



MCGPJ generator

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Radiative corrections for pion and kaon production at e^+e^- colliders of energies below 2 GeV

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ABSTRACT: Processes of electron–positron annihilation into charged pions and kaons are considered. Radiative corrections are taken into account exactly in the first order and within the leading logarithmic approximation in higher orders. A combined approach for accounting exact calculations and electron structure functions is used. An accuracy of the calculation can be estimated about 0.2%.

KEYWORDS: [Standard Model](#), [Electromagnetic Processes and Properties](#),

Large angle QED processes at e^+e^- colliders at energies below 3 GeV

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ABSTRACT: QED processes at electron–positron colliders are differential cross–sections for large–angle Bhabha scattering, annihilation into photons. Radiative corrections in the first order are taken into account. Logarithmic contributions are calculated in all orders by means of a method. An accuracy of the calculation can be estimated about 0.2%.

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Monte-Carlo generator for e^+e^- annihilation into lepton and hadron pairs with precise radiative corrections

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Abstract. Recently, various cross sections of e^+e^- annihilation into hadrons were accurately measured in the energy range from 0.37 to 1.39 GeV with the CMD-2 detector at the VEPP-2M collider. In the $\pi^+\pi^-$ channel a systematic uncertainty of 0.6% has been achieved. A Monte-Carlo Generator Photon Jets (MCGPJ) was developed to simulate events of Bhabha scattering as well as production of two charged pions, kaons and muons. Based on the formalism of structure functions, the leading logarithmic contributions related to the emission of photon jets in the collinear region are incorporated into the MC generator. Radiative corrections (RC) in the first order of α are accounted for exactly. The theoretical precision of the cross sections with RC is estimated to be better than 0.2%. Numerous tests of the program as well as a comparison with other MC generators and CMD-2 experimental data are presented.

1 Introduction

The cross sections of e^+e^- annihilation into hadrons are very important in various problems of particle physics and, in particular, they are required for the evaluation of the hadronic contribution to the anomalous magnetic moment

The goal of the new BNL experiment [5] is to measure the anomalous magnetic moment of the muon with the relative accuracy ~ 0.25 ppm. To reduce the current systematic error of the hadronic contribution to a_μ^{had} at least to the same level, the theoretical precision of the cross sections with radiative corrections (RC) should be better than 0.2% as it follows from a simple estimation: 60 ppm \times

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The MCGPJ generator is based on the papers from 1997
Theoretical support from Andrej Arbuzov and Eduard Kuraev (JINR)
From Novosibirsk it was lead by Gennadi Fedotovitch (BINP)
The code implementation by Alexey Sibidanov for CMD-2 experiment
F. Ignatov: maintenance and etc at CMD-3

Photons jet from initial/final e^+/e^- with collinear structure functions
+ exact NLO photon (pions in pointlike assumption)
VP table by NSK compilation
Declared precision $\sim 0.2\%$ for total cross section

Until now was only one available generator for $e^+e^- \rightarrow \pi^+\pi^- (\gamma)$
sufficiently precise for the scan measurement

BabaYaga 3.5 - doesn't have FSR, PS for ISR only
Phokhara - only NLO γ without FSR $\sim 1\%$ precision

$$e^+e^- \rightarrow e^+e^- (\gamma)$$

$$e^+e^- \rightarrow \mu^+\mu^- (\gamma)$$

$$e^+e^- \rightarrow \pi^+\pi^- (\gamma)$$

$$e^+e^- \rightarrow \tau^+\tau^- (\gamma)$$

$$e^+e^- \rightarrow K^+K^- (\gamma)$$

$$e^+e^- \rightarrow K_S K_L (\gamma)$$

$$e^+e^- \rightarrow \gamma\gamma(\gamma)$$

$$e^-e^- \rightarrow e^-e^- (\gamma)$$

Structure functions

Structure Function (SF) formalism based on paper:

E.Kuraev, V. Fadin, "On Radiative Corrections to $e^+ e^-$ Single Photon Annihilation at High-Energy"

[Sov.J.Nucl.Phys. 41 \(1985\) 466-472](#), [Yad.Fiz. 41 \(1985\) 733-742](#)

Used in most of $e^+e^- \rightarrow$ hadrons experimental measurements to take into account ISR radiative corrections

It consider one photon annihilation as Drell-Yan process with corresponding factorizations, Photon integrated emissions by probability $\mathcal{D}(z)$ function with help of DGLAP (...-Altarelli-Parisi-Lipatov)

evolution equations

$$d\sigma = \int dz_1 dz_2 \mathcal{D}(z_1) \mathcal{D}(z_2) \frac{d\tilde{\sigma}_0(z_1, z_2)}{|1 - \Pi(sz_1 z_2)|^2},$$

$$\mathcal{D}(z) = \delta(1-z) + \frac{\alpha}{2\pi} (L-1) P^{(1)}(z) + \left(\frac{\alpha}{2\pi}\right)^2 \frac{(L-1)^2}{2!} P^{(2)}(z) + \dots,$$

$$\mathcal{D}(z, s) = \mathcal{D}^\gamma(z, s) + \mathcal{D}^{e^+e^-}(z, s),$$

$$\begin{aligned} \mathcal{D}^\gamma(z, s) = & \frac{1}{2} b (1-z)^{\frac{b}{2}-1} \left[1 + \frac{3}{8} b + \frac{b^2}{16} \left(\frac{9}{8} - \frac{\pi^2}{3} \right) \right] \\ & - \frac{1}{4} b (1+z) + \frac{1}{32} b^2 \left(4(1+z) \ln \frac{1}{1-z} + \frac{1+3z^2}{1-z} \ln \frac{1}{z} - 5 - z \right), \quad b = \frac{2\alpha}{\pi} (L-1) \end{aligned}$$

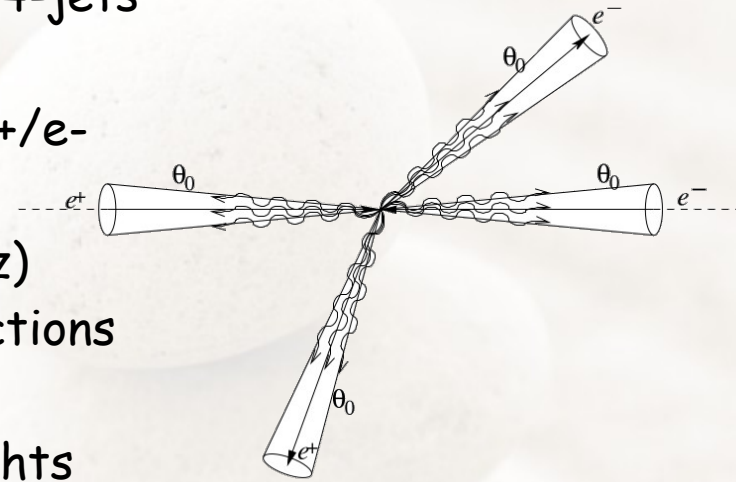
Includes next logarithmically enhanced corrections, but $\mathcal{D}(z)$ inclusive for photons in any directions

Generated as single photon "jet" collinear along lepton: **"collinear structure function"**

Jets + NLO photon matching

Have separate amplitudes either one-photon or factorized 4-jets

- × NLO photon ($\epsilon > \Delta$) is simulated upto narrow cone around e^+/e^-
- × NLO and $D(z)$ are matched by "compensators":
 - subtracting one photon out-of-cone contribution from $D(z)$
 - $\epsilon < \Delta$ jets are matched with one-photon soft+virtual corrections



Cones around leptons are helpful to deal with negative weights

Never was supposed to be used for ISR measurements:

With detected ISR photon at large angle - effectively just one-photon NLO amplitude
($D(z)$ - jets parts doesn't pass selections)

With undetected ISR photon at small angle - should be better but not sure how well
matched with selection by polar angle of SA photon system

N.B. for pure scan scenario NLO: jets can be switched off, but $\epsilon < \Delta$ jets vs one-photon matching need to be replaced by other piece of code - need to be checked consistency

Collinear jets limitation

Thanks to high statistics collected by **CMD-3**

It was observed a discrepancy in momentum distribution of experimental data vs theoretical spectra from **MCGPJ**

Important only for differential distributions in tails when two-photon kinematic selections play role.

Integrated cross section for scan scenario is unaffected at $\sim 0.06\%$.

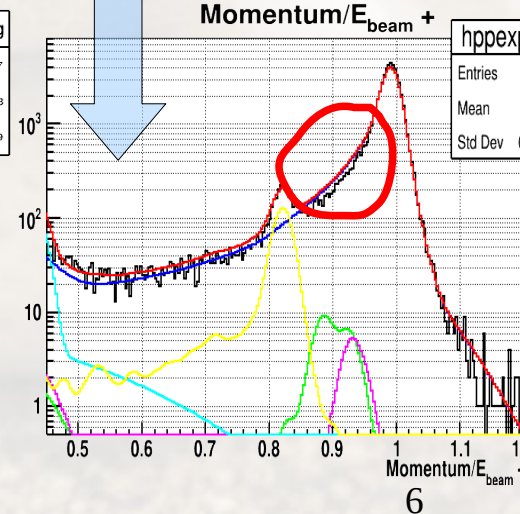
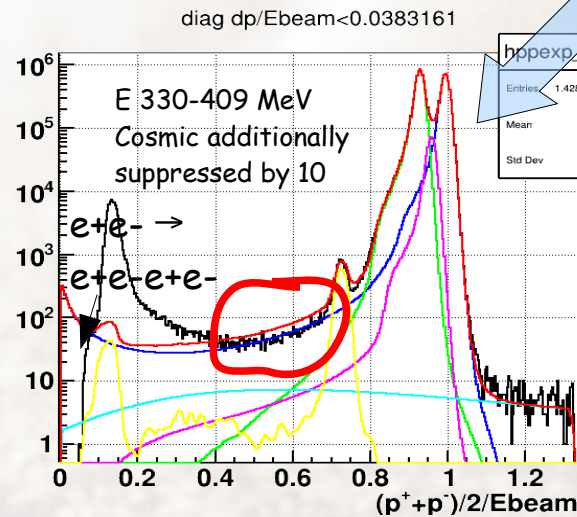
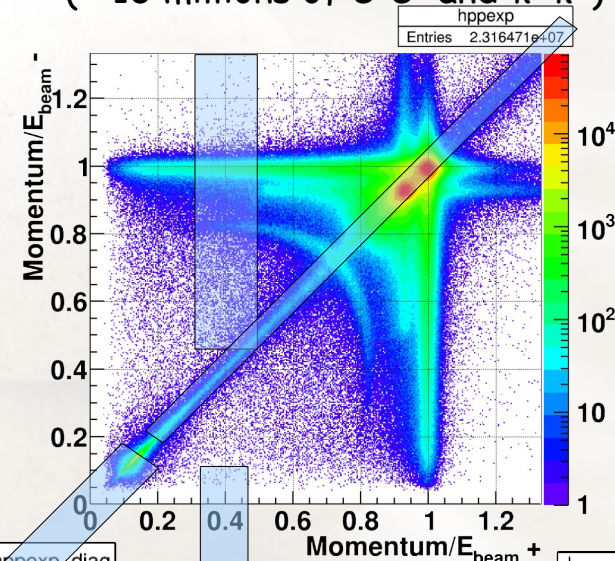
Comes from collinear jets approximation
photon jets angular distribution in one photon approximation
 (+ few other corrections):

$$f(c = \cos(\theta), x = \omega/E) \sim \frac{1}{pk} - \frac{x(1-x)}{1+(1-x)^2} \frac{m^2}{(pk)^2}$$

$$\sim \frac{1}{1-\beta c} - \frac{1-x}{1+(1-x)^2} * \frac{1-\beta^2}{(1-\beta c)^2}$$

MCGPJ Bhabha - jets with angles
 $\mu+\mu^- / \pi+\pi^-$ - in collinear SF approximation

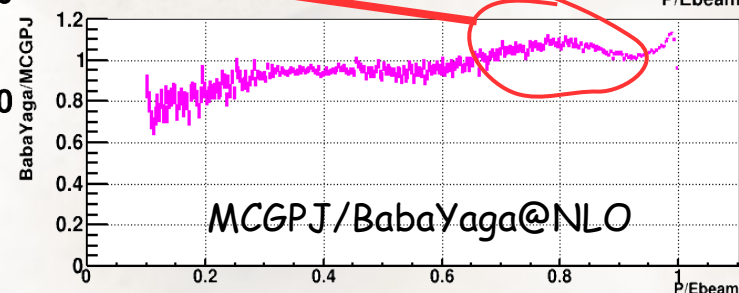
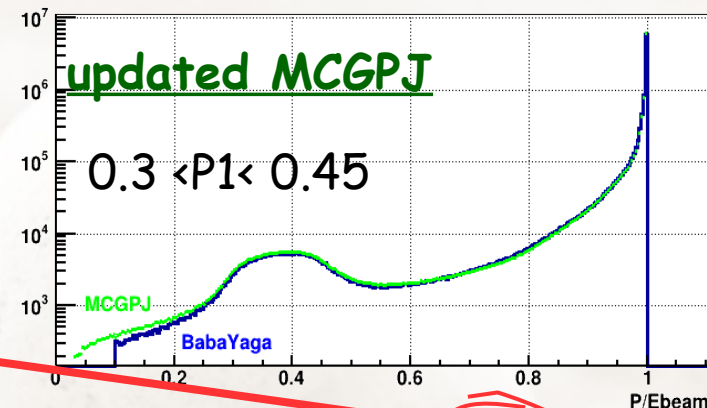
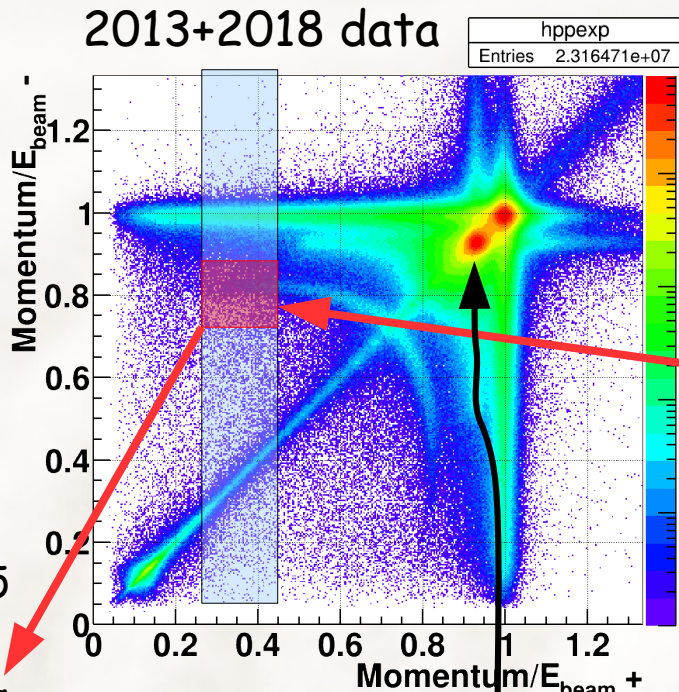
All events from RHO2013 scan
 (~ 10 millions of $e+e^-$ and $\pi+\pi^-$)



MCGPJ vs BabaYaga Bhabha spectrums

Updated MCGPJ/BabaYaga are inconsistent in tails at ~ 10% level for Bhabha

Can be looked region where no 2π events:
 $0.3 < P1 < 0.4$ & $0.75 < P2 < 0.85$
 (Ebeam < 375 MeV to suppress 3π)



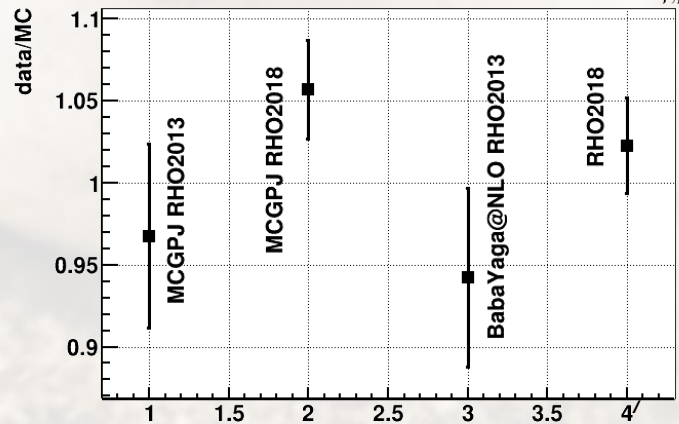
data/MC

MCGPJ 1.038 +/- 0.026

BabaYaga@NLO 1.006 +/- 0.026

It is necessary to have statistic ~ x10 more (or somehow to suppress 3π events)

for 2π analysis more crucial spectrum in another part, where pion peaks: $P1, P2 \sim 0.9 E_{beam}$



sQED limitations

Thanks to high statistics collected by CMD-3

It was observed a discrepancy in asymmetry from prediction

Integrated cross section for scan scenario is unaffected, but very important for study and control of systematics!

1% effect is disaster if we talk about ~0.1% precision

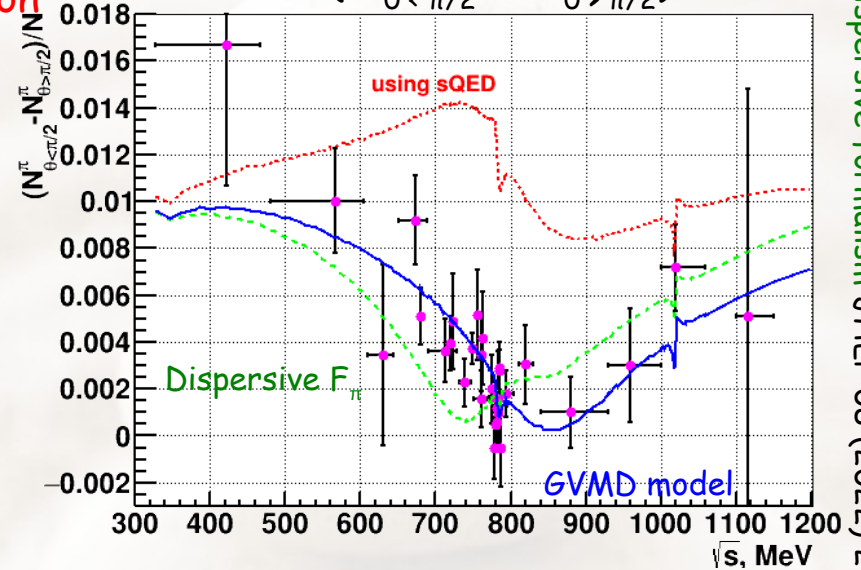
Comes from limitations of sQED approach

The theoretical model within GVMD was introduced, was confirmed by calculation in dispersive formalism

MCGPJ $\pi^+\pi^-$ - above sQED corrections can be used via pre-calculated tables $\delta_{FF}^V(s, \cos \theta)$ either from GVMD or dispersive paper

Phys.Rev.Lett. 132, 231903 (2024)

$$A = (N_{\theta < \pi/2} - N_{\theta > \pi/2})/N$$



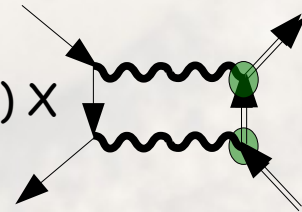
GVMD Phys.Lett.B 833 (2022) 137283
Dispersive formalism JHEP 08 (2022) 295

Implemented as correction to sQED:

$$d\sigma/dc = d\sigma_0/dc \times |F_{\pi}^2| \times (\delta_{sQED} + \delta_{FF})$$

$$\delta_{FF} \sim [F_{\pi}^{VMD}(q_1)F_{\pi}^{VMD}(q_2) - F_{\pi}^{VMD}(q)]/F_{\pi}^{VMD}(q) \times$$

δ_{FF} - IR finite, can be calculated separately as correction



Summary

Until now my recommendation list of generators for **the scan measurements**:

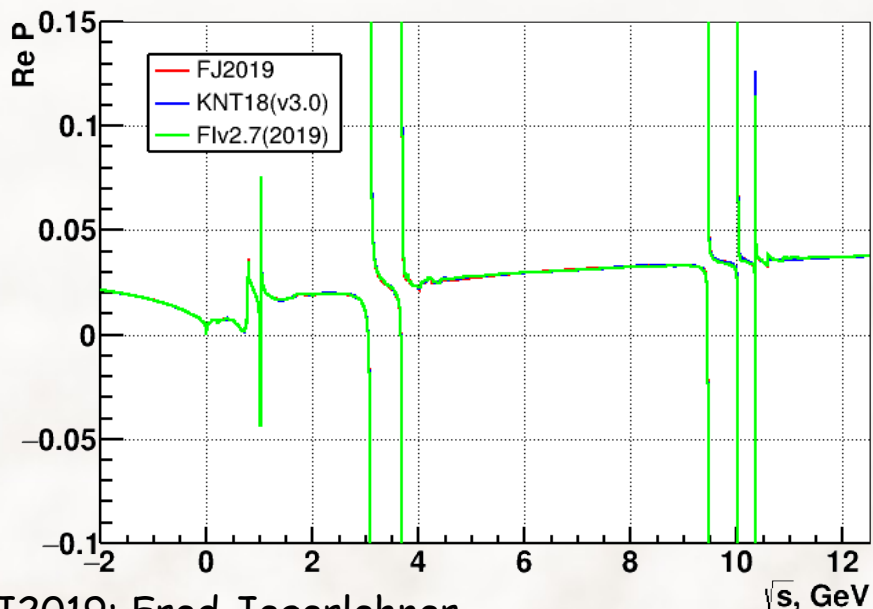
$e^+e^- \rightarrow e^+e^-(\gamma)$: BabaYaga@NLO (better consistency with data in
e⁺e⁻ asymmetry and momenta spectra)

$e^+e^- \rightarrow \mu^+\mu^-(\gamma)$: BabaYaga@NLO (differential cross section: parton shower γ with angles,
but no m_μ in FSR)

MCGPJ (integrated cross section - FSR with m_μ term)

$e^+e^- \rightarrow \pi^+\pi^-(\gamma)$: MCGPJ

Vacuum polarization



x FJ2019: Fred Jegerlehner

<http://www-com.physik.hu-berlin.de/~fjeger/software.html>

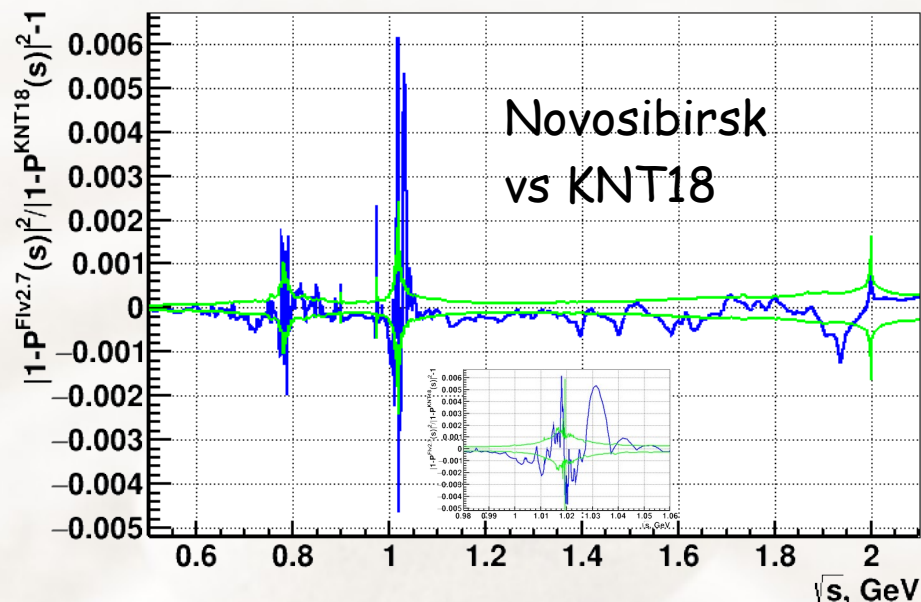
x KNT18(v3.0): A Keshavarzi, D Nomura, T Teubner

x FIv2.7(2019): Novosibirsk VP

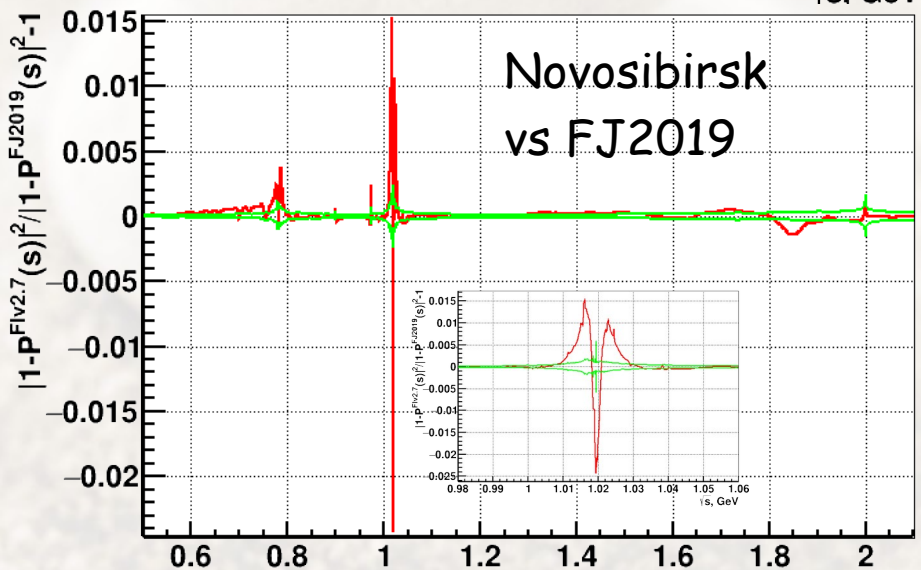
<https://cmd.inp.nsk.su/~ignatov/vpl/>

VP consistent at 0.05-0.1% outside of narrow resonances
At phi - statistical inconsistency ~0.5%, FJ up to 1.5-2.%

Fred is using dressed phi with PDG parameters
(should be bare M_ϕ , which is shifted by 254 keV)

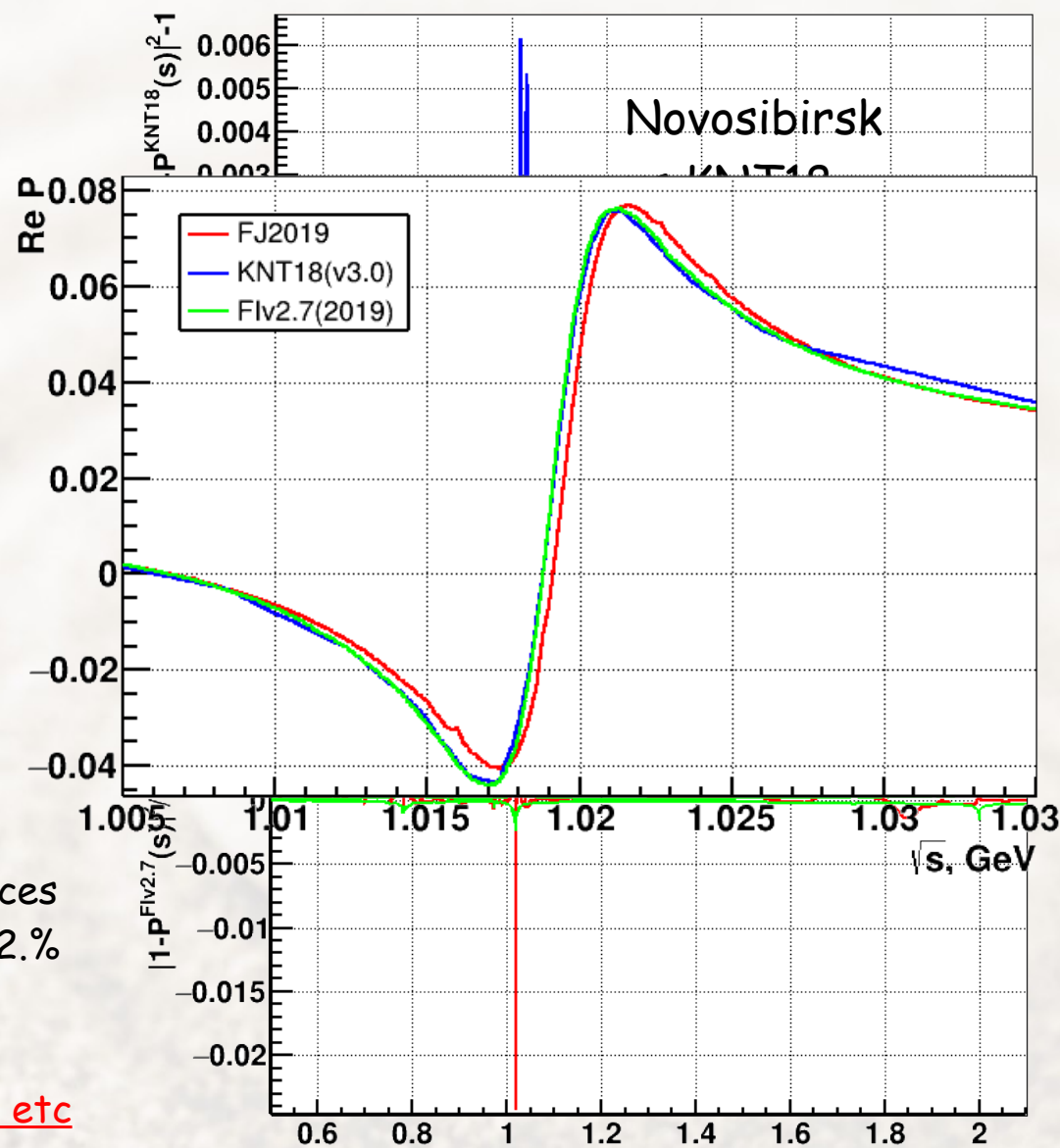
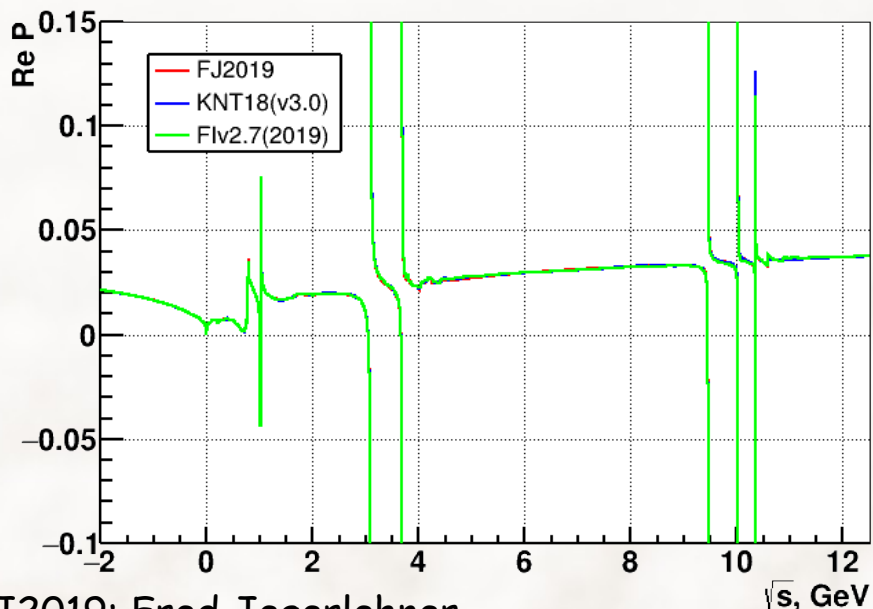


Novosibirsk
vs KNT18



Novosibirsk
vs FJ2019

Vacuum polarization



x FJ2019: Fred Jegerlehner

<http://www-com.physik.hu-berlin.de/~fjeger/software.html>

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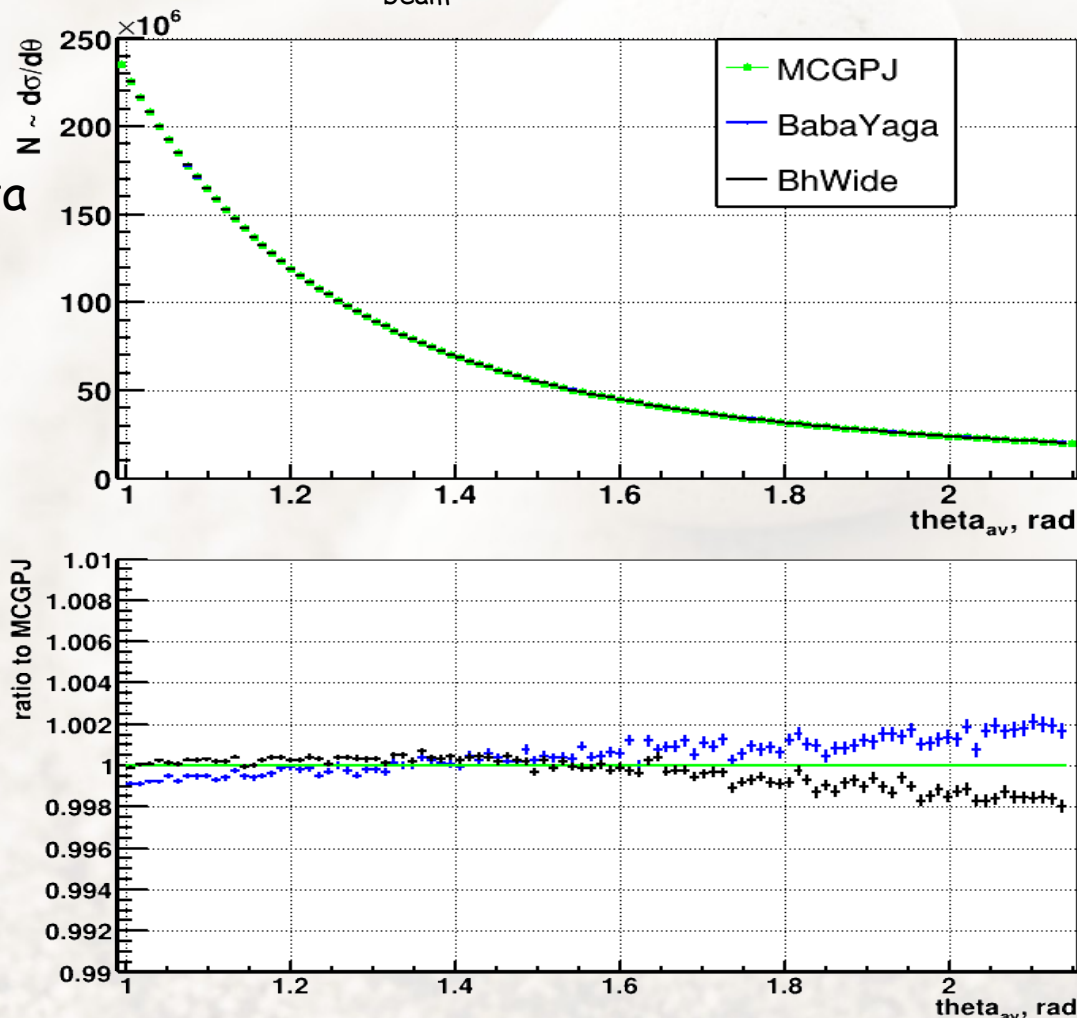
(should be bare M_ϕ , which is shifted by 254 keV)

Be careful with VP using at narrow resonances ϕ , J/ψ , etc

Other e+e- generators

Differential over angle spectrum comparison

$E_{\text{beam}} = 391.48 \text{ MeV}$



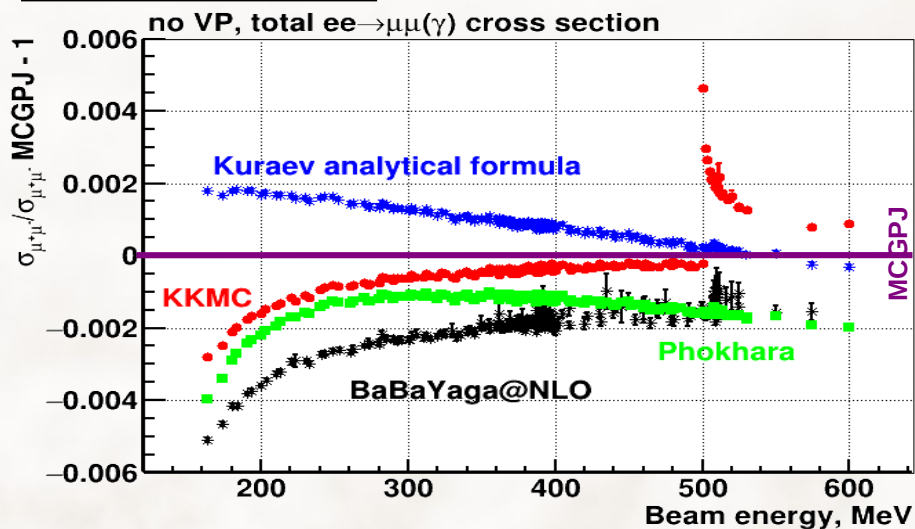
Differential cross section over theta consistent/or inconsistent at level $\sim 0.1-0.2\%$

But we are already sensitive to it in the asymmetry study with *CMD3* as shown in presentation yesterday

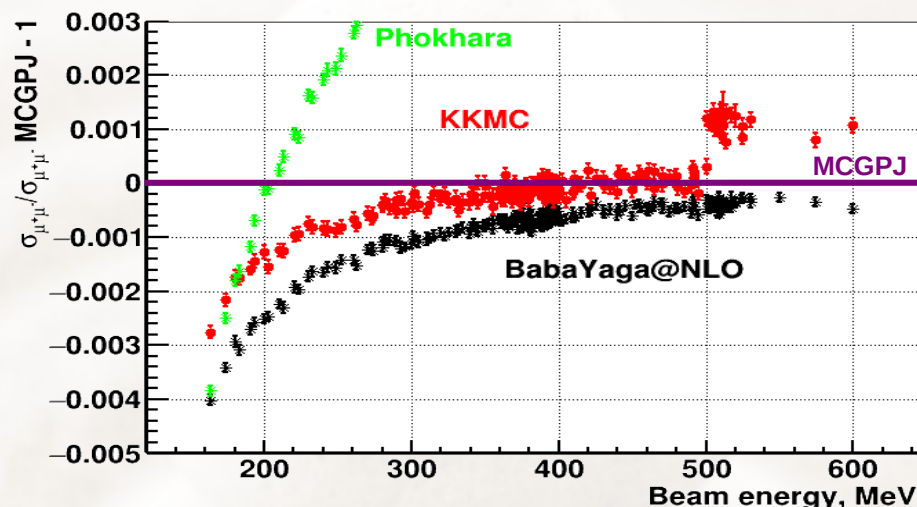
$e^+e^- \rightarrow \mu^+\mu^-(\gamma)$ cross-section

Comparison relative to MCGPJ, VP off

Total cross section



in CMD3 selection cuts



KKMC v 4.32, Phokhara v10.0, BabaYaga@NLO, MCGPJ
 KURAEV analytical formula for $e^+e^- \rightarrow \mu^+\mu^-(\gamma)$
 total cross-section: Phys.Rev.D72:114019,2005(arXiv:hep-ph/0505236)

KKMC was design for LEP energies
 MCGPJ for $\mu^+\mu^-$ is still without jets angular distribution
 Phokhara has limited precision for scanned mode (w/o ISR γ)

It is commonly used FSR correction in approx. with $E \gg M_\mu$:
 missed dependency $\delta_{FSR}^{virtual} \sim 2\alpha\pi/\beta_\mu$ with $\beta_\mu \rightarrow 0$

