

Comments on the $\pi\pi$ contribution to HVP

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MuONE meeting, MITP Mainz, June 4, 2024

Outline

Introduction: $(g - 2)_\mu$ in the Standard Model

Hadronic Vacuum Polarization contribution

- Data-driven approach

- Dispersive approach for the $\pi\pi$ contribution

- Lattice vs data-driven: intermediate window

Conclusions and Outlook

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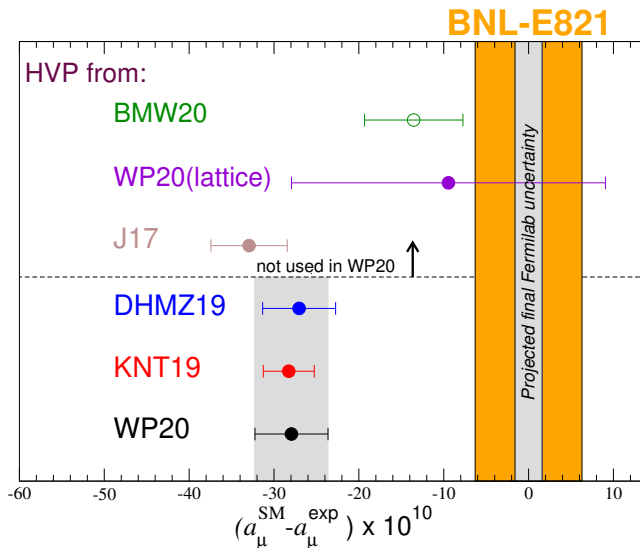
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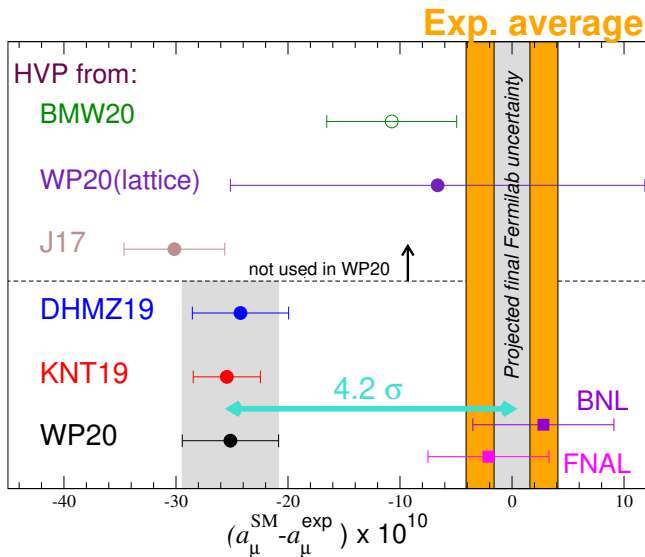
Present status of $(g - 2)_\mu$: experiment vs SM

Before



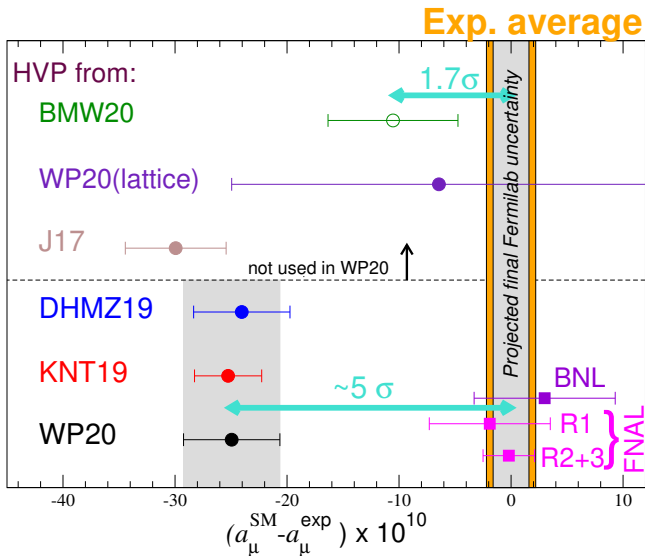
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After the 2021 Fermilab result



Present status of $(g - 2)_\mu$: experiment vs SM

After the 2023 Fermilab result



White Paper (2020): $(g - 2)_\mu$, experiment vs SM

Contribution	Value $\times 10^{11}$
HVP LO ($e^+ e^-$)	6931(40)
HVP NLO ($e^+ e^-$)	-98.3(7)
HVP NNLO ($e^+ e^-$)	12.4(1)
HVP LO (lattice, $udsc$)	7116(184)
HLbL (phenomenology)	92(19)
HLbL NLO (phenomenology)	2(1)
HLbL (lattice, uds)	79(35)
HLbL (phenomenology + lattice)	90(17)
QED	116 584 718.931(104)
Electroweak	153.6(1.0)
HVP ($e^+ e^-$, LO + NLO + NNLO)	6845(40)
HLbL (phenomenology + lattice + NLO)	92(18)
Total SM Value	116 591 810(43)
Experiment	116 592 059(22)
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	249(48)

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HVP NNLO ($e^+ e^-$)	12.4(1)
HVP LO (lattice BMW(20) , $udsc$)	7075(55)
HLbL (phenomenology)	92(19)
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T. Aoyama et al. Phys. Rep. 887 (2020) = WP(20)

Muon $g - 2$ Theory Initiative

Steering Committee:

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Aida El-Khadra (chair)

Martin Hoferichter

Laurent Lellouch

Christoph Lehner (vice-chair)

Tsutomu Mibe (J-PARC E34 experiment)

Lee Roberts (Fermilab E989 experiment)

Thomas Teubner

Hartmut Wittig

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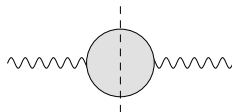
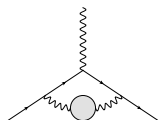
Muon $g - 2$ Theory Initiative

Plenary Workshops:

- ▶ 1st plenary meeting, Q-Center (Fermilab), 3-6 June 2017
- ▶ 2nd plenary meeting, Mainz, 18-22 June 2018
- ▶ 3rd plenary meeting, Seattle, 9-13 September 2019
- ▶ 4th plenary meeting, KEK (virtual), 28 June-02 July 2021
- ▶ 5th plenary meeting, Higgs Center Edinburgh, 5-9 Sept. 2022
- ▶ 6th plenary meeting, Bern, 4-8 Sept. 2023
- ▶ 7th plenary meeting, KEK, 9-13 Sept. 2024

Theory uncertainty comes from hadronic physics

- ▶ Hadronic contributions responsible for most of the theory uncertainty
- ▶ Hadronic vacuum polarization (HVP) is $\mathcal{O}(\alpha^2)$, dominates the total uncertainty, despite being known to $< 1\%$



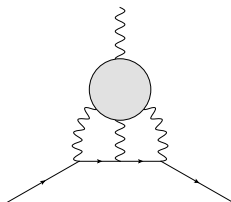
- ▶ unitarity and analyticity \Rightarrow dispersive approach
- ▶ \Rightarrow direct relation to experiment: $\sigma_{\text{tot}}(e^+e^- \rightarrow \text{hadrons})$
- ▶ e^+e^- Exps: BaBar, Belle, BESIII, CMD2/3, KLOE2, SND
- ▶ **alternative approach**: lattice, becoming competitive

(BMW, ETMC, Fermilab, HPQCD, Mainz, MILC, RBC/UKQCD)

\rightarrow talk by D. Giusti

Theory uncertainty comes from hadronic physics

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- ▶ Hadronic light-by-light (HLbL) is $\mathcal{O}(\alpha^3)$, known to $\sim 20\%$, second largest uncertainty (now subdominant)



- ▶ **earlier**: model-based—uncertainties difficult to quantify
- ▶ **recently**: dispersive approach \Rightarrow data-driven, systematic treatment
 → talk by M. Hoferichter
- ▶ lattice QCD is competitive

(Mainz, RBC/UKQCD)

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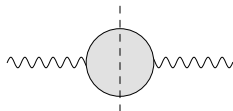
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HVP contribution: Master Formula

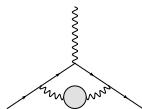
Unitarity relation: **simple**, same for all intermediate states



$$\text{Im}\bar{\Pi}(q^2) \propto \sigma(e^+e^- \rightarrow \text{hadrons}) = \sigma(e^+e^- \rightarrow \mu^+\mu^-)R(q^2)$$

Analyticity $\left[\bar{\Pi}(q^2) = \frac{q^2}{\pi} \int ds \frac{\text{Im}\bar{\Pi}(s)}{s(s-q^2)} \right] \Rightarrow$ **Master formula for HVP**

Bouchiat, Michel (61)



\Leftrightarrow

$$a_{\mu}^{\text{hvp}} = \frac{\alpha^2}{3\pi^2} \int_{s_{\text{th}}}^{\infty} \frac{ds}{s} K(s)R(s)$$

$K(s)$ known, depends on m_{μ} and $K(s) \sim \frac{1}{s}$ for large s

Comparison between DHMZ19 and KNT19

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	-0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	-0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	-0.12
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
$K_S K_L$	12.82(0.06)(0.18)(0.15)	13.04(19)	-0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	-0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
[1.8, 3.7] GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	-1.00
$J/\psi, \psi(2S)$	7.76(12)	7.84(19)	-0.08
[3.7, ∞) GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\text{HVP, LO}}$	694.0(1.0)(3.5)(1.6)(0.1) $_{\psi(0.7)}\text{DV+QCD}$	692.8(2.4)	1.2

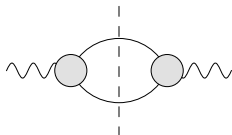
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For the dominant $\pi\pi$ channel more **theory input** can be used

The 2π contribution

For HVP the unitarity relation is **simple** and looks the same for all possible intermediate states, like 2π



$$\text{Im}\Pi(q^2) \propto \sigma(e^+e^- \rightarrow \pi^+\pi^-)$$

which implies

$$\bar{\Pi}_{2\pi}(q^2) = \frac{q^2}{\pi} \int_{4M_\pi^2}^{\infty} dt \frac{\alpha\sigma_\pi(t)^3 |F_V^\pi(t)|^2}{12t(t-q^2)}$$

de Trocóniz, Ynduráin (01,04), Leutwyler, GC (02,03), Anthanarayan et al. (13,16)

The pion vector form factor $F_V^\pi(t)$ also satisfies a dispersion relation

Analytic properties of pion form factors

Mathematical problem:

1. $F(t)$: analytic function except for a cut for $4M_\pi^2 \leq t < \infty$
2. $e^{-i\delta(t)}F(t) \in \mathbb{R}$ for $\text{Im}(t) \rightarrow 0^+$, with $\delta(t)$ a known function

Exact solution:

Omnès (58)

$$F(t) = P(t)\Omega(t) = P(t) \exp \left\{ \frac{t}{\pi} \int_{4M_\pi^2}^{\infty} \frac{dt'}{t'} \frac{\delta(t')}{t' - t} \right\},$$

$P(t)$ a polynomial \Leftrightarrow behaviour of $F(t)$ for $t \rightarrow \infty$
or presence of zeros

$\Omega(t)$ is called the Omnès function

Vector form factor of the pion

Pion vector form factor

$$\langle \pi^i(\mathbf{p}') | V_\mu^k(0) | \pi^l(\mathbf{p}) \rangle = i\epsilon^{ikl}(\mathbf{p}' + \mathbf{p})_\mu F_V^\pi(s) \quad s = (\mathbf{p}' - \mathbf{p})^2$$

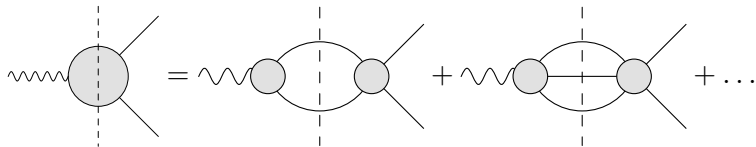
- ▶ normalization fixed by gauge invariance:

$$F_V^\pi(0) = 1 \quad \xrightarrow{\text{no zeros}} \quad P(t) = 1$$

→ talk by P. Stoffer

- ▶ $e^+e^- \rightarrow \pi^+\pi^-$ data \Rightarrow free parameters in $\Omega(t)$

Omnès representation including isospin breaking



Omnès representation including isospin breaking

- ▶ Omnès representation

$$F_V^\pi(s) = \exp \left[\frac{s}{\pi} \int_{4M_\pi^2}^{\infty} ds' \frac{\delta(s')}{s'(s'-s)} \right] \equiv \Omega(s)$$

- ▶ Split **elastic** ($\leftrightarrow \pi\pi$ phase shift, δ_1^1) from **inelastic** phase

$$\delta = \delta_1^1 + \delta_{\text{in}} \quad \Rightarrow \quad F_V^\pi(s) = \Omega_1^1(s) \Omega_{\text{in}}(s)$$

Eidelman-Lukaszuk: unitarity bound on δ_{in}

$$\sin^2 \delta_{\text{in}} \leq \frac{1}{2} \left(1 - \sqrt{1 - r^2} \right), \quad r = \frac{\sigma_{e^+e^- \rightarrow \neq 2\pi}^{l=1}}{\sigma_{e^+e^- \rightarrow 2\pi}} \Rightarrow s_{\text{in}} = (M_\pi + M_\omega)^2$$

- ▶ **$\rho - \omega$ -mixing** $F_V(s) = \Omega_{\pi\pi}(s) \cdot \Omega_{\text{in}}(s) \cdot G_\omega(s)$

$$G_\omega(s) = 1 + \epsilon \frac{s}{s_\omega - s} \quad \text{where} \quad s_\omega = (M_\omega - i\Gamma_\omega/2)^2$$

Free parameters

$$\Omega_1^1(s) \Rightarrow \begin{cases} \phi_0 = \delta_{\pi\pi}((0.8 \text{ GeV})^2) \\ \phi_1 = \delta_{\pi\pi}(68 M_\pi^2) \end{cases}$$

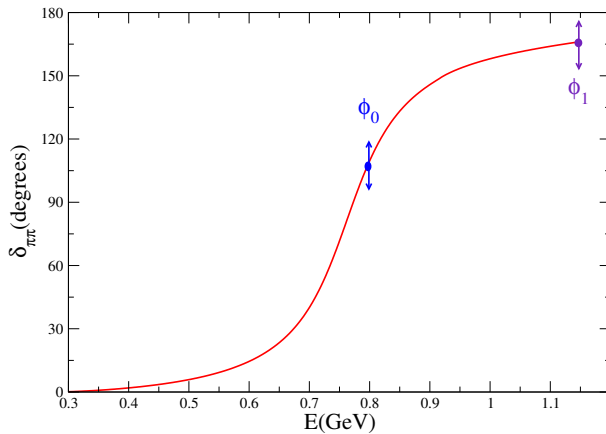
$$G_\omega(s) \Rightarrow \begin{cases} \epsilon & \omega - \rho \text{ mixing} \\ M_\omega \end{cases}$$

$$\Omega_{\text{in}}(s) \Rightarrow \begin{cases} c_1 \\ \vdots \\ c_P \end{cases} \quad \text{Im}\Omega_{\text{in}}(s) = 0 \quad s \leq s_{\text{in}}$$

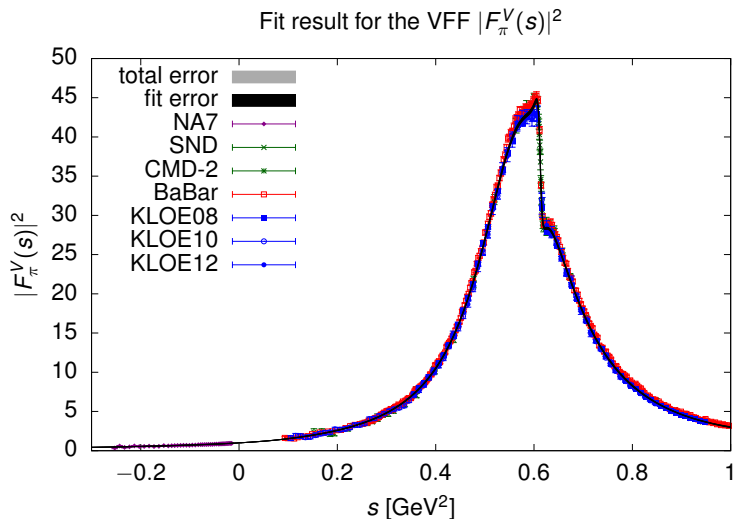
$$G_\omega(s) = 1 + \epsilon \frac{s}{s_\omega - s} \quad \text{where} \quad s_\omega = (M_\omega - i\Gamma_\omega/2)^2$$

$$\Omega_{\text{in}}(s) = 1 + \sum_{k=1}^N c_k (z(s)^k - z(0)^k) \quad z = \frac{\sqrt{s_{\pi\omega} - s_1} - \sqrt{s_{\pi\omega} - s}}{\sqrt{s_{\pi\omega} - s_1} + \sqrt{s_{\pi\omega} - s}}$$

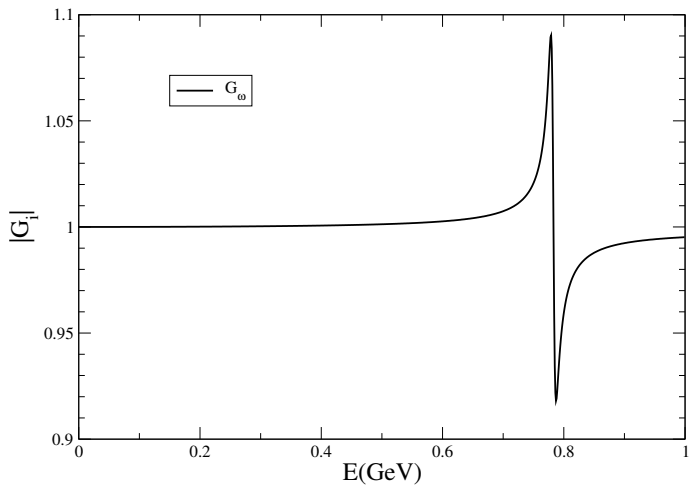
Free parameters



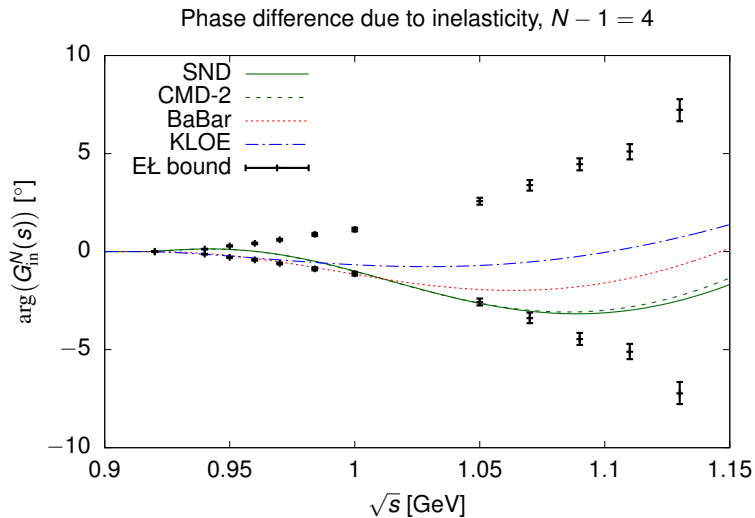
Fit results



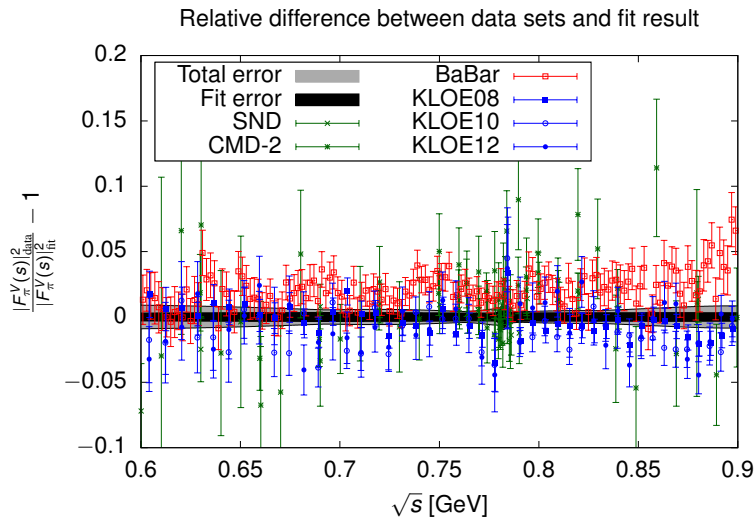
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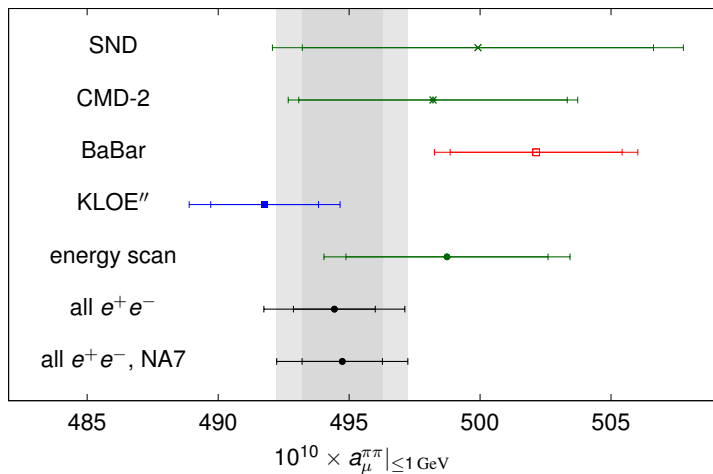


Fit results



Results for $(g - 2)_\mu$

Result for $a_\mu^{\pi\pi} |_{\leq 1 \text{ GeV}}$ from the VFF fits to single experiments and combinations



2π : comparison with the dispersive approach

The 2π channel can itself be described dispersively \Rightarrow more constrained theoretically

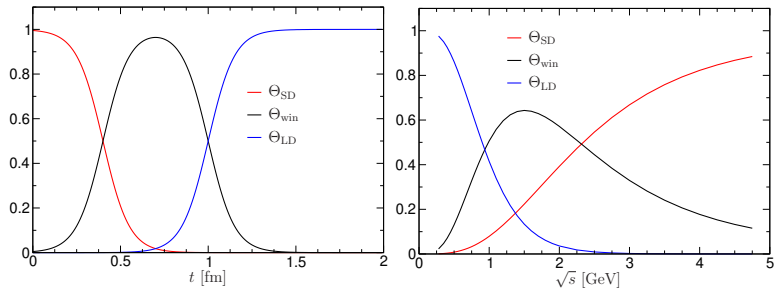
Ananthanarayan, Caprini, Das (19), GC, Hoferichter, Stoffer (18)

Energy range	ACD18	CHS18	DHMZ19	KNT19
< 0.6 GeV		110.1(9)	110.4(4)(5)	108.7(9)
< 0.7 GeV		214.8(1.7)	214.7(0.8)(1.1)	213.1(1.2)
< 0.8 GeV		413.2(2.3)	414.4(1.5)(2.3)	412.0(1.7)
< 0.9 GeV		479.8(2.6)	481.9(1.8)(2.9)	478.5(1.8)
< 1.0 GeV		495.0(2.6)	497.4(1.8)(3.1)	493.8(1.9)
[0.6, 0.7] GeV		104.7(7)	104.2(5)(5)	104.4(5)
[0.7, 0.8] GeV		198.3(9)	199.8(0.9)(1.2)	198.9(7)
[0.8, 0.9] GeV		66.6(4)	67.5(4)(6)	66.6(3)
[0.9, 1.0] GeV		15.3(1)	15.5(1)(2)	15.3(1)
≤ 0.63 GeV	132.9(8)	132.8(1.1)	132.9(5)(6)	131.2(1.0)
[0.6, 0.9] GeV		369.6(1.7)	371.5(1.5)(2.3)	369.8(1.3)
$[\sqrt{0.1}, \sqrt{0.95}]$ GeV		490.7(2.6)	493.1(1.8)(3.1)	489.5(1.9)

Present status of the window quantities

Weight functions for window quantities

RBC/UKQCD (18)

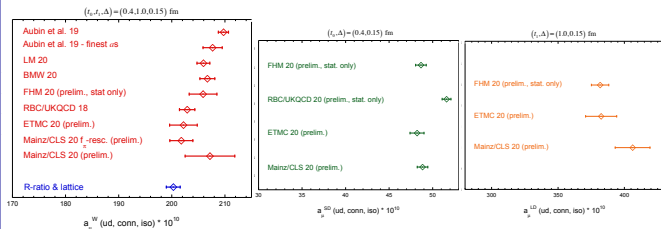


Present status of the window quantities

Lattice calculations of a_μ^{win} , circa 2021

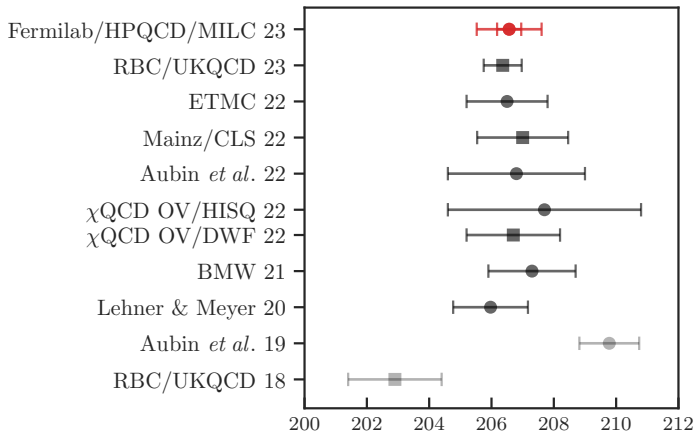
Summary: ud contribution

f	$a_\mu^{SD}(f) \cdot 10^{10}$	$a_\mu^W(f) \cdot 10^{10}$	$a_\mu^{LD}(f) \cdot 10^{10}$
ud	48.2 (0.8)	202.2 (2.6)	382.5 (11.7)



Present status of the window quantities

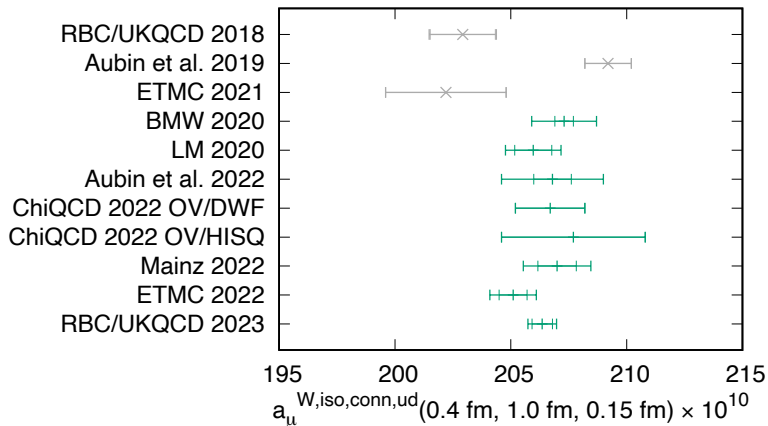
Now several lattice calculations confirm BMW's result



Fermilab Lattice-HPQCD-MILC (23)

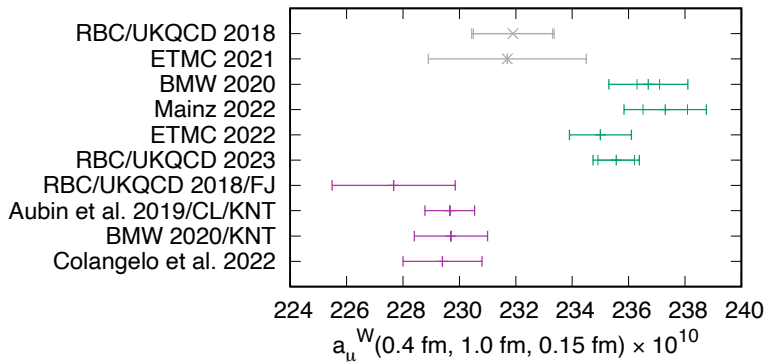
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RBC/UKQCD (23)

Individual-channel contributions to a_μ^{win}

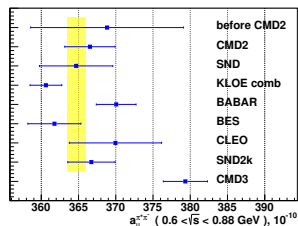
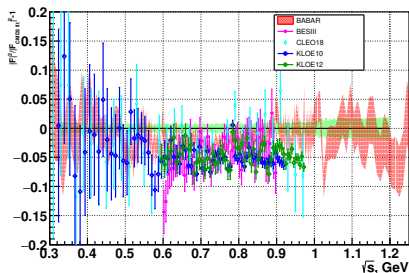
Channel	total	window
$\pi^+\pi^-$	504.23(1.90)	144.08(49)
$\pi^+\pi^-\pi^0$	46.63(94)	18.63(35)
$\pi^+\pi^-\pi^+\pi^-$	13.99(19)	8.88(12)
$\pi^+\pi^-\pi^0\pi^0$	18.15(74)	11.20(46)
K^+K^-	23.00(22)	12.29(12)
$K_S K_L$	13.04(19)	6.81(10)
$\pi^0\gamma$	4.58(10)	1.58(4)
Sum of the above	623.62(2.27)	203.47(78)
[1.8, 3.7] GeV (without $c\bar{c}$)	34.45(56)	15.93(26)
$J/\psi, \psi(2S)$	7.84(19)	2.27(6)
[3.7, ∞) GeV	16.95(19)	1.56(2)
WP(20) / GC, El-Khadra <i>et al.</i> (22)	693.1(4.0)	229.4(1.4)
BMWc	707.5(5.5)	236.7(1.4)
Mainz/CLS		237.3(1.5)
ETMc		235.0(1.1)
RBC/UKQCD		235.6(0.8)

Numbers for the channels refer to KNT19 — thanks to Alex Keshavarzi for providing them

$$\Delta a_\mu^{\text{HVP, LO}} = 14.4(6.8) (2.1\sigma), \quad \Delta a_\mu^{\text{win}} \sim 6.5(1.5) (\sim 4.3\sigma)$$

CMD-3 measurement of $e^+e^- \rightarrow \pi^+\pi^-$

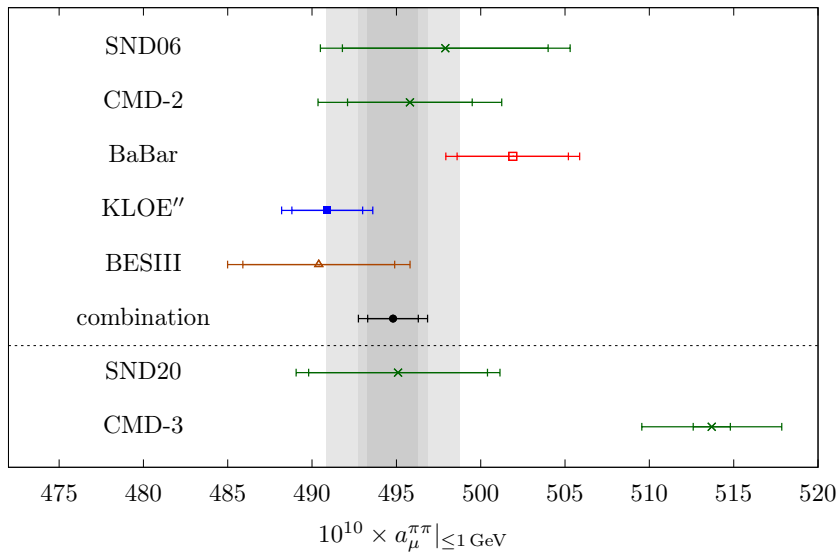
F. Ignatov et al., CMD-3, arXiv:2302.08834



The comparison of pion form factor measured in this work with the most recent ISR experiments (BABAR [21], KLOE [18, 19], BES [22]) is shown in Fig. 34. The comparison with the most precise previous energy scan experiments (CMD-2 [12, 13, 14, 15], SND [16] at the VEPP-2M and SND [23] at the VEPP-2000) is shown in Fig. 35. **The new result generally shows larger pion form factor in the whole energy range under discussion.** The most significant difference to other energy scan measurements, including previous CMD-2 measurement, is observed at the left side of ρ -meson ($\sqrt{s} = 0.6 - 0.75$ GeV), where it reach up to 5%, well beyond the combined systematic and statistical errors of the new and previous results. **The source of this difference is unknown at the moment.**

Preliminary analysis of the CMD-3 measurement

Work in progress, GC, Hoferichter and Stoffer (thanks for providing the plots)



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Work in progress, GC, Hoferichter and Stoffer (thanks for providing the plots)

$10^{10} \times$	$a_{\mu}^{\pi\pi} _{\leq 1\text{GeV}}$	$a_{\mu}^{\pi\pi, \text{win}} _{\leq 1\text{GeV}}$	χ^2/dof
SND06	497.9(6.1)(4.2)	139.6(1.8)(1.0)	1.09
CMD-2	495.8(3.7)(4.0)	139.4(1.0)(0.8)	1.01
BaBar	501.9(3.3)(2.2)	140.6(1.0)(0.7)	1.17
KLOE''	490.9(2.1)(1.7)	137.1(0.6)(0.4)	1.13
BESIII	490.4(4.5)(3.0)	137.8(1.3)(0.4)	1.01
SND20	495.1(5.3)(2.9)	139.2(1.5)(0.4)	1.88
CMD-3	513.7(1.1)(4.0)	144.0(0.3)(1.1)	1.09
Combination	494.8(1.5)(1.4)(3.4)	138.3(0.4)(0.3)(1.1)	1.21

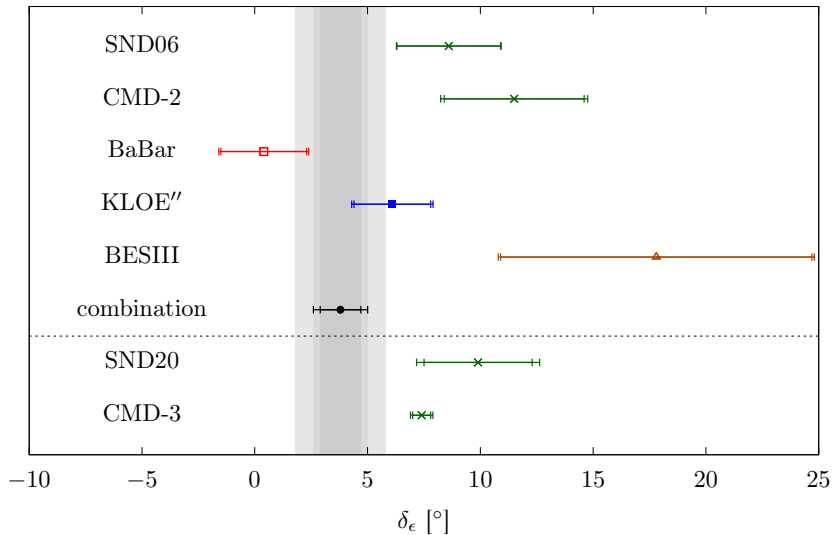
Combination: NA7 + all data sets other than SND20 and CMD-3

$$\Delta a_{\mu}^{\text{HVP, LO}}(\text{CMD-3-Comb.}) = 18.9(5.1), \quad \Delta a_{\mu}^{\text{win}}(\text{CMD-3-Comb.}) = 5.7(1.5)$$

$$\Delta a_{\mu}^{\text{HVP, LO}}(\text{BMW-WP20}) = 14.4(6.8), \quad \Delta a_{\mu}^{\text{win}}(\text{Lattice-WP20}) \sim 6.5(1.5)$$

Preliminary analysis of the CMD-3 measurement

Work in progress, GC, Hoferichter and Stoffer



Preliminary analysis of the CMD-3 measurement

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	M_ω (MeV)	$10^3 \cdot \text{Re } \epsilon_\omega$	$\delta_\epsilon [^\circ]$
SND06	782.12(33)(2)	2.03(5)(2)	8.6(2.3)(0.3)
CMD-2	782.65(33)(4)	1.90(6)(3)	11.5(3.1)(1.0)
BaBar	781.89(18)(4)	2.06(4)(2)	0.4(1.9)(0.6)
KLOE''	782.45(24)(5)	1.96(4)(2)	6.1(1.7)(0.6)
BESIII	783.07(61)(2)	2.03(19)(7)	17.8(6.9)(1.2)
SND20	782.34(28)(6)	2.07(5)(2)	9.9(2.4)(1.3)
CMD-3	782.33(6)(3)	2.08(1)(2)	7.4(4)(3)
Combination	782.07(12)(5)(8)	1.99(2)(2)(0)	3.8(0.9)(0.8)(1.6)

→ talk by P. Stoffer

Outline

Introduction: $(g - 2)_\mu$ in the Standard Model

Hadronic Vacuum Polarization contribution

- Data-driven approach

- Dispersive approach for the $\pi\pi$ contribution

- Lattice vs data-driven: intermediate window

Conclusions and Outlook

Conclusions

- ▶ Data-driven evaluation of the HVP contribution (WP20):
0.6% error \Rightarrow dominates the theory uncertainty
- ▶ Dominant contribution to HVP: $\pi\pi$ (<1 GeV). WP20 based on:
CMD-2, SND06, BaBar, KLOE
New puzzle: measurement by CMD-3 significantly higher!
- ▶ Recent lattice calculation [BMW(20)] has reached a similar precision
but differs from the dispersive one (=from e^+e^- data).
If confirmed \Rightarrow discrepancy with experiment \searrow below 2σ
- ▶ Intermediate window of BMW has been confirmed by other lattice
collaborations (Aubin et al., Mainz, ETMc, RBC/UKQCD, Fermilab-HPQCD-MILC)
and disagrees with data-driven [other than CMD-3, which would agree]

Outlook

- ▶ The Fermilab experiment aims to reduce the BNL uncertainty by a **factor four** \Rightarrow potential **7σ** discrepancy
- ▶ Improvements on the SM theory/data side:
 - ▶ Situation for HVP data-driven **urgently needs to be clarified**:
 - Thorough scrutiny of the new **CMD-3** result
 - Forthcoming measur./analyses: **BaBar**, **Belle II**, **BESIII**, **KLOE**, **SND**
 - Model-independent evaluation of **RadCorr** underway (but cannot be the culprit)
 - **MuonE** will provide an alternative way to measure HVP
 - ▶ HVP lattice:
calculations with precision \sim **BMW** for $a_{\mu}^{\text{HVP, LO}}$ are awaited