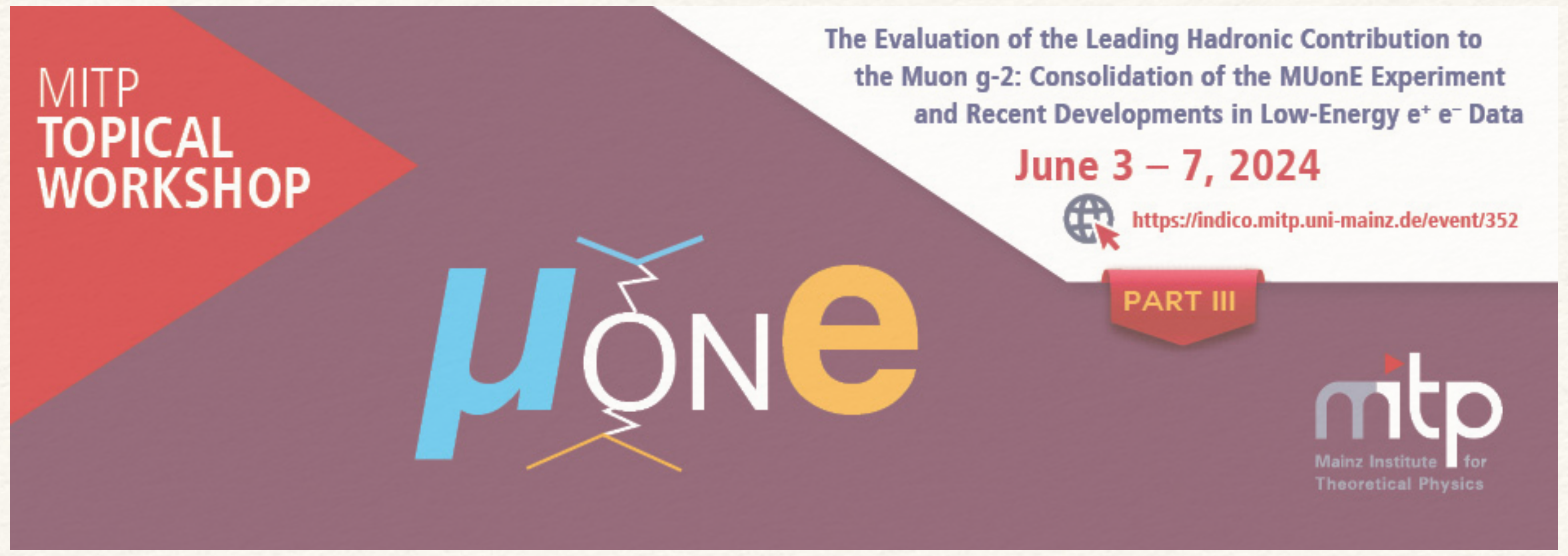

Jonathan Ronca

LoopIn: Loop Integrals for virtual corrections

In collaboration with: M. Bigazzi, G. Brunello G. Crisanti, T. Dave, M. K. Mandal, P. Mastrolia, S. Smith, W. J. Torres Bobadilla

ROMA TRE UNIVERSITÀ DEGLI STUDI

INFN Istituto Nazionale di Fisica Nucleare SEZIONE DI ROMA TRE

06 June 2024

Motivation

$e^+e^- \to \mu^+\mu^-$ virtuals @ NNLO QED
$e^+\mu^- \to e^+\mu^-$ virtuals @ NNLO QED
$q\bar{q} \to t\bar{t}$ virtuals @ NNLO QCD

PHYSICAL REVIEW LETTERS 128, 022002 (2022)

Two-Loop Four-Fermion Scattering Amplitude in QED

R. Bonciani,[1,*] A. Broggio,[2,†] S. Di Vita,[3,4] A. Ferroglia,[5,6,‡] M. K. Mandal,[7,8,§] P. Mastrolia,[8,7,∥] L. Mattiazzi,[7,8,¶] A. Primo,[9,**] J. Ronca,[10,††] U. Schubert,[11,‡‡] W. J. Torres Bobadilla,[12,§§] and F. Tramontano[10,∥∥]

JHEP
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Two-loop scattering amplitude for heavy-quark pair production through light-quark annihilation in QCD

Manoj K. Mandal,[a] Pierpaolo Mastrolia,[a,b] Jonathan Ronca[c] and William J. Torres Bobadilla[d]

JHEP
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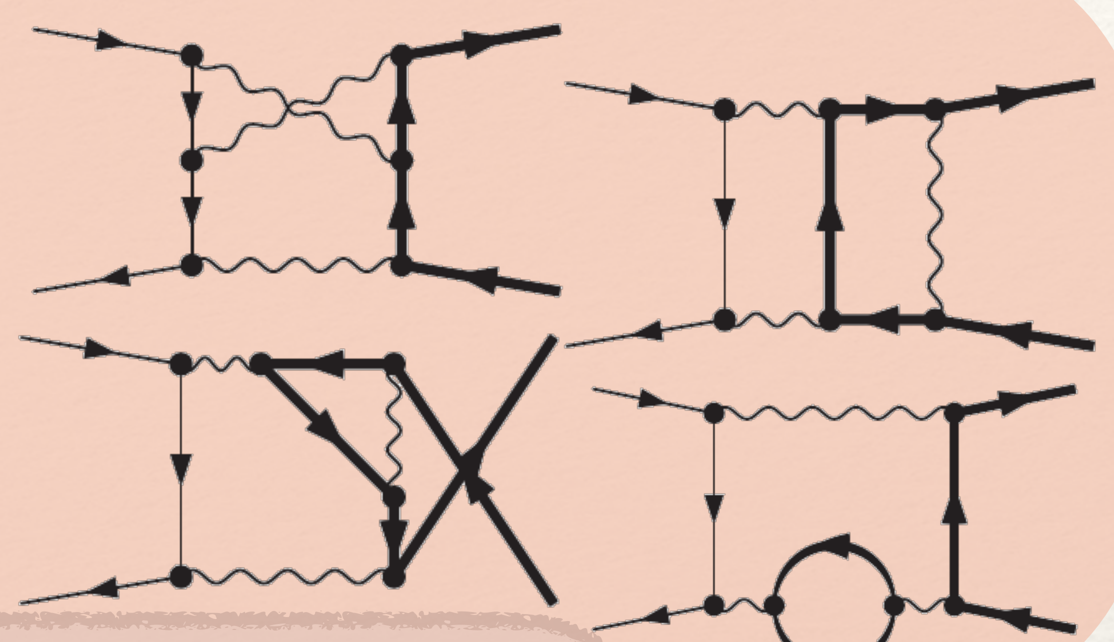
Muon-electron scattering at NNLO

A. Broggio,[a] T. Engel,[b,c,d] A. Ferroglia,[e,f] M.K. Mandal,[g,h] P. Mastrolia,[i,g] M. Rocco,[b] J. Ronca,[j] A. Signer,[b,c] W.J. Torres Bobadilla,[k] Y. Ulrich[l] and M. Zoller[b]

Process:
- NNLO SM QCD
- Four-point two-loop scattering
- Three mass scales
- IR and UV divergent



Method:
- Constructing double-virtual interferences
- Adaptive integrand decomposition
- Integration-by-parts identities
- Differential equation method for Master Integrals
- Magnus Exponential to expose the canonical basis
- dlog form of the differential matrix
- Analytical expansion in terms of GPLs

AIDA Framework

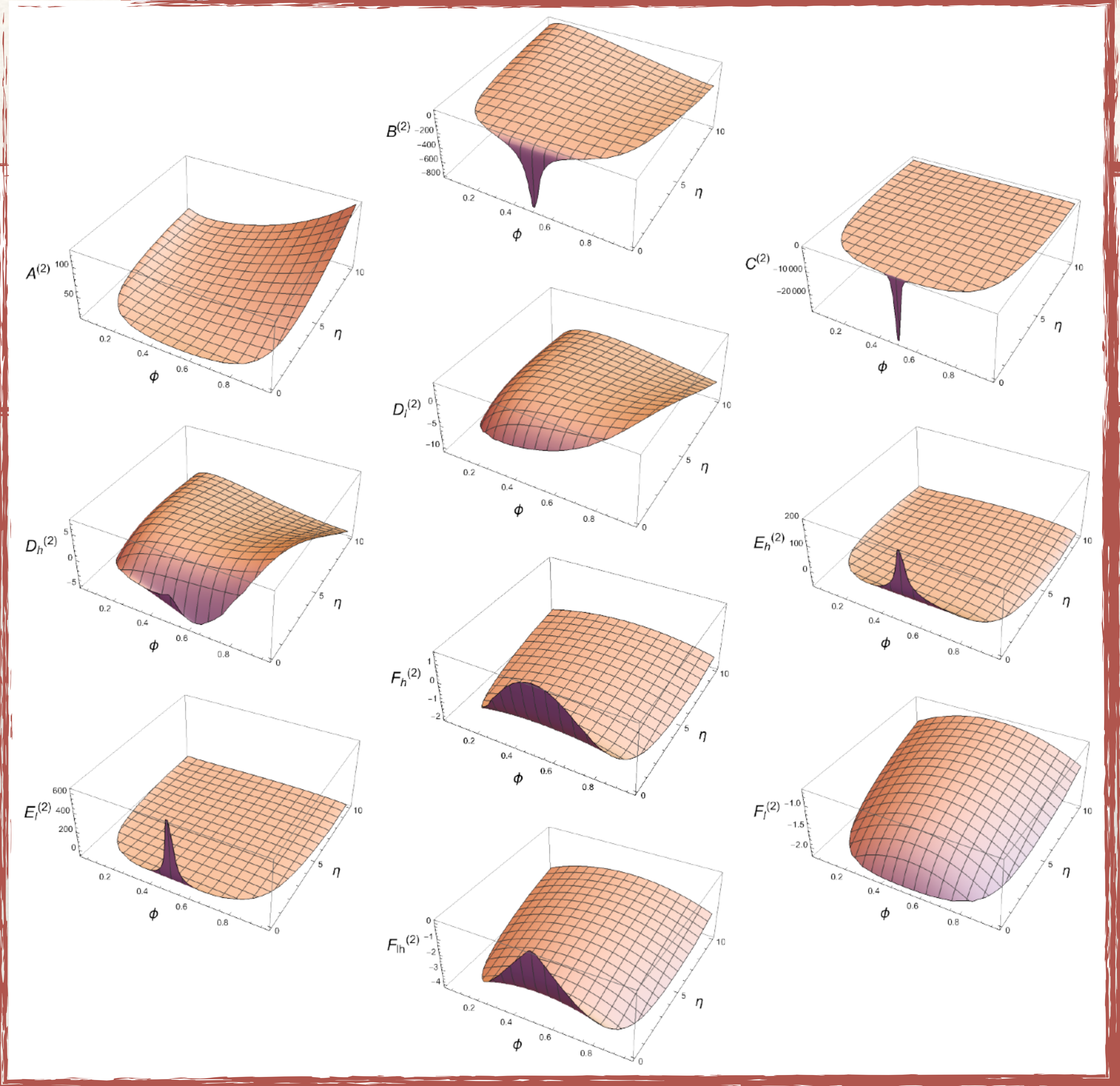
Tools:
- Mathematica
- FeynCalc
- AIDA + IBPs
- handyG
- PolyLogTools

Running:
- Mathematica package: O(10) sec/pt
- Within McMule: O(0.1) sec/pt

[JR - IRNTerascale@LNF - 2024]

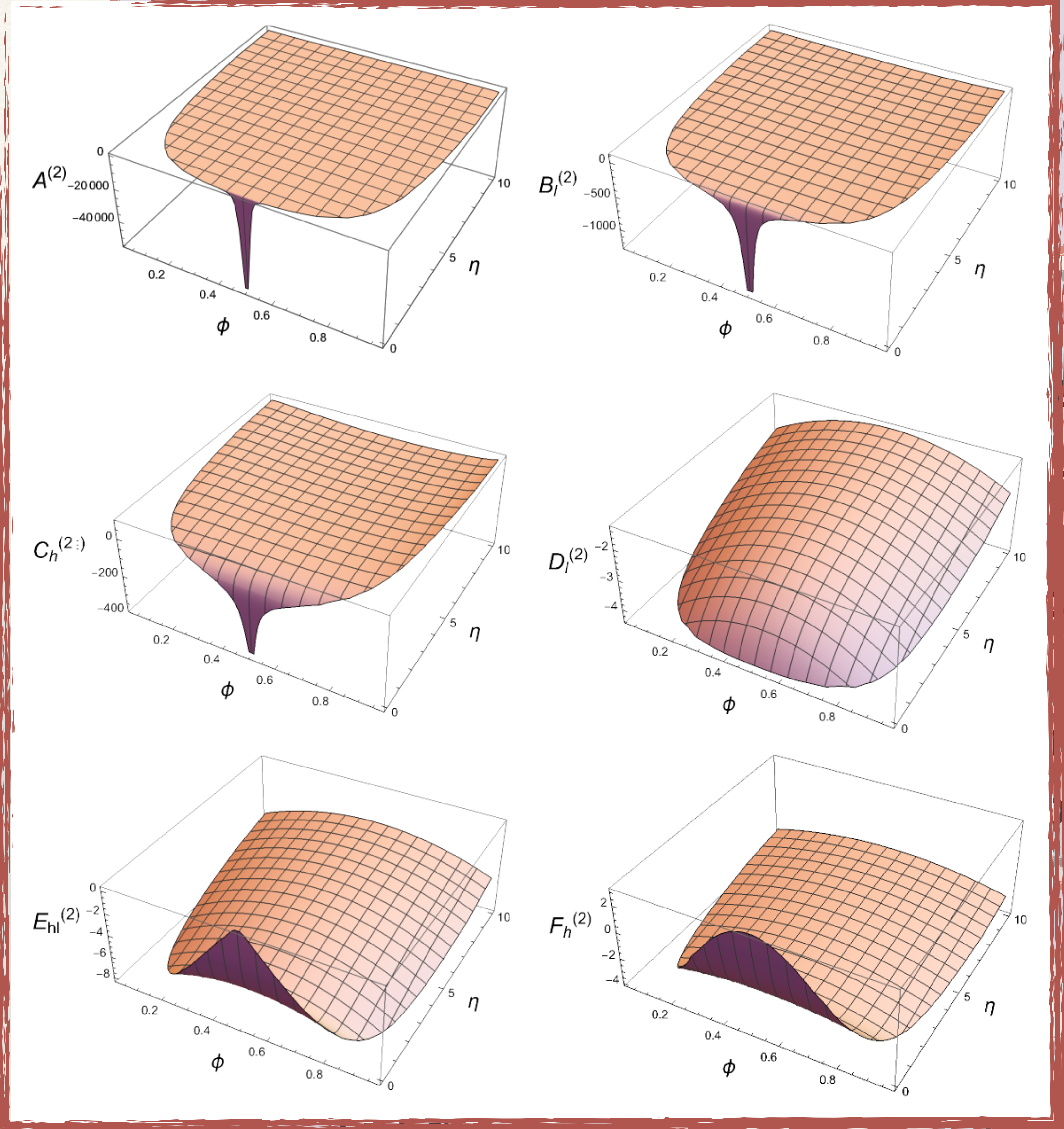
Motivation

$e^+e^- \to \mu^+\mu^-$ virtuals @ NNLO QED



Process:

- NNLO SM QCD



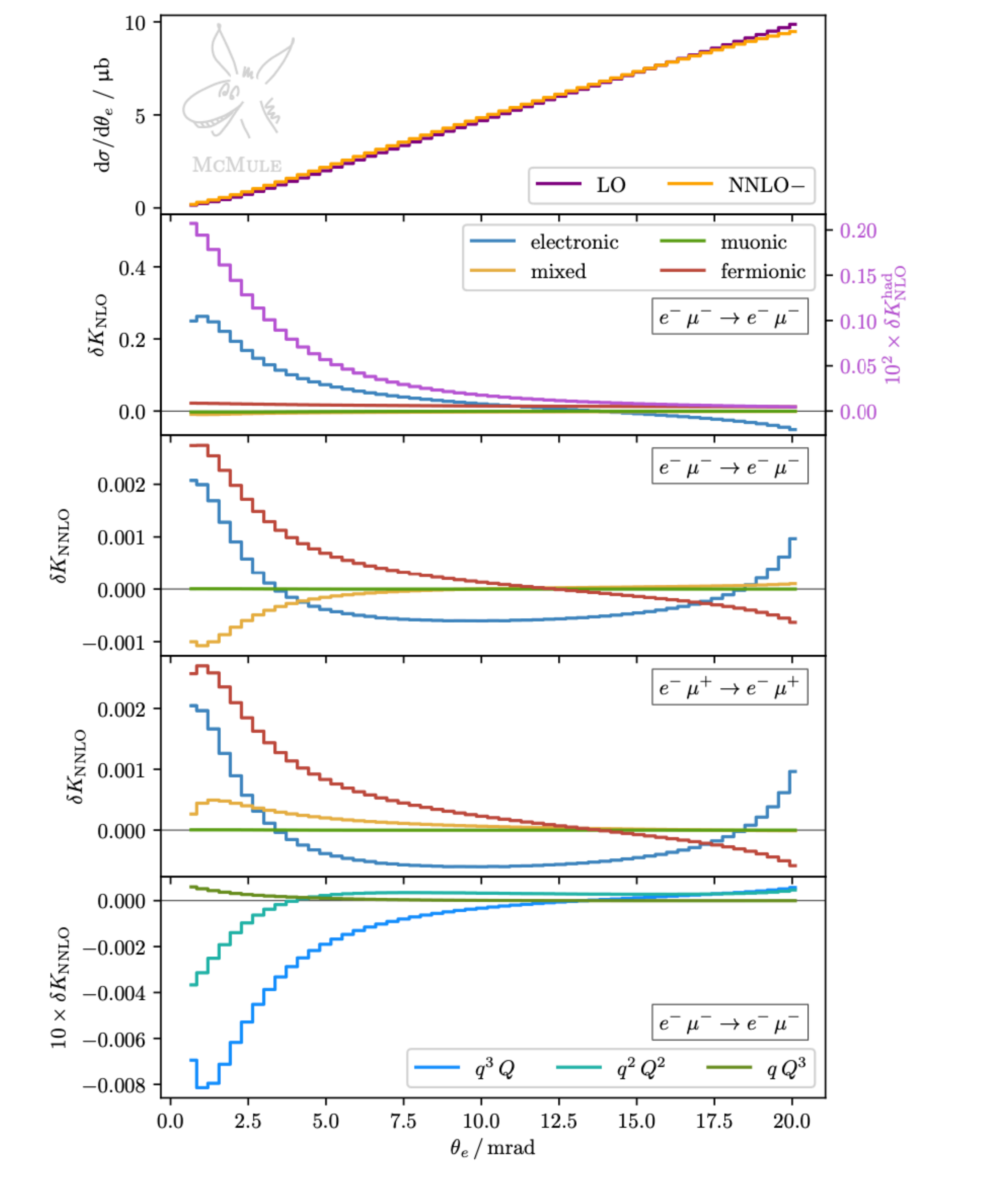


Figure 3: From top to bottom: (i) differential cross section w.r.t. θ_e for S1 at LO (violet), and NNLO (orange) for negative muons; (ii) NLO K factor for negative muons (positive muons have a sign flip for the mixed photonic correction); (iii) NNLO K factor for negative muons; (iv) NNLO K factor for positive muons; (v) NNLO K factor for disentangled mixed photonic corrections, for negative muons. In panels (ii)–(iv) the correction is split into photonic, i.e. electronic, mixed and muonic, and fermionic, including leptonic and hadronic. The hadronic correction at NLO corresponds to the signal of the experiment and is shown separately in purple in panel (ii).

Muon-electron scattering at NNLO

A. Broggio,[a] T. Engel,[b,c,d] A. Ferroglia,[e,f] M.K. Mandal,[g,h] P. Mastrolia,[i,g] M. Rocco,[b] J. Ronca,[j] A. Signer,[b,c] W.J. Torres Bobadilla,[k] Y. Ulrich[l] and M. Zoller[b]

- AIDA + IBPs
- handyG
- PolyLogTools

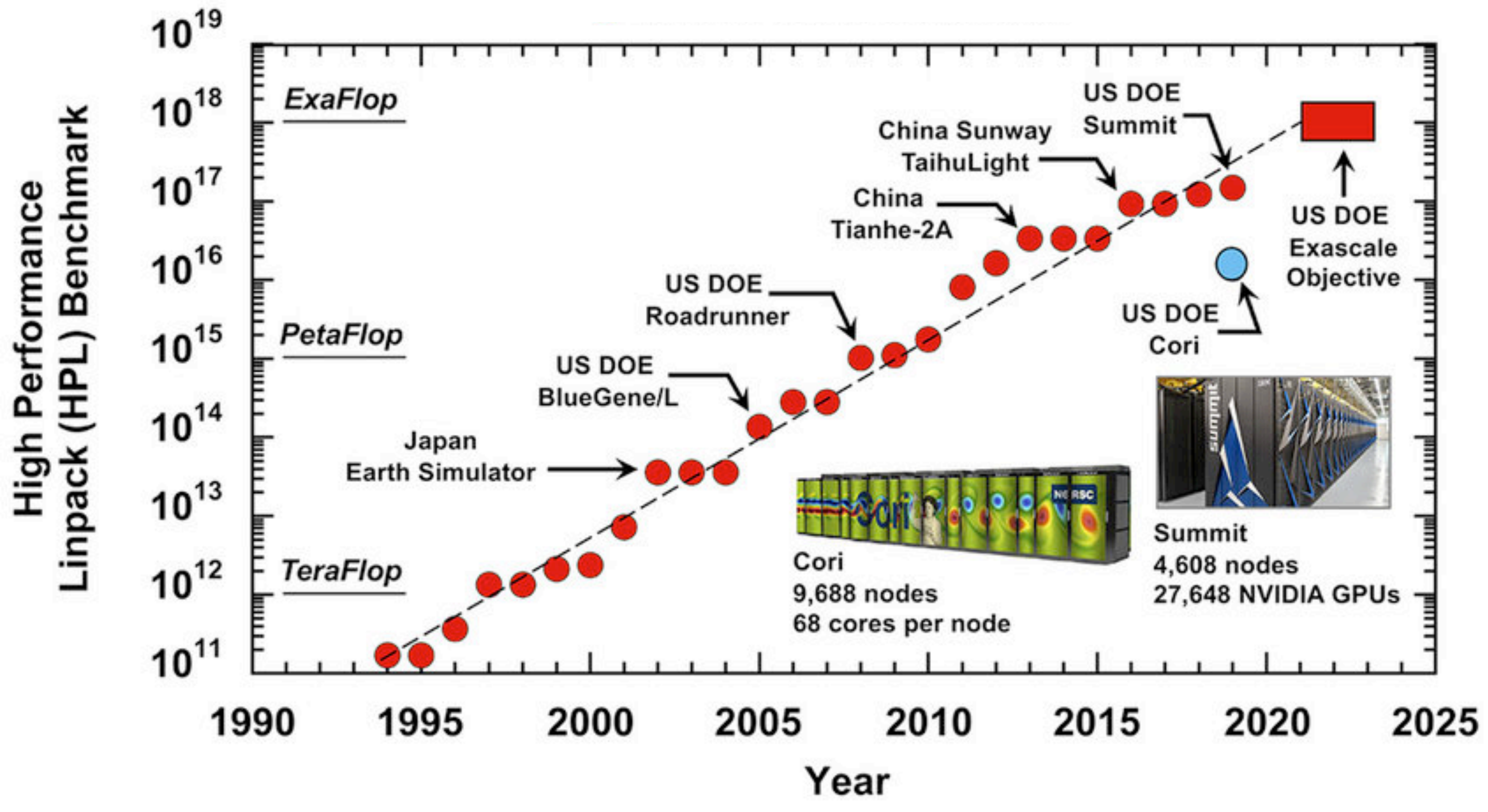
- Mathematica package: O(10) sec/pt
- Within McMule: O(0.1) sec/pt

[JR - IRNTerascale@LNF - 2024]

Motivation

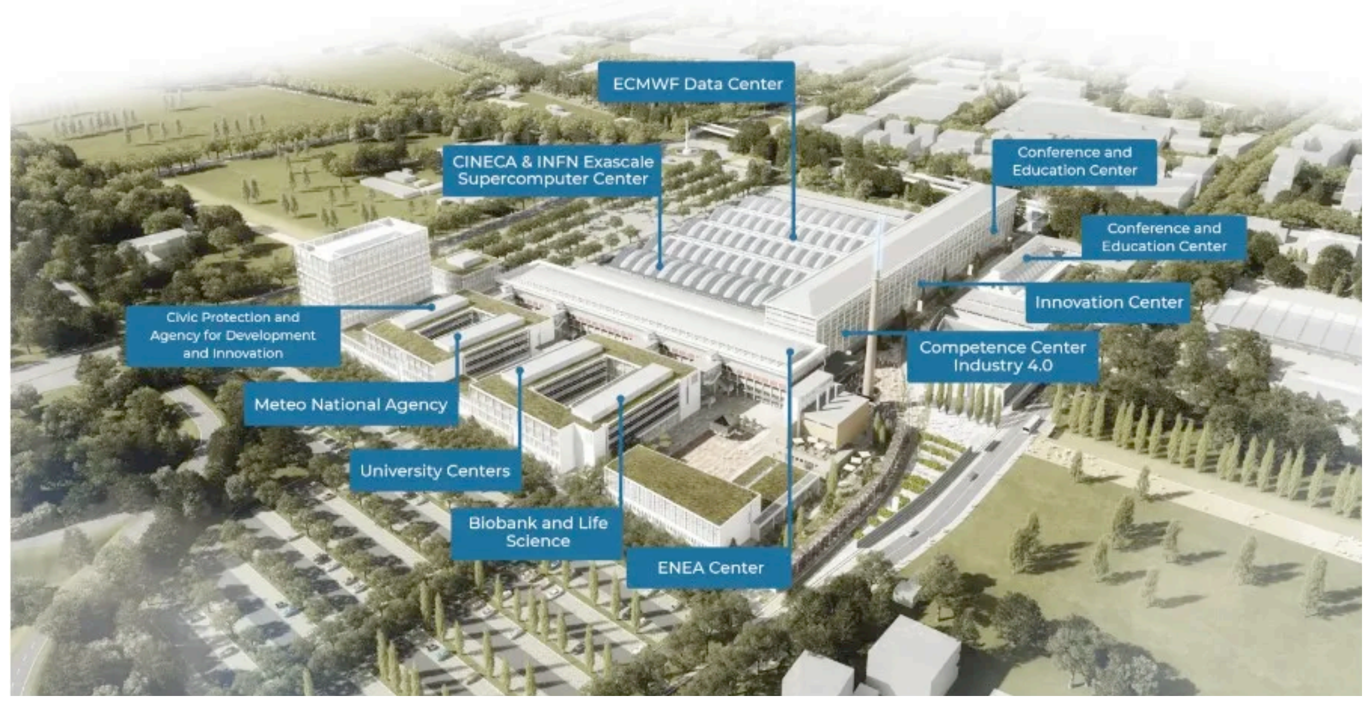
PNRR e SuperCalcolo Exascale : 1,000,000,000,000,000,000 Flops

- Advancement of HPC developments



- Tecnopolo *Cineca* a Bologna

UNA DATA VALLEY ITALIANA



- Centro Nazionale HPC, BD e QC

[Leonardo at Cineca] [video link]

Motivation

CN1.HPC.Spoke2 / Fundamental Research & Space Economy

- Spoke2 & Work Packages
  - WP1. Theoretical Physics
  - WP2. Experimental Particle Physics
  - WP3. Experimental Astro-Particle Physics
  - WP4. Boosting the Computational Performances
  - WP5. Architectural Support

Motivation

CN1.HPC.Spoke2 / Fundamental Research & Space Economy

- Spoke2 & Work Packages
  - WP1. Theoretical Physics

a. Development of algorithms, codes and computational strategies for the simulation of physical theories and models, towards pre-Exascale and Exascale architectures.

b. Theoretical research projects in domains already using HPC solutions, such as:

i. lattice field theory (flavour physics, QCD phase diagrams, hadronic physics, interactions beyond the Standard Model, machine learning in quantum field theories, electromagnetic effects in hadronic processes);

ii. collider physics phenomenology;

iii. gravitational waves, cosmology and astroparticle physics (neutron-star physics, primordial universe, dark matter and energy, neutrino physics);

iv. nuclear physics;

v. physics of complex systems (fluid dynamics, disordered systems, quantitative biology);

vi. condensed matter in low dimensional systems;

vii. quantum systems (entanglement, quantum simulations, quantum information).

Motivation

Usecase HPC.spoke2.WP1 /ADVANCED CALCULUS FOR PRECISION PHYSICS

Nodes: UNIBO - UNICAL - UNIMIB - UNIPD

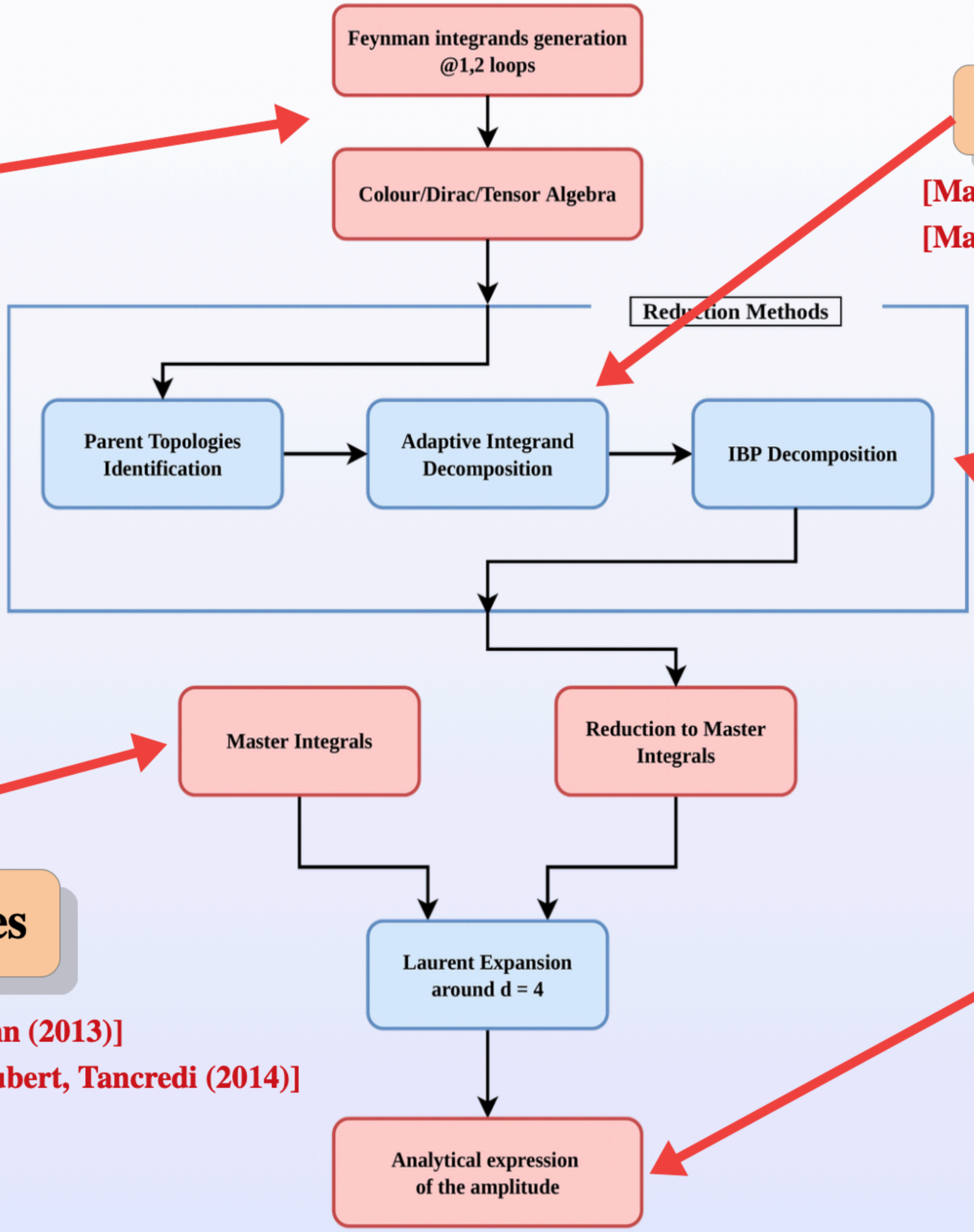
- **Five research directions:**
  - 1. Models & Diagrams
  - 2. Amplitudes & Integrals
  - 3. Cross Sections & Events
  - 4. Physics at Colliders
  - 5. Beyond Colliders

The software developed in this research program will have a major impact on Collider Phenomenology, as well as on Cosmology and Mathematics.

- Standard Model Physics
- Beyond Standard Model Physics
- Parton Distributions Functions
- Higgs boson and Heavy Particles Physics
- Effective Field Theories for Quantum and Classical Physics
- Scattering Amplitudes
- Physics of the Universe and Gravitational Waves Physics
- Computational Algebraic Geometry

Motivation

The AIDA framework



FeynCalc, FeynArts

[Mertig, Böhm, Denner (1991)]
[Shtabovenko, Mertig, Orellana (2016)]
[Hahn (2001)]

AIDA

[Mastrolia, Peraro, Primo, Torres (2016)]
[Mastrolia, Peraro, Primo, Torres, JR (To be published)]

Reduze, Kira

[Chetyrkin, Tkachev (1981)] [Laporta (2000)]
[von Manteuffel, Studerus (2012)]
[Maierhöfer, Usovitsch, Uwer (2018)]

In-house Mathematica routines

[Kotikov (1990)] [Gehrmann, Remiddi (1999)] [Henn (2013)]
[Argeri, Di Vita, Mastrolia, Mirabella, Schlenk, Schubert, Tancredi (2014)]

HandyG, GiNaC, PolyLogTools

[Bauer, Frink, Kreckel (2000)]
[Naterop, Signer, Ulrich (2019)]
[Duhr, Dulat (2019)]

Motivation

AIDA framework

- Mathematica framework (user-friendly)
- From the generation of the amplitude to its IBP reduction
- Implementation of the Adaptive Integrand Decomposition (AID)
- Can work with helicity amplitudes, form factors and interferences

What is missing?

- Parallelization
- Designed for up to 2L processes

What we are aiming for LoopIn to be?

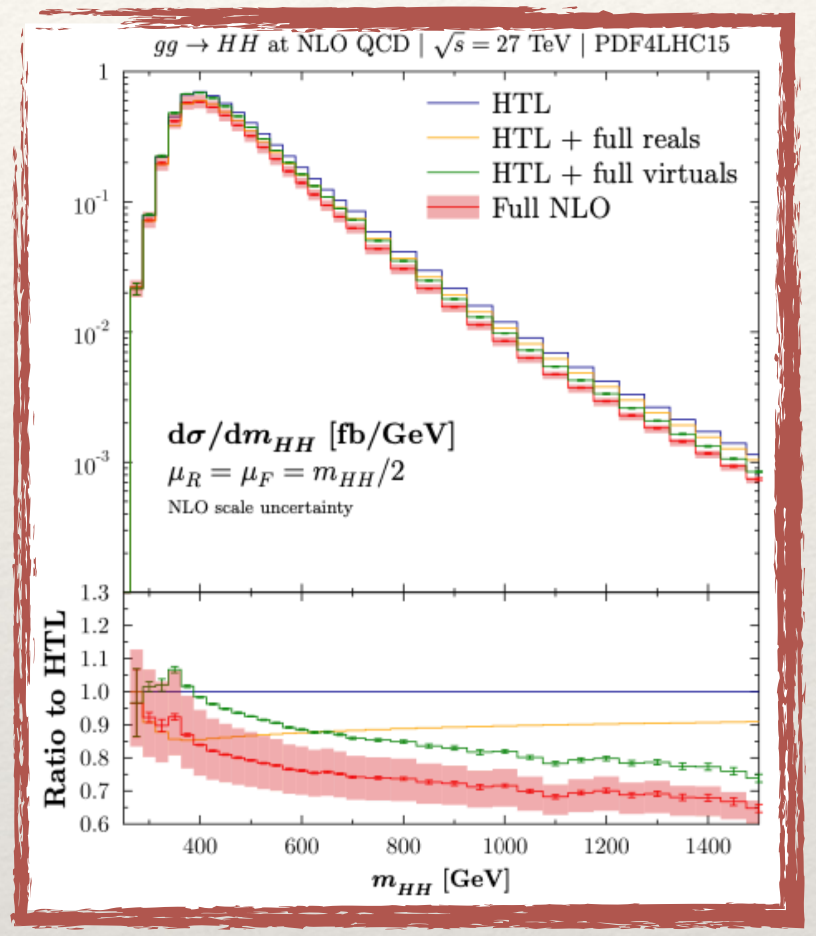
- Mathematica front-end (user-friendly)
- Minimal number of inputs
- From the generation of the amplitude to its numerical evaluation
- Modular: LoopIn has to be able to be interfaced with any code
- Flexible: User can manipulate the IO of LoopIn (with care)
- Every module of LoopIn will produce its own output
- Parallelizable
- Designed for any number of loop

From Cross Section to Amplitudes in pQFT

Total cross section

$$\sigma(1,2 \to p) = C_{\text{norm}} \left[\sum_{j=0}^{N_{\text{ord}}} \left(\frac{\alpha}{\pi}\right)^j \sigma_{\text{N}^j\text{LO}} \right] + \mathcal{O}\left(\alpha^{(N_{\text{ord}}+1)}\right)$$

$$\sqrt{s} = 13\,\text{TeV}: \quad \sigma_{\text{tot}} = 27.73(7)^{+4\%}_{-18\%}\,\text{fb},$$
$$\sqrt{s} = 14\,\text{TeV}: \quad \sigma_{\text{tot}} = 32.81(7)^{+4\%}_{-18\%}\,\text{fb},$$
$$\sqrt{s} = 27\,\text{TeV}: \quad \sigma_{\text{tot}} = 127.0(2)^{+4\%}_{-18\%}\,\text{fb},$$
$$\sqrt{s} = 100\,\text{TeV}: \quad \sigma_{\text{tot}} = 1140(2)^{+3\%}_{-18\%}\,\text{fb}$$



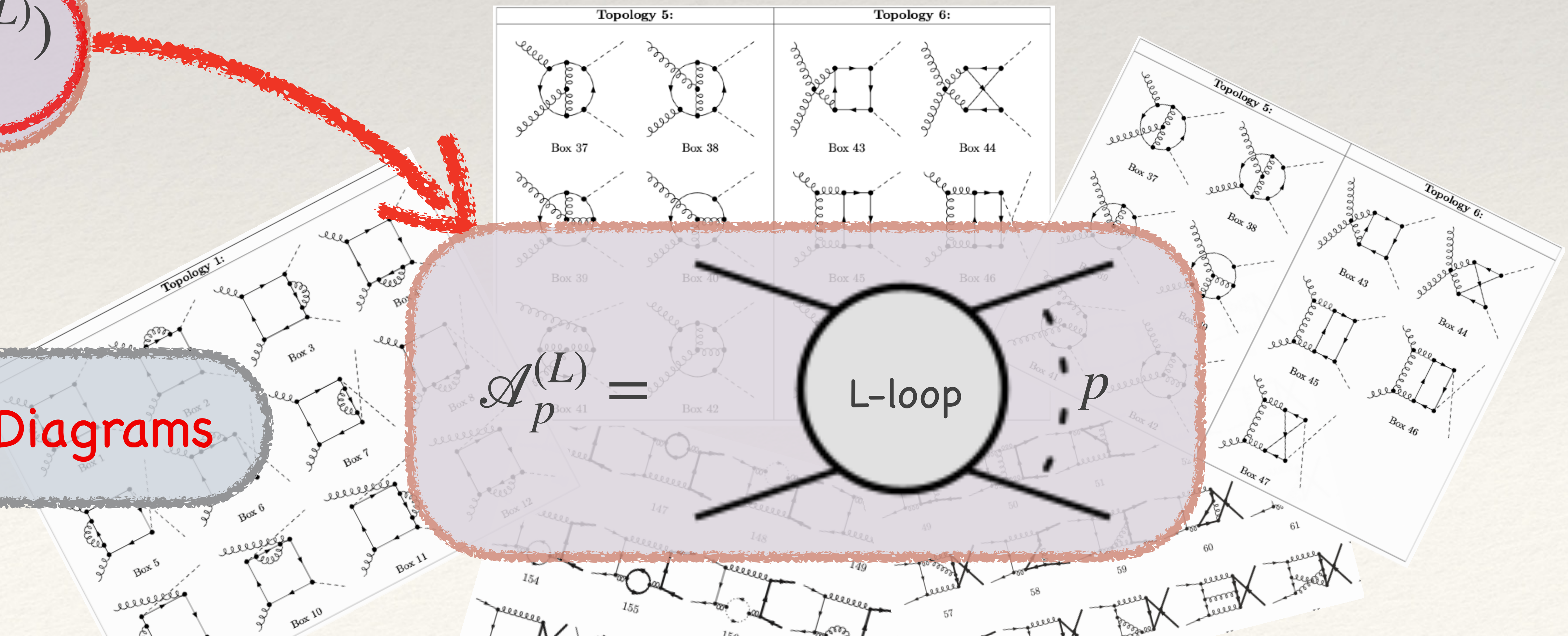
NjLO contribution

$$\sigma_{\text{N}^j\text{LO}} = \sum_{i=0}^{j} \int d\Phi_{(p+i)} \mathcal{M}^{(i,j-i)}_{p+i}$$

Interference terms

$$\mathcal{M}^{(m,L)}_p = \frac{1}{\text{N}_{\text{st.}}} \sum_{\text{st.}} \text{Re}\left(\mathcal{A}^{(m)*}_p \mathcal{A}^{(L)}_p\right)$$

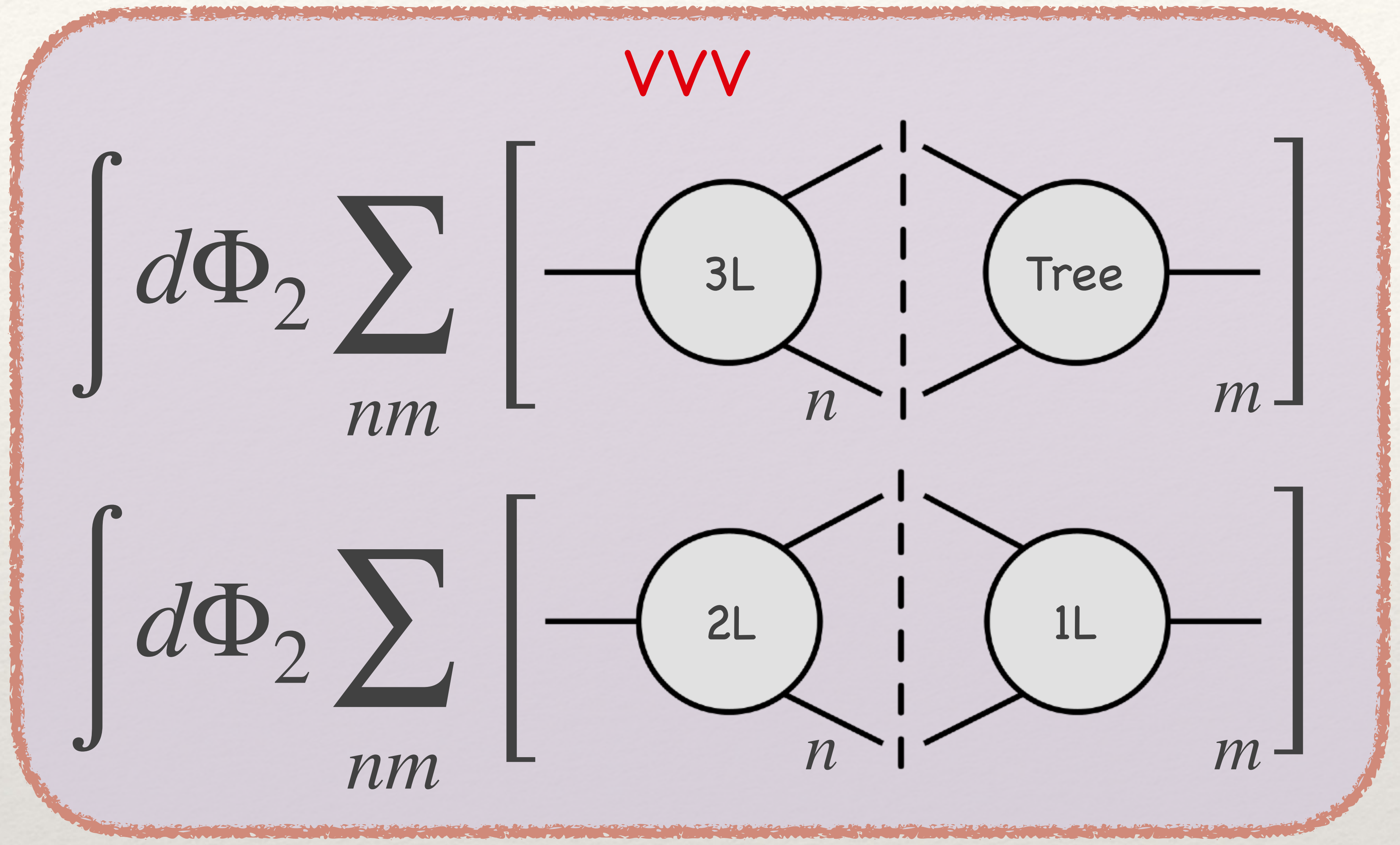
Feynman Diagrams

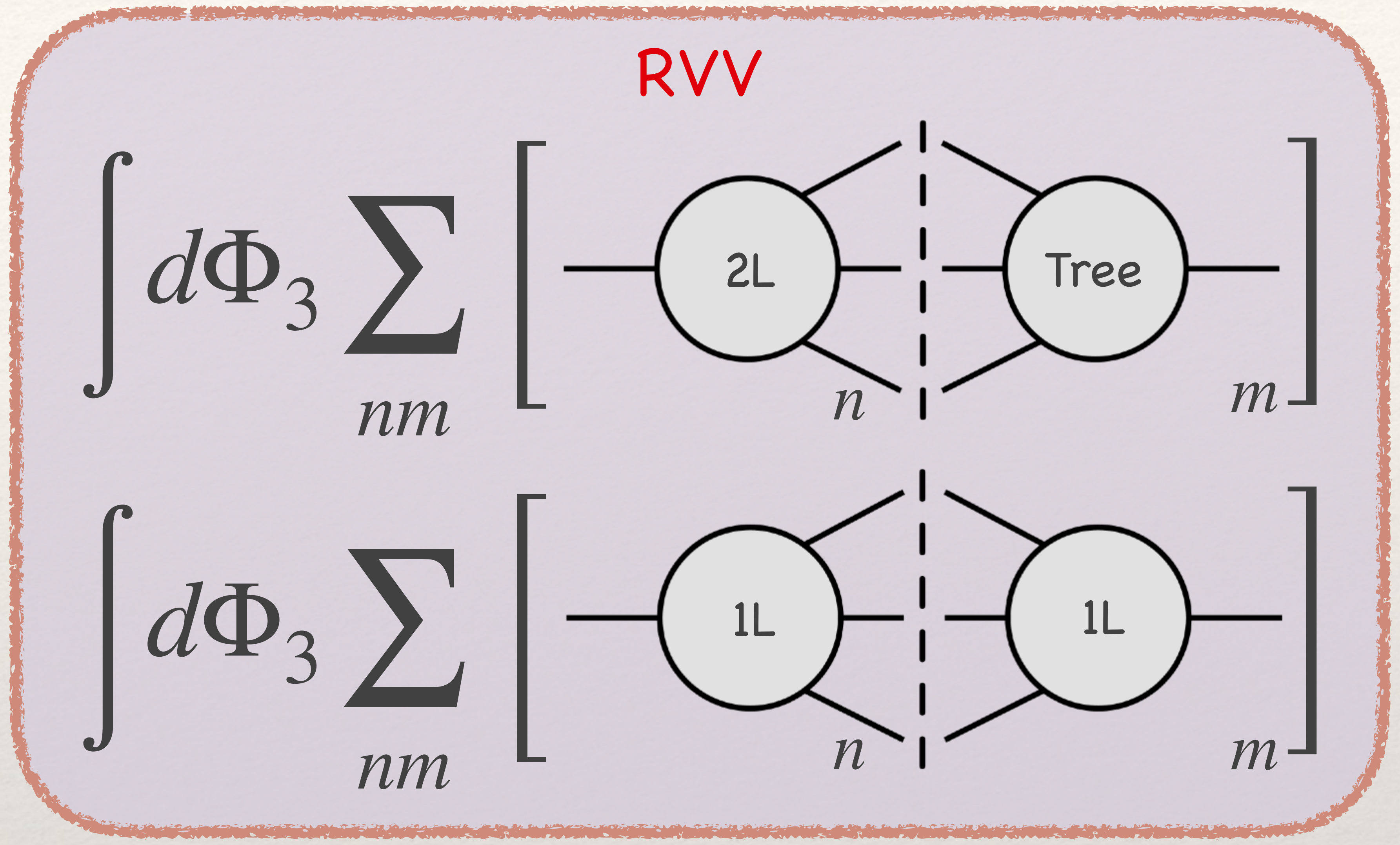


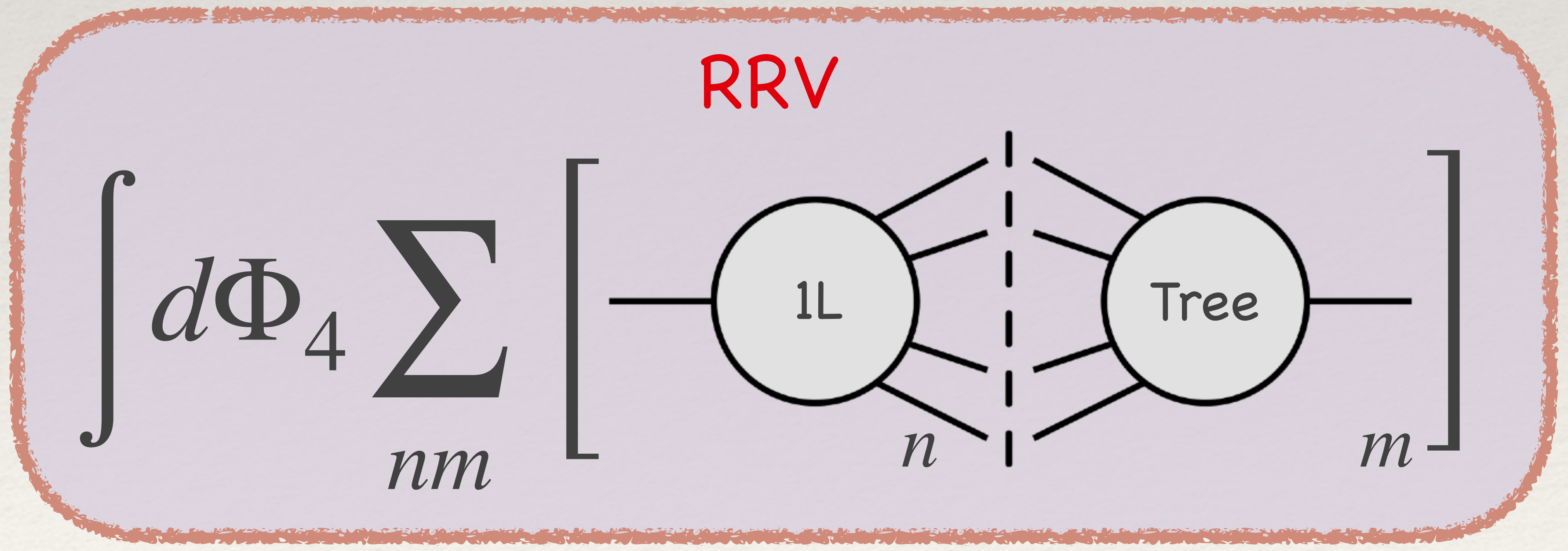
$$\mathcal{A}^{(L)}_p = \quad \text{[L-loop]} \quad p$$

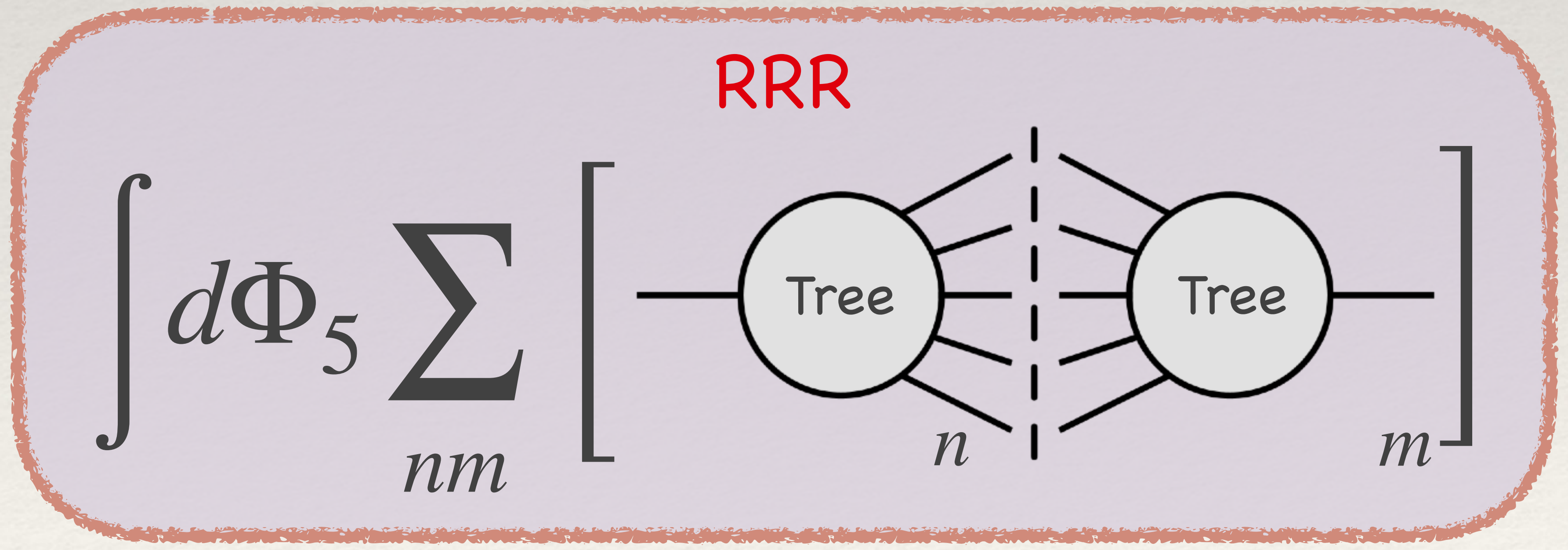
From Cross Section to Amplitudes in pQFT

Example: $\gamma^* \to \bar{l}l$ @ N^{3}LO QED









LoopIn

Mathematica

Scattering Process

$1,2 \to p$
$\mathrm{N}^j\mathrm{LO}$ <Model>
mL – interference

Phase-space point

$(\boldsymbol{s}_N, \boldsymbol{m}_N^2)$

LoopIn

Mathematica

Scattering Process

$1,2 \to p$
$\mathrm{N}^j\mathrm{LO}$ <Model>
mL – interference

Phase-space point

$(\boldsymbol{s}_N, \boldsymbol{m}_N^2)$

LoopIn


Mathematica
Scattering Process
$1,2 \to p$
N^jLO <Model>
mL – interference
Phase-space point
$(\boldsymbol{s}_N, \boldsymbol{m}_N^2)$
"Number"
$\mathscr{M}_p^{(m,L)}(\boldsymbol{s}_N, \boldsymbol{m}_N^2)$

LoopIn: setting up the calculation

How do we setup the calculation of each contribution to the cross section?

Process

$\gamma^* \to \bar{l} l \gamma$

N^{2}LO QED

$(\text{tree}, 2L)$ – interference

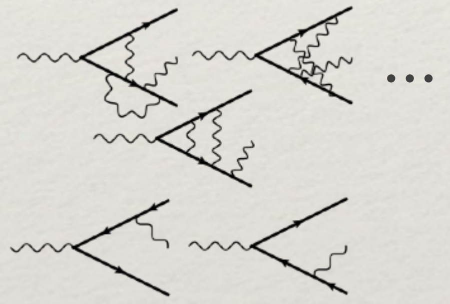
$\mathscr{A}_p^{(L)} =$ p-point n-loop Feynman amplitude

$(\mathscr{M}_p^{(L)})_{ij} =$ Interference between i-th $\mathscr{A}_p^{(0)}$ term And j-th $\mathscr{A}_p^{(n)}$ term

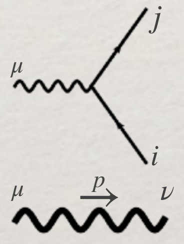
Drawing Feynman Diagrams

$\mathscr{A}_4^{(2)} =$ …

$\mathscr{A}_4^{(0)} =$



Feynman Rules

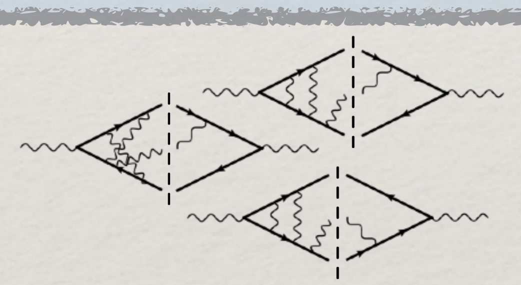


$\to ie\gamma^\mu \delta_{ij}$

$\to \frac{-ig^{\mu\nu}}{p^2 - i\epsilon}$

Building Interference terms

$\mathscr{M}_4^{(2)} =$



Summing over external states

$$\sum_h \epsilon_i^{(h)*} \epsilon_j^{(h)} \to g^{\mu\nu}$$

$$\sum_s \bar{\psi}^{(s_j)}(p_j) \cdots \psi^{(s_i)}(p_i) \bar{\psi}^{(s_i)}(p_i) \cdots \psi^{(s_j)}(p_j) \to \text{Tr}(\not{p}_j \cdots \not{p}_i \cdots)$$

Numerator Algebra

$$p_i^\mu p_j^\nu g_{\mu\nu} = p_i \cdot p_j$$

$$\text{Tr}(\cdots) = \sum_{ij} c_{ij} (p_i \cdot p_j)$$

Interference = combination of Integrals

$$(\mathscr{M}_p^{(L)})_{ij} = \sum_{\bar{\alpha}_n \bar{\beta}_r} c_{\bar{\alpha}_n \bar{\beta}_r} I_{\bar{\alpha}_n}^{\bar{\beta}_r}(\boldsymbol{s}, \boldsymbol{m}^2; d)$$

Feynman Integrals

$$I_{\bar{\alpha}_n}^{\bar{\beta}_r}(\boldsymbol{s}, \boldsymbol{m}^2; d) = \int \prod_{l=1}^{L} d^d k_l \frac{D_{n+1}^{\beta_1} \cdots D_{n+r}^{\beta_r}}{D_1^{\alpha_1} \cdots D_n^{\alpha_n}}$$

$D_i = q_i^2 - m_i^2 + i\epsilon$

q_i is a combination of loop and external momenta

- Each interference term can be expressed as a linear combination of Feynman Integrals
- Feynman integrals depend will depend on invariants, masses and space-time dimension*

Exporting all interference terms

*dimensional regularization is implicitly assumed during the whole discussion

Calculating Amplitudes: Decomposition

On the notation

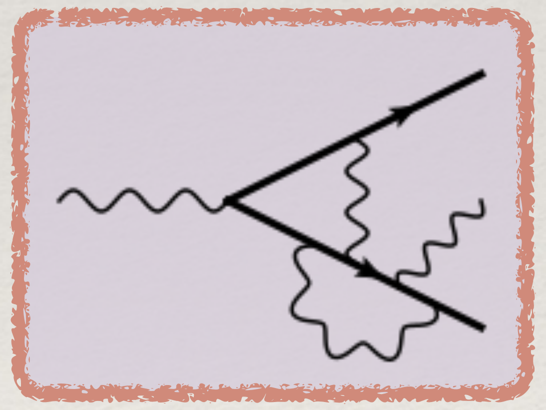
Feynman Integrals $\sim$ Graph topology with power indices

Indices

Topology

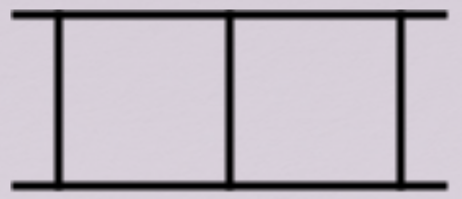
$$I^{\bar{\beta}_r}_{\bar{\alpha}_n}\left(\boldsymbol{s}, \boldsymbol{m}^2; d\right) = \int \prod_{l=1}^{L} d^d k_l \frac{D_{n+1}^{\beta_1} \cdots D_{n+r}^{\beta_r}}{D_1^{\alpha_1} \cdots D_n^{\alpha_n}}$$

Diagram



$\leftrightarrow$

Topology

$T =$  $= \begin{matrix} (k_1, m_\mu) \\ (k_1 - p_1, m_\mu) \\ (k_1 + p_2, 0) \\ (k_2 - p_2, m_\mu) \\ (k_2 + p_1, 0) \\ (k_1 + k_2, m_\mu) \\ (k_2 - p_2 - p_3, m_\mu) \end{matrix}$

Integral notation

$$I^{\bar{\beta}_r}_{\bar{\alpha}_n}\left(\boldsymbol{s}, \boldsymbol{m}^2; d\right) = \mathrm{Int}(T, \alpha_1, \ldots, \alpha_n, \beta_1, \ldots, \beta_r)$$

```
{{k[1], mt}
 {k[2], mt}
 {k[1] + p[3], 0}
 {k[1] + k[2] + p[3], mt}
 {k[1] - p[1] - p[2] + p[3], 0}
 {k[2] + p[1] + p[2], mt}            → topology11,
 {k[1], 0}
 {k[2], mt}
 {k[1] + p[3], mt}
 {k[1] + k[2] + p[3], 0}
 {k[1] - p[1] - p[2] + p[3], mt}
 {k[2] + p[1] + p[2], mt}            → topology19,

 {k[1], mt}
 {k[2], 0}
 {k[1] + p[3], 0}
 {k[1] + k[2] + p[3], 0}
 {k[1] - p[1] - p[2] + p[3], 0}
 {k[2] + p[1] + p[2], 0}             → topology6,
 {k[1], 0}
 {k[2], 0}
 {k[1] + p[3], mt}
 {k[1] + k[2] + p[3], mt}
 {k[1] - p[1] - p[2] + p[3], mt}
 {k[2] + p[1] + p[2], 0}             → topology36,

 {k[1], mt}
 {k[2], mt}
 {k[1] + p[3], 0}
 {k[1] + k[2] + p[3], mt}
 {k[2] - p[1] - p[2] + p[3], 0}
 {k[1] + p[1] + p[2], mt}            → topology49,
 {k[1], 0}
 {k[2], 0}
 {k[1] + k[2], 0}
 {k[2] - p[1] - p[2], 0}
 {k[2] - p[1], 0}
 {k[1] + p[1], 0}                    → topology65,

 {k[1], 0}
 {k[2], 0}
 {k[1] + p[2], 0}
 {k[2] - p[1] - p[2], 0}
 {k[1] - p[1], 0}
 {k[1] + k[2] - p[1], 0}             → topology66,
 {k[1], 0}
 {k[2], 0}
 {k[1] + k[2], 0}
 {k[2] + p[2], 0}
 {k[1] + k[2] + p[1] + p[2] - p[3], mt}
 {k[1] + k[2] + p[1] + p[2], 0}
 {k[1] + p[1], 0}                    → topology73}
```

- Every integral coming from the same diagram will belong to the same topology
- Indices $\bar{\alpha}_n$ and $\bar{\beta}_r$ characterize every individual integral
- Denominator basis might be not closed: Automatic choice of ISPs

Grouping Diagrams

[courtesy of Tom Dave]

[Crisanti,Dave,Smith:WIP]

Lee-Pomeransky Polynomials[1] are a way of representing the graphical properties of a Feynman Diagram as a polynomial.

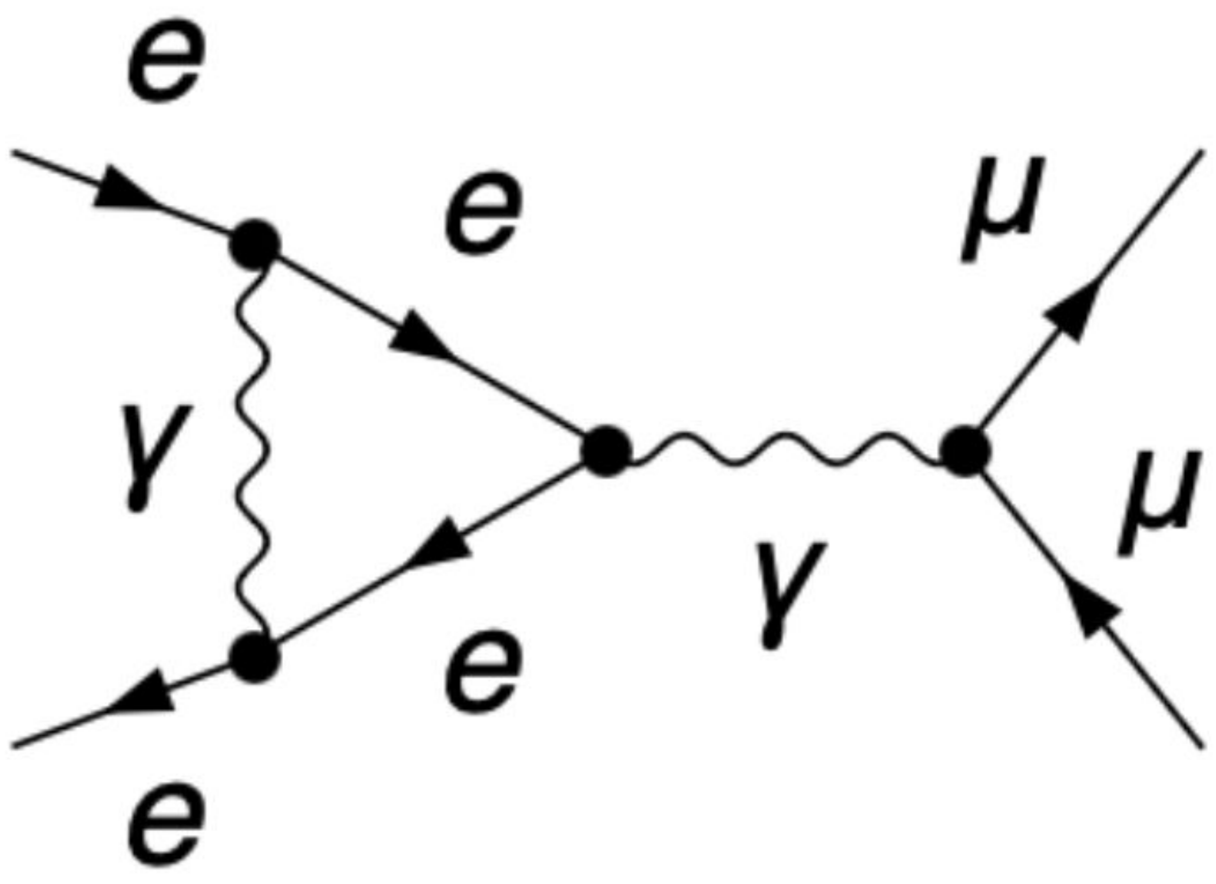
Whilst initially constructed from the momenta of the propagators within the diagram, the result only depends on variables that represent those propagators rather than the momenta within them.

[1]R.N. Lee (2013)- http://dx.doi.org/10.1007/JHEP11(2013)165

Grouping Diagrams

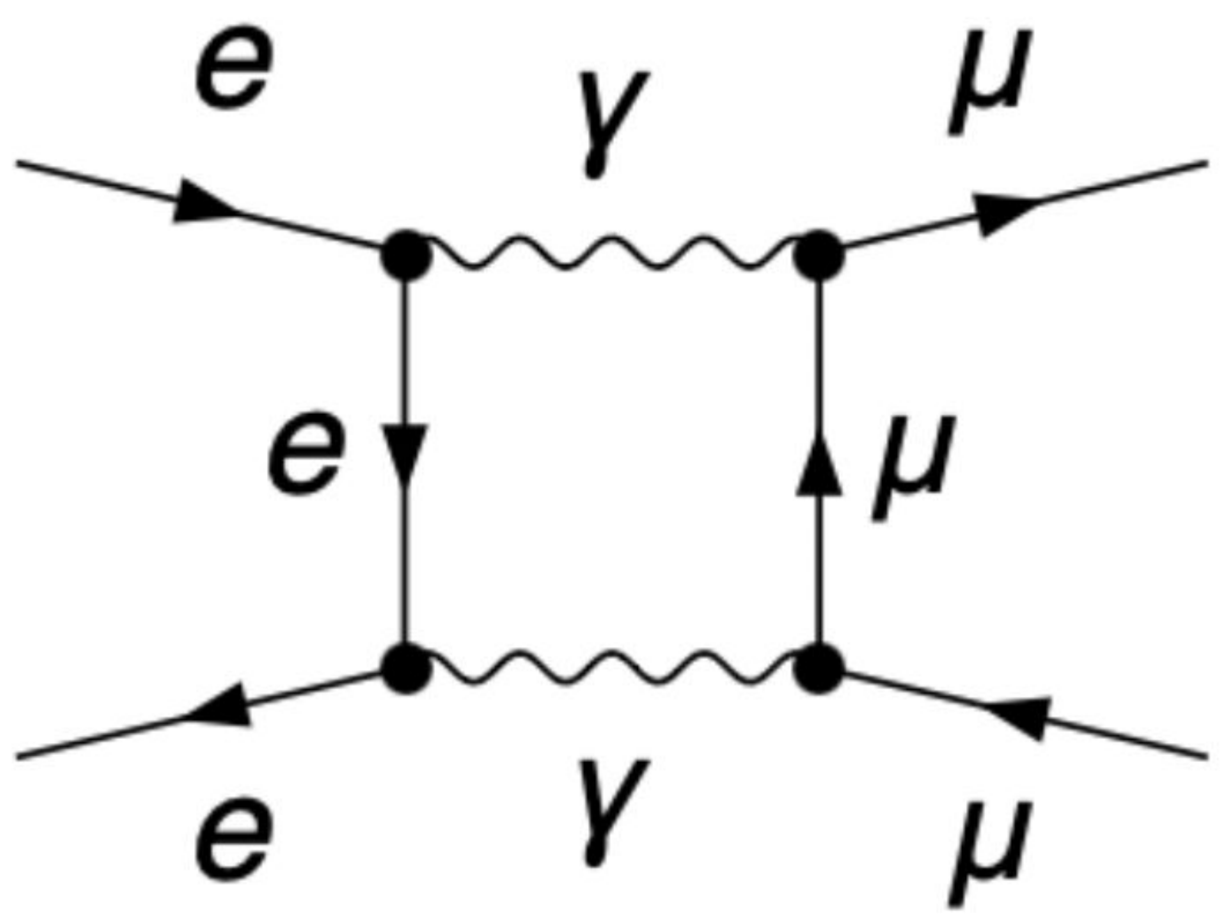
[courtesy of Tom Dave]

[Crisanti,Dave,Smith:WIP]



Propagators: (q), $(q+p_2)$ and $(q+p_2+p_3+p_4)$.

Polynomial: $s(x_2 \cdot x_3) + x_1 + x_2 + x_3$



Propagators: (q), $(q+p_2+p_3+p_4)$, $(q+p_3+p_4)$ and $(q+p_4)$.

Polynomial: $s(x_3 \cdot x_1) + t(x_2 \cdot x_4) + x_1 + x_2 + x_3 + x_4$

Grouping Diagrams

[courtesy of Tom Dave]

[Crisanti,Dave,Smith:WIP]

[Crisanti,Dave,Smith:WIP]

To consider a pinch, we can set one of the variables to zero.

We can then consider permutations of the variables.

Starting from a diagram with maximal propagators (Parent), if find pinches and a permutation of the variables that match to a diagram with fewer propagators (Sub-Diagram). The Sub-Diagram is a member of the Parent Diagram's family.

Example from the previous diagrams:

$$s(x_3 \cdot x_1) + t(x_2 \cdot x_4) + x_1 + x_2 + x_3 + x_4$$

$x_1 \to x_3$, $x_2 \to x_1$ and $x_3 \to x_2$ then set $x_4 = 0$

→

$$s(x_2 \cdot x_3) + x_1 + x_2 + x_3$$

The same as the Triangle diagram.

Grouping Diagrams

[courtesy of Tom Dave]

[Crisanti,Dave,Smith:WIP]

[Crisanti,Dave,Smith:WIP]

We were able to group 148 diagrams into 20 families, as seen in the table below:

Family	Diagrams
47	47, 48, 93, 94, 95, 96, 97, 98, 99, 100
49	34, 49, 106, 107, 108, 111, 112, 133, 134, 136, 137, 143, 144
50	50, 113, 132
51	51, 116, 117, 118, 120, 127, 128, 129
52	20, 52, 121, 122, 124, 138, 145, 146
53	5, 6, 7, 8, 43, 44, 45, 46, 53, 54, 55, 56, 57, 85, 86, 87, 88, 89, 90, 91, 92
58	1, 2, 3, 4, 36, 37, 38, 39, 40, 58, 101, 102, 103, 104, 135, 140
59	59, 60, 61, 62
63	63, 64, 65, 66, 67
68	9, 10, 11, 12, 22, 23, 24, 25, 26, 68, 123, 142
69	15, 16, 17, 18, 19, 69, 114, 139
70	29, 30, 31, 32, 33, 70, 130, 141
71	13, 71
72	72, 110
73	14, 35, 42, 73, 105, 109, 131, 147
74	74, 75, 76, 77
78	27, 78
79	79, 126
80	80, 81, 82, 83
84	21, 28, 41, 84, 115, 119, 125, 148

Table 1: Two Loop Groupings for $e^+e^- \to \gamma\gamma$

LoopIn

Scattering Process
$1,2 \to p$
N^jLO <Model>
mL – interference
Phase-space point
(s_N, m_N^2)
Mathematica
Qgraph
Drawing diagrams
FeynArts
FORM
Interferences
Summing over ext. state
Algebra
FeynCalc
Finding Topologies
Grouping diagrams
In-house
???
"Number"
$\mathscr{M}_p^{(m,L)}(s_N, m_N^2)$

Integration-by-parts identities

Is there a way to **reduce** the number of integrals we need to evaluate?

Typical 2-loop processes get contribution from $\sim 10^4 - 10^6$ integrals

Feynman Integrals

$$I^{\bar{\beta}_r}_{\bar{\alpha}_n}(s, m^2; d) = \int \prod_{l=1}^{L} d^d k_l \frac{D_{n+1}^{\beta_1} \cdots D_{n+r}^{\beta_r}}{D_1^{\alpha_1} \cdots D_n^{\alpha_n}}$$

Fact: Feynman integrals are **invariant over loop momenta shifts**

Integration-by-part Identities (IBPs)

$$k_l \to A_{li} k_i + B_{lj} p_j \implies \int \prod_{l=1}^{L} d^d k_l \frac{d}{dk_j^\mu} \left(v_i^\mu \frac{D_{n+1}^{\beta_1} \cdots D_{n+r}^{\beta_r}}{D_1^{\alpha_1} \cdots D_n^{\alpha_n}} \right) = 0$$

[Chetyrkin,Tkachov:1981]
[Laporta:hep-ph/0102033]

Using $\bar{\alpha}_r, \bar{\beta}_s$ as seeds: **GIGANTIC** system of equation

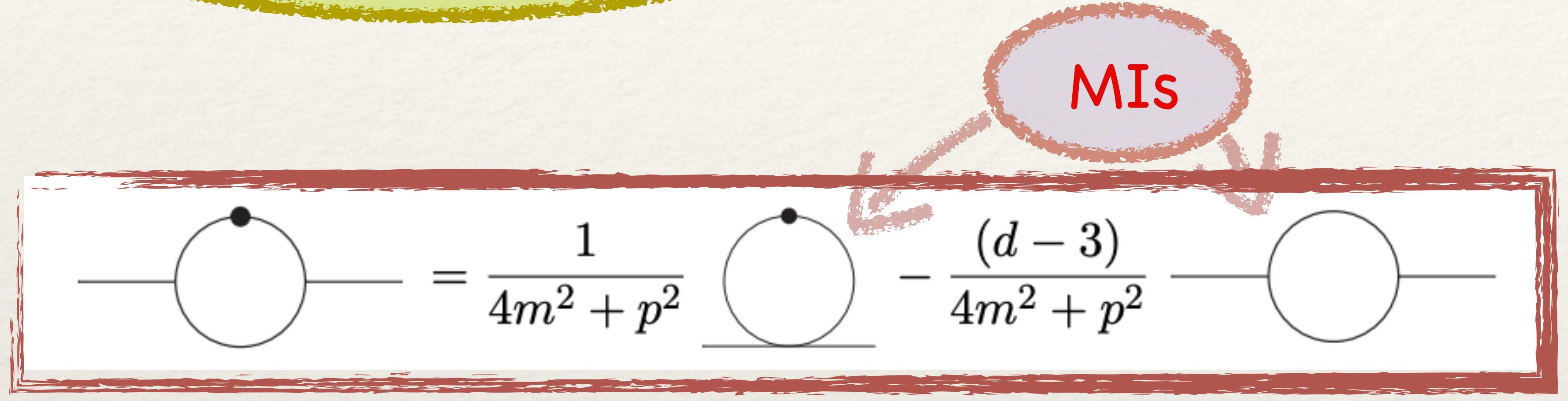
$$\sum_{\bar{\alpha}_s, \bar{\beta}_r} b_{\bar{\alpha}_s, \bar{\beta}_r} \mathrm{Int}(T, \alpha_1, \ldots, \alpha_n, \beta_1, \ldots, \beta_r) = 0$$

```
-Int[T, 1, 0, 0, 0, 0, -1, 0, 0, 0] α0 +
  Int[T, 1, 0, 0, 0, 0, 0, 0, -1, 0] α0 + Int[T, -1, 1, 0, 0, 0, 0, 0, 0, 0] α1 +
  Int[T, 0, 1, 0, 0, -1, 0, 0, 0, 0] α1 - Int[T, 0, 1, 0, 0, 0, -1, 0, 0, 0] α1 +
  Int[T, -1, 0, 1, 0, 0, 0, 0, 0, 0] α2 + Int[T, 0, 0, 1, -1, 0, 0, 0, 0, 0] α2 -
  Int[T, 0, 0, 1, 0, 0, -1, 0, 0, 0] α2 + Int[T, 0, 0, 0, 0, 0, 0, 0, 0, 0] (α0 - α5) +
  Int[T, -1, 0, 0, 0, 0, 1, 0, 0, 0] α5 - Int[T, 0, 0, 0, 0, 0, 1, 0, -1, 0] α5 +
  Int[T, -1, 0, 0, 0, 0, 0, 0, 0, 1] β8 - Int[T, 0, 0, 0, -1, 0, 0, 0, 0, 1] β8 -
  Int[T, 0, 0, 0, 0, 0, -1, 0, 0, 1] β8 + Int[T, 0, 0, 0, 0, 0, 0, -1, 0, 1] β8 +
  Int[T, 0, 0, 0, 0, 0, 0, 0, -1, 1] β8 - t Int[T, 0, 0, 0, 0, 0, 0, 0, 0, 1] β8 == 0
```

Example of an IBP operator

Integration-by-parts identities

Is there a way to **reduce** the number of integrals we need to evaluate?




```
{
diag3[1,0,1,0] →
 + 0
,
diag3[-1,1,1,1] →
 + diag3[0,1,1,1]*(((s-3*m2)*t+m2^2)/(s-4*m2))
 + diag3[0,0,1,1]*((-t-m2)/(s-4*m2))
 + diag3[0,1,1,0]*((2*t+s-2*m2)/(s-4*m2))
 + diag3[0,1,0,0]*(((-d+2)*t+(-d+2)*m2)/(((2*d-6)*m2)*s+(-8*d+24)*m2^2))
,
diag3[0,1,0,1] →
 + diag3[0,1,0,0]*((d-2)/((2*d-6)*m2))
,
diag3[1,-1,1,1] →
 + diag3[0,0,1,1]*((((d-2)*s+(-2*d+6)*m2)*t+((-d+4)*m2)*s+(2*d-6)*m2^2)/((d-4)*t^2+((-2*d
 + diag3[1,0,0,1]*(((d-4)*t^2+(-2*s+2*m2)*t+(-d+2)*m2^2)/((d-4)*t^2+((-2*d+8)*m2)*t+(d-4)
,
diag3[1,0,1,1] →
 + diag3[0,0,1,1]*((2*d-6)/((d-4)*t+(-d+4)*m2))
 + diag3[1,0,0,1]*((-2*d+6)/((d-4)*t+(-d+4)*m2))
,
diag3[1,1,-1,1] →
 + diag3[1,1,0,1]*(((s+m2)*t^2+((m2)*s-2*m2^2)*t+m2^3)/(t^2+(-2*m2)*t+m2^2))
 + diag3[1,0,0,1]*((t^2+(2*s-2*m2)*t+m2^2)/(t^2+(-2*m2)*t+m2^2))
 + diag3[0,1,0,0]*((((-3*d+8)*s)*t+((d-4)*m2)*s)/(((2*d-6)*m2)*t^2+((-4*d+12)*m2^2)*t+(2*
,
diag3[1,1,0,0] →
 + diag3[0,1,0,0]*(1/m2)
```

Example of a reductions table

Not all identities in the IBPs system are **independent**

Rank of the system = # of **Master Integral**

Choice of **Master Integral**

[Chetyrkin,Tkachov:1981]
[Laporta:hep-ph/0102033]

+

Gauss elimination on IBPs system

⇒ **Integral Reduction**

After reduction:

$$(\mathcal{M}_p^{(L)})_{ij} = \sum_n c_n J_n\left(s, m^2; d\right)$$

$$\#I_{\bar{\alpha}_n}^{\bar{\beta}_r} \sim O(10^4 - 10^6)$$
$$\downarrow$$
$$\#J_n \sim O(10^2 - 10^3)$$

- Number of integrals drastically decreases
- The choice of MIs is not unique

Integration-by-parts identities

Interference

$$(\mathscr{M}_p^{(L)})_{ij} = \sum_{\bar{\alpha}_n \bar{\beta}_r} c_{\bar{\alpha}_n \bar{\beta}_r} \, I_{\bar{\alpha}_n}^{\bar{\beta}_r}\left(\boldsymbol{s}, \boldsymbol{m}^2; d\right)$$

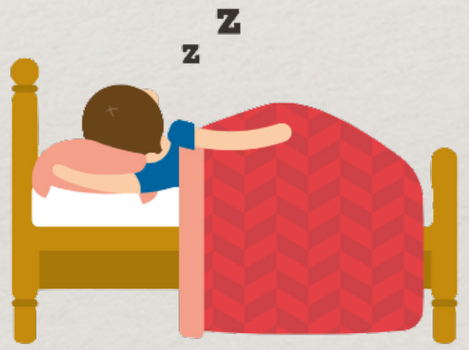
Reading Integrals From Interferences

```
Int[T, -1, 1, 2, 1, 1, 0, 0, 0, 0]
Int[T, -1, 2, 0, 1, 0, 1, 0, 0, 0]
Int[T, -1, 2, 1, 1, 1, 0, 0, 0, 0]
Int[T, 0, 1, 0, 1, 0, 1, 0, 0, 0]
Int[T, 0, 1, 0, 2, 0, 1, 0, -1, 0]
Int[T, 0, 1, 1, 0, 2, 0, 0, 0, 0]
Int[T, 0, 1, 1, 1, 1, 0, 0, 0, 0]
Int[T, 0, 1, 1, 1, 2, 0, -1, 0, 0]
Int[T, 0, 1, 1, 1, 2, 0, 0, -1, 0]
Int[T, 0, 1, 1, 1, 2, 0, 0, 0, 0]
Int[T, 0, 1, 1, 2, 1, 0, 0, -1, 0]
Int[T, 0, 1, 2, 0, 0, 1, 1, 0, 0]
Int[T, 0, 1, 2, 1, 1, 1, 1, 0, 0]
Int[T, 0, 1, 2, 1, 1, 1, 2, 0, 0]
Int[T, 0, 2, -1, 1, 0, 1, 0, 0, 0]
Int[T, 0, 2, 0, 1, 0, 1, 0, 0, -1]
```

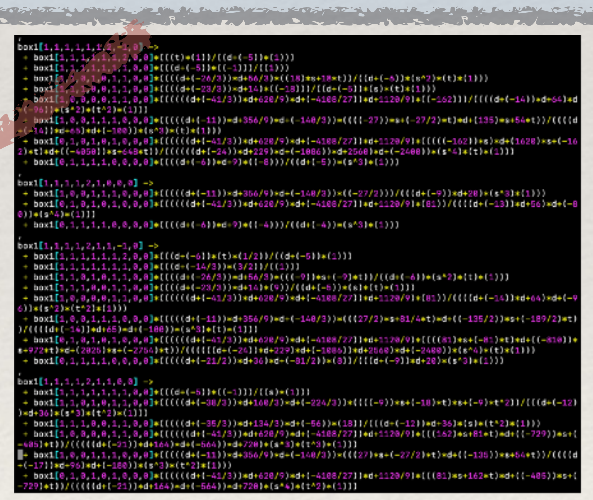
Creating Inputs for the IBP reducer

```
integralfamilies:
  - name: "T"
    loop_momenta: [k1,k2]
    top_level_sectors: [127]
    propagators:
      - [ k1, m2 ]
      - [ k1-p1, m2 ]
      - [ k1+p2, 0 ]
      - [ k2-p2, m2 ]
      - [ k2+p1, 0 ]
      - [ k1+k2, m2 ]
      - [ k2-p2-p3, m2 ]
      - [ k2, 0 ]
      - [ k1 + p3, 0 ]
```

Running IBPs (Possibly in parallel)



Reducer Output



Reduced Interference

$$(\mathscr{M}_p^{(L)})_{ij} = \sum_{n} c_n \, J_n\left(\boldsymbol{s}, \boldsymbol{m}^2; d\right)$$

LoopIn

Scattering Process
$1{,}2 \to p$
N^jLO <Model>
mL – interference
Phase-space point
(s_N, m_N^2)
Mathematica
Qgraph
Drawing diagrams
FeynArts
FORM
Interferences
Summing over ext. state
Algebra
FeynCalc
Finding Topologies
Grouping diagrams
In-house
Kira
Blade
FiniteFlow
IBP reductions
FIRE
LiteRED
???
"Number"
$\mathscr{M}_p^{(m,L)}(s_N, m_N^2)$

Numerical Integration

How do we **evaluate** Master integrals?

Evaluating Masters

$$J_n\left(s_{ij} = \mathrm{Num.}, m_k = \mathrm{Num.}; d\right) = ??$$

MonteCarlo Integration Methods

- Sector Decomposition
- Tropical Integration
- Loop-Tree Duality

Numerical solution of Differential Equations

- Auxiliary-mass flow
- DEs solutions along paths

Numerical Integration

Numerical solution of Differential equations

Auxiliary mass flow
(AMFlow)

[Liu,Ma:2201.11669]

- Introducing a mass parameter η into propagators
- Numerical IBPs + DE system depending on η only
- Automatic Boundary condition at $\eta \to \infty$
- Propagating boundaries to $\eta \to 0$

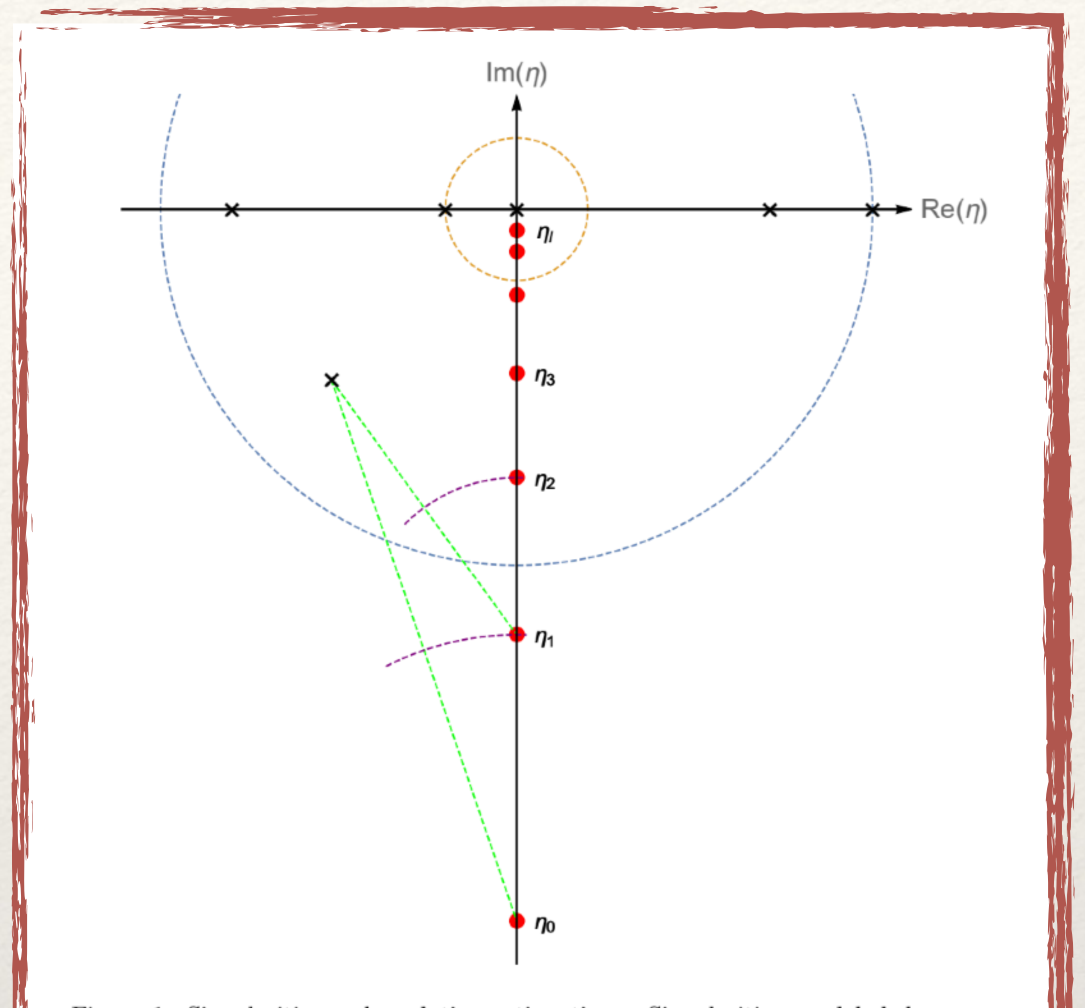


Figure 1: Singularities and analytic continuations. Singularities are labeled as crosses. Solid dots are points where to perform series expansions.

[Liu,Ma:2201.11669]

Series expansion methods
(DiffExp, SeaSyde)

[Hidding:2006.05510]
[Armadillo,Bonciani,Devoto,Rana,Vicini:2205.03345]

- Analytical IBPs + Differential equation system
- Boundary condition as input in Euclidean region
- Propagating boundary to physical region

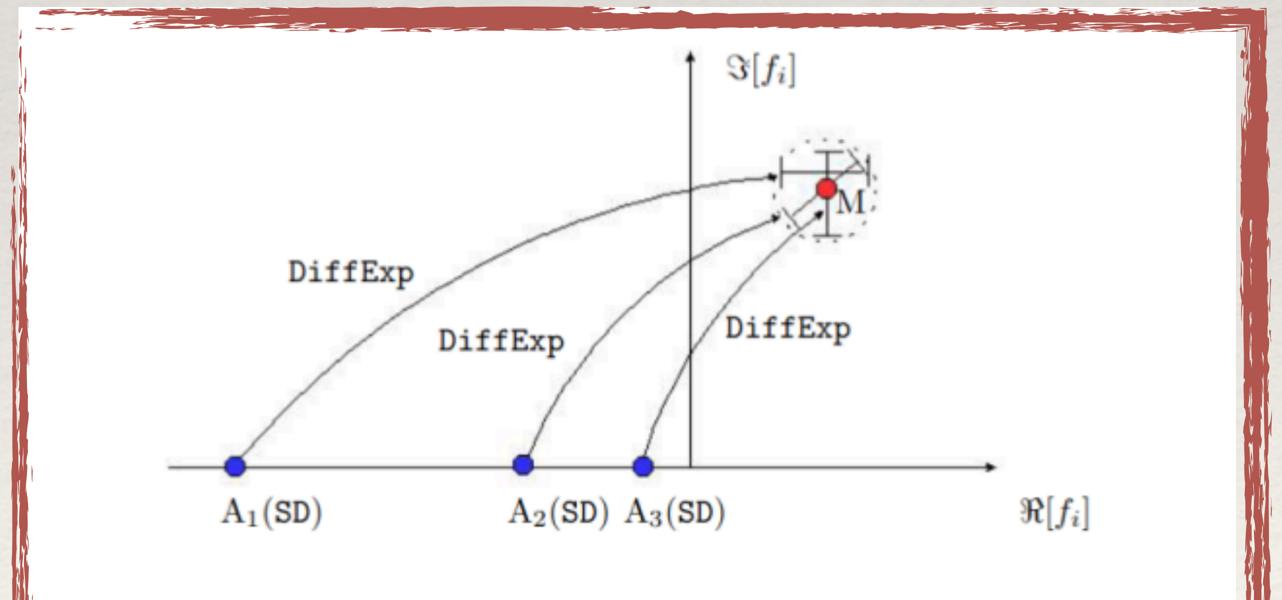
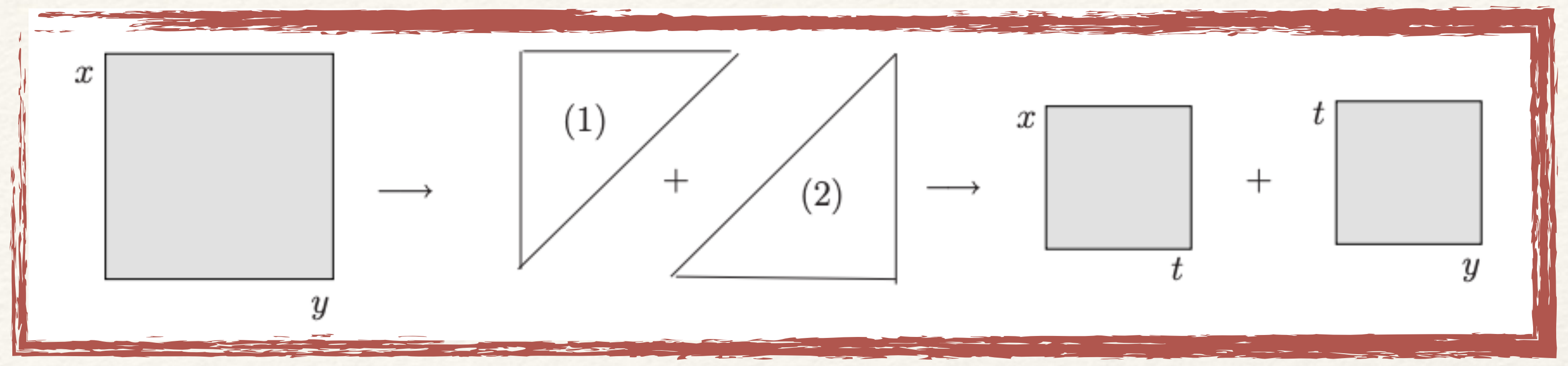


FIG. 1. Illustration of the DE transport method. The bound-

[Dubovyk,Freitas,Gluza,Grzanka,Hidding,Usovitsch:2201.0257]

Numerical Integration

MonteCarlo Integration methods

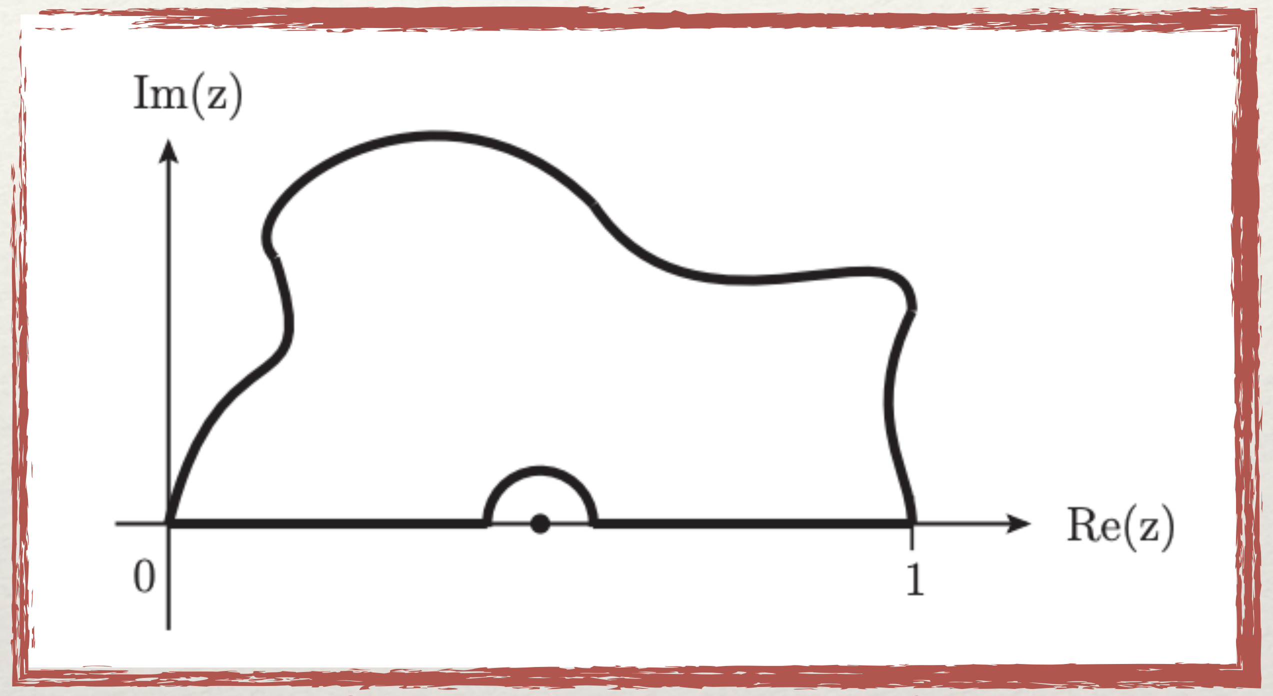


[Heinrich:0803.4177]

Sector Decomposition
(SecDec, pySecDec)

[Heinrich,Jones,Kerner,Magerya,Olsson,Schlenk:2305.19768]

- Feynman parametrization
- Splitting integration domain
- End-point subtraction of singularities and ϵ expansion
- contour deformation + expansion-by-region
- MonteCarlo integration of finite integrals



[Jones:GGI talk,13 Sept 2023]

Tropical integration
(FeynTrop)

[Borinski,Munch,Tellander:2302.08955]

- Feynman parameters + contour deformation $x_i \to x_i e^{-i\lambda \frac{d}{dx_i}\left(\frac{\mathcal{U}(\vec{x})}{\mathcal{F}(\vec{x})}\right)}$
- Tropical approximation of Symanzik Polynomial
- MonteCarlo integration improved with tropical sampling
- Improving sampling by geometrical insights

$$\left(\mathrm{Trop}(2x_1x_2 + 4x_3^2) = \max_{\mathrm{supp}}(x_1x_2, x_3^2)\right)$$

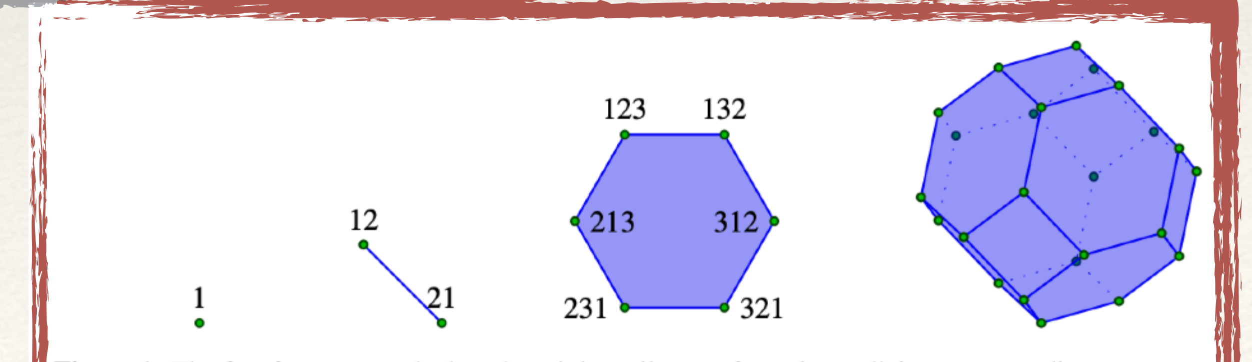


Figure 1: The first four permutohedra where it is easily seen from the explicit vertex coordinates (permutations) that all edges are parallel to $\mathbf{e}_i - \mathbf{e}_j$ for $i \neq j$.

[Borinsky,Munch,Tellander:2310.19890]

[courtesy of Marco Bigazzi]

AMFlow interface

Input: minimal set of MI

- Arranging MI according to the topologies.

- Generating a folder, for each topology, with a script to run AMFlow in parallel.

- Creating a Bash script for managing the AMFlow jobs.

AMFlow script

- Loading AMFlow and setting the IBPs reducer;
- Setting the AMFlowInfo parameters (by loading the information from parent folders and files);
- Loading ps-point by Bash script;
- Listing MI to evaluate and setting precision;
- Evaluating MI;
- Exporting the results.

Bash script

- **Inputs**:
  - Number of parallel processes;
  - AMFlow scripts paths;
  - List of ps-points;

- **Controls**:
  - Number of parallel processes;
  - Avoiding double execution of the same AMFlow script on different ps-points.

LoopIn

Scattering Process
1,2 → p
N^jLO <Model>
mL – interference
Phase-space point
(s_N, m_N^2)
Mathematica
Qgraph
Drawing diagrams
FeynArts
FORM
Interferences
Summing over ext. state
Algebra
FeynCalc
Finding Topologies
Grouping diagrams
In-house
Kira
Blade
FiniteFlow
IBP reductions
FIRE
LiteRED
AMFlow
pySecDec
DiffExp
Numerical Evaluation
SeaSyde
FeynTrop
"Number"
ℳ_p^(m,L)(s_N, m_N^2)

LoopIn: current status

Codes references

QGraph

[Nogueira:1991]

FeynArts

[Küblbeck,Böhm,Denner:1991]

[Hahn:hep-ph/0012260]

FeynCalc

[Mertig,Böhm,Denner:1991]

[Shtabovenko,Mertig,Orellana:2312.14089]

FORM

[Vermaseren:math-ph/0010025]

Kira

[Maierhöfer,Usovitsch,Uwer:1705.05610]

[Klappert,Lange,Maierhöfer,Usovitsch:2008.06494]

LiteRED

[Lee:1212.2685]

FIRE

[Smirnov:2311.02370]

FiniteFlow

[Peraro:1905.08019]

Blade

[Guan,Liu,Ma,Wu:2405.14621]

pySecDec

[Carter,Heinrich:1011.5493]

[Heinrich,Jones,Kerner,Magerya,Olsson,Schlenk:2305.19768]

FeynTrop

[Borinski,Munch,Tellander:2302.08955]

AMFlow

[Liu,Ma:2201.11669]

DiffExp

[Hidding:2006.05510]

SeaSyde

[Armadillo,Bonciani,Devoto,Rana,Vicini:2205.03345]

Summary

We present a novel code for automated evaluation of Scattering Amplitudes

LoopIn: **Loop In**tegrals for virtual amplitudes

Features:

- Automated framework for evaluation of scattering amplitudes in pQFT++
- Designed for parallelization
- Modular structure, easily upgradable
- Tested on many 1L and 2L virtual correction in QED/QCD (by M. Bigazzi)

Current status: alpha

Near future:

- tackling 3L QED processes

Long term:

- Robust grouping implementation
- Form factors and helicity amplitudes

LOOPIN

LOOPIN

Thank you for your attention!