

Monte Carlo codes overview and news for Strong2020



MCMULE
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for the MCMULE team

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Monte Carlo for **MU**ons and other **LE**ptons (mule-tools.gitlab.io)

- integrator (generator WIP) for fixed-order QED up to NNLO
- use QCD methods: FKS^ℓ subtraction with massive fermions

$$\underbrace{\int d\Phi_\gamma}_{\text{divergent and complicated}} \text{ (diagram with grey blob)} = \underbrace{\int d\Phi_\gamma}_{\text{complicated but finite}} \left(\text{diagram with grey blob} - \text{diagram with green blob} \right) + \underbrace{\int d\Phi_\gamma}_{\text{divergent but easy}} \text{ (diagram with green blob)}$$

- **challenge** virtual amplitudes with $m \neq 0 \implies$ massification
- **challenge** numerical instabilities \implies next-to-soft stabilisation
- for details: [\[see talk by Marco\]](#)



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- **challenge** virtual amplitudes with $m \neq 0 \implies$ massification

$$\mathcal{A}(m) = \left(\prod_j \sqrt{Z(m)} \right) \times S \times \mathcal{A}(m=0) + \mathcal{O}(m)$$

- **challenge** numerical instabilities \implies next-to-soft stabilisation
- for details: [\[see talk by Marco\]](#)



Monte Carlo for MUons and other LEptons (mule-tools.gitlab.io)

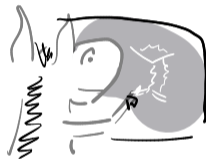
- integrator (generator WIP) for fixed-order QED up to NNLO
- use QCD methods: FKS^ℓ subtraction with massive fermions
- challenge virtual amplitudes with $m \neq 0 \implies$ massification
- challenge numerical instabilities \implies next-to-soft stabilisation

$$\begin{array}{c}
 \text{Diagram with wavy line} \\
 \xrightarrow{E_\gamma \rightarrow 0} \\
 \underbrace{\frac{1}{E_\gamma^2} \mathcal{E} \text{ Diagram}}_{\text{eikonal}} + \underbrace{\frac{1}{E_\gamma} (\mathcal{D} + \mathcal{S}) \text{ Diagram}}_{\text{next-to-soft}} + \mathcal{O}(E_\gamma^0)
 \end{array}$$

- for details: [\[see talk by Marco\]](#)

$$e^+e^- \rightarrow l^+l^- \quad l \in \{e, \mu\}$$

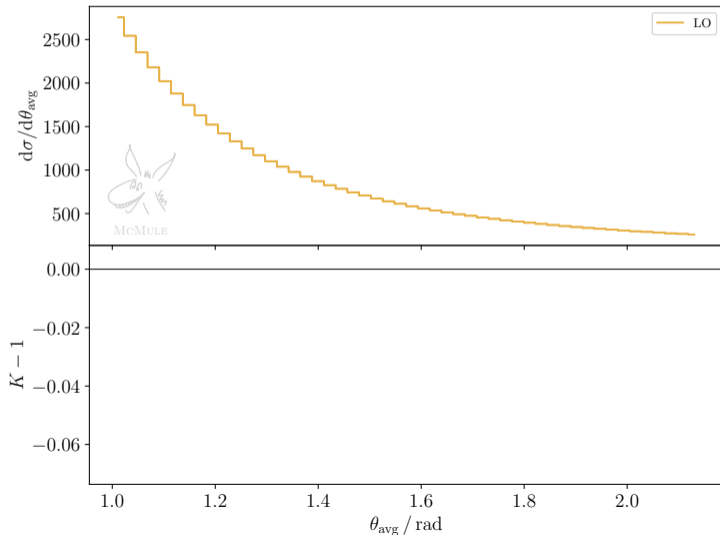
- LO, NLO with full mass dependence
- NNLO with massification & next-to-soft stabilisation
 $l = e$ [McMule 21] + missing fermionic corrections
 $l = \mu$ crossed from $e\mu \rightarrow e\mu$ [see talk by Marco]
- HVP with alphaQED [Jegerlehner]



radiative process is a subset: $(ee \rightarrow XX\gamma @ \text{NLO}) \subset (ee \rightarrow XX @ \text{NNLO})$

$e^+e^- \rightarrow e^+e^-$ @ CMD scenario with photonic* NNLO *fermionic underway

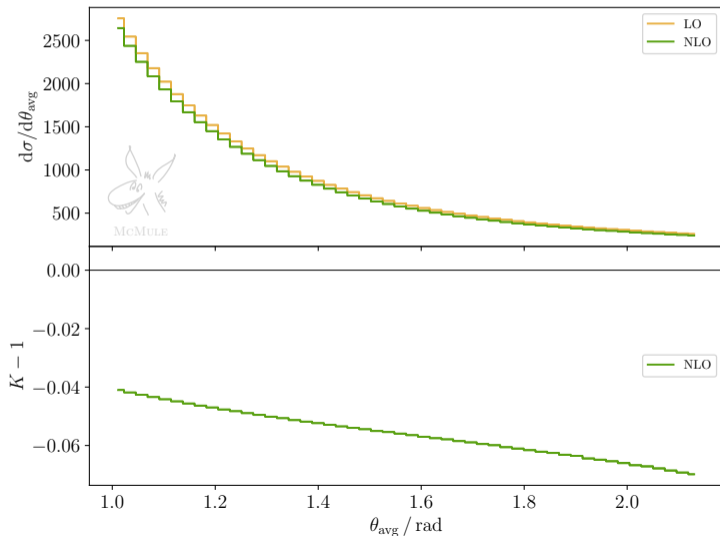
($\sqrt{s} = 0.7 \text{ GeV}$; $1 \text{ rad} \leq \theta_{\text{avg}} \leq \pi - 1 \text{ rad}$; $|\vec{p}_{\pm}| > 0.45\sqrt{s}/2$; $||\phi^+ - \phi^-| - \pi| < 0.15 \text{ rad}$; $|\theta^+ + \theta^- - \pi| < 0.25 \text{ rad}$)



$$\theta_{\text{avg}} = (\theta^- - \theta^+ + \pi)/2$$

$e^+e^- \rightarrow e^+e^-$ @ CMD scenario with photonic* NNLO *fermionic underway

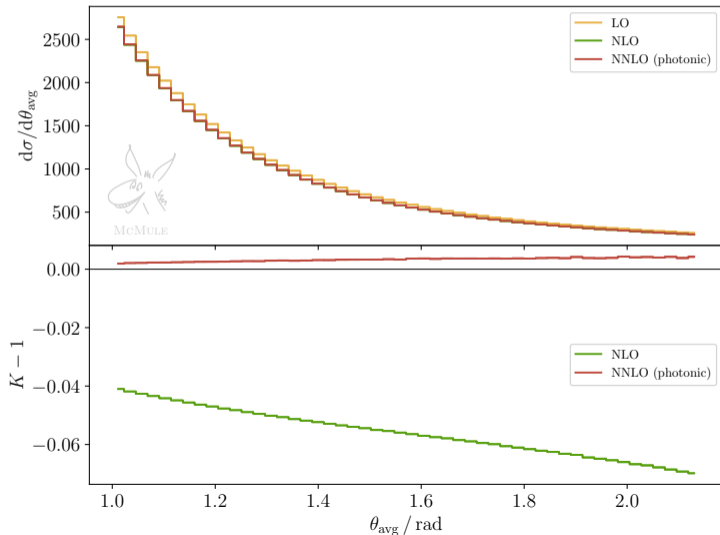
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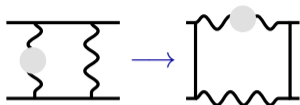
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fermionic corrections



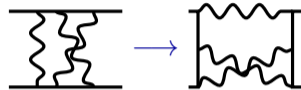
- contains HVP \rightarrow numerical
- previously (t -channel): hyperspherical [Fael 18], extension to s -channel unclear

$$\sigma \sim \int_0^\infty \Pi(-Q^2) K(Q^2) dQ^2$$

\implies now: dispersive with threshold subtraction

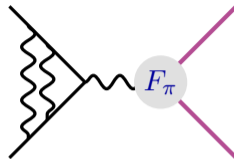
$$\sigma \sim \int_{4m_\pi^2}^\infty \text{Im}(\Pi(Q^2)) K'(Q^2) dQ^2$$

photonic corrections



- multivalued functions with non-trivial analytic continuation
- currently: hard-coded for CMD scenario
- future: include this in handyG

$$e^+e^- \rightarrow \pi^+\pi^-$$

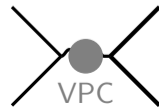
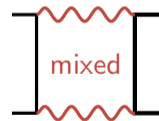
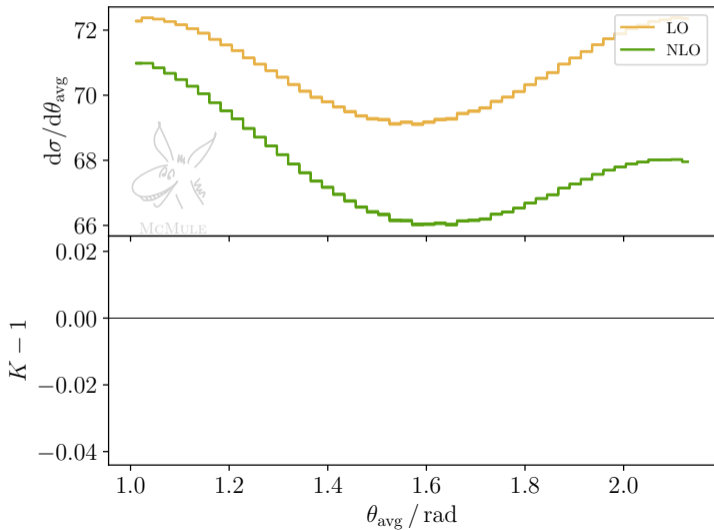


- we have electronic (ISC) NNLO for $ep \rightarrow ep$ with proton described by F_1 & F_2 [McMule 23]
 $\implies \pi$ can be obtained with $F_1, F_2 \sim f(F_\pi)$ & crossing
- soon: generic framework for NNLO $e^+e^- \rightarrow \gamma^* \rightarrow$ arbitrary nucleons

$$|\mathcal{A}|^2 \sim L_{\mu\nu}^{\text{NNLO}} \left(\sum_h j_X^\mu j_X^{*\nu} \right) \quad X \in \{\pi, {}^{12}\text{C}, p, {}^2\text{H}, \dots\}$$

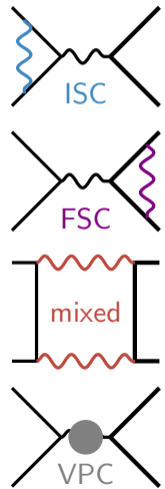
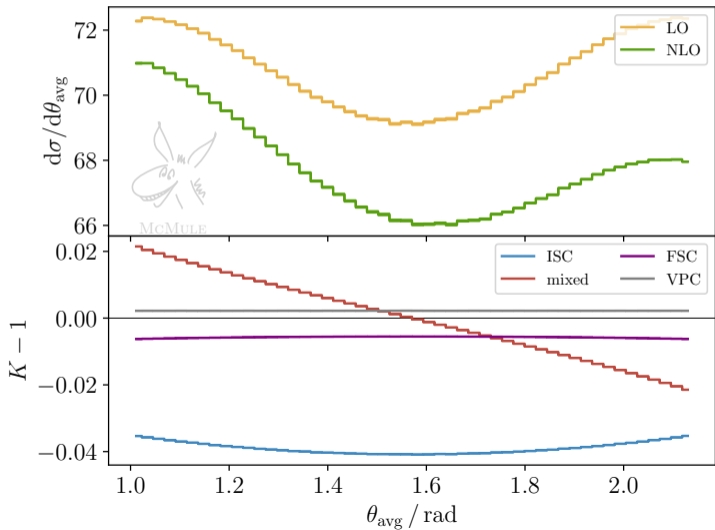
$e^+e^- \rightarrow \mu^+\mu^-$ @ CMD scenario *NNLO underway

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gauge-invariant split @ KLOE scenarios ($ee \rightarrow \mu\mu\gamma$)

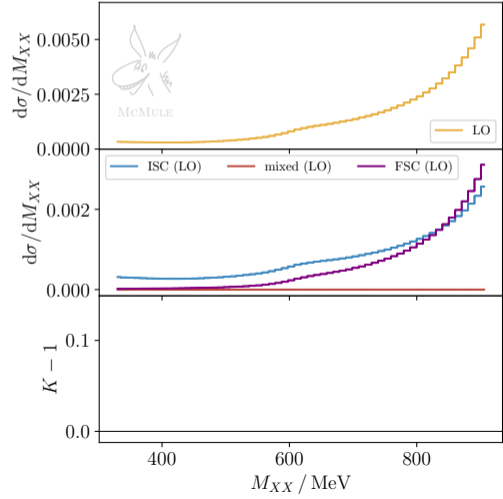
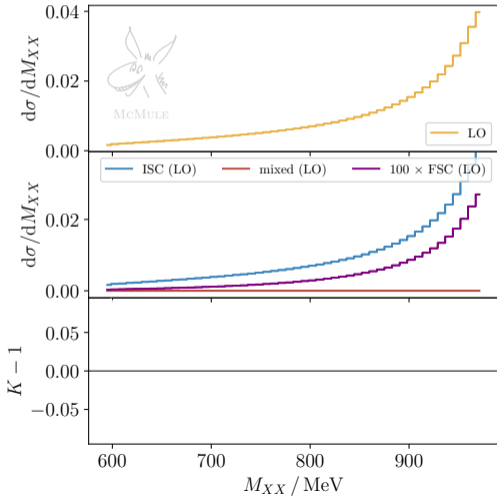
$$(50^\circ \leq \theta^\pm \leq 130^\circ; |p_\pm^z| > 90 \text{ MeV} \vee |p_\pm^\perp| > 160 \text{ MeV})$$

small angle (untagged) ($\vec{p}_{\tilde{\gamma}} = -(\vec{p}_+ + \vec{p}_-)$)

large angle (tagged)

$$(\theta_{\tilde{\gamma}} \leq 15^\circ \vee \theta_{\tilde{\gamma}} > 165^\circ; 0.35 \text{ GeV}^2 \leq M_{XX}^2 \leq 0.95 \text{ GeV}^2)$$

$$(50^\circ \leq \theta_\gamma \leq 130^\circ \wedge E_\gamma > 20 \text{ MeV}; 0.1 \text{ GeV}^2 \leq M_{XX}^2 \leq 0.85 \text{ GeV}^2)$$



gauge-invariant split @ KLOE scenarios ($ee \rightarrow \mu\mu\gamma$)

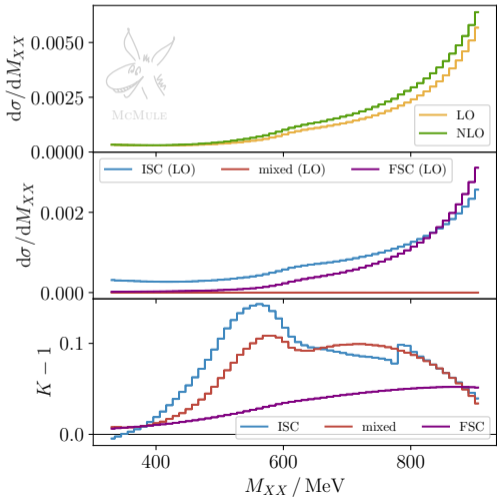
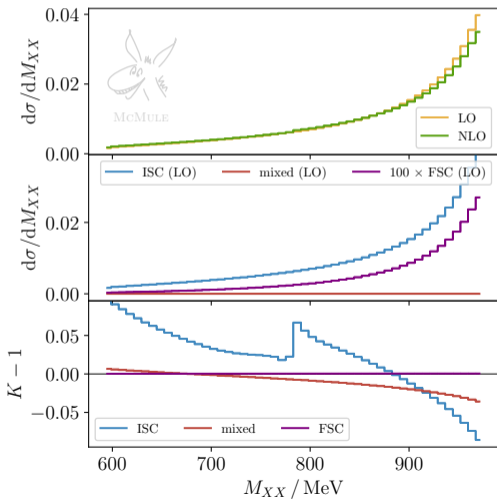
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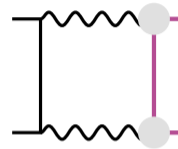
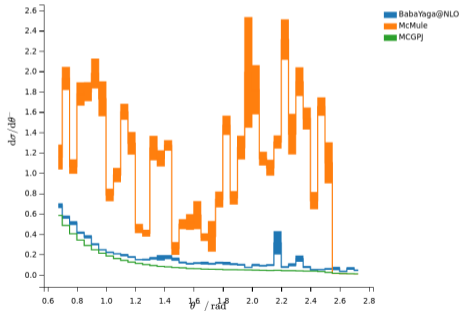
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to be investigated numerical instabilities for $ee \rightarrow ee\gamma$ in the B scenario

future include FSC/mixed for pions

Observable θ^-



future NNLO $ee \rightarrow \mu\mu\gamma$ from $pp \rightarrow 2j + \gamma$ [Badger, Czakon, Hartanto, Moodie, Peraro, Poncelet, Zoia 23]





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mule-tools.gitlab.io

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not pictured: P.Banerjee (IIT Guwahati), D.Moreno (PSI), D.Radic (PSI)