$e^+e^- \to \pi^+\pi^-$ at NLO matched with PS in BabaYaga@NLO

The Evaluation of the Leading Hadronic Contribution to the Muon $g-2$.
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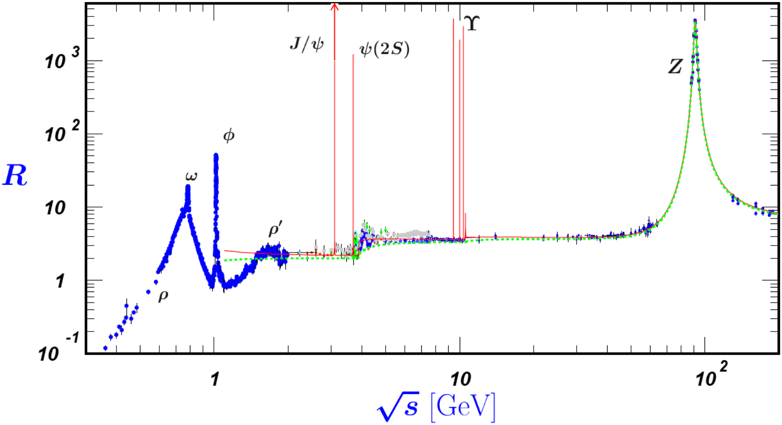
Outline

Introduction

Determination of the time-like Hadronic VP contribution to the g-2 of the muon

$$a_\mu^{\mathrm{HLO}} = \frac{\alpha}{\pi^2}\int_0^\infty \frac{\mathrm{d}s}{s} K(s) \left(\frac{\alpha(s)}{3} \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \right)$$

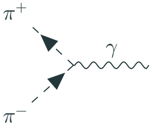


Scalar QED

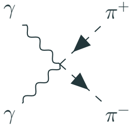
$e^+e^- \to \pi^+\pi^-$ can be described using scalar QED

sQED $\otimes$ QED Lagrangian

$$\mathcal{L}_{\text{sQED}}^{\text{int}} = -e\bar{\psi}\gamma^\mu\psi A_\mu - ieA_\mu\left(\phi^*\partial^\mu\phi - \partial^\mu\phi^*\phi\right) + e^2 A_\mu A^\mu \phi^*\phi$$



$= \quad -ie(p_- - p_+)$



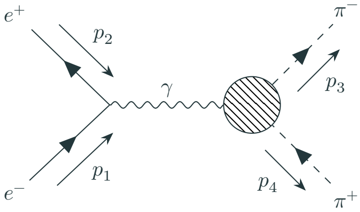
$= \quad 2ie^2 g^{\mu\nu}$

However pions do have an internal non perturbative structure $\Rightarrow$ Form Factor

- At **LO**, the VFF factorises
- At **NLO** the VFF model is crucial

Scalar QED

The tree level cross section is computed in the $F_\pi \times$ sQED approach



LO Cross section

$$\frac{\mathrm{d}\sigma^{\mathrm{LO}}}{\mathrm{d}\cos\theta} = \frac{\alpha^2\pi}{4s}\beta_\pi^3(1-\beta_e^2\cos^2\theta)|F_\pi(s)|^2$$

- Kinematics is kept **massive**
- The Form factor is factorised
- Symmetric for $\theta \to -\theta$

Forward-backward asymmetry

We can define the forward/backward (charge) asymmetry as

A_{FB}

$$A_{\text{FB}} = \frac{\sigma_B - \sigma_F}{\sigma_B + \sigma_F}$$

$$\sigma_F = \int_0^1 \mathrm{d}\sigma(\cos\theta), \quad \sigma_B = \int_{-1}^0 \mathrm{d}\sigma(\cos\theta)$$

The asymmetry is identically zero at LO but gets an NLO contribution from Initial Final State interference

$$\begin{aligned} A_{FB}^{\text{NLO}} =& A_{FB}^{\text{LO}} + \frac{\alpha}{\pi} A_{FB}^{\alpha} \\ =& 0 + \frac{\alpha}{\pi}\left(\frac{\sigma_B^{\text{odd}} - \sigma_F^{\text{odd}}}{\sigma^{\text{NLO}}}\right) \end{aligned}$$

Where the odd part is

$$\sigma^{\text{odd}} = \frac{\mathrm{d}\sigma_{\text{LO}}}{\mathrm{d}\cos\theta}(\delta_V + \delta_S) + \frac{\mathrm{d}\sigma_H}{\mathrm{d}\cos\theta}$$

NLO PS calculation in factorised sQED

NLO

The exact NLO cross section can be written as

NLO cross section

$$\sigma_{\text{NLO}} = \sigma_{2\to2} + \sigma_{2\to3} = \sigma_{\text{LO}} + \sigma_{\text{SV}} + \sigma_{\text{H}}\,,$$

where the splitting is given by

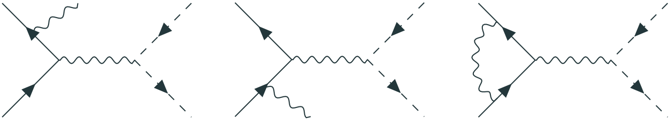
$$\sigma_{2\to2} = \frac{1}{F}\left\{\int \mathrm{d}\Phi_0 |\mathcal{M}_0|^2 + \int \mathrm{d}\Phi_0\, 2\text{Re}(\mathcal{M}_0^\dagger \mathcal{M}_{\text{V}}(\lambda))\right\} = \sigma_{\text{LO}} + \sigma_{\text{V}}(\lambda)\,,$$

$$\sigma_{2\to3} = \frac{1}{F}\left\{\mathrm{d}\Phi_1 |\mathcal{M}_{2\to3}|^2 + \int_{|\mathbf{k}|>\Delta E} \mathrm{d}\Phi_1 |\mathcal{M}_{2\to3}|^2\right\} = \sigma_{\text{soft}}(\lambda) + \sigma_{\text{H}}\,,$$

- $m_{\text{ph}}^2 = \lambda^2$ regularisation for IR divergences
- On-shell renormalisation of UV divergences
- Phase-space slicing for soft-hard bremsstrahlung

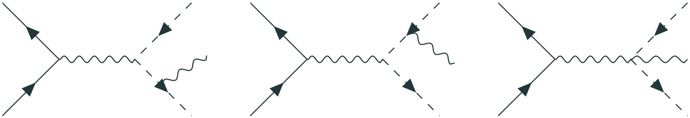
Initial State Radiation

$F_\pi(s) \times$

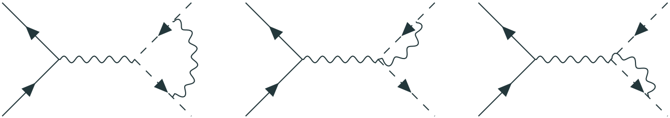


Final State Radiation

$F_\pi(s) \times$

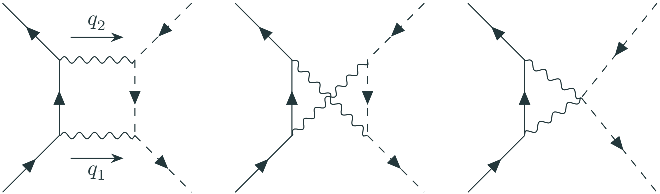


$F_\pi(s) \times$



Virtual IFI

For ISR and FSR the soft limit is clear. In IFI diagrams to which vertex we assign the form factor?



The $F_\pi \times$ sQED approach is justified because the IR divergence appears when

$$q_2 \to 0 \quad \Rightarrow \quad F(q_1^2) \to F(s), F(q_2^2) \to 1$$
$$q_1 \to 0 \quad \Rightarrow \quad F(q_2^2) \to F(s), F(q_1^2) \to 1$$

However the factorised prescription is valid only in the **soft limit**

The NLO is matched with a fully exclusive parton shower

PS master formula

$$d\sigma_{\text{matched}} = F_{\text{SV}}\, \Pi(\varepsilon, Q^2) \sum_{n=0}^{\infty} \frac{1}{n!} \left(\prod_{i=1}^{n} F_{\text{H},i} \right) |\mathcal{M}_n^{\text{PS}}|^2 d\Phi_n$$

For the $e^+e^- \to \pi^+\pi^-$ process, the Sudakov form factor $\Pi(\varepsilon, Q^2)$ is a combination of the scalar and spinor ones

$$\Pi(\varepsilon, Q^2) = \exp\left\{ -\frac{\alpha}{2\pi} \int_0^{1-\varepsilon} dz\, P(z) \int d\Omega_k\, \mathcal{I}(k) \right\} .$$

where the Altarelli Parisi vertex is given by

$$P_f(z) = \frac{1+z^2}{1-z}, \qquad P_s(z) = \frac{2\,z}{1-z} .$$

$$I_+^{\text{QED}}(\varepsilon) = \int_0^{1-\varepsilon} dz\, P_f(z) = -2\ln\varepsilon - \frac{3}{2} + 2\varepsilon - \frac{1}{2}\varepsilon^2 , \tag{1}$$

$$I_+^{\text{sQED}}(\varepsilon) = \int_0^{1-\varepsilon} dz\, P_s(z) = -2\ln\varepsilon - 2 + 2\varepsilon . \tag{2}$$

Handling the internal structure ot the pion

Dispersive Form factor[1]

Assuming the analiticity of $F_\pi(q^2)$ on the complex plane, one can write a dispersion relation

Dispersion Relation

$$F_\pi(q^2) = 1 - \frac{q^2}{\pi}\int_{4m_\pi^2}^{\infty} \mathrm{d}s' \, \frac{\mathrm{Im}F_\pi(s')}{s'}$$

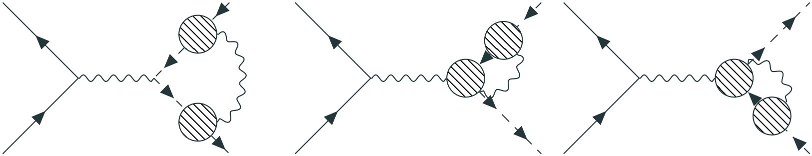
From analytical properties, one has the sum rule (satisfied for a certain cutoff Λ^2

$$\frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \frac{\mathrm{d}s'}{s'} \mathrm{Im}F_\pi(s') = 1.$$

[1]Gilberto Colangelo et al. "Radiative corrections to the forward-backward asymmetry in $e^+e^- \to \pi^+\pi^-$". In: *JHEP* 08 (2022), p. 295. DOI: 10.1007/JHEP08(2022)295. arXiv: 2207.03495 [hep-ph]

Inserting $F_\pi(q^2)$ in the FSR $\Rightarrow$ shift in renormalization constants.
The IR structure is unaffected.

$$2\text{Re}\frac{\mathcal{M}_0\mathcal{M}_{\text{FSR}}}{|\mathcal{M}_0|^2} = \frac{\alpha}{2\pi}\left(\text{Re}\delta^{\text{FSR}} + \delta Z_\phi\right)$$

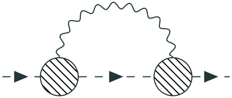


The tree level propagator is treated in the factorised approach $F_\pi(s) \times$ FSR. The other two VFF

$$\delta^{\text{FSR}} = \Bigg\{\delta^{\text{FSR}} - \frac{2}{\pi}\int\frac{\mathrm{d}s'}{s'}\text{Im}F_\pi(s')\delta^{\text{disp}}_{\text{FSR}}(s') + \frac{1}{\pi^2}\int\mathrm{d}s'\int\frac{\mathrm{d}s''}{s''}\frac{\text{Im}F_\pi(s')\text{Im}F_\pi(s'')}{s''-s'}\left(\delta^{\text{disp}}_{\text{FSR}}(s'')s'' - \delta^{\text{disp}}_{\text{FSR}}(s')s'\right)\Bigg\}$$

FSR: self energy

In the same way the self energy gets finite contributions

$\Sigma_\pi(p^2) \quad = $  $= e^2 \int \frac{\mathrm{d}^D q}{(2\pi)^D} \left\{ - \frac{(2p+q)^2 F_\pi^2(q^2)}{((q+p)^2 - m_\pi^2) q^2} \right\}$

$\int \mathrm{d}^D q \quad \longrightarrow \quad -\frac{\partial}{\partial p^2} \quad \longrightarrow \quad \int \frac{\mathrm{d}s'}{s'}$

$$\delta Z_\phi^{\text{point}} \equiv - \frac{\partial \Sigma_\pi(p^2, m_\pi^2, 0)}{\partial p^2}\Bigg|_{p^2 = m_\pi^2} \qquad \delta Z_\phi^{\text{disp}} \equiv - \frac{\partial \Sigma_\pi(p^2, m_\pi^2, s')}{\partial p^2}\Bigg|_{p^2 = m_\pi^2}$$

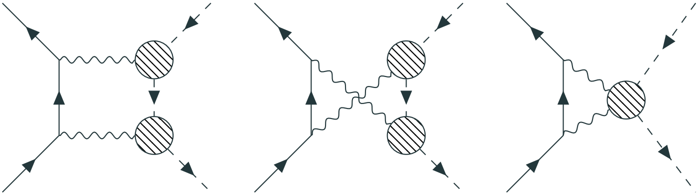
The complete counterterm in the dispersive approach reads

$$\delta Z_\phi = \Bigg\{ \delta Z_\phi^{\text{point}} - \frac{2}{\pi} \int \frac{\mathrm{d}s'}{s'} \mathrm{Im} F_\pi(s') \delta Z_\phi^{\text{disp}}(s') + \frac{1}{\pi^2} \int \mathrm{d}s' \int \frac{\mathrm{d}s''}{s''} \frac{\mathrm{Im} F_\pi(s') \mathrm{Im} F_\pi(s'')}{s'' - s'} \left(\delta Z_\phi^{\text{disp}}(s'') s'' - Z_\phi^{\text{disp}}(s') s' \right) \Bigg\}$$

Initial-Final interference

In the box we have two form factors evaluated at two different momenta

$$\begin{aligned}\delta_{\text{IFI}}^{\text{disp}} =&\delta(\lambda^2,\lambda^2) && \text{pole-pole}\\ &-\frac{1}{\pi}\int_{4m_\pi^2}^{\Lambda^2}\frac{\mathrm{d}s'}{s'}\,\mathrm{Im}F_\pi(s')\left[\delta(s',\lambda^2)+\delta(\lambda^2,s')\right] && \text{pole-dispersive}\\ &+\frac{1}{\pi^2}\int\int\frac{\mathrm{d}s'\mathrm{d}s''}{s's''}\,\mathrm{Im}F_\pi(s')\mathrm{Im}F_\pi(s'')\delta(s',s'') && \text{dispersive-dispersive}\end{aligned}$$



$$\frac{1}{|F_\pi(s)|^2}\frac{2\mathrm{Re}(\mathcal{M}_0^{\text{point}}\mathcal{M}_L^\dagger)}{|\mathcal{M}_0^{\text{point}}|^2}=\frac{1}{|F_\pi(s)|^2}\lim_{\lambda^2\to 0}\left(\mathrm{Re}F_\pi(s)\mathrm{Re}\delta+\mathrm{Im}F_\pi(s)\mathrm{Im}\delta\right)$$

Breaking down the correction

- **Pole Pole**

$$\delta(\lambda^2,\lambda^2) = \mathrm{Re}\delta^{\text{non-IR}}_{\text{pole-pole}} + \frac{2}{s}\delta(\lambda^2) + i\mathrm{Im}\delta_{\text{pole-pole}}$$

- **Pole Dispersive**

$$-\frac{2}{\pi}\int_{4m_\pi^2}^{\Lambda^2}\frac{\mathrm{d}s'}{s'}\mathrm{Im}F_\pi(s')\left(\underbrace{\mathrm{Re}\delta(s',\lambda^2) - \frac{1}{s-s'}\delta(\lambda^2)}_{\text{non-IR}} + \frac{1}{s-s'}\delta(\lambda^2) + i\mathrm{Im}\delta(s')\right)$$

The Box IR divergence from the pole-disp Box is given by

$$2\frac{\mathcal{M}_0^\dagger\mathcal{M}^\square(s,t,\lambda^2,s';q\to 0)}{|\mathcal{M}_0|^2} = \frac{\alpha}{\pi}\frac{2s}{s-s'}\left(m_e^2+m_\pi^2-t\right)\mathrm{C}_0\left(m_e^2,m_\pi^2,t,m_e^2,\lambda^2,m_\pi^2\right)$$

- **Dispersive Dispersive**

$$+\frac{1}{\pi^2}\int\int\frac{\mathrm{d}s'\mathrm{d}s''}{s's''}\mathrm{Im}F(s')\mathrm{Im}F(s'')\left(\mathrm{Re}\delta(s',s'') + i\mathrm{Im}\delta(s',s'')\right)$$

Pole Dispersive

$$
\begin{aligned}
\text{Re}\delta_{\text{IFI}}^{\text{disp}} = \text{Re}F_\pi(s)\Bigg(&\text{Re}\delta_{\text{pole-pole}}^{\text{non-IR}} + \frac{2}{s}\delta(\lambda^2) \\
&- \frac{2}{\pi}\text{PV}\int_{4m_\pi^2}^{\Lambda^2} \frac{\mathrm{d}s'}{s'}\text{Im}F_\pi(s')\left(\text{Re}\delta(s',\lambda^2) - \frac{1}{s-s'}\delta(\lambda^2)\right) \\
&+\frac{2}{s}\text{Re}F_\pi(s)\delta(\lambda^2) \quad -\frac{2}{s}\delta(\lambda^2) \\
&+ \frac{1}{\pi^2}\int\int \frac{\mathrm{d}s'\mathrm{d}s''}{s's''}\text{Im}F_\pi(s')\text{Im}F_\pi(s'')\text{Re}\delta(s',s'')\Bigg)
\end{aligned}
$$

The infrared divergence of the imaginary part comes from the integration over s'

$$
\begin{aligned}
\text{Im}\delta_{\text{tot}} = \text{Im}F_\pi(s)\Bigg(&\text{Im}\delta_{\text{pole-pole}} \\
&- \frac{2}{\pi}\int_{4m_\pi^2}^{s} \frac{\mathrm{d}s'}{s'}\left(\text{Im}F_\pi(s') - \text{Im}F_\pi(s)\right)\left(\text{Im}\delta(s')\right) \\
&-\frac{2}{\pi}\text{Im}F_\pi(s)\int_{4m_\pi^2}^{s}\frac{\mathrm{d}s'}{s'}\text{Im}\delta(s')\Bigg)
\end{aligned}
$$

After Passarino-Veltmann tensor reduction, the correction is proportional to

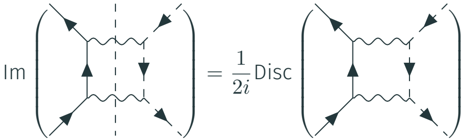
$$\begin{aligned}
&\mathsf{C}_0(m_e^2, m_e^2, s, s', m_e^2, s'')s(u-t) \\
&+\mathsf{C}_0(m_\pi^2, m_\pi^2, s, s', m_\pi^2, s'')\left(2m_\pi^2 - s\right)(t-u) \\
&\mathsf{D}_0(m_e^2, m_e^2, m_\pi^2, m_\pi^2, s, t, s', m_e^2, s'', m_\pi^2)\times \\
&\left(m_e^2 + m_\pi^2 - t\right)\left(m_e^2\left(2m_\pi^2 - t + u - s' - s''\right) + m_\pi^4 + (m_\pi - t)^2 + t\left(s' + s'' - u\right)\right) \\
&-\mathsf{D}_0(m_e^2, m_e^2, m_\pi^2, m_\pi^2, s, u, s', m_e^2, s'', m_\pi^2)\times \\
&\left(m_e^2 + m_\pi^2 - u\right)\left(m_e^2\left(2m_\pi^2 + t - u - s' - s''\right) + m_\pi^4 + (m_\pi - u)^2 + u\left(s' + s'' - t\right)\right)\Big\}
\end{aligned}$$

Pole dispersive: Imaginary part

We are always in the region $s > s'$. Imaginary part can be obtained cutting the 3- and 4-point functions

$$\mathrm{ImC_0}(m^2, m^2, s, s', m^2, \lambda^2) = \frac{\pi}{s\beta} \log \frac{1-\beta}{1+\beta}$$

To cut the 4-point diagram



$$\mathrm{ImD_0} = \frac{\pi}{\sqrt{\lambda(s,s',\lambda^2)(m_\pi^2 - m_e^2 - t)^2 - 4m_e^2 t) - 4\lambda^2 s' st}}$$

$$\log \frac{2\lambda^2 ss' + \lambda(s,s',\lambda^2)\left(m_e^2 + m_\pi^2 - t + \sqrt{\dfrac{\lambda(s,s',\lambda^2)(m_\pi^2 - m_e^2 - t)^2 - 4m_e^2 t) - 4\lambda^2 ss' t}{\lambda(s,s',\lambda^2)}}\right)}{2\lambda^2 ss' + \lambda(s,s',\lambda^2)\left(m_e^2 + m_\pi^2 - t - \sqrt{\dfrac{\lambda(s,s',\lambda^2)(m_\pi^2 - m_e^2 - t)^2 - 4m_e^2 t) - 4\lambda^2 ss' t}{\lambda(s,s',\lambda^2)}}\right)}$$

We neglect λ-polynomial terms in the $\log$ and $\mathcal{O}(\lambda^4)$

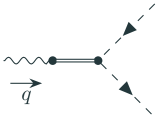
$$\mathrm{ImD}_0(t) = \frac{\pi}{\sqrt{(s-s')^2 - 2\lambda^2(s+s') - 4\lambda^2 s s' t/f}} \times \log\left(\frac{m_e^2 + m_\pi^2 - t + \sqrt{m_e^4 + (m_\pi^2 - t)^2 - 2m_e^2(m_\pi^2 + t)}}{m_e m_\pi}\right)$$

In the end, the IR divergence is reconstructed in the sum of the pole-dispersive contribution analytically cancels against the soft contribution

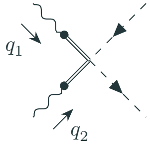
$$\delta_V^{\mathrm{IR}} = \left(\frac{(\mathrm{Re}F_\pi(s))^2 + (\mathrm{Im}F_\pi(s))^2}{|F_\pi(s)|^2}\right)\frac{2\alpha}{\pi}\log\frac{\lambda^2}{s}\log\frac{1-\beta\cos\theta}{1+\beta\cos\theta} = \delta_S^{\mathrm{IR}}$$

All the remaining integrals are kept numerical but are finite.
We achieve the IR cancellation event by event

In the GVMD approach, form factor is written as an additional propagator Andrea's Talk



$\times F_\pi(q^2)$



$\times F_\pi(q_1^2)F_\pi(q_2^2)$

GVMD Form factor

$$F_\pi(q^2) = \frac{1}{c_t}\sum_{v=0}^{n_r} c_v \frac{\Lambda_v^2}{\Lambda_v^2 - q^2}$$

$$\Lambda_v^2 = m_v^2 - i m_v \Gamma_v$$
$$c_v = |c_v| e^{i\phi_v}$$

[2] Fedor Ignatov and Roman N. Lee. "Charge asymmetry in $e^+e^- \to \pi^+\pi^-$ process". In: *Phys. Lett. B* 833 (2022), p. 137283. DOI: 10.1016/j.physletb.2022.137283. arXiv: 2204.12235 [hep-ph]

GVMD: Parameters fit

Parameters have been fitted with CMD-2 and CMD3-data

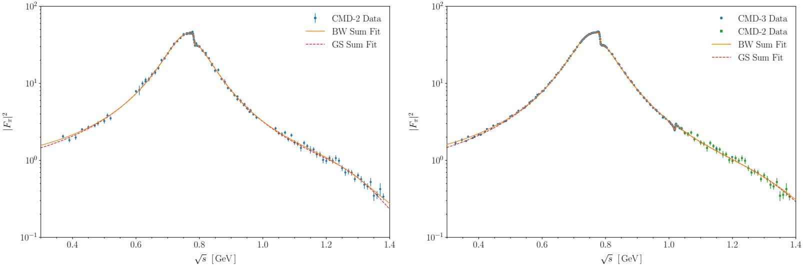
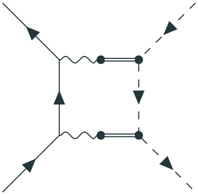


Figure 1: Fit of CMD-2 (left) and CMD-3 (right) form factors as a sum of BW functions, as required for the GVMD approach. The fit of both form factors with a more realistic sum of GS functions is reported for comparison.

In the one loop amplitudes, the GVMD form factor results in an additional propagator which can be simplified

$$\frac{1}{q^2-\lambda^2}\frac{1}{q^2-m^2} \to \frac{1}{m^2}\frac{1}{q^2-m^2} - \frac{1}{m^2}\frac{1}{q^2-\lambda^2}$$

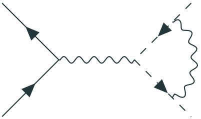


The correction for the IFI consists in an additional factor that vanishes in the soft limit

$$\delta^{\text{GVMD}} = \delta^{\text{point}} + \int \mathrm{d}q_1 \int \mathrm{d}q_2 \frac{F(q_1^2)F(q_2^2) - F(q^2)}{F(q^2)} \delta^{\text{point}}$$

Technical details and results

Coulomb correction[3]



$$\lim_{\beta_\pi \to 0} \delta_V^\pi = \frac{\alpha\pi}{2\beta_\pi}$$

$$\delta_V^\pi = \frac{\alpha}{\pi}\left\{\frac{\beta_\pi^2+1}{\beta_\pi}\left[\mathrm{Li}_2\left(\frac{1-\beta_\pi}{1+\beta_\pi}\right)+\frac{\pi^2}{3}\right]-2\right\}$$
$$-\frac{\alpha}{\pi}\frac{(\beta_\pi^2+1)}{2\beta_\pi}\log\left(\frac{1-\beta_\pi}{1+\beta_\pi}\right)\left[\frac{1}{2}\log\left(\frac{1-\beta_\pi^2}{4\beta_\pi^2}\right)+\log\left(\frac{\beta_\pi+1}{2\beta_\pi}\right)+2\right]+\text{IR terms}$$

The Sommerfeld factor is present in the vertex virtual correction. It can be resummed and matched as

$$\sigma^{\mathrm{NLO}} = \left(S-1+\delta_{SV}-\frac{z}{2}\right)\sigma^{\mathrm{LO}} \qquad S=\frac{z}{1-e^{-z}}, \qquad z=\frac{2\pi\alpha}{v}$$

[3]Sommerfeld, “Über die Beugung und Bremsung der Elektronen”,Arbuzov and Kopylova, “On relativization of the Sommerfeld-Gamow-Sakharov factor”

Form factors and cuts

The form factor can be chosen from the following list

Data	$\Lambda^2[\text{GeV}^2]$	$F_\pi \times$ sQED	GVMD	Disp
CMD-3	4	✓		✓
CMD-2	2	✓		✓
BW sum 2/3	$2/4$	✓	✓	✓
SND	1	✓		✓
Babar	9	✓		✓
Strong2020	9	✓		✓
BesIII	9	✓		✓
Kloe2	1	✓		✓
Phokhara	16	✓		✓
Bern	4	✓		✓

The cuts we have used are inspired by CMD-3. However one can compile from the `Makefile` the experimental setups of KLOE,BESIII, ecc

obs		$p_\pm$		ϑ_{avg}		$\delta\vartheta$		$\delta\phi$	
min	max	$0.45E$	$\beta_\pi\sqrt{s}/2$	$\pi\,\text{rad}$	$\pi - 1\,\text{rad}$	$0\,\text{rad}$	$0.25\,\text{rad}$	$0\,\text{rad}$	$0.15\,\text{rad}$

Table 1: Kinematical cuts that are inspired by the CMD-3 event selection criteria.

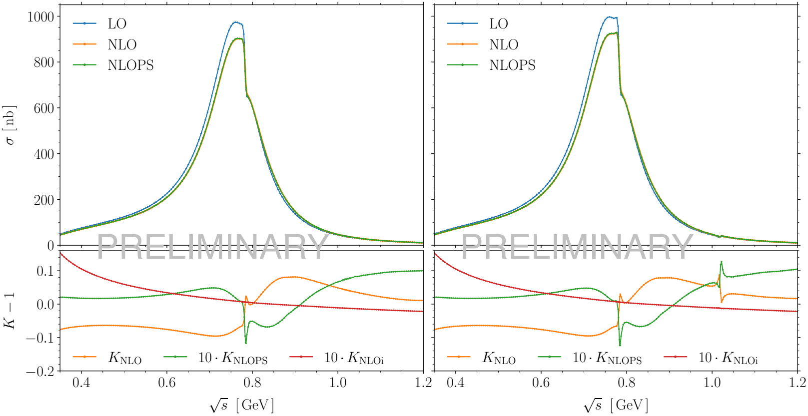


Figure 2: Total cross section as a function of the CoM energy. To the left, CMD-2 form factor. To the right CMD-3

Differential distributions

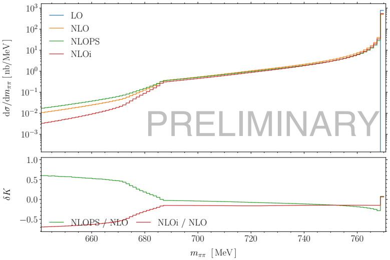


Figure 3: Invariant mass at $\sqrt{s} = 0.77$GeV

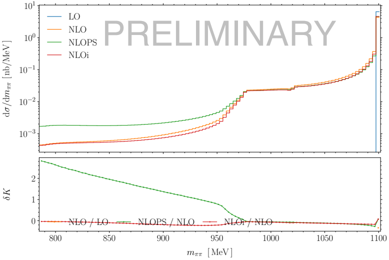
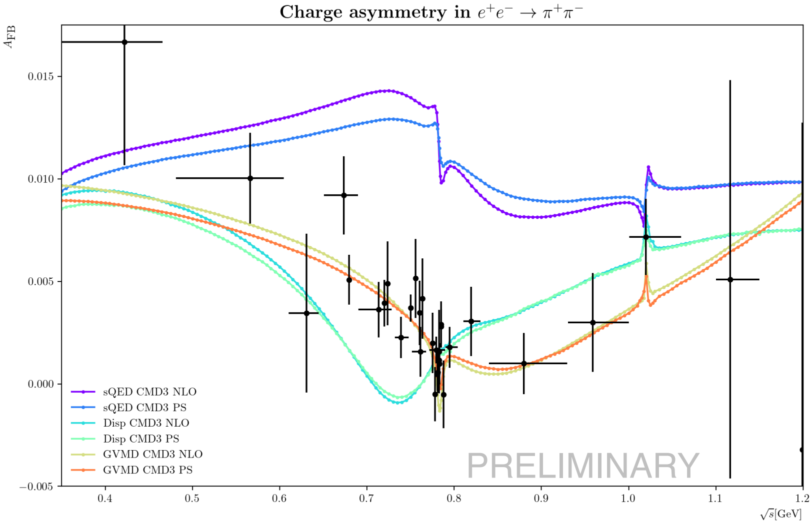


Figure 4: Invariant mass at $\sqrt{s} = 1.1$GeV

Asymmetry


Charge asymmetry in $e^+e^- \to \pi^+\pi^-$
A_{FB}
0.015
0.010
0.005
0.000
−0.005
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
$\sqrt{s}$[GeV]
sQED CMD3 NLO
sQED CMD3 PS
Disp CMD3 NLO
Disp CMD3 PS
GVMD CMD3 NLO
GVMD CMD3 PS
PRELIMINARY

State of BabaYaga

- As of today, we can generate $e^+e^- \to \pi^+\pi + -$ at NLO and NLO PS with **all masses**

Approach	NLO	NLO PS
$F_\pi \times$ sQED	✓	✓
GVMD	✓	✓
Dispersive	w.i.p.	w.i.p.

- In the next future, we will calculate $e^+e^- \to \pi^+\pi^-\gamma(+n\gamma)$
- Comparisons with other Monte Carlo generator will be done (hopefully) for Strong2020