
MITP Topical Workshop

RadioMonteCarLow 2

Radiative corrections and Monte Carlo tools for
low-energy hadronic cross sections in $e^+ e^-$ collisions

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MITP / MAINZ / 03-07 JUNE 2024

- **theoretical description** for $e^+ e^- \rightarrow \text{hadrons}$ at low energies $\sqrt{s} \lesssim 1 - 2 \text{ GeV}$
making also use of **radiative (return)** processes (some call them ISR processes)

- **main** processes

$$e^+ e^- \rightarrow \mu^+ \mu^- (+\gamma)$$

$$e^+ e^- \rightarrow e^+ e^- (+\gamma)$$

$$e^+ e^- \rightarrow \pi^+ \pi^- (+\gamma)$$

- **more** processes

$$e^+ e^- \rightarrow \gamma \gamma$$

$$e^+ e^- \rightarrow \pi^+ \pi^- \pi^0$$

$$e^+ e^- \rightarrow \pi^+ \pi^- \pi^0 \pi^0$$

$$e^+ e^- \rightarrow \pi^+ \pi^- \pi^+ \pi^-$$

- there are additional processes and $(e^+ e^-)$ in final state

- Strong2020: Radiative corrections and Monte Carlo tools for low-energy hadronic cross sections in $e^+ e^-$ collisions

WorkStop/ThinkStart in Zurich <https://indico.psi.ch/event/13707/>

- inspired by [0912.0749]

Eur. Phys. J. C (2010) 66: 585–686
DOI 10.1140/epjc/s10052-010-1251-4

THE EUROPEAN
PHYSICAL JOURNAL C

Review

Quest for precision in hadronic cross sections at low energy: Monte Carlo tools vs. experimental data

Working Group on Radiative Corrections and Monte Carlo Generators for Low Energies

- consolidate and implement the progress since 2010
produce report and repository of MC codes and comparisons by August 2024

Team: P. Beltrame, E. Budassi, C. Carloni Calame, G. Colangelo, L. Cotrozzi, A. Driutti,
 T. Engel, L. Flower, A. Gurgone, M. Hoferichter, F. Ignatov, S. Kollatzsch, B. Kubis,
 A. Kupsc, F. Lange, A. Lusiani, G. Montagna, S. Müller, O. Nicrosini, P. Petit Rosàs,
 J. Paltrinieri, F. Piccinini, A. Price, L. Punzi, M. Rocco, K. Schönwald, O. Shekhovtsova,
 A. Signer, A. Siódmok, G. Stagnitto, P. Stoffer, T. Teubner, W. Torres Bobadilla, F. Ucci,
 Y. Ulrich, G. Venanzoni

WP1:	QED for leptons at NNLO
WP2:	Form factor contributions at N^3LO
WP3:	Processes with hadrons
WP4:	Parton showers
WP5:	Experimental input

AFKQED	BHWIDE
Babayaga@NLO	MadGraph
KKMC	Photos
MCGPJ	...
McMule	
Phokhara	
Sherpa	

- are you still using your phone of 2010 ?
so why then should you use Monte Carlo codes of 2010 !
- **preserve**, further **develop**, and **make accessible** some well established codes for low-energy $e^+ e^-$
- get new (preferably young) people to join with new ideas/approaches
- cross fertilisation from huge effort made for LHC
- **open science approach**: what is in which generator, where can I get it
⇒ a public repository of all codes and all results
- a **community effort**, hopefully ongoing for many years to come

it worked marvellously for the MUonE theory initiative

Eur. Phys. J. C (2020) 80:591
<https://doi.org/10.1140/epjc/s10052-020-8138-9>

THE EUROPEAN
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Review

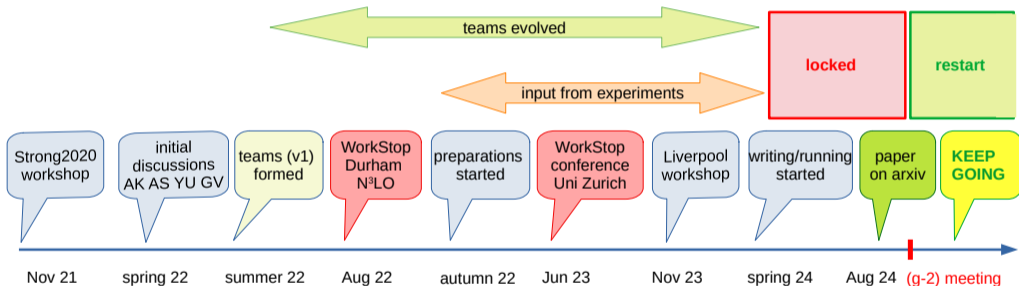
Theory for muon-electron scattering @ 10 ppm

A report of the MUonE theory initiative

P. Banerjee¹, C. M. Carloni Calame², M. Chiesa³, S. Di Vita⁴, T. Engel^{1,5}, M. Facl⁶, S. Laporta^{7,8}, P. Mastrolia^{7,8},
G. Montagna^{2,9}, O. Nicosini², G. Ossola¹⁰, M. Passera⁸, F. Piccinini², A. Primo⁵, J. Ronca¹¹, A. Signer^{1,5,a},
W. J. Torres Bobadilla¹¹, L. Trentadue^{12,13}, Y. Ulrich^{1,5}, G. Venanzoni¹⁴

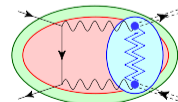
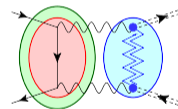
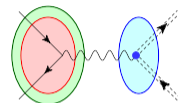
[2004.13663] was just the start of theory for MUonE !

we started just as a bunch of people, now we're in a **locked phase** (until Aug 2024)



09-13 September 2024 workshop of $(g - 2)$ Theory Initiative \Rightarrow fix point for our paper but **not** the end of our activities \Rightarrow **restart** after this, anyone can join again

$$e^+ e^- \rightarrow \pi^+ \pi^-$$



WP1: QED for leptons at NNLO

WP2: Form factor contributions at N³LO

WP3: Processes with hadrons

WP4: Parton showers / YFS

WP5: Experimental input

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Sec 2: experimental input

summary, reference

Sec 3: computational set up

set up 'language' to define
various contributions

Sec 4: precise description of MC codes

what is included and what not

Sec 5: MC comparison for scenarios

NOT an analysis, a realistic estimate
of importance of various contributions

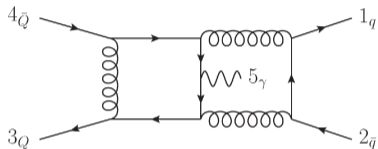
?? Sec 6 ?? luminosity processes

remainder of this talk / discussion

- everything I know (i.e. nothing) about [Section 2](#), experimental input \Rightarrow [Achim's talk](#)
- some remarks about [Section 3](#), computational set up
 - pure QED contributions [McMule / Mesmer / Matteo talks](#)
 - flag [Section 3.4](#), pions in the final state \Rightarrow [Peter's talk](#)
- flag [Section 4](#), description of codes \Rightarrow [Thursday's talks](#)
- flag [Section 5](#), Monte Carlo comparisons \Rightarrow [Yannick's talk](#)
- nothing about [Section 6](#), processes for luminosity, \Rightarrow likely to disappear
- timeline for completion of phase I by August 2024

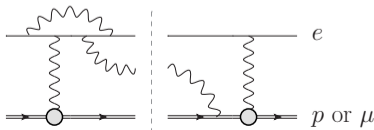
QCD@LHC

- e.g. [2304.06682] NNLO cross section for $pp \rightarrow jj\gamma$ (i.e. $2 \rightarrow 3$ massless)

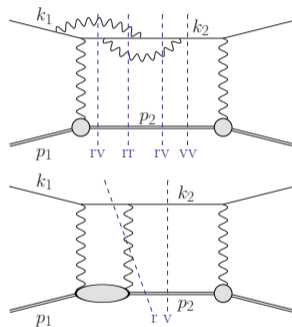


- automated one-loop codes, e.g. [OpenLoops](#)
- high-energy event generators e.g. [Sherpa](#)
- match NNLO to shower e.g. [MiNNLO_{PS}](#)

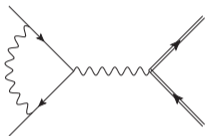
and, of course, [MUonE](#)



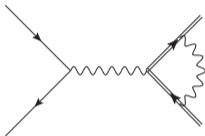
lepton-proton (similar situation / issues)



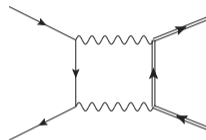
example: $e^+ e^- \rightarrow \mu^+ \mu^-$ at NLO, split into gauge invariant parts for computational and conceptual reasons



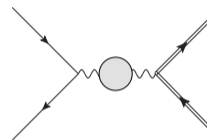
initial-state (ISC)



final-state (FSC)



mixed corrections



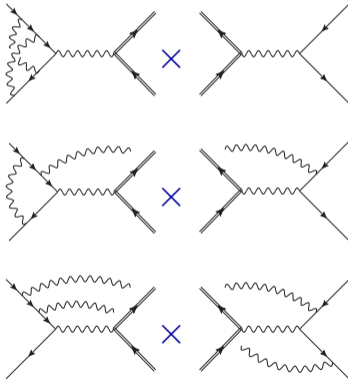
(H)VP corrections

$$\mathcal{A}_{mm}^{(1)}(q_e q_m q_l^2) = \mathcal{A}_{mm}^{(1)}(q_e^3 q_m) + \mathcal{A}_{mm}^{(1)}(q_e q_m^3) + \mathcal{A}_{mm}^{(1)}(q_e^2 q_m^2) + \mathcal{A}_{mm}^{(1)}(q_e q_m \Pi^{(1)})$$

$$d\sigma_{mm}^{(1)}(q_e^2 q_m^2 q_l^2) = \underbrace{d\sigma_{mm}^{(1)}(q_e^4 q_m^2)}_{\text{ISC}} + \underbrace{d\sigma_{mm}^{(1)}(q_e^2 q_m^4)}_{\text{FSC}} + \underbrace{d\sigma_{mm}^{(1)}(q_e^3 q_m^3)}_{\text{mixed}} + \underbrace{d\sigma_{mm}^{(1)}(q_e^2 q_m^2 \Pi^{(1)})}_{\text{VPC}}$$

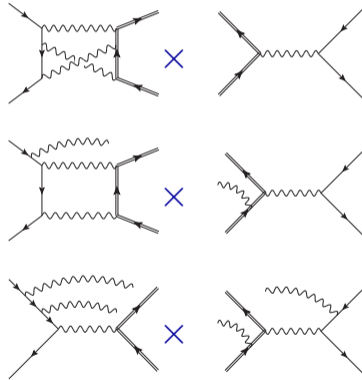
split for NNLO corrections to cross section

initial-state corrections



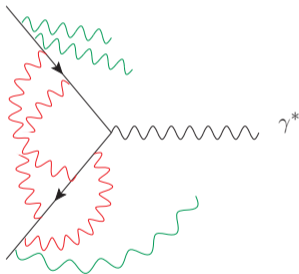
$$d\sigma_{mm}^{(2)}(q_e^6 q_m^2)$$

mixed corrections



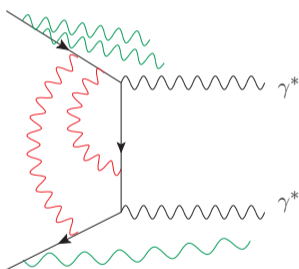
$$d\sigma_{mm}^{(2)}(q_e^4 q_m^6) \text{ and } d\sigma_{mm}(q_e^5 q_m^3) \dots$$

Buliding block $e^+ e^- \rightarrow \gamma^*$



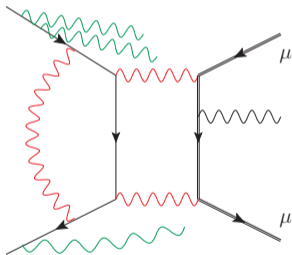
- **NNLO** available (used for $e\mu \rightarrow e\mu$ and $lp \rightarrow lp$) including real and virtual, **no approximation**
- moving towards **NNNLO**, \exists open questions
 - 3-loop known [2202.05276]
 - 2-loop $e^+ e^- \rightarrow \gamma^* \gamma$ **bottleneck**, need $m_e \neq 0$
 - 1-loop $e^+ e^- \rightarrow \gamma^* \gamma \gamma$ use tools, numerics?
 - 0-loop $e^+ e^- \rightarrow \gamma^* \gamma \gamma$ trivial but dangerous
- playground for combination with **shower/YFS**
- (dominant??) subset of N^3LO for MUonE
- can we use next-to-soft LBK to improve YFS ??

Buliding block $e^+ e^- \rightarrow \gamma^* \gamma^*$



- doubly virtual Compton scattering (gauge invariant)
- **NLO** doable, including real and virtual
- **NNLO** painful, would it be useful ?
 - if one $\gamma^* \rightarrow \gamma$ it is desperately wanted
 - combine with $\pi^+ \pi^-$ final state (??)
 - a (gauge-invariant) subset of N³LO for MUonE
- playground for combination with **shower/YFS**

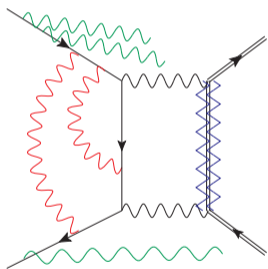
Process $e^+ e^- \rightarrow \mu^+ \mu^- \gamma$



- **NLO** done as part of NNLO MUonE (+ crossing)
- **NNLO** no show stoppers, but approximations
 - two-loop amplitude use 'massification' (drop m_e^2/Q^2 and m_μ^2/Q^2 effects)
 - one-loop available with full mass dependence
 - phase-space integration tricky but doable
- massification ok'ish for tagged well-isolated photon
- some subsets (OPE) can be done with full mass dependence, very useful for δ_{th} determination
- ??? full m_μ mass dependence for two-loop ???

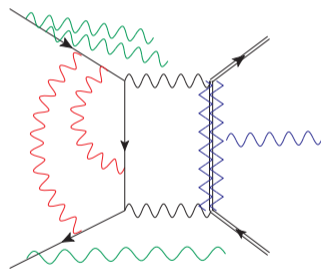
sewing together building blocks for $e^+ e^- \rightarrow \pi^+ \pi^-$

“ISC”



FsQED ok'ish if $F_\pi(q^2)$ in (!!) the loop

“FSC/mixed”



just adding form factors = (bad) fudge

the magnificent seven

- **AFKQED**: LO $2 \rightarrow 3$ + ISR “collinear structures”, FSR with **Photos** for $X \in \{\mu, \pi\}$
- **Babayaga@NLO**: NLO $2 \rightarrow 2$ + parton shower + “pions”
- **KKMC**: LO + YFS (CEEX) with $X = \mu$
- **MCGPJ**: NLO $2 \rightarrow 2$ + “collinear structures” for $X \in \{e, \mu, \pi\}$
- **McMule**: NNLO QED for $2 \rightarrow 2$ with $X \in \{e, \mu\}$, ISR for $X = \pi$
- **Phokhara**: NLO for $2 \rightarrow 3$ with $X \in \{\mu, \pi\}$, sQED for π
- **Sherpa**: (automated) YFS for $X \in \{\mu, \pi\}$ with matched NLO $2 \rightarrow 2$, sQED for π

Aim: for each code, make a precise statement what is included and how \Rightarrow **Thursday's talks**

possible further codes to be included in the future: BHWIDE, MadGraph, ????

scenarios: $e^+ e^- \rightarrow X^+ X^- (+\gamma)$ for $X \in \{e, \mu, \pi\}$, many observables \Rightarrow Yannick's talk
 always cut on $p_{\pm} > \text{something}$, selection of further cuts:

- CMD:** $e^+ e^- \rightarrow X^+(p_+) X^-(p_-)$ with $\sqrt{s} = 0.7 \text{ GeV}$
 cuts: $\supset \left| |\phi^+ - \phi^-| - \pi \right| < 0.15 \text{ rad}; \quad |\theta^+ + \theta^- - \pi| < 0.25 \text{ rad};$
- KLOE small angle (untagged):** $e^+ e^- \rightarrow X^+ X^- \gamma$ with $\sqrt{s} = 1.02 \text{ GeV}$
 cuts: \supset range of θ_{\pm} and M_{XX} ; set $\vec{p}_{\tilde{\gamma}} \equiv -(\vec{p}_+ + \vec{p}_-)$ and $\theta_{\tilde{\gamma}} \leq 15^\circ$ or $\theta_{\tilde{\gamma}} > 165^\circ$
- KLOE large angle (tagged):** $e^+ e^- \rightarrow X^+ X^- \gamma$ with $\sqrt{s} = 1.02 \text{ GeV}$
- BES III:** $e^+ e^- \rightarrow X^+ X^- \gamma$ with $\sqrt{s} = 4 \text{ GeV}$
- B:** $e^+ e^- \rightarrow X^+ X^- \gamma$ with $\sqrt{s} = 10 \text{ GeV}$
 cuts: \supset range of θ_{\pm} and $\exists \gamma$ within range of θ_{γ} and $E_{\gamma} > \text{something}$

	nr. pages	29 Apr 24	6 May 24	13 May 24	20 May 24	27 May 24	3 Jun 24	10 Jun 24	17 Jun 24	24 Jun 24	1 Jul 24
Intro	2										
Section 2.1	3	AK / GV / AD / LP / SM / AL									
Section 2.2	2	AK / GV / AD / LP / SM / AL									
Section 2.3	3	AK / GV / AD / LP / SM / AL									
preamble Section 3	1	AdS									
fixed order 3.1	5	AdS									
shower/YFS 3.2	5		CCC / FP / GM / ON / GS / FU			AdS / FL / KS / PS / TT					
HVP 3.3	2		TE								
external pions 3.4	8		PS / MH / GC / TT / BK								
AfkQED 4.1	4						SM / LC / FI				
Babayaga 4.2	4						CCC / GM / FP / ON / EB / AG / FU				
KKMC 4.3	4						JP / AnS				
MCGPJ 4.4	4						FI				
McMule 4.5	4	SK / MR		SK / MR / AdS / YU							
Phokhara 4.6	4						PPR / WTB / OS				
Sherpa 4.7	4						AP / LF				
input values 5.1	3			AG / AdS							
processes 5.2 – 5.6		YU / FI / WTB / CCC / SK / MR / JP / AnS / LC / AP / SM									
CMD Section 5.2	8	YU / FI / WTB / CCC / SK / MR / JP / AnS / LC / AP / SM									
KLOE untagged 5.3	8			YU / AdS / GV / SM							
KLOE tagged 5.4	8			YU / AdS / GV / SM							
BES III 5.5	8			YU / AdS / GV / SM							
B factory 5.6	8			YU / AdS / GV / SM							
nr. pages only a rough guideline		write first draft			consolidate text			coordinate runs, make plots			

most urgent tasks for MC responsible

- produce all observables for all possible scenarios for all MC codes **by yesterday** and check whether the results make sense !!
- convert the talks about the MC codes given on Thursday into a text on overleaf (roughly 4 pages) **by 20 June**

other urgent tasks

- finish Section 2
- finish Section 3

we will produce

- an updated on **Radiative corrections and Monte Carlo tools for low-energy hadronic cross sections in $e^+ e^-$ collisions** by **August 2024**