MESMER status and physics implementation

MITP - Mainz

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Overview

- The MESMER Event Generator
- Photonic contributions to μe scattering
- Virtual leptonic contributions to μe scattering
- Single π_0 production

Then, see Andrea's talk

MESMER

References

- Alacevich *et al.*, Muon-electron scattering at NLO, JHEP 02 (2019) 155
- Carloni Calame *et al.*, Towards muon-electron scattering at NNLO, JHEP 11 (2020) 028
- Budassi *et al.*, NNLO virtual and real leptonic corrections to muon-electron scattering, JHEP 11 (2021) 098
- Budassi *et al.*, Single π_0 production in μe scattering at MUonE, PLB 829 (2022) 137138
- Abbiendi *et al.*, Lepton pair production in muon-nucleus scattering, Phys. Lett. B 854 (2024), 138720.

What is MESMER?

MESMER (**M**uon **E**lectron **S**cattering with **M**ultiple **E**lectromagnetic **R**adiation)

- Fully differential Monte Carlo event generator.
- It generates fully exclusive events.
- It is used for high-precision simulation of μe scattering at low energies.
- It was developed for the MUonE experiment.

What can we do with MESMER?

- NLO QED (and weak) corrections: single real or virtual γ
- NNLO photonic corrections (double boxes are approximate)
- NNLO leptonic contributions
- $\mu e \to \mu e \ell^+ + \ell^-$, with $\ell = e, \mu$
- $\mu e \to \mu e \pi_0$, with $\pi_0 \to \gamma\gamma$
- $\mu X \to \mu X \ell^+ \ell^-$, with $\ell = e, \mu$.

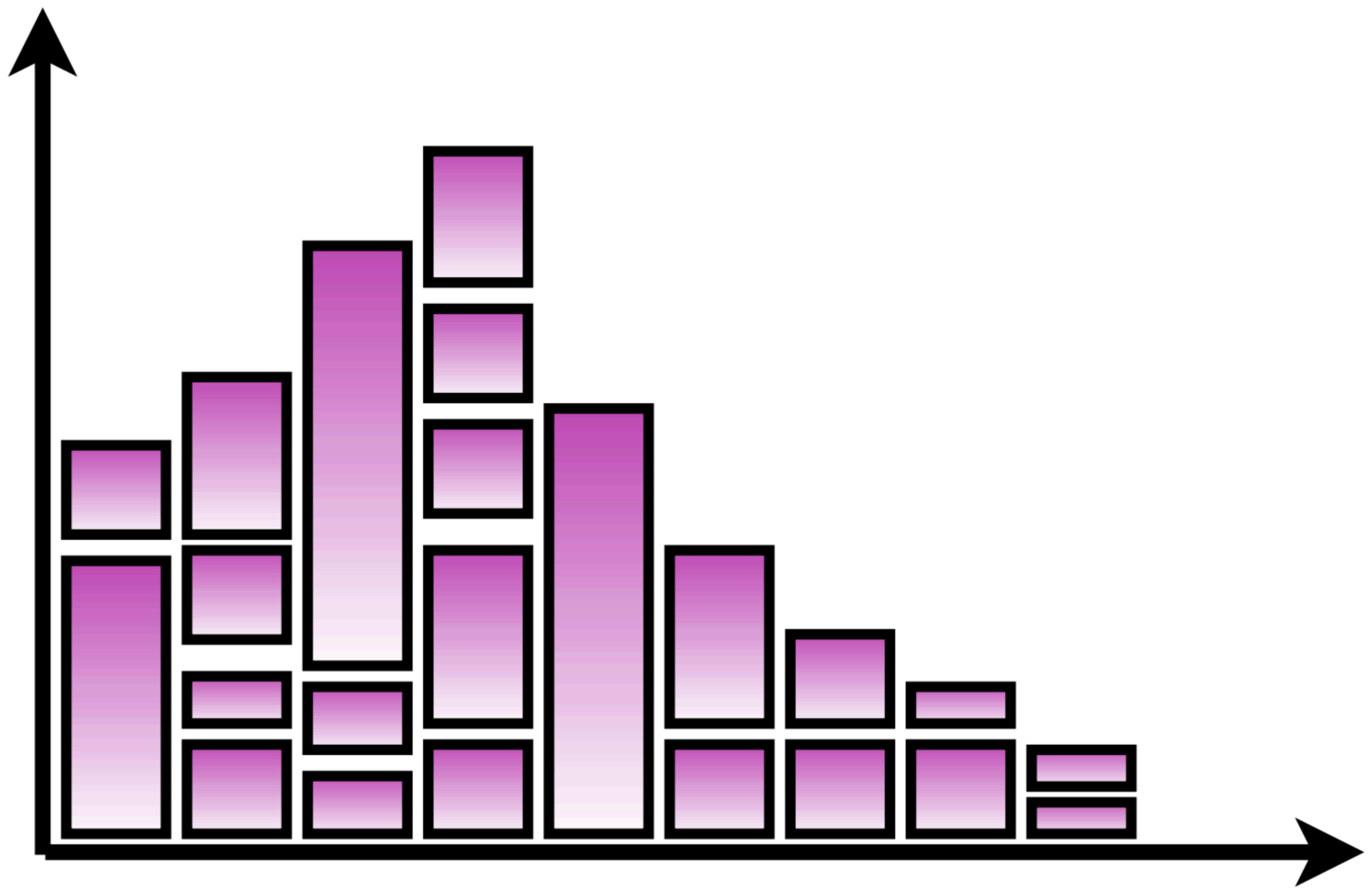
The coding

- The code is available at this Github repo.
- MESMER is written in Fortran 77.
- External libraries are used for one-loop integrals (`LoopTools`, `Collier`) and pseudo-random number generation (`RANLUX`).
- HVP is included with Jegerlehner's, Keshavarzi-Nomura-Teubner's and Ignatov's routines.

Inputs & outputs

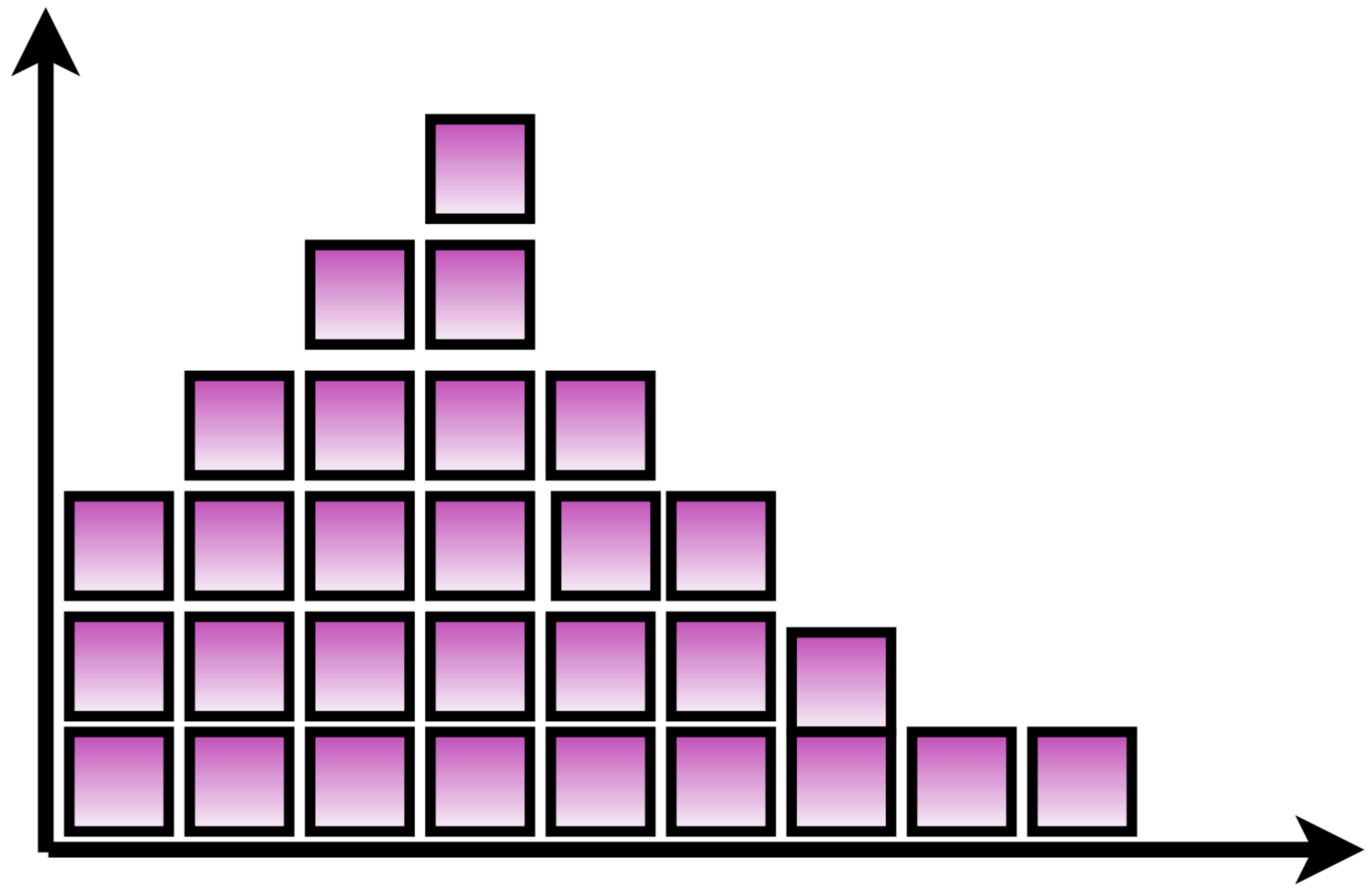
- Kinematical constraints can be imposed at generation level
- Kinematical cuts can be included and modified in the routine `cuts.F`, according to the user's needs
- A generic incoming muon momentum can be used as an input, on an event-by-event basis for realistic sims (beam profile)
- Events are written in specific output files with some control differential distributions
- A `C/C++` interface is provided to use the code as a library in a driver simulation program (`FairMUonE`)

Weights



- Events are generated with their weight
- It needs to be saved and carried throughout the calculation and detector simulation
- Generation is fast

- Events are not weighted and are distributed according to the cross section
- Generation is slow because of unweighting
- Tricky to guess a maximizer for weights before the generation.



Reweighting

- It is possible to calculate different reweighing r_i to study different contributions (*e.g.* VP, HVP parametrisations/inclusion, *etc.*)
- No need to re-generate another MC sample for different effects
- Exploitation of statistical correlation on the weights to reduce MC errors

$$\sigma = \sum_{i=0}^{n} \frac{w_i}{n} \qquad \sigma_{\text{reweighted}} = \sum_{i=0}^{n} \frac{w_i}{n} \times r_i; \quad r_i = \frac{w_i^{\text{reweighted}}}{w_i}$$

Physics in MESMER

μe scattering at NLO

- Real and virtual QED corrections to μe scattering are included
- Weak effects are calculated and weigh about $\sim 10^{-5}$ (LO) and $\lesssim 10^{-6}$ (NLO)
- The dependence on masses is kept. Radiative corrections are studied with **specific kinematical cuts** that mimic the MUonE experimental setup
  - Basic acceptance cuts:

$$\vartheta_e, \vartheta_\mu < 100\,\mathrm{mrad} \qquad E_e > 1\,\mathrm{GeV}$$

  - Acoplanarity cut:

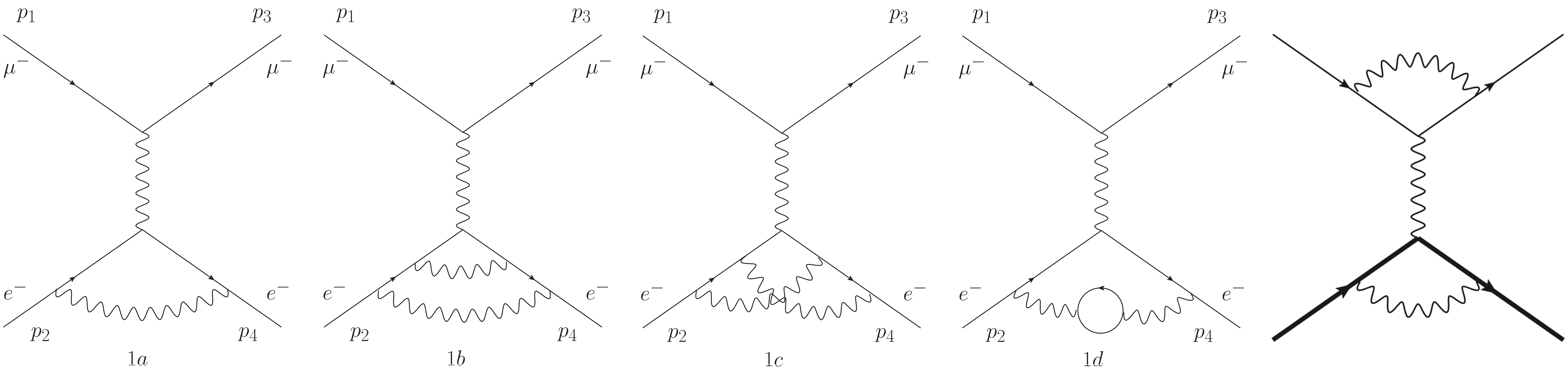
$$\xi \equiv \left| \pi - \left| \phi_e - \phi_\mu \right| \right| < 3.5\,\mathrm{mrad}$$

M. Alacevich, C. M. Carloni Calame et al., JHEP 02 (2019) 155.

μe scattering at NNLO: exact photonic I

- |NLO virtual diagrams|^2
- Virtual NNLO photonic contributions are included exactly for electron or muon leg emission. 2-loop QED vertex form factor taken from Mastrolia-Remiddi

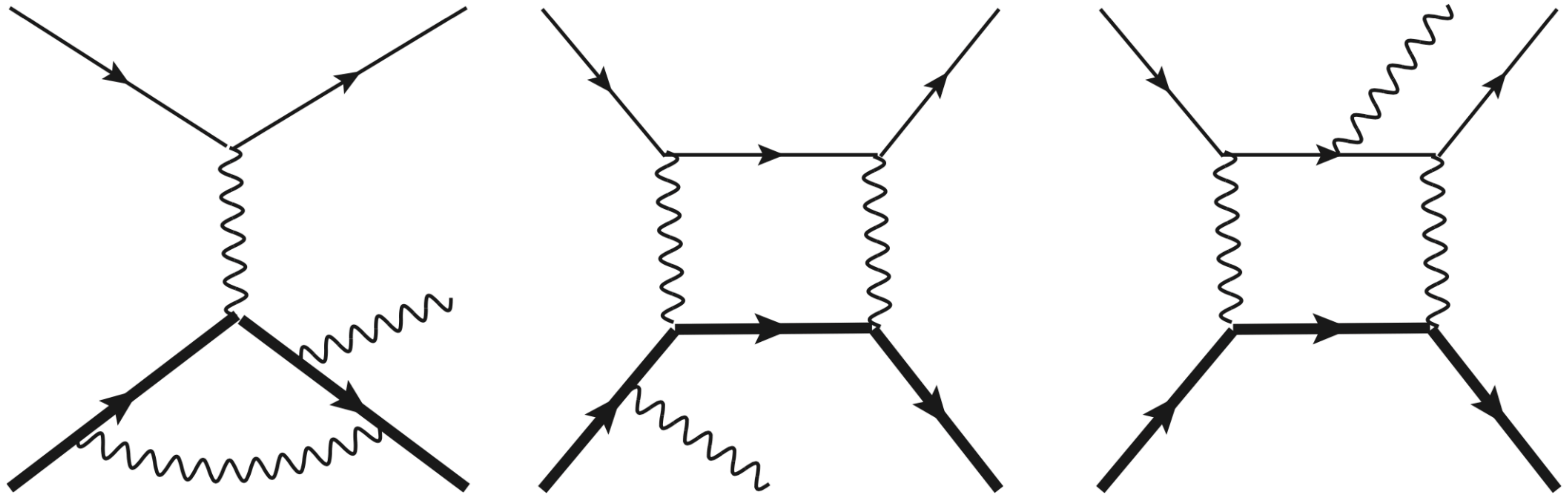
P. Mastrolia and E. Remiddi, Nucl.Phys.B 664 (2003), 341-356.



C. M. Carloni Calame, M. Chiesa, S. M. Hasan, G. Montagna, O. Nicrosini, and F. Piccinini, JHEP 11 (2020) 028

μe scattering at NNLO: exact photonic II

- Interference of LO $\mu e \to \mu e\gamma$ with



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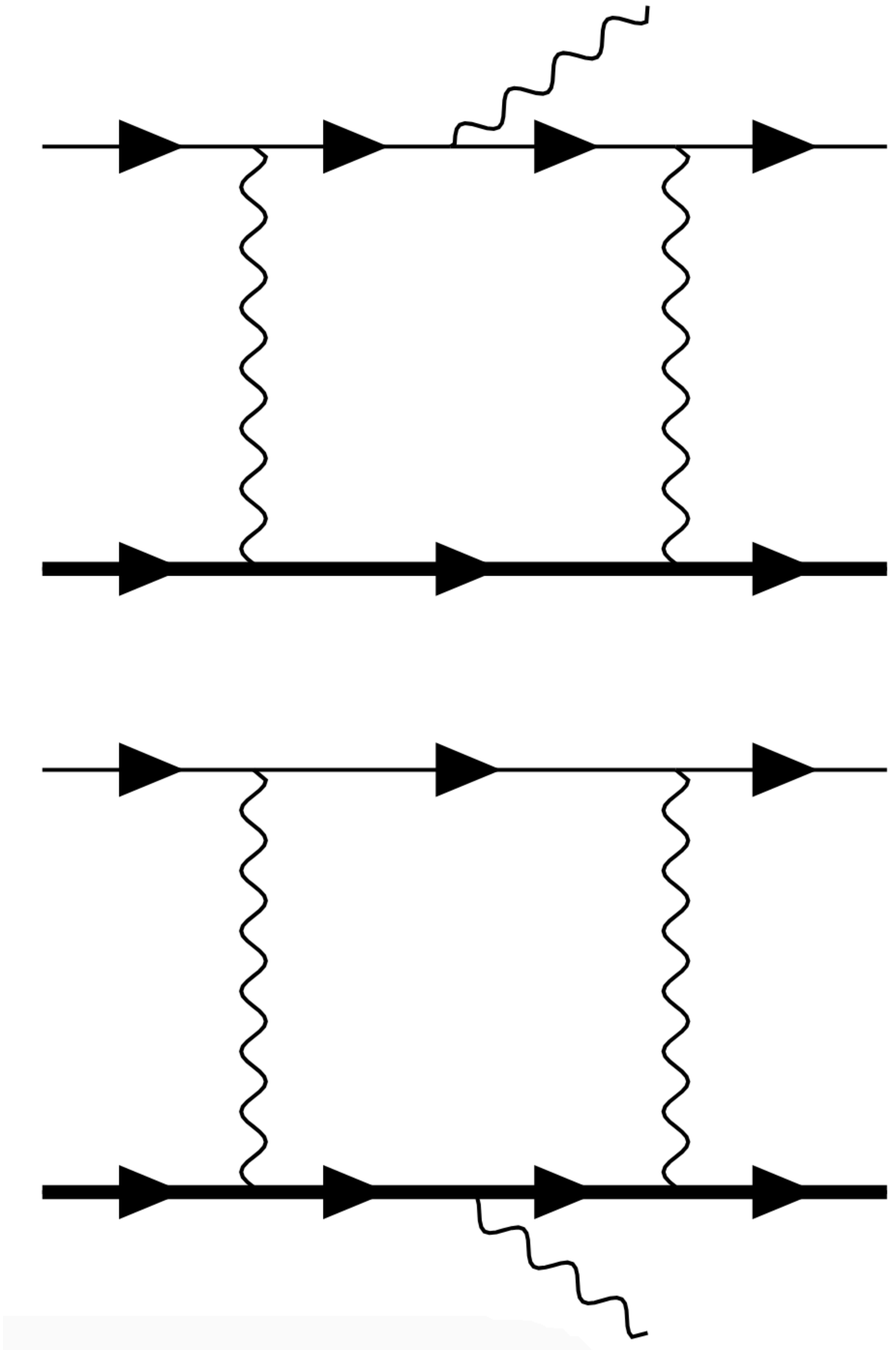
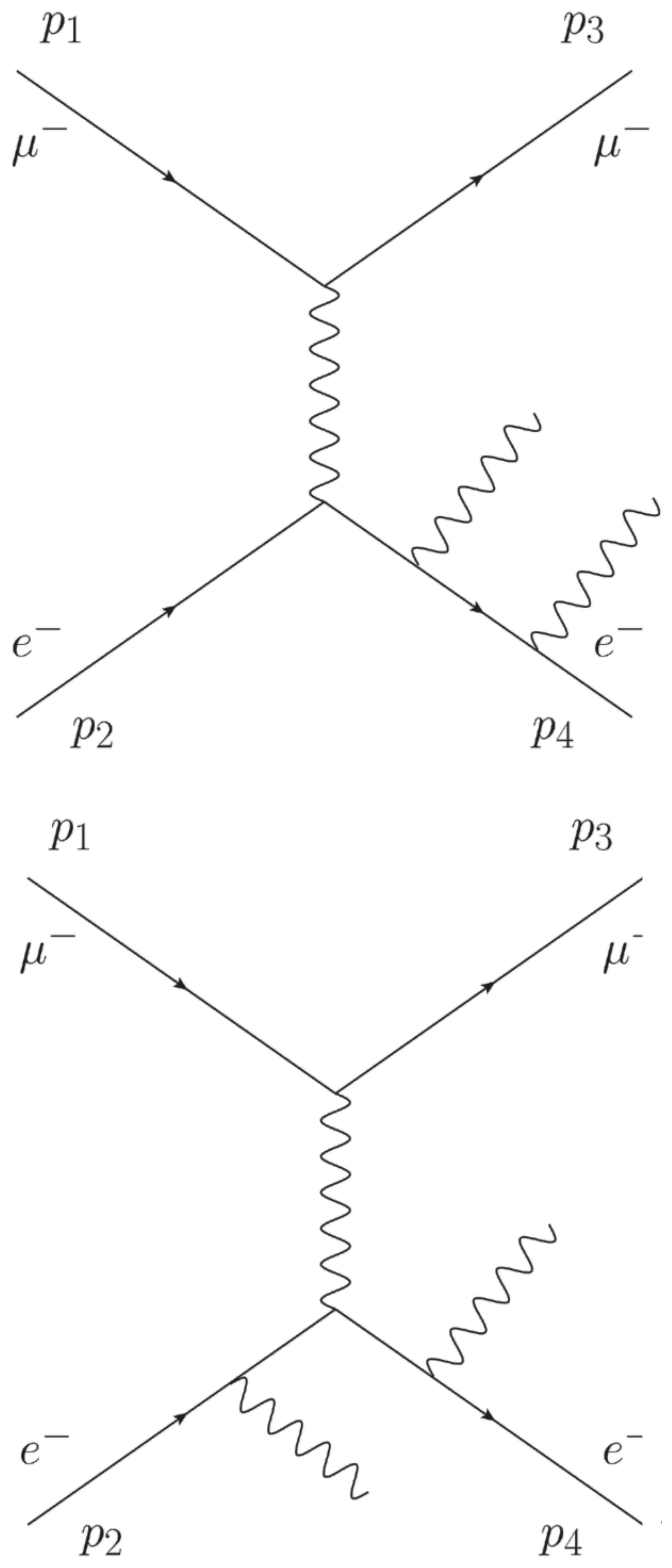
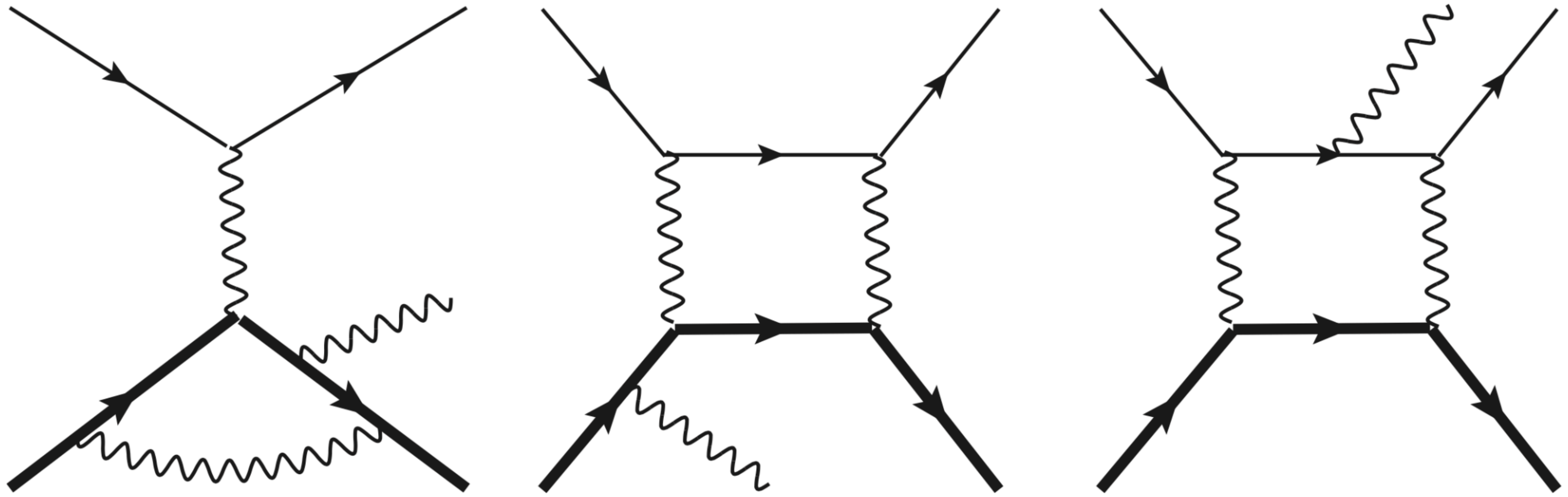




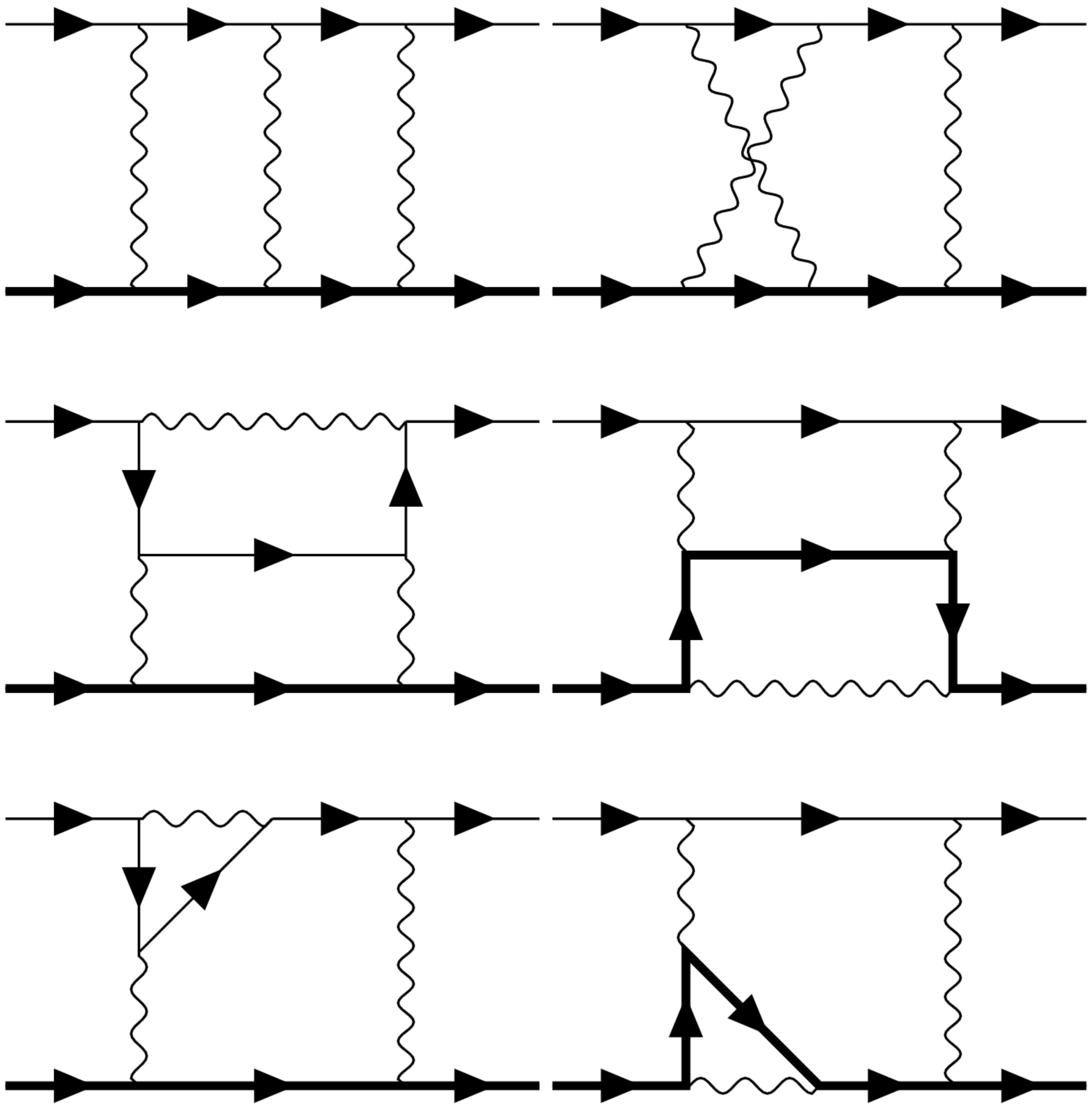


- 1-loop corrections to real photon emission exactly included (*e.g.* pentagons)
- Double real emission included exactly
- Small-photon mass prescription and slicing method
- Positive cross-checks with McMule

C. M. Carloni Calame, M. Chiesa, S. M. Hasan, G. Montagna, O. Nicrosini, and F. Piccinini, JHEP 11 (2020) 028

μe scattering at NNLO: approximate photonic I

- Of the two-loop virtual diagrams with a virtual photon insertion on top of NLO boxes, **only the IR part is included exactly**.
- The non-IR remnants are included approximately (YFS)



C. M. Carloni Calame, M. Chiesa, S. M. Hasan, G. Montagna, O. Nicrosini, and F. Piccinini, JHEP 11 (2020) 028

μe scattering at NNLO: approximate photonic II

D.R. Yennie, S, C. Frautschi, H. Suura, Annals Phys. 13 (1961), 379-452

Tree level

YFS IR factor

1-loop non-IR remnant

$$\mathcal{M}^{\alpha^0} = \mathcal{T}$$

$$\mathcal{M}^{\alpha^1} = Y\mathcal{T} + \mathcal{M}^{\alpha^1,R}$$

$$\mathcal{M}^{\alpha^2} = \frac{1}{2}Y^2\mathcal{T} + Y\mathcal{M}^{\alpha^1,R} + \mathcal{M}^{\alpha^2,R}$$

$$= -\frac{1}{2}Y^2\mathcal{T} + Y\mathcal{M}^{\alpha^1} + \mathcal{M}^{\alpha^2,R}$$

Exact *n*-loop amplitudes

2-loop non-IR remnant

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μe scattering at NNLO: approximate photonic II

$$\widetilde{\mathcal{M}}^{\alpha^2} = \underbrace{\mathcal{M}_e^{\alpha^2} + \mathcal{M}_\mu^{\alpha^2} + \mathcal{M}_{e\mu,\,1L\times 1L}^{\alpha^2}}_{\text{Exact}} + \underbrace{\frac{1}{2} Y_{e\mu}^2 \mathcal{T} + Y_{e\mu}\left(Y_e + Y_\mu\right)\mathcal{T} + (Y_e + Y_\mu)\mathcal{M}_{e\mu}^{\alpha^1,R} + Y_{e\mu}\mathcal{M}^{\alpha^1,R}}_{\text{YFS approximated}}$$

Exact

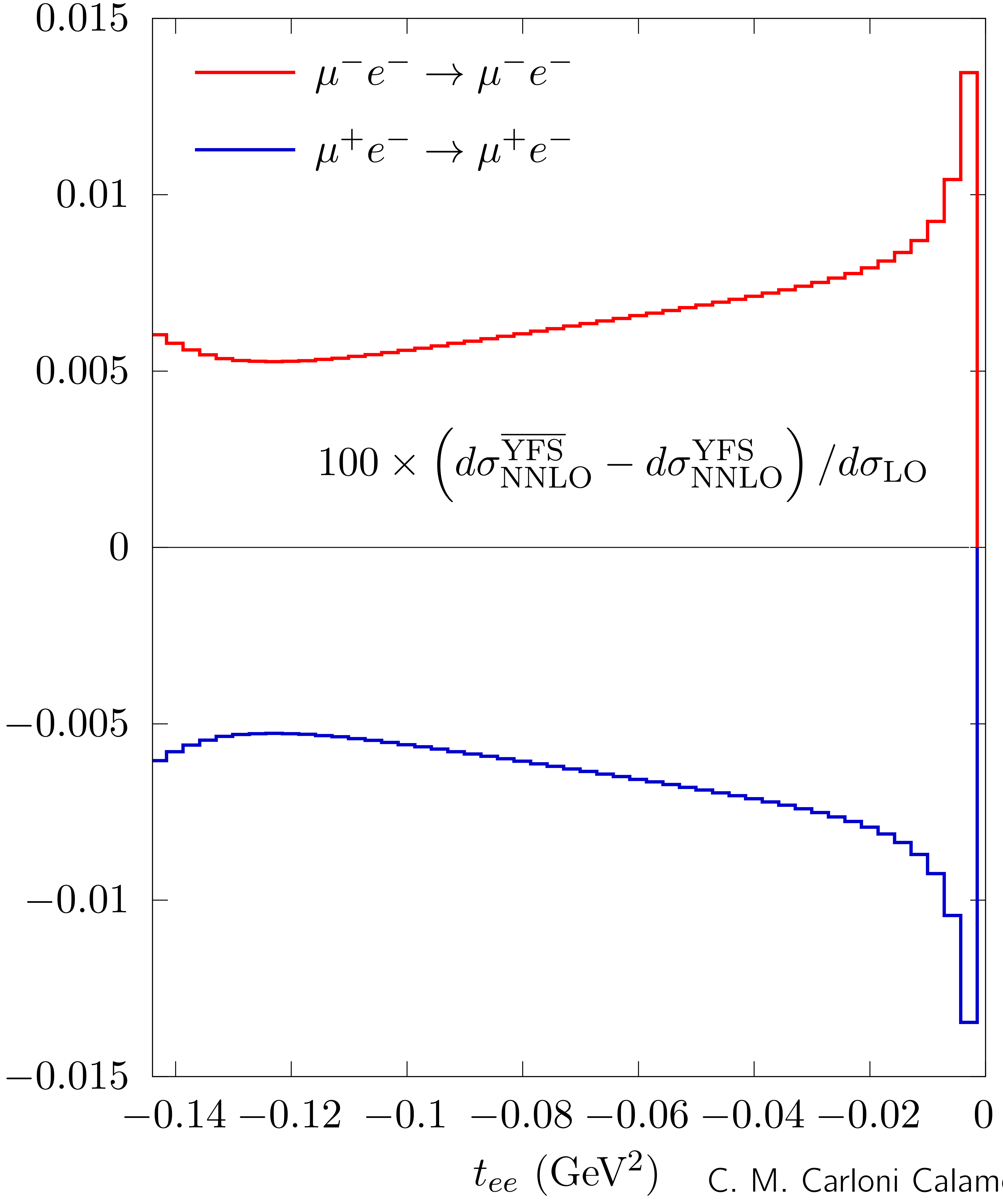
- $\mathcal{M}_{e\,(\mu)}^{\alpha^2}$: two virtual γ are both attached to the electron (muon) leg
- $\mathcal{M}_{e\mu,\,1L\times 1L}^{\alpha^2}$: 1 virtual γ is attached to each leg

YFS approximated

- This contains all the IR part
- It misses the non-IR remnant of the two loop diagrams with two γ connecting the e and μ legs
- $\mathcal{M}^{\alpha^2,R} \equiv 0$

$$Y_{ij} = \begin{cases} \frac{1}{8}\frac{\alpha}{\pi}Q_i^2\left[B_0\left(0, m_i^2, m_i^2\right) - 4m_i^2 C_0\left(m_i^2, 0, m_i^2, \lambda^2, m_i^2, m_i^2\right)\right] & \text{for } i = j \\ \frac{\alpha}{\pi}Q_iQ_j\vartheta_i\vartheta_j\left[p_i \cdot p_j\ C_0\left(m_i^2, (\vartheta_i p_i + \vartheta_i p_j)^2, m_j^2, \lambda^2, m_i^2, m_j^2\right) + \right. & \\ \qquad\qquad \left. + \frac{1}{4}B_0\left((\vartheta_i p_i + \vartheta_j p_j)^2, m_i^2, m_j^2\right)\right] & \text{for } i \neq j \end{cases}$$

YFS approximation: error estimation



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Educated guess

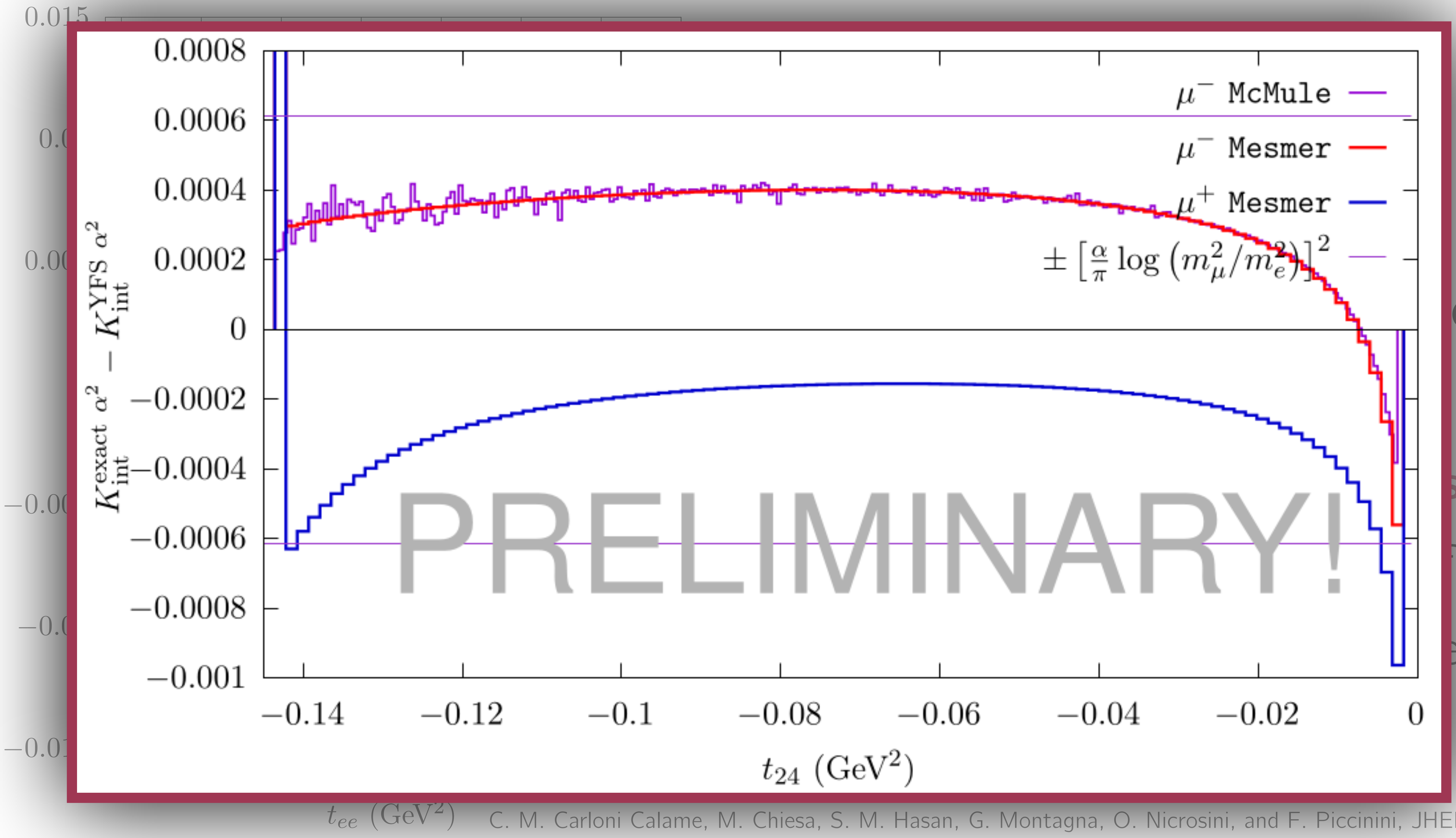
Square the difference between NLO-YFS and NLO-exact

$$\delta_{\mathrm{NNLO}} \sim \delta^2_{\mathrm{NLO}} = \left(\frac{\alpha}{\pi}\right)^2 \ln^2(m_\mu^2/m_e^2) = 6 \times 10^{-4}$$

Another educated guess

Turn off some non-IR terms different from $\mathscr{M}^{\alpha^2,R}$, *e.g.* some finite terms in the C_0 functions

YFS approximation: error estimation



NLO-

6×10^{-4}

fferent

erms in

t_{ee} (GeV^2)

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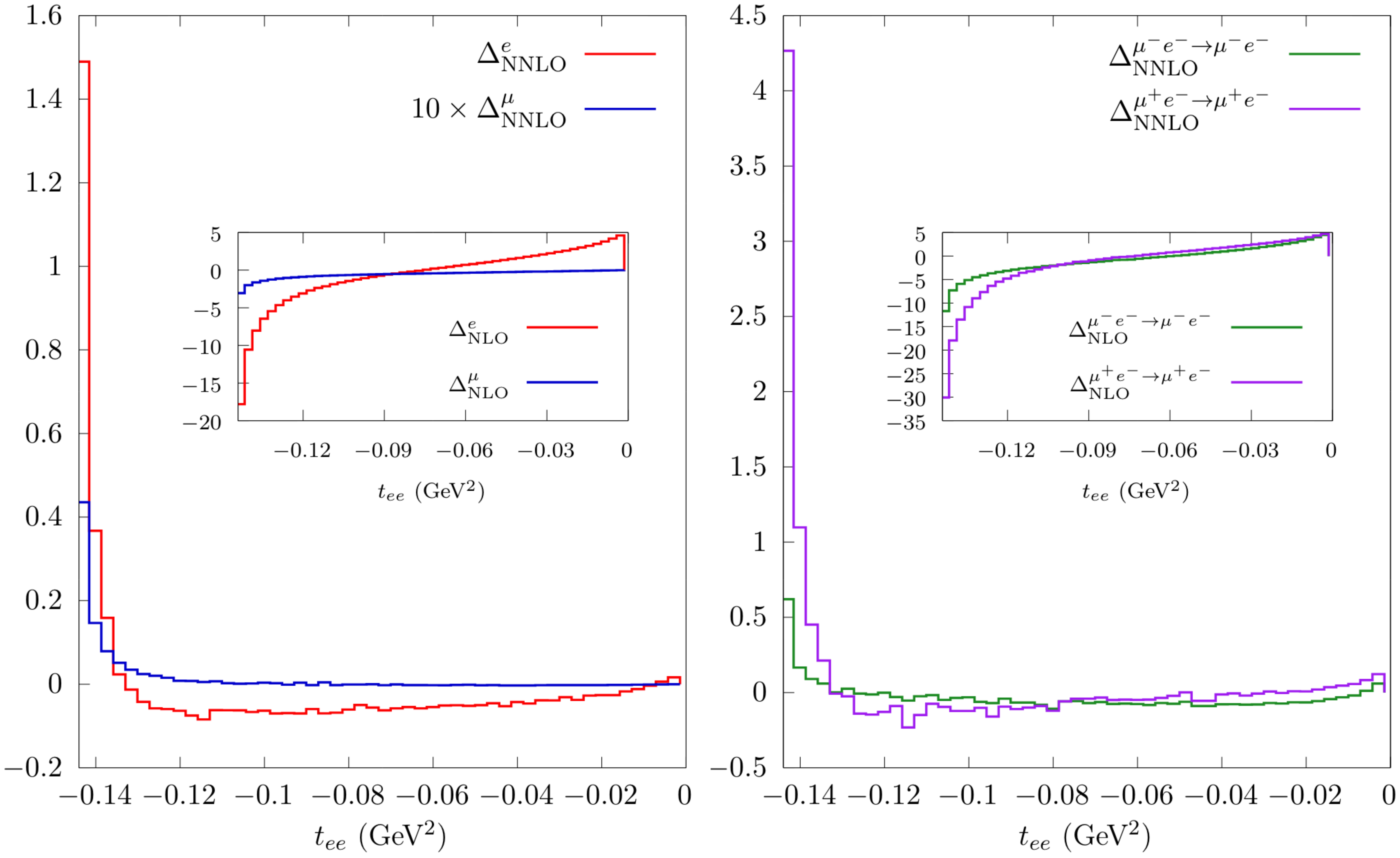
Results at NNLO photonic: t_{ee} I



Acceptance cut

$$\Delta_i^{\mathrm{NNLO}} = \frac{d\sigma_i^{\mathrm{NNLO}} - d\sigma_i^{\mathrm{NLO}}}{d\sigma_i^{\mathrm{LO}}} \times 100$$

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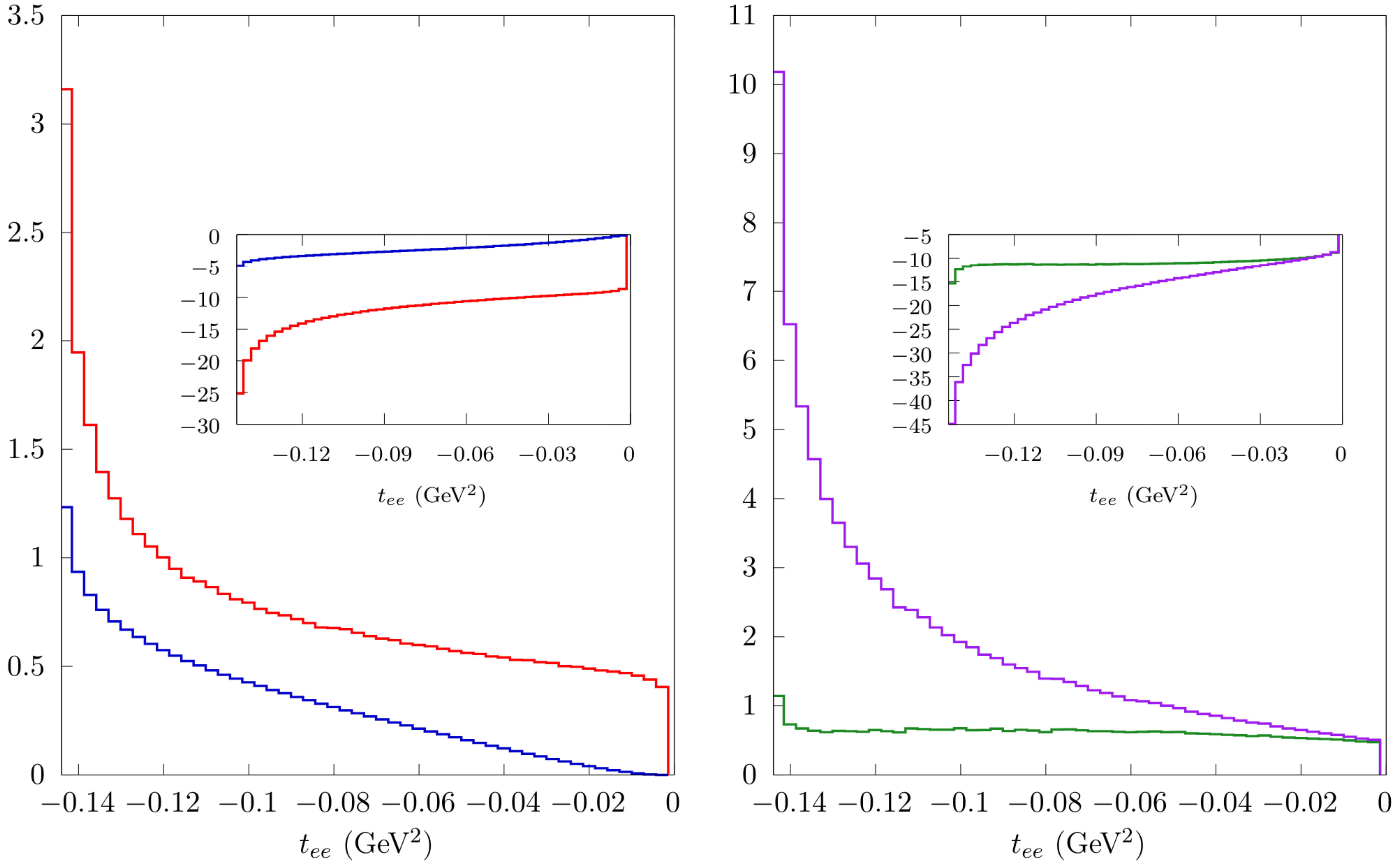
Results at NNLO photonic: t_{ee} II



Acoplanarity cut

$$\Delta_i^{\mathrm{NNLO}} = \frac{d\sigma_i^{\mathrm{NNLO}} - d\sigma_i^{\mathrm{NLO}}}{d\sigma_i^{\mathrm{LO}}} \times 100$$

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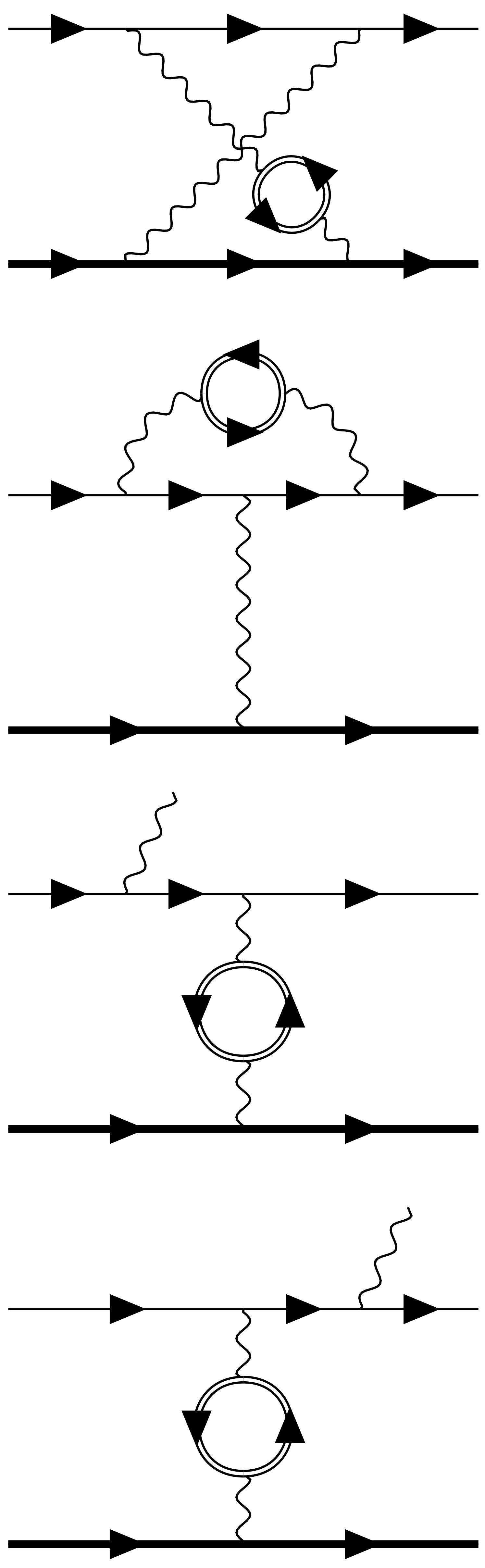
μe scattering at NNLO: leptonic

Complete fixed-order NNLO QED corrections with virtual leptonic pair

$$d\sigma^{\alpha^2}_{N_f} = d\sigma^{\alpha^2}_{\text{virt}} + d\sigma^{\alpha^2}_{\gamma} + d\sigma^{\alpha^2}_{\text{real}}$$

- Integration over z is performed **numerically** with MC techniques
- Master Integral techniques for a subset of such diagrams to cross-check results
- Interplay between real photon radiation and leptonic loop insertions
- **IR divergences** are cancelled by a sub-set of the virtual contributions. $\dfrac{-ig_{\mu\nu}}{q^2+i\varepsilon} \to -ig_{\mu\nu}\left(\dfrac{\alpha}{3\pi}\right)\displaystyle\int_{4m_\ell^2}^{\infty}\dfrac{dz}{z}\dfrac{1}{q^2-z+i\varepsilon}\left(1+\dfrac{4m_\ell^2}{2z}\right)\sqrt{1-\dfrac{4m_\ell^2}{z}}$



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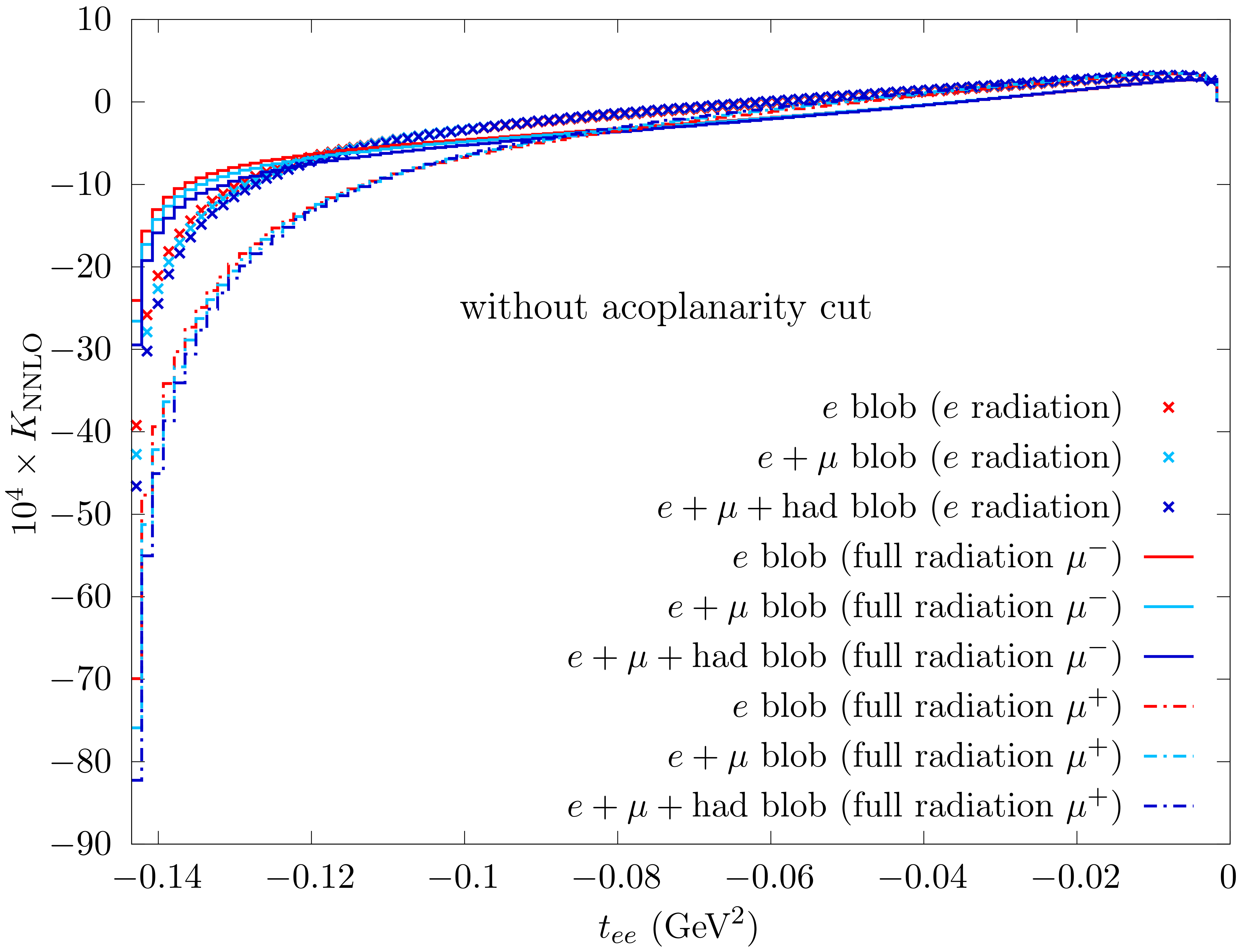
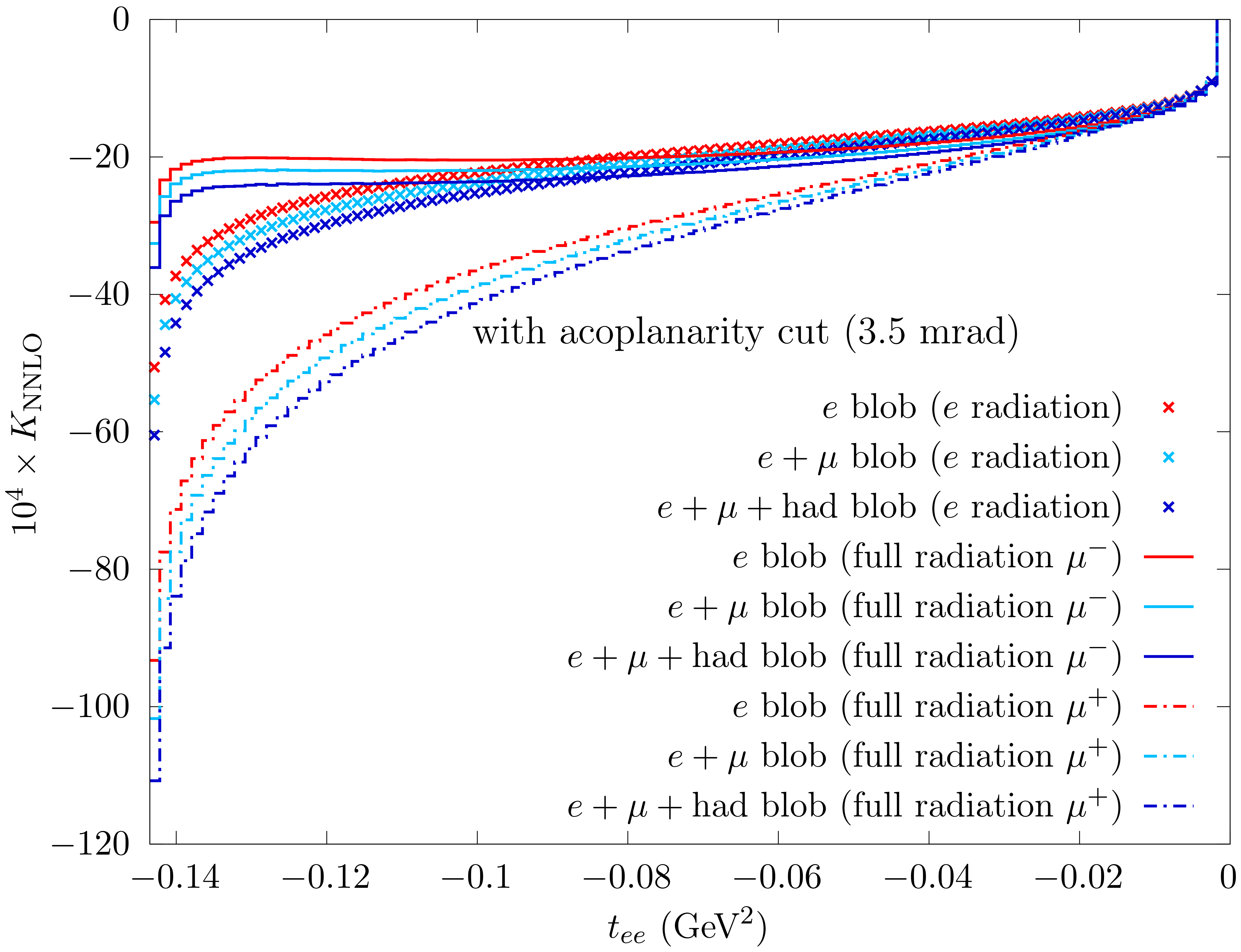
Results at NNLO leptonic: t_{ee}





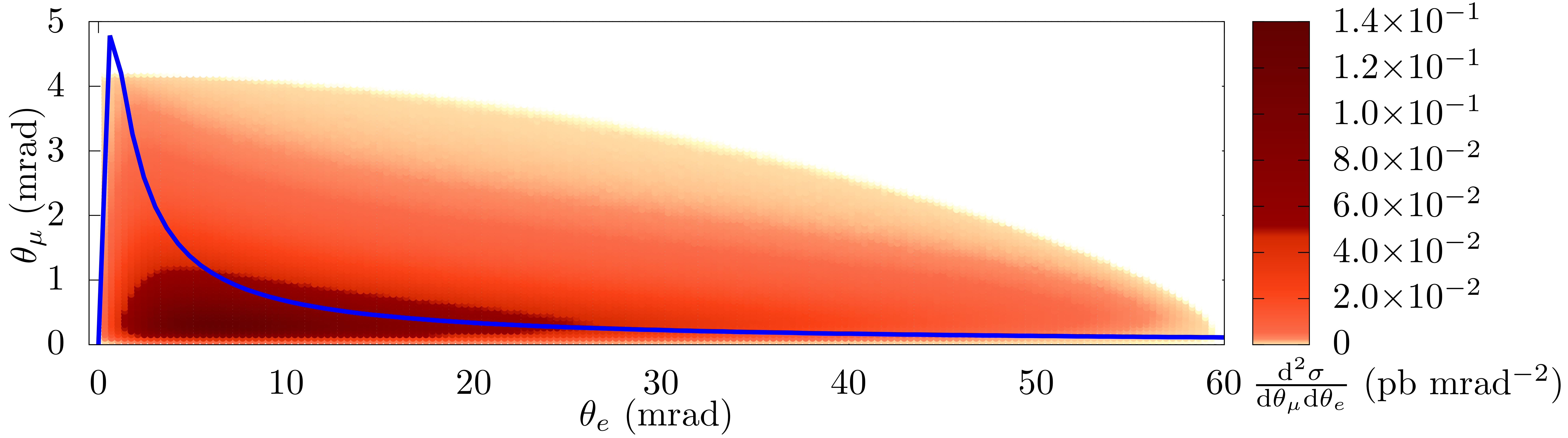
$$K_{\text{NNLO}} = \frac{d\sigma^{\alpha^2}_{N_f}}{d\sigma_{\text{NLO}}}$$

E. Budassi, C. M. Carloni Calame, M. Chiesa, C. L. Del Pio, S. M. Hasan, G. Montagna, O. Nicrosini, F. Piccinini. JHEP 11 (2021), 098

Backgrounds in MESMER

Single π_0 production: motivation

- Reliable estimation of background processes for MUonE: real and virtual hadronic contributions
- Virtual hadronic contributions had been studied previously
- Real hadronic pair production does not contribute given the limited MUoNE available phase space
- Potentially $\mu e \to \mu e \pi_0 \to \mu e \gamma\gamma$ is an important source of reducible background



E. Budassi, C. M. Carloni Calame, C. L. Del Pio, F. Piccinini. Phys. Lett. B 829 (2022) 137138.

Single π_0 production: theory

Interaction Lagrangian

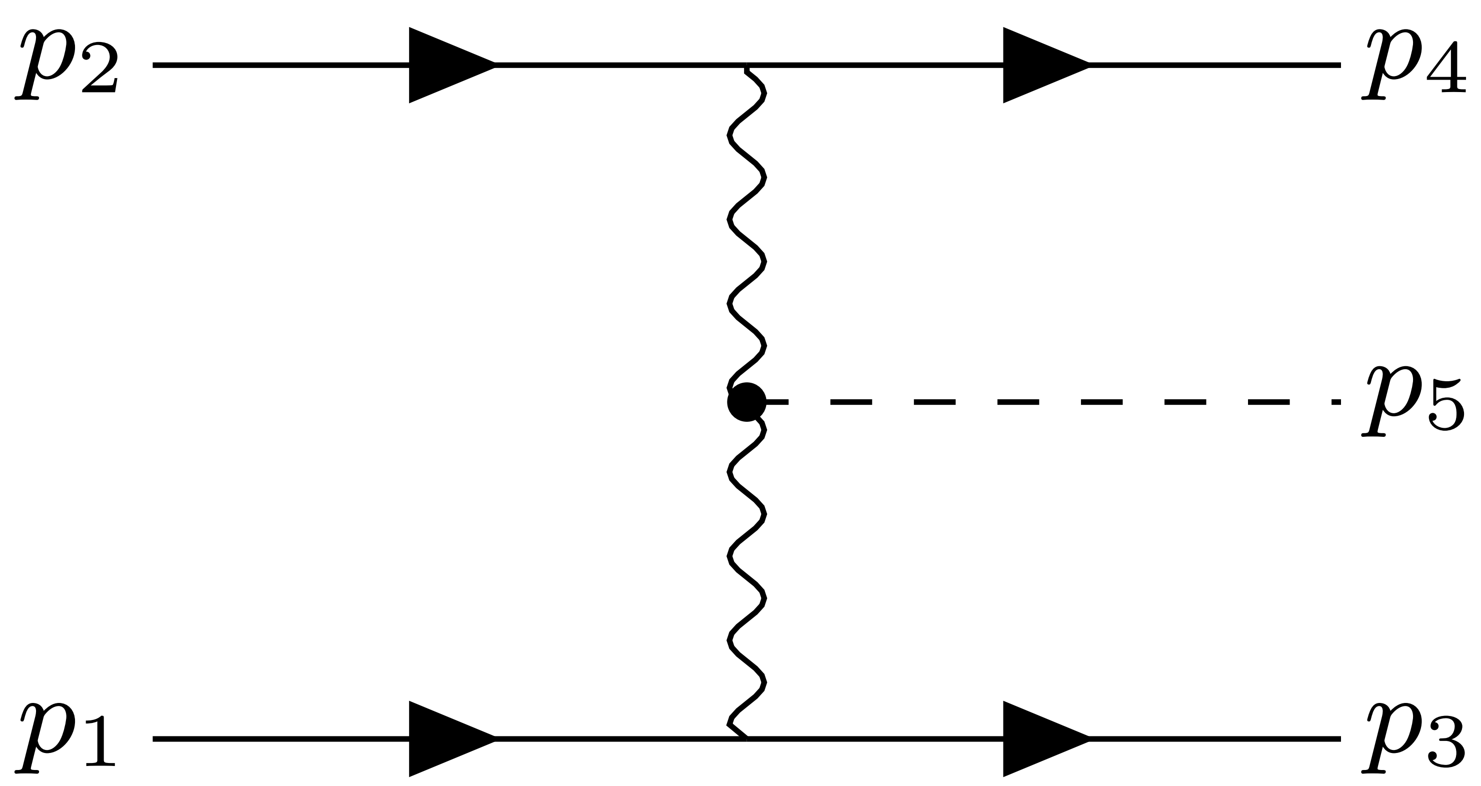
$$\mathscr{L}_{\rm I} = \frac{g}{2!}\varepsilon^{\mu\nu\kappa\lambda}F_{\mu\nu}F_{\kappa\lambda}\phi_\pi$$

where

$$g^2 = \frac{4\pi\Gamma_{\pi_0\to\gamma\gamma}}{m_{\pi_0}^3}, \qquad \Gamma_{\pi_0\to\gamma\gamma} = \frac{\alpha^2 m_{\pi_0}^3}{64\pi^3 f_\pi^2}$$

- Calculation of matrix element
- Calculation of the exact 3-body phase space
- Cross checks with EKHARA and older calculations
- Use of a form factor $F_{\pi_0\gamma^*\gamma^*}$

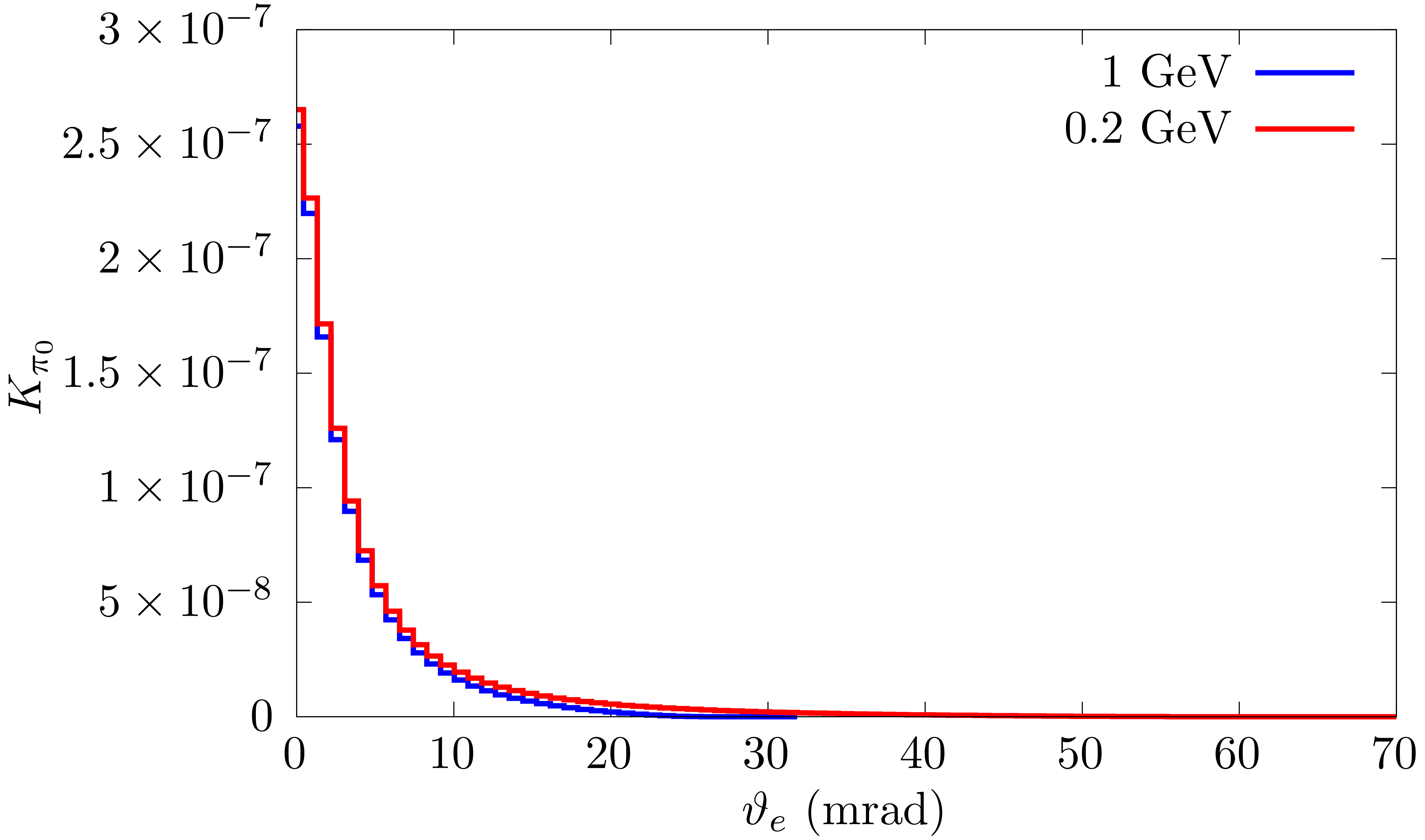
$$f_\pi = 0.092388\,\text{GeV}$$
$$\Gamma_{\pi_0\to\gamma\gamma} = 7.731\,\text{eV}$$
$$m_\pi = 134.9766\,\text{MeV}$$



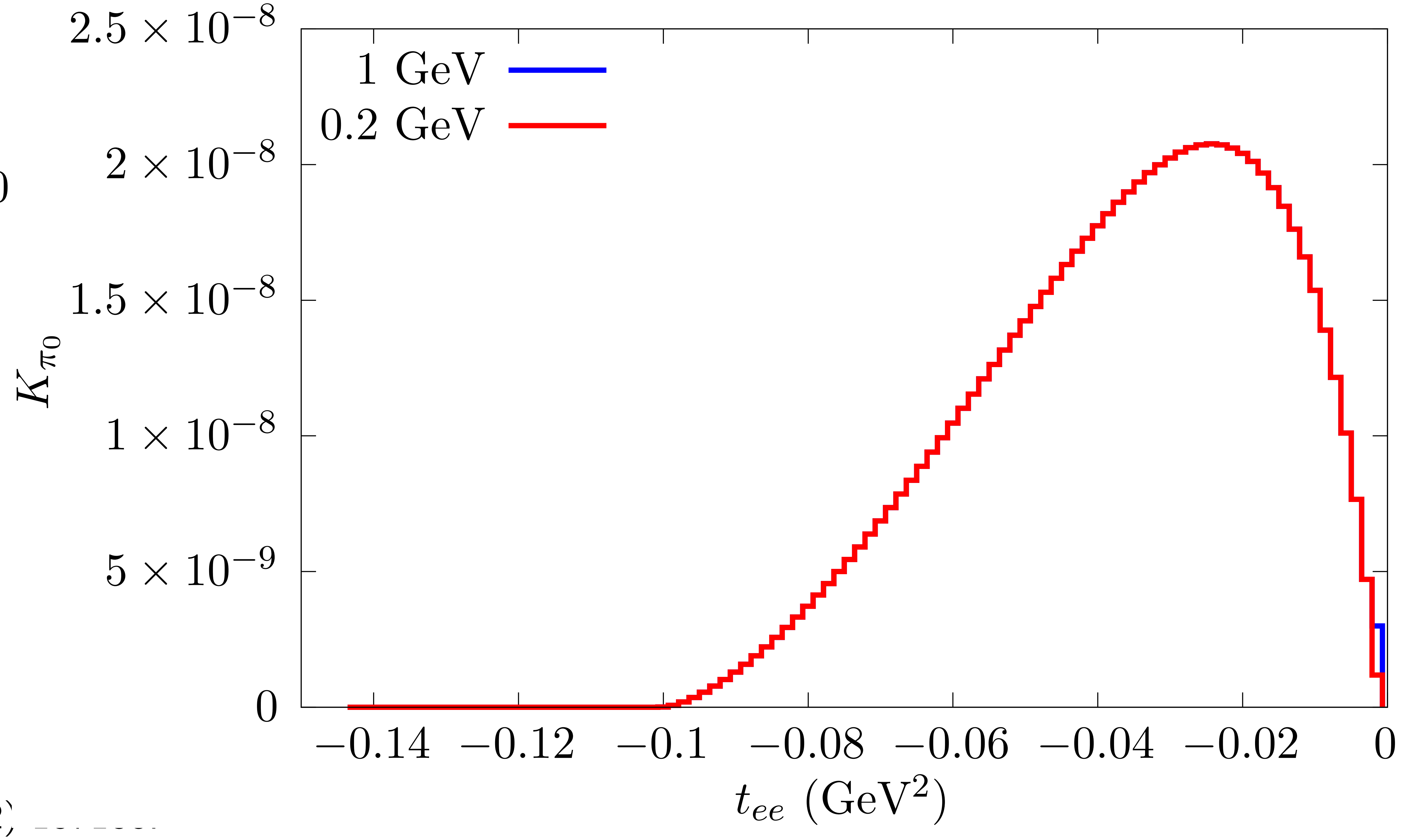
Brodsky, S. J., et al., Phys. Rev. D 4 (1971) 1532-1557
Czyz, H., et al., Phys. Rev. D 97 (1) (2018) 016006
Czyz, H., et al., Comput. Phys. Commun. 182 (2011) 1338-1349
Czyz, H., et al., Comput. Phys. Commun. 234 (2019) 245-255

E. Budassi, C. M. Carloni Calame, C. L. Del Pio, F. Piccinini. Phys. Lett. B 829 (2022) 137138.

Single π_0 production: phenomenology



$$K_{\pi_0} = \frac{d\sigma_{\pi_0}}{d\sigma_{\mathrm{LO}}}$$



Negligible contribution in differential distributions

E. Budassi, C. M. Carloni Calame, C. L. Del Pio, F. Piccinini. Phys. Lett. B 829 (2022)

Things-you-might-want-to-remember

- MESMER can be used for the MUonE experiment as a MC event generator
- It calculates $\mu e \to \mu e$ with radiative corrections in QED up to NNLO with full mass dependence (except some non-IR bits at NNLO which are approximated)
- It calculates NLO weak corrections
- Some background processes for MUonE are calculated with MESMER (Andrea's talk)
- Among those, π_0 production is negligible at MUoNE.