Status of MUonE theory

F. Piccinini

INFN, Sezione di Pavia (Italy)

The Evaluation of the Leading Contribution to the Muon $g-2$

Mainz, 3-7 June 2024

Finanziato dall'Unione europea NextGenerationEU | Ministero dell'Università e della Ricerca | Italiadomani PIANO NAZIONALE DI RIPRESA E RESILIENZA | INFN

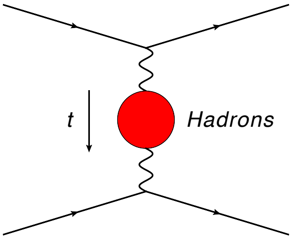
- ⋆ G. Abbiendi, C.M. Carloni Calame, U. Marconi, C. Matteuzzi, G. Montagna, O. Nicrosini, M. Passera, F. Piccinini, R. Tenchini, L. Trentadue, G. Venanzoni,
  Measuring the leading hadronic contribution to the muon g-2 via μe scattering
  Eur. Phys. J. C **77** (2017) no.3, 139 - arXiv:1609.08987 [hep-ph]
- ⋆ C. M. Carloni Calame, M. Passera, L. Trentadue and G. Venanzoni,
  A new approach to evaluate the leading hadronic corrections to the muon g-2
  Phys. Lett. B **746** (2015) 325 - arXiv:1504.02228 [hep-ph]

Master formula

$$a_\mu^{\rm HLO} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{\rm had}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

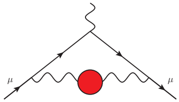
e.g. Lautrup, Peterman, De Rafael, Phys. Rept. 3 (1972) 193



- The hadronic VP correction to the running of α enters
  - ★ **$\Delta\alpha_{\rm had}(t)$ can be directly measured in a (single) experiment involving a space-like scattering process and $a_\mu^{\rm HLO}$ obtained through numerical integration**
    Carloni Calame, Passera, Trentadue, Venanzoni PLB 746 (2015) 325
  - ★ **A data-driven evaluation of $a_\mu^{\rm HLO}$, but with space-like data**

Kernel functions for a_μ^{HVP}

- LO: $\frac{\alpha}{\pi}(1-x)$



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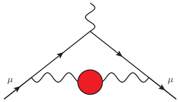
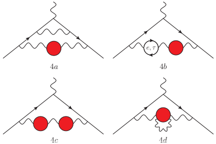


- NLO

E. Balzani, S. Laporta, M. Passera, Phys. Lett. B834 (2022) 137462

A.V. Nesterenko, J. Phys. G49 (2022) 5, 055001;

J. Phys. G50 (2022) 2, 029401



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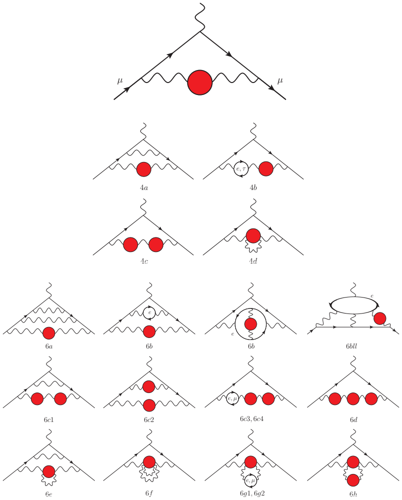

$\Longrightarrow$ talk by S. Laporta

Main challenge: precision on shapes of differential distributions at the 10ppm level

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Main sources of systematics on the theory side

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- **Radiative corrections to the signal**

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- **Predictions for Background processes**

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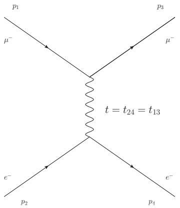
High precision Monte Carlo simulation tools required

First step towards precision: QED NLO and MC (2018)

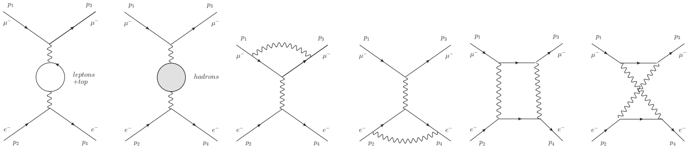


- analytical expression for tree level

$$\frac{d\sigma}{dt} = \frac{4\pi\alpha^2}{\lambda(s, m_\mu^2, m_e^2)} \left[\frac{(s - m_\mu^2 - m_e^2)^2}{t^2} + \frac{s}{t} + \frac{1}{2} \right]$$

- VP gauge invariant subset of NLO rad. corr.
- factorized over tree-level: $\alpha \to \alpha(t)$

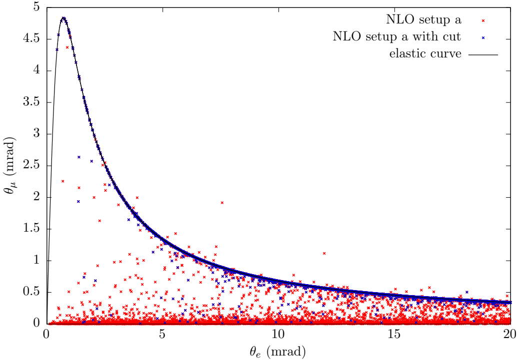
- QED NLO virtual diagrams and real emission diagrams with exact finite m_e and m_μ effects

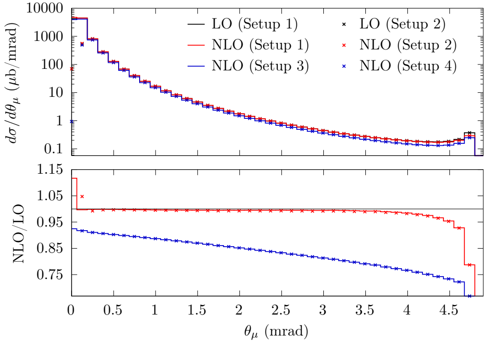


- **tree-level Z-exchange important** at the 10^{-5} level ($\sim tG_\mu/4\pi\alpha\sqrt{2}$ in the Fermi theory)
- SM weak RCs at most at a few 10^{-6} level, negligible

Alacevich et al. JHEP 02 (2019) 155

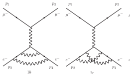
First realistic description of scattering events

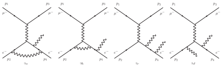




- many points fall out of the $2 \to 2$ correlation curve $\theta_\mu - \theta_e$ because of the radiative events
- NLO QED radiative corrections at the % level, enhanced by exclusive event selections

- **exact calculation of corrections along one lepton line with all finite mass effects**





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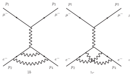

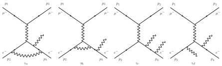

  - two independent calculations, with different IR singularities handling procedures (slicing and subtraction)

Carloni Calame et al., JHEP 11 (2020) 028,

P. Banerjee, T. Engel, A. Signer, Y. Ulrich, SciPost Phys. 9 (2020) 027

  - implemented in **`Mesmer`** and **`McMule`**, perfect numerical agreement

Second step, towards *photonic* radiative corrections at **NNLO** (2020)

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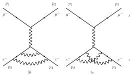

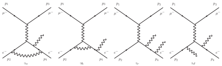

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  - implemented in **`Mesmer`** and **`McMule`**, perfect numerical agreement

- **NNLO with finite mass effects and approximate up-down interference in `Mesmer`**
  - interference of LO $\mu e \to \mu e$ amplitude with

+ many others

⇝ NNLO double-virtual amplitudes where at least 2 photons connect the e and μ lines are approximated according to the Yennie-Frautschi-Suura ('61) formalism to catch the IR divergent structure

Second step, *photonic* radiative corrections at **NNLO** (2023)

- **complete calculation of the amplitude $f^+f^- \to F^+F^-$ with $m_f = 0$, $m_F \neq 0$** R. Bonciani *et al.*, PRL 128 (2022)

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T. Engel, C. Gnendiger, A. Signer and Y. Ulrich, JHEP 02 (2019) 118

Y. Ulrich, *PoS* RADCOR2023 (2024) 077

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- **Next-to-soft stabilisation**, to obtain numerical stability in real-virtual corrections with soft and/or collinear photon configurations

T. Engel, A. Signer, Y. Ulrich, JHEP 04 (2022) 097; T. Engel, JHEP 07 (2023) 177

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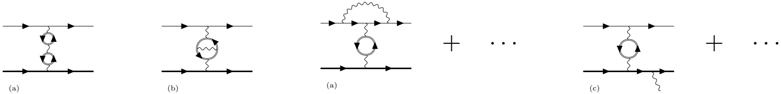
- with the above ingredients
  - **NNLO calculation neglecting terms of $\mathcal{O}(m_e^2/Q^2)$ in `McMule`**

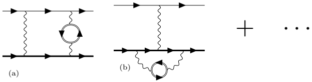
A. Broggio et al., JHEP 01 (2023) 112

$\Longrightarrow$ talk by M. Rocco

- any lepton (and hadron) in the VP blobs
- interfered with $\mu e \to \mu e$ or $\mu e \to \mu e \gamma$ amplitudes



- interfered with $\mu e \to \mu e$ amplitude



- 2-loop integral evaluated with dispersion relation techniques in **`Mesmer`**

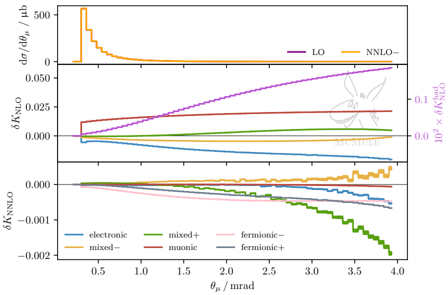
used e.g. in the past for Bhabha: Actis et al., Phys. Rev. Lett. 100 (2008) 131602; Carloni Calame et al., JHEP 07 (2011) 126

$$\frac{g_{\mu\nu}}{q^2 + i\epsilon} \to g_{\mu\nu} \frac{\alpha}{3\pi} \int_{4m_\ell^2}^{\infty} \frac{dz}{z} \frac{R_\ell(z)}{q^2 - z + i\epsilon} = g_{\mu\nu} \frac{\alpha}{3\pi} \int_{4m_\ell^2}^{\infty} \frac{dz}{z} \frac{1}{q^2 - z + i\epsilon} \left(1 + \frac{4m_\ell^2}{2z}\right) \sqrt{1 - \frac{4m_\ell^2}{z}}$$

- 2-loop integral evaluated (also) with hyperspherical method in **`McMule`** M. Fael, JHEP02 (2019) 027

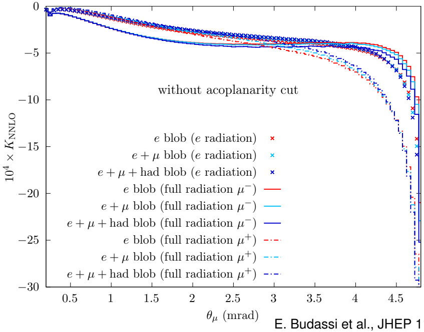
NNLO order of magnitude

McMule



A. Broggio et al., JHEP 01 (2023) 112

Mesmer



E. Budassi et al., JHEP 11 (2021) 098

NNLO order of magnitude

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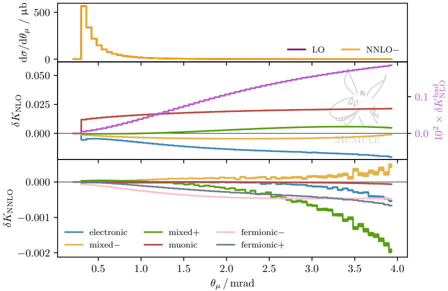
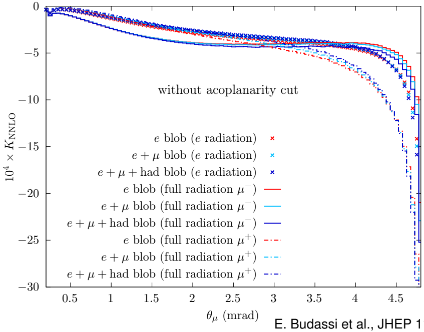


A. Broggio et al., JHEP 01 (2023) 112

Mesmer



E. Budassi et al., JHEP 11 (2021) 098

- **NNLO corrections at the $10^{-4} - 10^{-3}$ level**

NNLO order of magnitude

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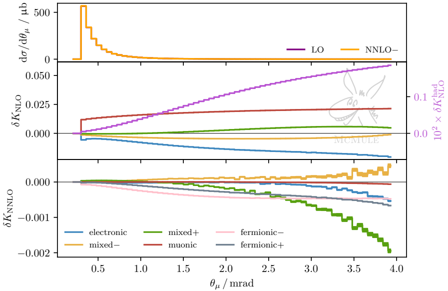

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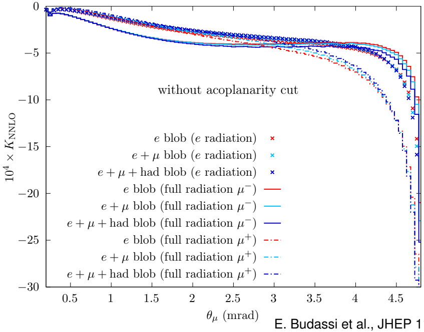


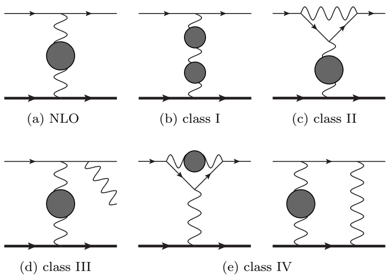
E. Budassi et al., JHEP 11 (2021) 098

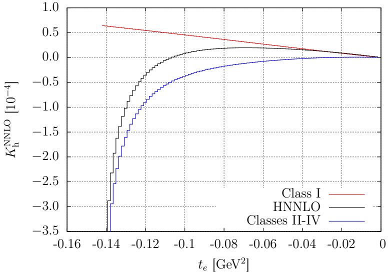
- **NNLO corrections at the $10^{-4} - 10^{-3}$ level**
- **eventually fixed order calculations need to be matched to resummation of higher order corrections, through PS techniques (e.g. BaBayaga) or YFS techniques (e.g. KKMC/SHERPA)**

$\Longrightarrow$ talk by A. Price

NNLO hadronic contributions (2019)

- using the dispersion relation approach





Fael, Passera, Phys. Rev. Lett. 122 (2019) 192001

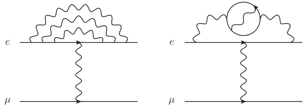
- corrections of the order of 10^{-4}
- hyperspherical integration method to calculate hadronic NNLO corrections, where the hadronic vacuum polarization is employed in the space-like region (used in **McMule**)

M. Fael, JHEP02 (2019) 027

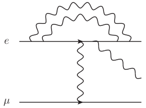
Towards N^{3}LO on the electron line

Y. Ulrich, N^{3}LO kick-off workstop/thinkstart, Durham, 3-5 August 2022

- All virtual (three loops)



- Single real emission (two loops)



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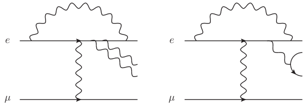
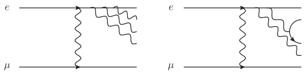


- Triple real



M. Fael, MUonE Collaboration Meeting, 16/05/2023, CERN

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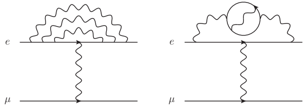

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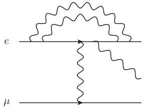

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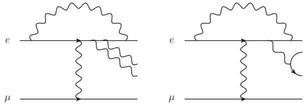

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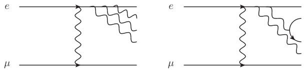

M. Fael, MUonE Collaboration Meeting, 16/05/2023, CERN

- **this contribution will allow improved perturbative predictions and more reliable theoretical uncertainty estimates**

Recent progress

- **the three-loop form factor with finite fermion mass is now available**

M. Fael, F. Lange, K. Schönwald, M. Steinhauser, Phys. Rev. Lett 128 (2022) 172003

M. Fael, F. Lange, K. Schönwald, M. Steinhauser, Phys. Rev.D 106 (2022) 034029

M. Fael, F. Lange, K. Schönwald, M. Steinhauser, Phys. Rev.D 107 (2023) 094017

- **All order subtraction scheme FKS$^\ell$ availale**

T. Engel, A. Signer, Y. Ulrich, JHEP 01 (2020) 085

- **very recent generalisation of the LBK theorem to multi-photon emission** $\Longrightarrow$ extension of next-to-soft stabilisation to multiple radiation

T. Engel, JHEP 03 (2024) 004

- **real-virtual-virtual corrections recently recalculated with $m_e \to 0$**

S. Badger, J. Krys, R. Moodle, S. Zoia, JHEP 11 (2023) 041

V.S. Fadin, R.N. Lee, JHEP 11 (2023) 148

Fixed target experiment $\Longrightarrow$ bound electron effects

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- **very recently estimated**

R. Plestid and M.B. Wise, arXiv:2403.12184

Fixed target experiment $\Longrightarrow$ bound electron effects

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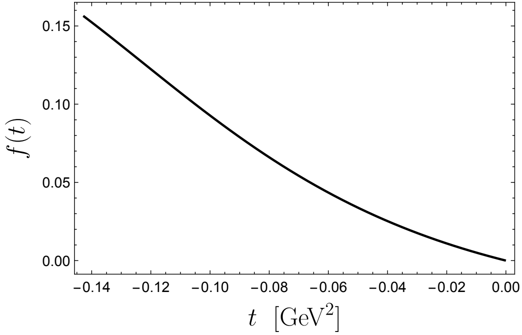
R. Plestid and M.B. Wise, arXiv:2403.12184

- for C

$$\frac{1}{\sigma}\frac{d\sigma}{dt} = \frac{1}{\sigma^0}\frac{d\sigma^0}{dt}\left(1 - K f(t)\right)$$

  - $K = 4.5 \cdot 10^{-4}$, scaling as $1/Z_A$



$\Longrightarrow$ talk by R. Plestid today

Backgrounds

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- **pion pair production forbidden** kinematically with the available $\sqrt{s}$

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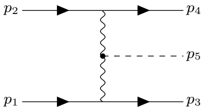
- **single π^0 production possible**



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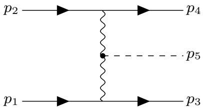

- **π^0 production** calculated and shown to be **well below 10^{-5}** w.r.t. $\mu e \to \mu e$

E. Budassi et al., PLB 829 (2022) 137138

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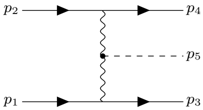

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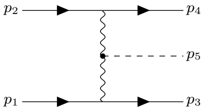
- **lepton pair production**

Backgrounds

- **pion pair production forbidden** kinematically with the available $\sqrt{s}$
- **single π^0 production possible**

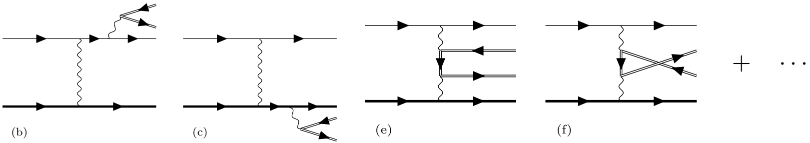


- **π^0 production** calculated and shown to be **well below 10^{-5}** w.r.t. $\mu e \to \mu e$

E. Budassi et al., PLB 829 (2022) 137138

- **lepton pair production**
  - $\mu^\pm e^- \to \mu^\pm e^- \ell^+ \ell^-$
  - $\mu^\pm N \to \mu^\pm N \ell^+ \ell^-$

- it also contributes at NNLO accuracy



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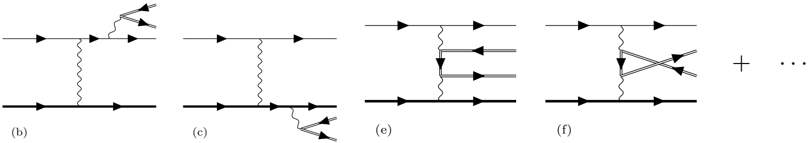

- the emission of an extra electron pair $\mu e \to \mu e\, e^+ e^-$ is potentially a dramatically large background, because of the presence of "peripheral" diagrams which develop powers of collinear logarithms upon integration

G. Racah, Il Nuovo Cimento 14 (1937) 83-113; L.D. Landau, E.M. Lifschitz, Phys. Z. Sowjetunion 6 (1934) 244; H.J. Bhabha, Proc. Roy. Soc. Lond. A152 (1935) 559;

R.N. Lee, A.A. Lyubyakin, V.A. Smirnov, Phys. Lett. B 848 (2024) 138408

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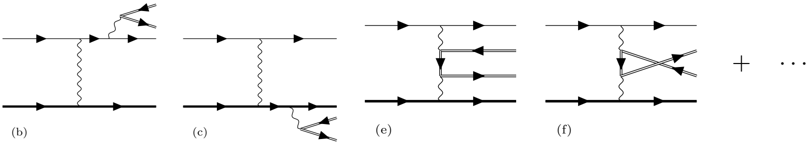

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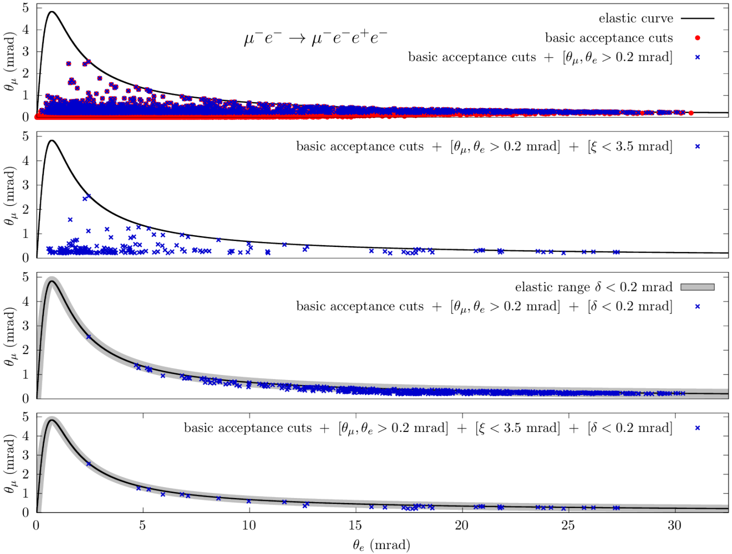
R.N. Lee, A.A. Lyubyakin, V.A. Smirnov, Phys. Lett. B 848 (2024) 138408

- **$\mu^\pm e^- \to \mu^\pm e^- \ell^+ \ell^-$ calculated with finite mass effects and implemented in `Mesmer`**

simulation of $5 \cdot 10^5$ points of $\mu^\pm e^- \to \mu^\pm e^- \ell^+ \ell^-$



Real pair emission from scattering on nucleus: $\mu^\pm N \to \mu^\pm N \ell^+ \ell^-$

G. Abbiendi et al., Phys. Lett B854 (2024) 138720

- **it can mimic the signal if one particle is not reconstructed or two tracks overlap within resolution**

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A.G. Bogdanov et al., IEEE transactions on nuclear science, 53, n. 2, April 2006

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$\Longrightarrow$ **a dedicated calculation implemented in the Monte Carlo generator `Mesmer`**

- **approximation: scattering on the external nucleus field**

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- **finite extension of the nucleus through a form factor**

$$F_Z(q) = \frac{1}{Ze} \int_0^\infty dr\, r^2 \rho_Z(r) \frac{\sin(qr)}{qr}$$

  - q : momentum transferred to the nucleus
  - ρ_Z : nuclear charged density

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- **different models for charge density**

J. Heeck, R. Szafron, Y. Uesaka, PRD 105 (2022) 053006

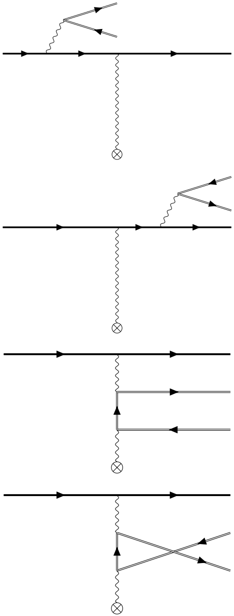
  - $F_Z(q) = 1$ (conservative)
  - 1 parameter Fermi model (1pF)

$$\rho_Z(r) = \frac{\rho_0}{1 + \exp \frac{r-c}{z}}$$

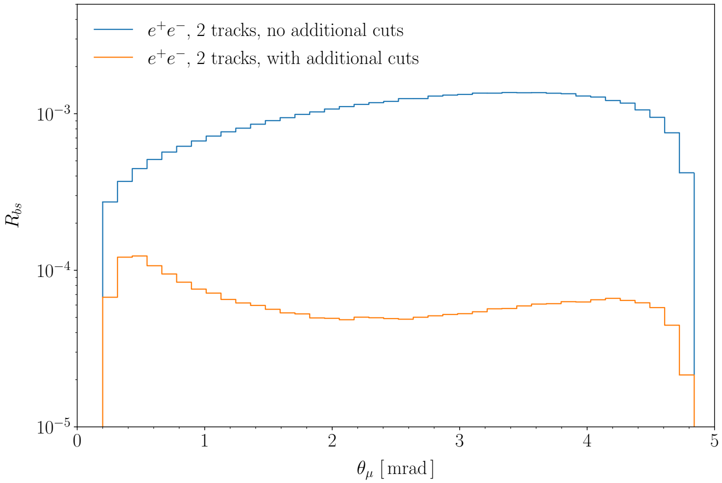
  - Fourier Bessel expansion (FB)

$$\begin{aligned} \rho_Z(r) &= \sum_k^n a_k\, j_0\left(\frac{k\pi r}{R}\right), \quad r \geq R \\ &= 0 \qquad\qquad\qquad\qquad > R \end{aligned}$$

  - modified-harmonic oscillator model



Background/signal ratio



G. Abbiendi, E. Budassi, C.M. Carloni Calame, A. Gurgone, F.P., Phys.Lett.B 854 (2024) 138720

$\Longrightarrow$ talk by A. Gurgone

Possible New Physics contamination in the $\Delta\alpha(t)$ determination?

A. Masiero, P. Paradisi and M. Passera, Phys. Rev. D102 (2020) 075013

P.S.B. Dev, W. Rodejohann, X.-J. Xu and Y. Zhang, JHEP 05 (2020) 053

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HVP determination with MUonE data will be robust against New Physics

Possible New Physics studies with MUonE (in complementary regions to $\Delta\alpha_h$)

- **interesting proposals for NP searches at MUonE (new light mediators) in $2 \to 3$ processes**
  - invisibly decaying light Z' in $\mu e \to \mu e Z'$

    Asai et al., Phys. Rev. D106 (2022) 5
    - a relevant background can be $\mu e \to \mu e \pi^0$, in addition to $\mu e \to \mu e \gamma$
  - long-lived mediators with displaced vertex signatures $\mu e \to \mu e A' \to \mu e e^+ e^-$

    Galon et al., Phys.Rev.D 107 (2023) 095003
  - through scattering off the target nuclei $\mu N \to \mu N X \to \mu N e^+ e^-$

    Grilli di Cortona and E. Nardi, Phys. Rev. D105 (2022) L111701

Summary

- **Given its precision requirements, MUonE represents a challenge for**
  - **QED corrections**
  - **background calculation**

- **at present we have two independent Monte Carlo tools, `Mesmer` and `McMule` featuring**
  - NLO QED corrections
  - NNLO QED corections from single lepton legs
  - YFS inspired approximation to the full NNLO QED in **`Mesmer`**
  - full NNLO QED with electron "massification" in **`McMule`**
  - pair production in **`Mesmer`**
    - $\mu^\pm e^- \to \mu^\pm e^- \ell^+ \ell^-$
    - $\mu^\pm N \to \mu^\pm N \ell^+ \ell^-$

- **efforts for N^{3}LO started**

- **work in progress for matching with higher order QED corrections**

Theoretical progress, thanks also to past

- MUonE theory workshops
  - Theory Kickoff Workshop, Padova, 4-5 September 2017
  - MITP Workshop, Mainz 19-23 February 2018
  - $2^{\rm nd}$ Workstop/ThinkStart, Zürich, 4-7 February 2019
  - N^{3}LO kick-off workstop/thinkstart IPPP Durham, 3-5 August 2022
  - MITP Workshop, Mainz 14-18 November 2022
- Five General MUonE Collaboration Meetings

A collection of references on calculation developments

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- Engel, JHEP 07 (2023) 177
- Badger et al., JHEP 11 (2023) 041
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- Ahmed et al., JHEP 01 (2024) 010
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