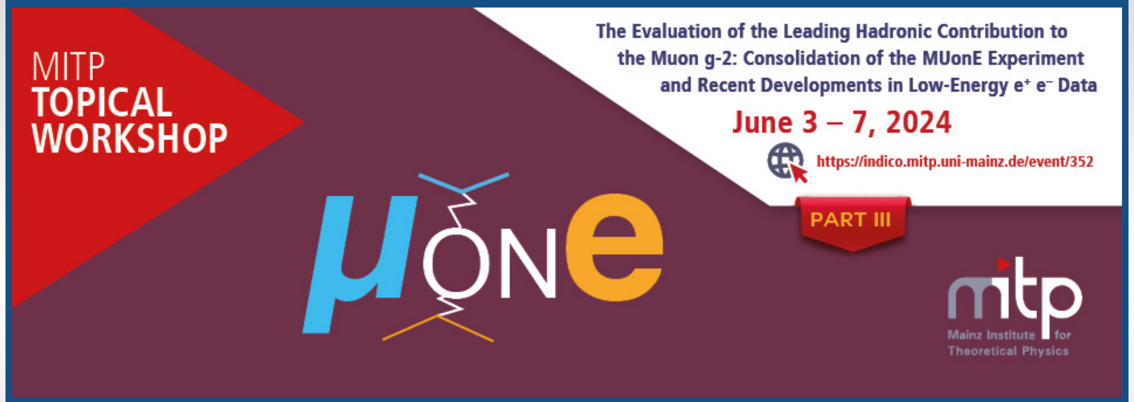
An overview of lattice QCD+QED progress for the HVP contribution to the muon g-2

Davide Giusti

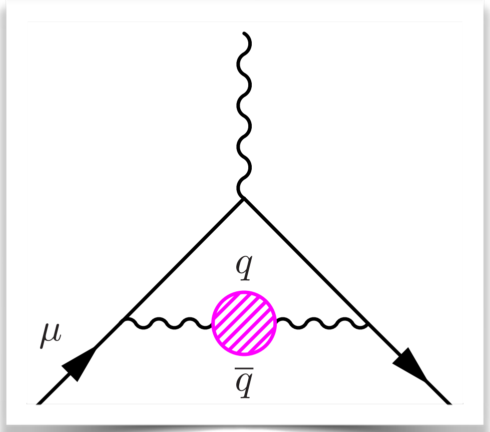
JÜLICH Forschungszentrum



Consolidation of the MUonE Experiment
Mainz

5th June 2024

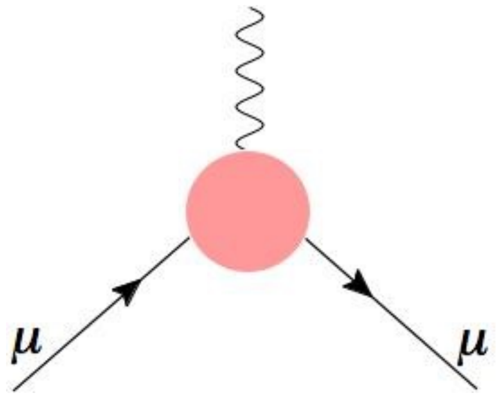
OUTLINE



- Introduction
- HVP from the lattice
- Window observables
- Connections to the MUonE experiment

Introduction

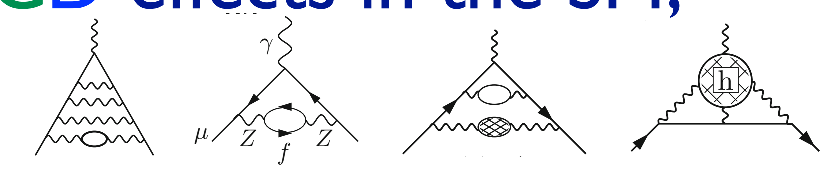
Muon magnetic anomaly



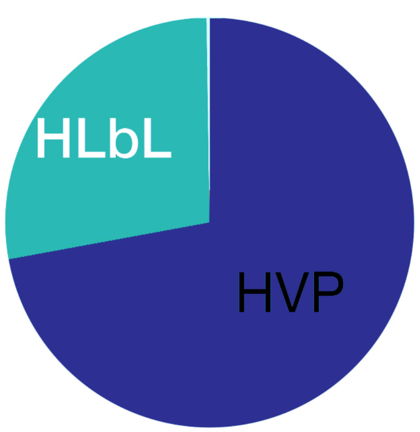
$$= (-i\,e)\,\bar{u}(p')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)\right]u(p)$$

muon anomalous magnetic moment: $a_\mu \equiv \frac{g_\mu - 2}{2} = F_2(0)$

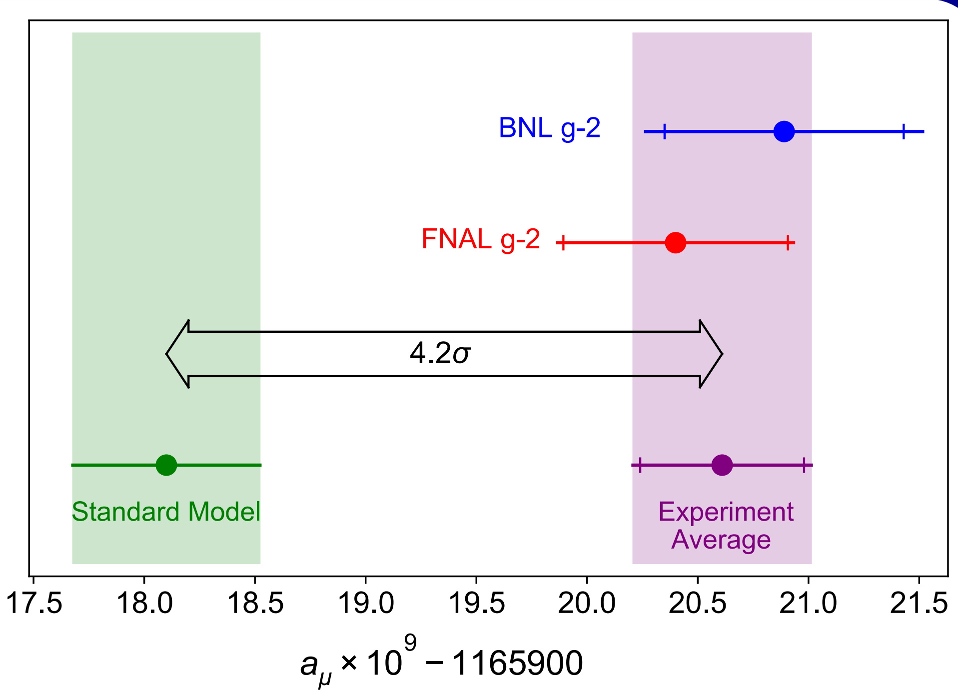
- is generated by quantum loops;
- receives contribution from QED, EW and QCD effects in the SM;
- is a sensitive probe of new physics



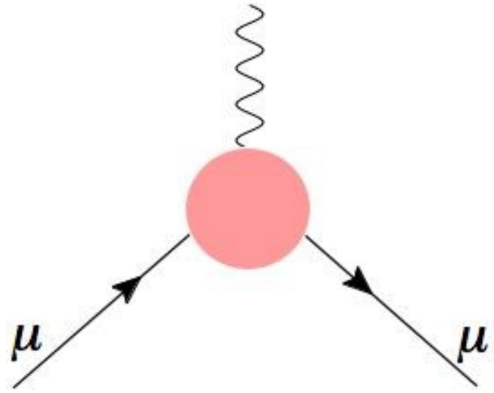
SM contributions to $a_\mu [\times 10^{10}]$	
5-loop QED	11 658 471.8931(104)
2-loop EW	15.36(10)
HVP LO	693.1(4.0)
HVP NLO	-9.83(7)
HVP NNLO	1.24(1)
HLbL	9.2(1.8)



Theory error dominated by hadronic physics



Muon magnetic anomaly



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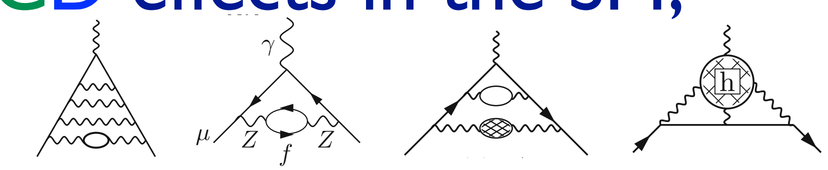

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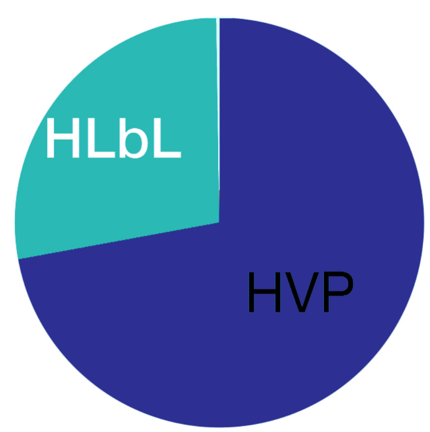
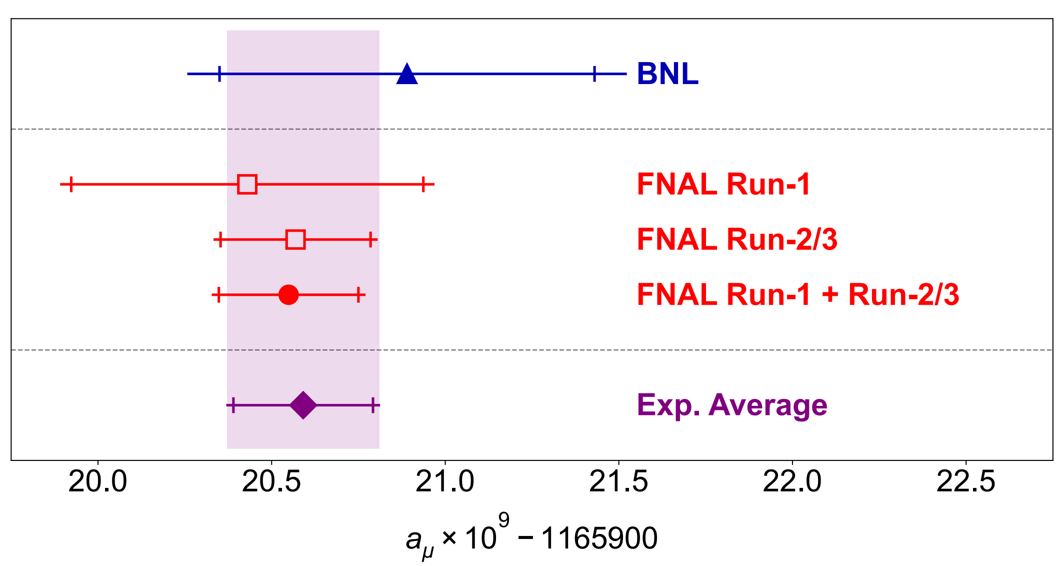




Theory error dominated by hadronic physics

Precision goal for Fermilab ×4 better implies knowing HVP at 0.2-0.3% accuracy

Muon g-2 2023

Hadronic contributions

$$a_\mu^{\text{exp}} - a_\mu^{\text{QED}} - a_\mu^{\text{EW}} = 718.9(4.1) \times 10^{-10} \stackrel{?}{=} a_\mu^{\text{had}}$$

Clearly right order of magnitude:

$$a_\mu^{\text{had}} = O\left(\left(\frac{\alpha}{\pi}\right)^2 \left(\frac{m_\mu}{M_\rho}\right)^2\right) = O\left(10^{-7}\right)$$

(already Gourdin & de Rafael '69 found $a_\mu^{\text{had}} = 650(50) \times 10^{-10}$)

Huge challenge: theory of strong interaction between quarks and gluons, QCD, hugely nonlinear at energies relevant for a_μ

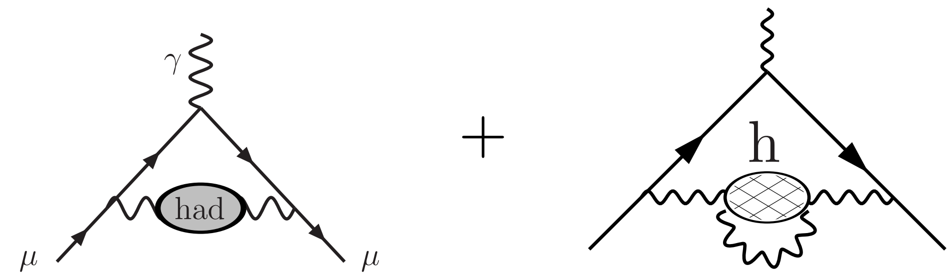
$\rightarrow$ perturbative methods used for electromagnetic and weak interactions do not work

$\rightarrow$ need nonperturbative approaches

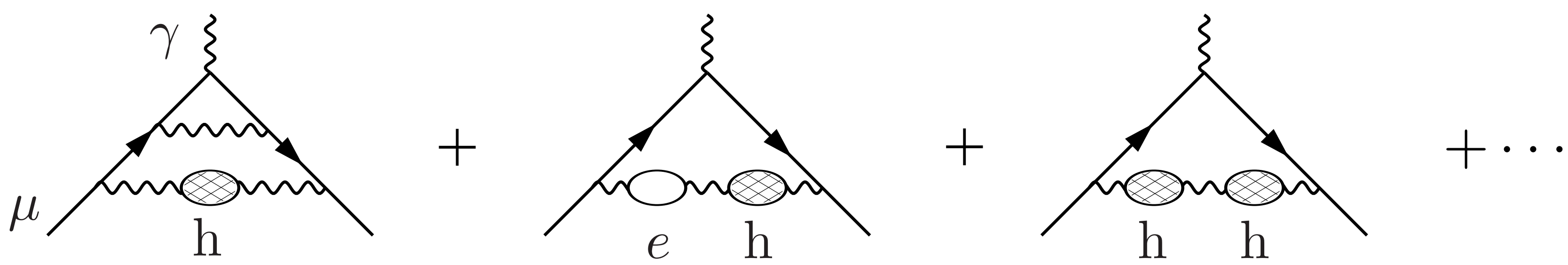
Write

$$a_\mu^{\text{had}} = a_\mu^{\text{LO-HVP}} + a_\mu^{\text{HO-HVP}} + a_\mu^{\text{HLbyL}} + O\left(\left(\frac{\alpha}{\pi}\right)^4\right)$$

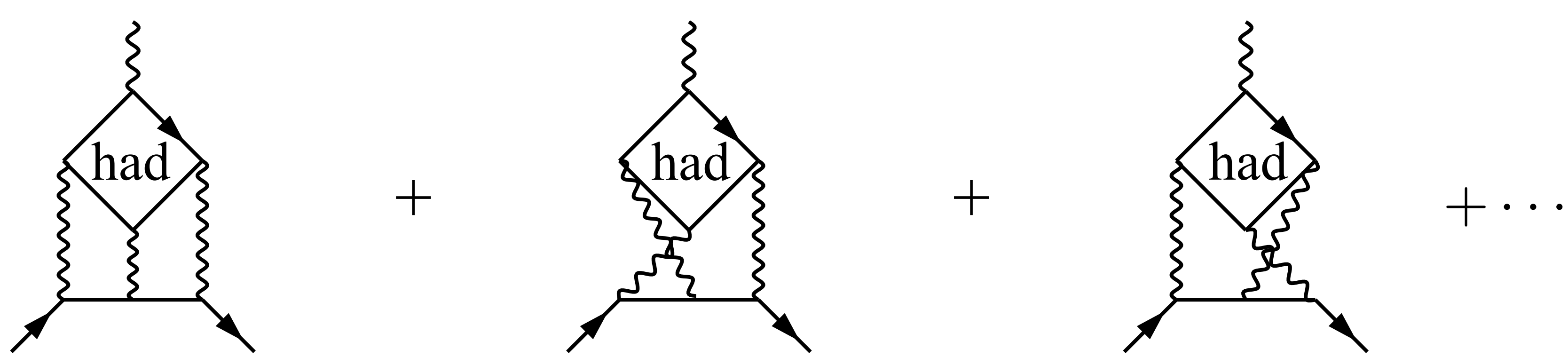
Hadronic contributions: diagrams



$\rightarrow a_\mu^{\text{LO-HVP}} = O\left(\left(\frac{\alpha}{\pi}\right)^2\right)$

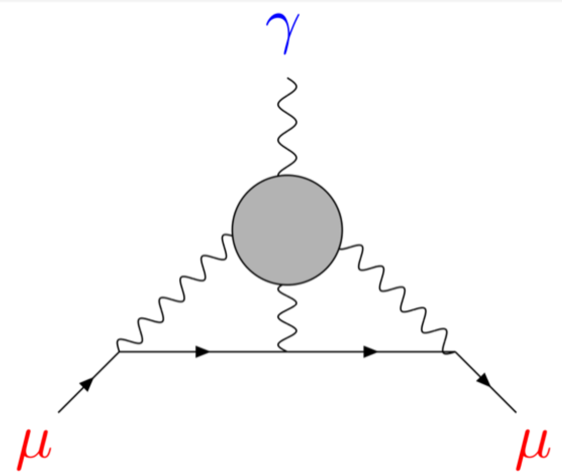


$\rightarrow a_\mu^{\text{NLO-HVP}} = O\left(\left(\frac{\alpha}{\pi}\right)^3\right)$

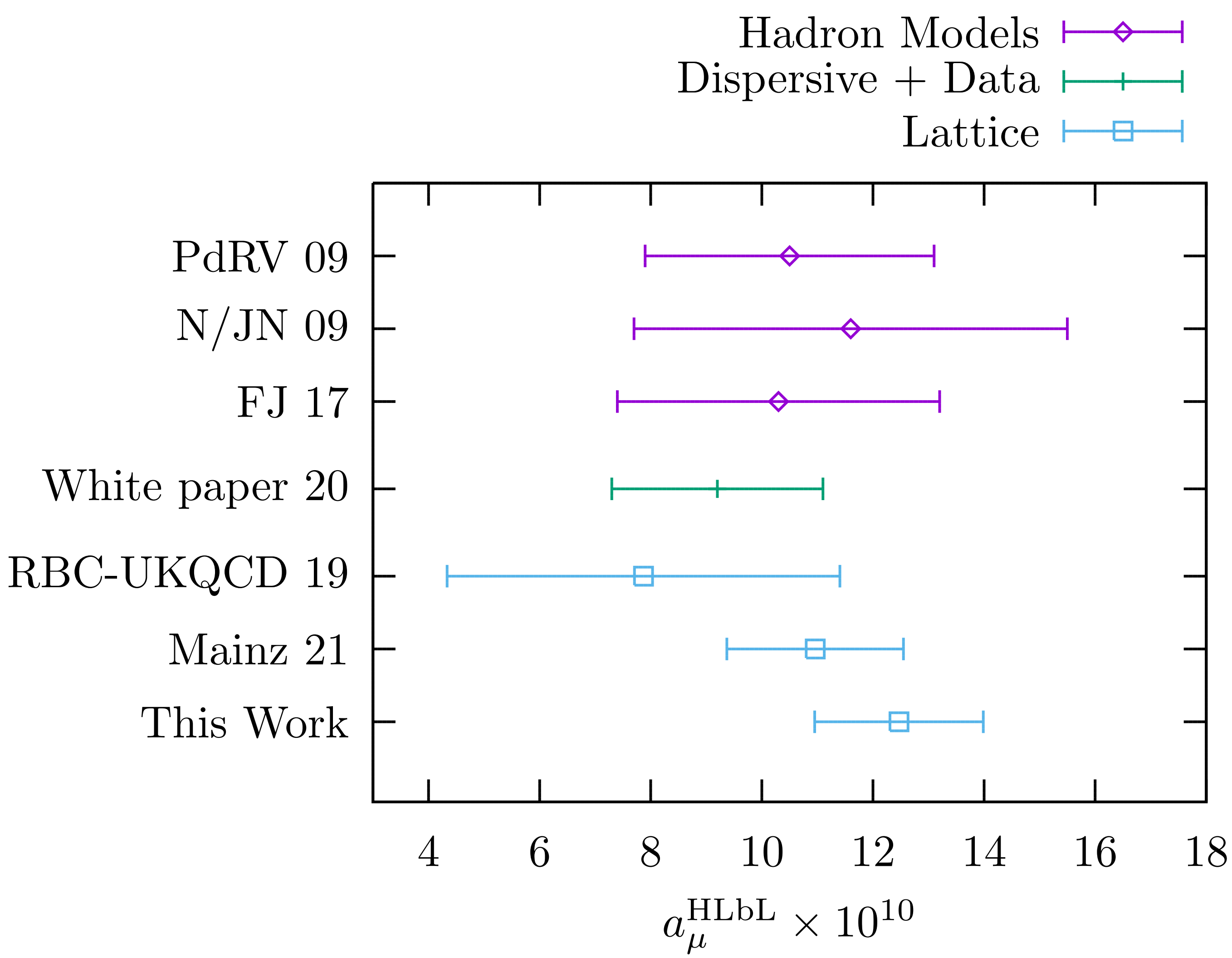


$\rightarrow a_\mu^{\text{HLbL}} = O\left(\left(\frac{\alpha}{\pi}\right)^3\right)$

Hadronic light-by-light



- HLbL much more complicated than HVP, but ultimate precision needed is $\simeq 10\%$ instead of $\simeq 0.2\%$
- For many years, only accessible to models of QCD w/ difficult to estimate systematics (Prades et al '09): $a_\mu^{\rm HLbL} = 10.5(2.6) \times 10^{-10}$
- Also, lattice QCD calculations were exploratory and incomplete
- Tremendous progress in past 5 years:
  - → Phenomenology: rigorous data driven approach [Colangelo, Hoferichter, Kubis, Procura, Stoffer,... '15-'20]
  - → Lattice: first two solid lattice calculations
- All agree w/ older model results but error estimate much more solid and will improve
- Agreed upon average w/ NLO HLbL and conservative error estimates [WP '20]
- $a_\mu^{\rm exp} - a_\mu^{\rm QED} - a_\mu^{\rm EW} - a_\mu^{\rm HLbL} = 709.7(4.5) \times 10^{-10} \stackrel{?}{=} a_\mu^{\rm HVP}$



[Blum et al '23]

Standard Model prediction vs Experiment

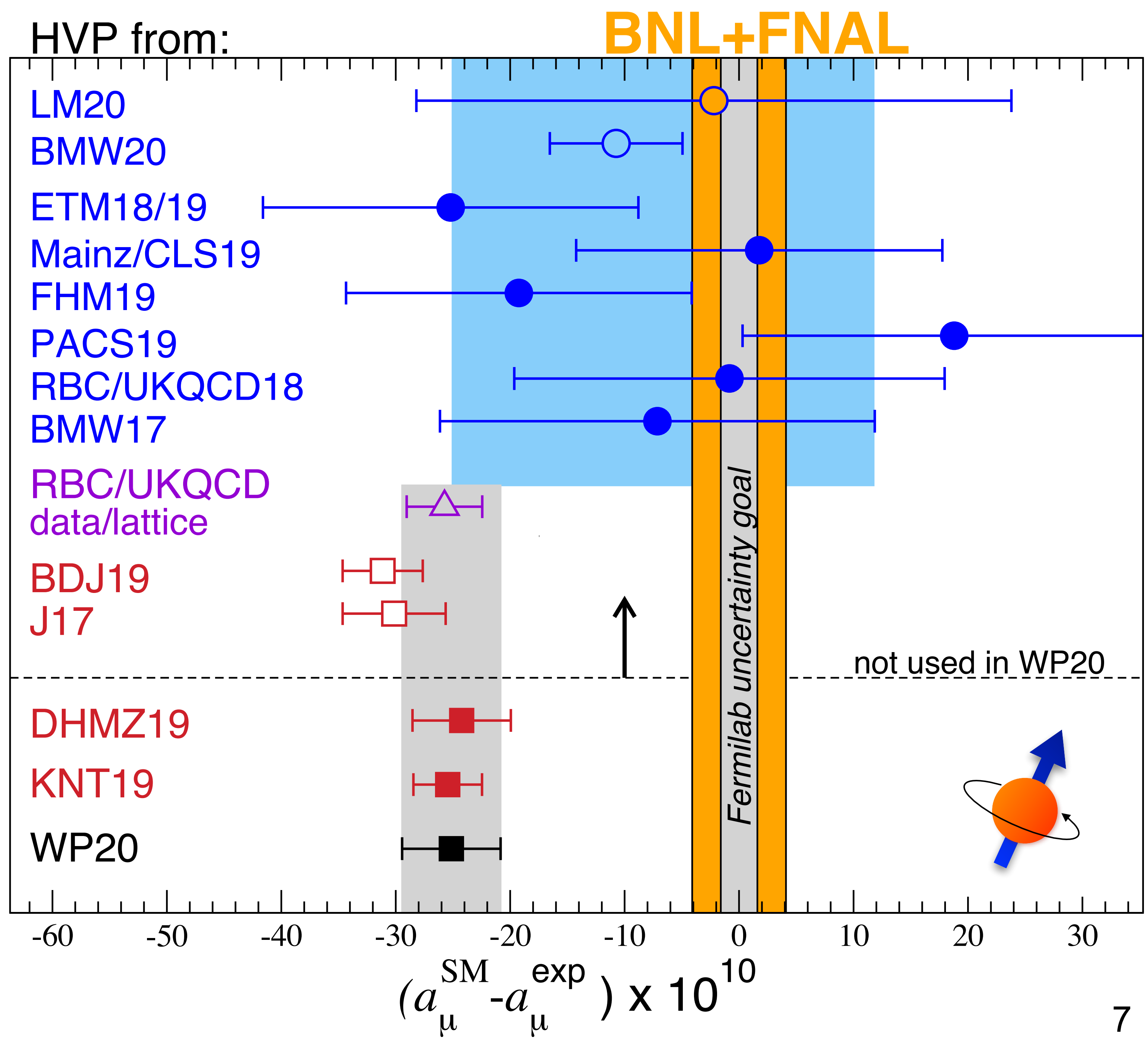
$$a_\mu^{\mathrm{SM}} \quad a_\mu^{\mathrm{HVP}} + \left[a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{Weak}} + a_\mu^{\mathrm{HLbL}}\right]$$

Lattice QCD + QED

hybrid: combine data & lattice

data driven

+ unitarity/analyticity constraints

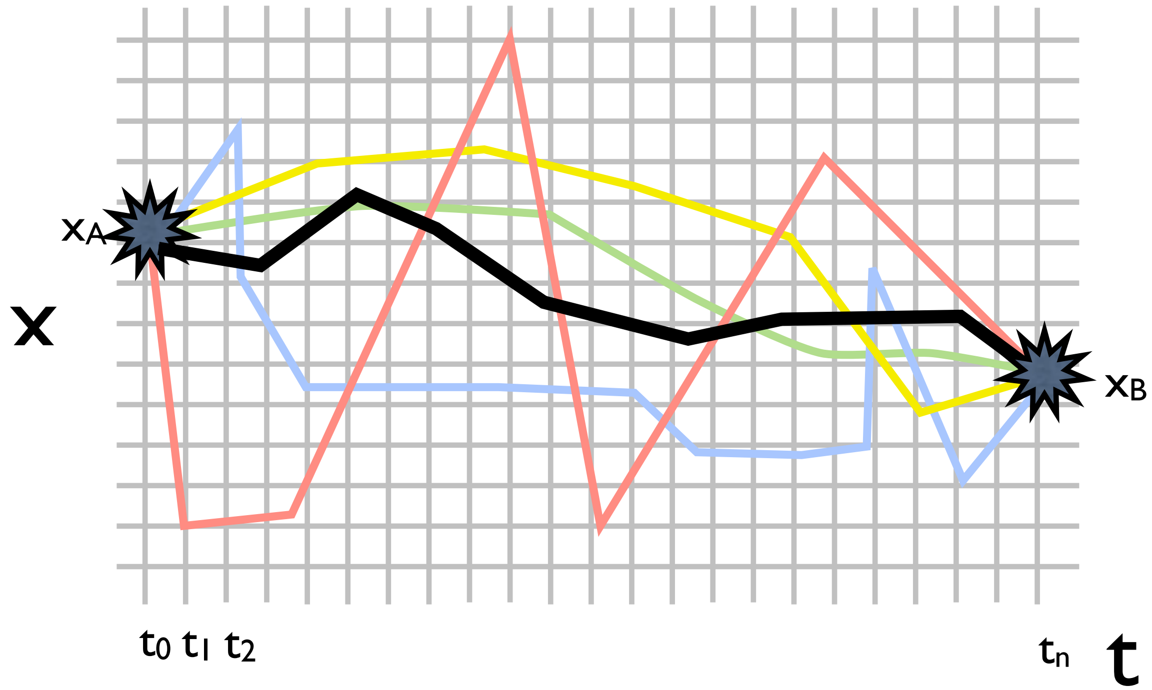


Small interlude:
Lattice QCD

Lattice QCD

Numerical first-principles approach to
non-perturbative QCD

- Discretise QCD onto 4D space-time lattice
- QCD equations ⟷ integrals over the values of quark and gluon fields on each site/link (QCD path integral)
- ~10^{12} variables (for state-of-the-art)

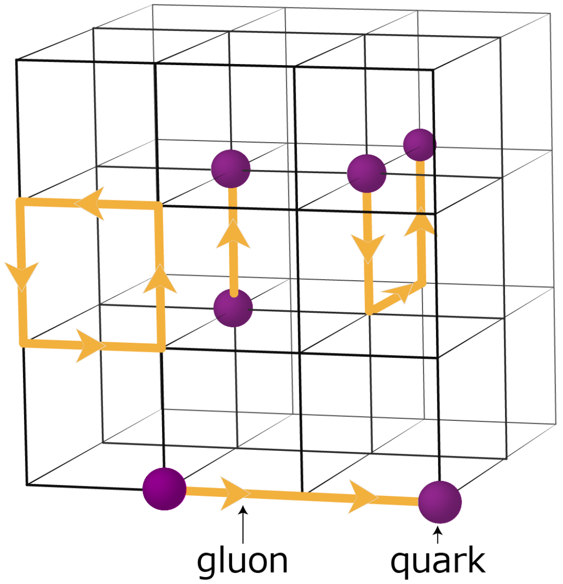


- Evaluate by importance sampling
- Paths near classical action dominate
- Calculate physics on a set (ensemble) of samples of the quark and gluon fields

Lattice QCD

Numerical first-principles approach to non-perturbative QCD

- Euclidean space-time $t \to i\tau$
- Finite lattice spacing a
- Volume $L^3 \times T = 64^3 \times 128$
- Boundary conditions



Approximate the QCD path integral by **Monte Carlo**

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A \mathcal{D}\overline{\psi} \mathcal{D}\psi \mathcal{O}[A, \overline{\psi}\psi] e^{-S[A,\overline{\psi}\psi]} \quad \Rightarrow \quad \langle \mathcal{O} \rangle \simeq \frac{1}{N_{\mathrm{conf}}} \sum_{i}^{N_{\mathrm{conf}}} \mathcal{O}([U^i])$$

with field configurations U^i distributed according to $e^{-S[U]}$

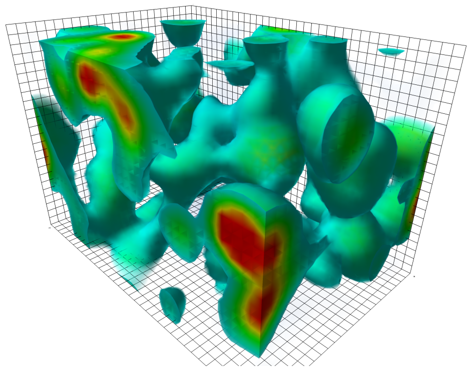
Lattice QCD

Workflow of a lattice QCD calculation

1. Generate field configurations via Hybrid Monte Carlo
   - Leadership-class computing
   - ~100K cores or 1000GPUs, 10's of TF-years
   - O(100-1000) configurations, each ~10-100GB

2. Compute propagators
   - Large sparse matrix inversion
   - ~few 100s GPUs
   - 10x field config in size, many per config

3. Contract into correlation functions
   - ~few GPUs
   - O(100k-1M) copies



Lattice QCD action density, Leinweber, CSSM, Adelaide, 2003

Hadrons are emergent phenomena of statistical average over background gluon configurations

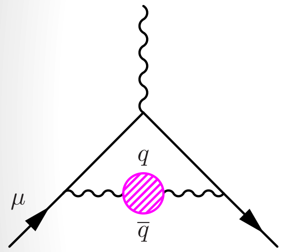
- 1 year on supercomputer ~ 100k years on laptop

Challenges of a full lattice calculation

To make contact with experiment need:

- **A valid approximation to the SM**
  - $\rightarrow$ at least u, d, s in the sea w/ $m_u = m_d \ll m_s$ (N_f=2+1) $\Rightarrow \sigma \sim 1\%$
  - $\rightarrow$ better also include c (N_f=2+1+1) & $m_u \leq m_d$ & EM $\Rightarrow \sigma \sim 0.1\%$
- **u & d w/ masses well w/in $SU(2)$ chiral regime** : $\sigma_\chi \sim (M_\pi/4\pi F_\pi)^2$
  - $\rightarrow$ $M_\pi \sim 135\,\mathrm{MeV}$ or many $M_\pi \leq 400\,\mathrm{MeV}$ w/ $M_\pi^{\mathrm{min}} < 200\,\mathrm{MeV}$ for $M_\pi \rightarrow 135\,\mathrm{MeV}$
- **a $\rightarrow$ 0** : $\sigma_a \sim (a\Lambda_{\mathrm{QCD}})^n,\ (am_q)^n,\ (a|\vec{p}|)^n$ w/ $a^{-1} \sim 2 \div 4\,\mathrm{fm}$
  - $\rightarrow$ at least 3 a's $\leq 0.1\,\mathrm{fm}$ for $a \rightarrow 0$
- **L $\rightarrow \infty$** : $\sigma_L \sim (M_\pi/4\pi F_\pi)^2 \times e^{-LM_\pi}$ for stable hadrons, $\sim 1/L^n$ for resonances, QED, ...
  - $\rightarrow$ many L w/ $(LM_\pi)^{\mathrm{max}} \gtrsim 4$ for stable hadrons & better otherwise to allow for $L \rightarrow \infty$
- These requirements $\Rightarrow O(10^{12})$ **dofs** that have to be integrated over
- **Renormalization** : best done nonperturbatively
- **A signal** : $\sigma_{\mathrm{stat}} \sim 1/\sqrt{N_{\mathrm{meas}}}$, reduce w/ $N_{\mathrm{meas}} \rightarrow \infty$

HVP from the lattice

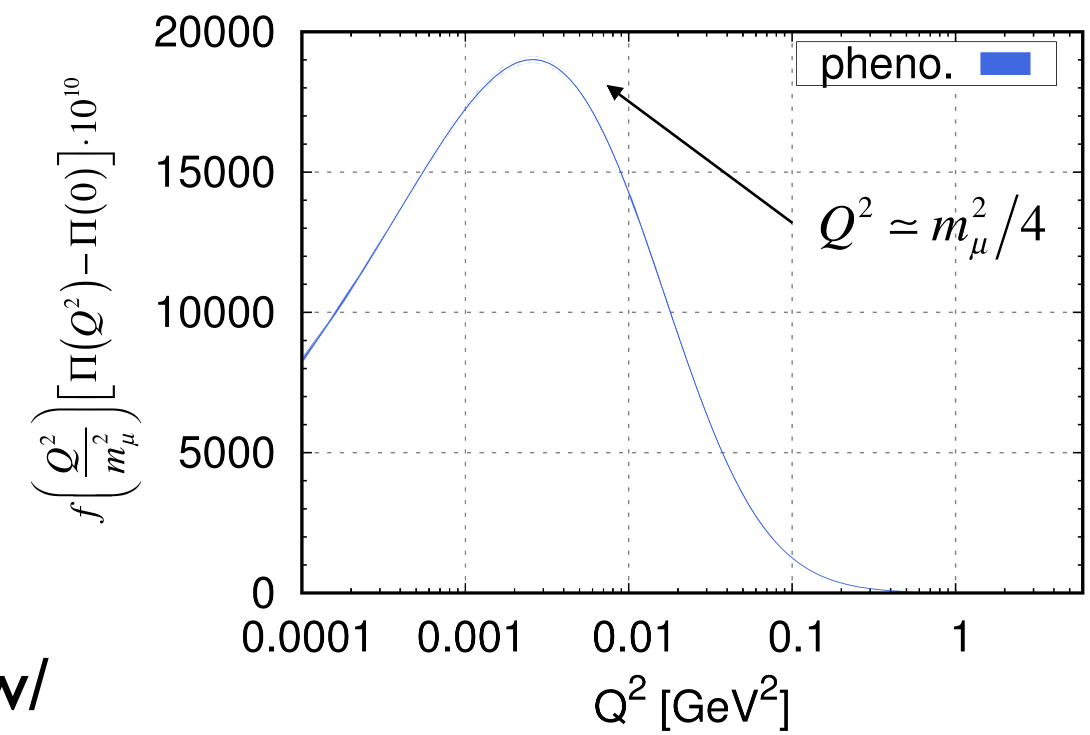


HVP from LQCD

$$\Pi_{\mu\nu}(Q) = \int d^4x\, e^{iQ\cdot x} \langle J_\mu(x) J_\nu(0) \rangle = \left[\delta_{\mu\nu} Q^2 - Q_\mu Q_\nu \right] \Pi(Q^2)$$

$$a_\mu^{\text{HVP,LO}} = 4\alpha_{em}^2 \int_0^\infty dQ^2 \, \frac{1}{m_\mu^2} f\left(\frac{Q^2}{m_\mu^2} \right) \left[\Pi(Q^2) - \Pi(0) \right]$$

B. E. Lautrup *et al.*, 1972



F. Jegerlehner, "alphaQEDc17"

FV & $a \neq 0$: A. discrete momenta ($Q_{\min} = 2\pi/T > m_\mu/2$); B. $\Pi_{\mu\nu}(0) \neq 0$ in FV contaminates $\Pi(Q^2) \sim \Pi_{\mu\nu}(Q)/Q^2$ for $Q^2 \to 0$ w/ very large FV effects; C. $\Pi(0) \sim \ln(a)$

↓

Time-Momentum Representation

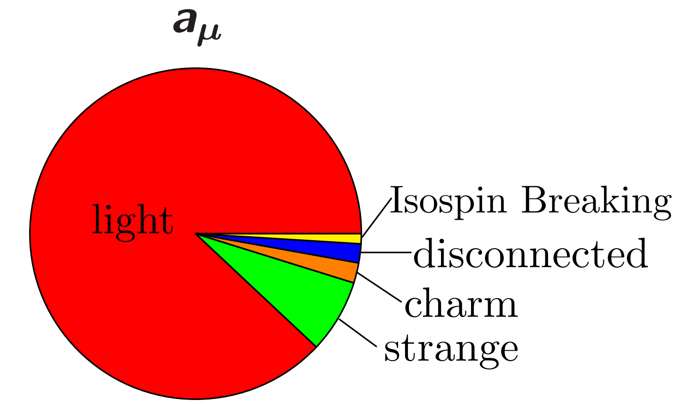
$$a_\mu^{\text{HVP,LO}} = 4\alpha_{em}^2 \int_0^\infty dt \; \tilde{f}(t) \, V(t)$$

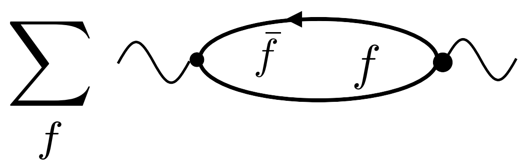
$$V(t) \equiv \frac{1}{3} \sum_{i=1,2,3} \int d\vec{x} \, \langle J_i(\vec{x},t) J_i(0) \rangle$$

D. Bernecker and H. B. Meyer, 2011

Time-Momentum Representation

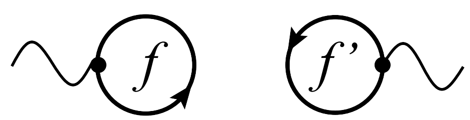
- No reliance on exp. data, except for hadronic quantities used to calibrate the simulation ($M_\pi, M_K, M_{nucl}, \ldots$)
- Can perform an explicit quark flavor separation of $a_\mu^{\rm HVP,LO}$



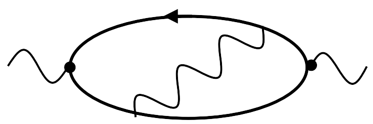


light-quark connected $a_\mu^{\rm HVP,LO}({\rm ud}) \sim 90\,\%$ of total

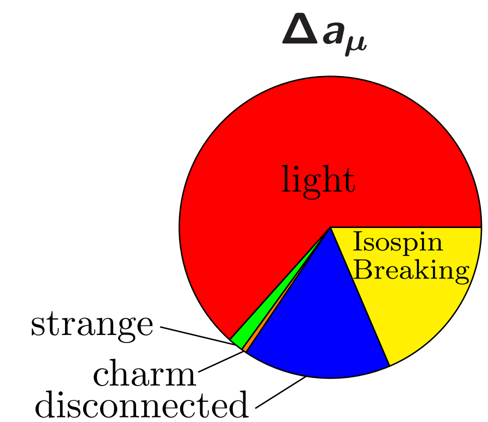
s,c-quark connected $a_\mu^{\rm HVP,LO}({\rm s,c}) \sim 8\,\%\,, 2\,\%$ of total



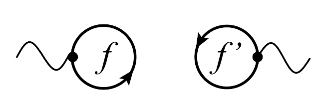
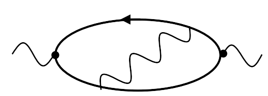
disconnected $a_{\mu,disc}^{\rm HVP,LO} \sim 2\,\%$ of total

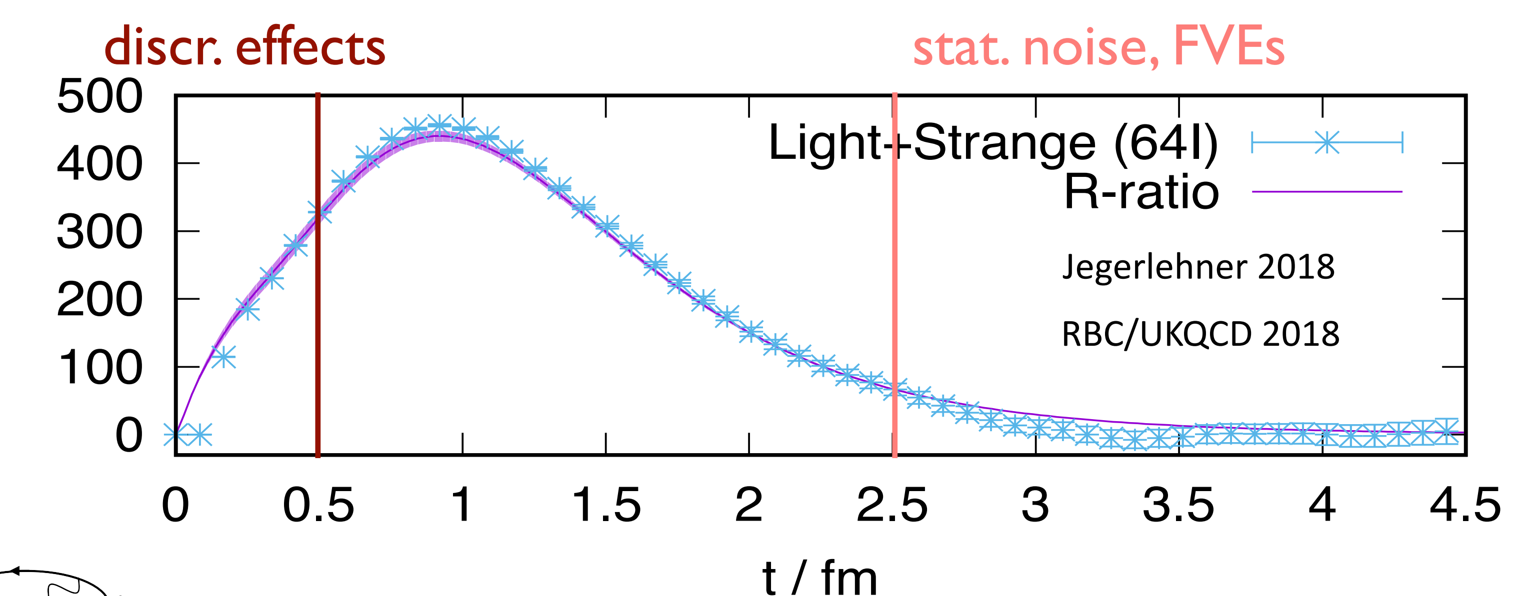


IB ($m_u \neq m_d + {\rm QED}$) $\delta a_\mu^{\rm HVP,LO} \sim 1\,\%$ of total



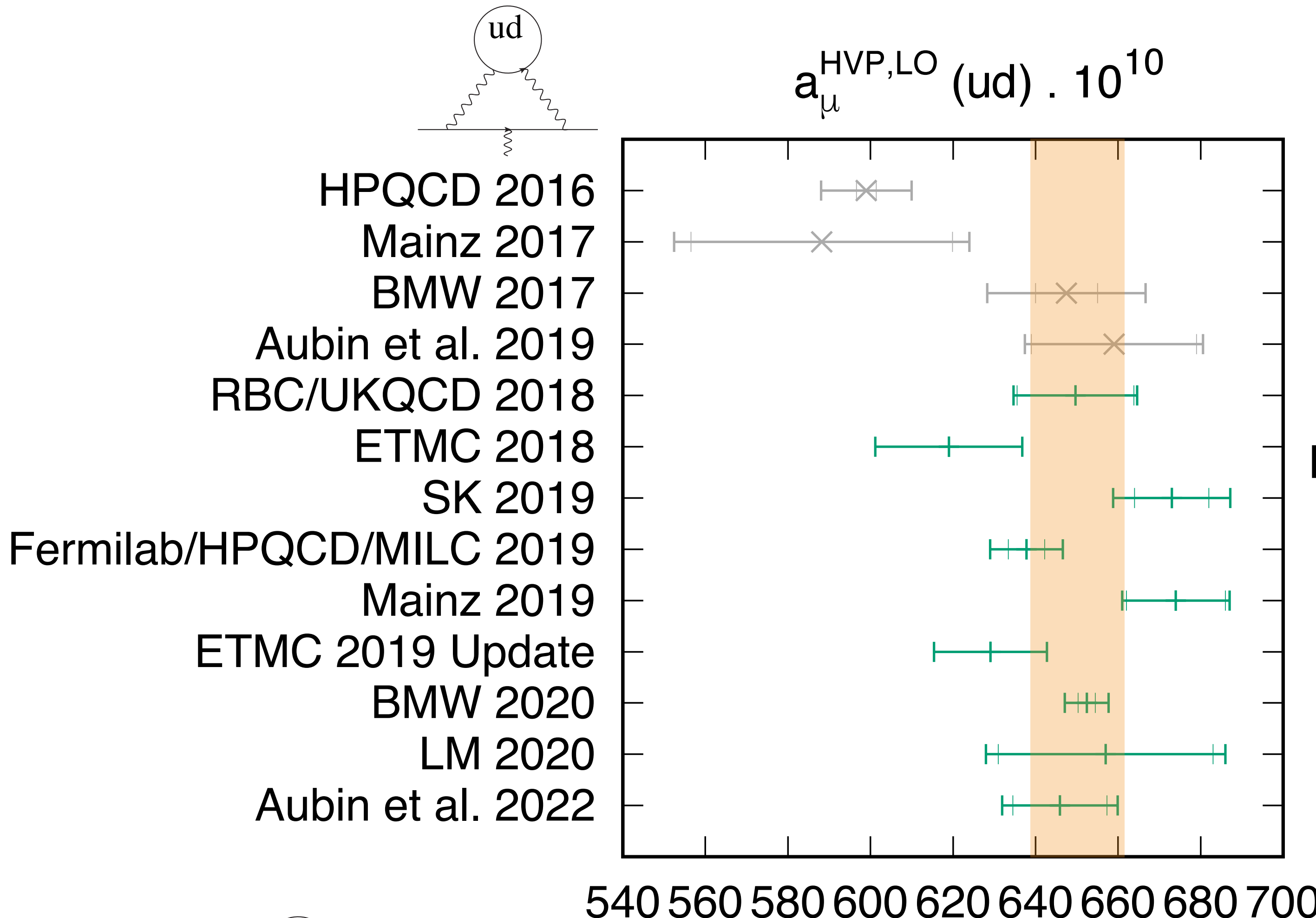
Challenges:

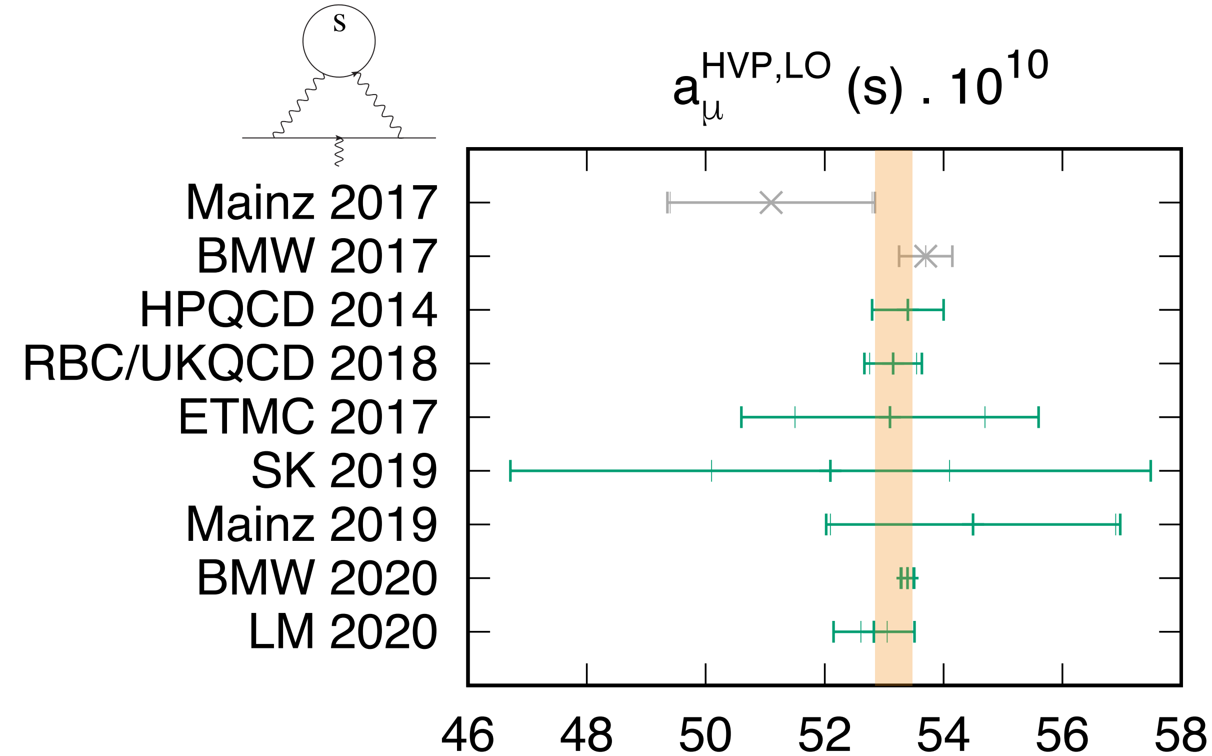
- sub-percent stat. precision
  exp. growing StN ratio in $V(t)$ as $t \to \infty$
- correct for FVEs, control discr. effects (scale setting and continuum extrap.)
- quark-disconn. diagrams 
  control stat. & stochastic noise
- isospin-breaking: $m_u \neq m_d,\ \alpha_{em} \neq 0$ 

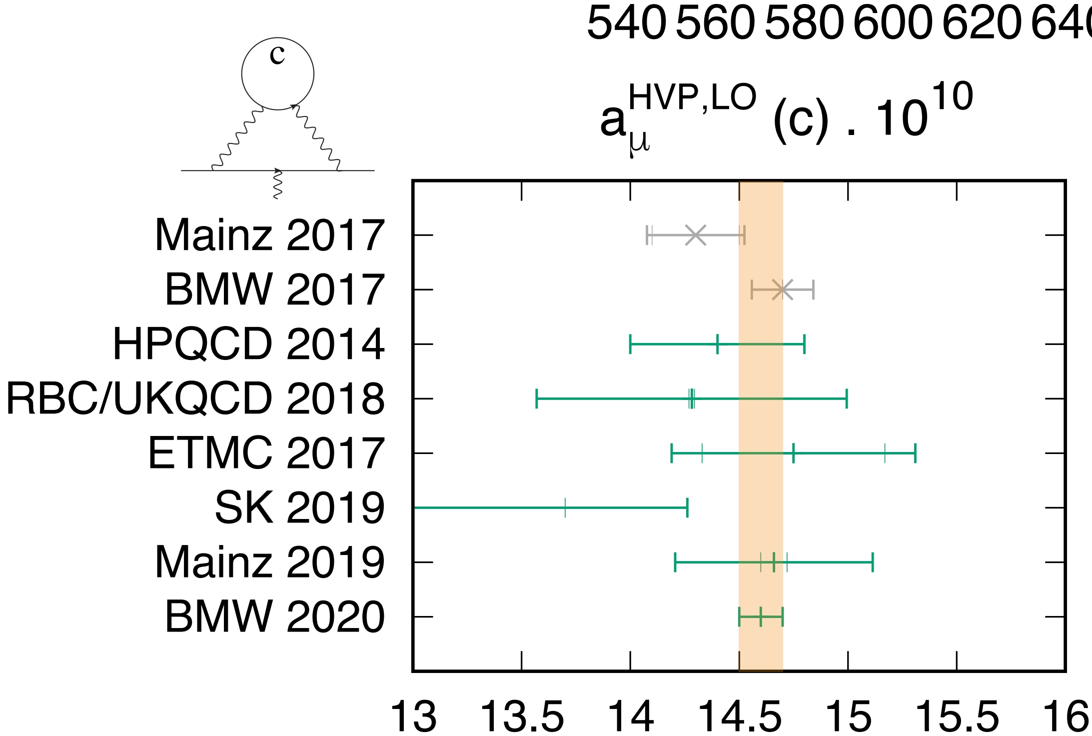


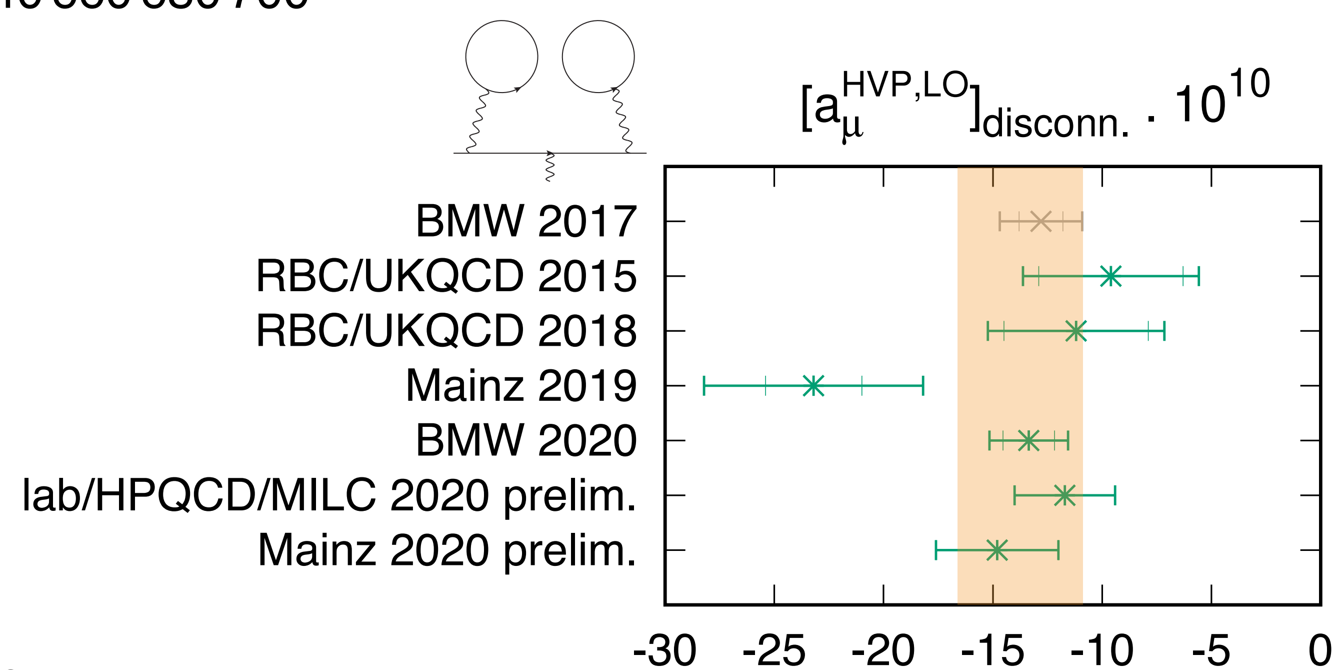
Results for each contribution

WP '20

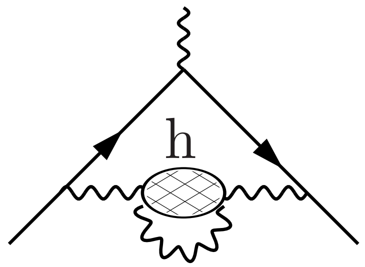


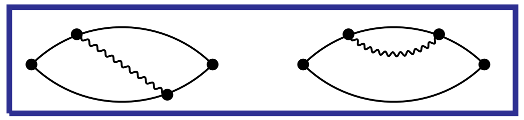




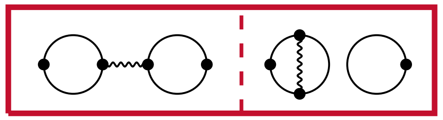


Isospin-breaking contributions

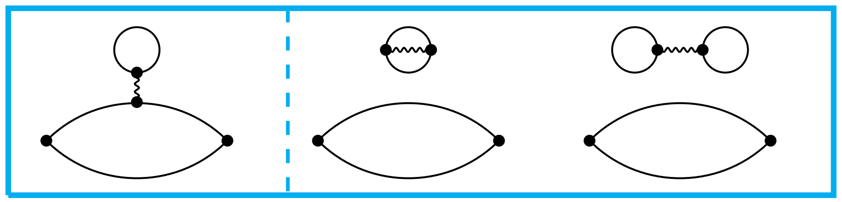




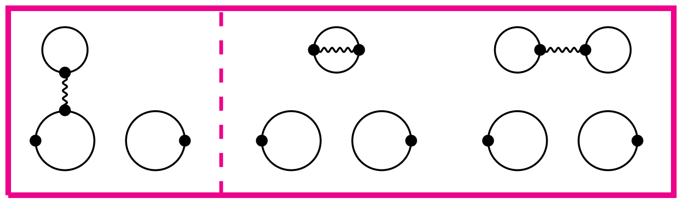
BMW $-1.27(40)(33)$
RBC/UKQCD $5.9(5.7)(1.7)$
ETM $1.1(1.0)$



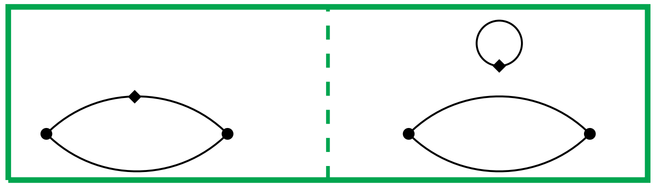
$-0.55(15)(11)$ BMW
$-6.9(2.1)(2.0)$ RBC/UKQCD



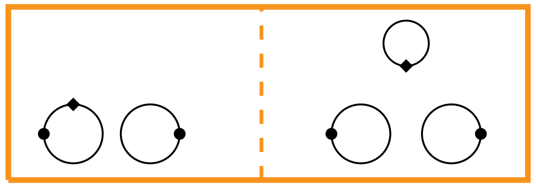
$-0.0095(86)(99)$ $0.42(20)(19)$ BMW



$0.011(24)(14)$ $-0.047(33)(23)$ BMW



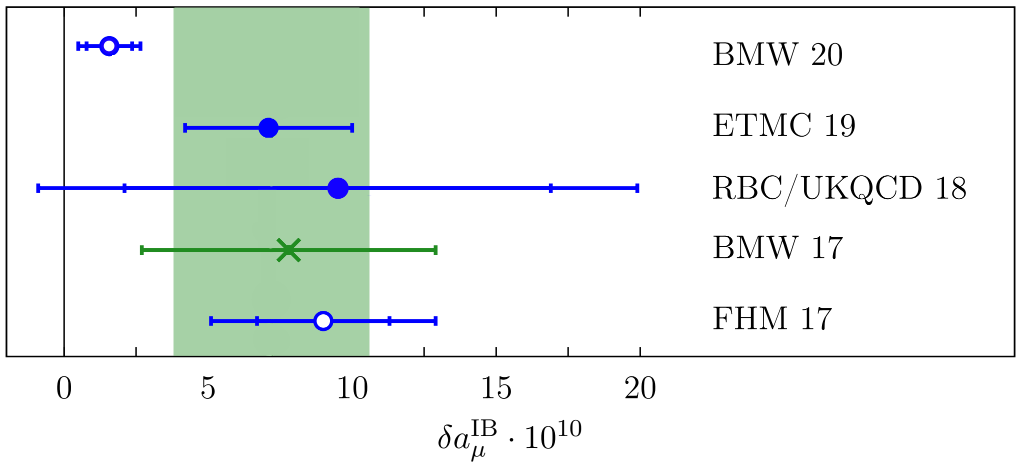
$6.59(63)(53)$ BMW
$10.6(4.3)(6.8)$ RBC/UKQCD
$6.0(2.3)$ ETM
$7.7(3.7)$ $9.0(2.3)$ FHM
$9.0(0.8)(1.2)$ LM



$-4.63(54)(69)$ BMW

BMW [arXiv:2002.12347]
RBC/UKQCD [Phys.Rev.Lett. 121 (2018) 2, 022003]
ETM [Phys. Rev. D 99, 114502 (2019)]
FHM [Phys.Rev.Lett. 120 (2018) 15, 152001]
LM [Phys.Rev.D 101 (2020) 074515]

WP '20

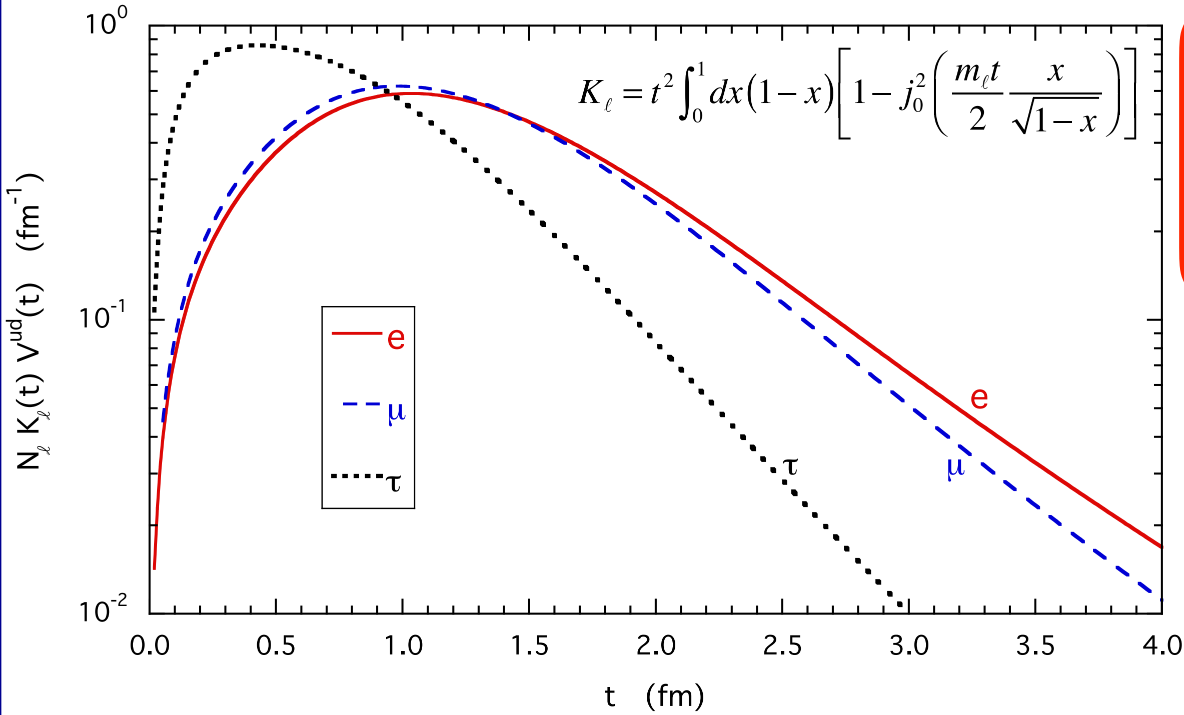


- Small overall value due to large cancellations
- Large statistical uncertainties
- More precise calculations are in progress

Ratios of the HVP contributions to the lepton g-2

Ratio electron/muon

DG and S. Simula 2020



$$R_{e/\mu} \equiv \left(\frac{m_\mu}{m_e}\right)^2 \frac{a_e^{\rm HVP}}{a_\mu^{\rm HVP}}$$

- numerator and denominator share the same hadronic input
- hadronic uncertainties strongly correlated (~ 98%) and largely cancel out

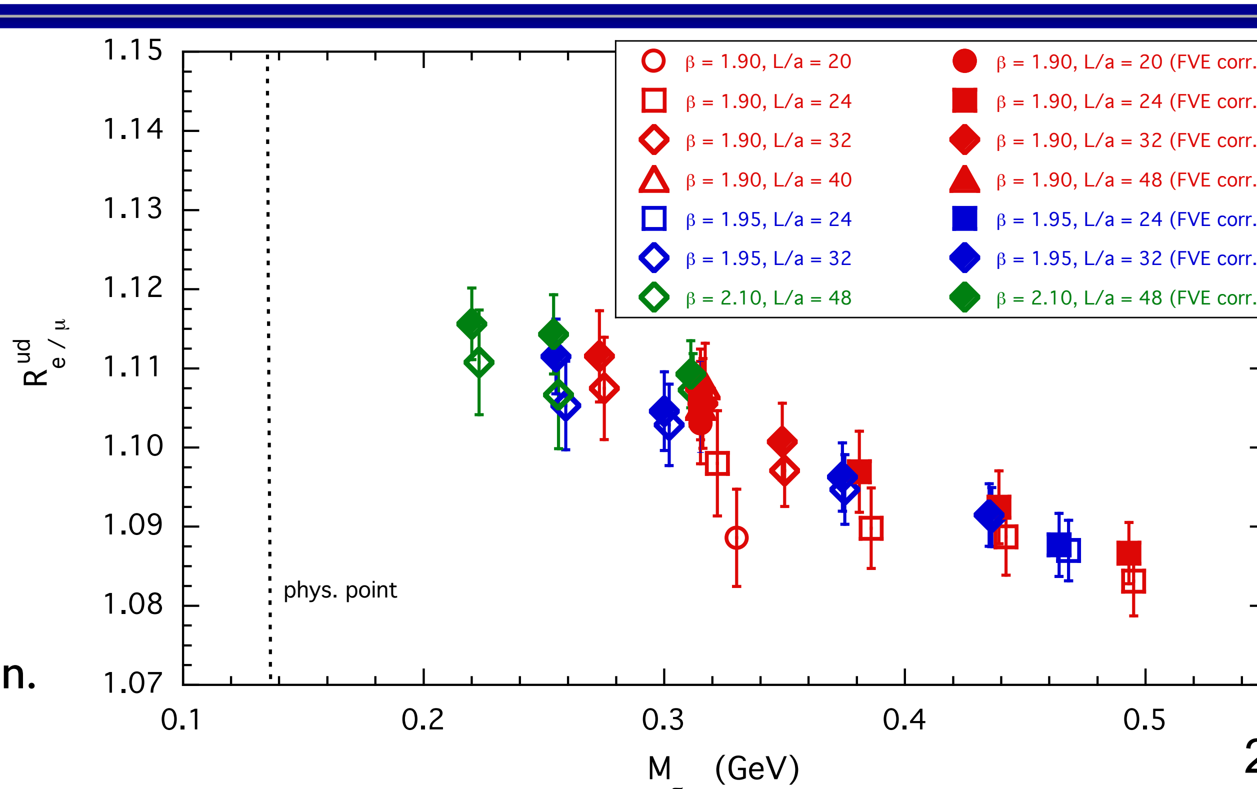
$$R_{e/\mu} \equiv R_{e/\mu}^{ud} \cdot \widetilde{R}_{e/\mu}$$

$$R_{e/\mu}^{ud} \equiv \left(\frac{m_\mu}{m_e}\right)^2 \frac{a_e^{\rm HVP}(ud)}{a_\mu^{\rm HVP}(ud)}$$

$$\widetilde{R}_{e/\mu} \equiv \frac{1 + \sum_{j=s,c,IB,disc} \frac{a_e^{\rm HVP}(j)}{a_e^{\rm HVP}(ud)}}{1 + \sum_{j=s,c,IB,disc} \frac{a_\mu^{\rm HVP}(j)}{a_\mu^{\rm HVP}(ud)}}$$

$R_{e/\mu}^{ud}$

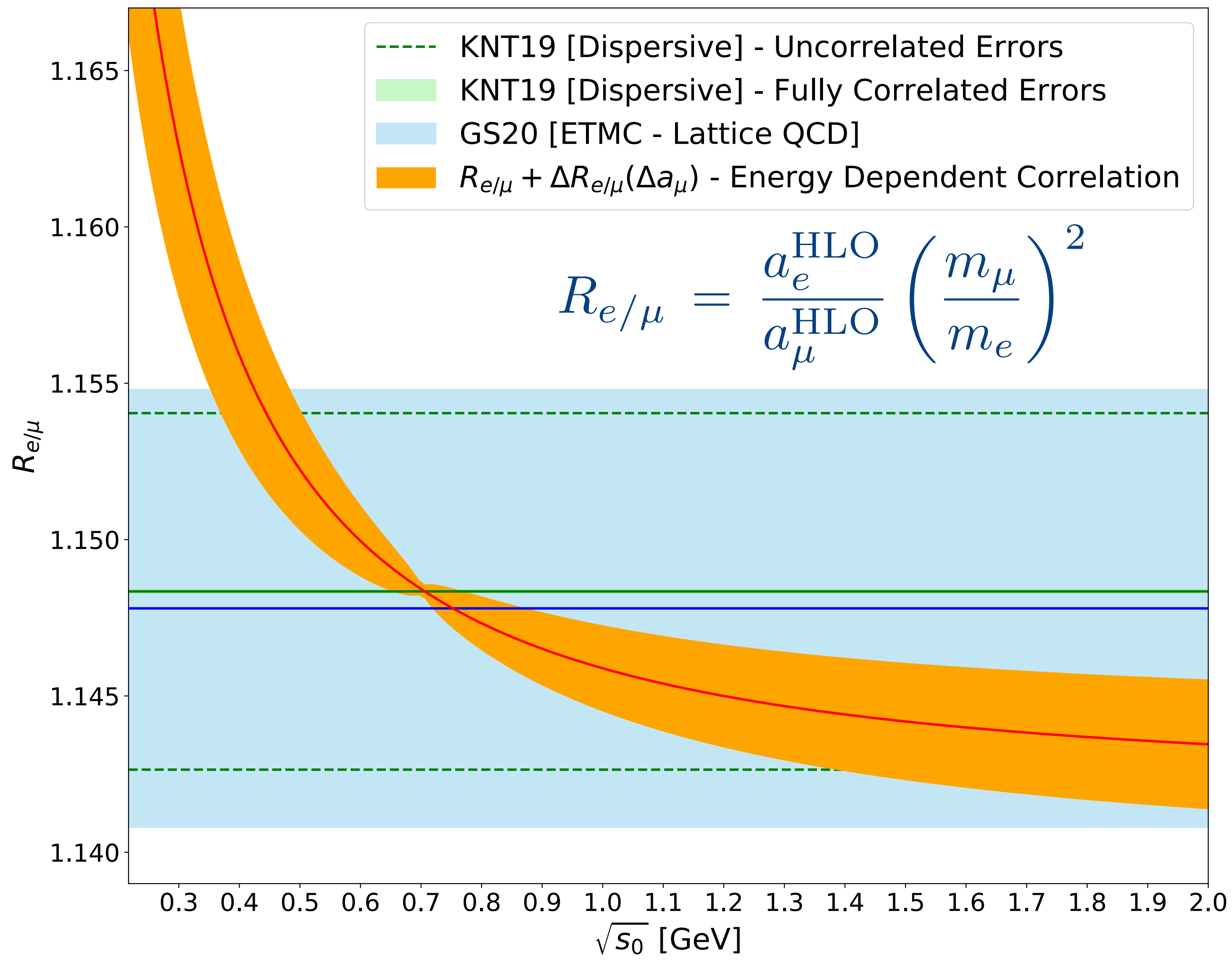
- Precision of the data ≈ 4 times better than the individual HVP terms
- Discretization and scale setting errors play a minor role
- Non-trivial pion mass dependence
- Visible FVEs, removed using the analytic representation. The correction does not exceed ~ 1.3%



Trying to accommodate the g-2 discrepancy

Shift of the e/μ g-2 scaled HLO ratio

e/μ



Good agreement between lattice [Giusti & Simula 2020] and KNT19.
Possible future bounds on very low energy shifts Δσ(s)?

Keshavarzi, Marciano, MP, Sirlin, PRD 2020

Window observables

Windows "on the g-2 mystery"

Restrict integration over Euclidean time to sub-intervals

→ reduce/enhance sensitivity to systematic effects

$$a_\mu^{\mathrm{HVP,LO}} = a_\mu^{SD} + a_\mu^{W} + a_\mu^{LD}$$

$$a_\mu^{SD}(f; t_0, \Delta) \equiv 4\alpha_{em}^2 \int_0^\infty dt\, \tilde{f}(t) V^f(t) \left[1 - \Theta\left(t, t_0, \Delta\right)\right]$$

$$a_\mu^{W}(f; t_0, t_1, \Delta) \equiv 4\alpha_{em}^2 \int_0^\infty dt\, \tilde{f}(t) V^f(t) \left[\Theta\left(t, t_0, \Delta\right) - \Theta\left(t, t_1, \Delta\right)\right]$$

$$a_\mu^{LD}(f; t_1, \Delta) \equiv 4\alpha_{em}^2 \int_0^\infty dt\, \tilde{f}(t) V^f(t)\, \Theta\left(t, t_1, \Delta\right)$$

$$\Theta\left(t, t', \Delta\right) = \frac{1}{1 + e^{-2(t-t')/\Delta}}$$

"Standard" choice:

$$t_0 = 0.4 \text{ fm} \quad t_1 = 1.0 \text{ fm}$$

$$\Delta = 0.15 \text{ fm}$$

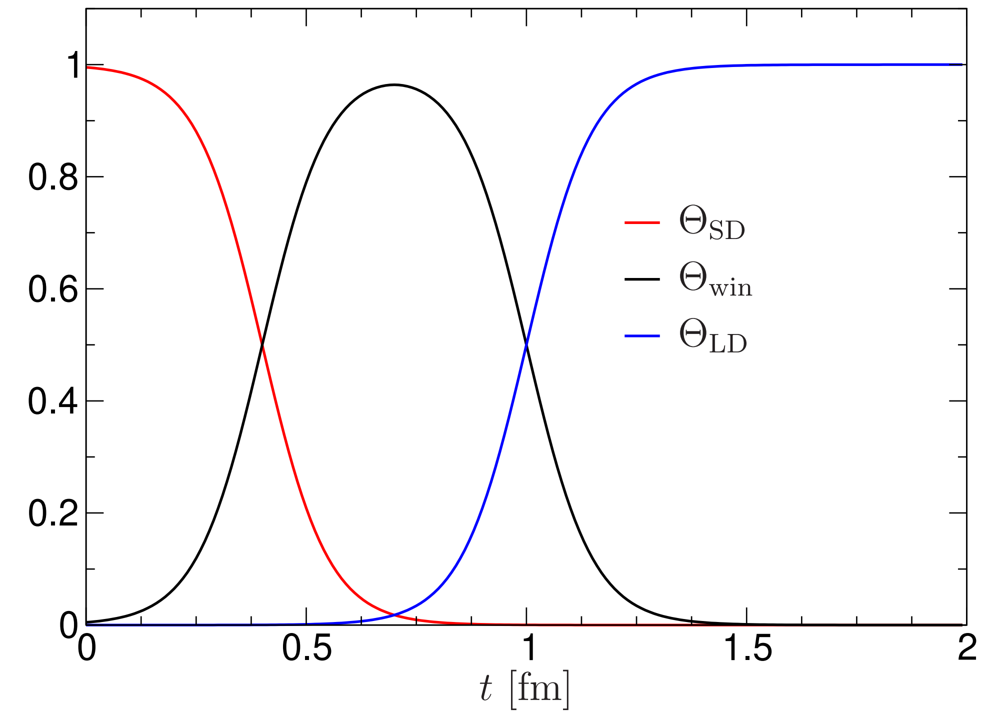
RBC/UKQCD 2018

Intermediate window

- Reduced FVEs
- Much better StN ratio

→ Precision test of different lattice calculations

→ Commensurate uncertainties compared to dispersive evaluations



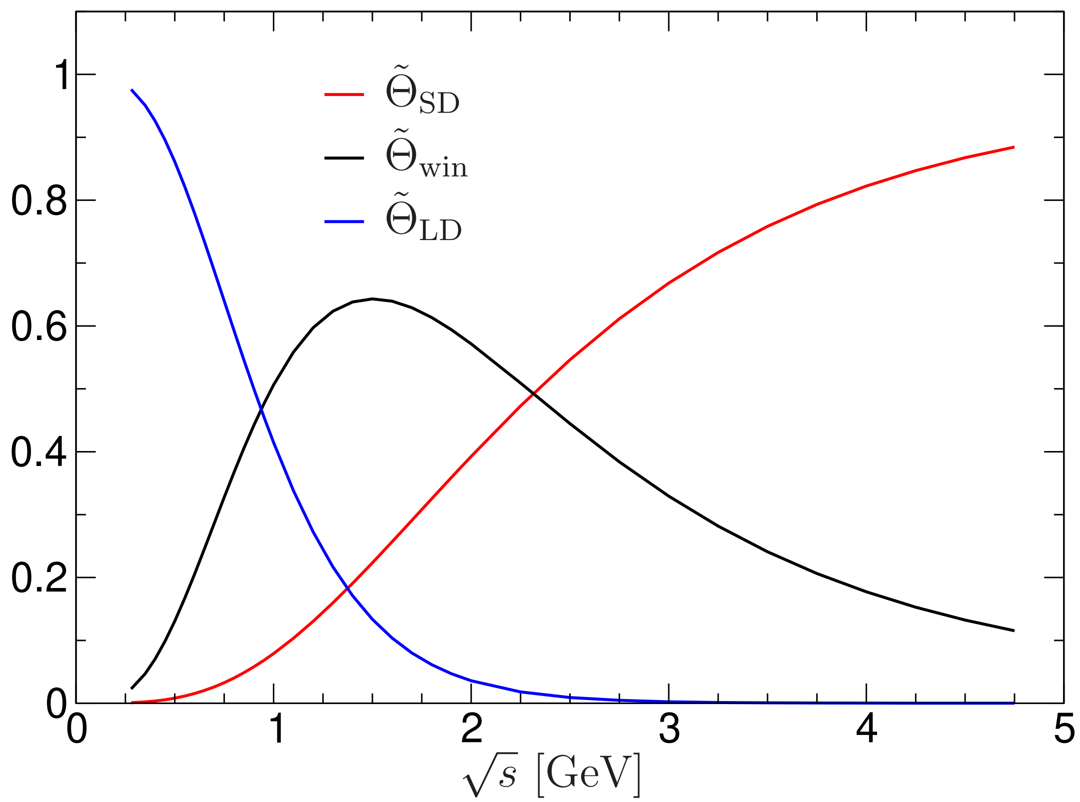
Comparison with R-ratio

$$V(t) = \frac{1}{12\pi^2}\int_{M_{\pi^0}}^{\infty} d(\sqrt{s})\, R(s)\, s\, e^{-\sqrt{s}t}$$

$$R(s) = \frac{3s}{4\pi\alpha_{em}^2}\sigma(s, e^+e^- \to hadrons)$$

Insert $V(t)$ into the expression for TMR

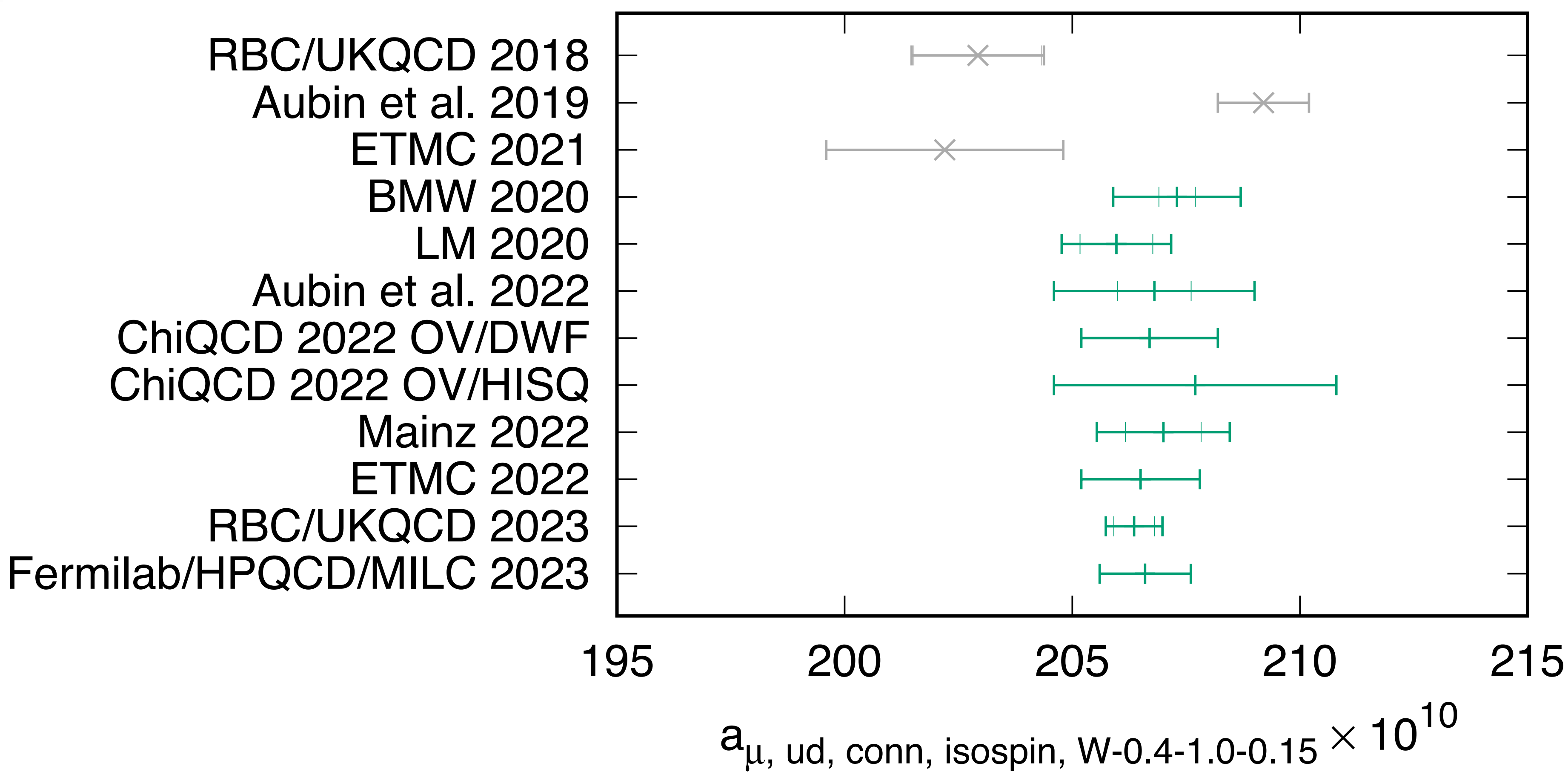
$$a_{\mu,win}^{\mathrm{HVP,LO}} = 4\alpha_{em}^2 \int_{M_{\pi^0}}^{\infty} d(\sqrt{s}) R(s)\, \frac{1}{12\pi^2}\, s \int_0^{\infty} dt\, \tilde{f}(t)\, \Theta_{win}(t)\, e^{-\sqrt{s}t}$$



Colangelo *et al.* 2022

	$a_{\mathrm{SD}}^{\mathrm{HVP}}$	$a_{\mathrm{int}}^{\mathrm{HVP}}$	$a_{\mathrm{LD}}^{\mathrm{HVP}}$	$a_{\mathrm{total}}^{\mathrm{HVP}}$
All channels	68.4(5)	229.4(1.4)	395.1(2.4)	693.0(3.9)
	[9.9%]	[33.1%]	[57.0%]	[100%]
2π below 1.0 GeV	13.7(1)	138.3(1.2)	342.3(2.3)	494.3(3.6)
	[2.8%]	[28.0%]	[69.2%]	[100%]
3π below 1.8 GeV	2.5(1)	18.5(4)	25.3(6)	46.4(1.0)
	[5.5%]	[39.9%]	[54.6%]	[100%]
White Paper [1]	–	–	–	693.1(4.0)
RBC/UKQCD [24]	–	231.9(1.5)	–	715.4(18.7)
BMWc [36]	–	236.7(1.4)	–	707.5(5.5)
BMWc/KNT [7, 36]	–	229.7(1.3)	–	–
Mainz/CLS [99]	–	237.30(1.46)	–	–
ETMC [100]	69.33(29)	235.0(1.1)	–	–

Results for the intermediate window



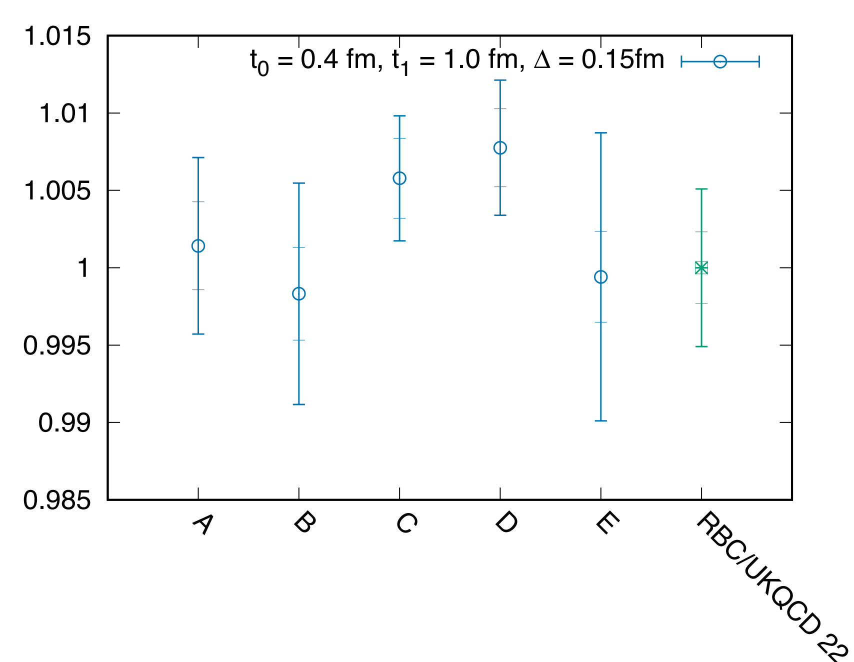
Blinding

- 2 analysis groups for ensemble parameters (not blinded)
- 5 analysis groups for vector-vector correlators (blinded, to avoid bias towards other lattice/R-ratio results)
- Blinded vector correlator $C_b(t)$ relates to true correlator $C_0(t)$ by

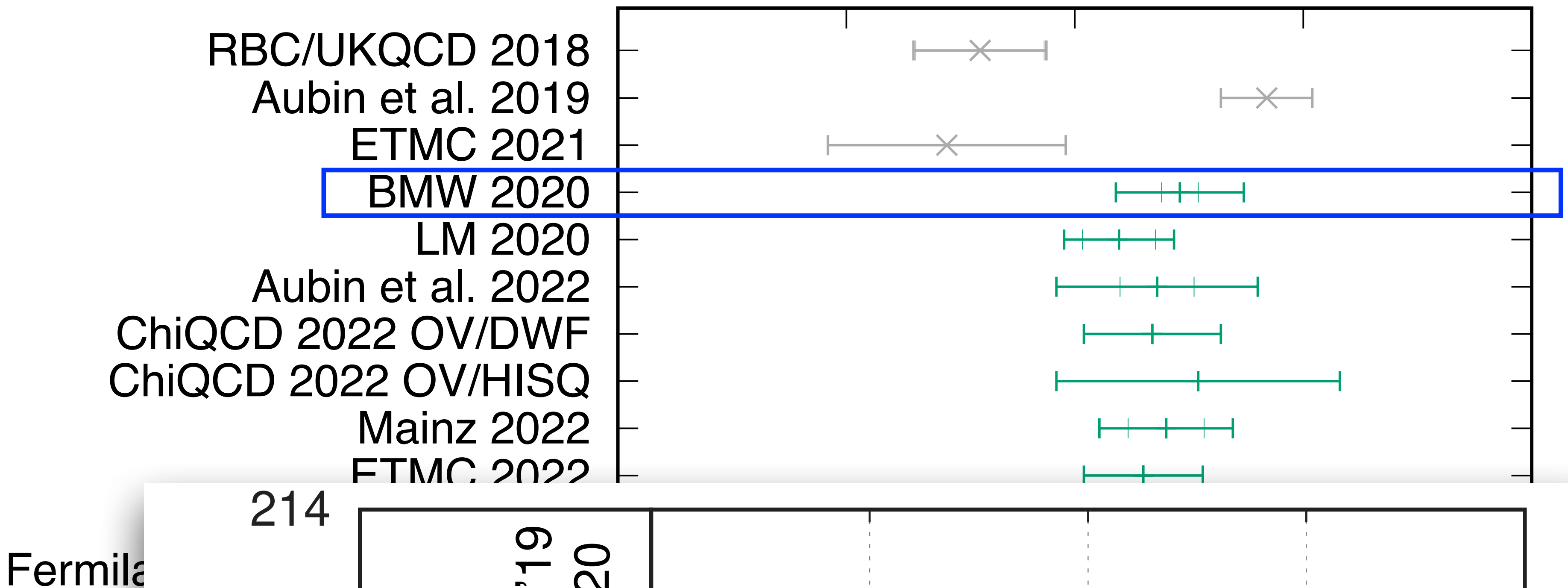
$$C_b(t) = (b_0 + b_1 a^2 + b_2 a^4) C_0(t) \qquad (1)$$

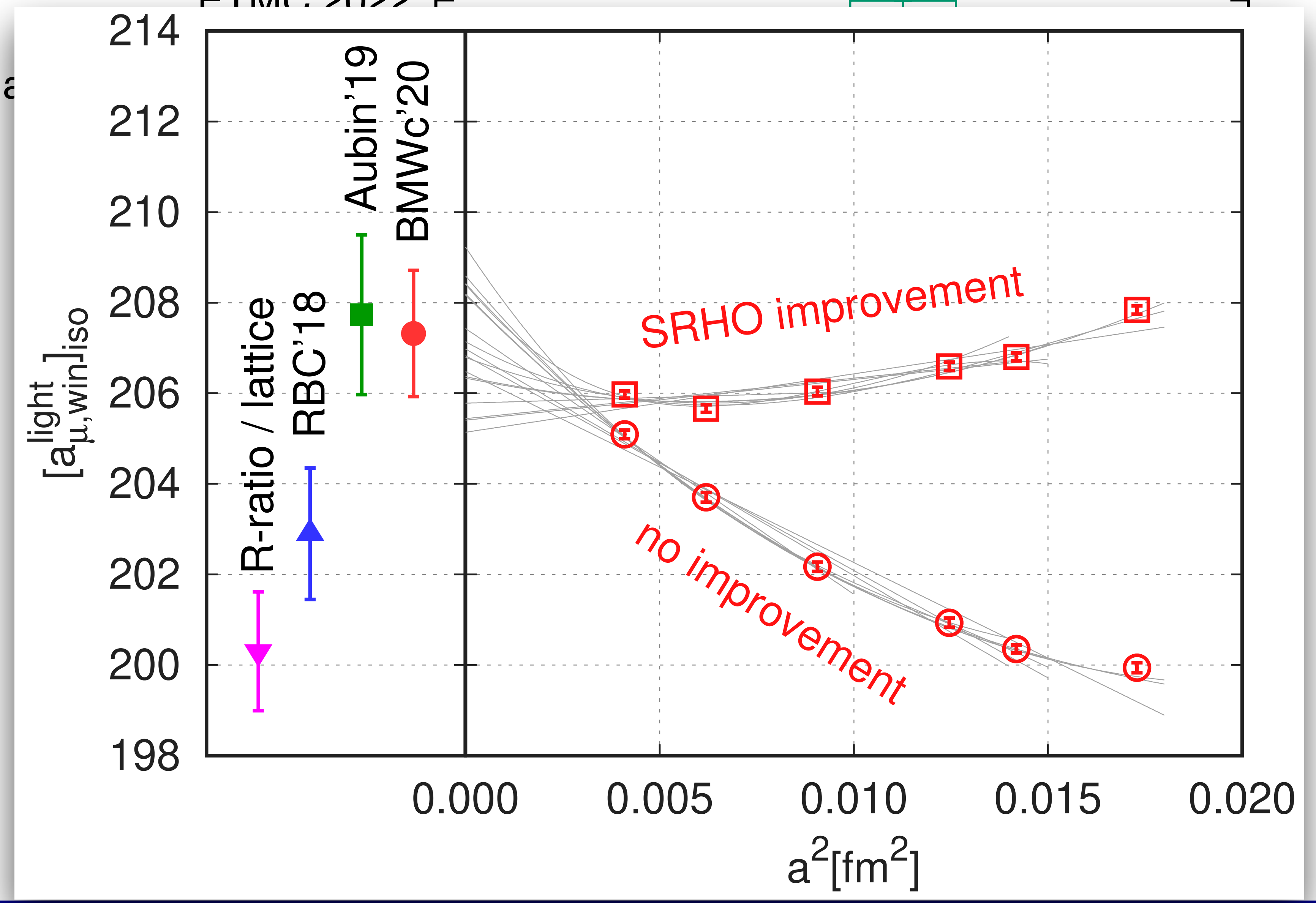
with appropriate random b_0, b_1, b_2, different for each analysis group. This prevents complete unblinding based on previously shared data on coarser ensembles.

Relative unblinding (standard window)

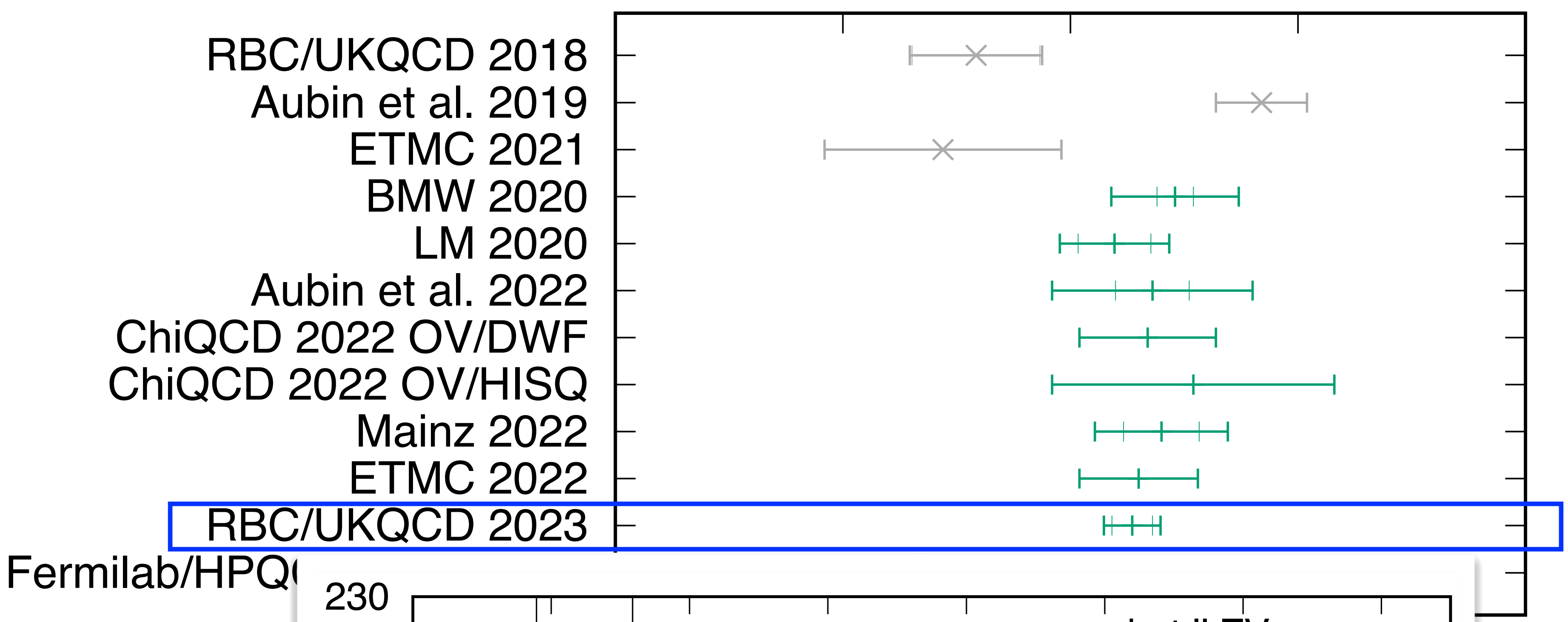


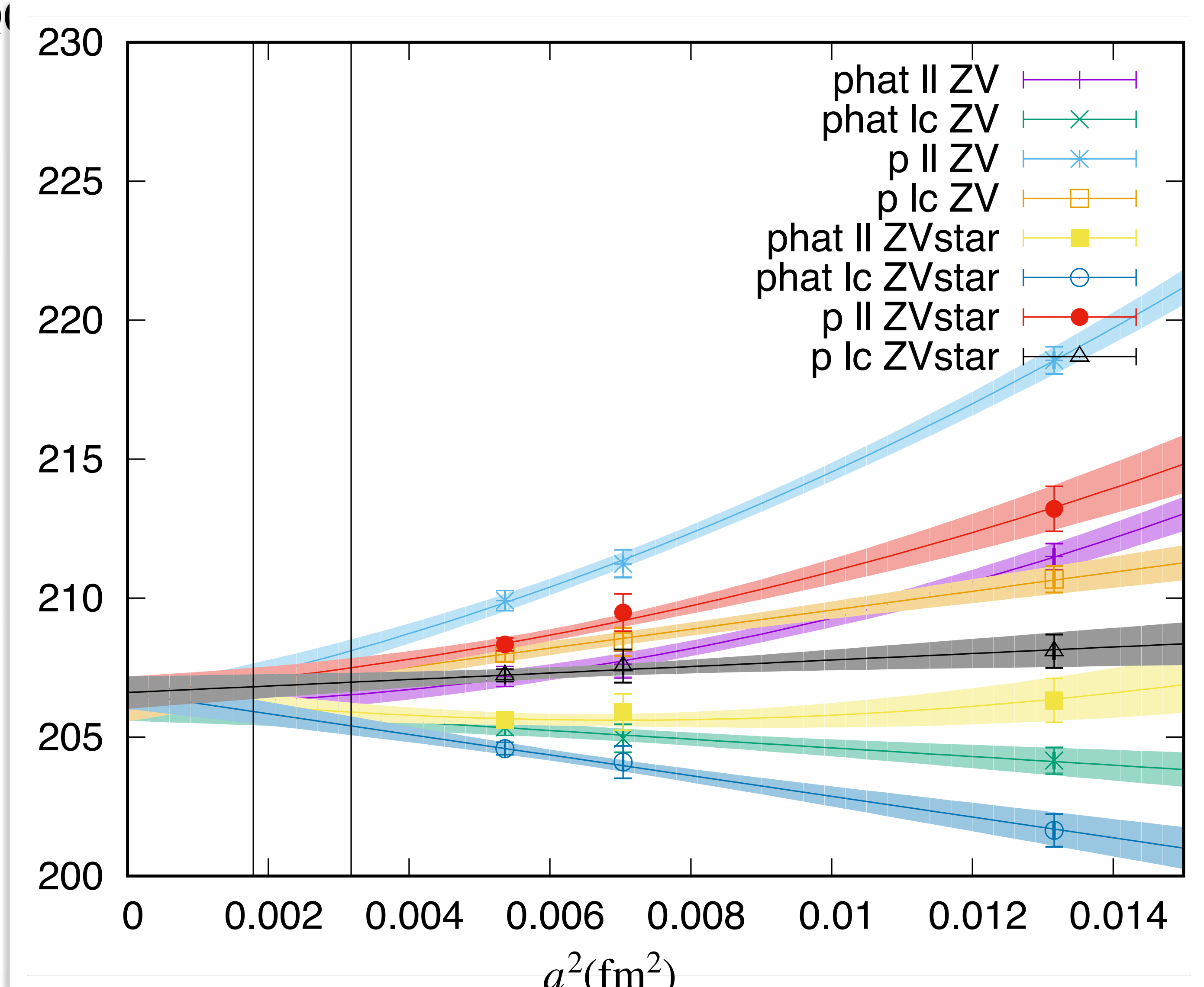
Results for the intermediate window





Results for the intermediate window



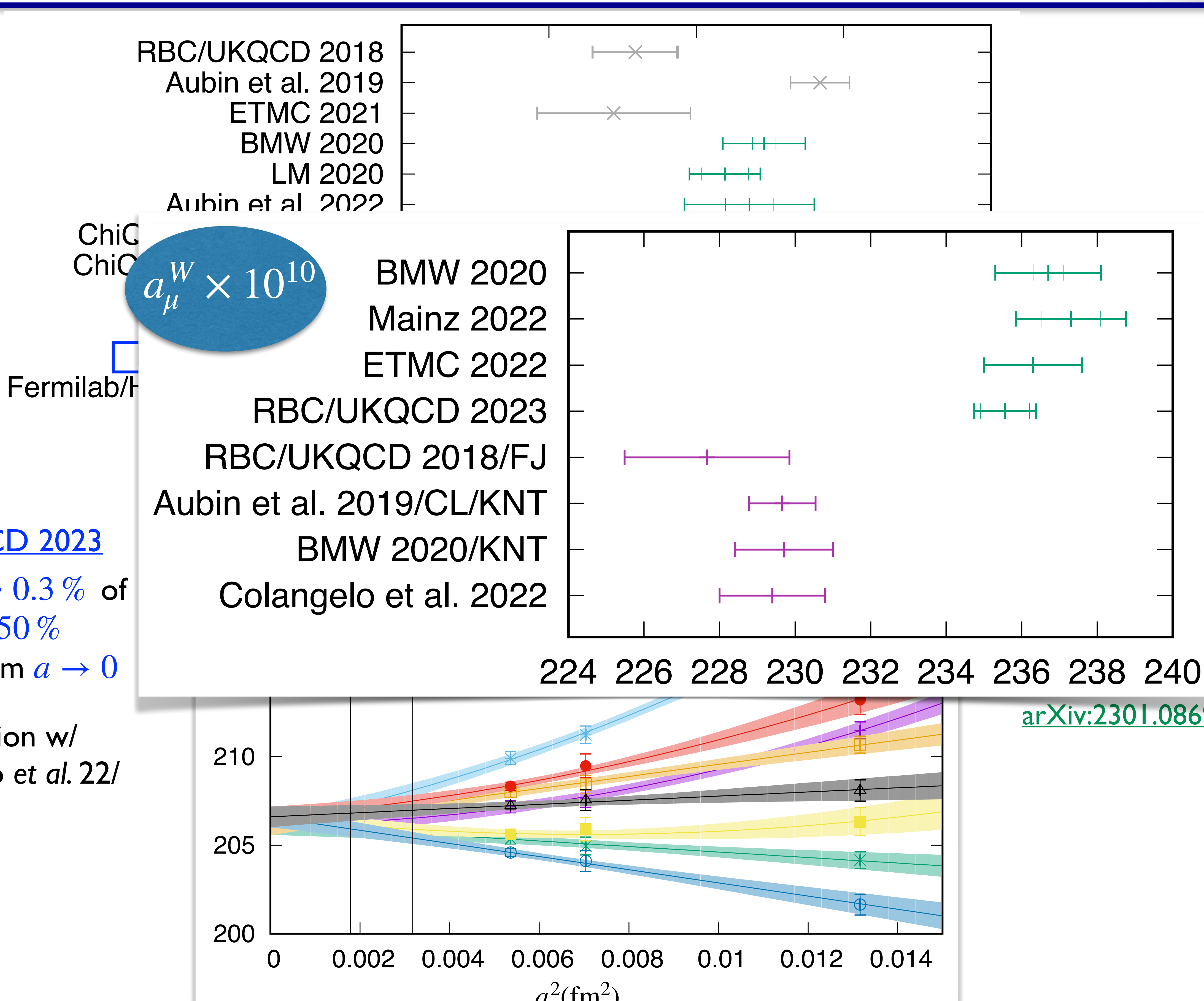


RBC/UKQCD 2023

- tot. err. $\sim 0.3\,\%$ of which $\sim 50\,\%$ comes from $a \to 0$
- 3.9σ tension w/ Colangelo *et al.* 22/ Lat

Blum, DG *et al.* 2023
arXiv:2301.08696

Results for the intermediate window



RBC/UKQCD 2023

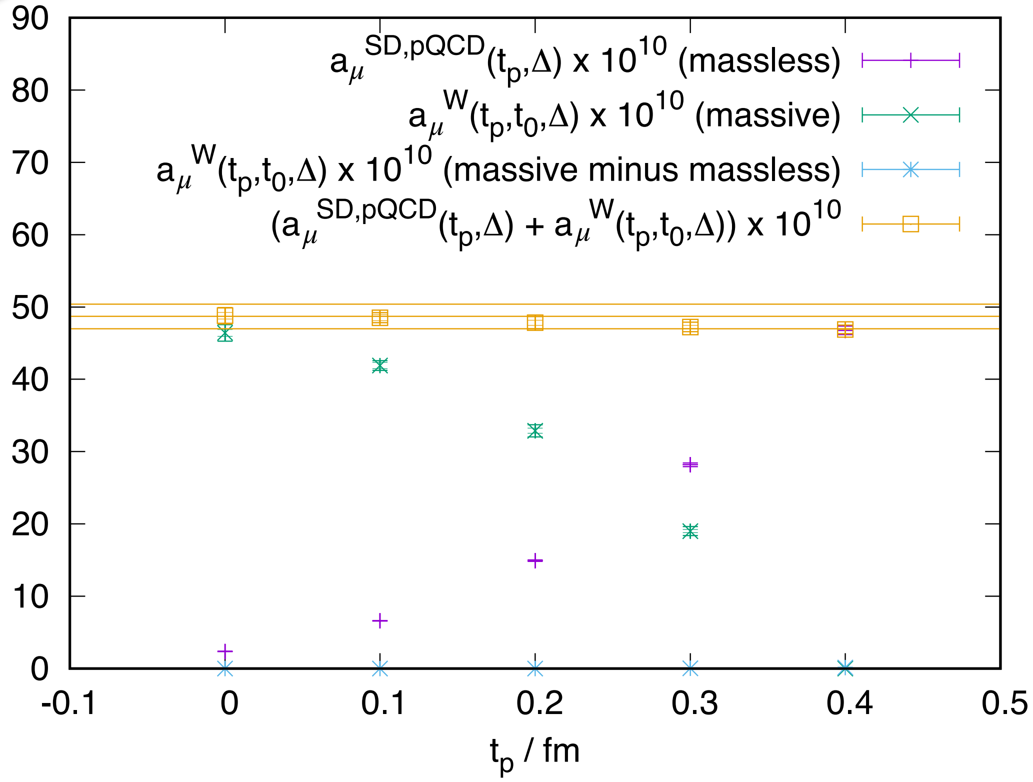
- tot. err. $\sim 0.3\,\%$ of which $\sim 50\,\%$ comes from $a \to 0$
- 3.9σ tension w/ Colangelo *et al.* 22/ Lat

023
arXiv:2301.08696

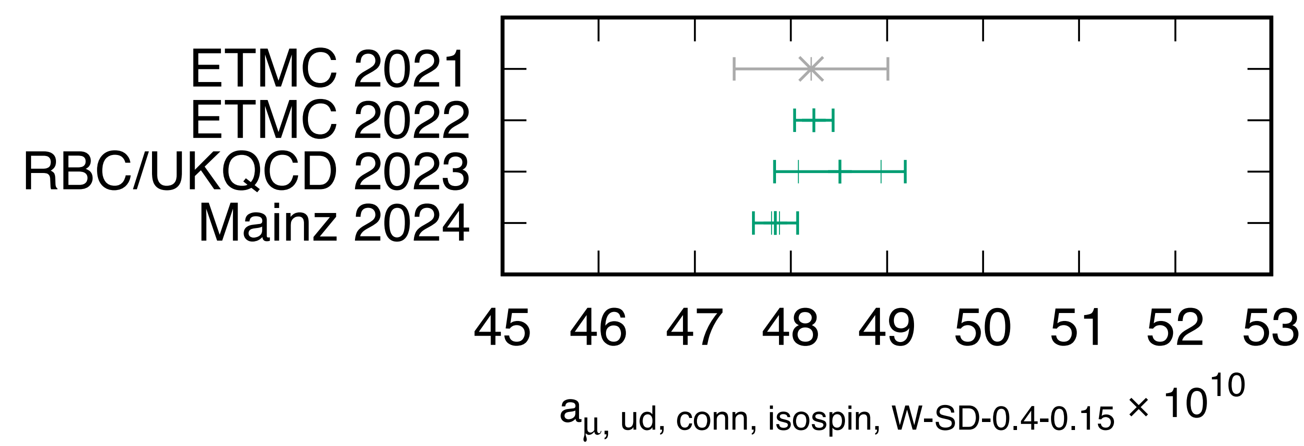
Other windows

SD

plot from RBC/UKQCD '23



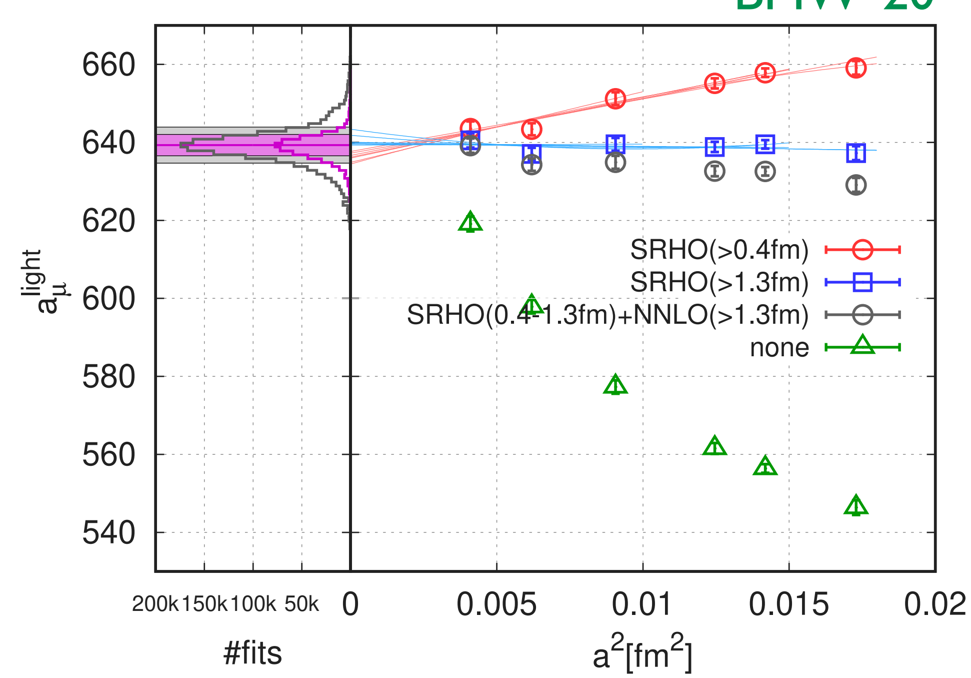
- dominated by perturbation theory
- large cutoff effects



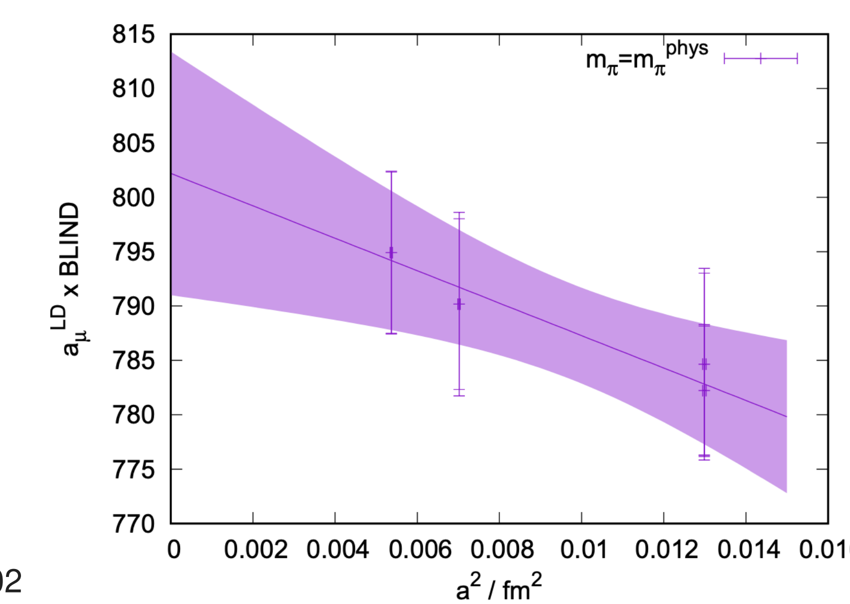
LD

- large FV effects + StN problem
- continuum limit in BMW20 calculation is non trivial
- sub-percent accuracy goal

BMW '20



RBC/UKQCD - blind, preliminary



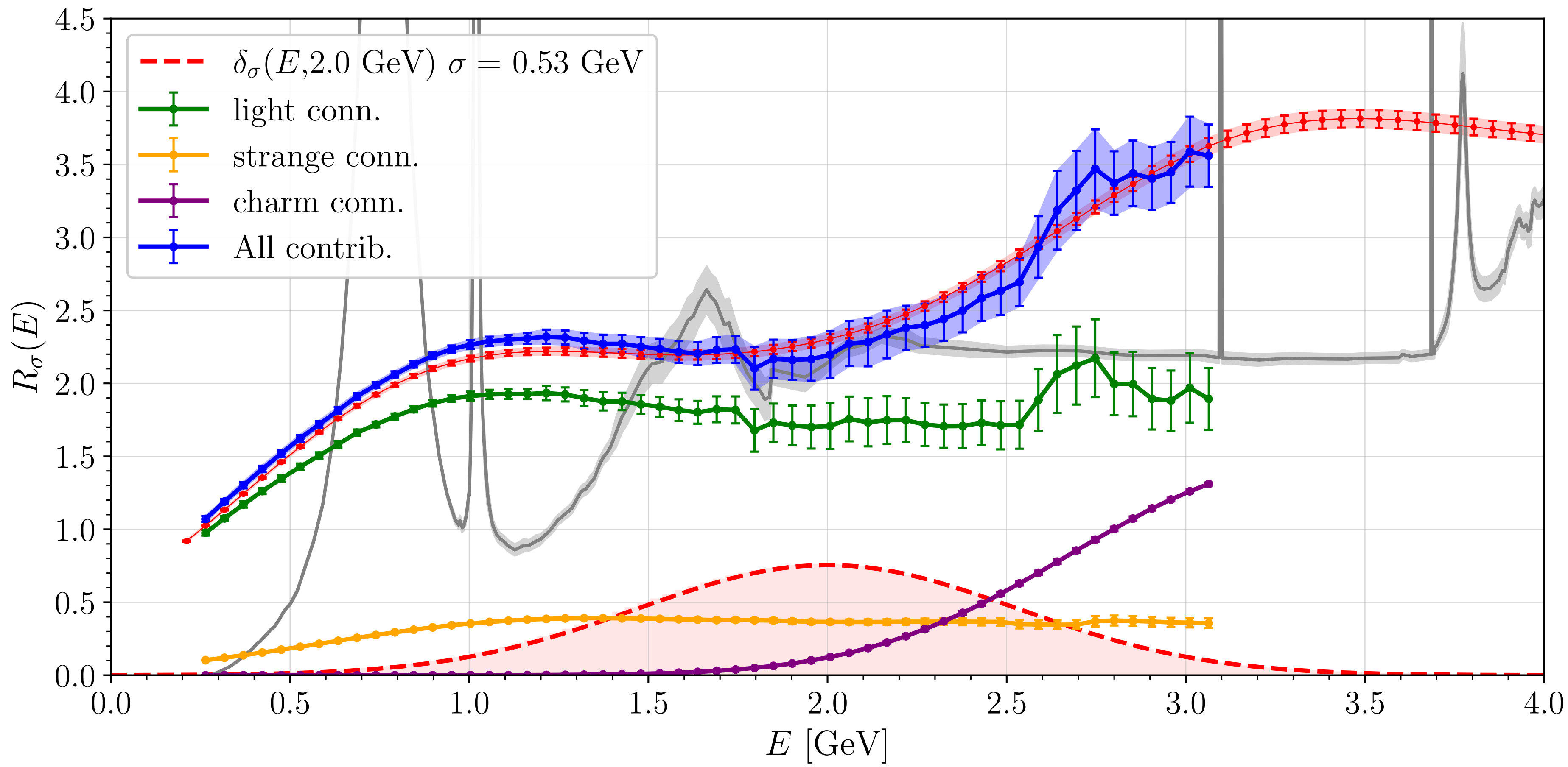
talk by C. Lehner @ Moriond 2024

Probing the R-ratio on the lattice

$R_\sigma(E)$: preliminary results

$$R_\sigma(E) = \int_{2M_\pi}^{\infty} \mathrm{d}\omega \delta_\sigma(\omega, E) R(\omega) \qquad \delta_\sigma(\omega, E) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(\omega-E)^2}{2\sigma^2}}$$

$R_\sigma(E)$ from e^+e^- data

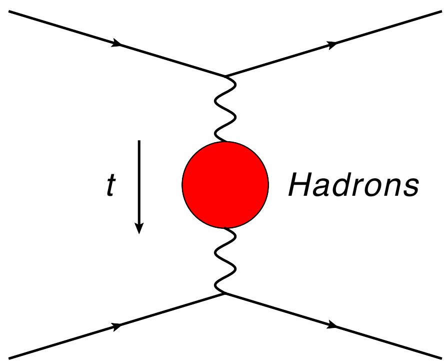


- Uncertainty coming mostly from light quark contributions, strange & charm ones are very precise
- Disconnected contributions are tiny and cannot be appreciated on this scale

Alexandrou *et al.* 2023

Connections to the MUonE Experiment

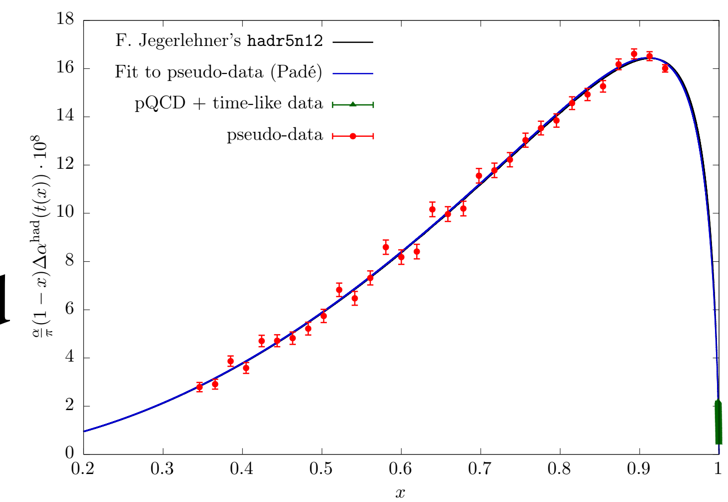
MUonE



B. E. Lautrup *et al.* 1972

$$a_\mu^{\rm HVP} = \frac{\alpha_{em}}{\pi} \int_0^1 dx (1-x)\, \Delta\alpha_{em}^{\rm HVP}\left[t(x)\right]$$

$$\sigma(\mu e \to \mu e)$$



$$t(x) \equiv -\frac{x^2}{1-x} m_\mu^2$$

$x \in [0.93,\ 1]$ not experimentally reached

LQCD

DG and S. Simula 2019

$$\left[a_\mu^{\rm HVP}\right]_> = 4\alpha_{em}^2 \int_0^\infty dt\, \widetilde{f}_>(t) V(t) \quad \Rightarrow \quad \left[a_\mu^{\rm HVP}\right]_> = 92(2)\cdot 10^{-10}$$

quark-connected terms only

Uncertainty $(\simeq 2\cdot 10^{-10})$ close to the experimental statistical target $(\simeq 0.3\%)$ of $\left[a_\mu^{\rm HVP}\right]_<$

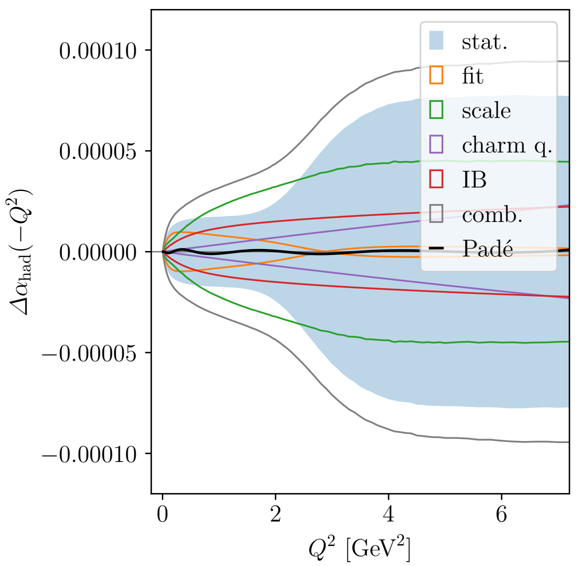
Hadronic running of α_{em} from the lattice

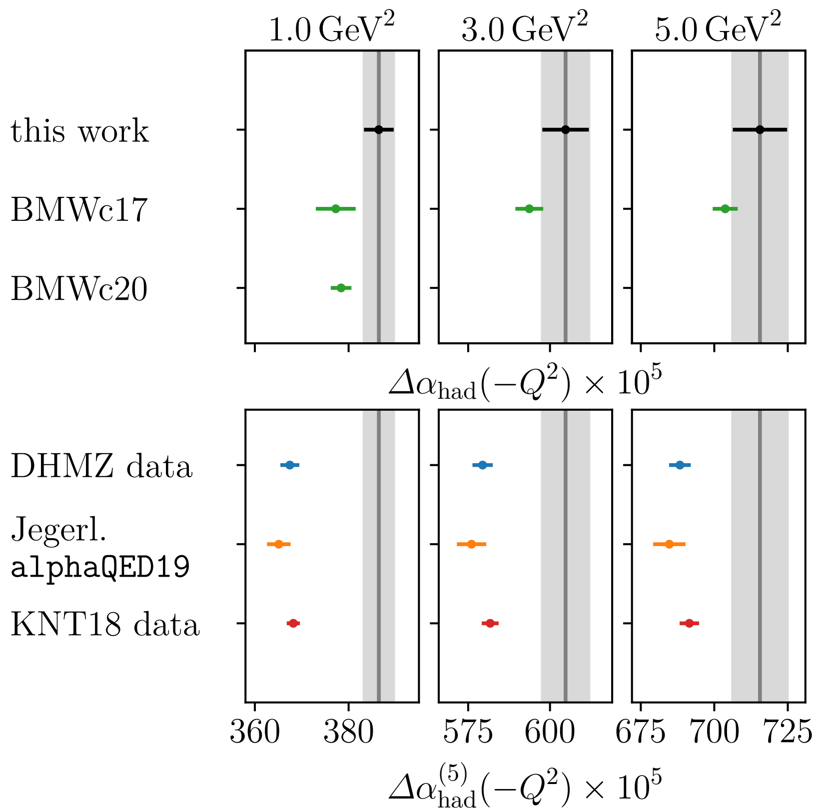
Lattice result for the hadronic running of α

[Cè et al., arXiv:2203.08676]

Starting point: Results for $\Delta\alpha_{\rm had}(-Q^2)$ for Euclidean momenta $0 \leq Q^2 \leq 7\,\mathrm{GeV}^2$ **[T. San José, TUE 17:10]**

Rational approximation:

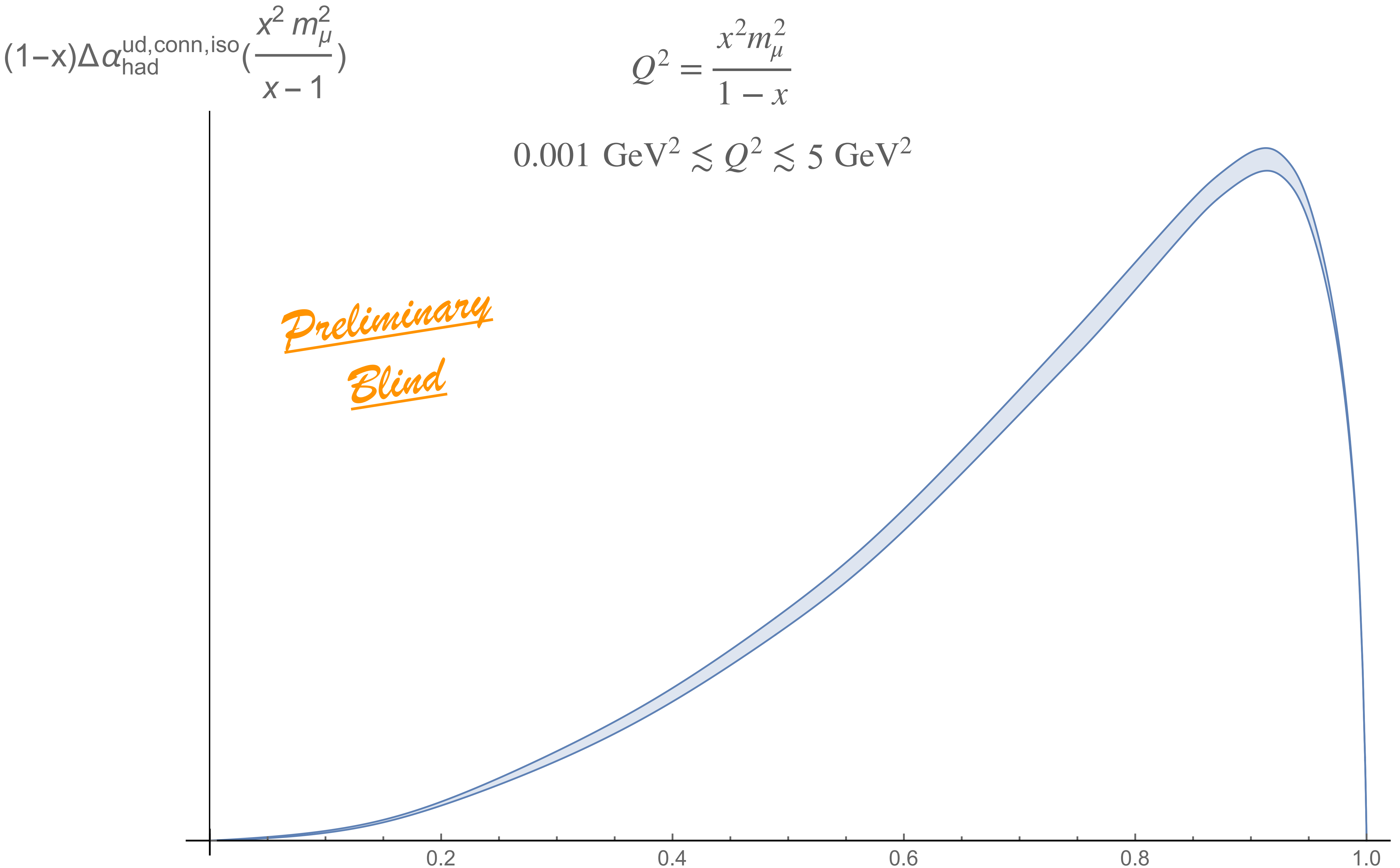




- Mainz/CLS and BMWc (2017) differ by 2–3% at the level of 1–2σ
- Tension between Mainz/CLS and phenomenology by $\sim 3\sigma$ for $Q^2 \gtrsim 3\,\mathrm{GeV}^2$
- Tension increases to $\gtrsim 5\sigma$ for $Q^2 \lesssim 2\,\mathrm{GeV}^2$ (smaller statistical error due to ansatz for continuum extrapolation)

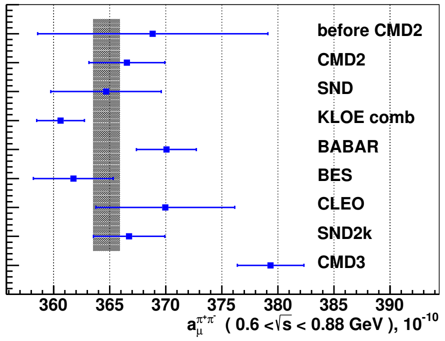
Systematic uncertainties from fit ansatz, scale setting, charm quenching, isospin-breaking and missing bottom quark contribution (five flavour theory) included in error budget

Hadronic running of α_{em} from the lattice



Summary and Outlook

- Tremendous progress in lattice calculations of HVP (and HLbL!) contributions
- Sub-percent calculation by BMW must be checked and impressive efforts from various lattice collaborations are in progress
- An update of the White Paper is aimed for late 2024
- Benchmark quantities (windows) crucial for checking the internal consistency of lattice calculations. For a_μ^W a new puzzle arises: remarkable agreement between lattice calculations but significant tension with dispersive prediction
- Extend calculation of window quantities to individual flavor and quark-disconnected contributions. Reach better precision for isospin-breaking contr.
- Extend comparison with phenomenological analyses to understand discrepancies. Clarify tensions in $\pi^+\pi^-$ BaBar, KLOE, CMD3



- $\mu e \to \mu e$ experiment MUonE very important for experimental cross-check and complementarity w/ LQCD