Status of the hadronic corrections to the muon $g-2$: HLbL

b UNIVERSITÄT BERN

AEC ALBERT EINSTEIN CENTER FOR FUNDAMENTAL PHYSICS

Martin Hoferichter

Albert Einstein Center for Fundamental Physics,

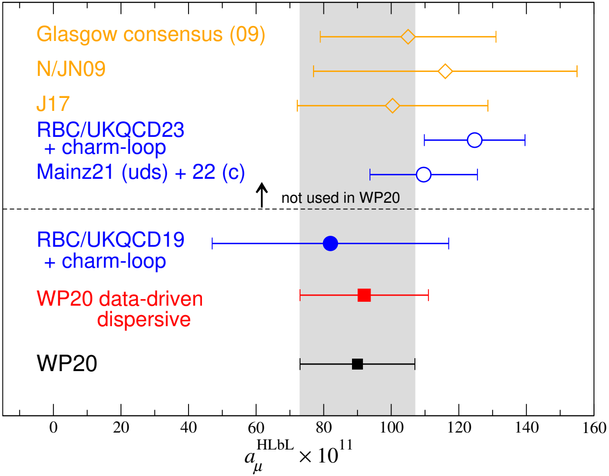
Institute for Theoretical Physics, University of Bern

June 5, 2024

The Evaluation of the Leading Hadronic Contribution to the Muon $g-2$: Consolidation of the MUonE Experiment and Recent Developments in Low Energy e^+e^- Data

Mainz, Germany

HLbL scattering: status



Lattice QCD

- Mainz 2021, 2022:

$$a_\mu^{\text{HLbL}}[uds] = 107(16) \times 10^{-11}$$

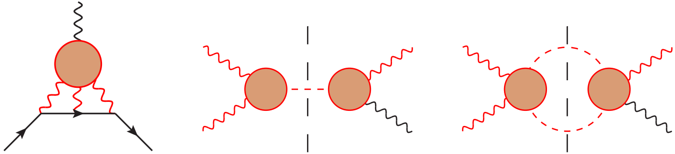
$$a_\mu^{\text{HLbL}}[c] = 2.8(5) \times 10^{-11}$$

- RBC/UKQCD 2023:

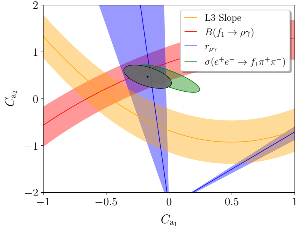
$$a_\mu^{\text{HLbL}}[uds] = 122(15)\times 10^{-11}$$

- Good agreement between lattice QCD and phenomenology at $\simeq 20 \times 10^{-11}$
- Need another factor of 2 for final Fermilab precision see talk by M. Knecht

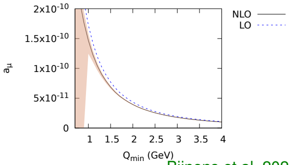
HLbL scattering: data-driven, dispersive evaluations



- Organized in terms of **hadronic intermediate states**, in close analogy to HVP Colangelo et al. 2014, ...
- Leading channels implemented with **data input for $\gamma^*\gamma^* \to$ hadrons**, e.g., $\pi^0 \to \gamma^*\gamma^*$
- Uncertainty dominated by subleading channels
  $\hookrightarrow$ **axial-vector mesons** $f_1(1285)$, $f_1(1420)$, $a_1(1260)$
- Optimized HLbL basis MH, Stoffer, Zillinger 2024
- Matching to short-distance constraints



MH, Kubis, Zanke 2023

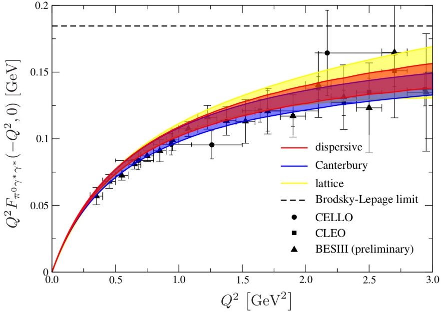


Bijnens et al. 2021

HLbL scattering: white paper details

Contribution	PdRV(09)	N/JN(09)	J(17)	Our estimate
π^0, η, η'-poles	114(13)	99(16)	95.45(12.40)	93.8(4.0)
π, K-loops/boxes	$-19(19)$	$-19(13)$	$-20(5)$	$-16.4(2)$
S-wave $\pi\pi$ rescattering	$-7(7)$	$-7(2)$	$-5.98(1.20)$	$-8(1)$
subtotal	88(24)	73(21)	69.5(13.4)	69.4(4.1)
scalars	–	–	–	} $-1(3)$ (scalars + tensors)
tensors	–	–	1.1(1)	
axial vectors	15(10)	22(5)	7.55(2.71)	6(6)
u, d, s-loops / short-distance	–	21(3)	20(4)	15(10)
c-loop	2.3	–	2.3(2)	3(1)
total	105(26)	116(39)	100.4(28.2)	92(19)

HLbL scattering: pseudoscalar poles



- Pion pole from data MH et al. 2018, Masjuan, Sánchez-Puertas 2017 and lattice Gérardin et al. 2019

$$a_\mu^{\pi^0\text{-pole}}\Big|_{\text{dispersive}} = 63.0^{+2.7}_{-2.1}\times 10^{-11} \qquad a_\mu^{\pi^0\text{-pole}}\Big|_{\text{Canterbury}} = 63.6(2.7)\times 10^{-11}$$

$$a_\mu^{\pi^0\text{-pole}}\Big|_{\text{lattice+PrimEx}} = 62.3(2.3)\times 10^{-11} \qquad a_\mu^{\pi^0\text{-pole}}\Big|_{\text{lattice}} = 59.7(3.6)\times 10^{-11}$$

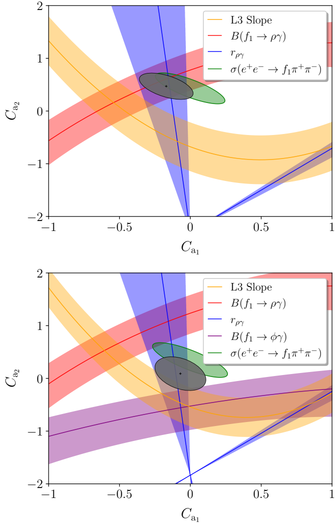
- Singly-virtual results agree well with BESIII measurement
- Same program in progress for η, η' poles
- New lattice results indicate some tension in $\gamma\gamma$ width

Determination of axial-vector TFFs

- Three independent TFFs, accessible in
  - $e^+e^- \to e^+e^- f_1$ (space-like)
  - $f_1 \to \rho\gamma$, $f_1 \to \phi\gamma$
  - $f_1 \to e^+e^-$
  - $\boldsymbol{e^+e^- \to f_1\pi^+\pi^-}$

  $\hookrightarrow$ global analysis in VMD parameterizations
- Constraint from $e^+e^- \to f_1\pi^+\pi^-$ for the first time allows for unambiguous solutions
- Most information available for f_1

  $\hookrightarrow$ f_1' and a_1 from $U(3)$ symmetry
- Analysis of consequences for HLbL in progress



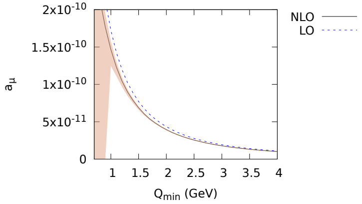
MH, Kubis, Zanke 2023

Short-distance contributions

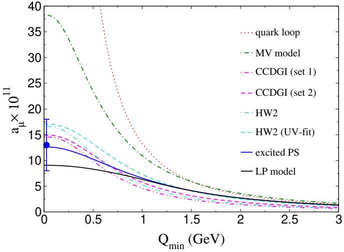
- **Higher-order short-distance constraints**
  - Two-loop α_s corrections
  - Higher-order OPE corrections
  - Higher-order terms in Melnikov–Vainshtein limit
- **Implementation of SDCs**
  - Large-N_c Regge models Colangelo . . .
  - Holographic QCD Leutgeb, Rebhan, Cappiello, . . .
  - Interpolants Lüdtke, Procura

$\hookrightarrow$ reasonable agreement on longitudinal component

- **Transverse component/axial-vectors**
  - SDCs MH, Stoffer 2020
  - Implementation of axial-vectors, new HLbL basis, new dispersive formalism



Bijnens, Hermansson-Truedsson, Laub, Rodríguez-Sánchez 2021



Colangelo, Hagelstein, MH, Laub, Stoffer 2021

New insights on HLbL tensor

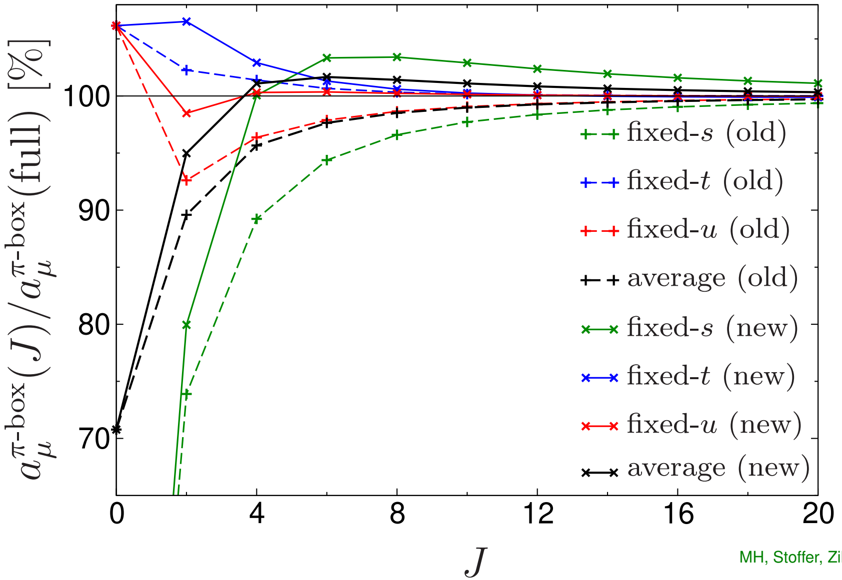
- Recall discussions with MV about the **definition of the pion pole**

$$\frac{F_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2)F_{\pi^0\gamma^*\gamma^*}(q_3^2,0)}{q_3^2-M_\pi^2} \quad \text{vs.} \quad \frac{F_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2)F_{\pi^0\gamma^*\gamma^*}(M_\pi^2,0)}{q_3^2-M_\pi^2}$$

- Comparison in Colangelo, Hagelstein, MH, Laub, Stoffer 2019:
  - First variant: dispersion relation in four-point kinematics
  - Second variant: dispersion relation in $g-2$ ("triangle") kinematics
- Triangle variant looks attractive because of SDCs, but very complicated in low-energy region due to missing $2\pi, \ldots$ cuts
- **Kinematic singularities**
  - Disappear in four-point kinematics only for the entire HLbL tensor due to sum rules
    $\hookrightarrow$ higher partial waves, axial-vectors, tensors
  - For axial-vectors: can find a basis manifestly free of kinematic singularities
    $\hookrightarrow$ ideal for axial-vectors, also good for pion box; not possible for tensors

$\hookrightarrow$ complementary information from triangle kinematics Lüdtke, Procura, Stoffer 2023

Saturation of the pion box in new basis



MH, Stoffer, Zillinger 2024

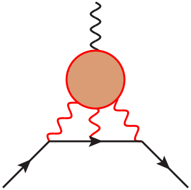
HLbL dispersion relation in triangle vs. four-point kinematics

	DR in four-point kinematics					
triangle-DR	π^0, η, η'	2π	S	A	T	$\dots$
π^0, η, η'		$\times$	$\times$	$\times$	$\times$	$\times$
2π	$\times$		$\times$	$\times$	$\times$	$\times$
V						
S	$\times$	$\times$		$\times$	$\times$	$\times$
A	$\times$	$\times$	$\times$		$\times$	$\times$
T	$\times$	$\times$	$\times$	$\times$		$\times$
$\dots$						$\dots$

Lüdtke, Procura, Stoffer 2023

Status of the HLbL contribution to $(g-2)_\mu$

- **Hadronic light-by-light scattering (lattice)**:
  - Two complete calculations (RBC/UKQCD, Mainz), ETMC, BMWc working on it
  - Lattice calculations for $P=\pi^0,\eta,\eta'$ TFFs
    $\hookrightarrow$ long-distance tail
- **Hadronic light-by-light scattering (phenomenology)**:
  - Dominant intermediate states largely done, dispersive results for η, η' poles to come
  - Axial-vector states in new optimized basis
  - Matching to short-distance constraints
  - Tensor states in alternative dispersive approach in triangle kinematics



Seventh plenary workshop of the Muon $g-2$ Theory Initiative