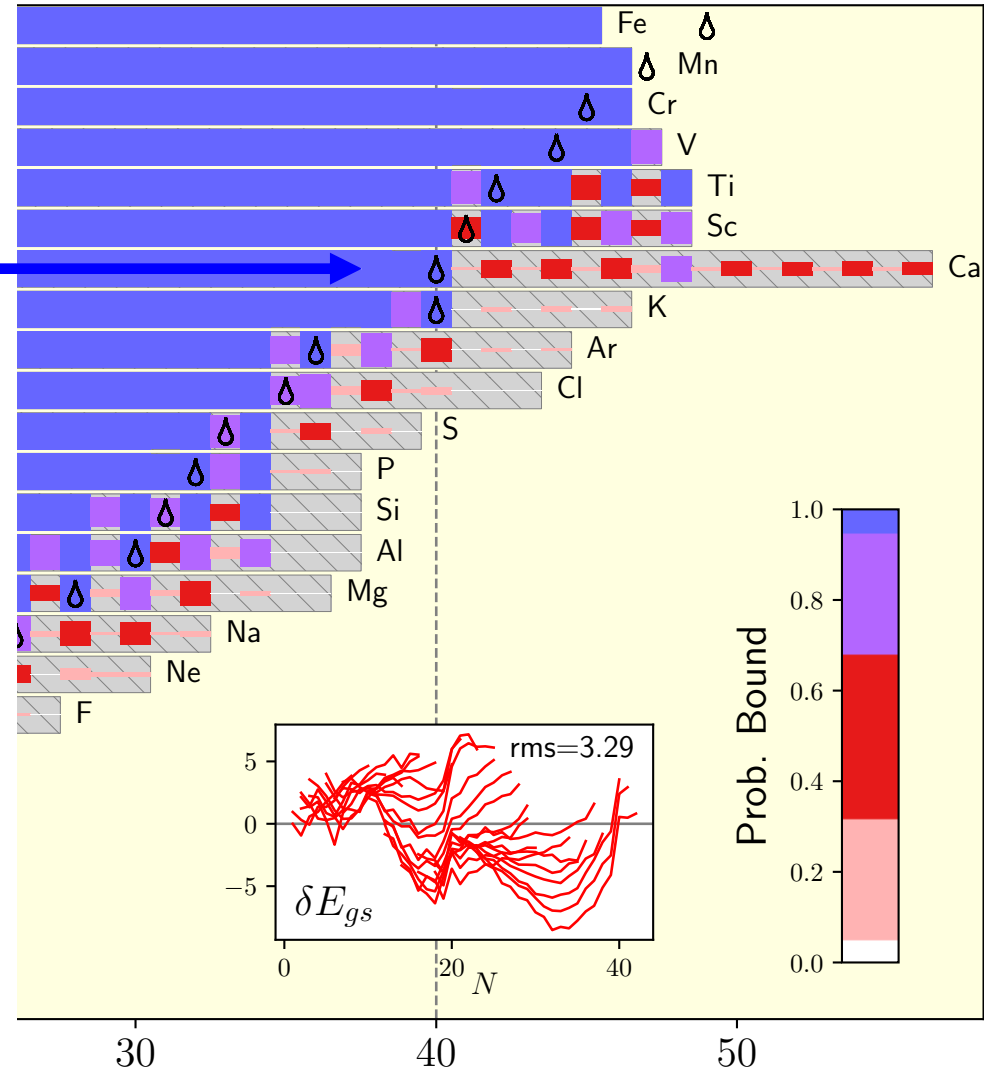
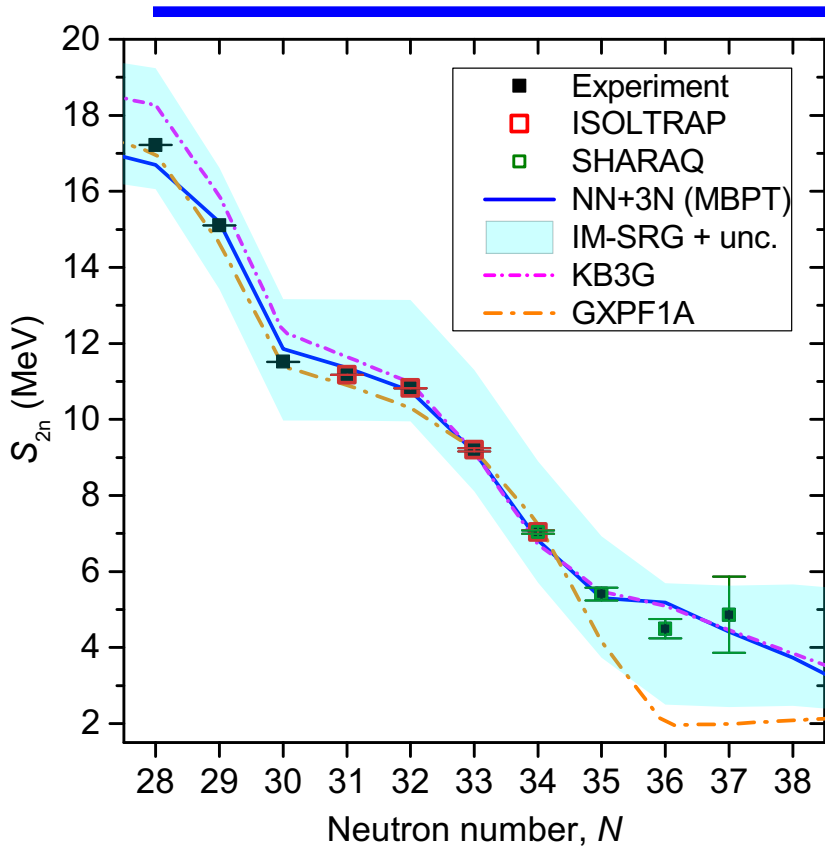
# Highlights from ISOLDE/CERN and RIBF/RIKEN

active exp-theory collaborations

pioneering  $^{51-57}\text{Ca}$  masses

Wienholtz et al., Nature (2013)

Michimasa et al., PRL (2019)



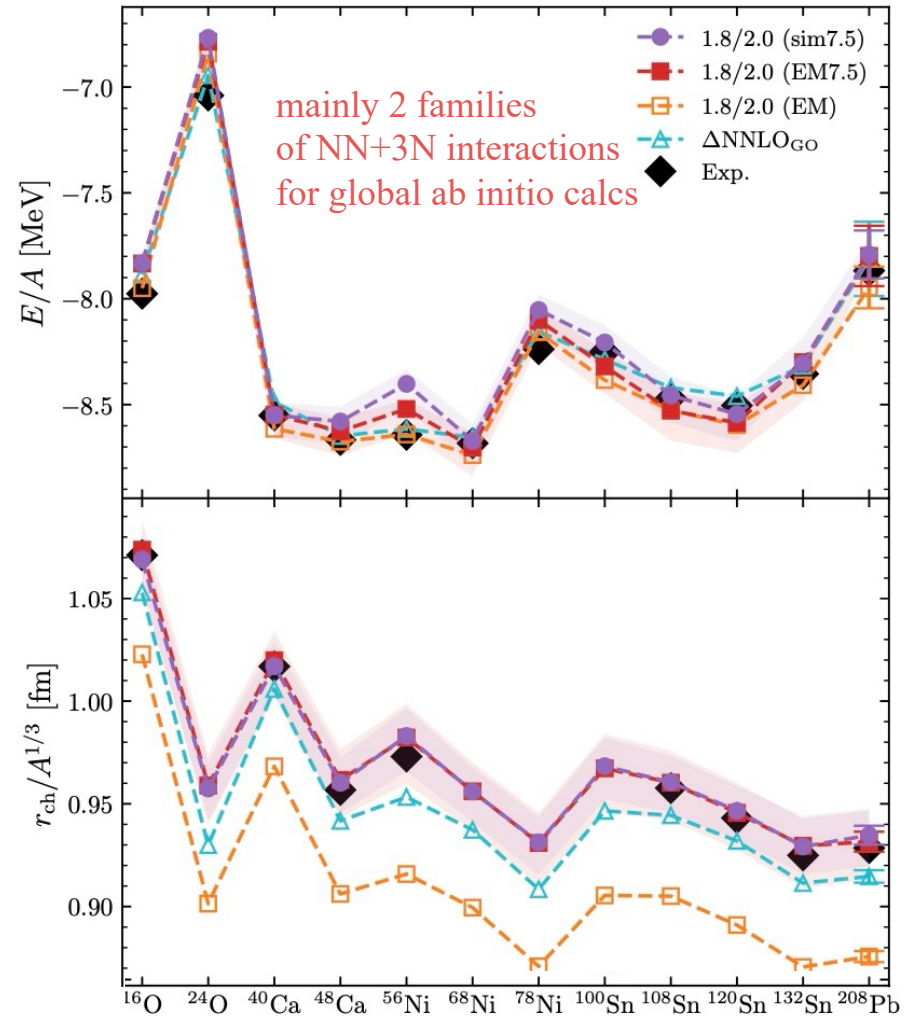
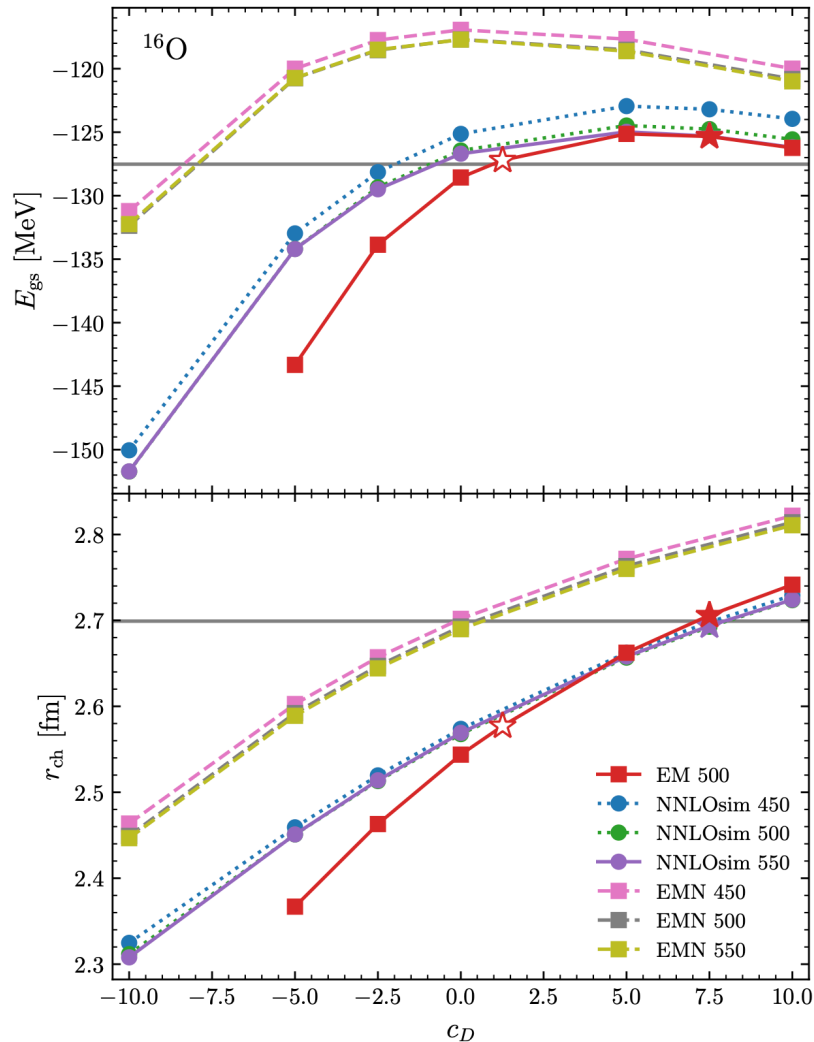
importance of 3N interactions  
for neutron-rich shell structure

# New chiral low-resolution interactions

Arthuis, Hebeler, AS, arXiv:2401.06675, see Kai Hebeler's talk

based on SRG-evolved NN interactions, 3N couplings fit to  ${}^3\text{H}$  and  ${}^{16}\text{O}$

accurate for ground-state properties from  ${}^{16}\text{O}$  to  ${}^{208}\text{Pb}$



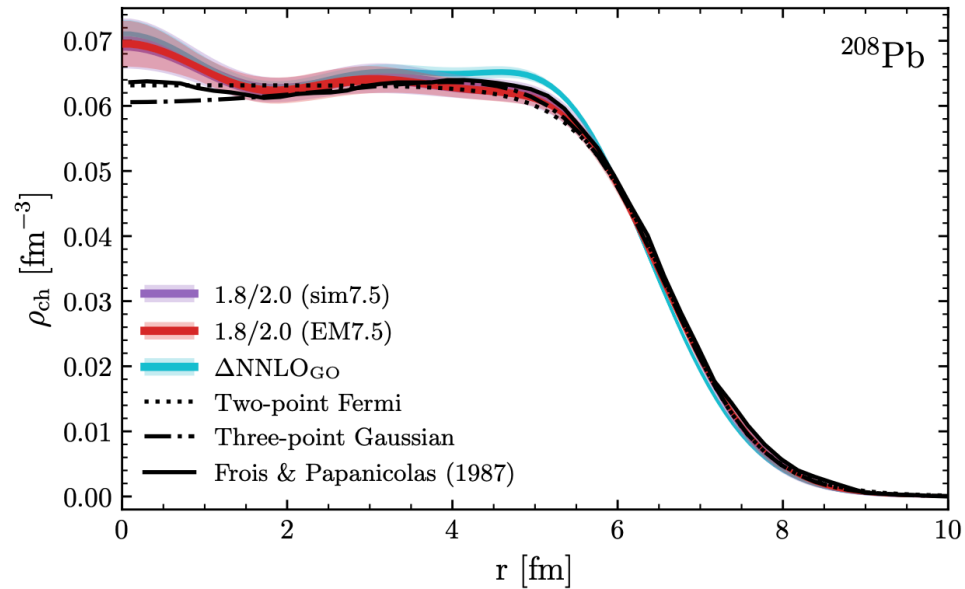
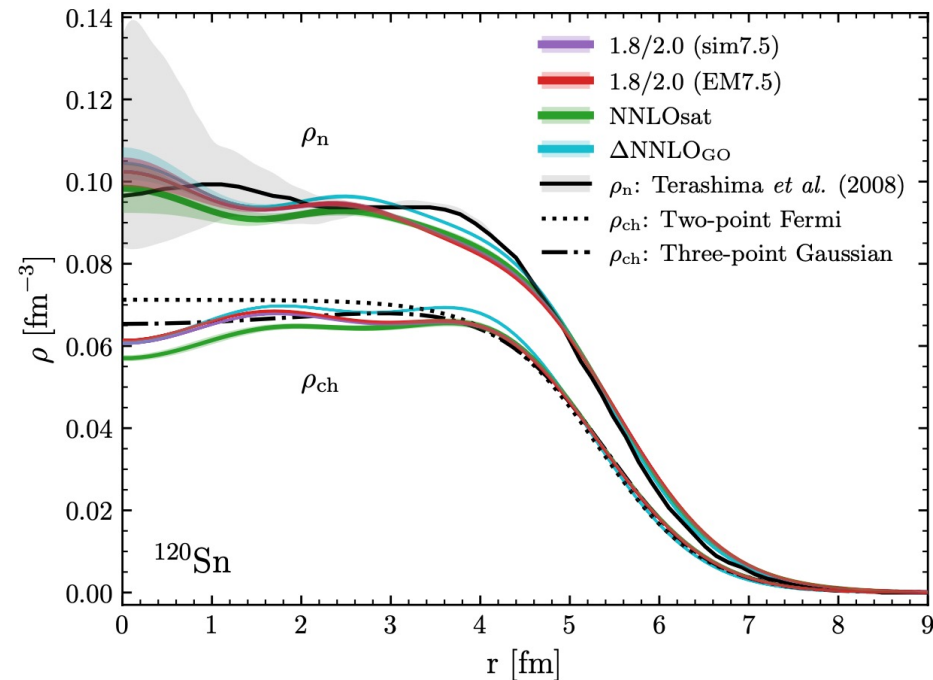
# Neutron/proton density distributions in medium-heavy nuclei

Arthuis, Hebeler, AS, arXiv:2401.06675

based on SRG-evolved NN interactions, 3N couplings fit to  $^3\text{H}$  and  $^{16}\text{O}$

accurate for ground-state properties from  $^{16}\text{O}$  to  $^{208}\text{Pb}$

very good agreement for density distributions in  $^{120}\text{Sn}$  and  $^{208}\text{Pb}$



# Neutron skins

Arthuis, Hebeler, AS, arXiv:2401.06675

very model independent

based on new (and previous)  
chiral NN+3N interactions

dependence of neutron skin

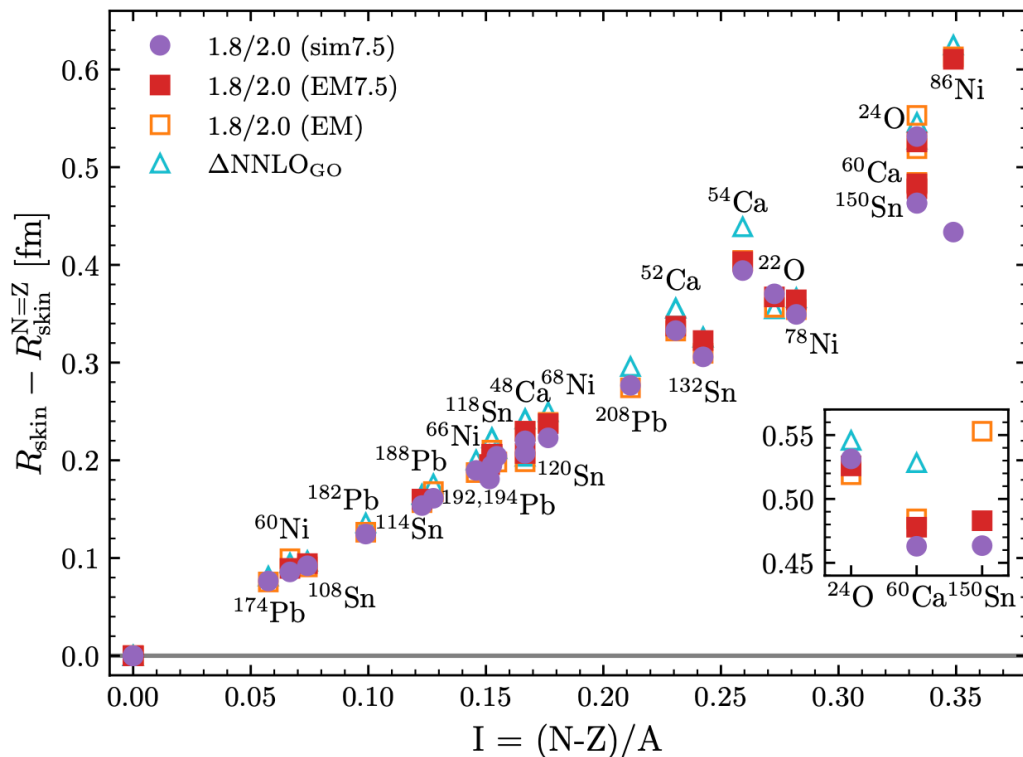
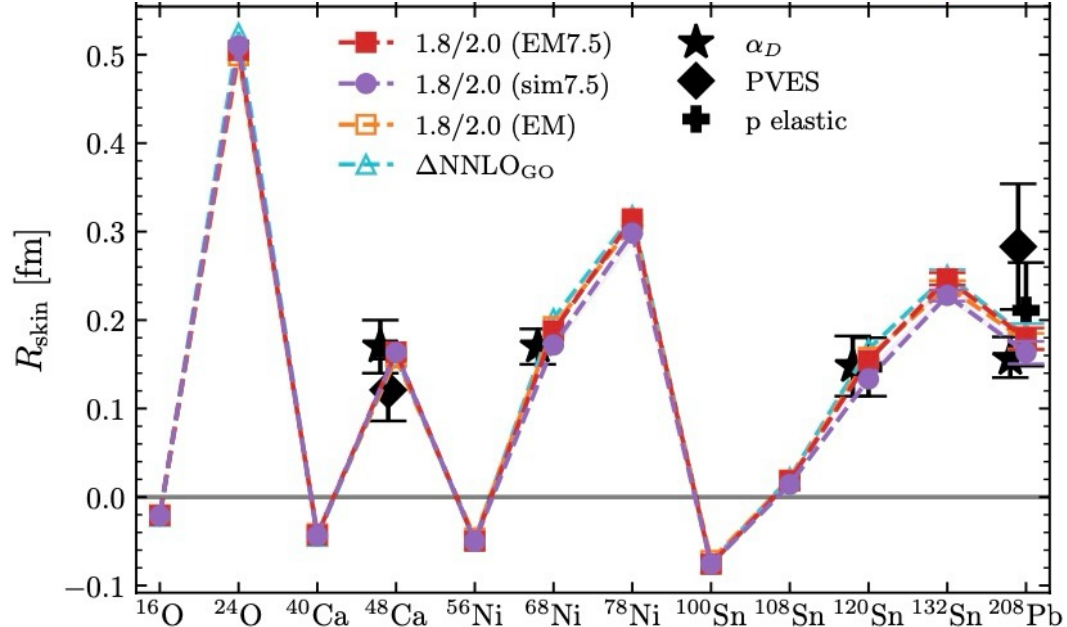
corrected for Coulomb is

linear in isospin

see also Novario et al., PRL (2023)

interesting predictions

for extreme n-rich nuclei



# EFT uncertainties for SRG-evolved interactions

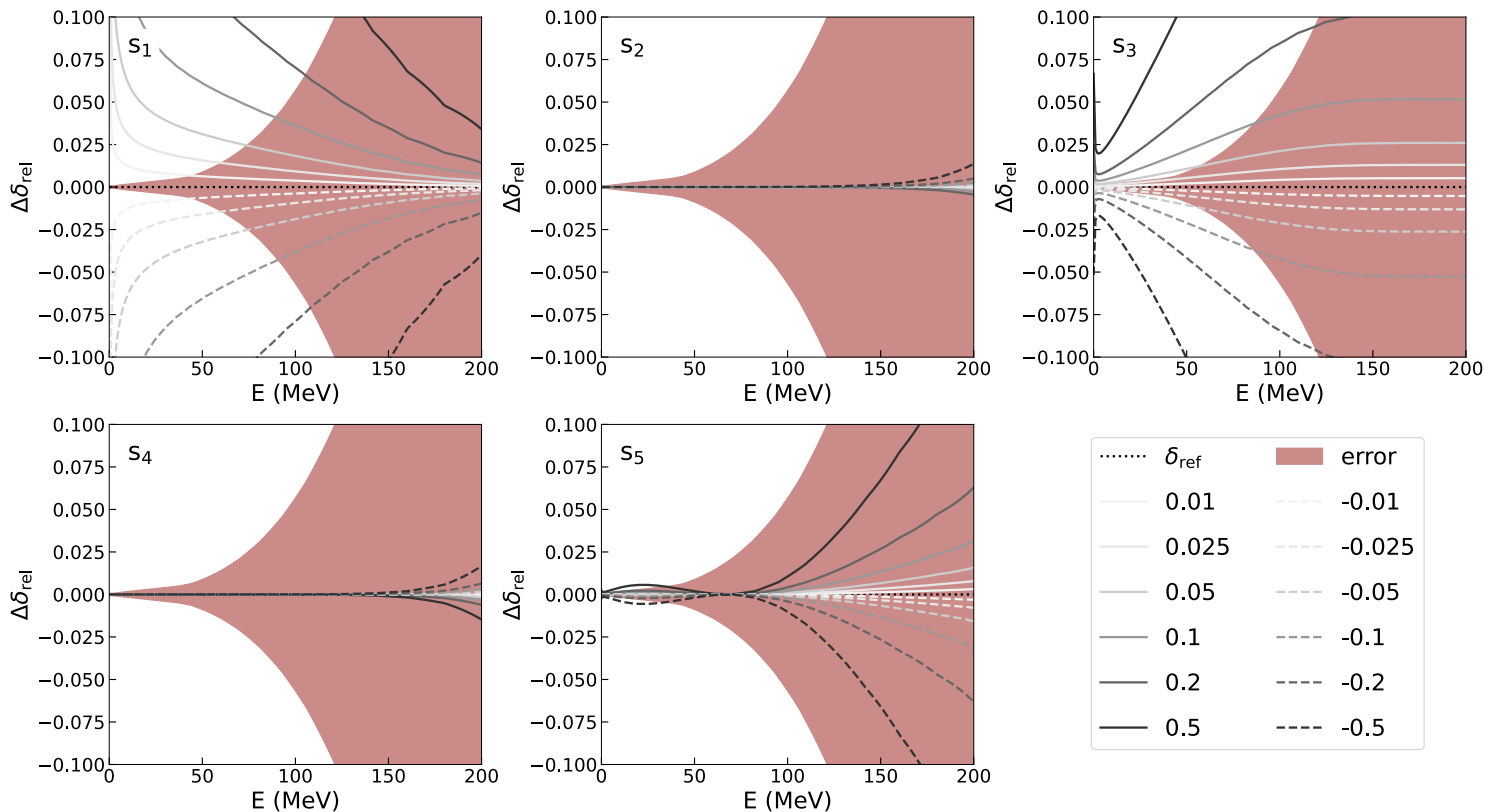
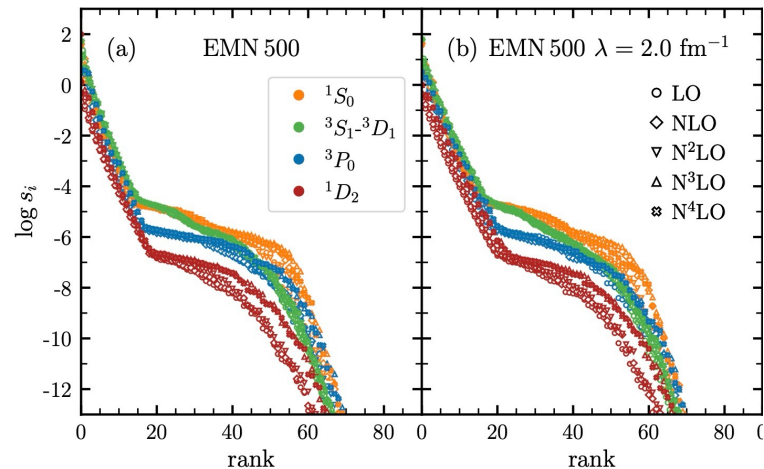
work of Tom Plies and Matthias Heinz, preliminary

use singular value decomposition (SVD)

as operator basis see Tichai et al., PLB (2021)

consider lowest 5 singular values/operators

effectively 3 generate phase shift variation

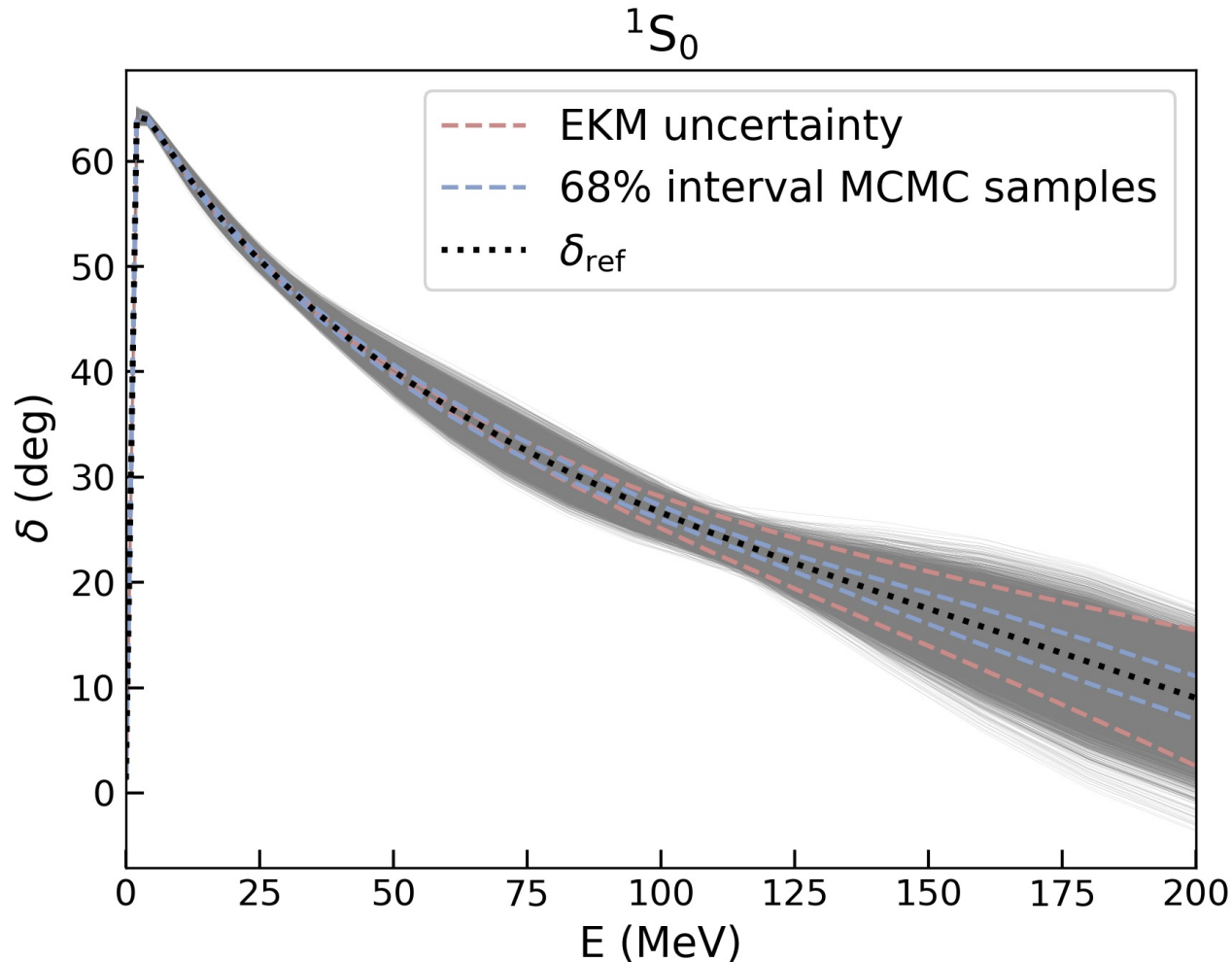




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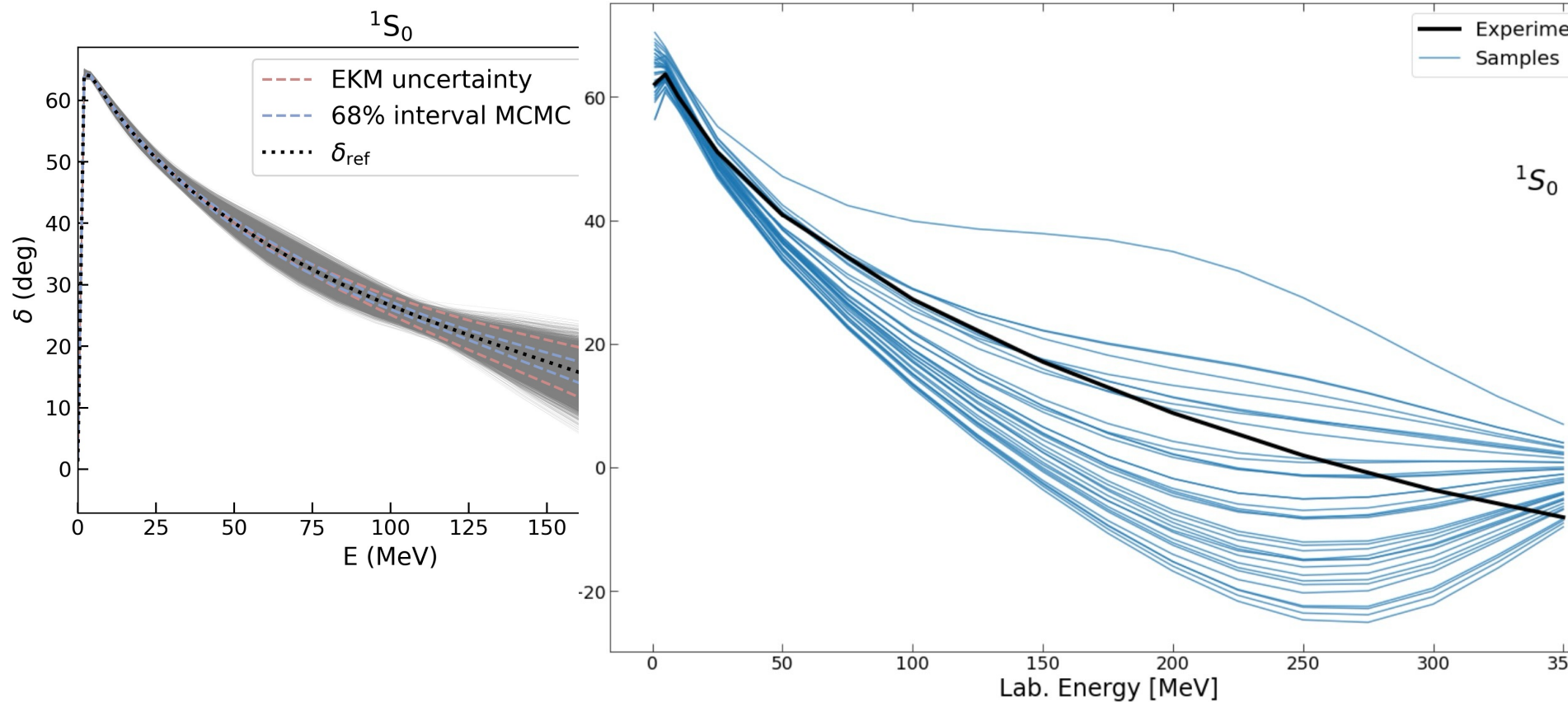
generate range of low-resolution NN interactions from random draws among 3 singular values with likelihood given by EKM uncertainties



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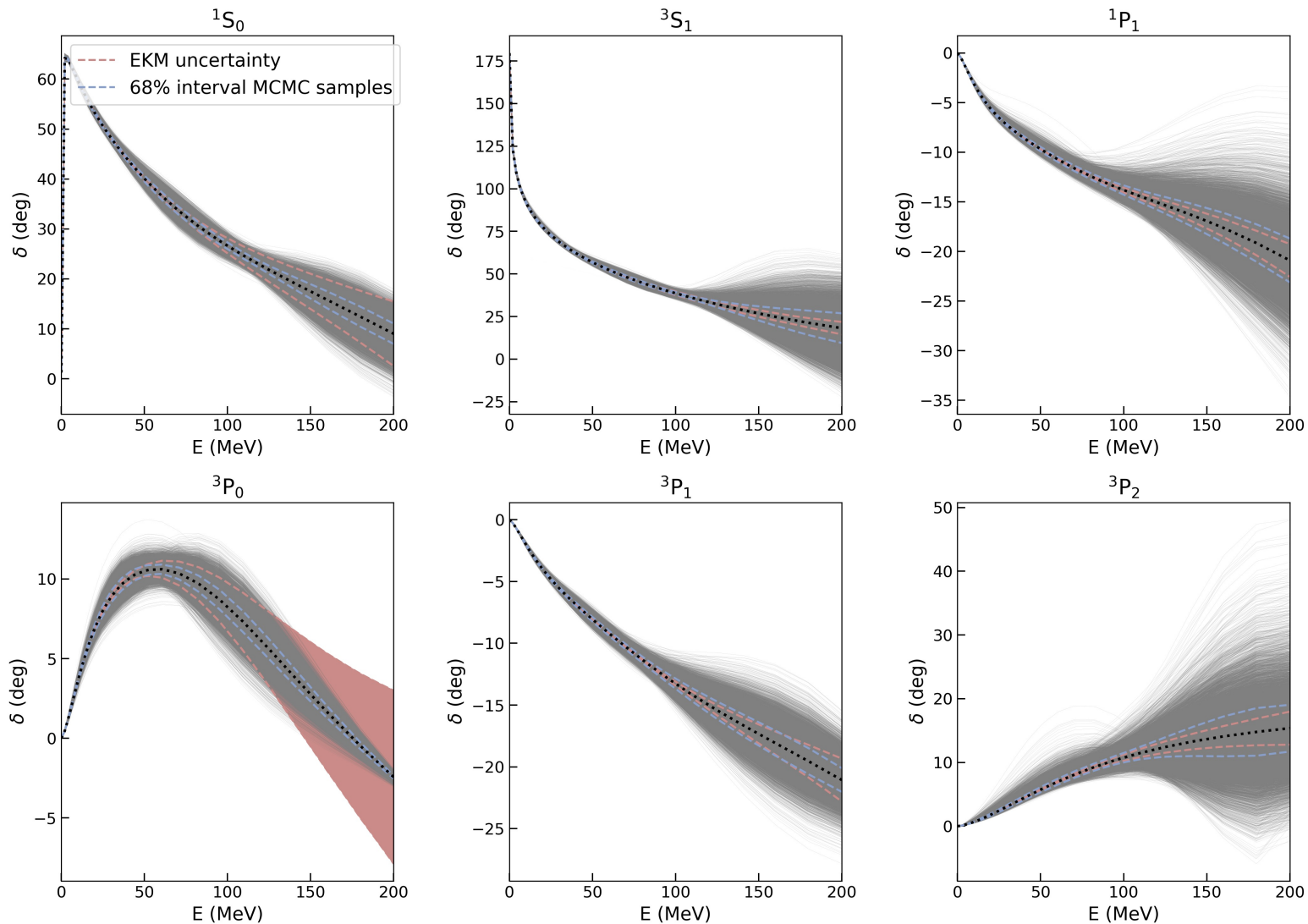
comparison to nonimplausible  $\Delta N^2\text{LO}$  interactions Ekström, Forssen et al.

# EFT uncertainties for SRG-evolved interactions

work of Tom Plies and Matthias Heinz, preliminary

generate range of low-resolution NN interactions

here: SRG-evolved EMN, S and P waves, higher partial waves unvaried



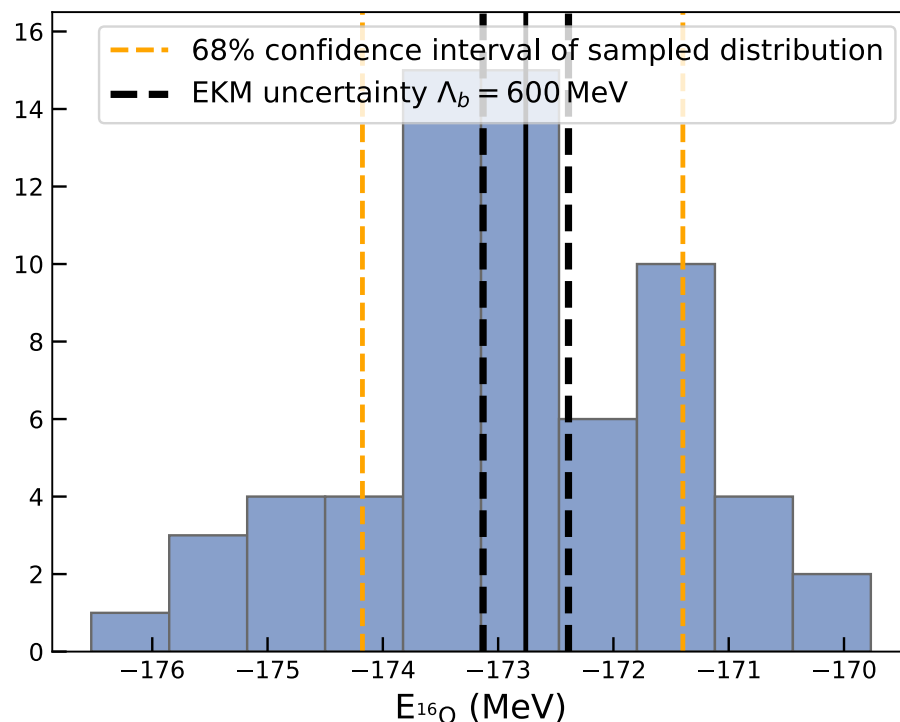
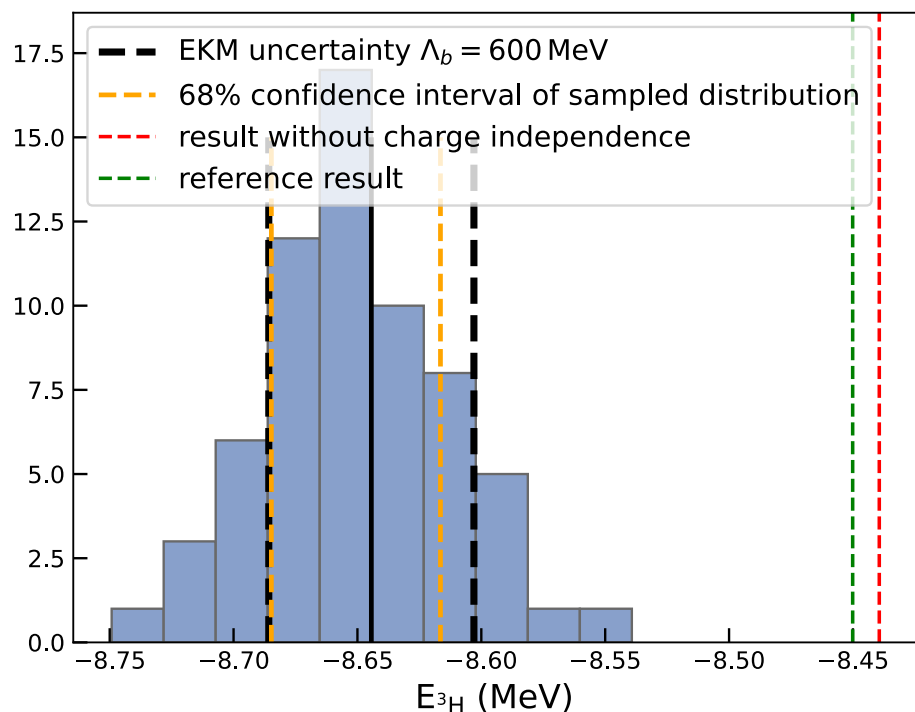
# EFT uncertainties for SRG-evolved interactions

work of Tom Plies and Matthias Heinz, preliminary

generate range of low-resolution NN interactions

here: SRG-evolved EMN, S and P waves, higher partial waves unvaried

resulting posterior distributions for  ${}^3\text{H}$  and  ${}^{16}\text{O}$  ground-state energies



${}^3\text{H}$  uncertainties agree well with EKM,  ${}^{16}\text{O}$  broader (good!)

next steps: improve sampling and include 3N interactions

# Isotope shifts in $^{168,170,172,174,176}\text{Yb}$

Door, Yeh, Heinz, Kirk, Lyu, Miyagi et al., arXiv:2403.07792

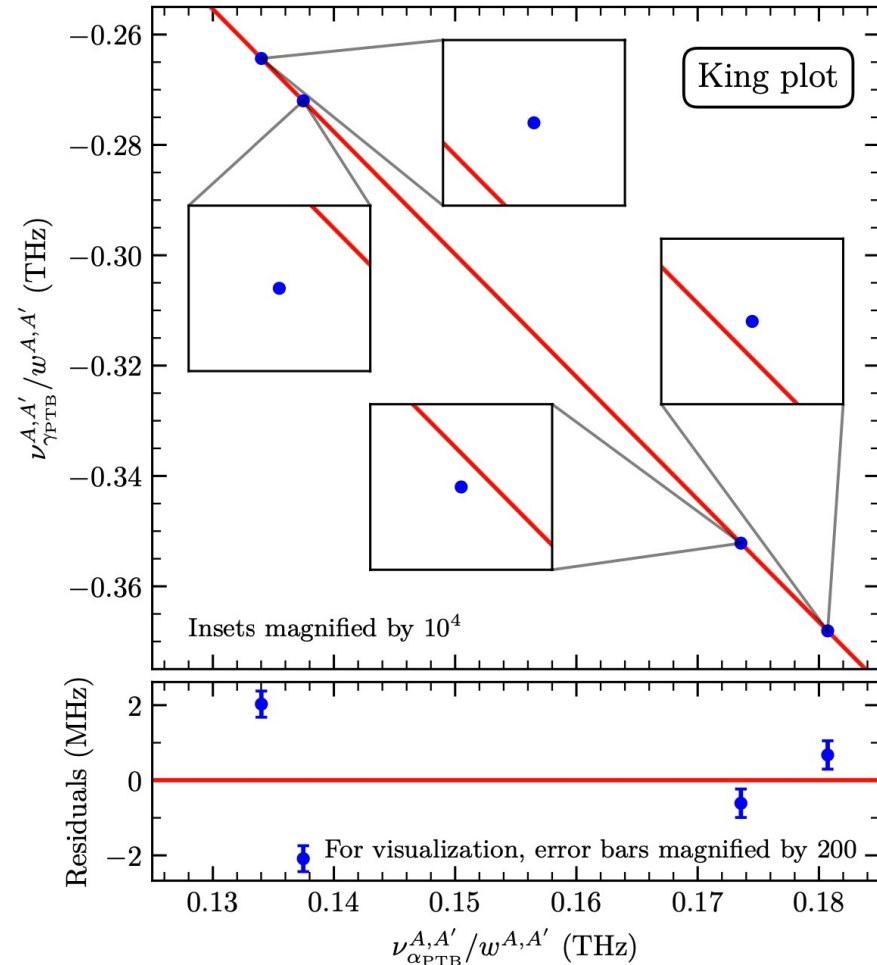
isotope shifts of atomic transitions

$$\nu_{\tau}^{A,A'} = \nu_{\tau}^A - \nu_{\tau}^{A'} \approx \underbrace{K_{\tau} w^{A,A'}}_{\text{mass shift}} + \underbrace{F_{\tau} \delta\langle r^2 \rangle^{A,A'}}_{\text{field shift}}$$

leading terms give linear King plot

nonlinearities from higher-order  
Standard Model and BSM

$$\nu_{\tau,\text{nonlin.}}^{A,A'} = \underbrace{G_{\tau}^{(2)} (\delta\langle r^2 \rangle^2)^{A,A'} + G_{\tau}^{(4)} \delta\langle r^4 \rangle^{A,A'}}_{\text{higher-order nuclear structure}} + \underbrace{\frac{\alpha_{\text{NP}}}{\alpha_{\text{EM}}} D_{\tau} h^{A,A'}}_{\text{possible new boson}} + \dots$$



laser spectroscopy (PTB) + Penning trap mass measurements (MPIK)  
show clear nonlinearities

# Isotope shifts in $^{168,170,172,174,176}\text{Yb}$

Door, Yeh, Heinz, Kirk, Lyu, Miyagi et al., arXiv:2403.07792

nonlinearity decomposition suggests one dominant contribution

$$\nu_{\tau, \text{nonlin.}}^{A, A'} = G_{\tau}^{(2)} (\delta \langle r^2 \rangle^2)^{A, A'} + G_{\tau}^{(4)} \delta \langle r^4 \rangle^{A, A'}$$

higher-order nuclear structure

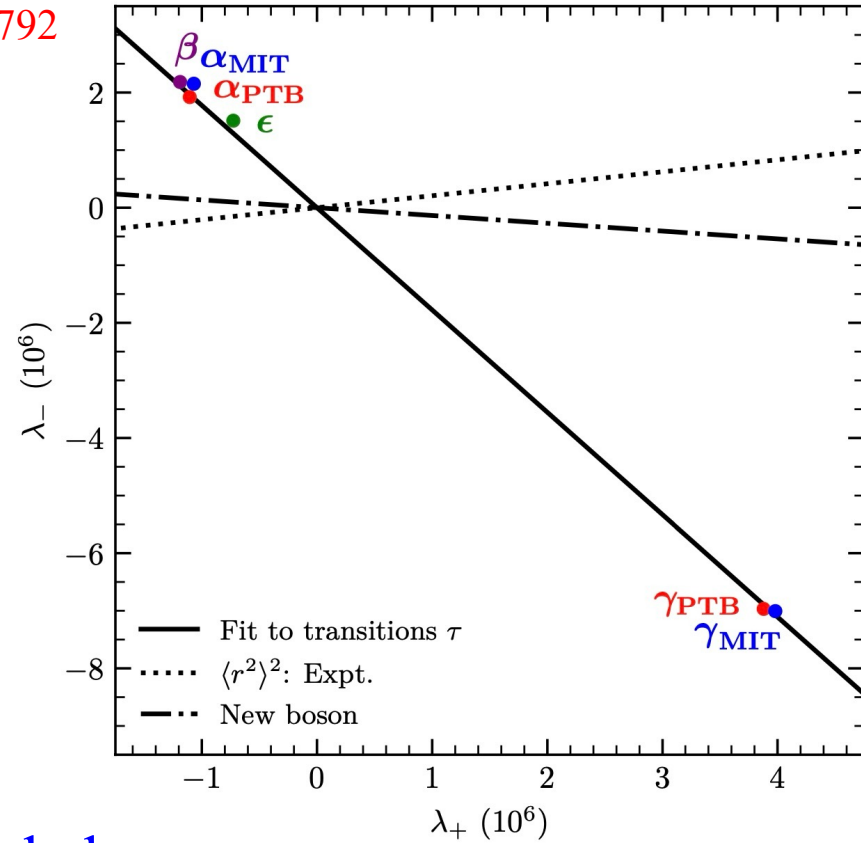
$$+ \frac{\alpha_{\text{NP}}}{\alpha_{\text{EM}}} D_{\tau} h^{A, A'} + \dots$$

possible new boson

→ not quadratic field shift

→ not new boson

theory predictions of quartic radius needed



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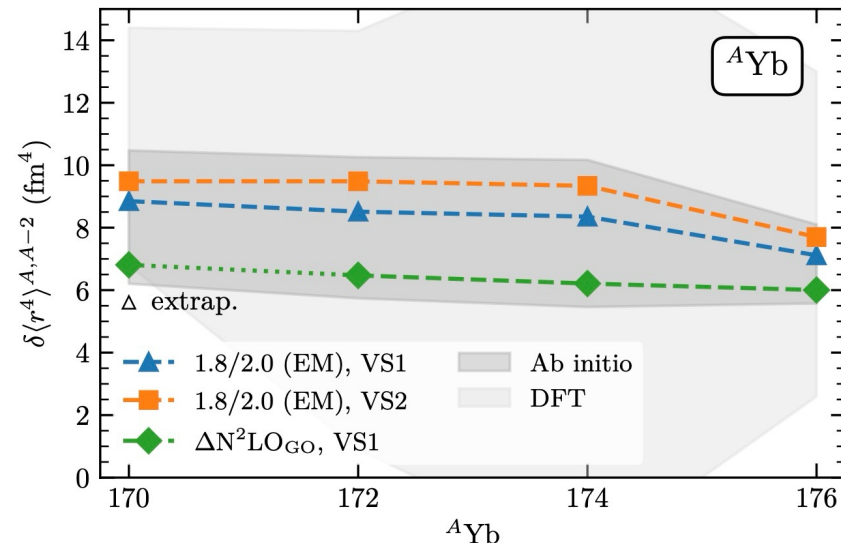
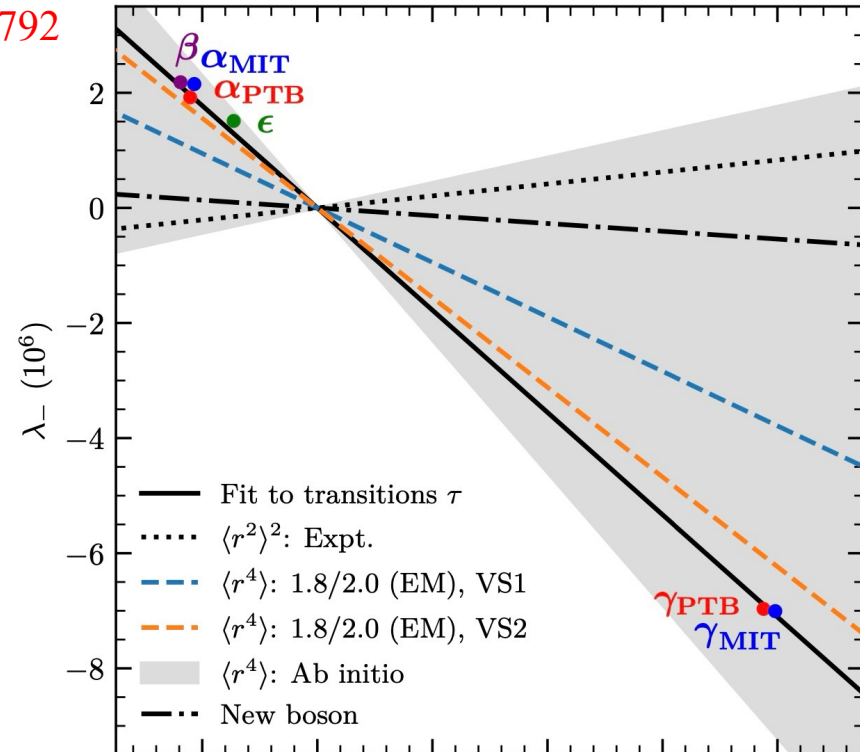
VS-IMSRG calculations,

uncertainty estimates from:

1.8/2.0 (EM),  $\Delta\text{N}^2\text{LO}_{\text{GO}}$  interactions,

two valence spaces,

many-body estimated from IMSRG(3)



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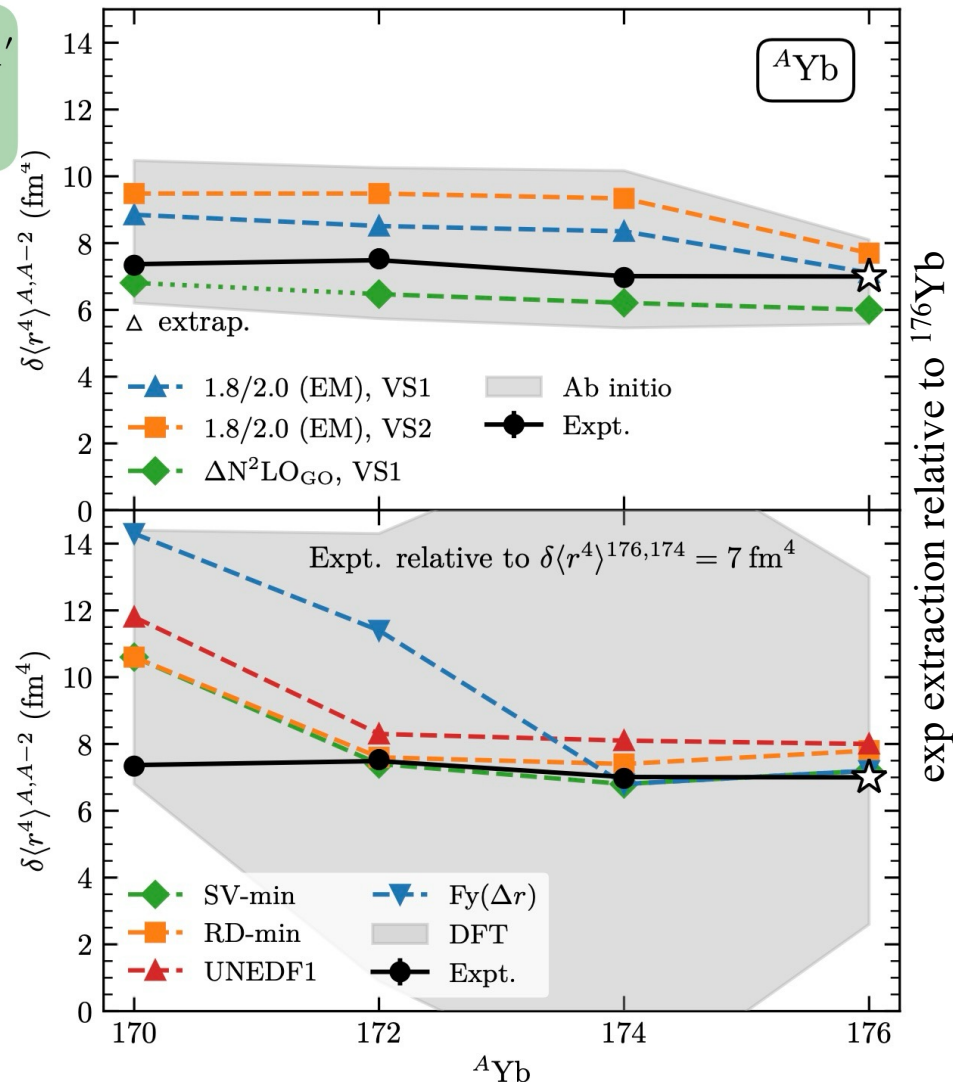
higher-order nuclear structure

$$+ \frac{\alpha_{\text{NP}}}{\alpha_{\text{EM}}} D_{\tau} h^{A, A'} + \dots$$

possible new boson

- not quadratic field shift
- not new boson
- dominant nonlinearity from quartic radius term

NEW: extract quartic radius from experimental data:  
observable related to deformation, trends consistent with ab initio





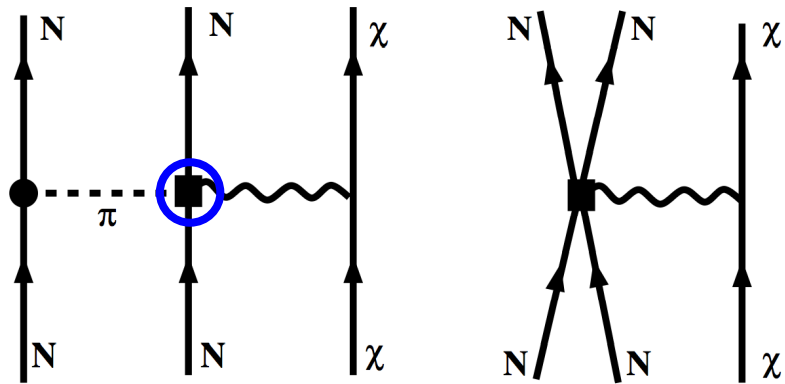
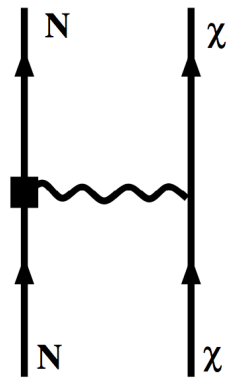
# Chiral EFT for coupling to electroweak interactions

axial-vector currents (beta decays)  
one-body currents at  $Q^0$  and  $Q^2$

	NN	3N	4N
LO $\mathcal{O}\left(\frac{Q^0}{\Lambda^0}\right)$			
NLO $\mathcal{O}\left(\frac{Q^2}{\Lambda^2}\right)$			
N <sup>2</sup> LO $\mathcal{O}\left(\frac{Q^3}{\Lambda^3}\right)$			
N <sup>3</sup> LO $\mathcal{O}\left(\frac{Q^4}{\Lambda^4}\right)$			
	+ ... (2011) ...	+ ... (2006) ...	+ ... (2006) ...

derived in (1994/2002)

+ two-body currents at  $Q^3$



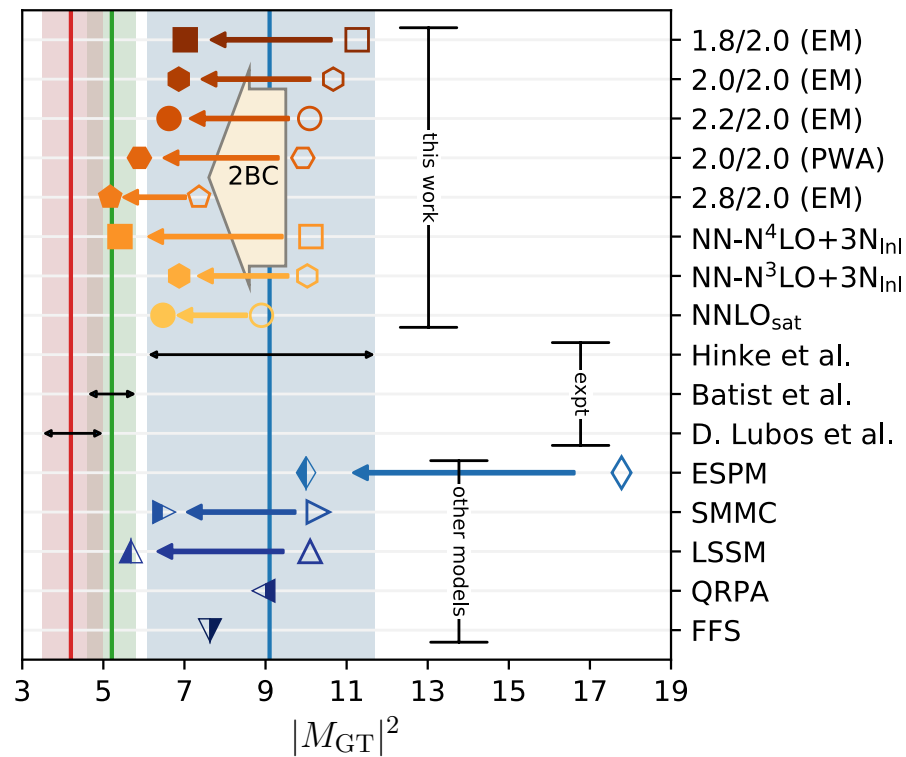
same couplings in forces and currents

# Chiral EFT for coupling to electroweak interactions

consistent electroweak one- and two-body currents

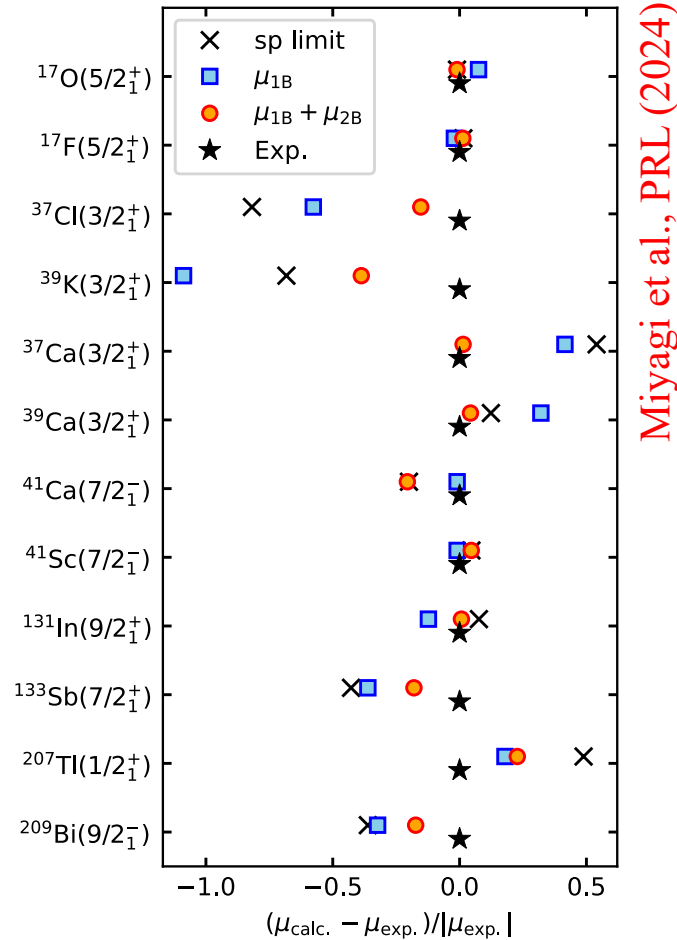
## Gamow-Teller beta decay of $^{100}\text{Sn}$

Gysbers et al., Nature Phys. (2019)



## Magnetic moments of nuclei

Pastore et al. (2012-)



two-body currents (2BC) key for quenching puzzle of beta decays

+ always improve magnetic moments

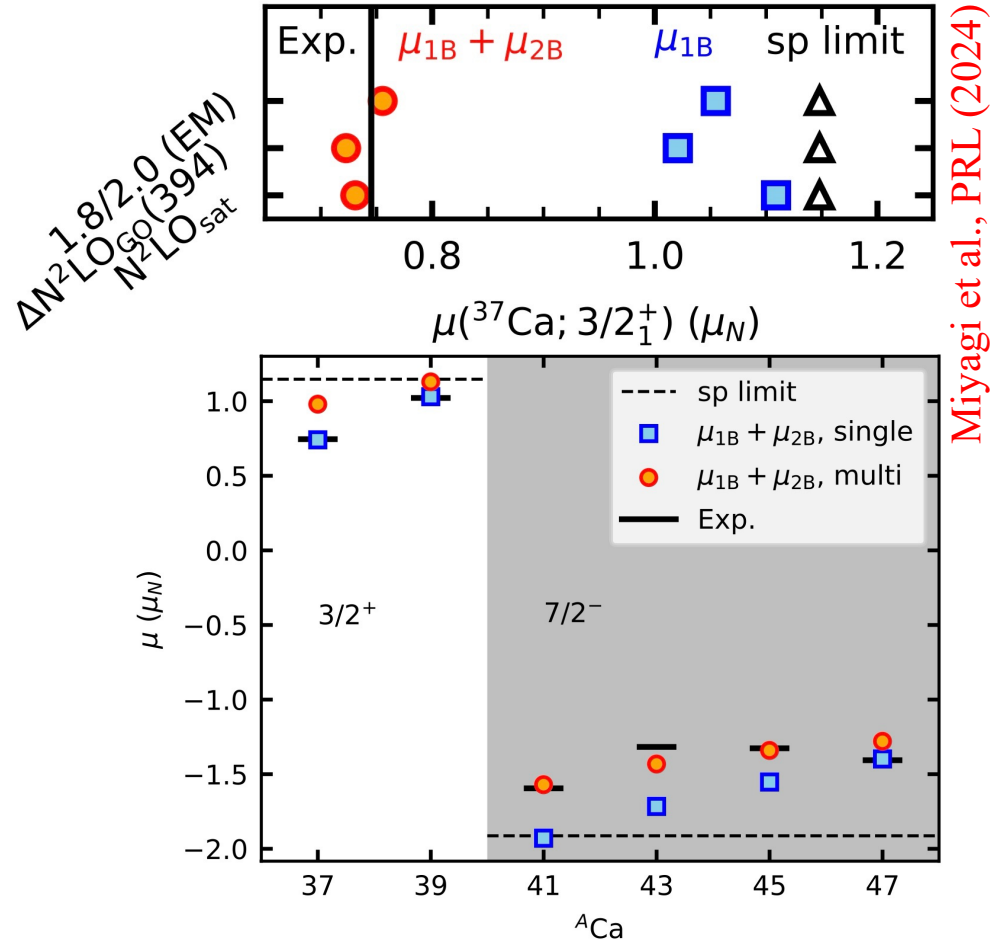
# Chiral EFT for coupling to electroweak interactions

consistent electroweak one- and two-body currents

## Magnetic moments of nuclei

Pastore et al. (2012-)

Interaction and many-body uncertainties explored, but need to work on order-by-order uncertainties

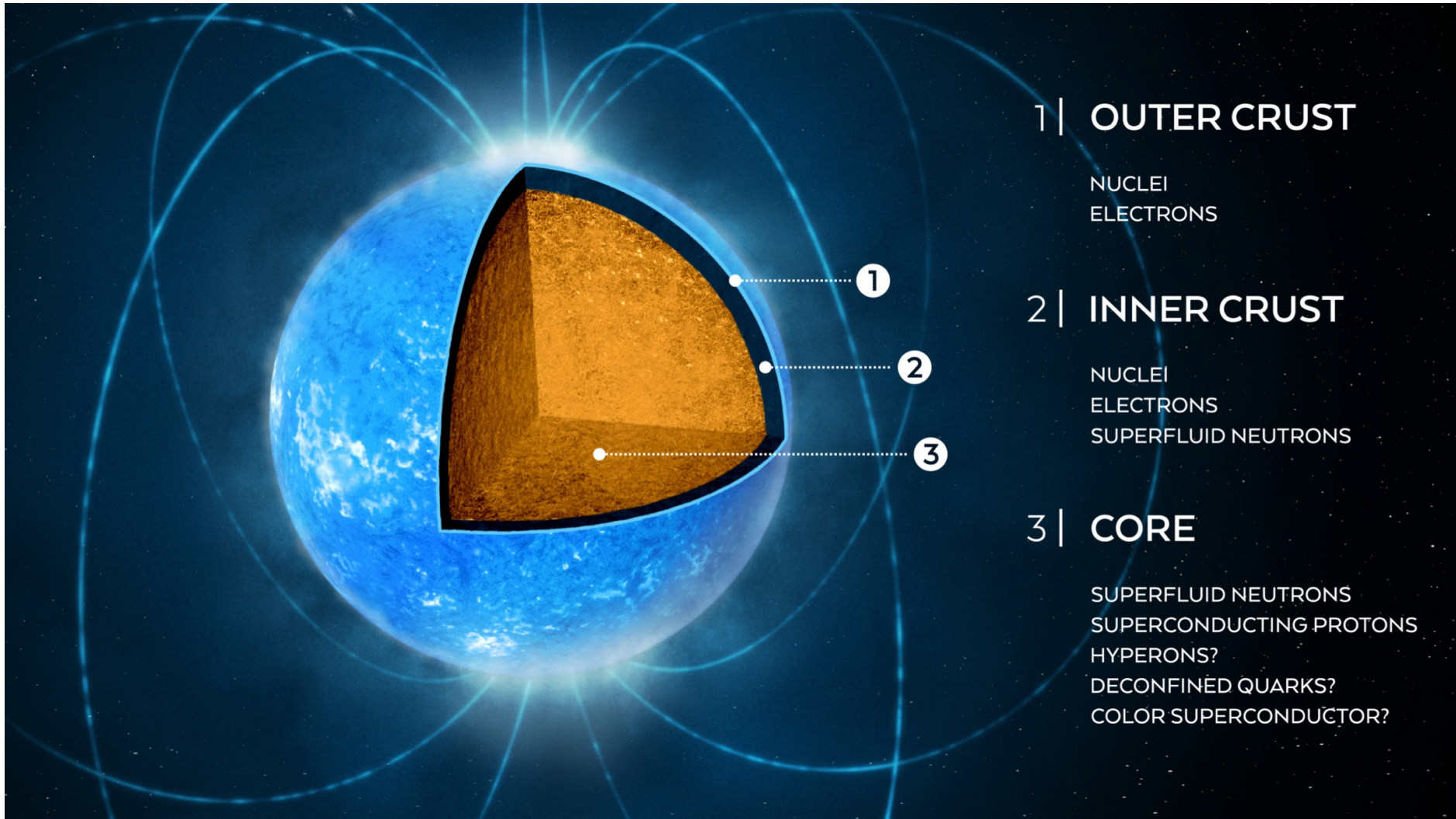


Miyagi et al., PRL (2024)

two-body currents (2BC) key for quenching puzzle of beta decays + always improve magnetic moments

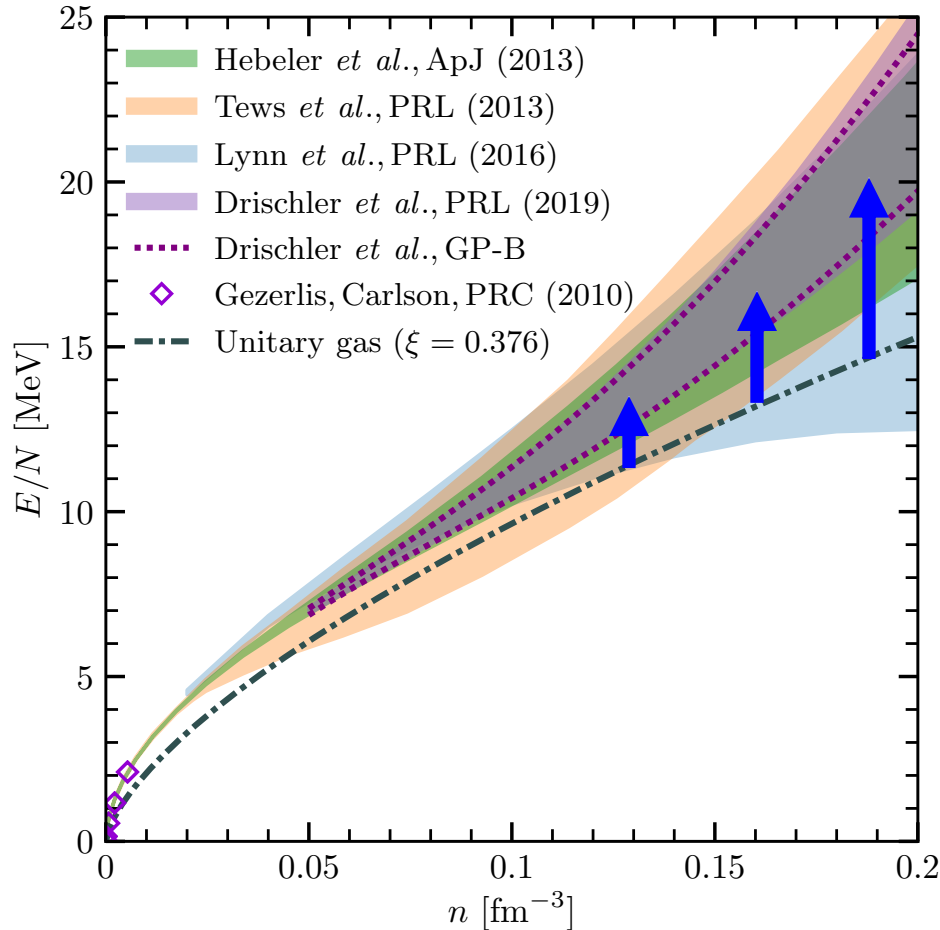
# Extreme matter in neutron stars

governed by the same strong interactions:  
chiral EFT sets pressure of first few km to inside



# Chiral EFT calculations of neutron matter

good agreement up to saturation density for neutron matter  
including NN, 3N, 4N interactions up to N<sup>3</sup>LO



comparison from Huth *et al.*, PRC (2021)

slope determines pressure of  
neutron matter

comparison to unitary Fermi gas  
measured with cold atoms

behavior very similar to  $0.1 \text{ fm}^{-3}$   
because neutrons have large  
scattering length  $a_s = -18.5 \text{ fm}$

stronger increase towards higher  
densities (EOS becomes stiffer)  
due to repulsive 3N forces

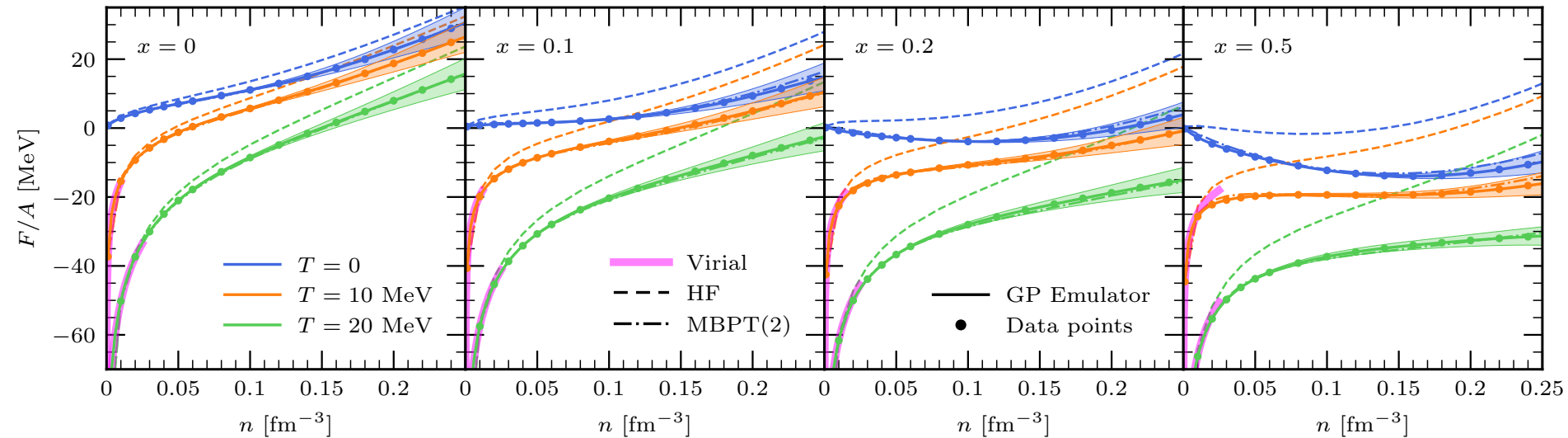
# EOS for arbitrary proton fraction and temperature

Keller, Hebeler, AS, PRL (2023)

based on chiral EFT NN+3N interactions to N<sup>3</sup>LO

order-by-order EFT uncertainties + (small) many-body uncertainties

see Hannah Götting's talk for Bayesian EFT uncertainties



excellent reproduction of free energy data by Gaussian process (GP)

agrees with model-indep. virial EOS Horowitz, AS, NPA (2006) at low densities

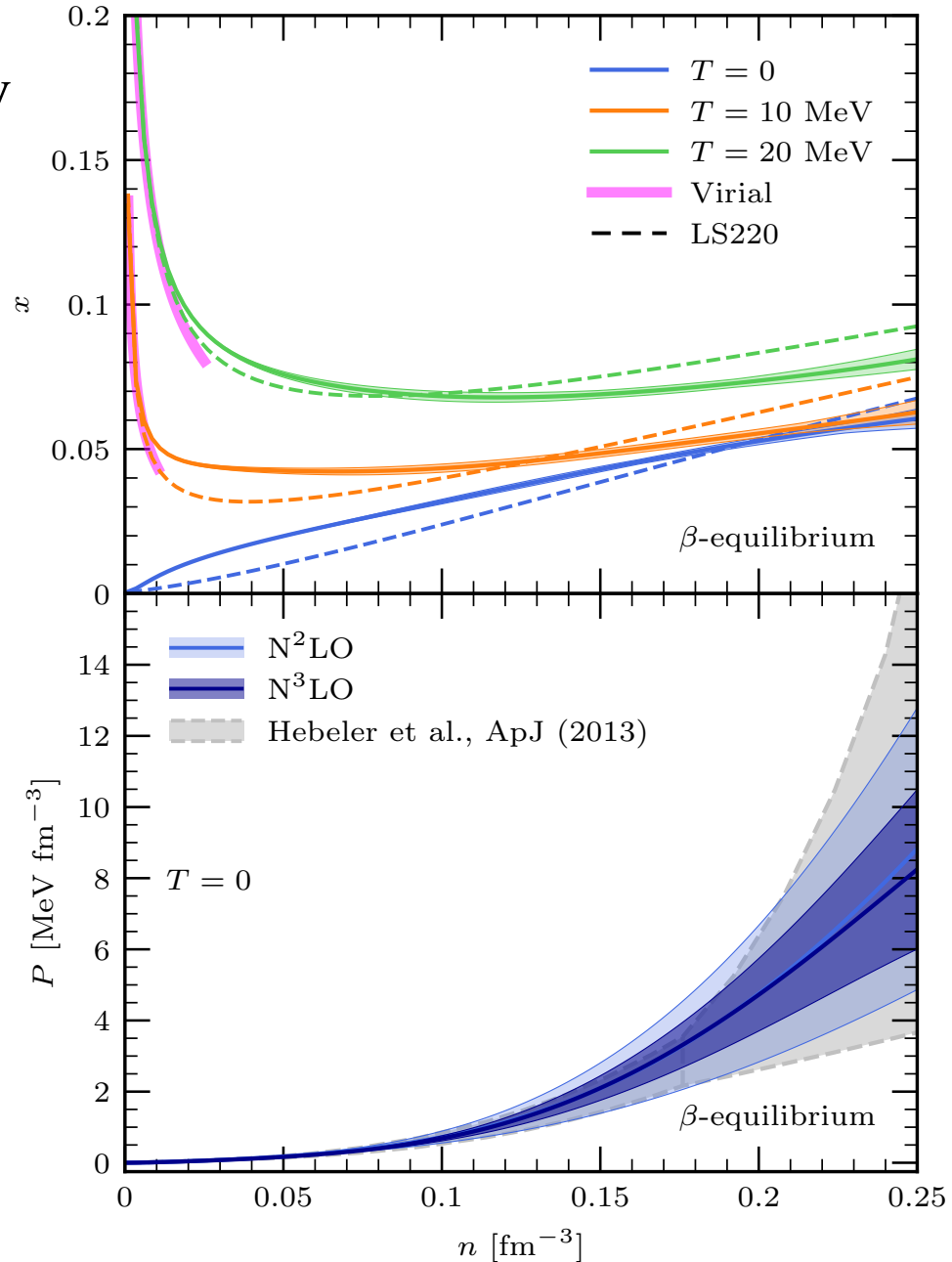
# EOS for neutron star matter in beta equilibrium

Keller, Hebeler, AS, PRL (2023)

use GP emulator to access arbitrary  
proton fraction,  
solve for beta equilibrium

EOS of neutron star matter  
at  $N^2\text{LO}$  and  $N^3\text{LO}$ ,  
no indication of EFT breakdown

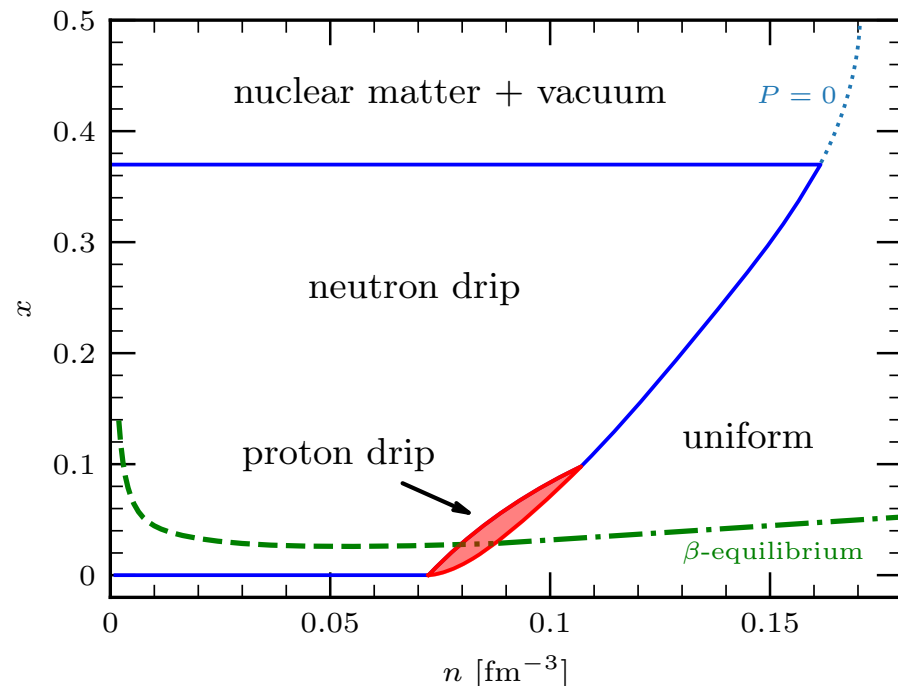
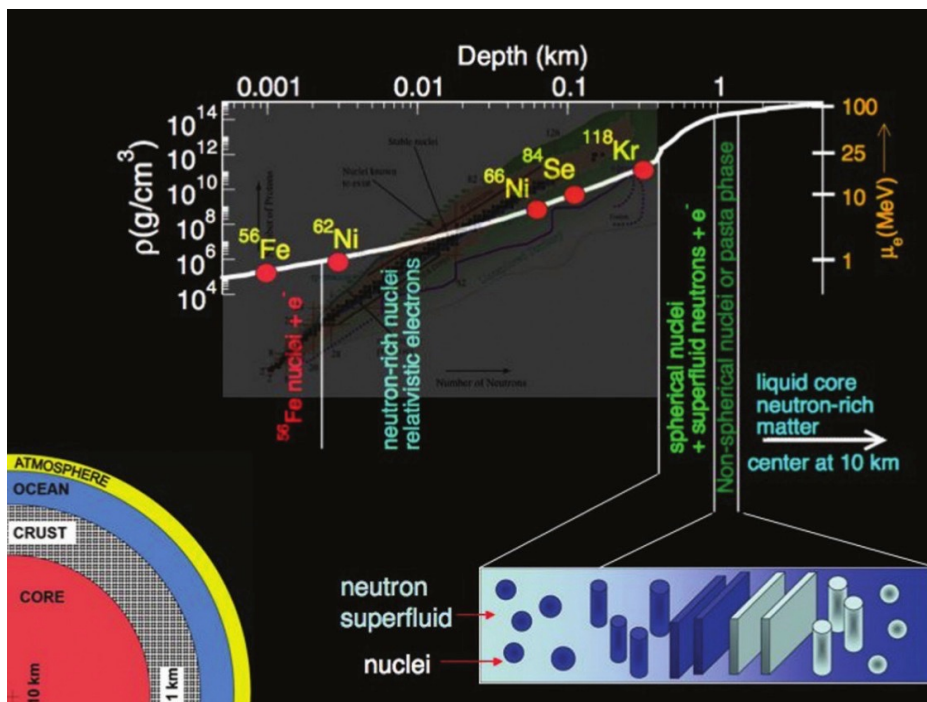
$N^3\text{LO}$  band prefers higher  
pressures, improvement over  
older calculations



# Subnuclear phase diagram of neutron star matter

~ 5% proton fraction in denser neutron matter

below  $\sim 0.5 n_0$  possible pasta phases: clusters/structures of high density surrounded by neutron (and proton) gas: neutron (proton) drip



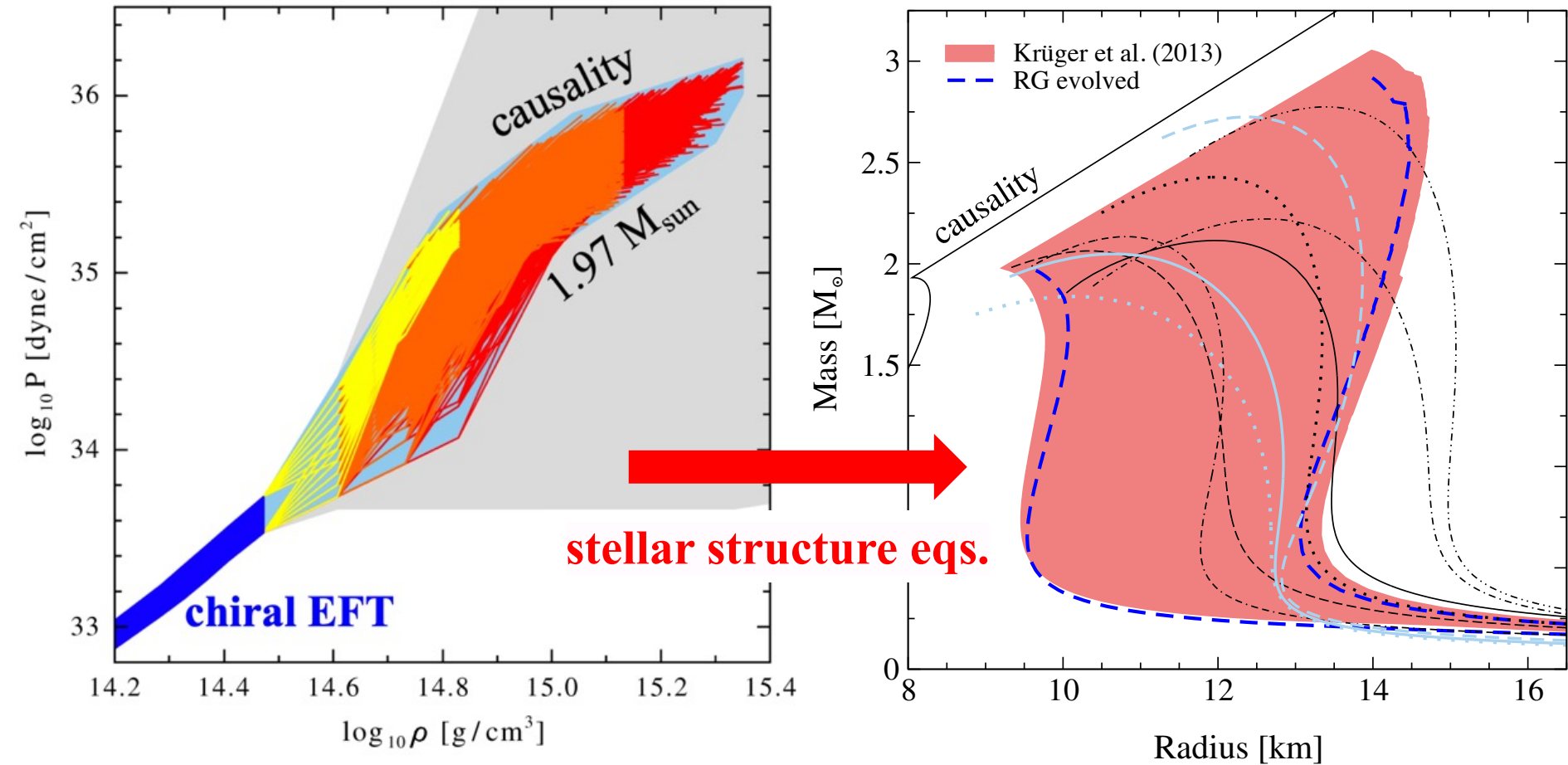
Keller, Hebeler, Pethick, AS, PRL (2024)

Challenging problem for uncertainty quantification due to correlated quantities, derivatives with larger mb uncertainties,...



# Impact on neutron stars Hebeler, Lattimer, Pethick, AS, PRL (2010), ApJ (2013)

constrain high-density EOS by causality, require to support  $2 M_{\text{sun}}$  star

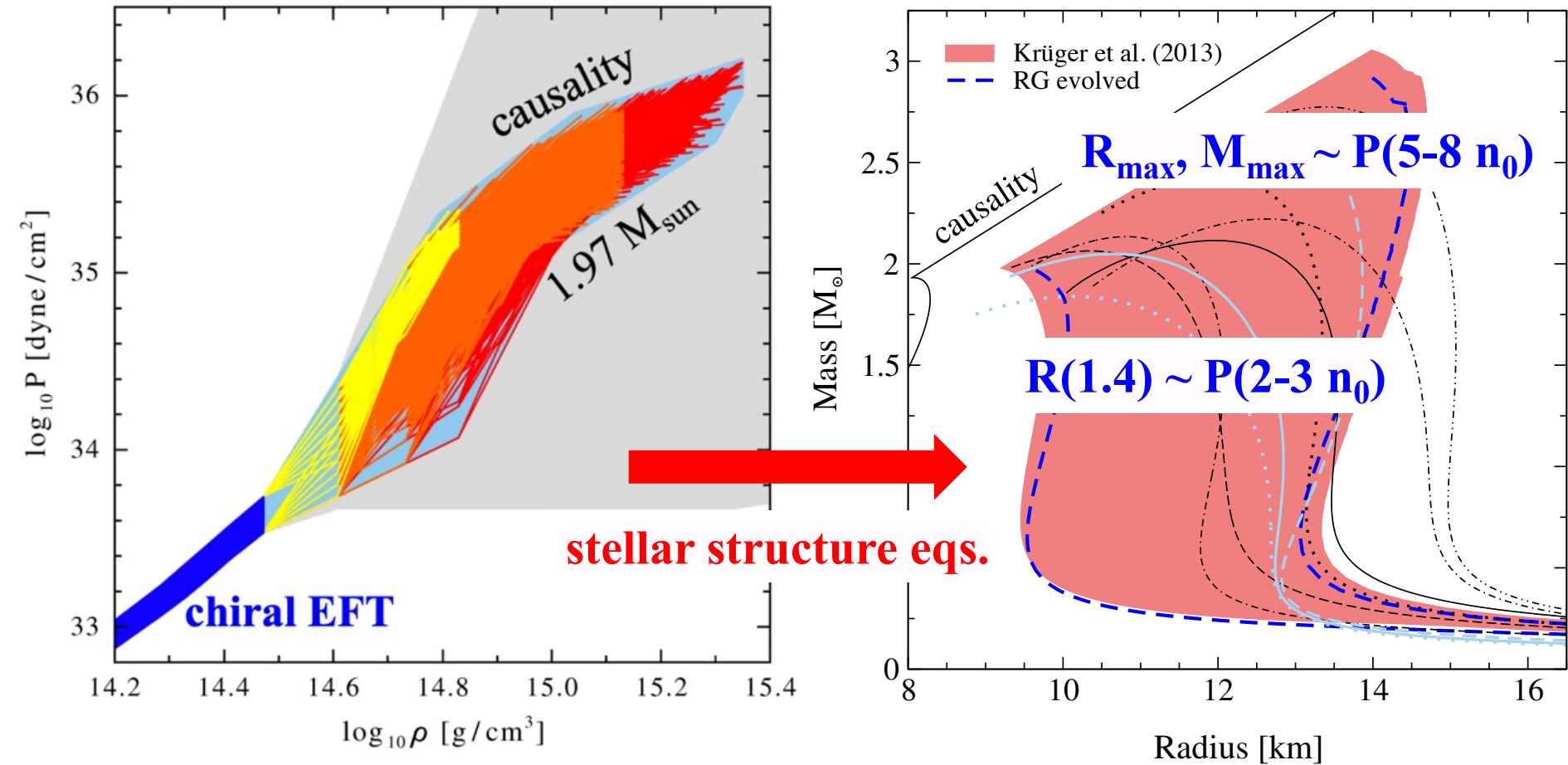


predicts neutron star radius: 9.7 - 13.9 km for  $M=1.4 M_{\text{sun}}$

1.8 - 4.4  $n_0$  modest central densities

# Impact on neutron stars Hebeler, Lattimer, Pethick, AS, PRL (2010), ApJ (2013)

constrain high-density EOS by causality, require to support  $2 M_{\text{sun}}$  star



stellar structure eqs.

predicts neutron star radius: 9.7 - 13.9 km for  $M=1.4 M_{\text{sun}}$

1.8 - 4.4  $n_0$  modest central densities

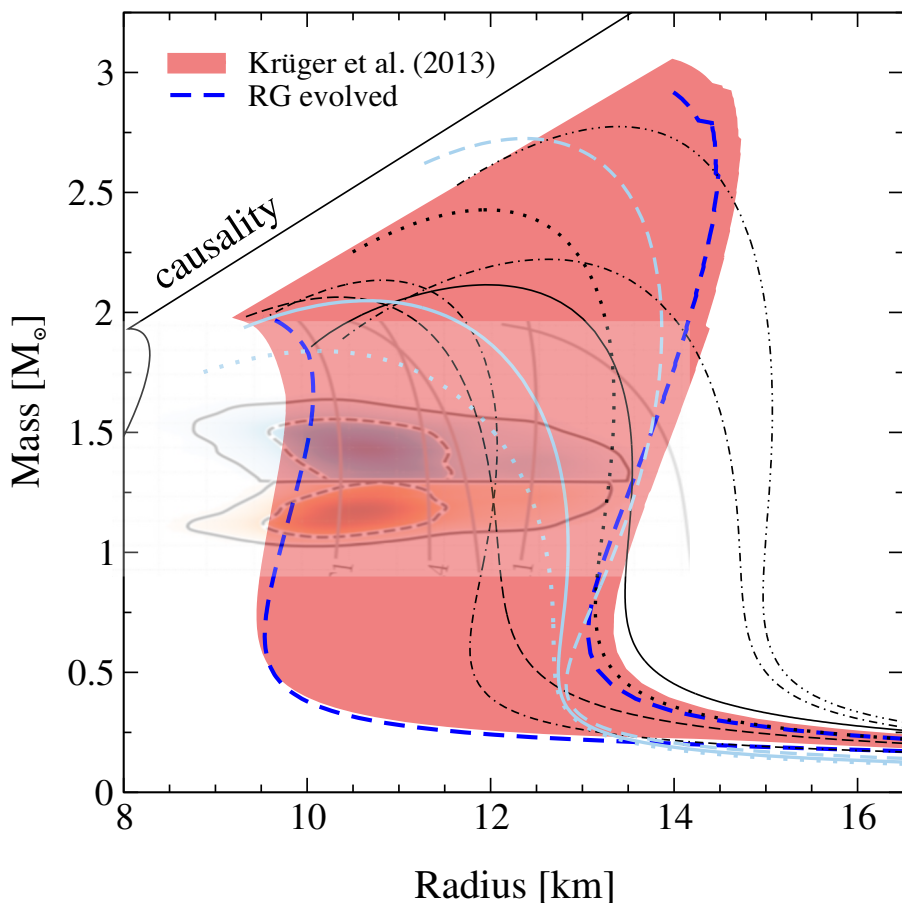
# Neutron star radius

chiral EFT + general EOS extrapolation based on causality +  $2 M_{\text{sun}}$  stars

predicted neutron star radii: **9.7 - 13.9 km** for  $M=1.4 M_{\text{sun}}$

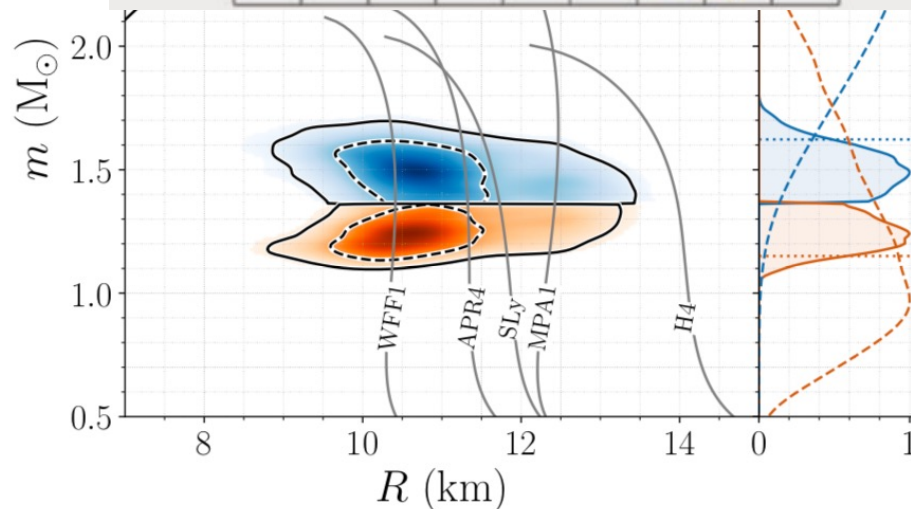
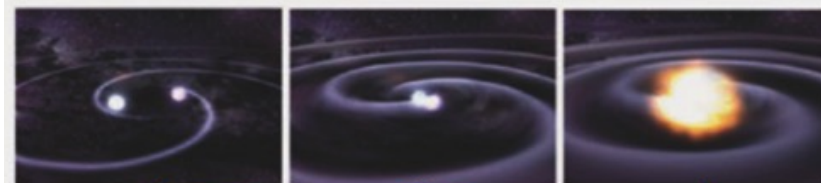
Hebeler, Lattimer, Pethick, AS, PRL (2010), ApJ (2013)

consistent with LIGO/Virgo

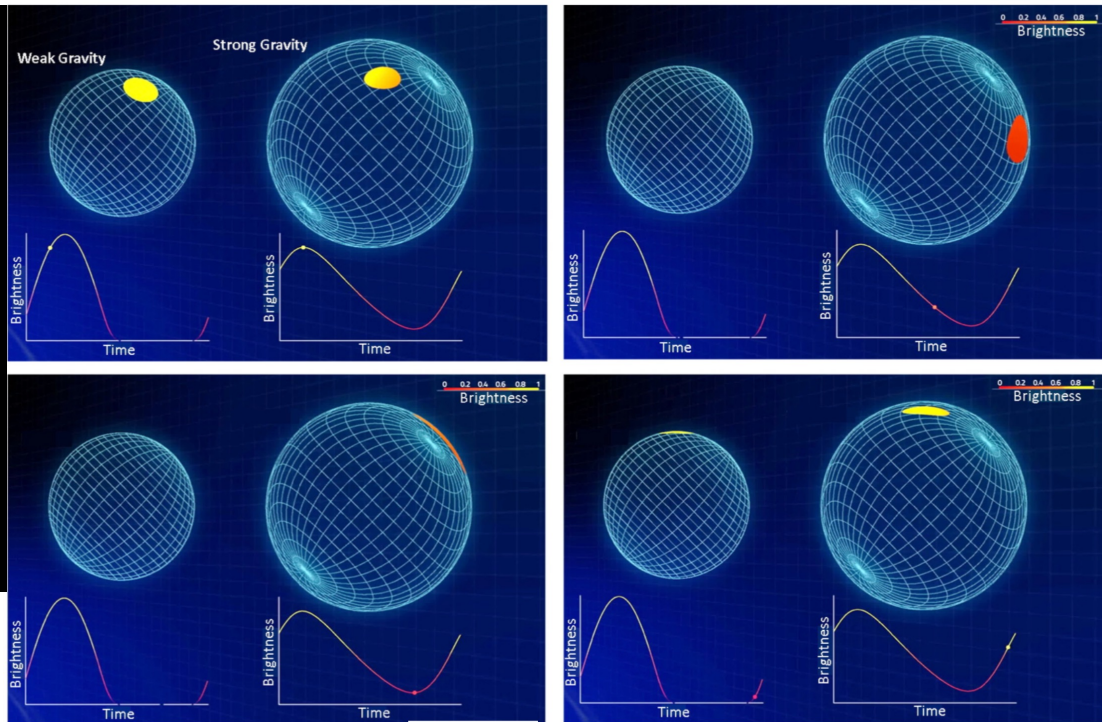
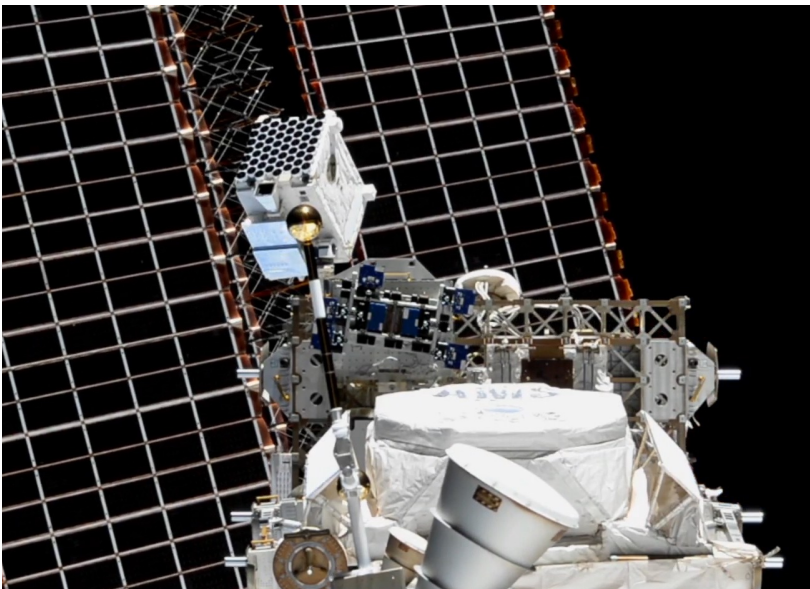


GW170817: Measurements of neutron star radii and equation of state

The LIGO Scientific Collaboration and The Virgo Collaboration



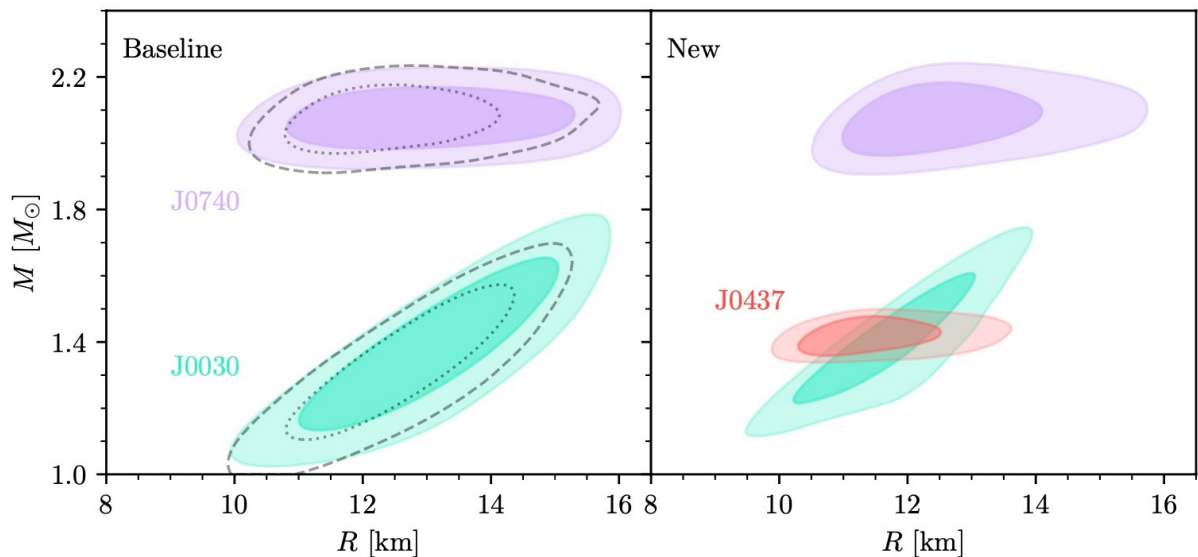
# New NICER results for J0437



Neutron star radius from pulse profile modeling

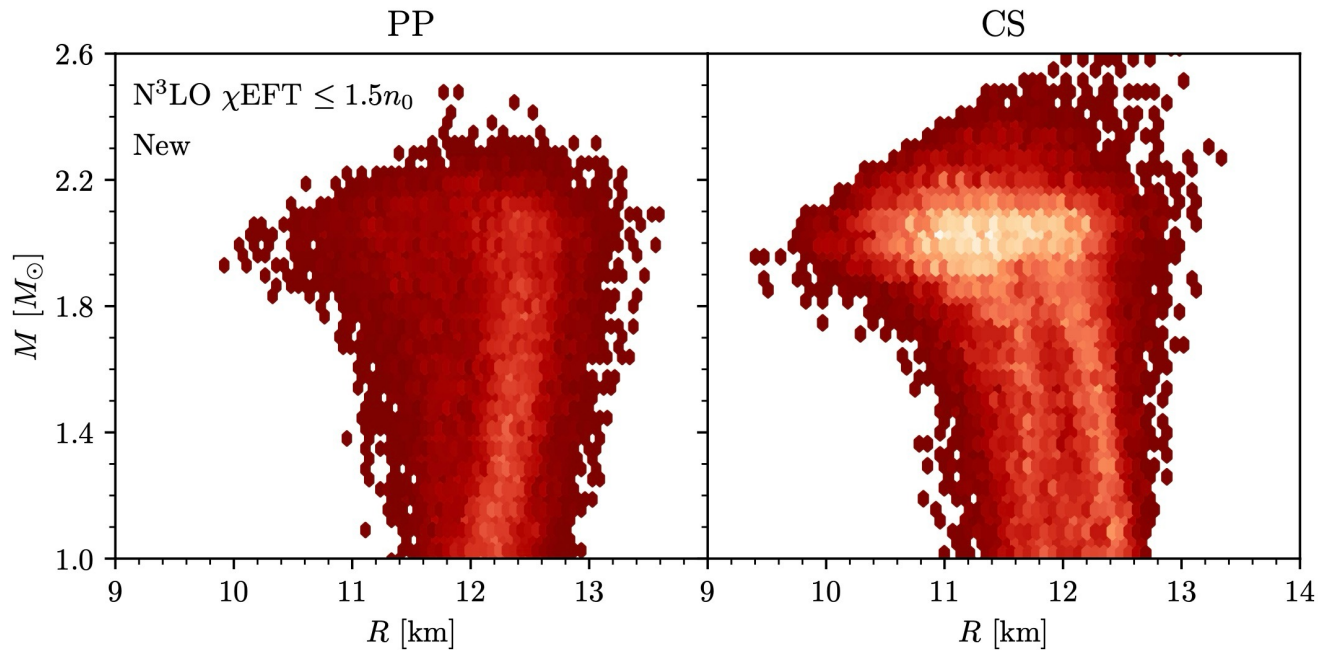
here: Amsterdam analysis  
*ApJL* (2019), (2021), submitted (2024)

J0030 and J0740  
similar results from  
Illinois-Maryland analysis  
*ApJL* (2019), (2021)

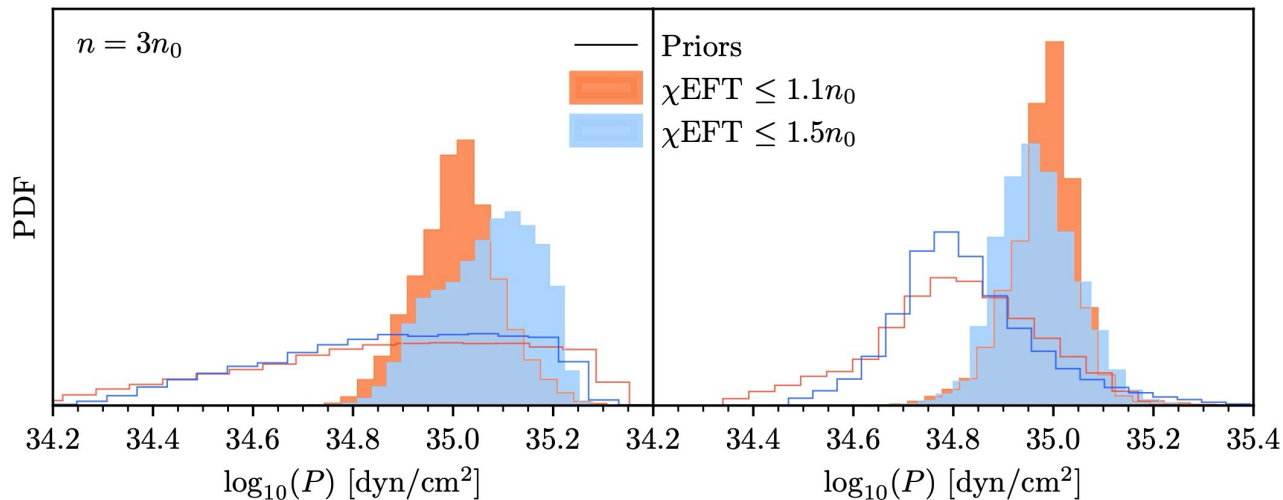


# Combined LIGO/Virgo and NICER constraints

Raaijmakers et al., ApJL (2020), (2021), Rutherford, Mendes, Svensson et al., submitted (2024)



neutron star radius  
 $\sim 12 \pm 1$  km



pressure posteriors  
at  $3 n_0$   
 $\rightarrow$  astro prefers  
higher pressures

# Summary

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Chiral EFT interactions + powerful many-body methods

→ great progress for ab initio calcs of nuclei and dense matter

reliable EOS up to  $\sim 1-2 n_0$  with controlled uncertainties

→ key for multimessenger era of neutron stars,  
high-density constraints from astrophysics

## Discussion points

Interaction uncertainties have focused on EFT truncation.

What about other systematic choices (such as regulators, fit choices,...)?

Status of  $A_y$  puzzle. Is this solved with EFT uncertainties?

Suite of few- and many-body observables for interaction optimization?

Many-body uncertainties beyond expert assessment.

More systematic exploration/development of accurate  $NN+3N$  needed.

EFT uncertainties for electroweak matrix elements/responses with external momentum.





# Impact of PREX and $^{208}\text{Pb}$ dipole polarizability

Essick, Landry, AS, Tews, PRL, PRC (2021)

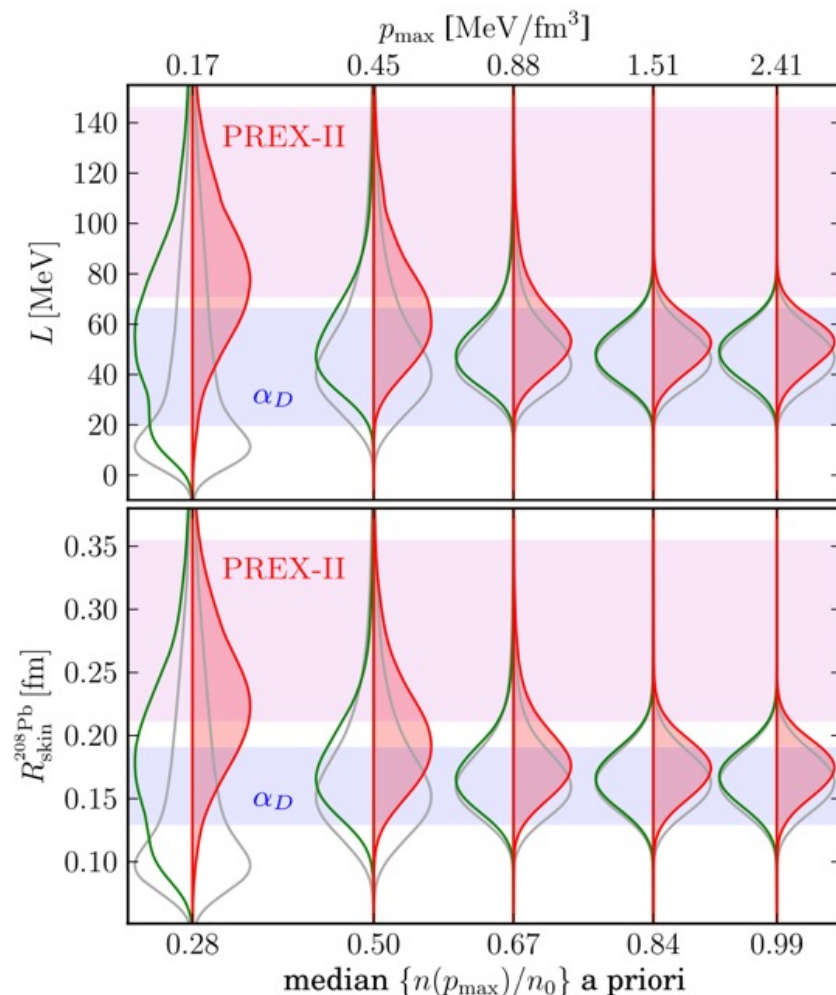
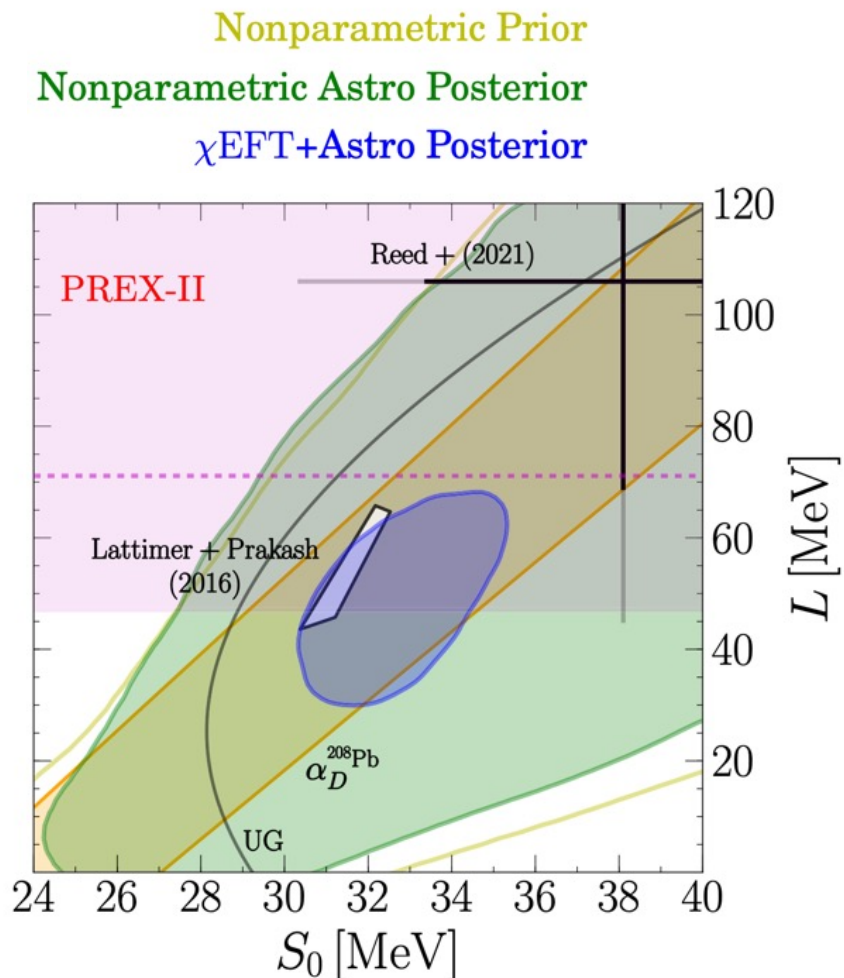


FIG. 2. Prior (gray, unshaded), Astro posterior (green, left-unshaded), and Astro + PREX-II posterior (red, right-shaded)

$^{208}\text{Pb}$  dipole polarizability Tamii et al., PRL (2021)

very consistent with  $\chi\text{EFT+Astro}$  posterior