Recent progress of chiral three-nucleon forces and applications to nuclei and matter

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Outline

Uncertainties related to chiral three-nucleon interactions:

I. Chiral expansion and different regularization schemes

II. Fixing of low-energy couplings + SRG evolution

III. Inclusion of 3NFs in many-body calculations

3NFs in different regularization schemes

	momentum space	coordinate space
nonlocal	**<u>nonlocal MS</u>**	
regulators:		
long-range	$f_\Lambda^{\rm long}(\mathbf{p},\mathbf{q}) = \exp\left[-((\mathbf{p}^2 + 3/4\,\mathbf{q}^2)/\Lambda^2)^n\right]$	
short-range	$f_\Lambda^{\rm short}(\mathbf{p},\mathbf{q}) = f_\Lambda^{\rm long}(\mathbf{p},\mathbf{q}) = f_R(\mathbf{p},\mathbf{q})$	
regularization:	$\left\langle \mathbf{p}'\mathbf{q}'\vert V_{3N}^{\rm reg}\vert\mathbf{p}\mathbf{q}\right\rangle = f_R(\mathbf{p}',\mathbf{q}')\,\langle \mathbf{p}'\mathbf{q}'\vert V_{3N}\vert\mathbf{p}\mathbf{q}\rangle\, f_R(\mathbf{p},\mathbf{q})$	
local	**<u>local MS</u>**	**<u>local CS</u>**
regulators:		
long-range	$f_\Lambda^{\rm long}(\mathbf{Q}_i) = \exp\left[-(\mathbf{Q}_i^2/\Lambda^2)^2\right]$	$f_R^{\rm long}(\mathbf{r}) = 1 - \exp\left[-(r^2/R^2)^n\right]$
short-range	$f_\Lambda^{\rm short}(\mathbf{Q}_i) = f_\Lambda^{\rm long}(\mathbf{Q}_i) = f_\Lambda(\mathbf{Q}_i)$	$f_R^{\rm short}(\mathbf{r}) = \exp\left[-(r^2/R^2)^n\right]$
regularization:	$\left\langle \mathbf{p}'\mathbf{q}'\vert V_{3N}^{\rm reg}\vert\mathbf{p}\mathbf{q}\right\rangle = \langle \mathbf{p}'\mathbf{q}'\vert V_{3N}\vert\mathbf{p}\mathbf{q}\rangle \prod_i f_R(\mathbf{Q}_i)$	$V_{3N}^{\pi,\rm reg}(\mathbf{r}_{ij}) = f_R^{\rm long}(\mathbf{r}_{ij})V_{3N}^{\pi}(\mathbf{r}_{ij})$ $\delta^{\rm reg}(\mathbf{r}_{ij}) = \alpha_n f_R^{\rm short}(\mathbf{r}_{ij})$
semilocal	**<u>semilocal MS</u>**	**<u>semilocal CS</u>**
regulators:		
long-range	$f_\Lambda^{\rm long}(\mathbf{Q}_i) = \exp\left[-(\mathbf{Q}_i^2 + m_\pi^2)/\Lambda^2\right]$	$f_R^{\rm long}(\mathbf{r}) = (1 - \exp\left[-r^2/R^2\right])^n$
short-range	$f_\Lambda^{\rm short}(\mathbf{p}) = \exp\left[-\mathbf{p}^2/\Lambda^2\right]$	$f_\Lambda^{\rm short}(\mathbf{p}) = \exp\left[-\mathbf{p}^2/\Lambda^2\right]$
regularization:	$\left\langle \mathbf{p}'\mathbf{q}'\vert V_{3N}^{{\rm reg},\pi}\vert\mathbf{p}\mathbf{q}\right\rangle = f_R^{\rm long}(\mathbf{Q}_i)\,\langle \mathbf{p}'\mathbf{q}'\vert V_{3N}\vert\mathbf{p}\mathbf{q}\rangle$ $\left\langle \mathbf{p}'\mathbf{q}'\vert V_{3N}^{{\rm reg},\delta}\vert\mathbf{p}\mathbf{q}\right\rangle = f_\Lambda^{\rm short}(\mathbf{p}'_\delta)\left\langle \mathbf{p}'\mathbf{q}'\vert V_{3N}^{\delta}\vert\mathbf{p}\mathbf{q}\right\rangle f_\Lambda^{\rm short}(\mathbf{p}_\delta)$	$V_{3N}^{\pi,\rm reg}(\mathbf{r}_{ij}) = f_R^{\rm long}(\mathbf{r}_{ij})V_{3N}^{\pi}(\mathbf{r}_{ij})$ $\delta(\mathbf{r}_{ij}) \xrightarrow{FT} V_{3N}^{\delta}$ $\left\langle \mathbf{p}'\mathbf{q}'\vert V_{3N}^{{\rm reg},\delta}\vert\mathbf{p}\mathbf{q}\right\rangle = f_\Lambda^{\rm short}(\mathbf{p}'_\delta)\left\langle \mathbf{p}'\mathbf{q}'\vert V_{3N}^{\delta}\vert\mathbf{p}\mathbf{q}\right\rangle f_\Lambda^{\rm short}(\mathbf{p}_\delta)$

KH, Phys. Rept. 890, 1 (2021)

3NFs in different regularization schemes

	momentum space	coordinate space
nonlocal *regulators:* long-range short-range *regularization:*	**nonlocal MS** N^2LO N^3LO $f_\Lambda^{\text{long}}(\mathbf{p},\mathbf{q}) = \exp\left[-((\mathbf{p}^2 + 3/4\,\mathbf{q}^2)/\Lambda^2)^n\right]$ $f_\Lambda^{\text{short}}(\mathbf{p},\mathbf{q}) = f_\Lambda^{\text{long}}(\mathbf{p},\mathbf{q}) = f_R(\mathbf{p},\mathbf{q})$ $\left\langle \mathbf{p}'\mathbf{q}'\vert V_{3\text{N}}^{\text{reg}}\vert\mathbf{pq}\right\rangle = f_R(\mathbf{p}',\mathbf{q}')\,\langle \mathbf{p}'\mathbf{q}'\vert V_{3\text{N}}\vert\mathbf{pq}\rangle\, f_R(\mathbf{p},\mathbf{q})$	
local *regulators:* long-range short-range *regularization:*	**local MS** N^2LO N^3LO $f_\Lambda^{\text{long}}(\mathbf{Q}_i) = \exp\left[-(\mathbf{Q}_i^2/\Lambda^2)^2\right]$ $f_\Lambda^{\text{short}}(\mathbf{Q}_i) = f_\Lambda^{\text{long}}(\mathbf{Q}_i) = f_\Lambda(\mathbf{Q}_i)$ $\left\langle \mathbf{p}'\mathbf{q}'\vert V_{3\text{N}}^{\text{reg}}\vert\mathbf{pq}\right\rangle = \langle \mathbf{p}'\mathbf{q}'\vert V_{3\text{N}}\vert\mathbf{pq}\rangle \prod_i f_R(\mathbf{Q}_i)$	**local CS** N^2LO N^3LO $f_R^{\text{long}}(\mathbf{r}) = 1 - \exp\left[-(r^2/R^2)^n\right]$ $f_R^{\text{short}}(\mathbf{r}) = \exp\left[-(r^2/R^2)^n\right]$ $V_{3\text{N}}^{\pi,\text{reg}}(\mathbf{r}_{ij}) = f_R^{\text{long}}(\mathbf{r}_{ij})V_{3\text{N}}^{\pi}(\mathbf{r}_{ij})$ $\delta^{\text{reg}}(\mathbf{r}_{ij}) = \alpha_n f_R^{\text{short}}(\mathbf{r}_{ij})$
semilocal *regulators:* long-range short-range *regularization:*	**semilocal MS** N^2LO N^3LO $f_\Lambda^{\text{long}}(\mathbf{Q}_i) = \exp\left[-(\mathbf{Q}_i^2 + m_\pi^2)/\Lambda^2\right]$ $f_\Lambda^{\text{short}}(\mathbf{p}) = \exp\left[-\mathbf{p}^2/\Lambda^2\right]$ $\left\langle \mathbf{p}'\mathbf{q}'\vert V_{3\text{N}}^{\text{reg},\pi}\vert\mathbf{pq}\right\rangle = f_R^{\text{long}}(\mathbf{Q}_i)\,\langle \mathbf{p}'\mathbf{q}'\vert V_{3\text{N}}\vert\mathbf{pq}\rangle$ $\left\langle \mathbf{p}'\mathbf{q}'\vert V_{3\text{N}}^{\text{reg},\delta}\vert\mathbf{pq}\right\rangle = f_\Lambda^{\text{short}}(\mathbf{p}'_\delta)\left\langle \mathbf{p}'\mathbf{q}'\vert V_{3\text{N}}^{\delta}\vert\mathbf{pq}\right\rangle f_\Lambda^{\text{short}}(\mathbf{p}_\delta)$	**semilocal CS** N^2LO N^3LO $f_R^{\text{long}}(\mathbf{r}) = \left(1 - \exp\left[-r^2/R^2\right]\right)^n$ $f_\Lambda^{\text{short}}(\mathbf{p}) = \exp\left[-\mathbf{p}^2/\Lambda^2\right]$ $V_{3\text{N}}^{\pi,\text{reg}}(\mathbf{r}_{ij}) = f_R^{\text{long}}(\mathbf{r}_{ij})V_{3\text{N}}^{\pi}(\mathbf{r}_{ij})$ $\delta(\mathbf{r}_{ij}) \xrightarrow{FT} V_{3\text{N}}^{\delta}$ $\left\langle \mathbf{p}'\mathbf{q}'\vert V_{3\text{N}}^{\text{reg},\delta}\vert\mathbf{pq}\right\rangle = f_\Lambda^{\text{short}}(\mathbf{p}'_\delta)\left\langle \mathbf{p}'\mathbf{q}'\vert V_{3\text{N}}^{\delta}\vert\mathbf{pq}\right\rangle f_\Lambda^{\text{short}}(\mathbf{p}_\delta)$

Violation of chiral symmetry at N3LO? Non-renormalizable? *Epelbaum, Krebs*

Development of large cutoff interactions for QMC calculations in progress *Tews et al.*
→ Reduction of cutoff artifacts.

Derivation of consistently regularised currents at this order very hard, switched to SMS

Formal derivation 'basically' finished.
Implementation in progress… (hard!)

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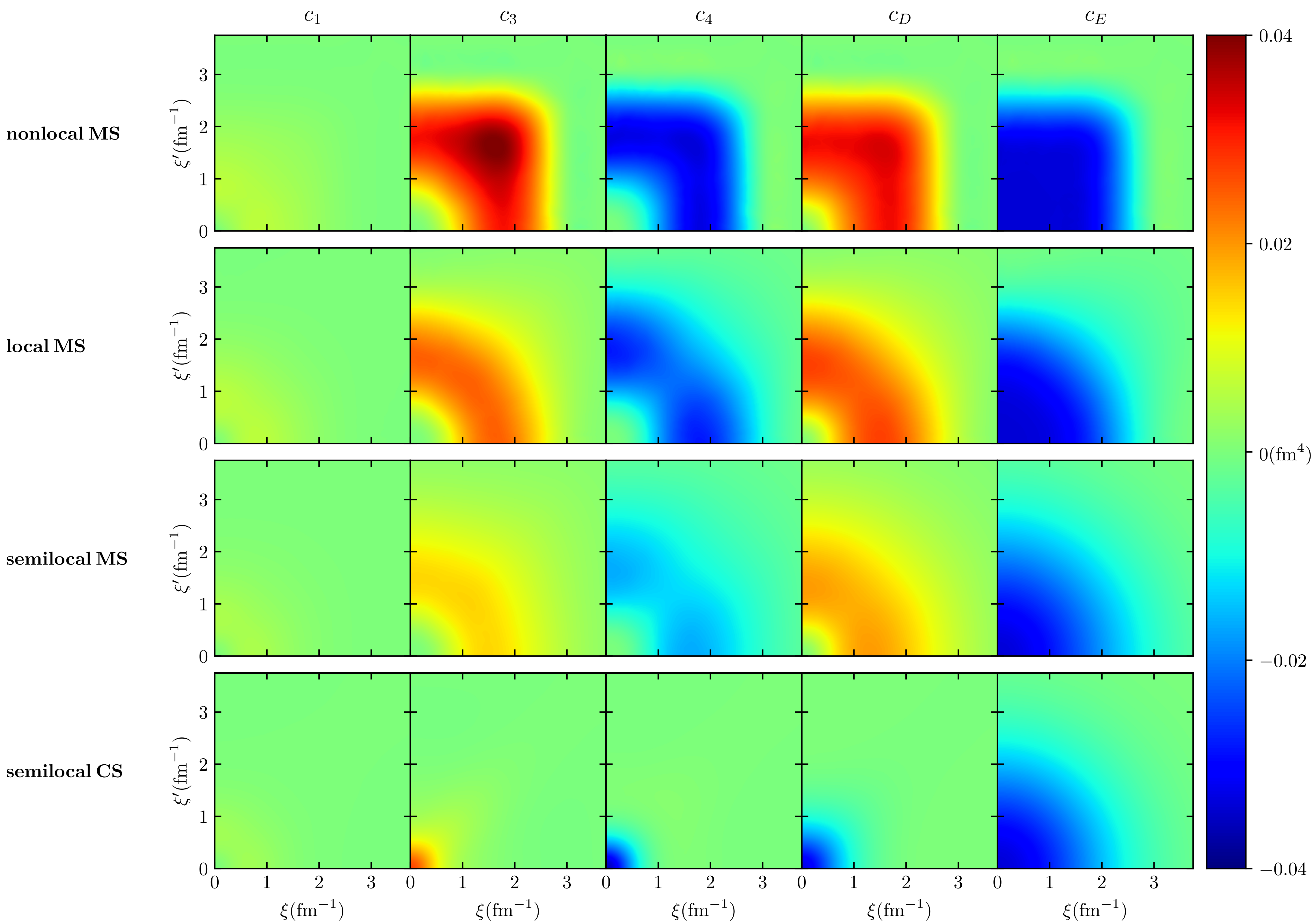
Illustration of 3NFs in different regularization schemes



$$\xi^2 = p^2 + 3/4q^2 \quad \tan\theta = p/(\sqrt{3}/2q) = \frac{\pi}{4}$$

KH, Phys. Rept. 890, 1 (2021)

Uncertainty II: Fitting of low-energy couplings + SRG evolution

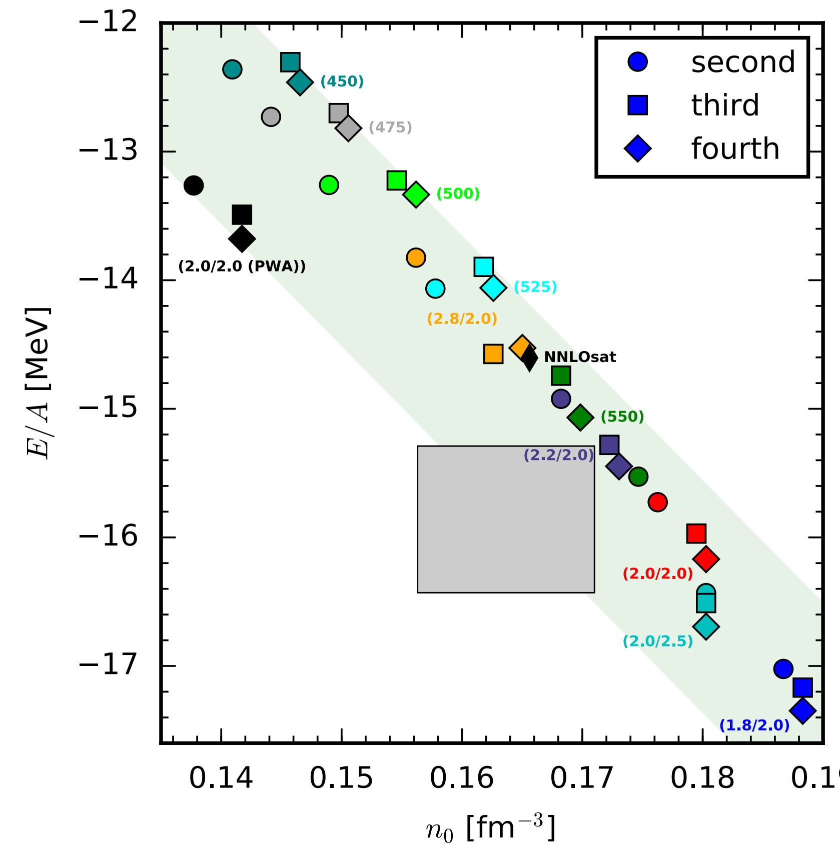
- choice of observables
- separate or simultaneous fits of NN and 3N LECs?
- using few-body and/or many-body observables?
- using bare or low-resolution interactions for fits?
- low resolution interactions:
  - evolve NN interactions to lower scales via the RG
  - fit the 3N LECs at a low cutoff scale ($\Lambda_{3N} \sim 2\,\mathrm{fm}^{-1}$)
  - e.g., use ^3H binding energy and ^4He radius to fix c_D and c_E

	$V_{\mathrm{low}\,k}$		SRG	
Λ or λ/Λ_{3NF} [fm^{-1}]	c_D	c_E	c_D	c_E
1.8/2.0 (EM c_i's)	+1.621	−0.143	+1.264	−0.120
2.0/2.0 (EM c_i's)	+1.705	−0.109	+1.271	−0.131
2.0/2.5 (EM c_i's)	+0.230	−0.538	−0.292	−0.592
2.2/2.0 (EM c_i's)	+1.575	−0.102	+1.214	−0.137
2.8/2.0 (EM c_i's)	+1.463	−0.029	+1.278	−0.078
2.0/2.0 (EGM c_i's)	−4.381	−1.126	−4.828	−1.152
2.0/2.0 (PWA c_i's)	−2.632	−0.677	−3.007	−0.686

PRC 83, 031301 (2011)

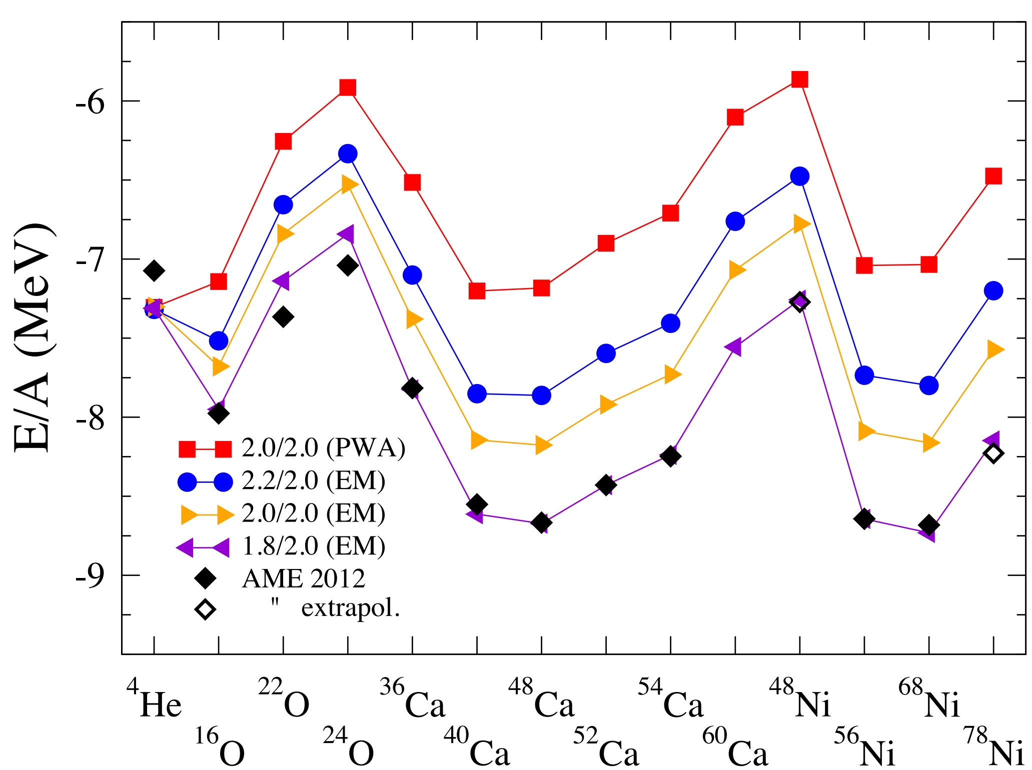
Low-resolution fits versus consistent NN+3N evolution

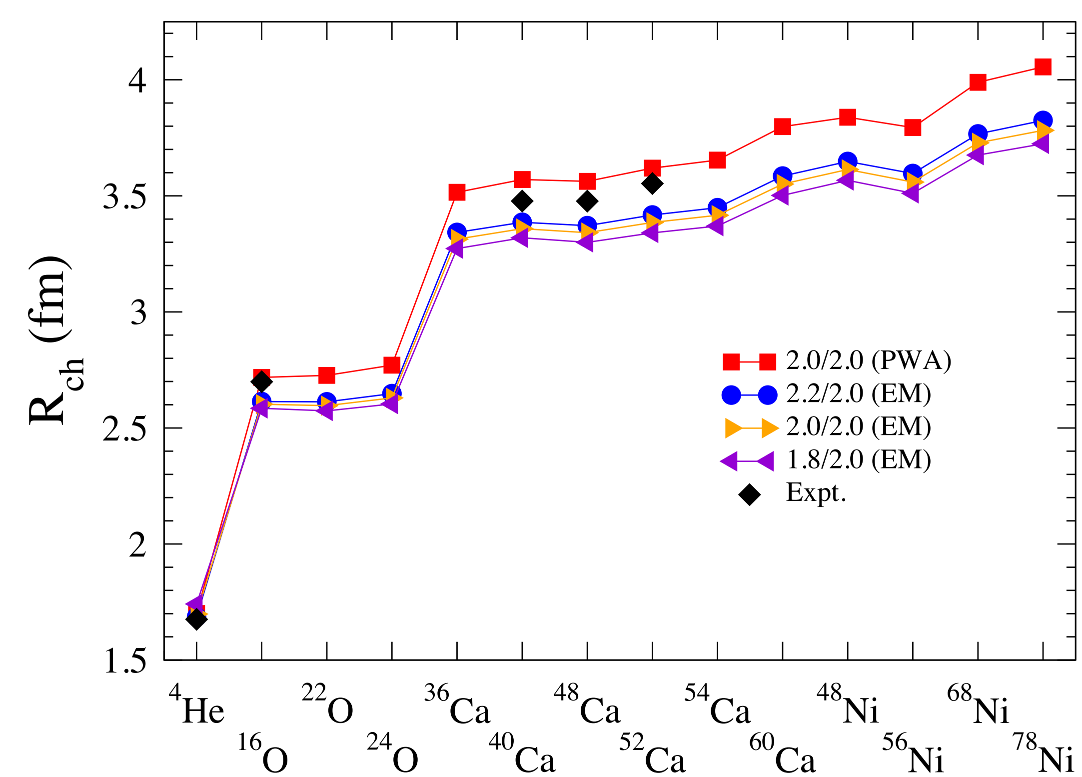
	NN SRG evolution + 3N fits				
λ_{SRG} (fm^{-1})	Λ_{3NF} (fm^{-1})	c_D	c_E	$r_{^3\text{H}}$ (fm)	$E_{^4\text{He}}$ (MeV)
∞	2.0	+1.5	0.114	1.601	−28.64(4)
2.8	2.0 [116]	+1.278	−0.078	1.604	−28.75(2)
2.6	2.0	+1.26	−0.099	1.605	−28.77(2)
2.4	2.0	+1.265	−0.115	1.606	−28.80(2)
2.2	2.0 [116]	+1.214	−0.137	1.608	−28.86(2)
2.0	2.0 [116]	+1.271	−0.131	1.612	−28.95(2)
1.8	2.0 [116]	+1.264	−0.120	1.617	−29.11(2)
1.6	2.0	+1.25	−0.075	1.626	−29.42(2)
∞	2.5	−1.45	−0.633	1.604	−28.65(4)
2.8	2.5	−1.35	−0.735	1.606	−28.84(2)
2.6	2.5	−1.2	−0.75	1.606	−28.85(2)
2.4	2.5	−1.0	−0.725	1.607	−28.89(2)
2.2	2.5	−0.7	−0.675	1.609	−28.95(2)
2.0	2.5 [116]	−0.292	−0.592	1.612	−29.05(2)
1.8	2.5	0.05	−0.503	1.617	−29.21(2)
1.6	2.5	0.55	−0.353	1.626	−29.48(2)



KH, Bogner, Furnstahl, Nogga, Schwenk, PRC 83, 031301(2011)
KH, Phys. Repts. 890, 1 (2021)

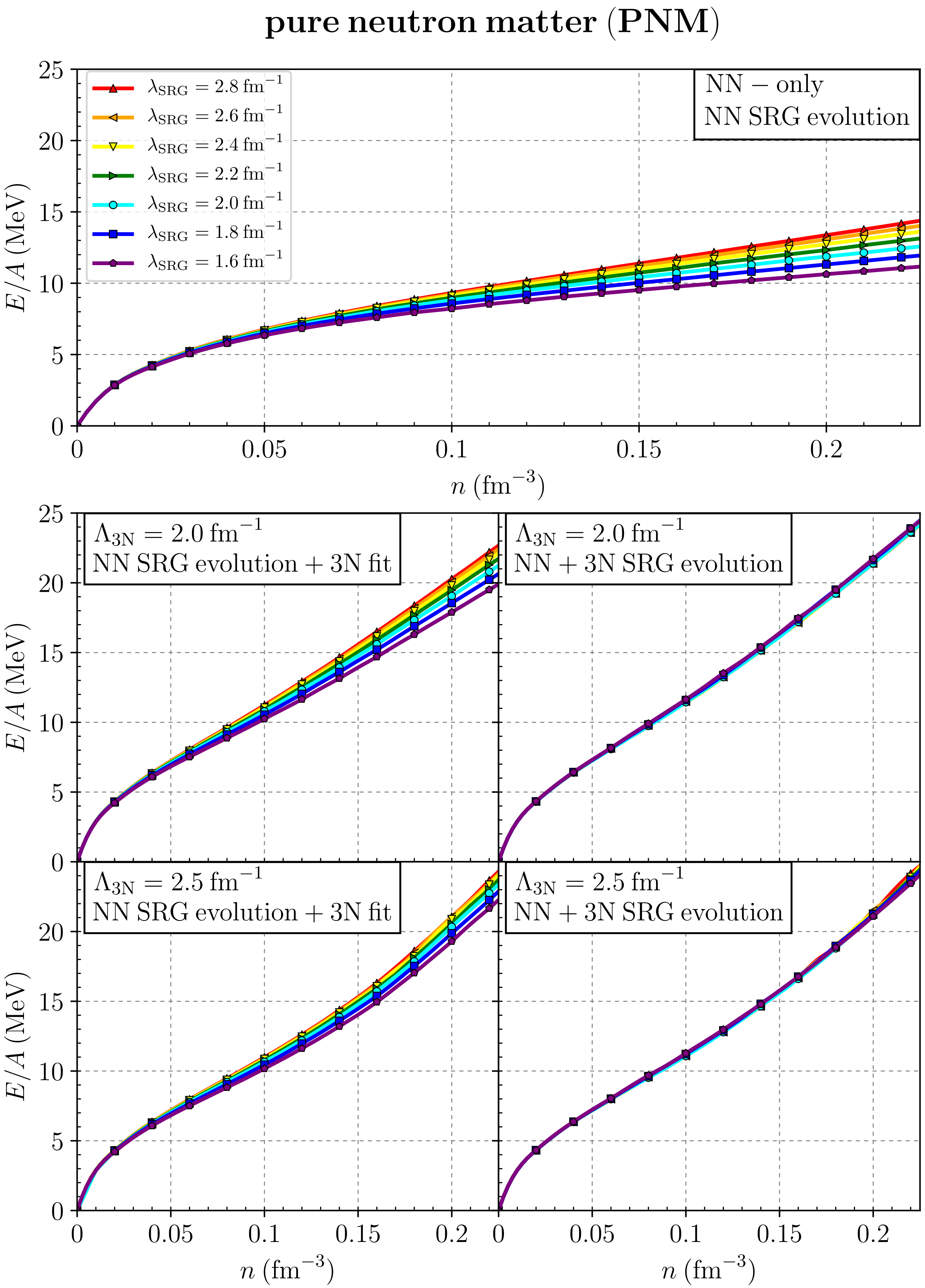
Drischler, KH, Schwenk, PRL 122, 042501 (2019)





Simonis, Stroberg, KH, Holt, Schwenk, PRC 96, 014303 (2017)

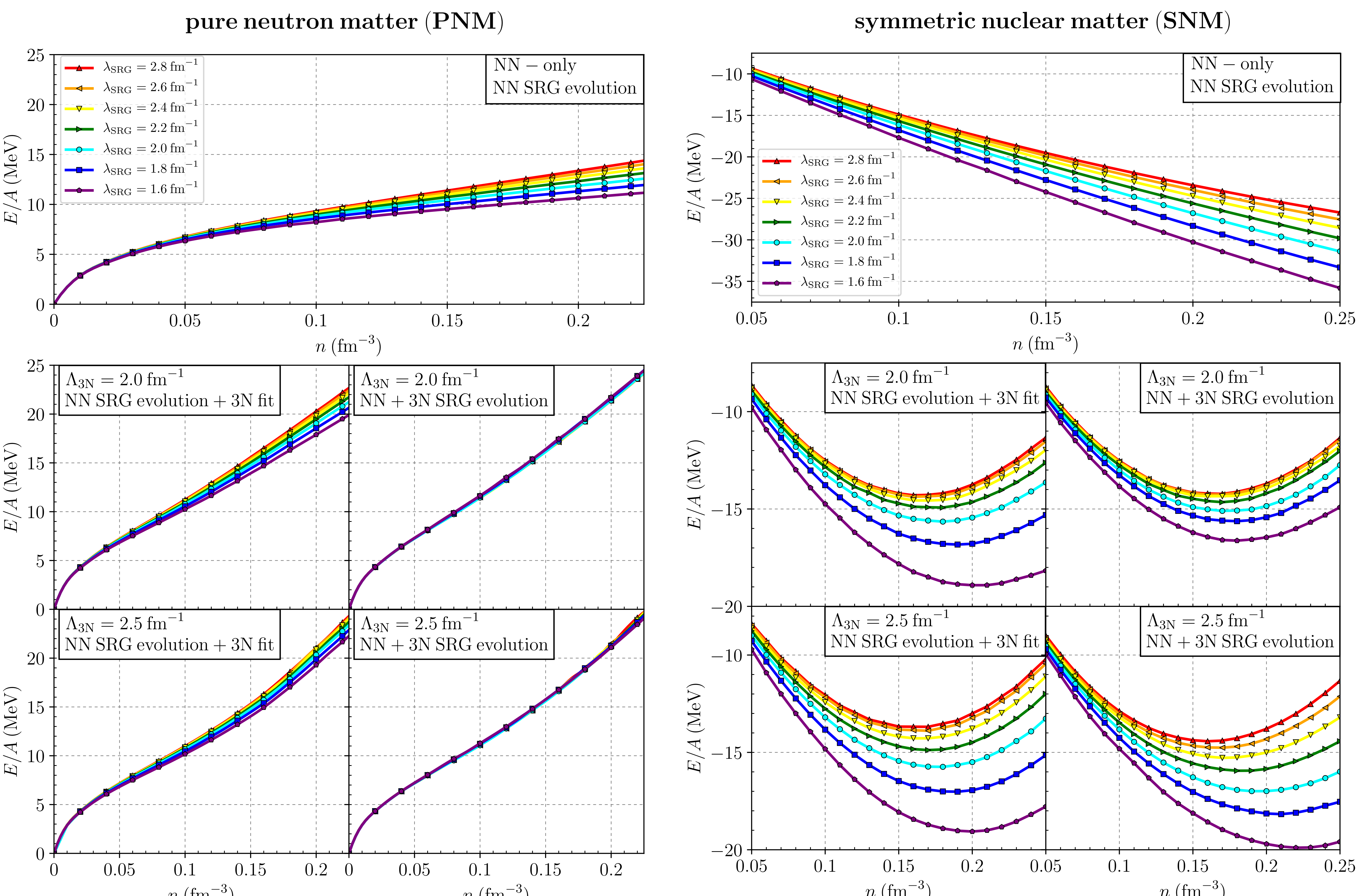
Low-resolution fits versus consistent NN+3N evolution



		NN+3N SRG evolution		
$\lambda_{\rm SRG}$ ($\rm fm^{-1}$)	$\Lambda_{\rm 3NF}$ ($\rm fm^{-1}$)	$E_{^3\rm H}$ (MeV)	$r_{^3\rm H}$ (fm)	$E_{^4\rm He}$ (MeV)
∞	2.0	−8.482	1.601	−28.64(4)
2.8	2.0 [116]	−8.482	1.605	−28.72(2)
2.6	2.0	−8.481	1.606	−28.73(2)
2.4	2.0	−8.481	1.608	−28.73(2)
2.2	2.0 [116]	−8.480	1.611	−28.74(2)
2.0	2.0 [116]	−8.479	1.615	−28.75(2)
1.8	2.0 [116]	−8.478	1.622	−28.76(2)
1.6	2.0	−8.476	1.635	−28.79(2)
∞	2.5	−8.482	1.604	−28.65(4)
2.8	2.5	−8.482	1.608	−28.75(2)
2.6	2.5	−8.482	1.609	−28.76(2)
2.4	2.5	−8.482	1.610	−28.77(2)
2.2	2.5	−8.481	1.613	−28.77(2)
2.0	2.5 [116]	−8.481	1.617	−28.77(2)
1.8	2.5	−8.480	1.625	−28.77(2)
1.6	2.5	−8.478	1.638	−28.77(2)

KH, Phys. Rept. 890, 1 (2021)

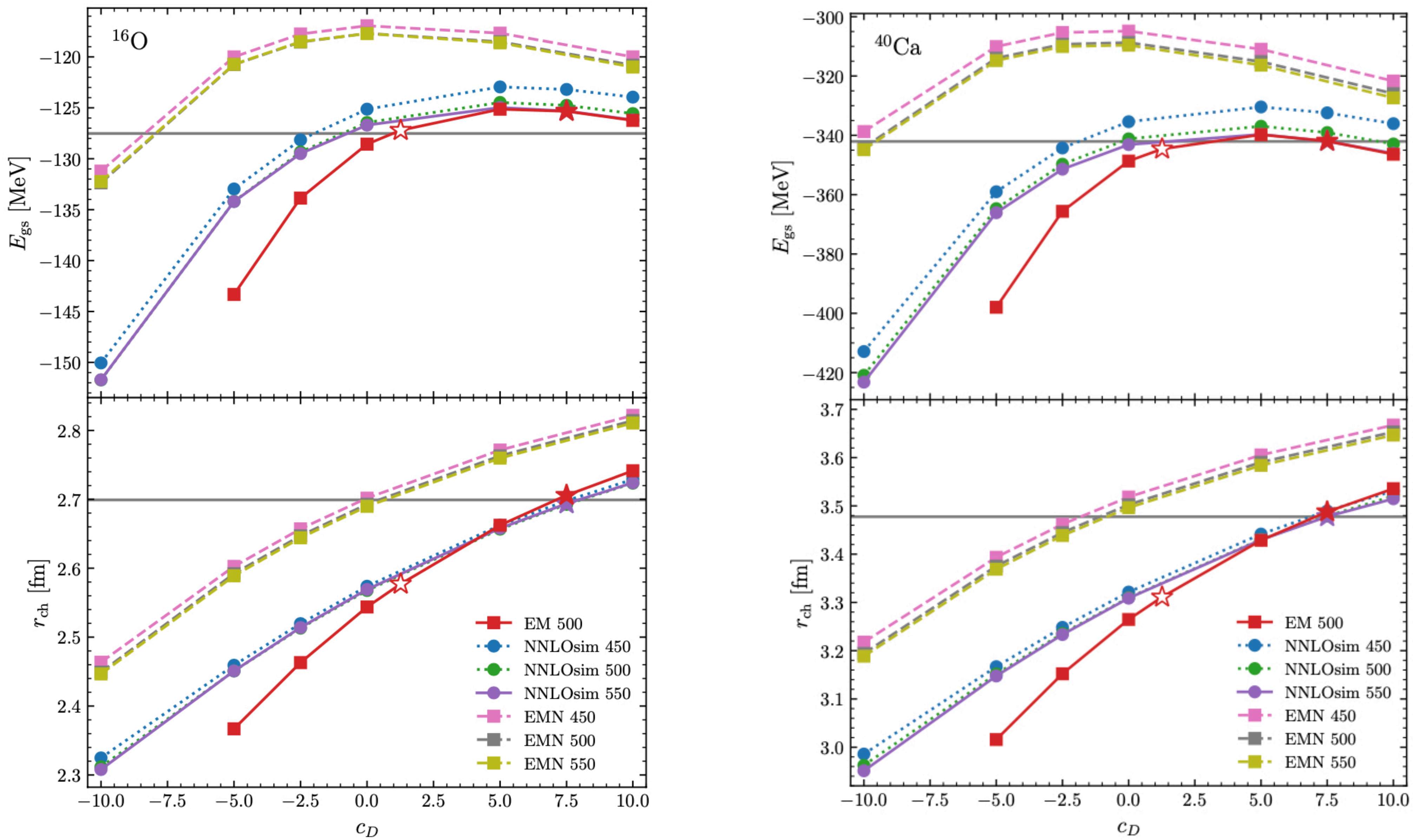
Low-resolution fits versus consistent NN+3N evolution



KH, Phys. Rept. 890, 1 (2021)

New and improved 'magic' interactions

Arthuis, KH, Schwenk, arXiv:2401.06675



- Fit to c_D and c_E to ^{3}H energy and **both** binding energies and radius of ^{16}O using EM500 and NNLO_{sim} 550 low-resolution NN interactions
- Simultaneous reproduction of experimental binding energies and charge radii *see talk by Achim for more details and results*

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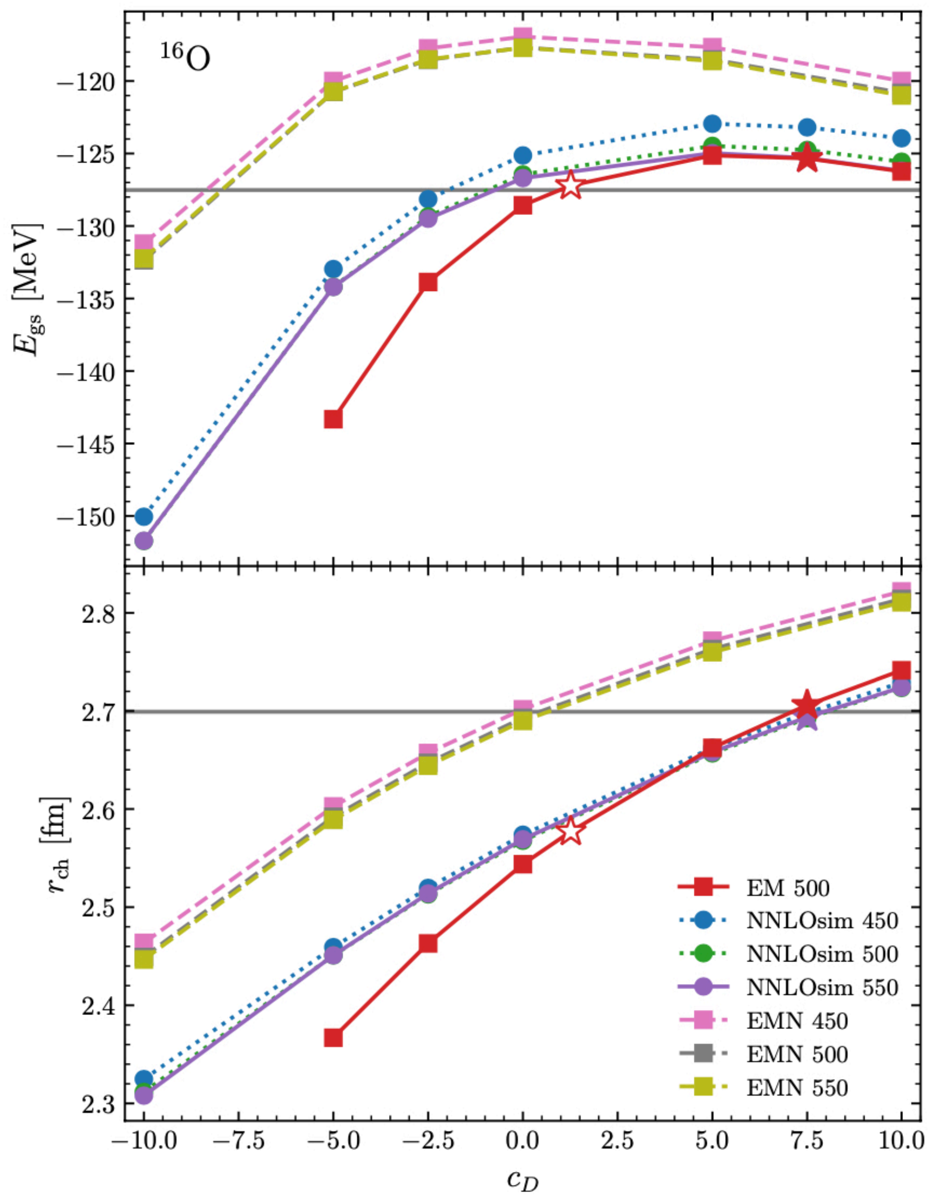
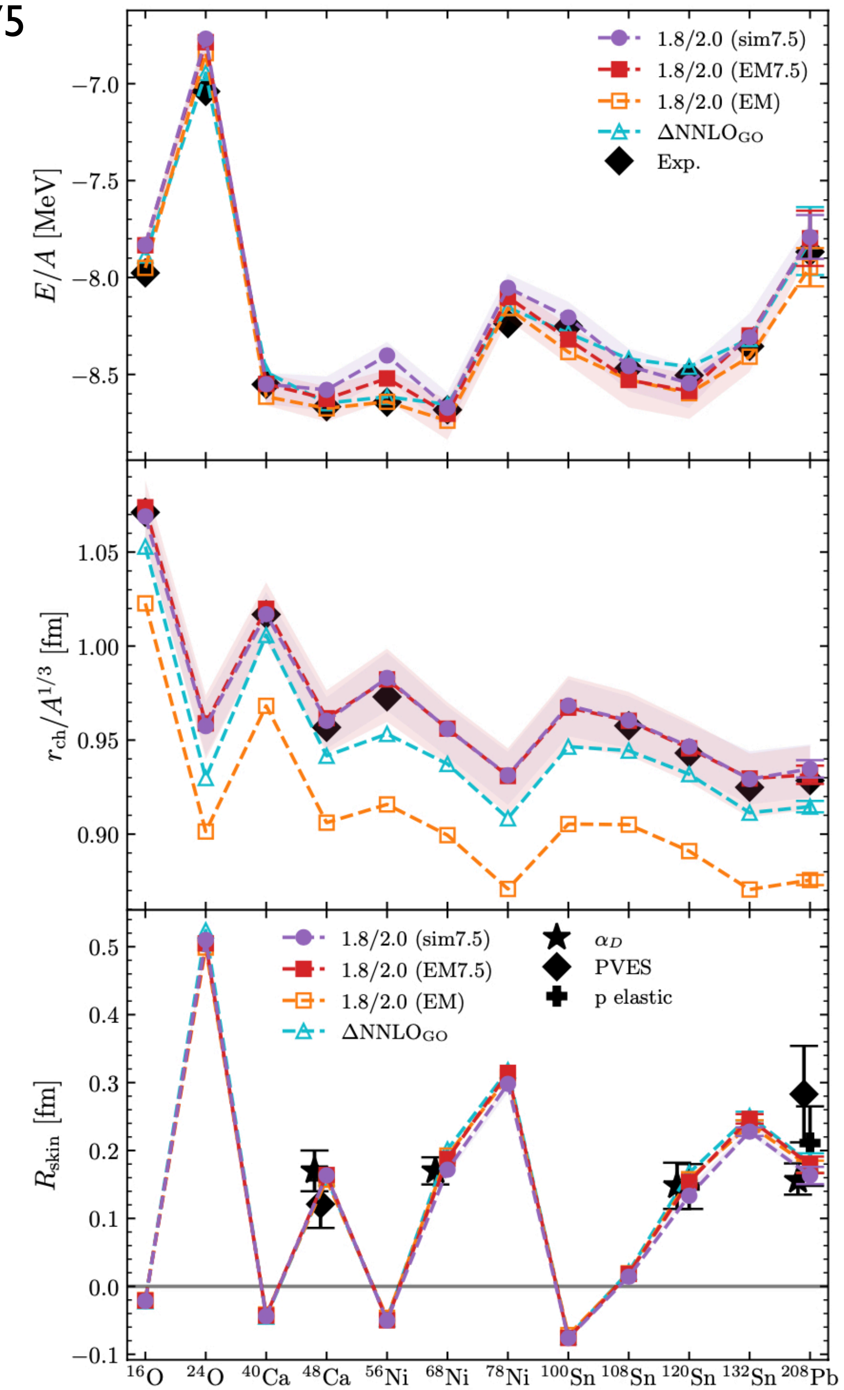




- Fit to c_D and c_E to ^{3}H energy and **both** binding energies and radius of ^{16}O using EM500 and NNLO$_{sim}$ 550 low-resolution NN interactions
- Simultaneous reproduction of experimental binding energies and charge radii *see talk by Achim for more details and results*

Uncertainty III: Inclusion of 3NF in many-body calculations

A general Hamiltonian consisting of kinetic energy, NN and 3N interactions:

$$\hat{H} = \hat{T}_{\text{rel}} + \hat{V}_{\text{NN}} + \hat{V}_{3\text{N}}$$

$$\hat{T}_{\text{rel}} = \sum_{ij} \langle i|T|j\rangle\, \hat{a}_i^\dagger \hat{a}_j,$$

$$\hat{V}_{\text{NN}} = \frac{1}{(2!)^2} \sum_{ijkl} \langle ij|V_{\text{NN}}^{\text{as}}|kl\rangle\, \hat{a}_i^\dagger \hat{a}_j^\dagger \hat{a}_l \hat{a}_k,$$

$$\hat{V}_{3\text{N}} = \frac{1}{(3!)^2} \sum_{ijklmn} \langle ijk|V_{3\text{N}}^{\text{as}}|lmn\rangle\, \hat{a}_i^\dagger \hat{a}_j^\dagger \hat{a}_k^\dagger \hat{a}_n \hat{a}_m \hat{a}_l$$

can be exactly rewritten in a normal-ordered form:

$$\hat{H} = \Gamma_{\text{HF}}^{(0)} + \hat{\Gamma}_{\text{HF}}^{(1)} + \hat{\Gamma}_{\text{HF}}^{(2)} + \hat{\Gamma}_{\text{HF}}^{(3)}$$

$$\Gamma_{\text{HF}}^{(0)} = \sum_i n_i \langle i|T|i\rangle + \frac{1}{2}\sum_{ij} n_i n_j \langle ij|V_{\text{NN}}^{\text{as}}|ij\rangle + \frac{1}{6}\sum_{ijk} n_i n_j n_k \langle ijk|V_{3\text{N}}^{\text{as}}|ijk\rangle,$$

$$\hat{\Gamma}_{\text{HF}}^{(1)} = \sum_{ij}\left[\langle i|T|j\rangle + \sum_k n_k \langle ik|V_{\text{NN}}^{\text{as}}|jk\rangle + \frac{1}{2}\sum_{kl} n_k n_l \langle ikl|V_{3\text{N}}^{\text{as}}|jkl\rangle\right] N(\hat{a}_i^\dagger \hat{a}_j),$$

$$\hat{\Gamma}_{\text{HF}}^{(2)} = \sum_{ijkl}\left[\langle ij|V_{\text{NN}}^{\text{as}}|kl\rangle + \sum_m n_m \langle ijm|V_{3\text{N}}^{\text{as}}|klm\rangle\right] N(\hat{a}_i^\dagger \hat{a}_j^\dagger \hat{a}_l \hat{a}_k),$$

$$\hat{\Gamma}_{\text{HF}}^{(3)} = \sum_{ijklmn} \langle ijk|V_{3\text{N}}^{\text{as}}|lmn\rangle N(\hat{a}_i^\dagger \hat{a}_j^\dagger \hat{a}_k^\dagger \hat{a}_n \hat{a}_m \hat{a}_l)\,.$$

Traditional normal ordering framework for 3N interactions

1. transformation to Jacobi HO basis plus antisymmetrization

$$\langle p'q'\alpha'|V_{3N}^{(i),\mathrm{reg}}|pq\alpha\rangle \rightarrow \langle N'n'\alpha'|V_{3N}^{(\mathrm{as,reg}}|Nn\alpha\rangle$$

2. transformation to single particle basis

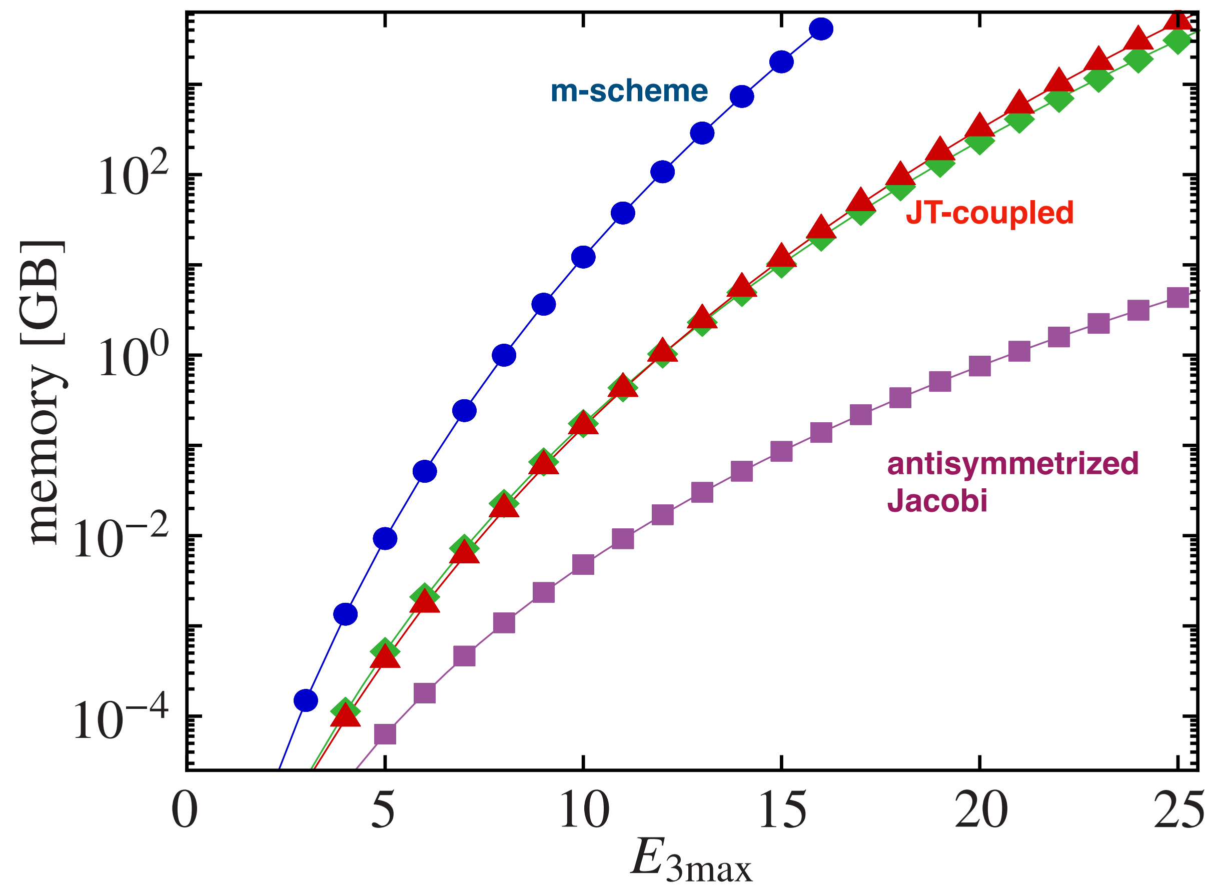
$$\langle N'n'\alpha'|V_{3N}^{\mathrm{as,\ reg}}|Nn\alpha\rangle \rightarrow \langle 1'2'3'|V_{3N}^{\mathrm{as,\ reg}}|123\rangle$$

3. Normal ordering with respect to some reference state

$$\langle 1'2'|\overline{V}|12\rangle = \sum_{3} \bar{n}_3 \langle 1'2'3|V_{3N}^{\mathrm{as}}|123\rangle$$

- severe memory limitations for handling of single-particle matrix elements with increasing E3max
- Significant optimisations possible when storing only those matrix elements needed for normal ordering

Miyagi at al., PRC 105, 1 (2022)



Roth at al., PRC 90 024325 (2014)

Novel normal ordering framework for 3N interactions

1. Use momentum space and expand reference state in HO basis:

$$\langle \mathbf{k}_1'\mathbf{k}_2'|\overline{V}|\mathbf{k}_1\mathbf{k}_2\rangle = \sum_{n_3 l_3 m_3} \overline{n}_3 \langle \mathbf{k}_1'\mathbf{k}_2'\gamma_3|V_{3\mathrm{N}}^{\mathrm{as}}|\mathbf{k}_1\mathbf{k}_2\gamma_3\rangle$$

$$= \int d\mathbf{k}_3 d\mathbf{k}_3' \langle \mathbf{k}_1'\mathbf{k}_2'\mathbf{k}_3'|V_{3\mathrm{N}}^{\mathrm{as}}|\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3\rangle \sum_{n_3 l_3 m_3} \overline{n}_3 \langle \gamma_3|\mathbf{k}_3'\rangle\langle \mathbf{k}_3|\gamma_3\rangle$$

2. Rewrite interaction in Jacobi momentum basis:

$$\langle \mathbf{p}'\mathbf{P}'|\overline{V}|\mathbf{p}\mathbf{P}\rangle = \int d\mathbf{k}_3 d\mathbf{k}_3' \langle \mathbf{p}'\mathbf{q}'|V_{3\mathrm{N}}^{\mathrm{as}}|\mathbf{p}\mathbf{q}\rangle \delta(\mathbf{P}+\mathbf{k}_3-\mathbf{P}'-\mathbf{k}_3') \sum_{n_3 l_3 m_3} \bar{n}_3 \langle \gamma_3|\mathbf{k}_3'\rangle\langle \mathbf{k}_3|\gamma_3\rangle$$

3. Decomposition in Jacobi partial wave momentum states:

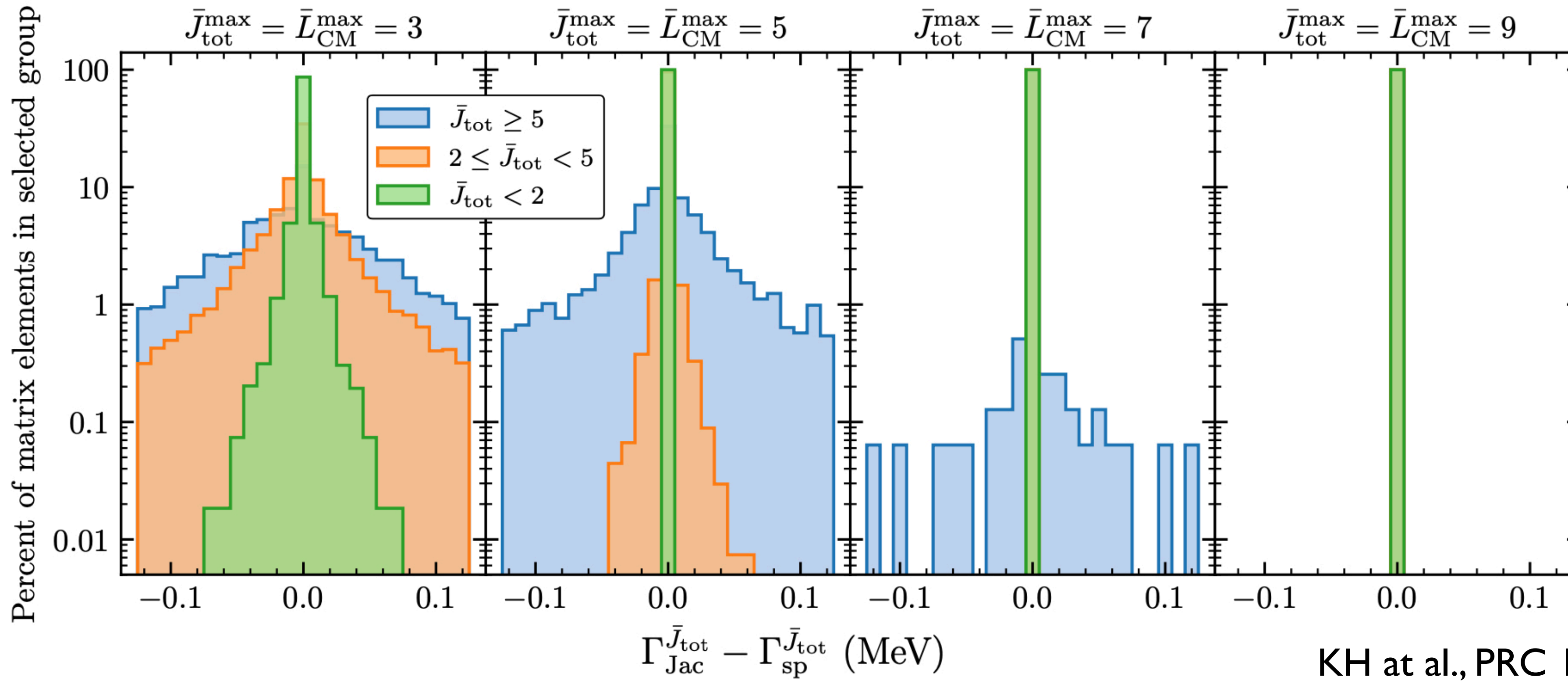
$$\langle p'P'L'M'L'_{cm}M'_{cm}|\overline{V}|pPLML_{cm}M_{cm}\rangle$$
$$= \int d\hat{\mathbf{p}} d\hat{\mathbf{P}} d\hat{\mathbf{p}}' d\hat{\mathbf{P}}' Y^*_{L'_{cm}M'_{cm}}(\hat{\mathbf{P}}') Y^*_{L'M'}(\hat{\mathbf{p}}') \langle \mathbf{p}'\mathbf{P}'|\overline{V}|\mathbf{p}\mathbf{P}\rangle Y_{L_{cm}M_{cm}}(\hat{\mathbf{P}}) Y_{LM}(\hat{\mathbf{p}})$$

4. transform matrix elements to Jacobi HO basis

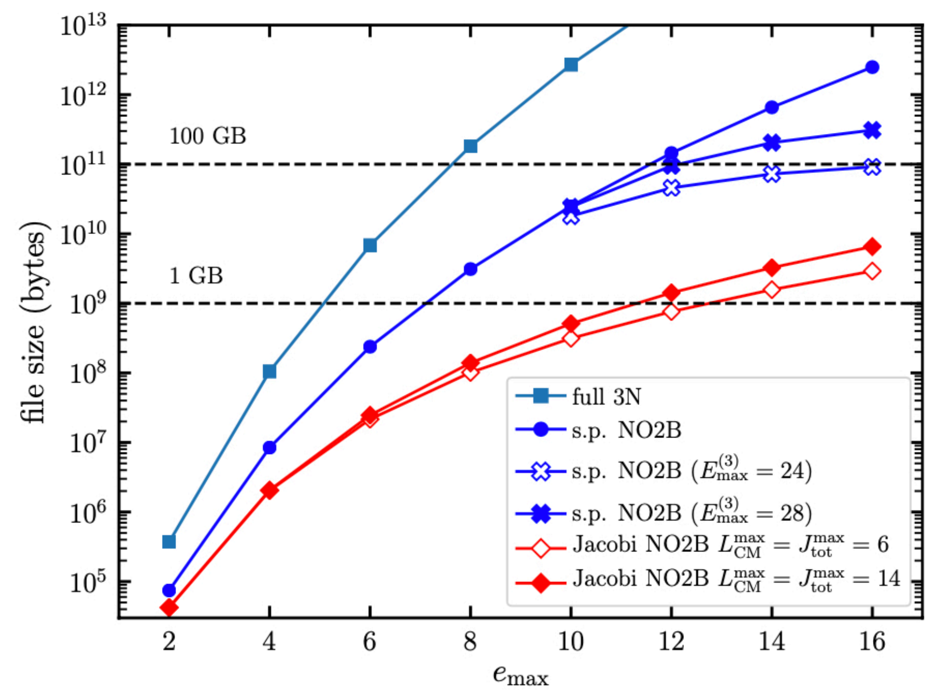
$$\langle p'P'L'M'L'_{cm}M'_{cm}|\overline{V}|pPLML_{cm}M_{cm}\rangle$$
$$\rightarrow \langle n_p' N_P' L'M'L'_{cm}M'_{cm}|\overline{V}|n_p N_P LML_{cm}M_{cm}\rangle$$

5. transformation to single-particle HO basis via generalized Talmi transformation (taking into account L_{cm} dependence)

Novel normal ordering framework for 3N interactions



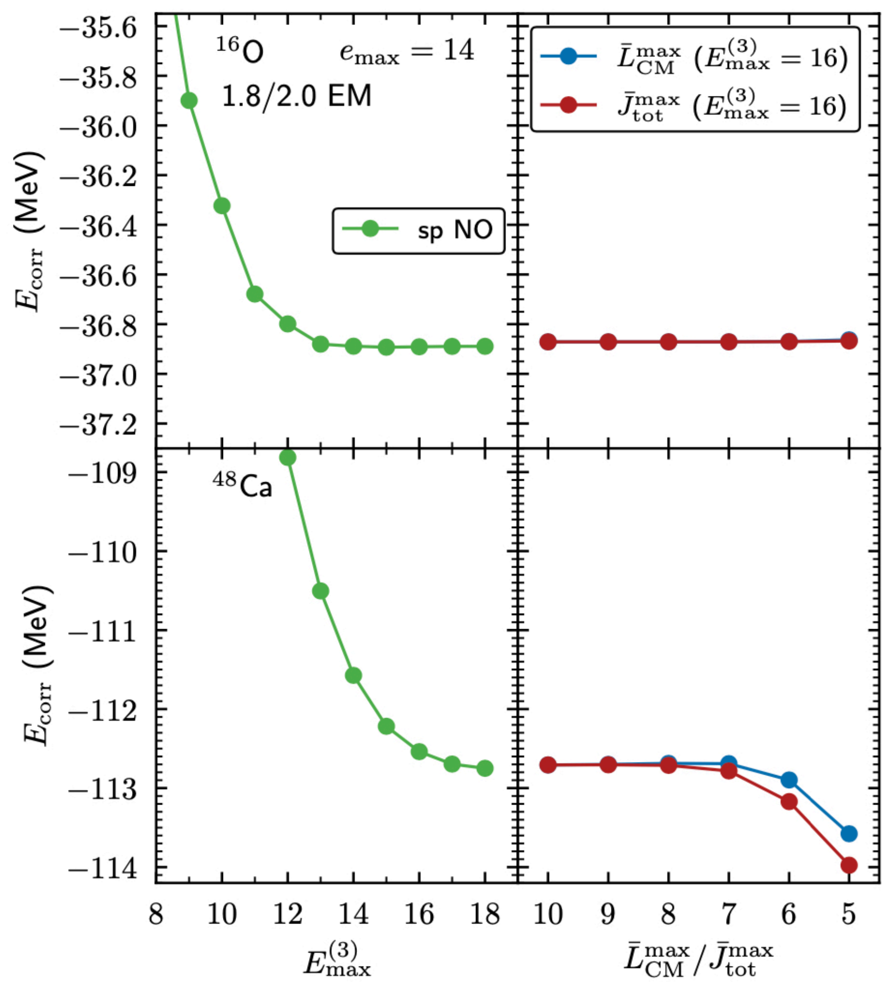
KH at al., PRC 107, 024310 (2023)

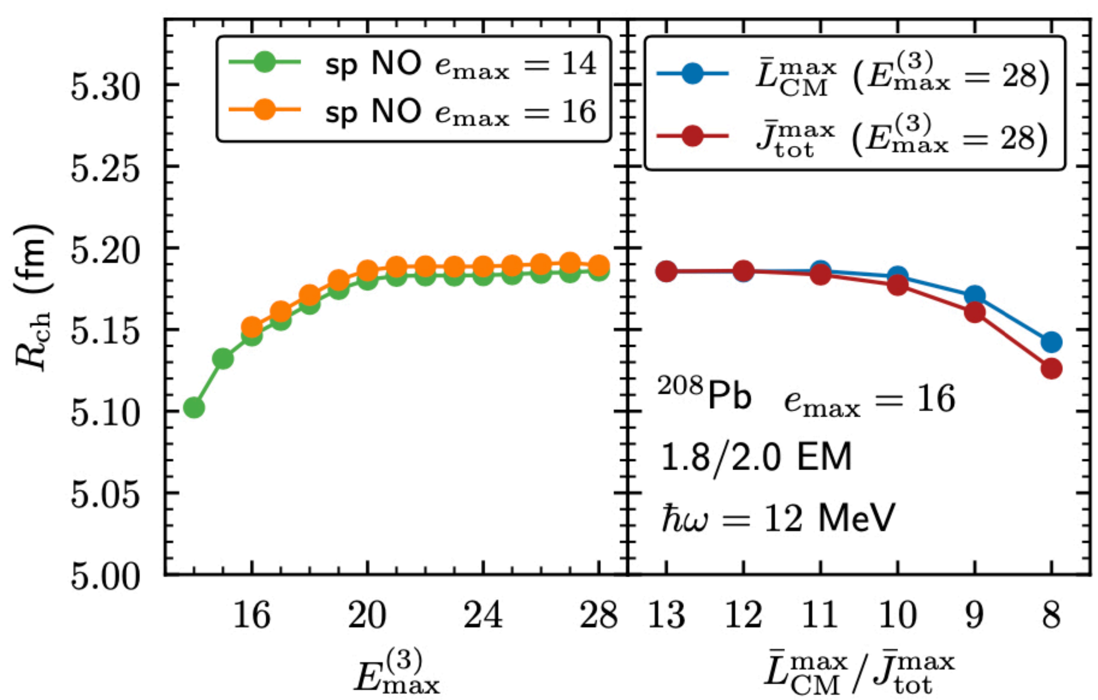


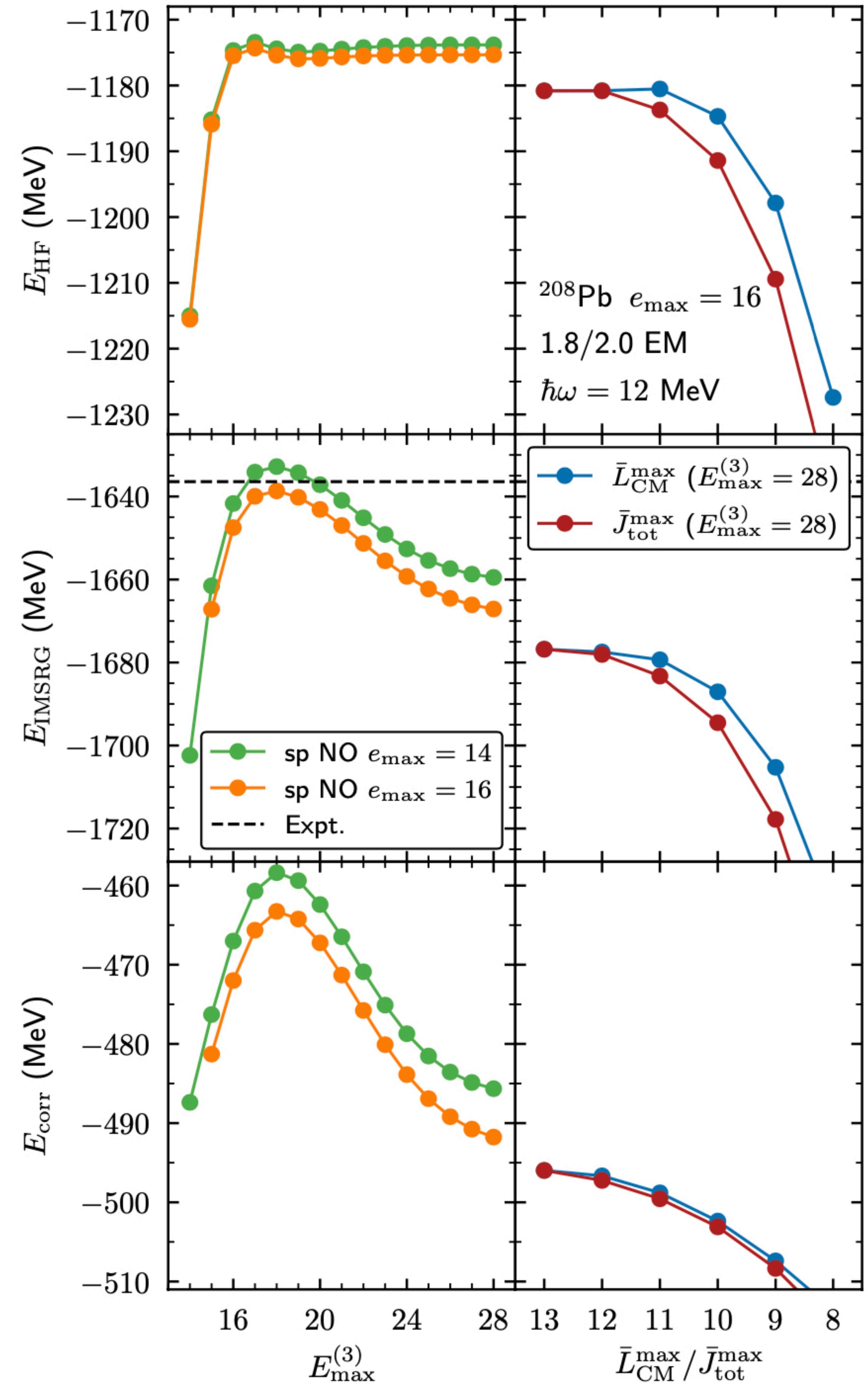
- at no stage single-particle 3N HO matrix elements needed
- N_{max} can be increased straightforwardly
- basis size and storage space determined by J_{max} and $L_{cm,max}$,

Novel normal ordering framework for 3N interactions

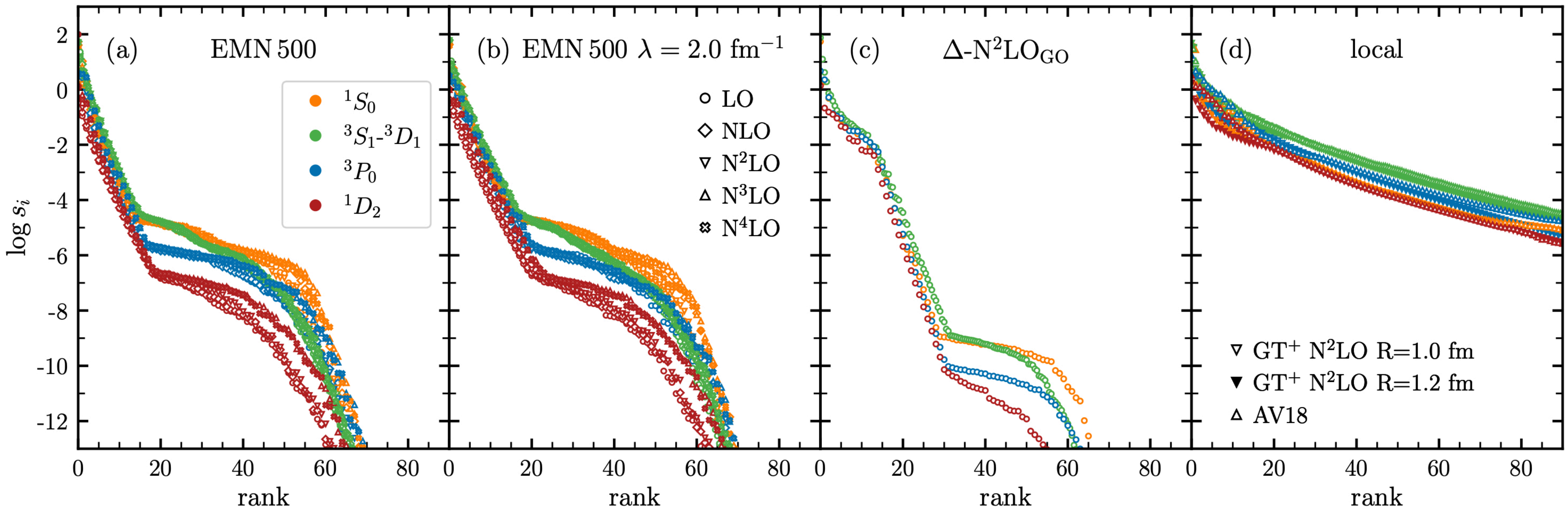
KH at al., PRC 107, 024310 (2023)



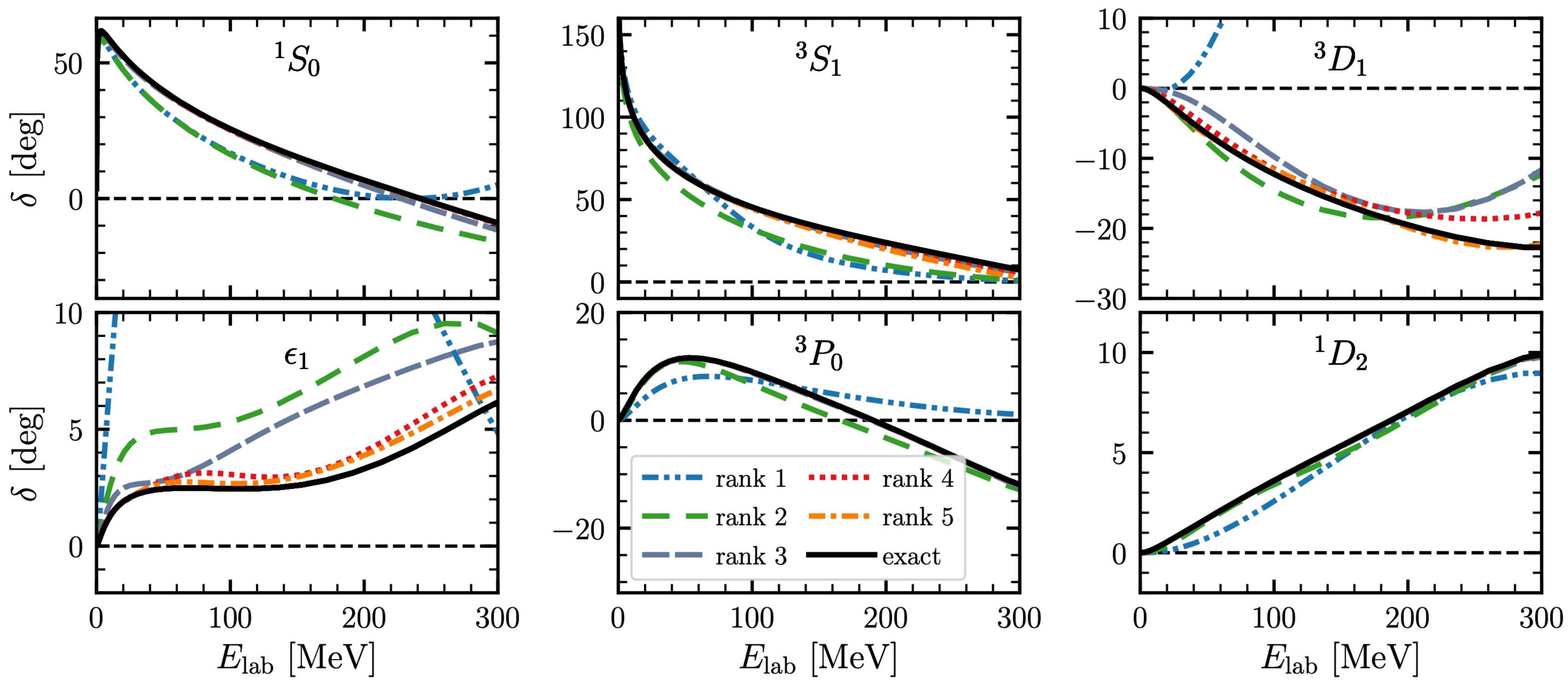


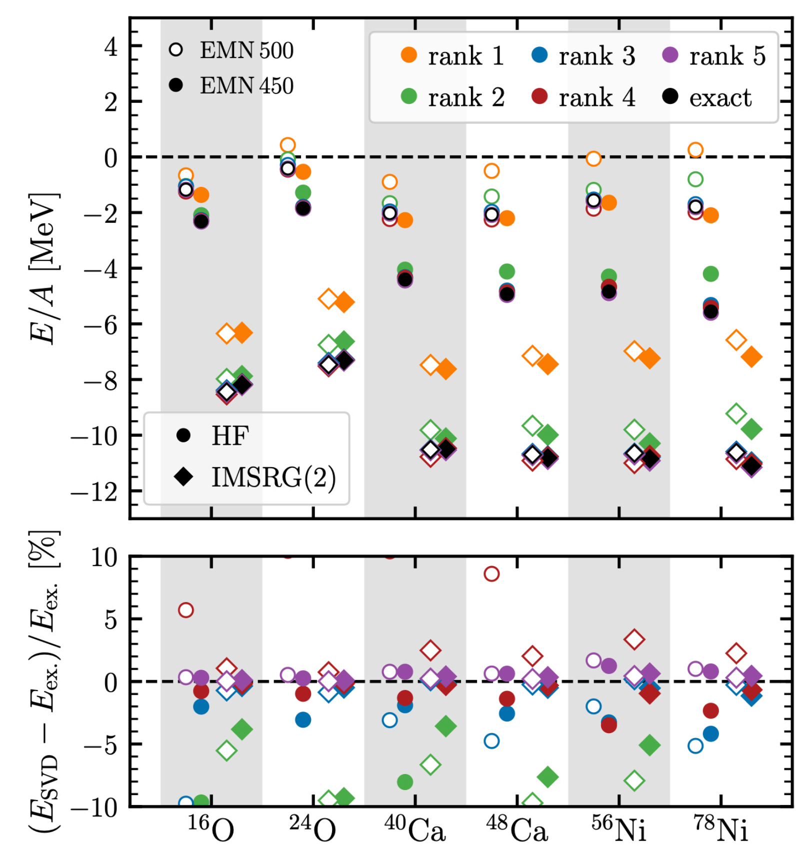


Singular value decomposition of NN interactions



Tichai et al., PLB 821, 136623 (2021)

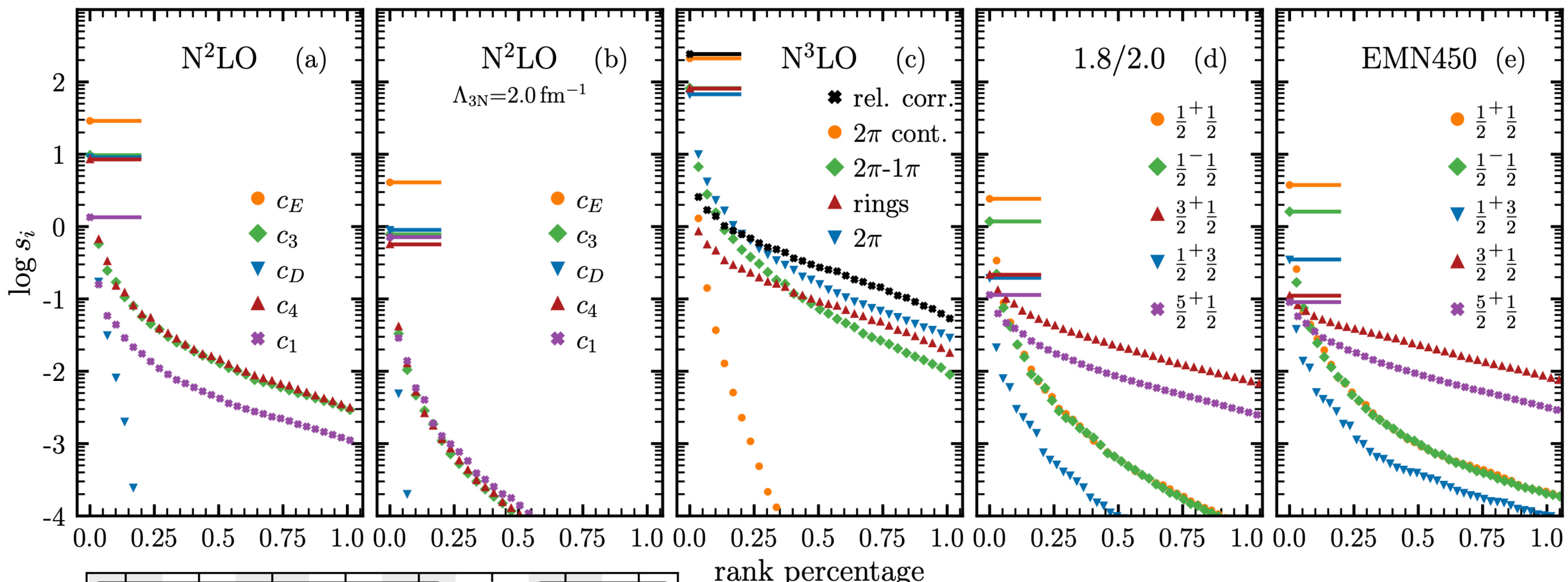


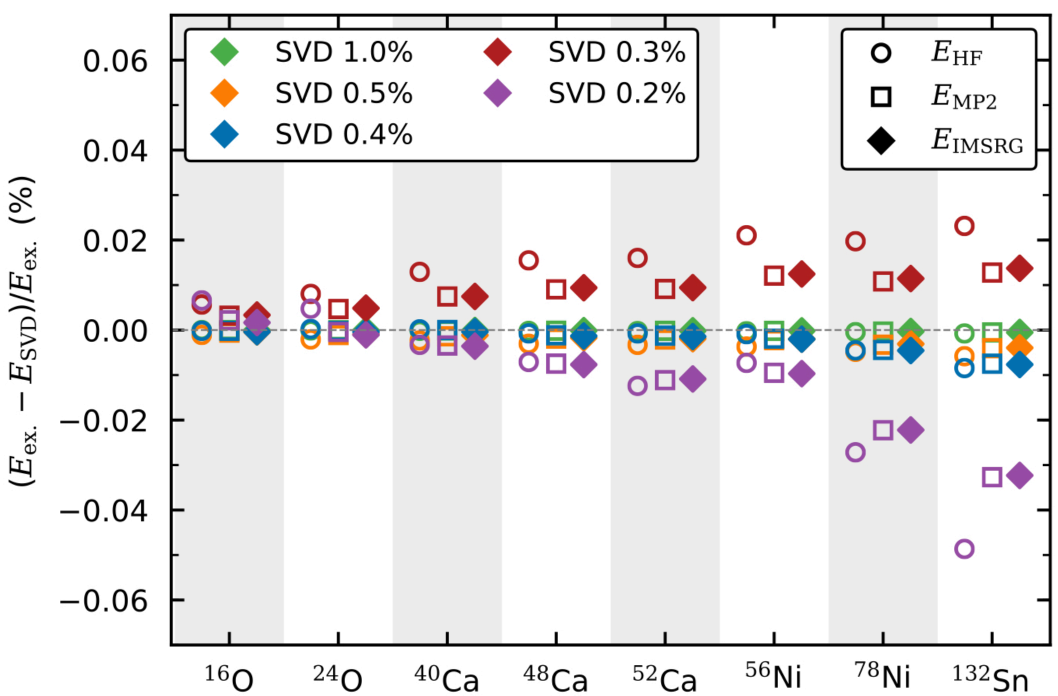


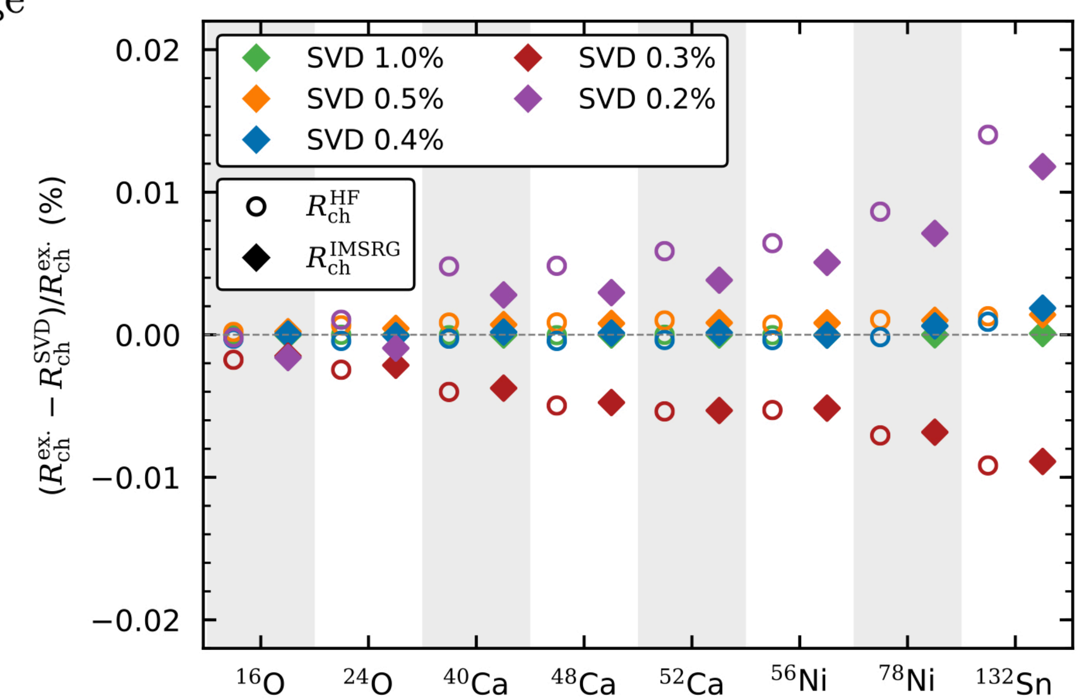
- excellent agreement with full results for phase shifts and binding energies for very low number of ranks (= number of retained singular values)

(Randomized) singular value decomposition of 3N interactions

Tichai et al., arXiv:2307.15572



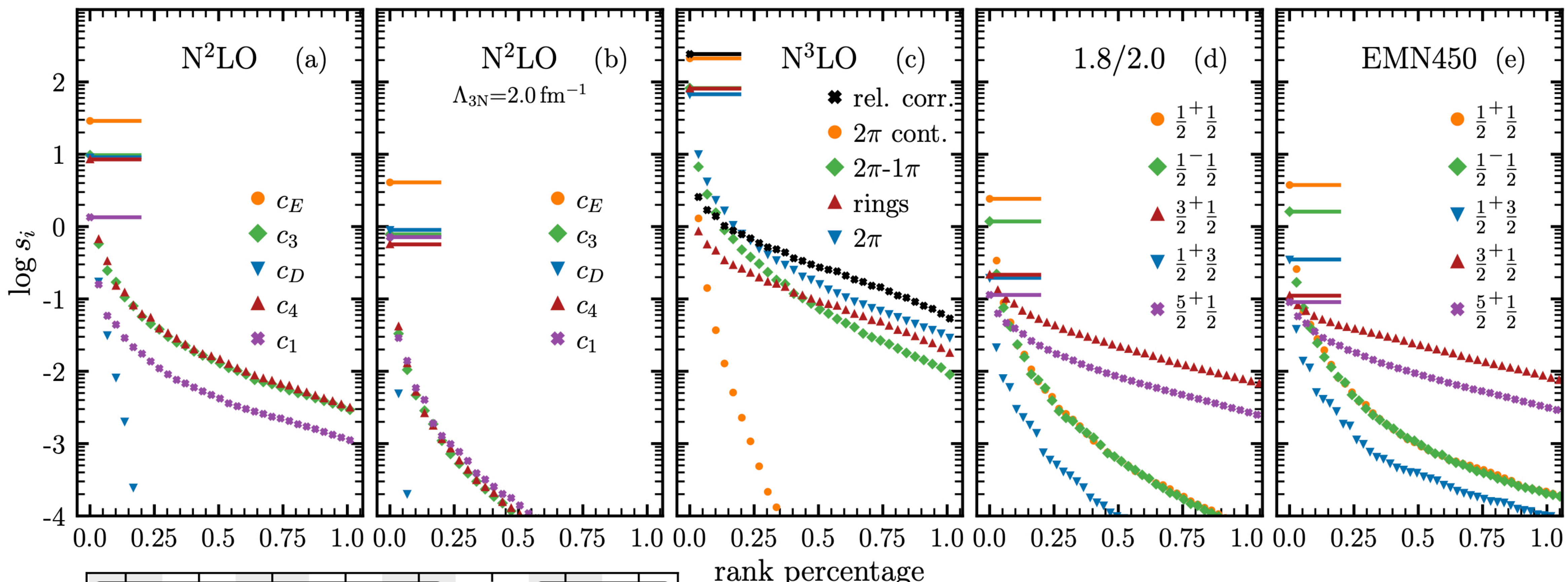


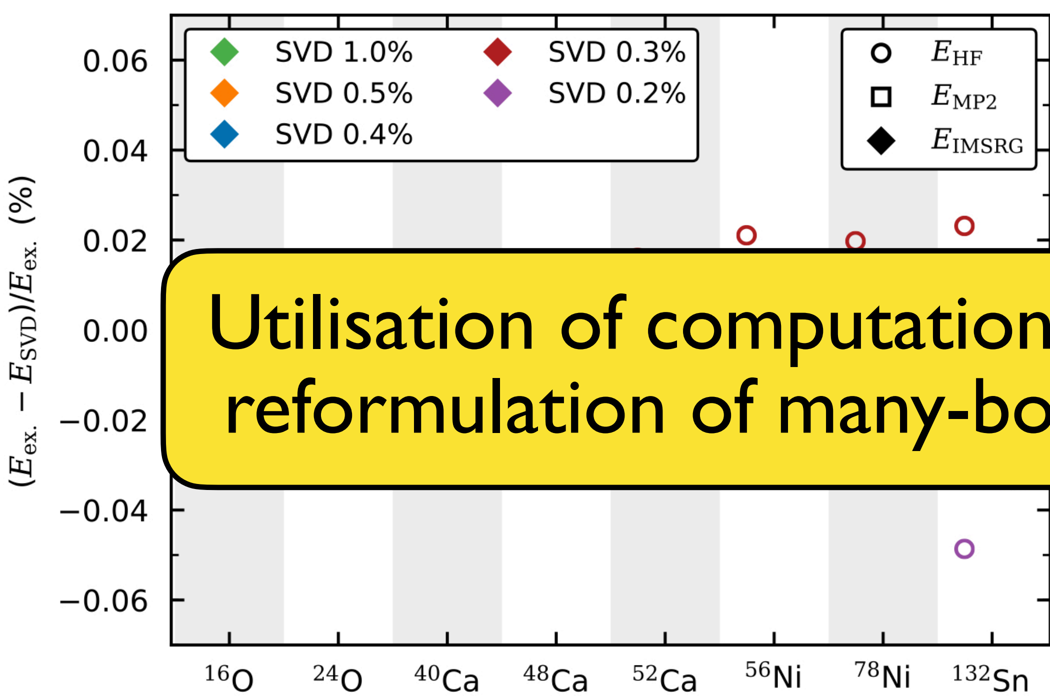


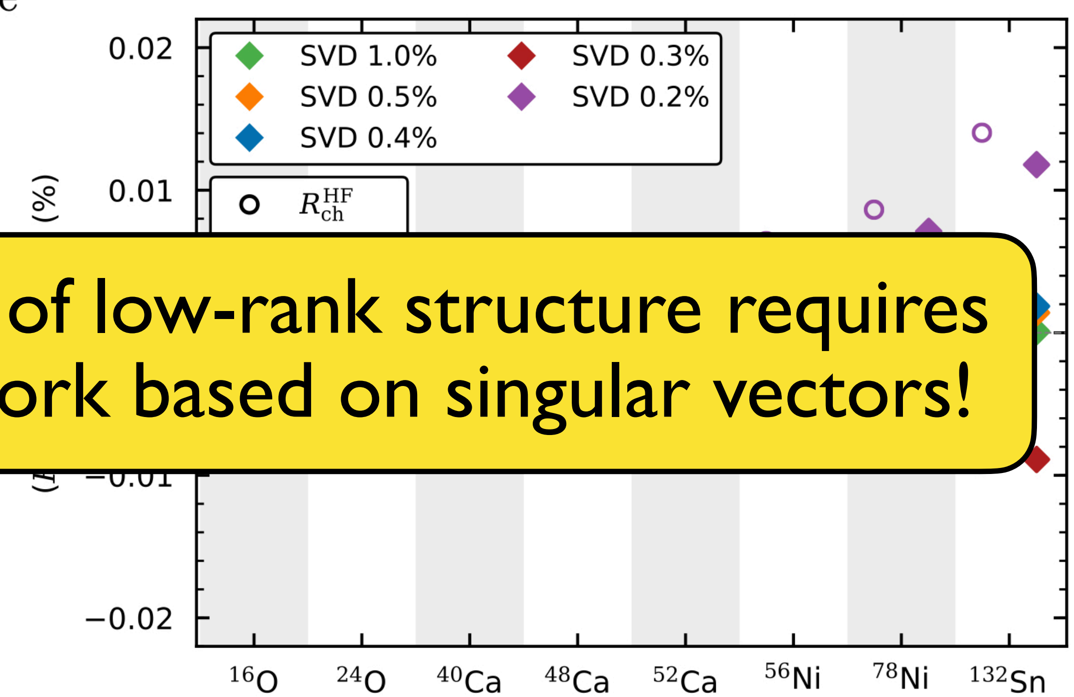
- again good agreement with full results for binding energies and charge radii for very low number of ranks (NN interactions not SVD-decomposed)

(Randomized) singular value decomposition of 3N interactions

Tichai et al., arXiv:2307.15572







Utilisation of computational benefits of low-rank structure requires reformulation of many-body framework based on singular vectors!

- again good agreement with full results for binding energies and charge radii for very low number of ranks (NN interactions not SVD-decomposed)

Which contributions should a comprehensive uncertainty estimate contain?

I. Power counting scheme

II. Chiral expansion

III. Regularization schemes

IV. Different fitting strategies for low-energy couplings

V. Truncation in many-body expansions/SRG evolution

VI. Basis truncations

VII.…?