Simulation of lepton pair production with MESMER

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Based on arXiv:2109.14606 and arXiv:2401.06077

The Evaluation of the Leading Hadronic Contribution to the Muon $g-2$

MITP Topical Workshop – 04 June 2024

UNIVERSITÀ DI PAVIA

Introduction

- **Lepton pair production**, both in **muon-electron** ($\propto Z$) and **muon-nucleus** ($\propto Z^2$) scattering is the main physical background of MUonE experiment

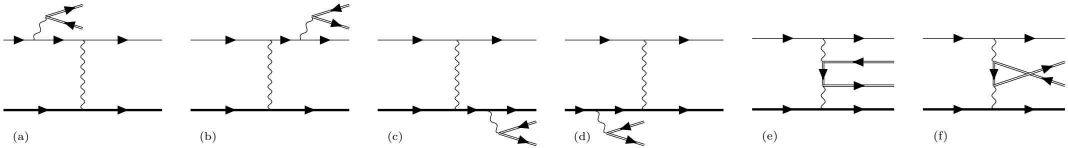
- There are four possible processes:
  - $\mu^\pm e^- \to \mu^\pm e^- e^+ e^-$
  - $\mu^\pm e^- \to \mu^\pm e^- \mu^+ \mu^-$
  - $\boldsymbol{\mu^\pm N \to \mu^\pm N\, e^+ e^-}$
  - $\mu^\pm N \to \mu^\pm N\, \mu^+ \mu^-$

- They can resemble the elastic signal if ony **two tracks** are detected

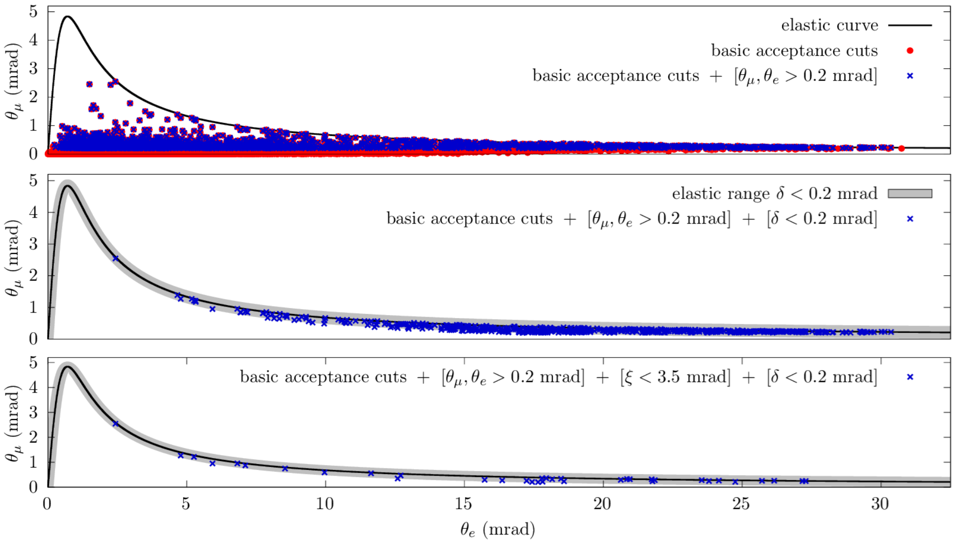
- All processes are implemented in MESMER at tree-level with exact kinematics:
  - E. Budassi et al., *NNLO virtual and real leptonic corrections to muon-electron scattering*, JHEP 11 (2021) 098, [2109.14606]
  - G. Abbiendi et al., *Lepton pair production in muon-nucleus scattering*, Phys. Lett. B 854 (2024) 138720, [2401.06077]

Lepton pair production in muon-electron scattering

- Process computed at tree level in the context of NNLO leptonic corrections to muon-electron scattering
- Exact $2 \to 4$ phase space sampled with a multi-channel importance sampling
- Event selection criteria ($E_\mu = 150$ GeV):
  - One muon-like track: $\theta_\mu < 4.84$ mrad, $E_\mu < 10.28$ GeV
  - One electron-like track: $\theta_e < 100$ mrad, $E_e > 1$ GeV
  - Minimum scattering angles: $\theta_e, \theta_\mu > 0.2$ mrad
  - Elasticity cut: $\delta = \sqrt{(\theta_e - \theta_e^{el})^2 + (\theta_\mu - \theta_\mu^{el})^2} < 0.2$ mrad
  - Acoplanarity: $\xi = |\pi - |\phi_e - \phi_\mu|| < 3.5$ mrad

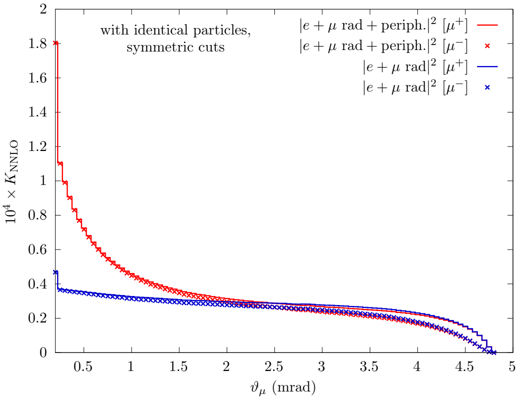


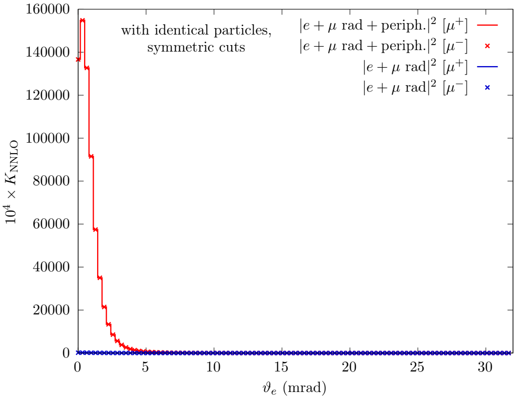
Numerical results for $\mu^\pm e^- \to \mu^\pm e^- e^+ e^-$ (1)



Numerical results for $\mu^\pm e^- \to \mu^\pm e^- e^+ e^-$ (2)

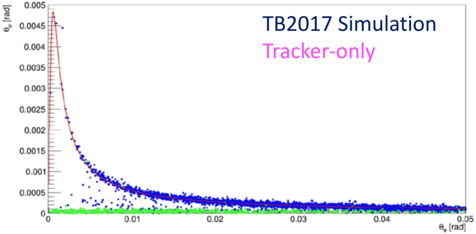
$$K_{\rm NNLO} = {\rm d}\sigma(\mu^\pm e^- \to \mu^\pm e^- e^+ e^-)/{\rm d}\sigma(\mu^\pm e^- \to \mu^\pm e^-)$$

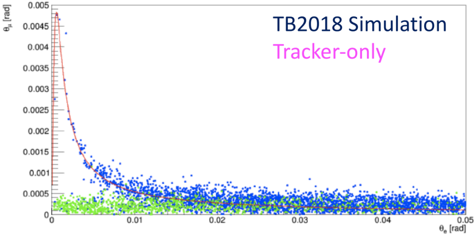




Lepton pair production in muon-nucleus scattering

- **Lepton pair production in μN scattering** is expected to the MUonE dominant background:
  - ↪ Two-track events are caused by only **one** undetected particle instead of two as in μe scattering
  - ↪ The cross section is $\propto \boldsymbol{Z^2}$ instead of $\propto Z$
- In GEANT4 the muon deflection is neglected $(\theta_\mu = 0)$
  - ↪ But for MUonE we expect $\mathcal{O}(m_\mu/E_\mu) \sim 0.66$ mrad
- The process $\boldsymbol{\mu^\pm N \to \mu^\pm N \ell^+ \ell^-}$ with $\ell = \{e, \mu\}$ is now implemented in MESMER for $N = \{\text{Be, C, Fe}\}$ without any approximation on the angular variables
- The agreement between MESMER and GEANT4 is about **1-5%** for the inclusive cross section





[From Giovanni Abbiendi]

The calculation in a nutshell

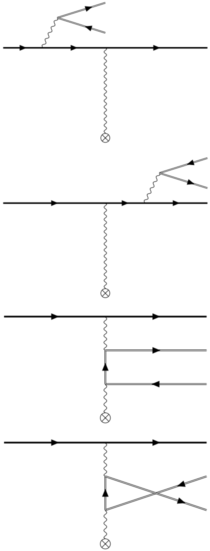
- The process is described as a scattering of a muon in an **external Coulomb field**, generated by a nucleus at rest
- Nuclear recoil is neglected and therefore only the energy is conserved
- The cross section is highly enhanced for $\theta_\mu \to 0$ since $\mathrm{d}\sigma/\mathrm{d}\theta_\mu \sim 1/|Q|^4$

  $\hookrightarrow$ Phase-space sampling based on a multi-channel importance sampling
- The finite size of the nucleus is accounted by using the **e.m. form factor**

$$F(Q) = \frac{4\pi}{Ze} \int_0^\infty \mathrm{d}r \, r^2 \rho(r) \frac{\sin(Qr)}{Qr}$$

Q : momentum transferred to the nucleus, ρ : nuclear charge density

- Good approximation for small angles: $\theta_\mu < 4.84$ mrad $\longrightarrow$ $Q^2 < 0.6$ GeV^2
- Different models for ρ are used to evaluate the theoretical uncertainty on the model



Form factor models

- One parameter Fermi model (1pF):

$$\rho(r) = \frac{\rho_0}{1 + \exp\frac{r-c}{z}} \qquad \longleftarrow \quad \text{Standard model for Be, C, Fe}$$

- Modified harmonic oscillator model (MHO):

$$\rho(r) = \rho_0' \left(1 + \omega \frac{r^2}{a^2}\right) \exp\left(-\frac{r^2}{a^2}\right) \qquad \longleftarrow \quad \text{Alternative model for Be}$$

- Fourier-Bessel expansion (FB):

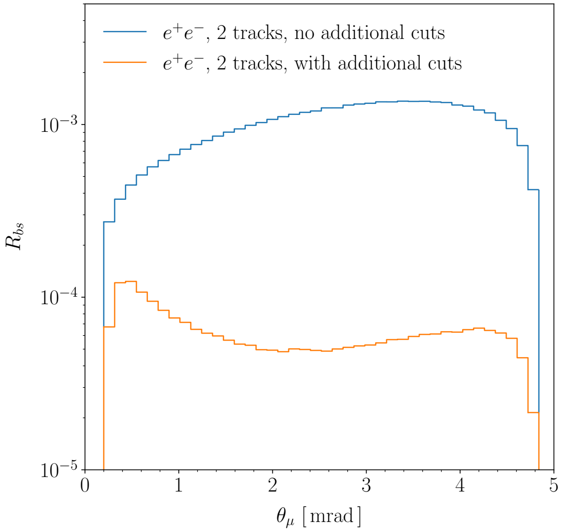
$$\rho(r) = \begin{cases} \sum_{k=1}^{n} a_k \, j_0\left(\frac{k\pi r}{R}\right) & r \leq R \\ 0 & r > R \end{cases} \qquad \longleftarrow \quad \text{Alternative model for C}$$

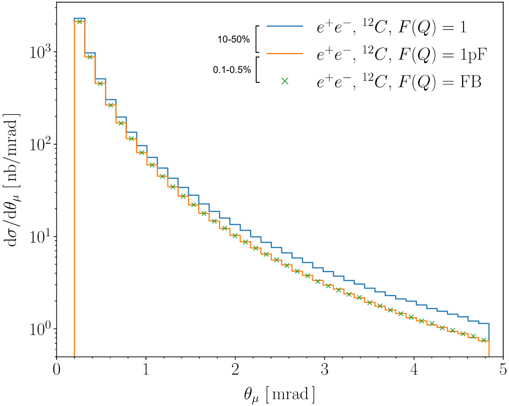
with $j_0(x) = sin(x)/x$

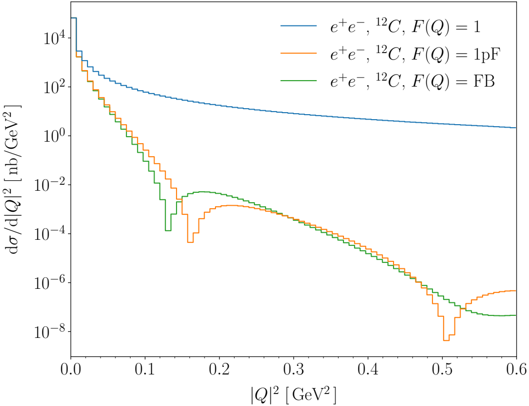
J. Heeck, R. Szafron, Y. Uesaka, *Isotope dependence of muon decay in orbit*, Phys. Rev. D 105 (5) (2022) 053006

Background-to-signal ratio for $\mu^+ C \to \mu^+ C\, e^+ e^-$

- The background/signal ratio R_{bs} is $\sim 10^{-3}$ with only acceptance cuts and $\sim \mathbf{7 \cdot 10^{-5}}$ with both acoplanarity and elasticity cuts
- Even a conservative uncertainty of $\sim 10\%$ on the model should allow a background subtraction with an accuracy below the target precision of 10^{-5}
- QED corrections can be enhanced up to $\sim 10\%$
- Uncertainty on background simulation can be taken under control thanks to data:
  1. Measure 3-tracks events that can be unambiguously identified as pair events
  2. Obtain the **ratio** between 2-track and 3-track from MC simulation
  3. Extract the expected 2-track events





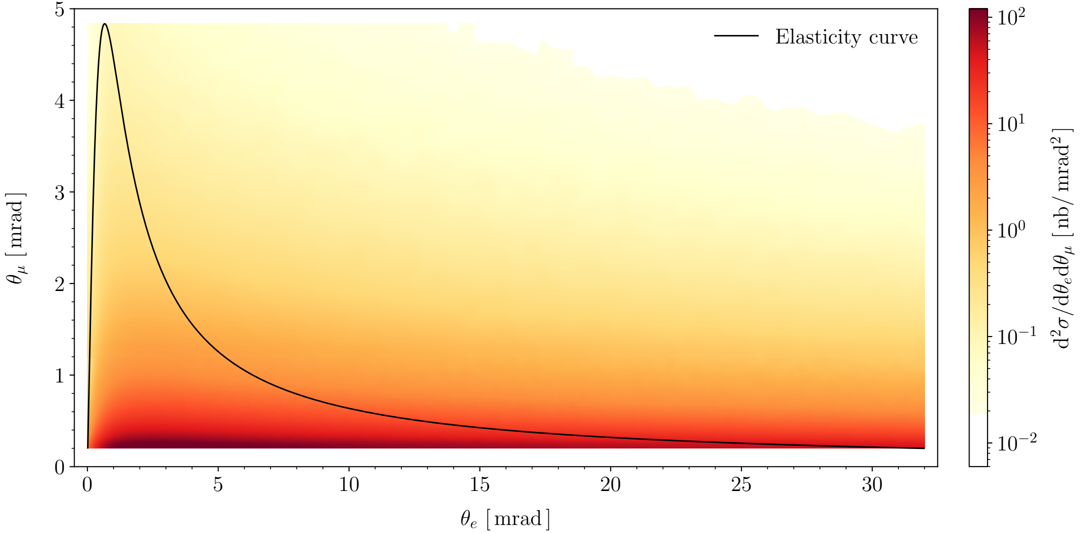


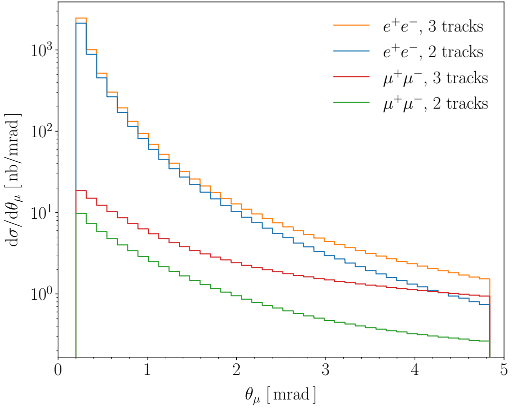
- Cuts: $E_e > 0.2$ GeV, $E_\mu > 10.2$ GeV, $\theta_e < 32$ mrad, $\theta_\mu < 4.8$ mrad, $\boldsymbol{\theta_\mu > 0.2}$ **mrad**, $|Q|^2 < 0.6$ GeV^2

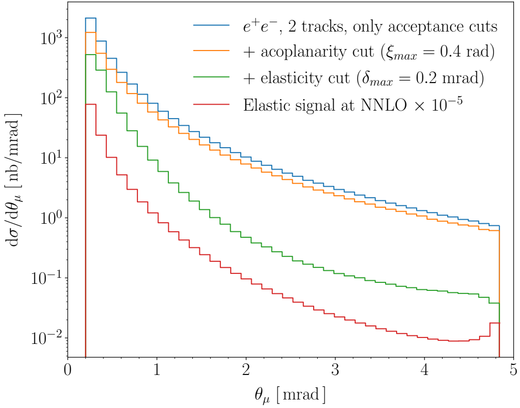
- Only the events with **one muon track** and **one electron track** in the acceptance region are selected

  $\hookrightarrow$ Only one $e^\pm$ is accepted, or both $e^\pm$ are accepted but their tracks overlap, i.e. $\theta_{ee} < 20\ \mu$rad

Numerical results for $\mu^+ C \to \mu^+ C\, e^+ e^-$ (2)







Two additional cuts to reject the background (only for two-track events):

- Acoplanarity: $|\pi - |\phi_e - \phi_\mu|| < 400$ mrad
- Elasticity: $\delta_{elastic}(\theta_e, \theta_\mu) < 0.2$ mrad

Background-to-signal ratio for $\mu^+ C \to \mu^+ C\, e^+ e^-$

- The background/signal ratio R_{bs} is $\sim 10^{-3}$ with only acceptance cuts and $\sim \mathbf{7 \cdot 10^{-5}}$ with both acoplanarity and elasticity cuts
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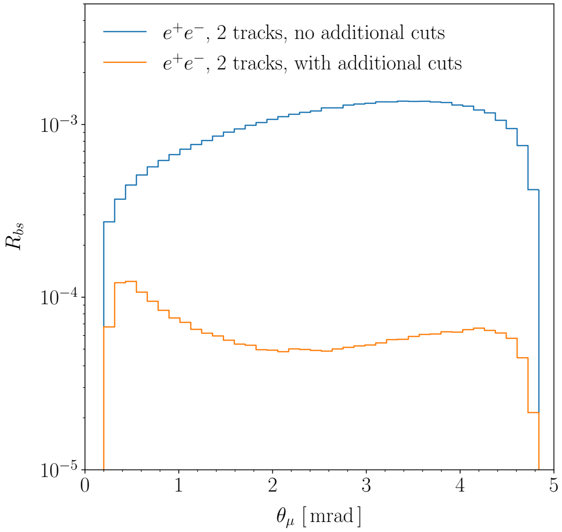


Conclusions

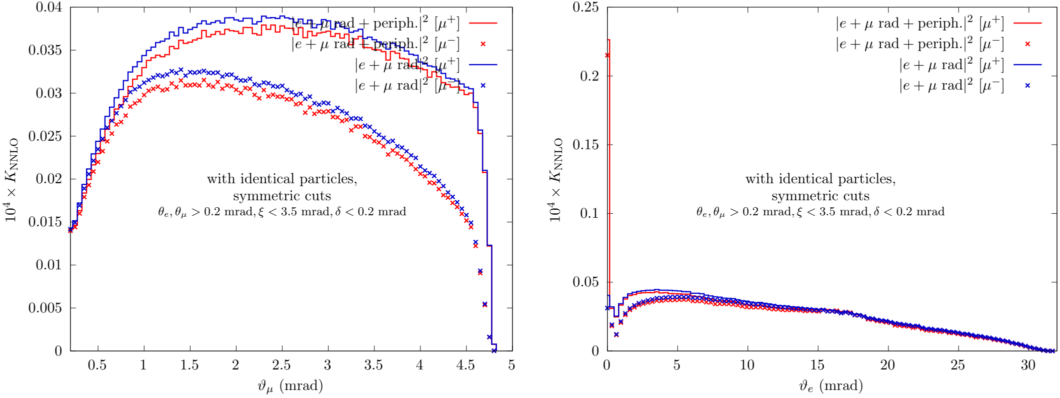
- The processes $\mu^\pm e^- \to \mu^\pm e^- \,\ell^+\ell^-$ and $\mu^\pm N \to \mu^\pm N\,\ell^+\ell^-$ with $\ell = \{e, \mu\}$ are implemented in MESMER
- The computation of $\mu^\pm N \to \mu^\pm N\,\ell^+\ell^-$ can be improved by going beyond the external field approximation and by including the dominant QED corrections
- GEANT4 collaboration is working to implement $\mu^\pm N \to \mu^\pm N\,\ell^+\ell^-$ with $\theta_\mu \neq 0$ starting from MESMER
- Strategies for background subtraction are under studies by experimental colleagues
- Bremsstrahlung $\mu^\pm N \to \mu^\pm N\,\gamma$ can trigger non-prompt pair production $\gamma\, N' \to N'\ell^+\ell^-$

  $\hookrightarrow$ Work in progress to include it in MESMER

Backup

Numerical results for $\mu^\pm e^- \to \mu^\pm e^- e^+ e^-$ (3)

$$K_{\rm NNLO} = {\rm d}\sigma(\mu^\pm e^- \to \mu^\pm e^- e^+ e^-)/{\rm d}\sigma(\mu^\pm e^- \to \mu^\pm e^-)$$



Implementation in MESMER

- The calculation has been included in MESMER in two separate channels:
  $\hookrightarrow$ "`nphot 1020`" generates $\mu^\pm N \to \mu^\pm N\, e^+ e^-$ with $N =$ Be, C, Fe
  $\hookrightarrow$ "`nphot 1021`" generates $\mu^\pm N \to \mu^\pm N\, \mu^+ \mu^-$ with $N =$ Be, C, Fe

- It works both in standalone and embedded mode

- The code requires a **cut on the minimum $\boldsymbol{\theta_\mu}$** to obtain a numerical convergence
  $\hookrightarrow$ We suggest $\theta_\mu > 0.1$ mrad to avoid any instability

- The code is available on the muesli gitlab repository:
  $\hookrightarrow$ `https://gitlab.cern.ch/muesli/nlo-mc/mesmer-dev`

- Don't hesitate to contact the Pavia group for any clarification or suggestion!

How to run the program

To run the program in standalone mode:

- Download the latest MESMER distribution from the Muesli gitlab repository
- Compile the code with the usual `make` command to get the `mesmer` executable
- Run the executable with the new input card using the command `./mesmer < input_nuclear`
- The output (distributions, events, cross sections) is saved in the chosen path

It is also possible to run the code in embedded mode

Example of input data card

```
Ebeam 160.
nphot 1020
nev 1e4
wnorm 1.
path test-run
store yes no
run
10.23d0        !  Muon min energy in LAB [GeV]
0.2d-3         !  Muon min angle in LAB [rad]
4.84d-3        !  Muon max angle in LAB [rad]
0.2d0          !  Electron min energy in LAB [GeV]
0.d0           !  Electron min angle in LAB [rad]
0.032d0        !  Electron max angle in LAB [rad]
2              !  Min number of electrons passing the cuts (1 or 2)
0.774d0        !  |Q| max transferred to the nucleus [GeV]
1              !  Muon charge (1 or -1)
6              !  Atomic number (Z=4 for Be, Z=6 for C, Z=26 for Fe)
1              !  Form factor?  (1:  yes, 0:  no, -1:  alt)
```