PAUL SCHERRER INSTITUT PSI

The Evaluation of the Leading Hadronic Contribution to a_μ

Consolidation of MUonE Experiment and Recent Developments in Low Energy e^+e^- Data

McMule for MUonE

Marco Rocco for McMule

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Mainz, 4th June 2024

formalities

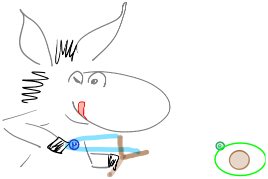
- higher-order predictions and comparison with precision experiments
- focus on $2 \to 2$ low-energy QED+ scattering processes
- **input**: matrix elements by us or others (at NNLO + first visits at N3LO)
- **output**: physical cross section for any physical observable at fixed order
- at present an integrator, generator features under testing

McMule

Monte Carlo for MUons and other LEptons

code → `https://mule-tools.gitlab.io/`

docs → `https://mcmule.readthedocs.io/`



phenomenology

basic setup

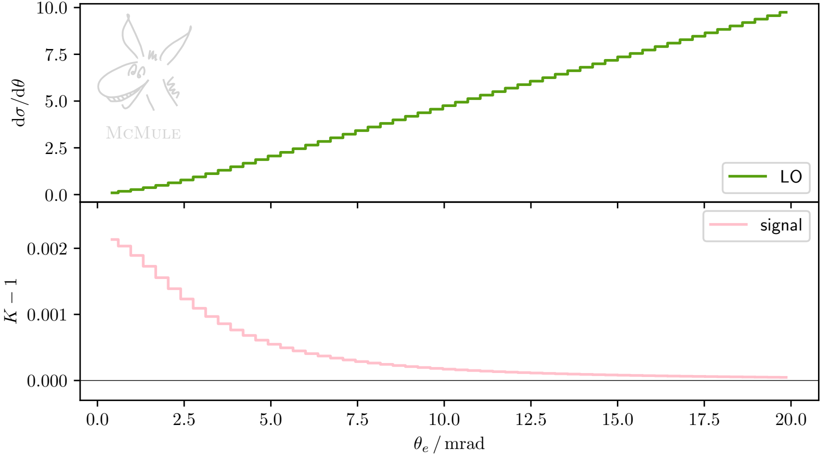
$$\mu e \to \mu e \quad @\ \mathrm{NNLO}$$

- kinematical setup mimics MUonE:

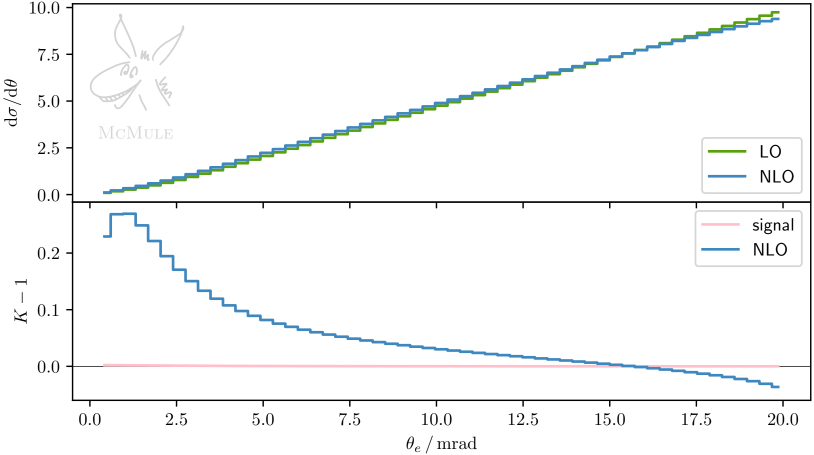
$$\texttt{S1} \quad :: \quad E_{\mu,i} = 160\,\mathrm{GeV} \qquad E_{e,f} > 1\,\mathrm{GeV} \qquad \theta_{\mu,f} > 0.3\,\mathrm{mrad}$$

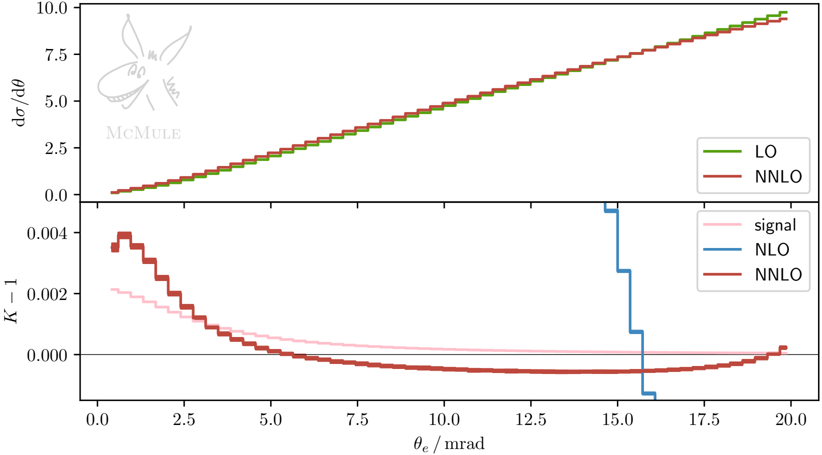
- results for different kinematical scenarios and any IR safe observable
- no mass is neglected

S1 θ_e spectrum



S1 θ_e spectrum

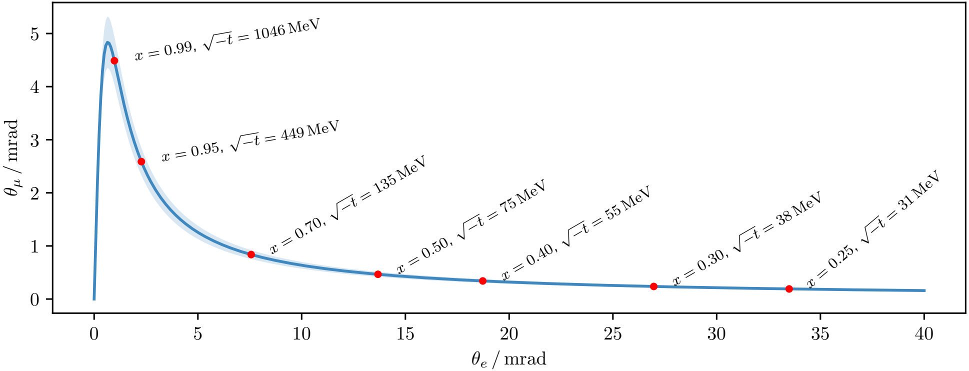




10.0
7.5
5.0
2.5
0.0
dσ/dθ
McMule
LO
NNLO
0.004
0.002
0.000
K − 1
signal
NLO
NNLO
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
θe / mrad

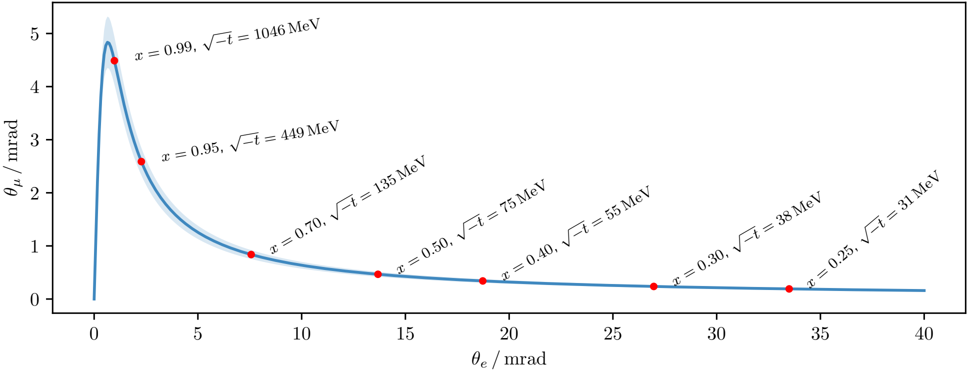
how to handle hard radiation?

- elasticity veto! $\to 0.9 < \frac{\theta_{\mu,f}}{\theta^{\text{el}}_{\mu,f}} < 1.1$ (S2)
-

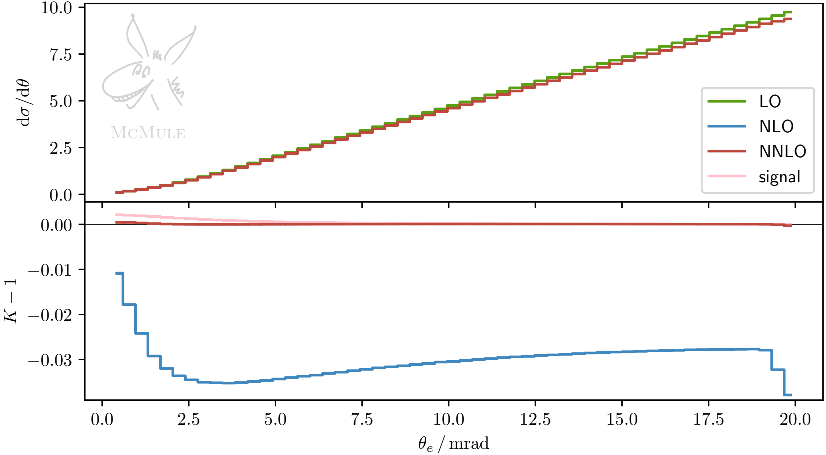


how to handle hard radiation?

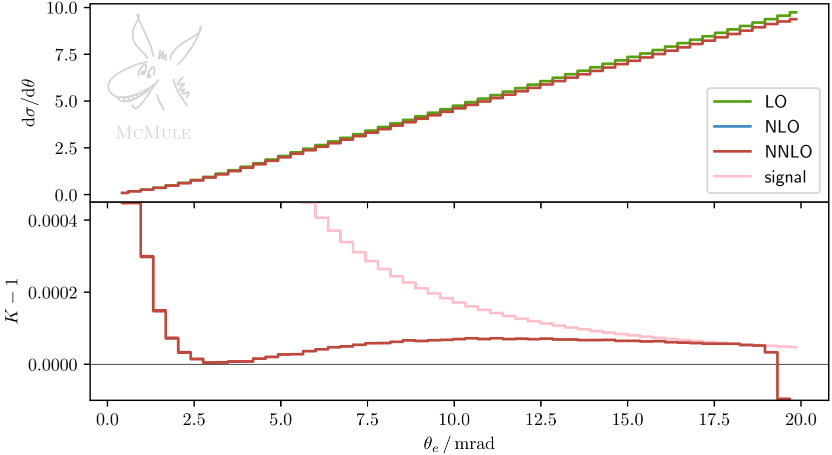
- elasticity veto! $\rightarrow 0.9 < \frac{\theta_{\mu,f}}{\theta^{\mathrm{el}}_{\mu,f}} < 1.1$ (S2)
- something else?



S2 θ_e spectrum

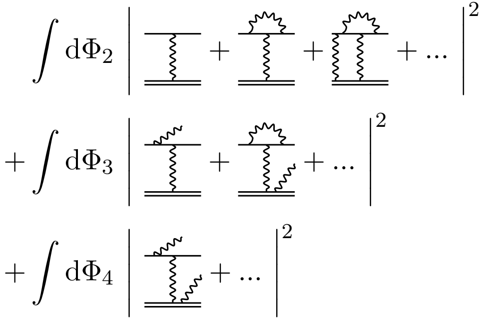


S2 θ_e spectrum

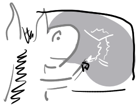


a peek in the ~~stable~~ theory

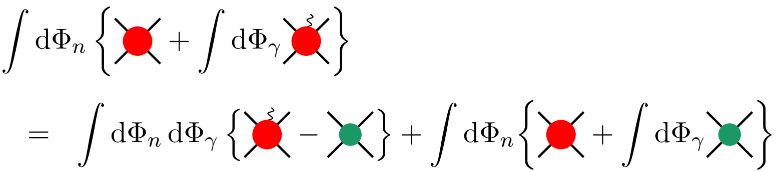
McMule at NNLO :: 3 pillars



1. fully-differential PS integration
   $\rightarrow$ FKS$^\ell$
2. virtual amplitudes with massive particles
   $\rightarrow$ one-loop: OpenLoops
   $\rightarrow$ two-loop: massification
3. numerical instabilities due to pseudo-singularities
   $\rightarrow$ next-to-soft stabilisation



local subtraction of infrared divergences



- ◇ exploits exponentiation of soft singularities [YFS 61]
- ◇ works at all orders in QED [Engel, Signer, Ulrich 19]

- ◇ singularities are dealt with locally → stable numerical integration
- ◇ subtraction makes negative-weighted events much more frequent
- ◇ theory error: 0

- *photonic* and *fermionic* ($\rightarrow$ hyperspherically [Fael 18]) corrections
- photonic at NNLO are split as

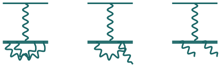
$$\mathrm{d}\sigma^{(2)} = \int \mathrm{d}\Phi_n \, \mathcal{M}_n^{(2)} + \int \mathrm{d}\Phi_{n+1} \, \mathcal{M}_{n+1}^{(1)} + \int \mathrm{d}\Phi_{n+2} \, \mathcal{M}_{n+2}^{(0)}$$

- for each part identify gauge-invariant subsets based on lepton charges (q for electron, Q for muon)

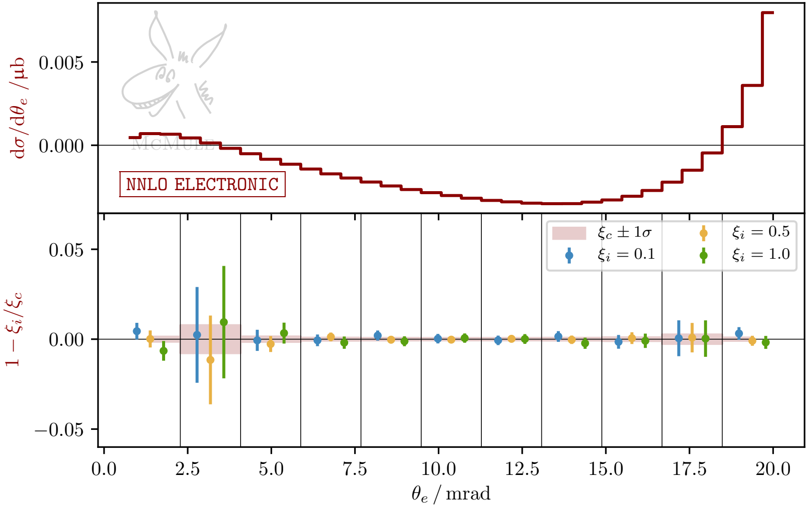
  ◇ $q^6 \, Q^2$ :: *electronic*

  ◇ $\{ q^5 \, Q^3, \, q^4 \, Q^4, \, q^3 \, Q^5 \}$ :: *mixed*

  ◇ $q^2 \, Q^6$ :: *muonic*



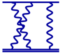
validation: ξ_c (in)dependence for electronic corrections



full 2-loop amplitude with $M \neq 0$, $m = 0 \rightarrow$ [Bonciani et al. 21]

full 2-loop amplitude with $M \neq 0$, $m \neq 0 \rightarrow$ [??]

◇ exploit scale hierarchy $m^2 \ll M^2, Q^2$, expand in $m^2/Q^2 \sim 0$



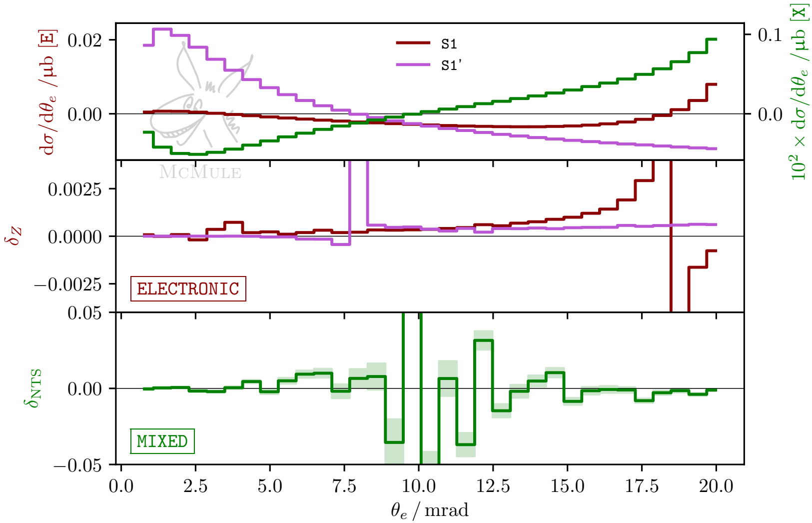
$$\sim A \log^2 \frac{m^2}{Q^2} + B \log \frac{m^2}{Q^2} + C + \mathcal{O}\Big(\frac{m^2}{Q^2}\Big)$$

◇ massification: $\mathcal{A}_{mM}(m) = \mathcal{S} \times Z \times Z \times \mathcal{A}_{mM}(0) + \mathcal{O}(m)$

[Penin 06, Becher, Melnikov 07; Engel, Gnendiger, Signer, Ulrich 18]

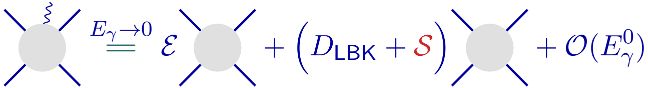
◇ theory error: $\mathcal{O}(10^{-3})\,@\,\mathrm{NNLO} \sim \mathcal{O}(10^{-6})$

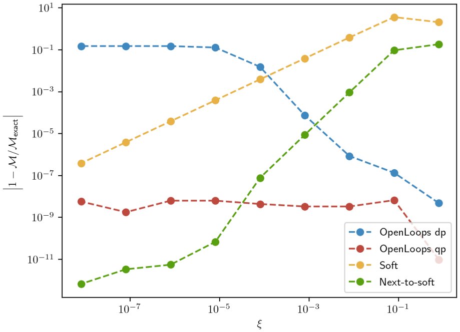
validation: massification



OpenLoops [Buccioni, Pozzorini, Zoller 18, Buccioni et al. 19]

LBK theorem [LBK 58-61, Engel, Signer, Ulrich 21, 2xEngel 23]





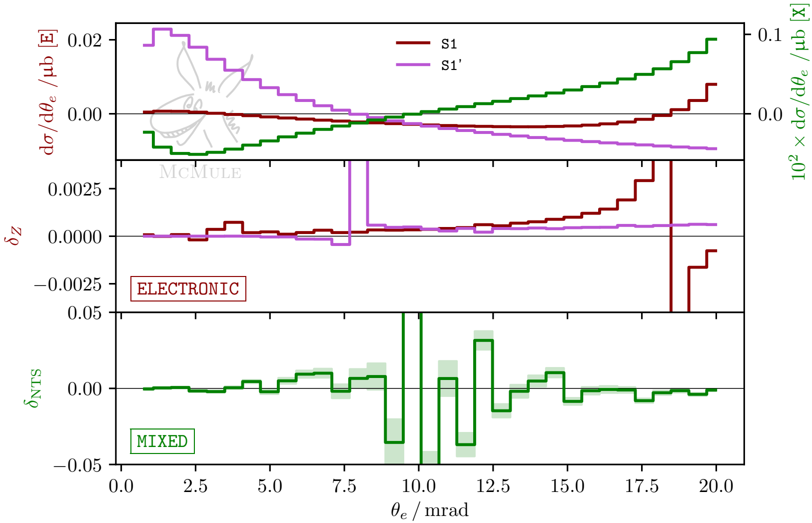
◇ introduce NTS stabilisation [McMule 21, 22]

– if $E_\gamma < E_{\text{NTS}} \sim 10^{-3}\sqrt{s}/2$
switch to the expansion above

– theory error:
$\mathcal{O}(10^{-2})\,@\,\text{NNLO} \sim \mathcal{O}(10^{-5})$

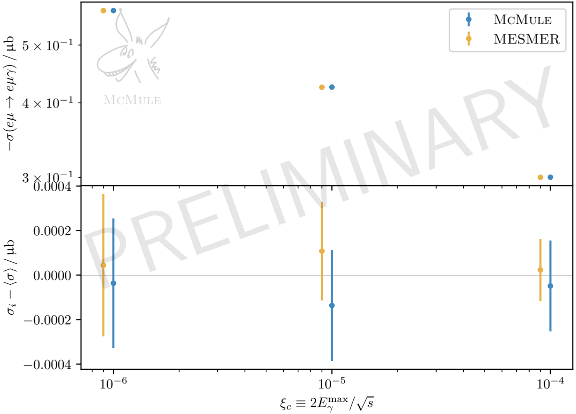
– nice(r) carbon footprint

validation: NTS stabilisation



(FKS$^\ell$ + DIMREG) vs (slicing $+ m_\gamma$)

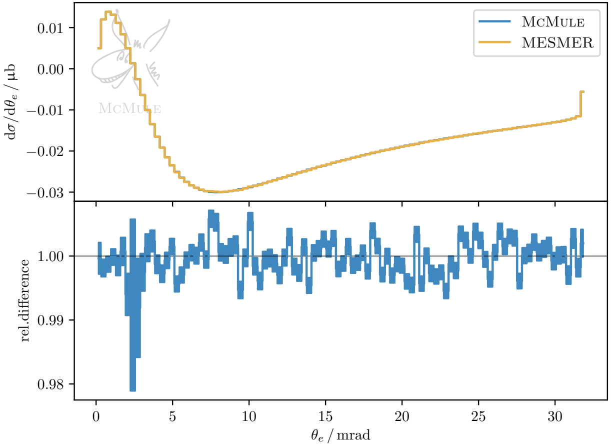
$e\,\mu \to e\,\mu\,\gamma$ @ NLO with $\xi_c = \omega_s = 10^{-\{6,5,4\}}$ (Mesmer as in [Carloni et al. 20])



validation: comparison McMule vs Mesmer (1)

(FKS$^\ell$ + DIMREG) vs (slicing $+ m_\gamma$)

$e\,\mu \to e\,\mu\,\gamma$ @ NLO with $\xi_c = \omega_s = 10^{-\{6,5,4\}}$ (Mesmer as in [Carloni et al. 20])



(massification) vs (YFS approximation)

- McMule + massified VV ($m_e = 0$, [Mastrolia et al 18-22]):

  $-2.50(1) \cdot 10^{-2}\,\mu\text{b}$

- McMule + YFS approximation for VV (courtesy of Carlo):

  $6.45(1) \cdot 10^{-2}\,\mu\text{b}$

- Mesmer:

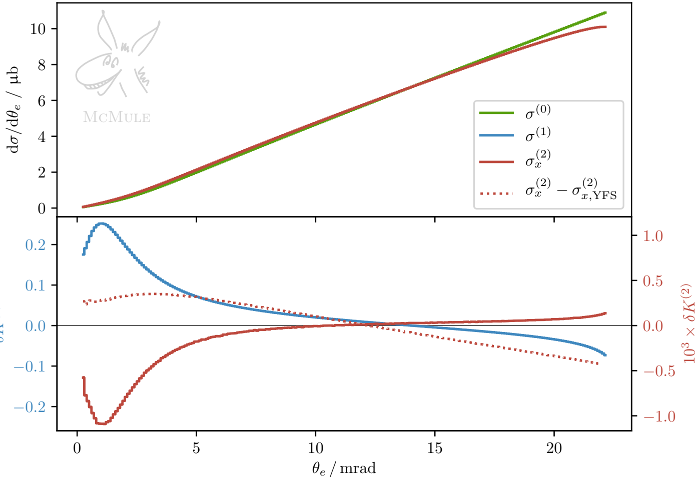
  $6.47(3) \cdot 10^{-2}\,\mu\text{b}$

  – error estimate (LO units): $\left(\frac{\alpha}{\pi}\right)^2 \log \frac{m_e^2}{m_\mu^2} \sim 6 \cdot 10^{-4}$

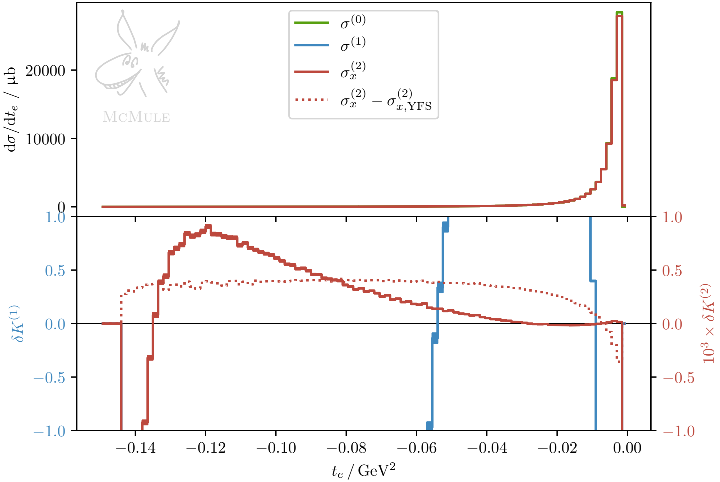
  [LO $= 1.214(1) \cdot 10^{2}\,\mu\text{b}$]

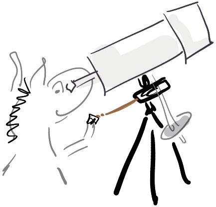
validation: comparison McMule vs Mesmer (2)

distributions: θ_e



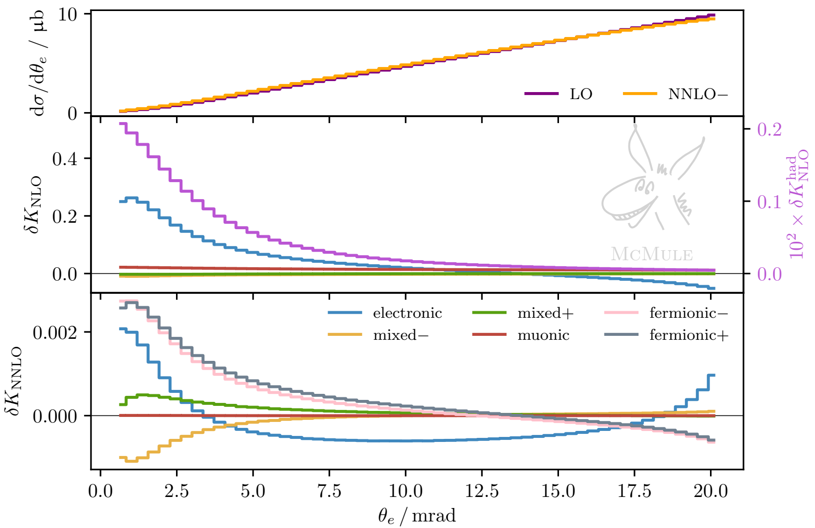
distributions:



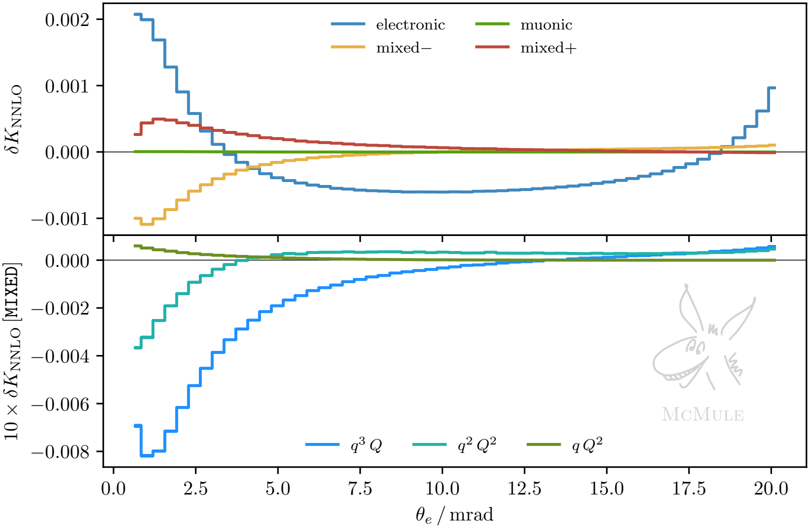


looking for another $\mathbb{N}$

S1 θ_e spectrum: a different perspective



zoom on the different perspective



Approximate N^{3}LO

VVV RVV RRV RRR

$$d\sigma^{(3)} = \int d\Phi_n \, \mathcal{M}_n^{(3)} + \int d\Phi_{n+1} \, \mathcal{M}_{n+1}^{(2)} + \int d\Phi_{n+2} \, \mathcal{M}_{n+2}^{(1)} + \int d\Phi_{n+3} \, \mathcal{M}_{n+3}^{(0)}$$

- IR subtraction with FKS$^\ell$
- $\mathcal{M}_{n+2}^{(1)}$ & $\mathcal{M}_{n+3}^{(0)}$ via OpenLoops + NTS stabilisation
- $\mathcal{M}_n^{(3)}$ via massification (if $m=0$ ✓) or eikonals or ?
- $\mathcal{M}_{n+1}^{(2)}$ via { NTS approximation / collinear approximation ← 2-loop massive splitting amplitude / massification }

[Tim Engel]

McMule is predominantly an integrator

- we can calculate $\sigma = \int \mathrm{d}\Phi\ \mathcal{M}\ S(\{p_i\})$
- measurement function S can be implemented numerically $\sim$ cuts, histograms
- event generators produce events (more or less) distributed according to $w = \mathrm{d}\Phi\ \mathcal{M}$
- trivial solution: dump every event $\{p_i\}$ and weight w to file ("garden hose approach")

[Yannick Ulrich]

why this isn't great

minimise $\{p_i\}$ to propagate through the expensive detector simulation

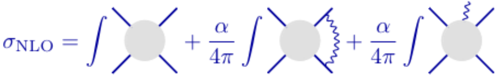
- the w can be negative beyond LO and span many orders of magnitude
- clever sampling can help but not fully solve the problem
- if $r \times N$ of N weights are negative, we need $\propto 1/(1-2r)^2$ events

$\Rightarrow$ reduce r as much as possible by cancelling negative weights as early as possible

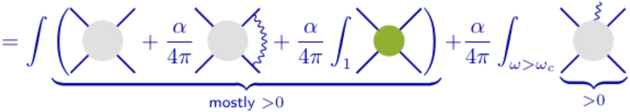
[Yannick Ulrich]

origin of negative weights

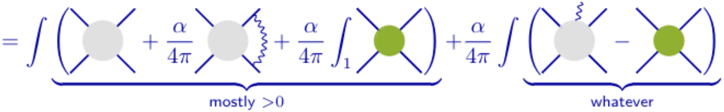
... at NLO for simplicity



- slicing: fairly few negative weights but numerically construct $\log \omega_c$



- subtraction: easier integration but lots and lots of negative weights
  ($\mathcal{O}(5\%)$ at NLO, more at NNLO)



[Yannick Ulrich]

can we get both

two observations

1. cross section $\sigma = \int_{\mathcal{C}} \mathrm{d}\sigma > 0$, irregardless of the size of integration region $\mathcal{C}$
2. experiments have a finite resolution

   (we already knew that because we can't see soft photons)

algorithm to remove negative weights [Andersen, Maier 21]

- pick an event with $w_i < 0$
- find nearby events until $\sum_{i\in\mathcal{C}} w_i > 0$
- if $\mathcal{C}$ gets too big (events become resolvable), abort (or add more events)
- else $w_i \to \frac{\sum_{j\in\mathcal{C}} w_j}{\sum_{j\in\mathcal{C}} |w_j|} w_i$

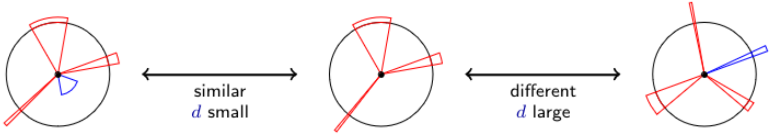
we can remove negative weights without biasing physical observables!

[Yannick Ulrich]

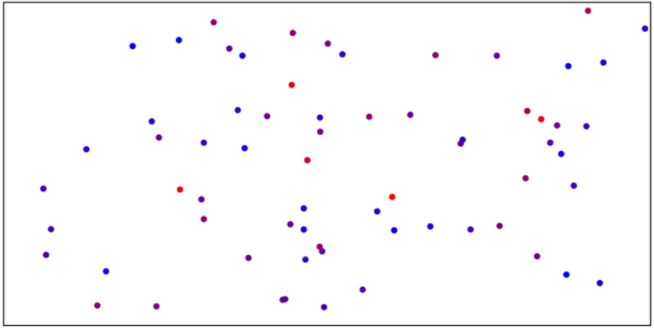
what does nearby mean?

we need to define a metric in event space $d(e_1, e_2) \geq 0$

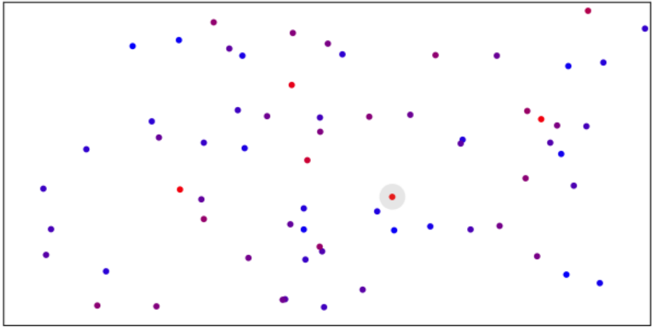
- doesn't really matter how we do this as long as IR safe
  (events with soft photons are near each other)
- ideally: events that look similar are closer to each other than those that don't
- MUonE example: $d(e_1, e_2) = \sqrt{\left|\theta_1^e - \theta_2^e\right|^2 + \left|\theta_1^\mu - \theta_2^\mu\right|^2}$
- can add ϕ and/or energy information, depending on analysis



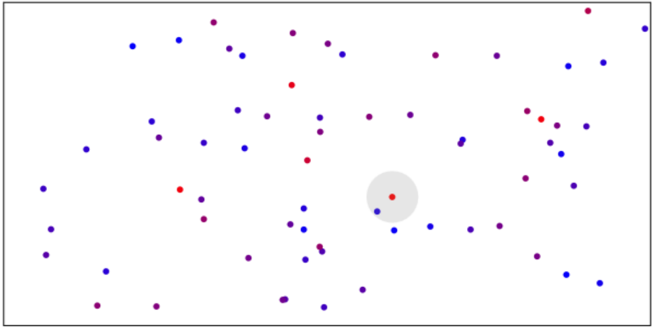
[Yannick Ulrich]



[Yannick Ulrich]

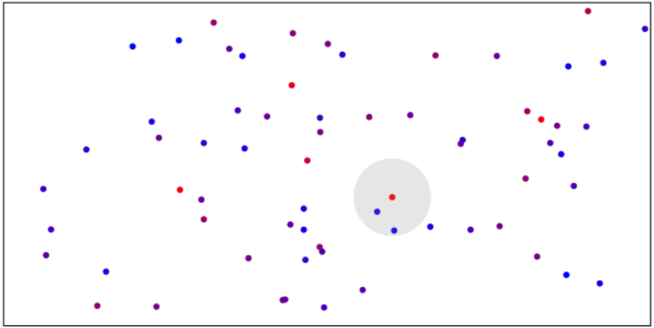


[Yannick Ulrich]



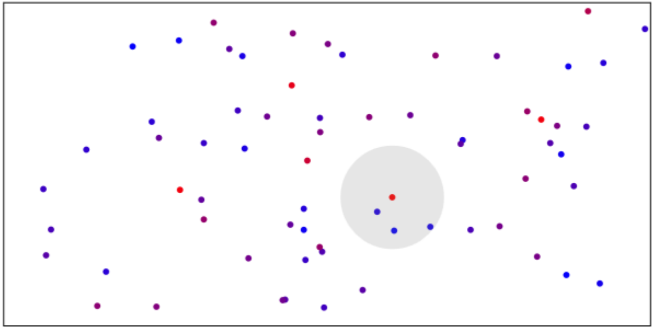
[Yannick Ulrich]

example

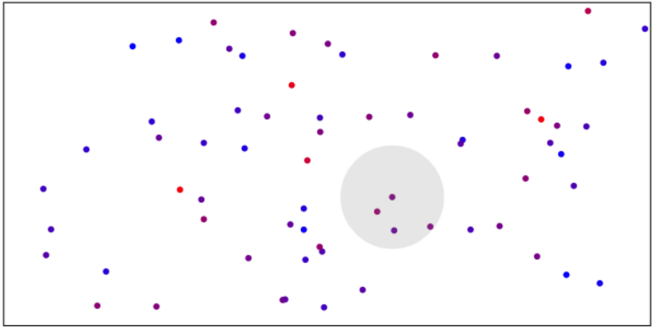


[Yannick Ulrich]

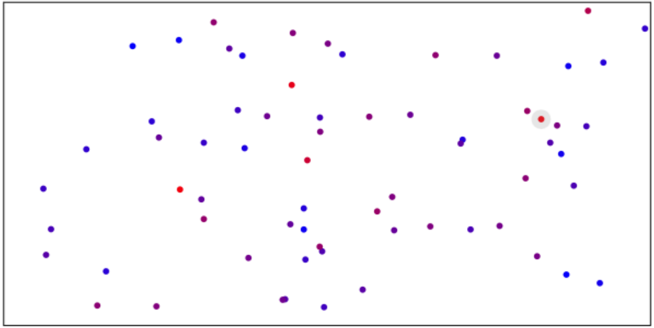
example



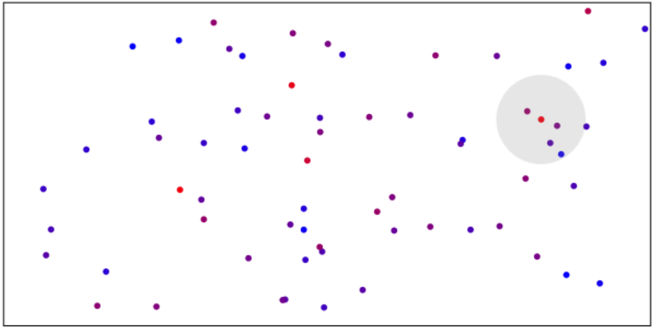
[Yannick Ulrich]



[Yannick Ulrich]

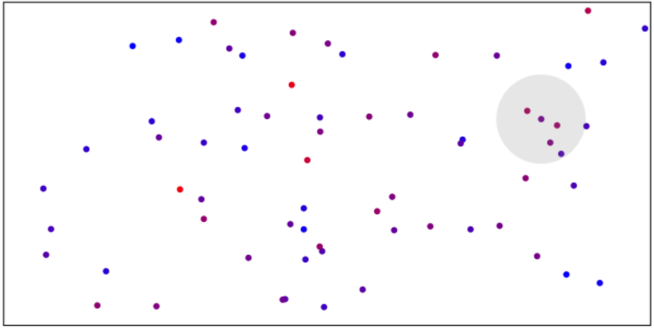


[Yannick Ulrich]

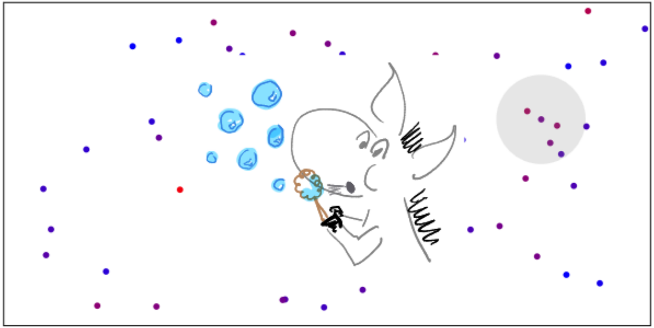


[Yannick Ulrich]

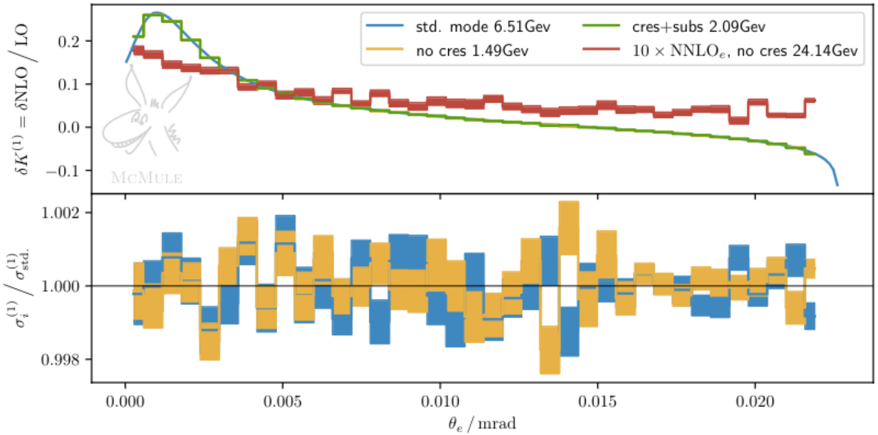
example



[Yannick Ulrich]



[Yannick Ulrich]



$r \approx 2 \times 10^{-2} \to 2 \times 10^{-5}$, $w_{\min}/\langle w \rangle = -10^5 \to -10^{-3}$, similar at NNLO

[Yannick Ulrich]

◇ NNLO with different external masses [2212.06481]

◇ naive theory error (missing higher orders) $\mathcal{O}(10^{-5})$

- two-masses two-loop to stress test massification?
- use YFS approximation at $\mathrm{N^3LO}$?
  - feasibility: can it be used for $2 \to 3$ kinematics?
  - cross-check for RVV computed with McMule cut-and-patch?
- deeper insights on collinear hierarchies at $\mathrm{N^3LO}$?
- resummation (analytic & parton shower) at NLL?
- ...