An alternative approach to extract a_{μ}^{HLO} from the MUonE data

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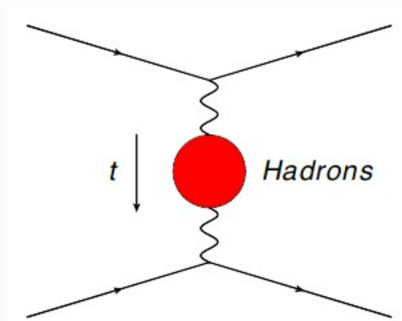
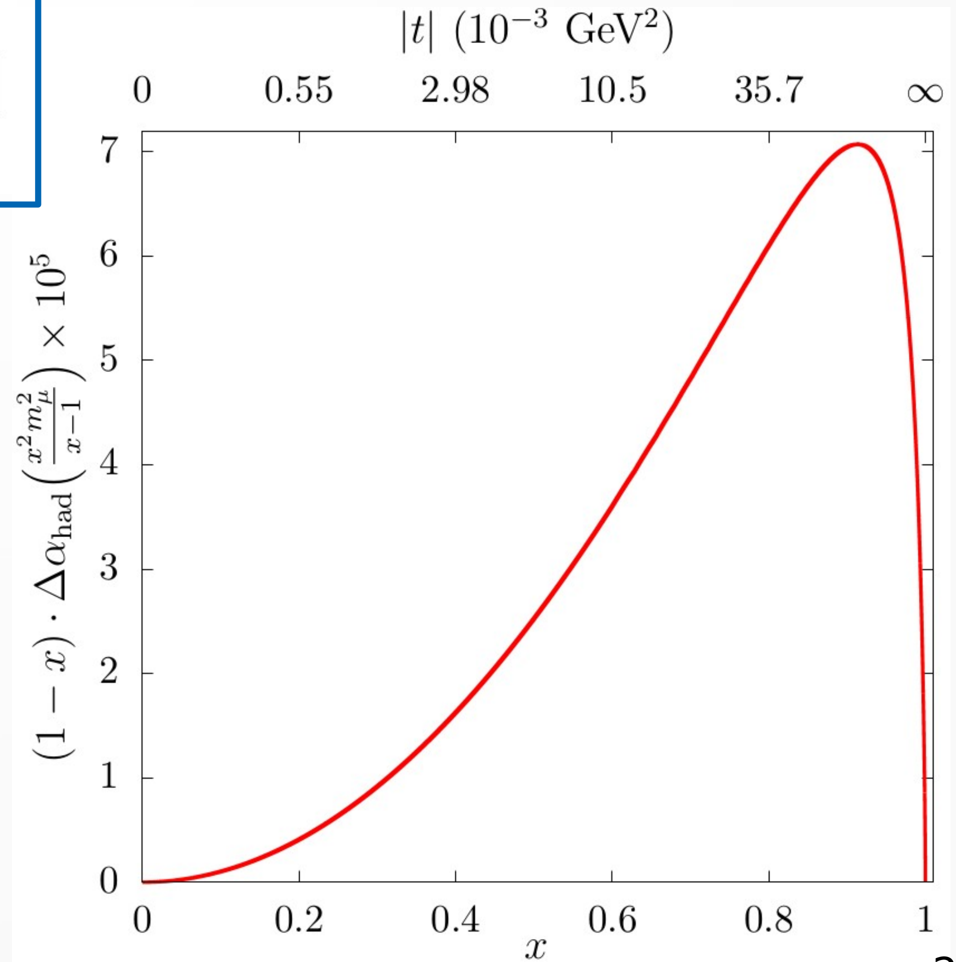
The Evaluation of the Leading Hadronic Contribution to the Muon g-2: Consolidation of the MUonE Experiment and Recent Developments in Low Energy e^+e^- Data
Mainz, 4th June 2024

Space-like integral

$$a_\mu^{HLO} = \frac{\alpha_0}{\pi} \int_0^1 dx(1-x)\Delta\alpha_{had}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$





$\Delta\alpha_{had}$ parameterization

Inspired from the 1 loop QED contribution of lepton pairs and top quark at $t < 0$

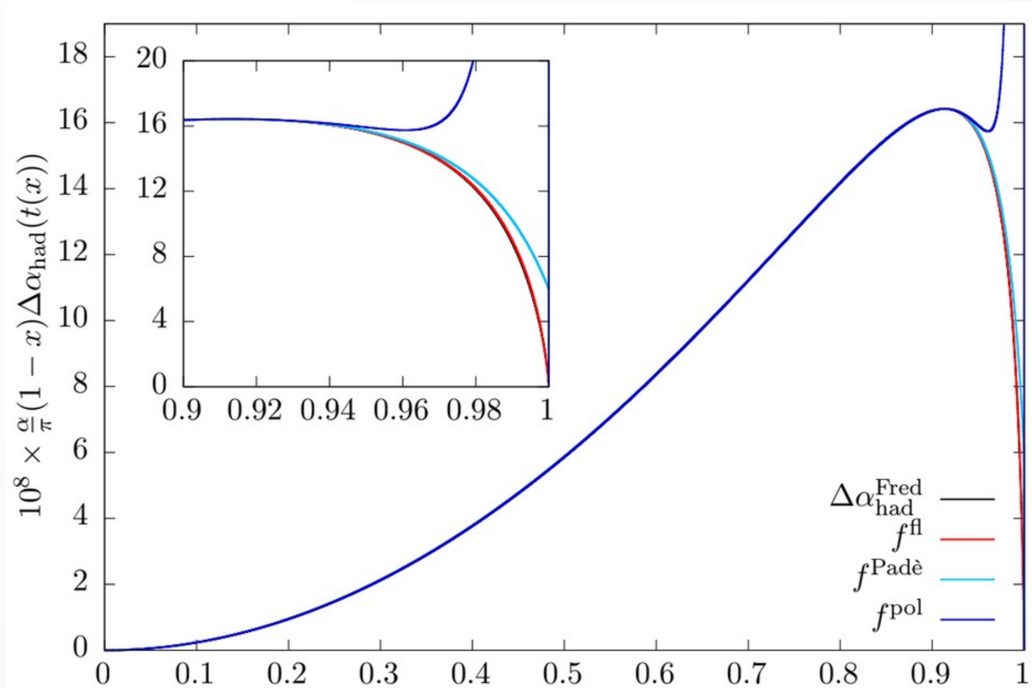
$$\Delta\alpha_{had}(t) = KM\left\{-\frac{5}{9} - \frac{4}{3}\frac{M}{t} + \left(\frac{4}{3}\frac{M^2}{t^2} + \frac{M}{3t} - \frac{1}{6}\right)\frac{2}{\sqrt{1-\frac{4M}{t}}}\ln\left|\frac{1-\sqrt{1-\frac{4M}{t}}}{1+\sqrt{1-\frac{4M}{t}}}\right|\right\}$$

2 parameters:
K, M

Allows to calculate the full value of a_μ^{HLO}

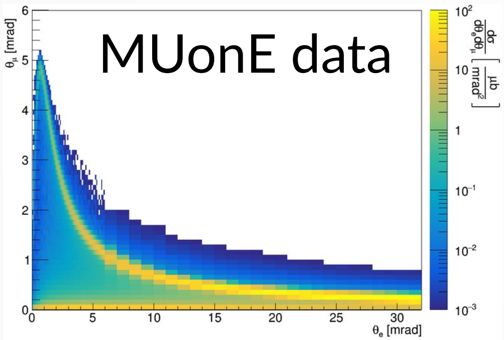
Dominant behaviour in the MUonE kinematic region:

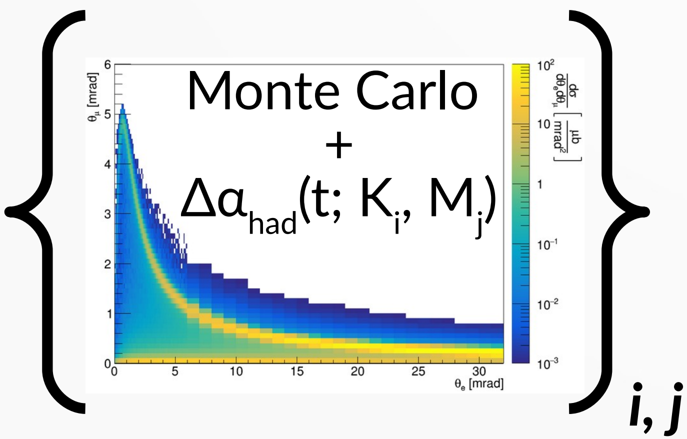
$$\Delta\alpha_{had}(t) \simeq -\frac{1}{15}Kt$$



Extraction of $\Delta\alpha_{had}(t)$

Extraction of $\Delta\alpha_{had}(t)$ through a template fit to the 2D (θ_e, θ_μ) distribution:





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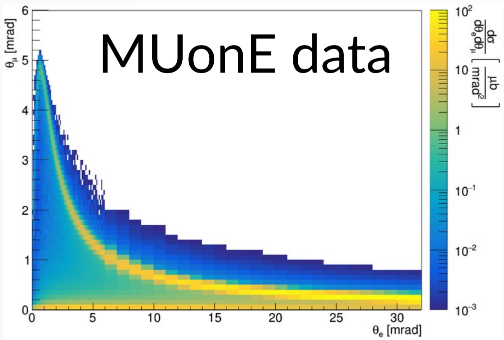
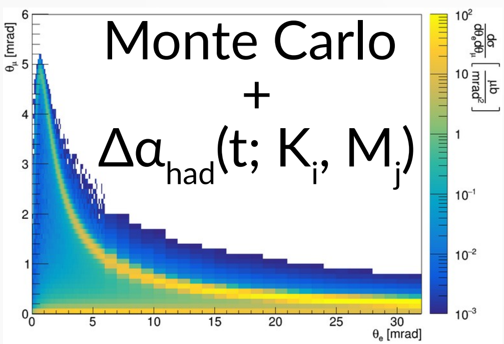
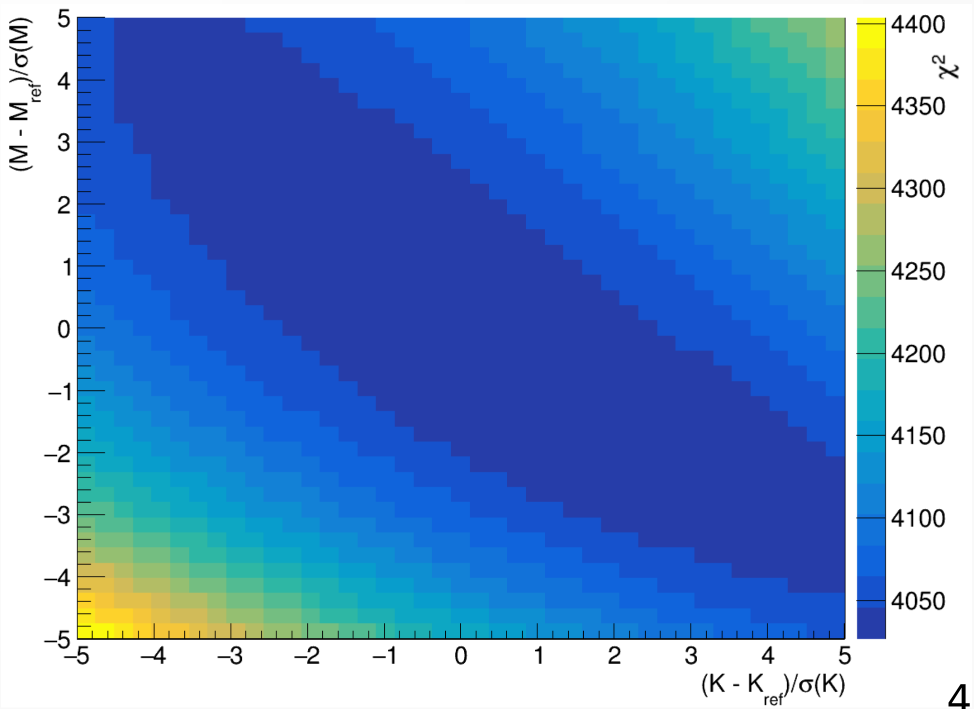




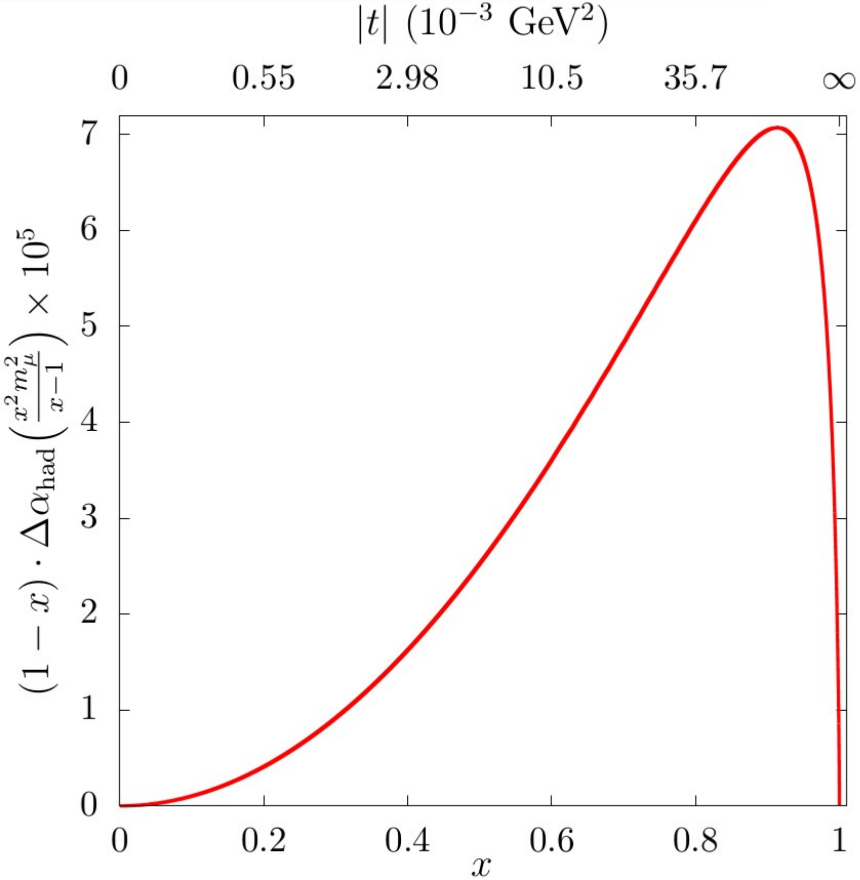
i, j

$$\chi^2 = \sum_i^{\text{bins}} \left(\frac{\text{data}_i - \text{templ}(K,M)_i}{\sigma_i^{\text{data}}} \right)^2$$



Compute a_μ^{HLO}

Input the best fit parameters in the MUonE master integral



(K_{best}, M_{best})

$$a_\mu^{HLO} = \frac{\alpha_0}{\pi}\int_0^1 dx(1-x)\Delta\alpha_{had}[t(x)]$$

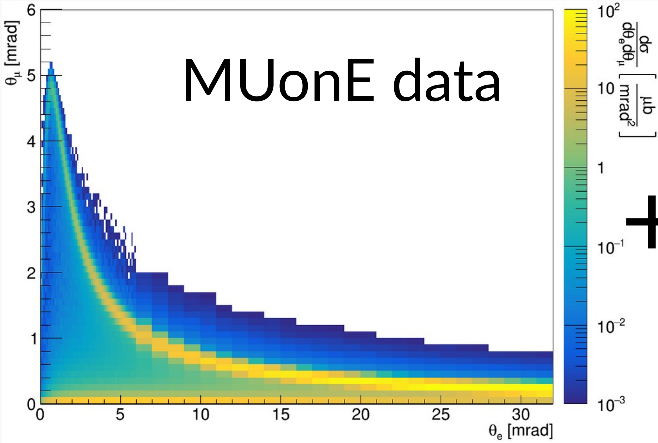
Results from a simulation with the expected final statistics (4×10^{12} elastic events):

$$a_\mu^{\mathrm{HLO}} = (688.8 \pm 2.4) \times 10^{-10}$$

(0.35% accuracy)

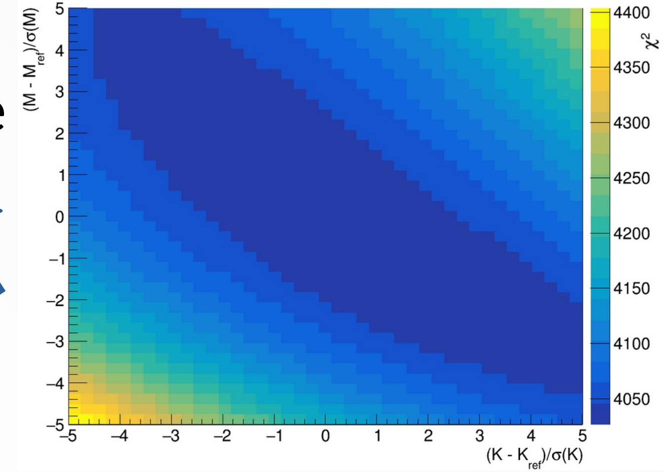
Input value

$$a_\mu^{\mathrm{HLO}} = 688.6 \times 10^{-10}$$



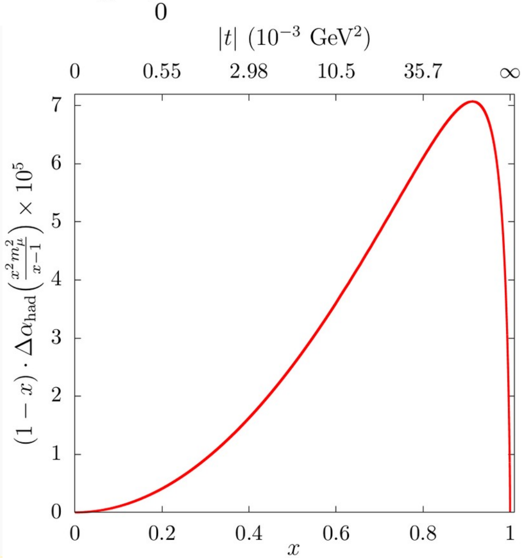
$+ \Delta\alpha_{had}(t; K, M)$

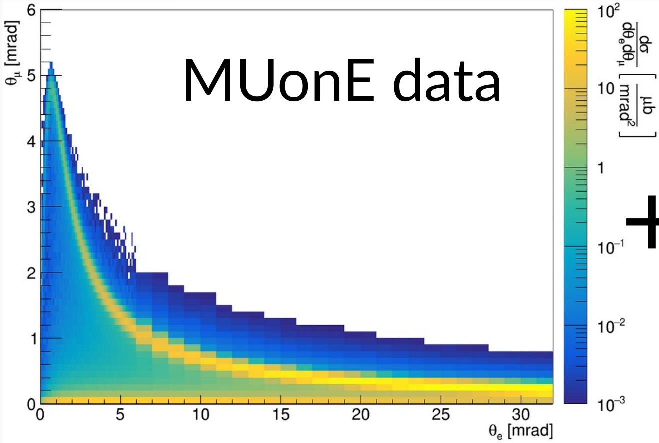
Template fit



$\Delta\alpha_{had}(t; K_{best}, M_{best})$

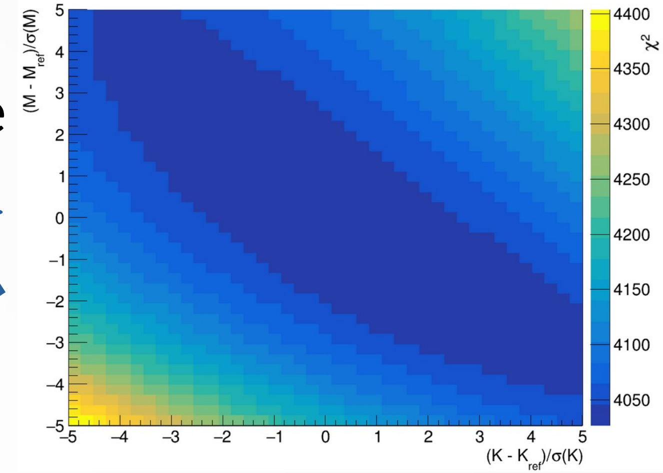
$$a_\mu^{HLO} = \frac{\alpha_0}{\pi} \int_0^1 dx(1-x)\Delta\alpha_{had}[t(x)]$$





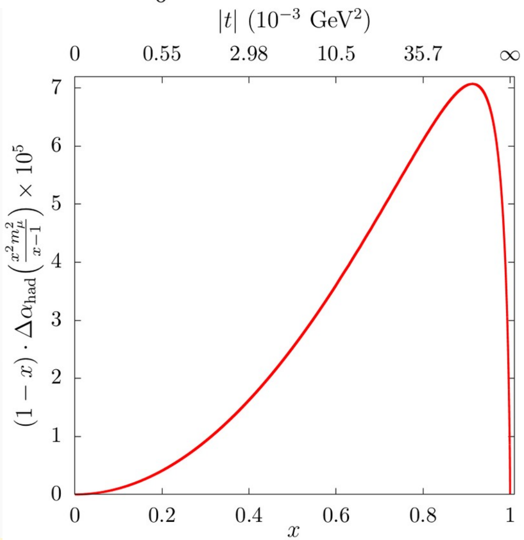
+ $\Delta\alpha_{had}(t; K, M)$

Template fit



$\Delta\alpha_{had}(t; K_{best}, M_{best})$

$$a_\mu^{HLO} = \frac{\alpha_0}{\pi} \int_0^1 dx(1-x)\Delta\alpha_{had}[t(x)]$$



?

Can we compute a_μ^{HLO} in a different way using MUonE data?

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Letter

An alternative evaluation of the leading-order hadronic contribution to the muon $g-2$ with MUonE

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A R T I C L E I N F O

Editor: G.F. Giudice

A B S T R A C T

We propose an alternative method to extract the leading-order hadronic contribution to the muon $g-2$, a_μ^{HLO}, with the MUonE experiment. In contrast to the traditional method based on the integral of the hadronic contribution to the running of the electromagnetic coupling, $\Delta\alpha_{had}$, in the space-like region, our approach relies on the computation of the derivatives of $\Delta\alpha_{had}(t)$ at zero squared momentum transfer t. We show that this approach allows to extract $\sim 99\%$ of the total value of a_μ^{HLO} from the MUonE data, while the remaining $\sim 1\%$ can be computed combining perturbative QCD and data on e^+e^- annihilation to hadrons. This leads to a competitive evaluation of a_μ^{HLO} which is robust against the parameterization used to model $\Delta\alpha_{had}(t)$ in the MUonE kinematic region, thanks to the analyticity properties of $\Delta\alpha_{had}(t)$, which can be expanded as a polynomial at $t \sim 0$.

An alternative method to compute a_μ^{HLO} with MUonE

Based on:

S. Bodenstein et al, Phys. Rev. D 85 (2012)

C.A. Dominguez et al, Phys. Rev. D 96 (2017)

Start from traditional dispersive integral:

$$a_\mu^{\text{HLO}} = \frac{\alpha^2}{3\pi^2} \int_{s_{\text{th}}}^{\infty} \frac{ds}{s} K(s) R(s)$$

$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)s/m_\mu^2}$$

$$R(s) \propto \sigma(e^+e^- \to \text{hadrons})$$

An alternative method to compute $a_\mu^{\rm HLO}$ with MUonE

Based on:
S. Bodenstein et al, Phys. Rev. D 85 (2012)
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Start from traditional dispersive integral:

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$$R(s) \propto \sigma(e^+e^- \to \text{hadrons})$$

$$s_{\rm th} = m_{\pi^0}^2$$

$$s_0 \gtrsim (2\,\text{GeV})^2$$

$$\frac{\alpha^2}{3\pi^2}\int_{s_{\rm th}}^{s_0} \frac{ds}{s} K(s) R(s) \quad + \quad \frac{\alpha^2}{3\pi^2}\int_{s_0}^{\infty} \frac{ds}{s} K(s) R(s)$$

pQCD

$$-\text{Im}\Pi_{had}(s) = \frac{\alpha}{3} R(s)$$

Low energy integral

$$\int_{s_{\rm th}}^{s_0} \frac{ds}{s} K(s) \frac{{\rm Im}\Pi_{had}(s)}{\pi} =$$

$$\int_{s_{\rm th}}^{s_0} \frac{ds}{s} [K(s) - K_1(s)] \frac{{\rm Im}\Pi_{had}(s)}{\pi} + \int_{s_{\rm th}}^{s_0} \frac{ds}{s} K_1(s) \frac{{\rm Im}\Pi_{had}(s)}{\pi}$$

Low energy integral

$$\int_{s_{\rm th}}^{s_0} \frac{ds}{s} K(s) \frac{{\rm Im}\Pi_{had}(s)}{\pi} =$$

$$\int_{s_{\rm th}}^{s_0} \frac{ds}{s} [K(s) - K_1(s)] \frac{{\rm Im}\Pi_{had}(s)}{\pi} + \int_{s_{\rm th}}^{s_0} \frac{ds}{s} K_1(s) \frac{{\rm Im}\Pi_{had}(s)}{\pi}$$

$$K_1(s) = a_0 s + \sum_{n=1}^{3} \frac{a_n}{s^n}$$

$K_1(s)$ approximates $K(s)$ for $s < s_0$.

Meromorphic function:

no cuts, poles in $s = 0$.

Two different techniques to get $K_1(s)$:

1) Least squares minimization

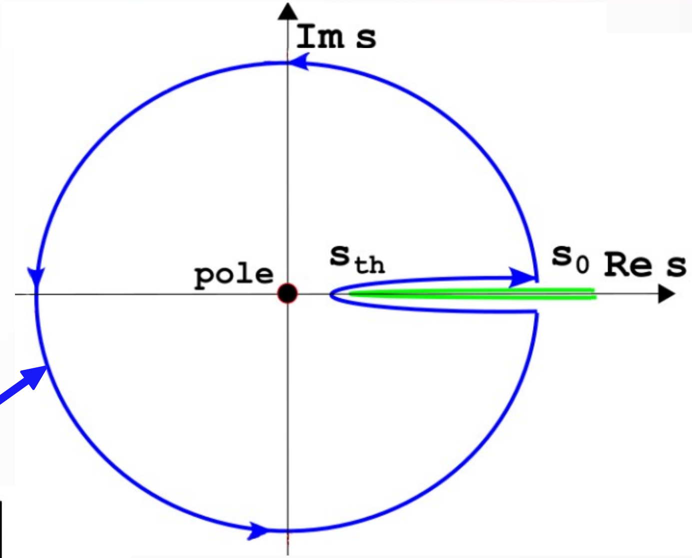
2) Minimize $\int_{s_{\rm th}}^{s_0} \frac{ds}{s} |K(s) - K_1(s)| R(s)$

Low energy integral

Use Cauchy's theorem

$$\int_{s_{\rm th}}^{s_0} \frac{ds}{s} K_1(s) \frac{{\rm Im}\Pi_{had}(s)}{\pi} =$$

$$\text{Res}\left[\Pi_{had}(s)\frac{K_1(s)}{s}\right]_{s=0} - \frac{1}{2\pi i}\oint_{|s|=s_0} \frac{ds}{s} K_1(s)\Pi_{had}(s)\Big|_{\rm pQCD}$$



$$\text{Res}\left[\Pi_{had}(s)\frac{K_1(s)}{s}\right]_{s=0} = \sum_{n=1}^{3} \frac{c_n}{n!}\frac{d^{(n)}}{ds^n}\Pi_{had}(s)\Big|_{s=0} = \sum_{n=1}^{3} \frac{c_n}{n!}\frac{d^{(n)}}{dt^n}\Delta\alpha_{had}(t)\Big|_{t=0}$$

From MUonE

High energy integral

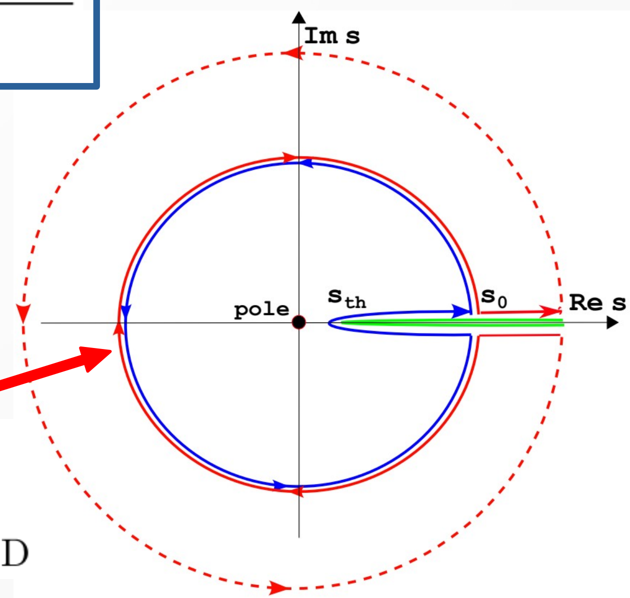
Similar strategy for the high energy part

$$\int_{s_0}^{\infty}\frac{ds}{s}K(s)\frac{\mathrm{Im}\Pi_{had}(s)}{\pi} =$$

$$\int_{s_0}^{\infty}\frac{ds}{s}[K(s)-\tilde{K}_1(s)]\frac{\mathrm{Im}\Pi_{had}(s)}{\pi} + \boxed{\int_{s_0}^{\infty}\frac{ds}{s}\tilde{K}_1(s)\frac{\mathrm{Im}\Pi_{had}(s)}{\pi}}$$

$$\tilde{K}_1(s) = K_1(s) - c_0 s$$

$$\int_{s_0}^{\infty}\frac{ds}{s}\tilde{K}_1(s)\frac{\mathrm{Im}\Pi_{had}(s)}{\pi} = \frac{1}{2\pi i}\oint_{|s|=s_0}\frac{ds}{s}\tilde{K}_1(s)\Pi_{had}(s)\Big|_{\mathrm{pQCD}}$$



Compute a_μ^{HLO}

Rearranging the previous equations...

$$a_\mu^{\mathrm{HLO}} = a_\mu^{\mathrm{HLO\ (I)}} + a_\mu^{\mathrm{HLO\ (II)}} + a_\mu^{\mathrm{HLO\ (III)}} + a_\mu^{\mathrm{HLO\ (IV)}}$$

$$a_\mu^{\mathrm{HLO\ (I)}} = -\frac{\alpha}{\pi}\sum_{n=1}^{3}\frac{c_n}{n!}\frac{d^{(n)}}{dt^n}\Delta\alpha_{had}(t)\bigg|_{t=0}$$

$$a_\mu^{\mathrm{HLO\ (II)}} = \frac{\alpha}{\pi}\frac{1}{2\pi i}\oint_{|s|=s_0}\frac{ds}{s}c_0 s\ \Pi_{had}(s)\bigg|_{\mathrm{pQCD}}$$

$$a_\mu^{\mathrm{HLO\ (III)}} = \frac{\alpha^2}{3\pi^2}\int_{s_{\mathrm{th}}}^{s_0}\frac{ds}{s}[K(s) - K_1(s)]R(s)$$

$$a_\mu^{\mathrm{HLO\ (IV)}} = \frac{\alpha^2}{3\pi^2}\int_{s_0}^{\infty}\frac{ds}{s}[K(s) - \tilde{K}_1(s)]R(s)$$

Compute a_μ^{HLO}

Rearranging the previous equations...

$$a_\mu^{\mathrm{HLO}} = a_\mu^{\mathrm{HLO\ (I)}} + a_\mu^{\mathrm{HLO\ (II)}} + a_\mu^{\mathrm{HLO\ (III)}} + a_\mu^{\mathrm{HLO\ (IV)}}$$

99%

$$a_\mu^{\mathrm{HLO\ (I)}} = -\frac{\alpha}{\pi} \sum_{n=1}^{3} \frac{c_n}{n!} \frac{d^{(n)}}{dt^n} \Delta\alpha_{had}(t)\Big|_{t=0}$$

MUonE

$$a_\mu^{\mathrm{HLO\ (II)}} = \frac{\alpha}{\pi} \frac{1}{2\pi i} \oint_{|s|=s_0} \frac{ds}{s} c_0 s\ \Pi_{had}(s)\Big|_{\mathrm{pQCD}}$$

$$a_\mu^{\mathrm{HLO\ (III)}} = \frac{\alpha^2}{3\pi^2} \int_{s_{\mathrm{th}}}^{s_0} \frac{ds}{s} [K(s) - K_1(s)] R(s)$$

$$a_\mu^{\mathrm{HLO\ (IV)}} = \frac{\alpha^2}{3\pi^2} \int_{s_0}^{\infty} \frac{ds}{s} [K(s) - \tilde{K}_1(s)] R(s)$$

Compute a_μ^{HLO}

Rearranging the previous equations…

$$a_\mu^{\mathrm{HLO}} = a_\mu^{\mathrm{HLO\ (I)}} + a_\mu^{\mathrm{HLO\ (II)}} + a_\mu^{\mathrm{HLO\ (III)}} + a_\mu^{\mathrm{HLO\ (IV)}}$$

99%

$$a_\mu^{\mathrm{HLO\ (I)}} = -\frac{\alpha}{\pi}\sum_{n=1}^{3}\frac{c_n}{n!}\frac{d^{(n)}}{dt^n}\Delta\alpha_{had}(t)\Big|_{t=0}$$

MUonE

1%

$$a_\mu^{\mathrm{HLO\ (II)}} = \frac{\alpha}{\pi}\frac{1}{2\pi i}\oint_{|s|=s_0}\frac{ds}{s}c_0 s\ \Pi_{had}(s)\Big|_{\mathrm{pQCD}}$$

$$a_\mu^{\mathrm{HLO\ (III)}} = \frac{\alpha^2}{3\pi^2}\int_{s_{\mathrm{th}}}^{s_0}\frac{ds}{s}[K(s)-K_1(s)]R(s)$$

$$a_\mu^{\mathrm{HLO\ (IV)}} = \frac{\alpha^2}{3\pi^2}\int_{s_0}^{\infty}\frac{ds}{s}[K(s)-\tilde{K}_1(s)]R(s)$$

Time-like data + pQCD

$a_\mu^{\text{HLO (I)}}$ from MUonE data

$$a_\mu^{\text{HLO (I)}} = -\frac{\alpha}{\pi}\sum_{n=1}^{3}\frac{c_n}{n!}\frac{d^{(n)}}{dt^n}\Delta\alpha_{had}(t)\Big|_{t=0}$$

The relevant quantities are the derivatives of $\Delta\alpha_{had}(t)$ at $t = 0$.

$a_\mu^{\mathrm{HLO\ (I)}}$ from MUonE data

$$a_\mu^{\mathrm{HLO\ (I)}} = -\frac{\alpha}{\pi}\sum_{n=1}^{3}\frac{c_n}{n!}\frac{d^{(n)}}{dt^n}\Delta\alpha_{had}(t)\Big|_{t=0}$$

The relevant quantities are the derivatives of $\Delta\alpha_{had}(t)$ at t = 0.

Try different parameterizations to fit MUonE data

(max 3 fit parameters, due to the statistics collected by MUonE)

$$\Delta\alpha_{had}(t) = KM\left\{-\frac{5}{9}-\frac{4}{3}\frac{M}{t}+\left(\frac{4}{3}\frac{M^2}{t^2}+\frac{M}{3t}-\frac{1}{6}\right)\frac{2}{\sqrt{1-\frac{4M}{t}}}\ln\left|\frac{1-\sqrt{1-\frac{4M}{t}}}{1+\sqrt{1-\frac{4M}{t}}}\right|\right\}$$

Lepton-like

$$\Delta\alpha_{\mathrm{had}}(t) = P_1 t\frac{1+P_2 t}{1+P_3 t}$$

Padé approxiamant

$$\Delta\alpha_{\mathrm{had}}(t) = P_1 t + P_2 t^2 + P_3 t^3$$

3° polynomial

a_μ^{HLO} (I) from MUonE data

Reconstruction approximants

D. Greynat, E. de Rafael, JHEP 2022 (5)

$$\Delta\alpha_{\mathrm{had}}(t) = \sum_{n=1}^{\mathrm{N}} \mathscr{A}(n,\mathrm{L}) \left(\frac{\sqrt{1-\frac{t}{t_0}}-1}{\sqrt{1-\frac{t}{t_0}}+1} \right)^n + \sum_{p=1}^{\lfloor \frac{\mathrm{L}+1}{2} \rfloor} \mathscr{B}(2p-1)\, \mathrm{Li}_{2p-1} \left(\frac{\sqrt{1-\frac{t}{t_0}}-1}{\sqrt{1-\frac{t}{t_0}}+1} \right)$$

$$\Delta\alpha_{\mathrm{had}}(t) = A_1 \mathscr{S}_1 + A_2 \mathscr{S}_2 + A_3 \mathscr{S}_3 + B_1 \mathscr{L}_1$$

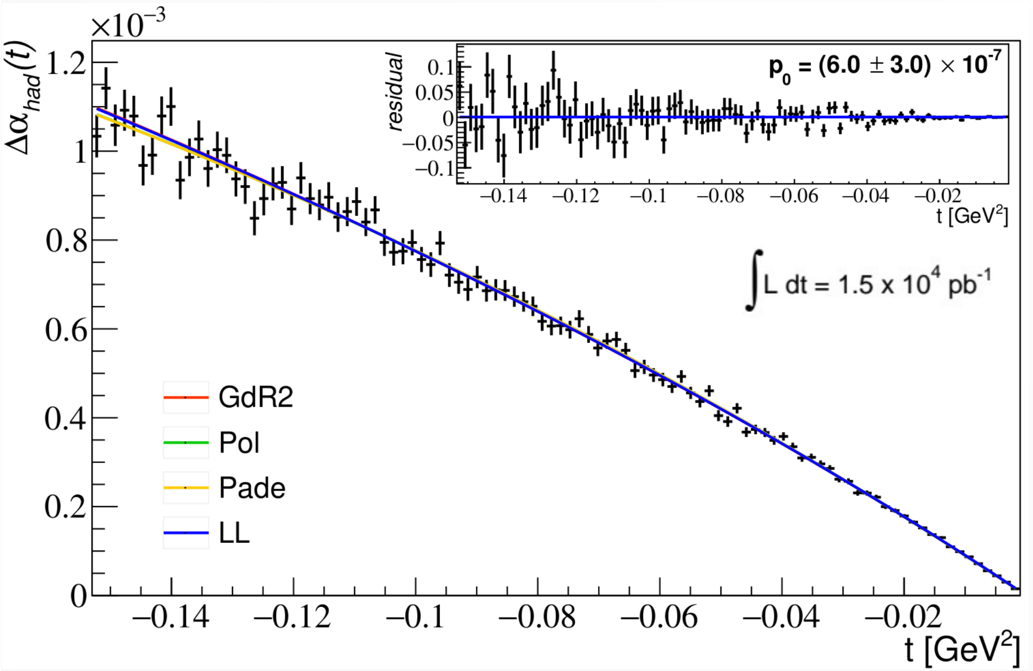
$$\mathscr{S}_i = \left(\frac{\sqrt{1-\frac{t}{t_0}}-1}{\sqrt{1-\frac{t}{t_0}}+1} \right) ; \qquad A_i = \mathscr{A}(i,1) \quad i = 1,\ 2,\ 3$$

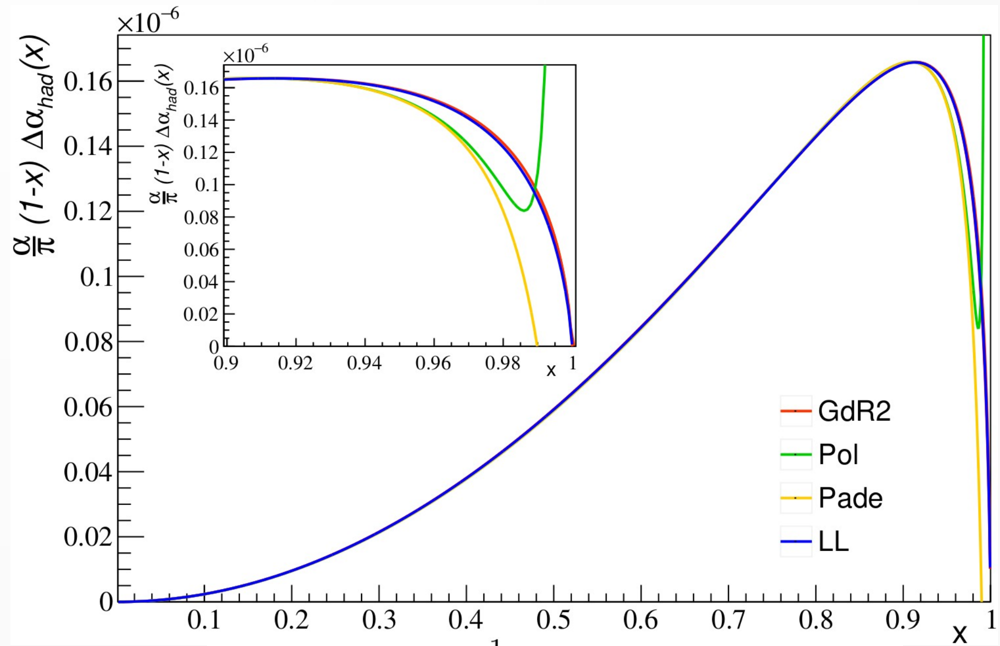
$$\mathscr{L}_1 = \mathrm{Li}_1 \left(\frac{\sqrt{1-\frac{t}{t_0}}-1}{\sqrt{1-\frac{t}{t_0}}+1} \right) ; \qquad B_1 = \mathscr{B}(1)$$

Tested L = 1, N = 3
Several variants with different number of free parameters

Fit the MUonE data

Simplified fit: simulate the MUonE signal using time-like compilations of $\Delta\alpha_{had}$. Error bars according to the MUonE final statistics.





$$a_\mu^{HLO} = \frac{\alpha_0}{\pi} \int_0^1 dx(1-x)\Delta\alpha_{had}[t(x)]$$

Results: $a_\mu^{\text{HLO (I)}}$

$$a_\mu^{\text{HLO (I)}} = -\frac{\alpha}{\pi}\sum_{n=1}^{3}\frac{c_n}{n!}\frac{d^{(n)}}{dt^n}\Delta\alpha_{had}(t)\Big|_{t=0}$$

Minimization I	$a_\mu^{\text{HLO (I)}}$ (10^{-10})							
s_0 values	LL	Padé	Pol	GdR1	GdR2	GdR3	GdR4	GdR5
$(1.8\ \text{GeV})^2$	688.7±2.2	688.7±2.9	688.9±2.9	688.2±2.2	688.0±2.2	688.0±2.2	687.0±2.3	688.0±2.6
$(2.5\ \text{GeV})^2$	691.7±2.2	691.6±3.0	691.8±3.0	691.0±2.2	690.8±2.2	690.8±2.2	689.8±2.3	690.9±2.9
$(12\ \text{GeV})^2$	696.3±2.2	696.3±3.0	696.3±3.2	695.4±2.2	695.3±2.2	695.2±2.2	694.1±2.3	695.3±3.7
Minimization II	$a_\mu^{\text{HLO (I)}}$ (10^{-10})							
s_0 values	LL	Padé	Pol	GdR1	GdR2	GdR3	GdR4	GdR5
$(1.8\ \text{GeV})^2$	688.5±2.2	688.1±4.2	689.8±3.3	688.3±2.1	688.4±2.1	688.6±2.2	687.1±2.1	688.4±5.8
$(2.5\ \text{GeV})^2$	689.5±2.2	689.1±4.2	690.8±3.3	689.3±2.1	689.4±2.1	689.6±2.2	688.1±2.1	689.4±5.7
$(12\ \text{GeV})^2$	690.3±2.1	689.9±4.6	691.6±3.6	689.8±2.1	690.1±2.2	690.2±2.2	688.6±2.1	690.0±5.9

$a_\mu^{\text{HLO (I)}}$ ~ 99% of the total value.

($a_\mu^{\text{HLO}} = 695.1\times10^{-10}$ input from time-like data).

Results: $a_\mu^{\mathrm{HLO\ (II,\ III,\ IV)}}$

$$a_\mu^{\mathrm{HLO\ (II)}} = \frac{\alpha}{\pi}\frac{1}{2\pi i}\oint_{|s|=s_0}\frac{ds}{s}c_0 s\ \Pi_{had}(s)\Big|_{\mathrm{pQCD}}$$

$$a_\mu^{\mathrm{HLO\ (III)}} = \frac{\alpha^2}{3\pi^2}\int_{s_{\mathrm{th}}}^{s_0}\frac{ds}{s}[K(s)-K_1(s)]R(s)$$

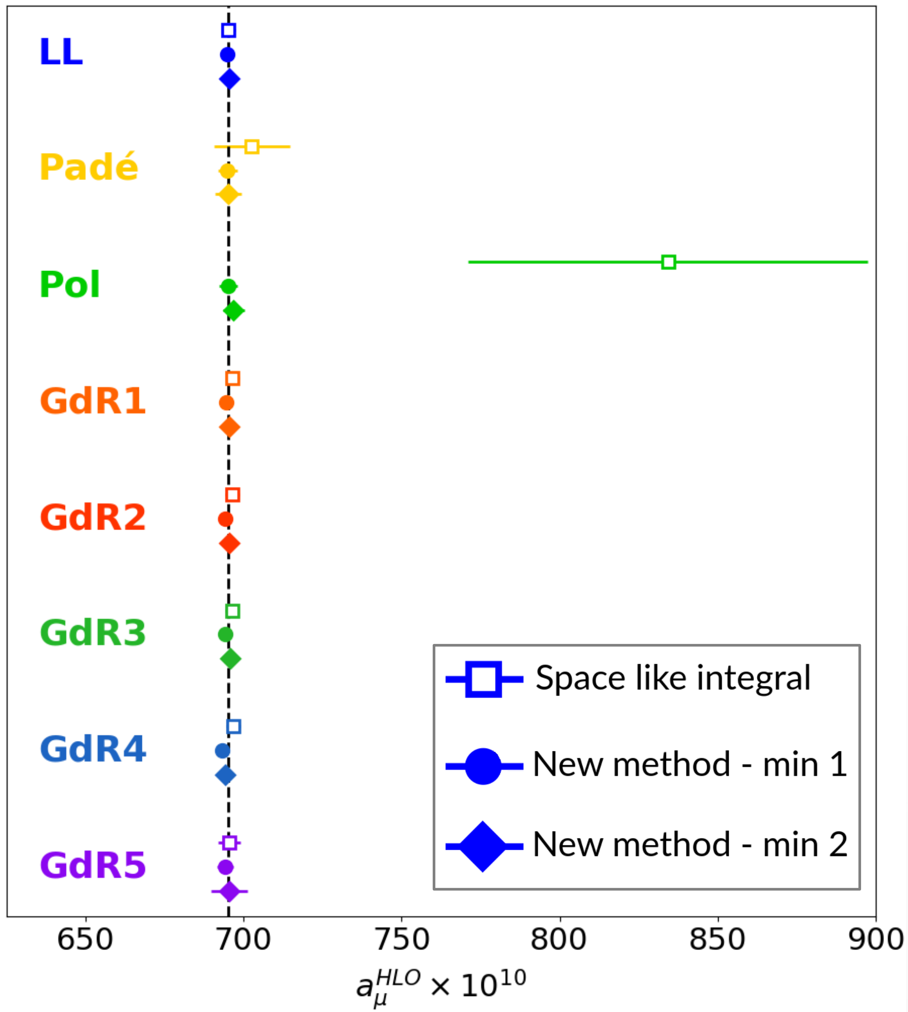
$$a_\mu^{\mathrm{HLO\ (IV)}} = \frac{\alpha^2}{3\pi^2}\int_{s_0}^{\infty}\frac{ds}{s}[K(s)-\tilde{K}_1(s)]R(s)$$

Minimization I			
s_0 values	$a_\mu^{\mathrm{HLO\ (II)}}$ (10^{-10})	$a_\mu^{\mathrm{HLO\ (III)}}$ (10^{-10})	$a_\mu^{\mathrm{HLO\ (IV)}}$ (10^{-10})
$(1.8\ \mathrm{GeV})^2$	2.94±0.04	0.43±0.01	2.95±0.05
$(2.5\ \mathrm{GeV})^2$	1.84±0.01	-0.34±0.01	1.79±0.02
$(12\ \mathrm{GeV})^2$	0.208±0.001	-1.695±0.035	0.079±0.001
Minimization II			
s_0 values	$a_\mu^{\mathrm{HLO\ (II)}}$ (10^{-10})	$a_\mu^{\mathrm{HLO\ (III)}}$ (10^{-10})	$a_\mu^{\mathrm{HLO\ (IV)}}$ (10^{-10})
$(1.8\ \mathrm{GeV})^2$	3.23±0.04	0.91±0.02	3.00±0.05
$(2.5\ \mathrm{GeV})^2$	2.54±0.01	1.52±0.02	1.96±0.02
$(12\ \mathrm{GeV})^2$	0.360±0.001	4.85±0.05	0.096±0.001

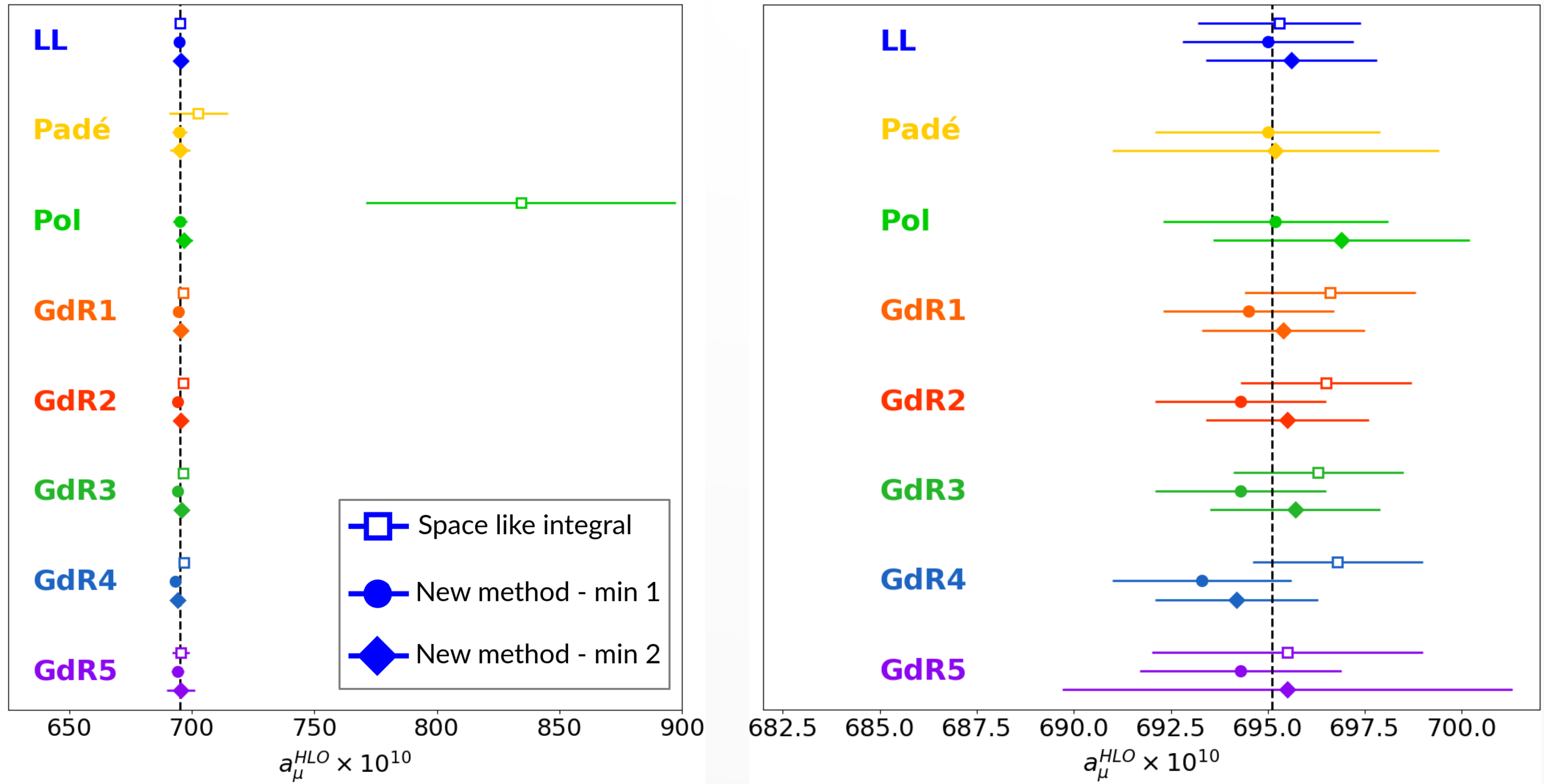
$a_\mu^{\mathrm{HLO\ (II+III+IV)}}$ ~ 1% of the total value.

($a_\mu^{\mathrm{HLO}} = 695.1\times10^{-10}$ input from time-like data).

Results: a_μ^{HLO}



Results: a_μ^{HLO}



Conclusions

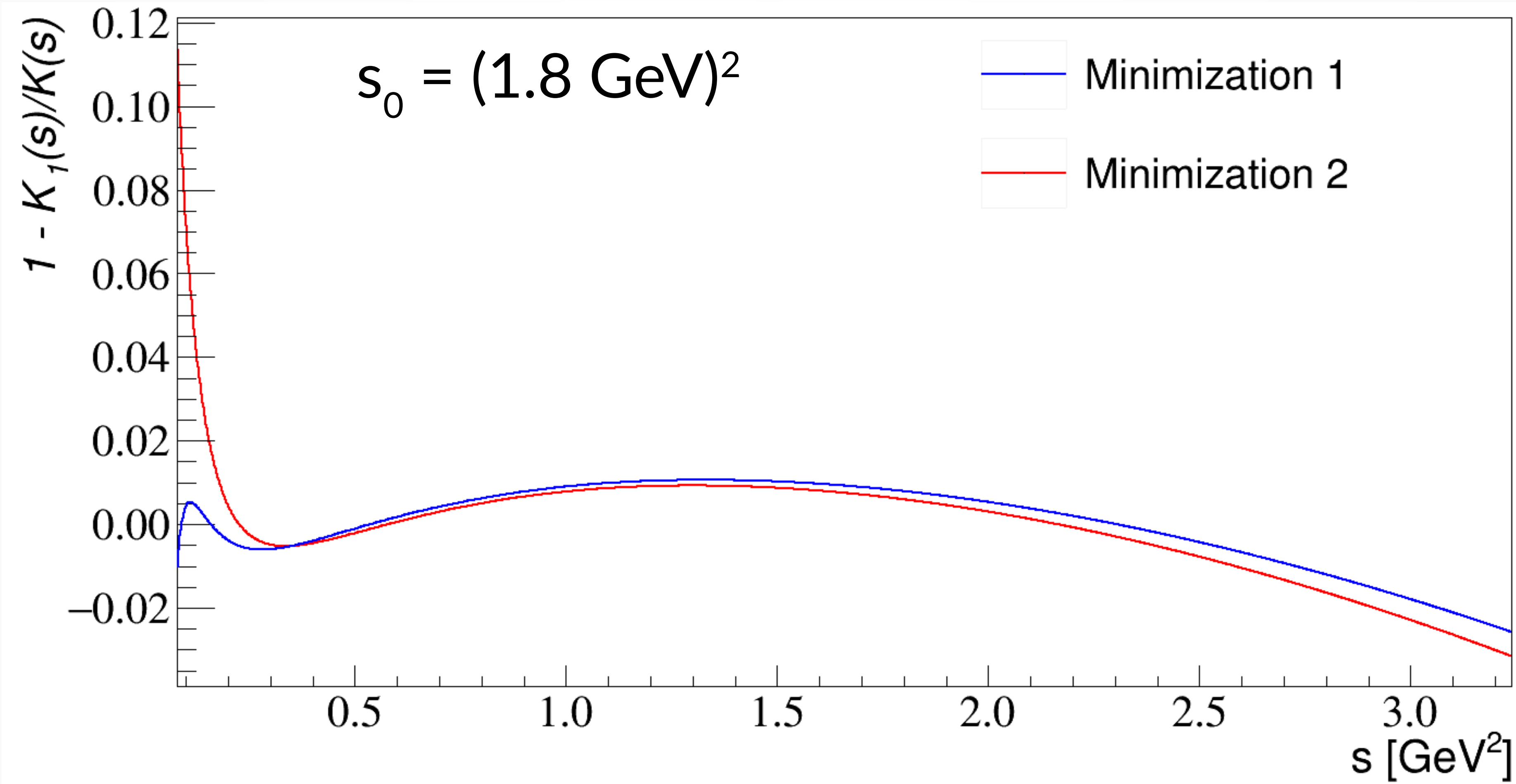
- Alternative method to calculate a_μ^{HLO} with MUonE data:
  less sensitive to the parameterization chosen to model $\Delta\alpha_{\mathrm{had}}(\mathrm{t})$
  in the MUonE kinematic range.
  Comparable uncertainty to the space-like integral method.

- No need to change the template fit workflow:
  ”simply” generate template distributions
  using different parameterizations.

- From the analysis point of view, fit functions with ≤ 3 free parameters (possibly with low correlation) should be preferred.

BACKUP

Difference $K_1(s)$ - $K(s)$



Results: $a_\mu^{\mathrm{HLO\ (II,\ III,\ IV)}}$

$$a_\mu^{\mathrm{HLO\ (II)}} = \frac{\alpha}{\pi}\frac{1}{2\pi i}\oint_{|s|=s_0}\frac{ds}{s}c_0 s\ \Pi_{had}(s)\Big|_{\mathrm{pQCD}}$$

$$a_\mu^{\mathrm{HLO\ (III)}} = \frac{\alpha^2}{3\pi^2}\int_{s_{\mathrm{th}}}^{s_0}\frac{ds}{s}[K(s)-K_1(s)]R(s)$$

$$a_\mu^{\mathrm{HLO\ (IV)}} = \frac{\alpha^2}{3\pi^2}\int_{s_0}^{\infty}\frac{ds}{s}[K(s)-\tilde{K}_1(s)]R(s)$$

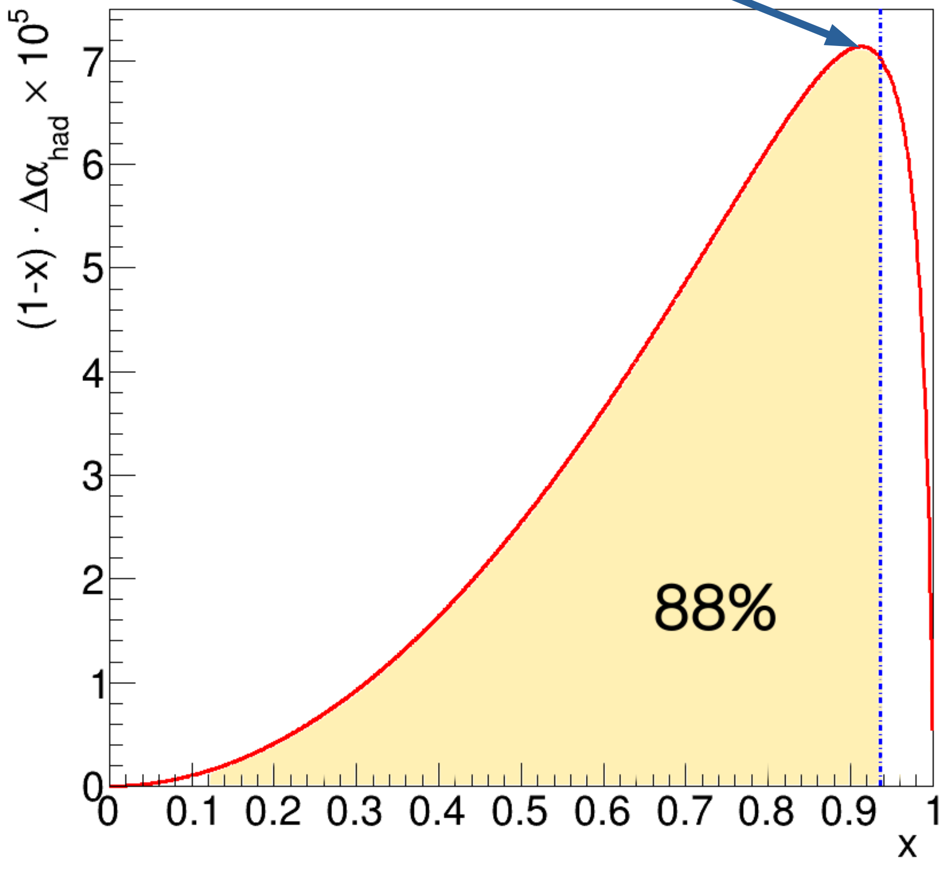
Minimization I			
s_0 values	$a_\mu^{\mathrm{HLO\ (II)}}$ (10^{-10})	$a_\mu^{\mathrm{HLO\ (III)}}$ (10^{-10})	$a_\mu^{\mathrm{HLO\ (IV)}}$ (10^{-10})
$(1.8\ \mathrm{GeV})^2$	2.94±0.04	0.43±0.01	2.95±0.05
$(2.5\ \mathrm{GeV})^2$	1.84±0.01	-0.34±0.01	1.79±0.02
$(12\ \mathrm{GeV})^2$	0.208±0.001	-1.695±0.035	0.079±0.001
Minimization II			
s_0 values	$a_\mu^{\mathrm{HLO\ (II)}}$ (10^{-10})	$a_\mu^{\mathrm{HLO\ (III)}}$ (10^{-10})	$a_\mu^{\mathrm{HLO\ (IV)}}$ (10^{-10})
$(1.8\ \mathrm{GeV})^2$	3.23±0.04	0.91±0.02	3.00±0.05
$(2.5\ \mathrm{GeV})^2$	2.54±0.01	1.52±0.02	1.96±0.02
$(12\ \mathrm{GeV})^2$	0.360±0.001	4.85±0.05	0.096±0.001

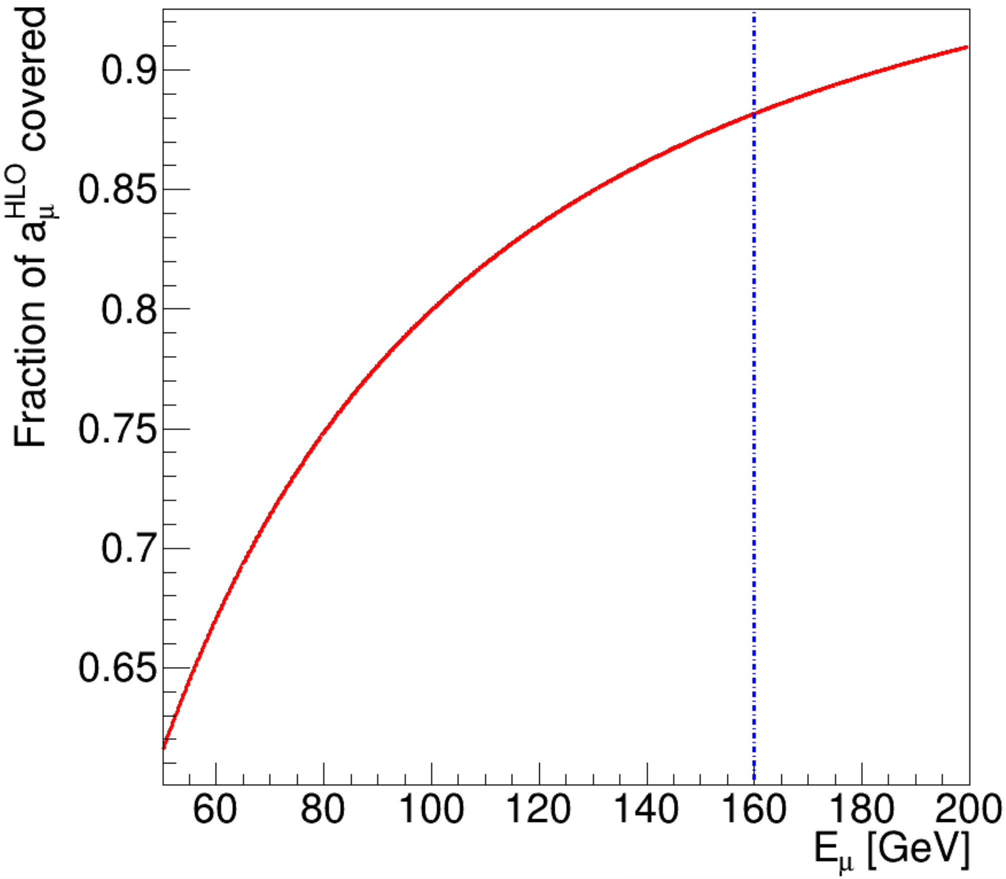
$a_\mu^{\mathrm{HLO\ (II+III+IV)}}$ ~ 1% of the total value.

($a_\mu^{\mathrm{HLO}} = 695.1\times10^{-10}$ input from time-like data).

$x < 0.936$

$t_{peak} \sim -0.108\ \mathrm{GeV}^2$ $x_{peak} \sim 0.92$



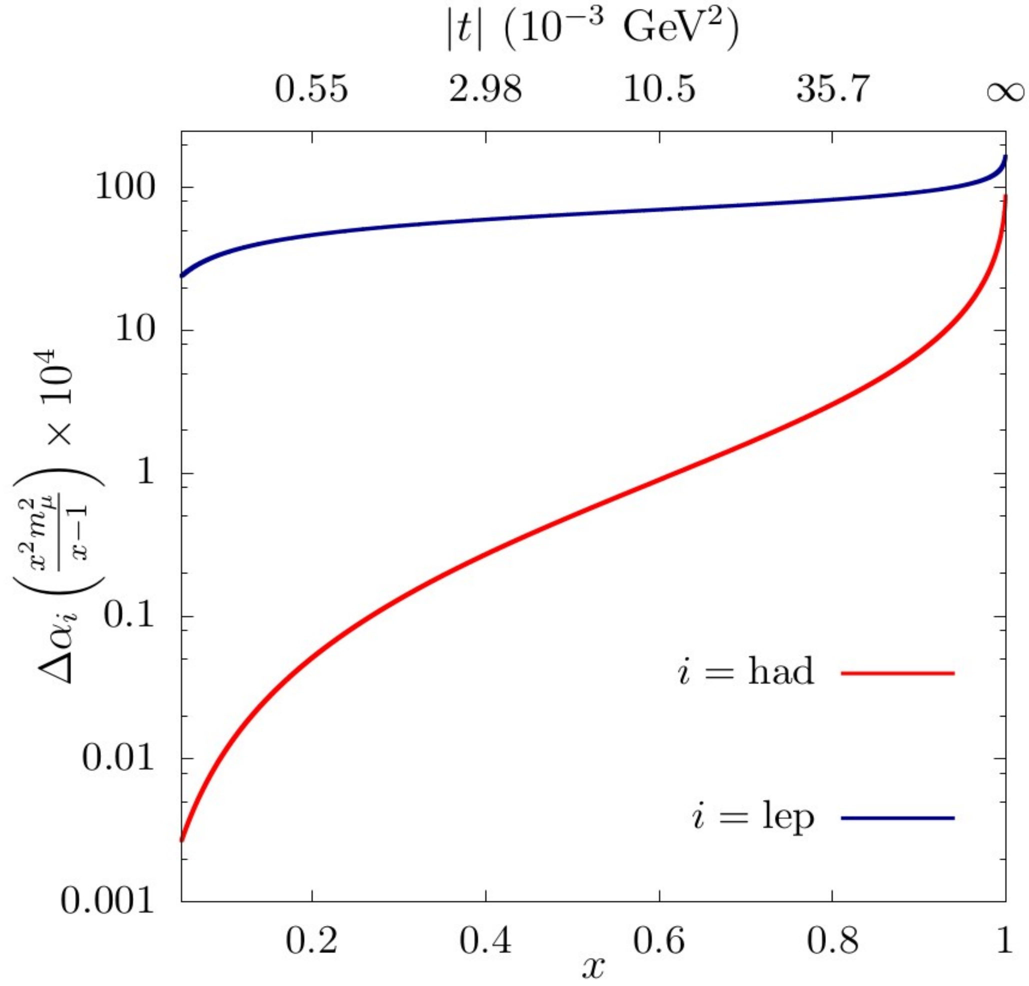


- 160 GeV muon beam on atomic electrons.

$$\sqrt{s} \sim 420\,\mathrm{MeV}$$

$$-0.153\,\mathrm{GeV}^2 < t < 0\,\mathrm{GeV}^2$$

$$\Delta\alpha_{had}(t) \lesssim 10^{-3}$$



$$R_{\text{had}} = \frac{d\sigma_{\text{data}}(\Delta\alpha_{\text{had}})}{d\sigma_{\text{MC}}(\Delta\alpha_{\text{had}} = 0)} \sim 1 + 2\Delta\alpha_{\text{had}}(t)$$

From theoretical calculation

To be measured

$\Delta\alpha_{\text{had}}(t) < 10^{-3}$