Implementation of pion form factor in event generators

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The Evaluation of the Leading Hadronic Contribution to the Muon $g-2$

MITP Topical Workshop – 05 June 2024

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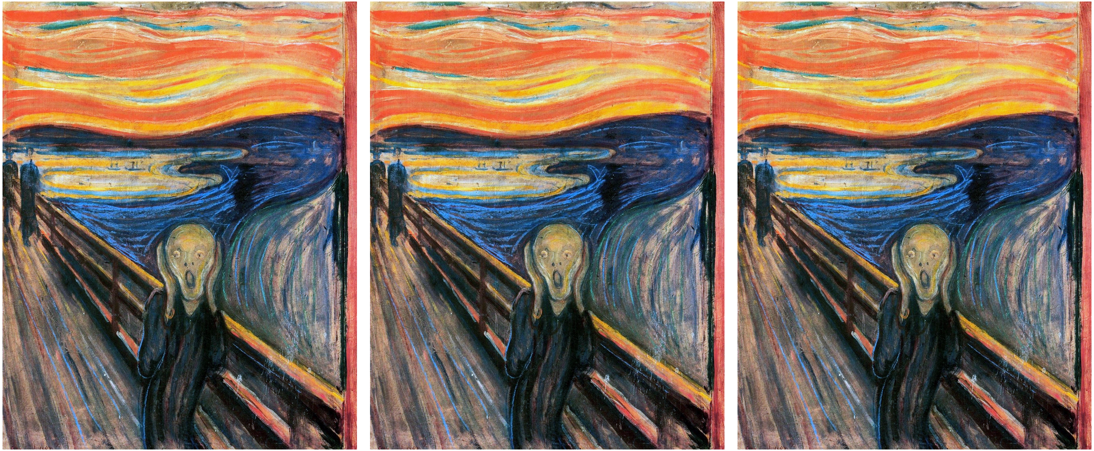
Pions have an internal structure



What do we need?

- **From experimentalists**: A pion form factor parametrisation $F_\pi(q^2)$ from data (*Achim's talk*)

  $\hookrightarrow$ At LO the only formal requirement is $F_\pi(0) = 1$

- **From theorists**: A method to insert $F_\pi(q^2)$ in loop diagrams
  - Factorised sQED (*Pau's talk*)

    $\hookrightarrow$ G. Rodrigo, H. Czyz, J. H. Kuhn, M. Szopa, *Radiative return at NLO and the measurement of the hadronic cross-section in electron–positron annihilation*, Eur.Phys.J.C 24 (2002) 71-82
  - Dispersive method (*Peter's and Francesco's talk*)

    $\hookrightarrow$ G. Colangelo, M. Hoferichter, J. Monnard, J. Ruiz de Elvira, *Radiative corrections to the forward–backward asymmetry in* $e^+e^- \to \pi^+\pi^-$, JHEP 08 (2022) 295
  - GVMD model (*Fedor's talk*)

    $\hookrightarrow$ F. Ignatov, R. N. Lee, *Charge asymmetry in* $e^+e^- \to \pi^+\pi^-$ *process*, PLB 833 (2022) 137283

- We need $F_\pi(q^2)$ not just $|F_\pi(q^2)|$ beyond factorised sQED

Formal requirements

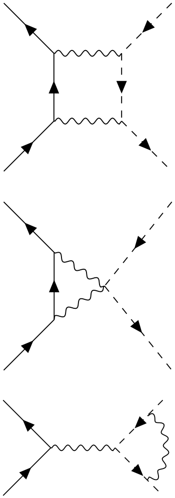
- **Factorised sQED**: Any $F_\pi(q^2)$ as long as $F_\pi(0) = 1$, only $|F_\pi|$ needed
- **Dispersive method**: Any $F_\pi(q^2)$ that respects the dispersive sum rule

$$F_\pi(q^2) = 1 - \frac{q^2}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'} \frac{\mathrm{Im} F_\pi(s')}{s' - q^2} \quad \longrightarrow \quad \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'} \mathrm{Im} F_\pi(s') = 1$$

- **GVMD model**: $F_\pi(q^2)$ must be written as a sum of BW functions

$$F_\pi(q^2) = \sum_{v=0}^{N} c_v \frac{\Lambda_v^2}{\Lambda_v^2 - q^2} \quad \text{with} \quad \sum_{v=0}^{N} c_v = 1$$

Writing $F_\pi(q^2)$ in a propagator-like form allows to solve the loop integrals with $F_\pi(q^2)$ in each sQED vertex with standard techniques



Pion form factor for Strong2020 comparisons (1)

- We need to fix a common F_π parametrisation to compare generators w/o differences induced by different F_π
- Agreement on a sum of Gounaris-Sakurai (GS) functions $\mathrm{BW}_v^{\mathrm{GS}}$, very common in experiments
  ↪ G. J. Gounaris, J. J. Sakurai, *Finite width corrections to the vector meson dominance prediction for $\rho \to e^+e^-$*, Phys. Rev. Lett. 21 (1968) 244-247

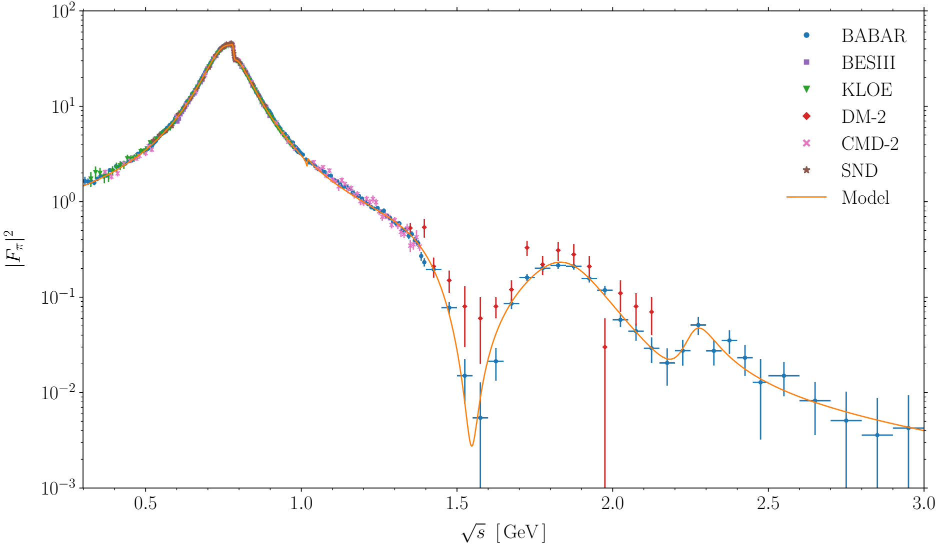
$$F_\pi(q^2) = \frac{\mathrm{BW}_\rho^{\mathrm{GS}}(q^2)\left[1 + (q^2/m_\omega^2)\, c_\omega\, \mathrm{BW}_\omega(q^2) + (q^2/m_\phi^2)\, c_\phi\, \mathrm{BW}_\phi(q^2)\right]}{1 + c_{\rho'} + c_{\rho''} + c_{\rho'''}} + \frac{c_{\rho'}\, \mathrm{BW}_{\rho'}^{\mathrm{GS}}(q^2) + c_{\rho''}\, \mathrm{BW}_{\rho''}^{\mathrm{GS}}(q^2) + c_{\rho'''}\, \mathrm{BW}_{\rho'''}^{\mathrm{GS}}(q^2)}{1 + c_{\rho'} + c_{\rho''} + c_{\rho'''}}$$

$$\mathrm{BW}_v^{\mathrm{GS}}(q^2) = \frac{m_v^2 + d(m_v)\, m_v\, \Gamma_v}{m_v^2 - q^2 + f(q^2, m_v, \Gamma_v) - i\, m_v\, \Gamma(q^2, m_v, \Gamma_v)} \qquad v = \rho, \rho', \rho'', \rho'''$$

$$\mathrm{BW}_v(q^2) = \frac{m_v^2}{m_v^2 - q^2 - i m_v \Gamma_v} \qquad v = \omega, \phi$$

A huge thanks to Fedor!

Pion form factor for Strong2020 comparisons (2)



Pion form factor for Strong2020 comparisons (3)

	ρ	ρ'	ρ''	ρ'''	ω	ϕ
m_V (MeV)	774.56	1485.9	1866.8	2264.5	782.48	1019.47
Γ_V (MeV)	148.32	373.60	303.34	113.27	8.55	4.25
$\lvert c_V \rvert$	-	0.14104	0.0614	0.0047	0.00158	0.00045
φ_V (rad)	-	3.7797	1.429	0.921	0.075	2.888

- *"We do not claim that this expression for $F_\pi(q^2)$ is a proper combination of all experimental data. It is simply a fixed parameterisation inspired by real data and mainly serves the purpose of allowing generator comparisons without impact from pion form factor variations."*

- Data from BaBar, BES III, CMD-2, DM-2, KLOE, SND, but not CMD-3

- The quality of the fit was sacrificed to ensure the dispersive sum rule (accuracy $\sim 10^{-5}$)

- Use it for the GVMD is not straightforward, MCGPJ approach for IFI:

$$|F_\pi(s)|^2 \left(F_\pi(q_1^2)F_\pi(q_2^2) - F_\pi(s)\right)/F_\pi(s) \quad \longrightarrow \quad |F_\pi^{\mathrm{GS}}(s)|^2 \left(F_\pi^{\mathrm{BW}}(q_1^2)F_\pi^{\mathrm{BW}}(q_2^2) - F_\pi^{\mathrm{BW}}(s)\right)/F_\pi^{\mathrm{BW}}(s)$$

Uncertainty from different approaches