The BabaYaga@NLO event generator

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The Evaluation of the Leading Contribution to the Muon $g-2$: Consolidation of the MUonE Experiment and Recent Developments in Low Energy e^+e^- Data

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"historical" BabaYaga authors
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+
L. Barzè, G. Balossini, C. Bignamini (in the past)
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⋆ Webpage

http://www.pv.infn.it/hepcomplex/babayaga.html

(or better ask the authors!)

⋆ **BabaYaga** core references:

- Barzè et al., Eur. Phys. J. C **71** (2011) 1680 — BabaYaga with dark photon
- Balossini et al., Phys. Lett. **663** (2008) 209 — BabaYaga@NLO for $e^+e^- \to \gamma\gamma$
- Balossini et al., Nucl. Phys. **B758** (2006) 227 — BabaYaga@NLO for Bhabha
- C.M. Carloni Calame et al., Nucl. Phys. Proc. Suppl. **131** (2004) 48 — BabaYaga@NLO
- C.M. Carloni Calame, Phys. Lett. B **520** (2001) 16 — improved PS BabaYaga
- C.M. Carloni Calame et al., Nucl. Phys. B **584** (2000) 459 — BabaYaga

⋆ Related work:

- S. Actis et al.
  "Quest for precision in hadronic cross sections at low energy: Monte Carlo tools vs. experimental data", Eur. Phys. J. C **66** (2010) 585
  Report of the Working Group on Radiative Corrections and Monte Carlo Generators for Low Energies
- C.M. Carloni Calame et al., JHEP **1107** (2011) 126
  NNLO massive pair corrections

Reference processes for luminosity

- Instead of getting the luminosity from machine parameters, it's more effective to exploit the relation

$$\sigma = \frac{N}{L} \quad \rightarrow \quad L = \frac{N_{\rm ref}}{\sigma_{\rm theory}} \qquad \frac{\delta L}{L} = \frac{\delta N_{\rm ref}}{N_{\rm ref}} \oplus \frac{\delta \sigma_{\rm theory}}{\sigma_{\rm theory}}$$

- Reference (normalization) processes are required to have a clean topology, high statistics and be calculable with high theoretical accuracy

⋆ Large-angle QED processes as $e^+e^- \to e^+e^-$ (Bhabha), $e^+e^- \to \gamma\gamma$, $e^+e^- \to \mu^+\mu^-$ are golden processes at flavour factories to achieve a typical precision at the level of $1 \div 0.1\%$

↪ QED radiative corrections are mandatory

↦ **BabaYaga** has been developed for high-precision simulation of QED processes at flavour factories (primarily for luminosity determination)

Overall accuracy of the MC: NLOPS

- ⋆ Typically exact $\mathcal{O}(\alpha)$ (NLO) photonic corrections are matched with higher-order leading logarithmic contributions [multiple photon corrections]

  [+ vacuum polarization, using a data driven routine for the calculation of the non-perturbative $\Delta\alpha^{(5)}_{\rm had}(q^2)$ hadronic contribution]

- ⋆ Common methods used to account for multiple photon corrections are the analytical collinear QED Structure Functions (SF), YFS exponentiation and QED Parton Shower (PS)

- The QED PS [implemented in BabaYaga/BabaYaga@NLO] is an exact MC solution of the QED DGLAP equation for the non-singlet electron SF $D(x, Q^2)$

$$Q^2 \frac{\partial}{\partial Q^2} D(x, Q^2) = \frac{\alpha}{2\pi} \int_x^1 \frac{dt}{t} P_+(t) D(\frac{x}{t}, Q^2)$$

- The PS solution can be cast into the form

$$D(x, Q^2) \; = \; \Pi(Q^2) \sum_{n=0}^{\infty} \; \int \frac{\delta(x - x_1 \cdots x_n)}{n!} \; \prod_{i=0}^{n} \left[\frac{\alpha}{2\pi} P(x_i) \; L \; dx_i \right]$$

  → $\Pi(Q^2) \equiv e^{-\frac{\alpha}{2\pi} L I_+}$ Sudakov form factor, $I_+ \equiv \int_0^{1-\epsilon} P(x) dx$, $L \equiv \ln Q^2/m^2$ collinear log, ϵ soft–hard separator and Q^2 virtuality scale

  → the kinematics of the photon emissions can be recovered → exclusive photons generation

- The accuracy is improved by matching exact NLO with higher-order leading log corrections
  - ⋆ theoretical error starts at $\mathcal{O}(\alpha^2)$ (NNLO) QED corrections, for all QED channels [Bhabha, $\gamma\gamma$ and $\mu^+\mu^-$]

Matching NLO and PS in BabaYaga@NLO

Exact $\mathcal{O}(\alpha)$ (NLO) soft+virtual (SV) corrections and hard-bremsstrahlung (H) matrix elements can be combined with QED PS *via* a matching procedure

- $d\sigma_{PS}^{\infty} = \Pi(Q^2,\varepsilon)\ \sum_{n=0}^{\infty} \frac{1}{n!}\ |\mathcal{M}_{n,PS}|^2\ d\Phi_n$
- $d\sigma_{PS}^{\alpha} = [1 + C_{\alpha,PS}]\,|\mathcal{M}_0|^2 d\Phi_2 + |\mathcal{M}_{1,PS}|^2 d\Phi_3 \equiv d\sigma_{PS}^{SV}(\varepsilon) + d\sigma_{PS}^{H}(\varepsilon)$
- $d\sigma_{\rm NLO}^{\alpha} = [1 + C_{\alpha}]\,|\mathcal{M}_0|^2 d\Phi_2 + |\mathcal{M}_1|^2 d\Phi_3 \equiv d\sigma_{\rm NLO}^{SV}(\varepsilon) + d\sigma_{\rm NLO}^{H}(\varepsilon)$
- $F_{SV} = 1 + (C_\alpha - C_{\alpha,PS}) \qquad F_H = 1 + \frac{|\mathcal{M}_1|^2 - |\mathcal{M}_{1,PS}|^2}{|\mathcal{M}_{1,PS}|^2}$

$$d\sigma_{\rm matched}^{\infty} = F_{SV}\ \Pi(Q^2,\varepsilon)\ \sum_{n=0}^{\infty} \frac{1}{n!}\ \left(\prod_{i=0}^{n} F_{H,i}\right)\ |\mathcal{M}_{n,PS}|^2\ d\Phi_n$$

$d\Phi_n$ is the exact phase space for n final-state particles

(2 fermions + an arbitrary number of photons)

Any approximation is confined into matrix elements

- F_{SV} and $F_{H,i}$ are infrared/collinear safe and account for missing $\mathcal{O}(\alpha)$ non-logs, avoiding double counting of leading-logs
- $\left[\sigma^{\infty}_{matched}\right]_{\mathcal{O}(\alpha)} = \sigma^{\alpha}_{\mathrm{NLO}}$
- resummation of higher orders LL (PS) contributions is preserved
- the cross section is still fully differential in the momenta of the final state particles (e^+, e^- and $n\gamma$)
  (F's correction factors are applied on an event-by-event basis)
- as a by-product, part of photonic $\alpha^2 L$ included by means of terms of the type $F_{SV\ |\ H,i} \otimes$ [leading-logs]

G. Montagna et al., **PLB** 385 (1996)

- the theoretical error is shifted to $\mathcal{O}(\alpha^2)$ (NNLO, 2 loop) not infrared, singly collinear terms: very naively and roughly (for photonic corrections)

$$\frac{1}{2}\alpha^2 L \equiv \frac{1}{2}\alpha^2 \log\frac{s}{m_e^2} \sim 5 \times 10^{-4}$$

Loosely and schematically, the corrections to the LO cross section can be arranged as (collinear log $L \equiv \log \frac{s}{m_e^2}$)

LO	α^0		
NLO	αL	α	
NNLO	$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$
h.o.	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^n$	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^{n-1}$	$\ldots$

Blue: Leading-Log PS, Leading-Log YFS, SF

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Red: matched PS, YFS, SF + NLO

Summary of QED (photonic) radiative corrections

Loosely and schematically, the corrections to the LO cross section can be arranged as (collinear log $L \equiv \log \frac{s}{m_e^2}$)

LO	90%		
NLO	10%	0.5%	
NNLO	0.5%	0.05%	0.01%
h.o.	0.01%	$\cdots$	$\cdots$

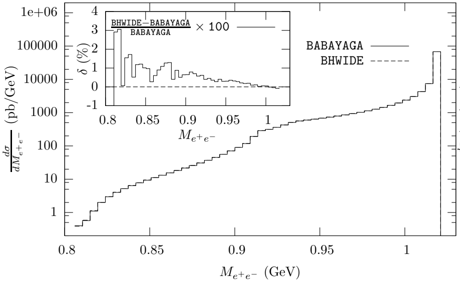
Tipically at flavour factories (on integrated Bhabha σ)

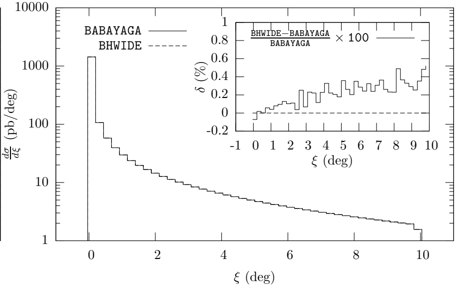
Estimating the theoretical accuracy

S. Actis et al. Eur. Phys. J. C **66** (2010) 585

"Quest for precision in hadronic cross sections at low energy: Monte Carlo tools vs. experimental data"

- **It is extremely important to compare independent calculations/implementations/codes, in order to**
  - ↦ asses the technical precision, spot bugs (with the same th. ingredients)
  - ↦ estimate the theoretical error when including partial/incomplete higher-order corrections
- E.g. comparison `BabaYaga@NLO` vs. `Bhwide` at KLOE





Without vacuum polarization, to compare QED corrections consistently

At the Φ and τ–charm factories (cross sections in nb)

By BabaYaga group, Ping Wang and A. Sibidanov

setup	BabaYaga@NLO	BHWIDE	MCGPJ	$\delta(\%)$
$\sqrt{s} = 1.02$ GeV, $20^\circ \leq \vartheta_\mp \leq 160^\circ$	6086.6(1)	6086.3(2)	—	0.005
$\sqrt{s} = 1.02$ GeV, $55^\circ \leq \vartheta_\mp \leq 125^\circ$	455.85(1)	455.73(1)	—	0.030
$\sqrt{s} = 3.5$ GeV, $\lvert\vartheta_+ + \vartheta_- - \pi\rvert \leq 0.25$ rad	35.20(2)	—	35.181(5)	0.050

$\mapsto$ Agreement well below 0.1%

At BaBar (cross sections in nb)

By A. Hafner and A. Denig

angular acceptance cuts	BabaYaga@NLO	BHWIDE	$\delta(\%)$
$15^\circ \div 165^\circ$	119.5(1)	119.53(8)	0.025
$40^\circ \div 140^\circ$	11.67(3)	11.660(8)	0.086
$50^\circ \div 130^\circ$	6.31(3)	6.289(4)	0.332
$60^\circ \div 120^\circ$	3.554(6)	3.549(3)	0.141

$\mapsto$ Agreement at the $\sim 0.1\%$ level

Theoretical accuracy, comparisons with NNLO

- NLO RC being included, the theoretical error starts at $\mathcal{O}(\alpha^2)$ (NNLO)
  ↪ anyway large NNLO RC already included by exponentiation (and by $\mathcal{O}(\alpha)$ PS $\times$ non-log-NLO)

⋆ The full set of NNLO QED corrections to Bhabha scattering are known

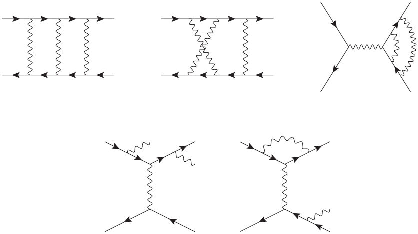
- `BabaYaga@NLO` formulae can be truncated at $\mathcal{O}(\alpha^2)$ to be consistently and systematically compared with all the classes of NNLO corrections

$$\sigma^{\alpha^2} = \sigma^{\alpha^2}_{\rm SV} + \sigma^{\alpha^2}_{\rm SV,H} + \sigma^{\alpha^2}_{\rm HH}$$

  - $\sigma^{\alpha^2}_{\rm SV}$: soft+virtual photonic corrections up to $\mathcal{O}(\alpha^2)$
    ↦ compared with the corresponding available NNLO QED calculation
  - $\sigma^{\alpha^2}_{\rm SV,H}$: one–loop soft+virtual corrections to single hard bremsstrahlung
    ↦ estimated relying on existing (partial) results
  - $\sigma^{\alpha^2}_{\rm HH}$: double hard bremsstrahlung
    ↦ compared with the exact $e^+e^- \to e^+e^-\gamma\gamma$ cross section, to register really negligible differences (at the 1×10^{-5} level)

NNLO Bhabha calculations

- Photonic corrections A. Penin, PRL **95** (2005) 010408 & Nucl. Phys. **B734** (2006) 185

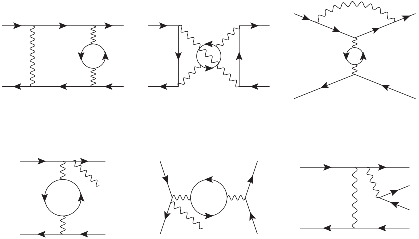


here real γ is "soft"

- Electron loop corrections

R. Bonciani *et al.*, Nucl. Phys. **B701** (2004) 121 & Nucl. Phys. **B716** (2005) 280

S. Actis, M. Czakon, J. Gluza and T. Riemann, Nucl. Phys. **B786** (2007) 26



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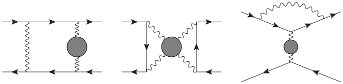
NNLO Bhabha calculations

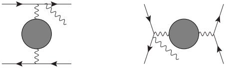
- Heavy fermion and hadronic loops

R. Bonciani, A. Ferroglia and A. Penin, PRL **100** (2008) 131601

S. Actis, M. Czakon, J. Gluza and T. Riemann, PRL **100** (2008) 131602

J.H. Kühn and S. Uccirati, Nucl. Phys. **B806** (2009) 300

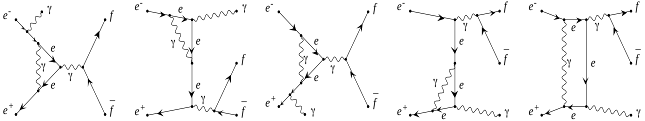




here real γ is "soft"

- One-loop soft+virtual corrections to single hard bremsstrahlung

S. Actis, P. Mastrolia and G. Ossola, Phys. Lett. **B682** (2010) 419



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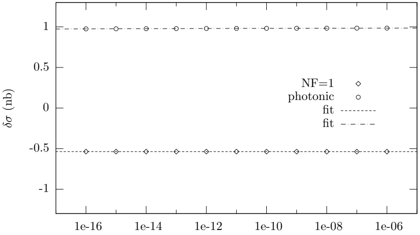
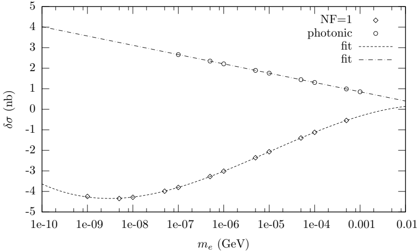
Comparison with NNLO calculation for $\sigma_{\rm SV}^{\alpha^2}$

Using realistic cuts for luminosity at KLOE

Comparison of $\sigma_{\rm SV}^{\alpha^2}$ calculation of `BabaYaga@NLO` with

- Penin (photonic): function of the logarithm of the soft photon cut–off (left plot) and a fictitious electron mass (right plot)





★ differences are infrared safe$^\varepsilon$, as expected

★ $\delta\sigma(\text{photonic})/\sigma_0 \propto \alpha^2 L$, as expected

- Numerically, for various selection criteria at the Φ and B factories

$$\sigma_{\rm SV}^{\alpha^2}(\text{Penin}) - \sigma_{\rm SV}^{\alpha^2}(\texttt{BabaYaga@NLO}) < 0.02\% \times \sigma_0$$

Summary

- ⋆ In the last ~25 years BabaYaga/BabaYaga@NLO has been developed for high-precision luminometry at flavour factories

- ⋆ It simulates QED processes
  $\hookrightarrow e^+e^- \to e^+e^- \quad (+n\gamma)$
  $\hookrightarrow e^+e^- \to \mu^+\mu^- \ (+n\gamma)$
  $\hookrightarrow e^+e^- \to \gamma\gamma \qquad (+n\gamma)$

  with multiple-photon emission in a QED Parton Shower framework, matched with exact NLO matrix elements

- ⋆ A theoretical precision at the 0.5×10^{-3} level is achieved (at least for Bhabha), with a systematic comparison to independent calculations/codes and assessing the size of missing higher-order corrections

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- Looking ahead: work in progress and future improvements
  - addition of pion final state for energy scan $\pi^+\pi^-(+n\gamma)$

    $\Longrightarrow$ talk by Francesco Pio Ucci
  - addition of radiative return channels
    - $\pi^+\pi^-\gamma(+n\gamma)$
    - $\mu^+\mu^-\gamma(+n\gamma))$
  - going beyond NLOPS
  - inclusion of weak corrections (for higher energies)