The standard model prediction of the muon g-2

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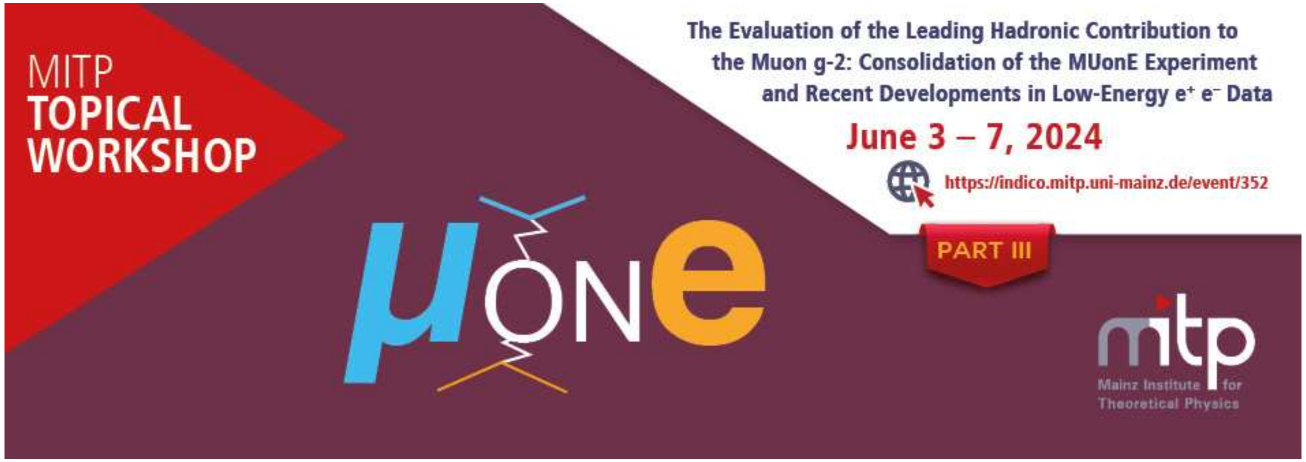


MITP
TOPICAL
WORKSHOP

The Evaluation of the Leading Hadronic Contribution to
the Muon g-2: Consolidation of the MUonE Experiment
and Recent Developments in Low-Energy e⁺ e⁻ Data

June 3 – 7, 2024

https://indico.mitp.uni-mainz.de/event/352

PART III

OUTLINE

Introduction

QED contribution

EW contribution

QCD contribution

Summary

OUTLINE

Introduction

QED contribution

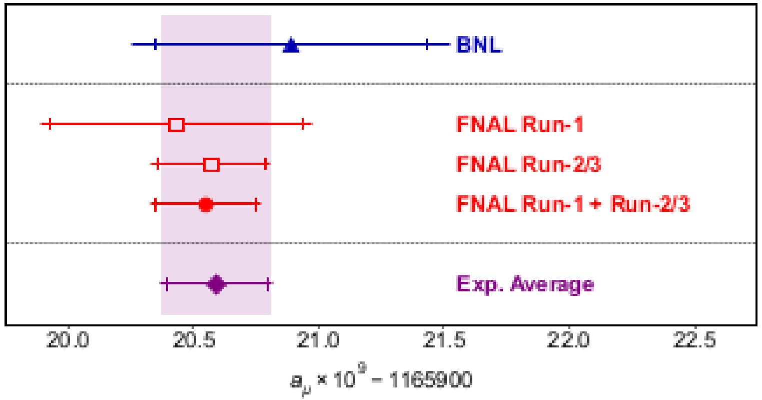
EW contribution

QCD contribution cf. also talks by G. Colangelo, D. Giusti, P. Stoffer

Summary

INTRODUCTION

Experimental situation



Analysis of data collected during run 1, run 2 and run 3 have been published

B. Abi et al. [Muon g-2 Coll.], PRL 126, 120801 (2021)

D. P. Aguillard et al. [Muon g-2 Coll.], PRL 131, 161802 (2023)

D. P. Aguillard et al. [Muon g-2 Coll.], arXiv:2402.15410 [hep-ex]

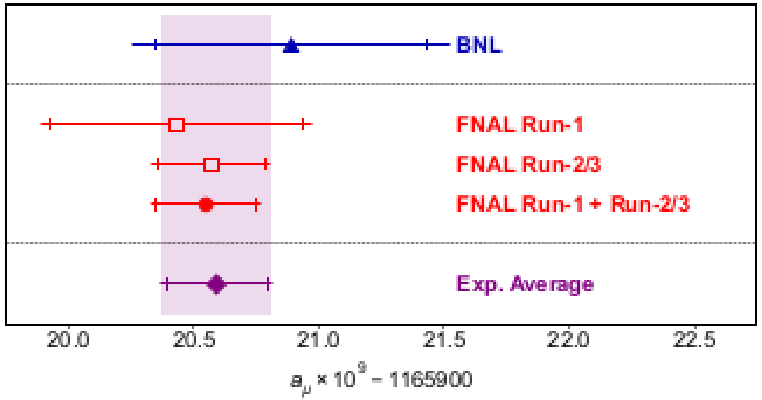
Confirmation of BNL result

Uncertainty already reduced by a factor ∼3

cf. talk by A. Driutti

INTRODUCTION

Experimental situation



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D. P. Aguillard et al. [Muon g-2 Coll.], arXiv:2402.15410 [hep-ex]

World-average value

$$a_\mu^{\rm exp;WA} \quad = \quad 116\,592\,059(22) \cdot 10^{-11} \quad [0.19\,{\rm ppm}]$$

INTRODUCTION

Situation on the theory side: g-2 Theory Initiative

Collect, assess, compare,... theory contributions to SM prediction of the muon g-2

Steering Committee

- Gilberto Colangelo (Bern)
- Michel Davier (Orsay) co-chair
- Aida El-Khadra (UIUC & Fermilab) chair
- Martin Hoferichter (Bern)
- Christoph Lehner (Regensburg University & BNL) co-chair
- Laurent Lellouch (Marseille)
- Tsutomu Mibe (KEK)
  J-PARC Muon g-2/EDM experiment
- Lee Roberts (Boston)
  Fermilab Muon g-2 experiment
- Thomas Teubner (Liverpool)
- Hartmut Wittig (Mainz)

https://muon-gm2-theory.illinois.edu

INTRODUCTION

Situation on the theory side: g-2 Theory Initiative

Working Groups and coordinators

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data-driven HVP

Achim Denig
Fedor Ignatov
Bogdan Malaescu

analytic HLbL

Hans Bijnens
Anton Rebhan

lattice HVP

Steve Gottlieb
Antonin Portelli

lattice HLbL

Luchang Jin
Harvey Meyer

INTRODUCTION

Situation on the theory side: g-2 Theory Initiative

Collect, assess, compare,... theory contributions to SM prediction of the muon g-2

Full and detailed account [up to June 15, 2020] given in the White Paper

T. Aoyama et al., Phys. Rep. 887, 1 - 166 (2020)

Needs to be updated (before final release of FNAL measurement)

QED contribution

QED contribution

loops with only photons and leptons

can be computed in perturbation theory (conceptually clear)

$$a_\mu{}^{\text{QED}} = C_\mu^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\mu^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\mu^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\mu^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\mu^{(10)}\left(\frac{\alpha}{\pi}\right)^5 + \cdots$$

need to go to high orders $\Delta a_\mu^{\text{exp}} \longrightarrow \sim 14 \cdot 10^{-11}$

$$(\alpha/\pi)^4 = 2.91\ldots \cdot 10^{-11} \qquad (\alpha/\pi)^5 = 6.76\ldots \cdot 10^{-14}$$

becomes technically challenging

multiflavour QED

$$C_\mu^{(2n)} = A_1^{(2n)} + A_2^{(2n)}(m_\mu/m_e) + A_2^{(2n)}(m_\mu/m_\tau) + A_3^{(2n)}(m_\mu/m_e, m_\mu/m_\tau)$$

QED contribution

- expressions for $A_1^{(2)}$, $A_1^{(4)}$, $A_2^{(4)}$, $A_1^{(6)}$, $A_2^{(6)}$, $A_3^{(6)}$ known analytically

- $A_1^{(8)}$ has also been evaluated! ($\longrightarrow a_e$) S. Laporta, Phys. Lett. B 772, 232 (2017)

- Mass dependent parts $A_2^{(8)}(m_\mu/m_e)$, $A_2^{(8)}(m_\mu/m_\tau)$, $A_3^{(8)}(m_\mu/m_e, m_\mu/m_\tau)$ evaluated numerically

T. Kinoshita and M. Nio, Phys. Rev. D 73, 053007 (2006); T. Aoyama et al., Phys. Rev. D 91, 033006 (2015)

and crossed-checked by independent QFT methods A. Kataev, Phys. Rev. D 86, 013019 (2012)

A. Kurz et al., Nucl. Phys. B 879, 1 (2014); Phys. Rev. D 92, 073019 (2015)

QED contribution

- $(\alpha/\pi)^5$

A few contributions are known analytically

S. Laporta, Phys. Lett. B 328, 522 (1994)
J.-P. Aguilar, D. Greynat, E. de Rafael, Phys. Rev. D 77, 093010 (2008)

Complete numerical results available

T. Kinoshita and M. Nio, Phys. Rev. D 73, 053007 (2006); T. Aoyama et al., Phys. Rev. D 78, 053005 (2008); D 78, 113006 (2008); D 81, 053009 (2010); D 82, 113004 (2010); D 83, 053002 (2011); D 83, 053003 (2011); D 84, 053003 (2011); D 85, 033007 (2012); Phys. Rev. Lett. 109, 111807 (2012); Phys. Rev. Lett. 109, 111808 (2012)

No systematic cross-checks even for mass-dependent contributions

New independent numerical evaluation of $A_1^{(10)}$ $(\longrightarrow a_e)$

S. Volkov, Phys. Rev. D 96, 096018 (2017); D 98, 076018 (2018); D 100, 096004 (2019); arXiv:2404.00649

$$A_1^{(10)}[\text{Volkov}] = 5.891(61) \quad \text{vs.} \quad A_1^{(10)}[\text{AHKN}] = 6.737(159)$$

discrepancy found in the contribution of graphs without fermion loops

QED contribution

$C_\mu^{(2)}$	0.5
$C_\mu^{(4)}$	$0.765\,857\,425(17)$
$C_\mu^{(6)}$	$24.050\,509\,96(32)$
$C_\mu^{(8)}$	$130.878\,0(61)$
$C_\mu^{(10)}$	$750.72(93)$

QED contribution

$C_\mu^{(2)}$	0.5
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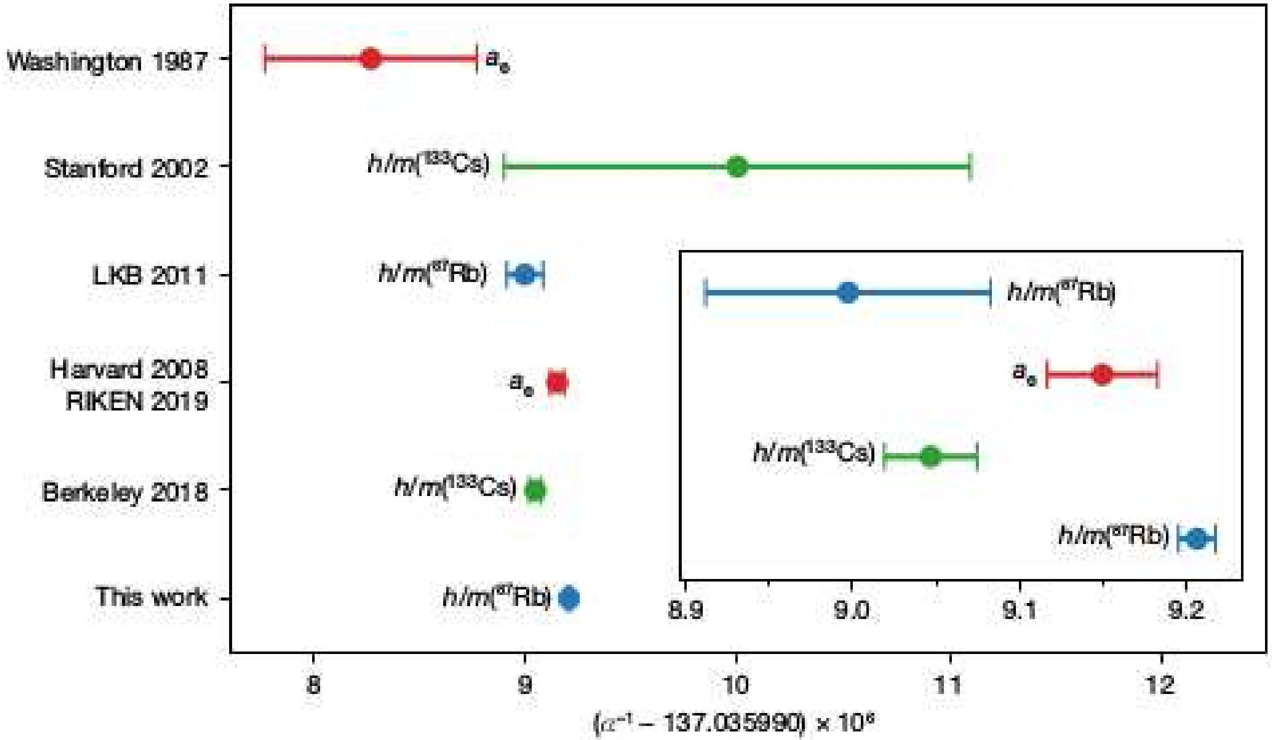
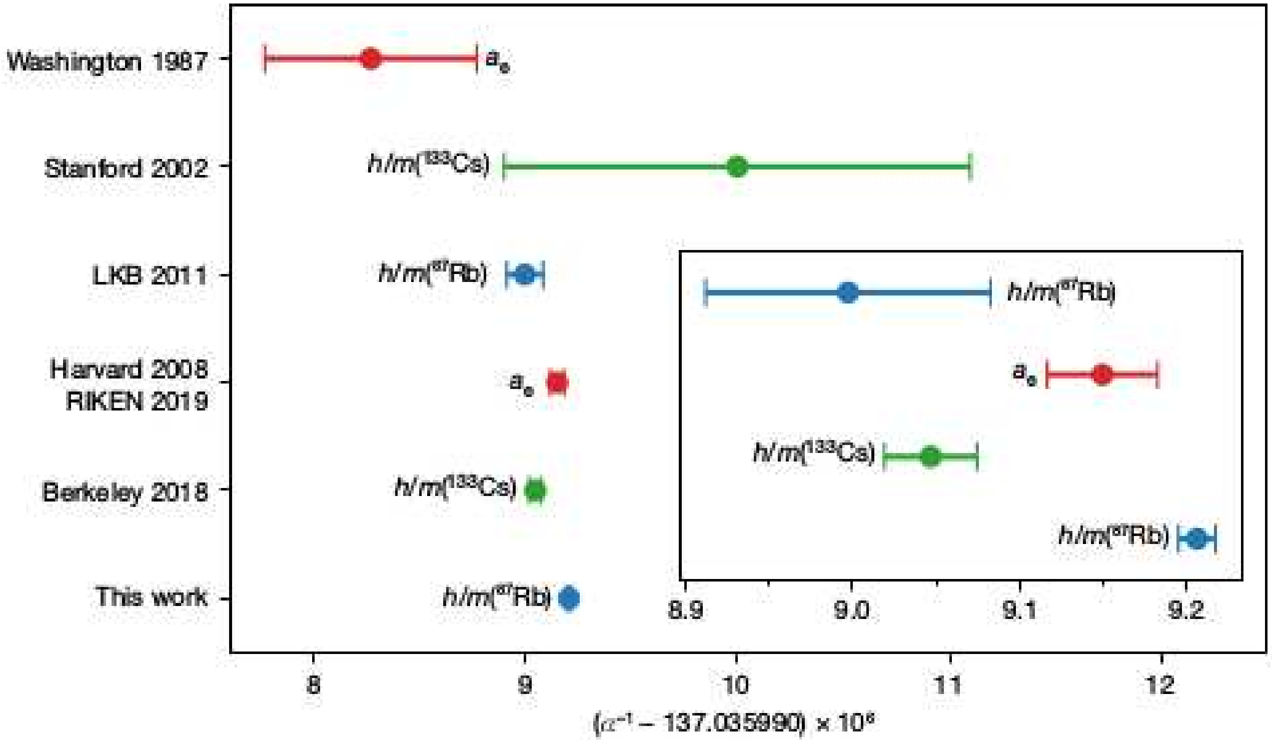
Requires an experimental determination of α with

$$\frac{\Delta\alpha}{\alpha} \sim \frac{\Delta a_\mu}{a_\mu} \sim 0.14\mathrm{ppm}$$



A. Wicht, J. M. Hensley, E. Sarajilic, S. Chu, Phys. Scr. T102, 82 (2002)
R. Bouchendira, P. Clade, S. Guellati-Khelifa, F. Nez and F. Biraben, Phys. Rev. Lett. 106, 080801 (2011)
R. H. Parker, C. Yu, W. Zhong, B. Estey, H. Müller, Science 360, 191 (2018)
L. Morel, Z. Yao, P. Cladé, S. Guellati-Khélifa, Nature 588, 61 (2020)

$$\alpha^2 = \frac{2R_\infty}{c} \cdot \frac{M_{\text{atom}}}{m_e} \cdot \frac{h}{M_{\text{atom}}}$$



⟶ existing tension/discrepancy between $\alpha(\mathrm{Cs18})$ and $\alpha(\mathrm{Rb20})$ (but also between $\alpha(\mathrm{Rb11})$ and $\alpha(\mathrm{Rb20})$) of no concern for a_μ

⟶ for a_μ the value of α could be provided by the qH effect

$$\alpha^{-1}[qH] = 137.036\,00300(270) \quad [19.7\mathrm{ppb}]$$

P. J. Mohr, B. N. Taylor, D. B. Newell, Rev. Mod. Phys. 80, 633 (2008)

QED contribution :

$C_\mu^{(2)}$	0.5
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$C_\mu^{(10)}$	$750.72(93)$

n	$C_\mu^{2n}(\alpha/\pi)^n \cdot 10^{11}$
1	$116\,140\,973.321(23)$
2	$413\,217.6258(70)$
3	$30\,141.90233(33)$
4	$381.004(17)$
5	$5.0783(59)$
$a_\mu^{\rm QED}(\rm Cs18)$	$116\,584\,718.931(7)_{\rm mass}(17)_{\alpha^4}(6)_{\alpha^5}(100)_{\alpha^6}(23)_{\alpha(\rm Cs18)} \cdot 10^{-11}$

QED contribution

$C_\mu^{(2)}$	0.5
$C_\mu^{(4)}$	$0.765\,857\,425(17)$
$C_\mu^{(6)}$	$24.050\,509\,96(32)$
$C_\mu^{(8)}$	$130.878\,0(61)$
$C_\mu^{(10)}$	$750.72(93)$

- $a_\mu^{\mathrm{QED}}(\mathrm{Cs18}) = 116\,584\,718.931(7)_{\mathrm{mass}}(17)_{\alpha^4}(6)_{\alpha^5}(100)_{\alpha^6}(23)_{\alpha(Cs19)} \cdot 10^{-11}$

- $$a_\mu^{\mathrm{exp;WA}} - a_\mu^{\mathrm{QED}}(\mathrm{Cs18}) = 7341(22) \cdot 10^{-11}$$

- QED provides more than 99.99% of the total value, without uncertainties at this level of experimental precision

- No uncertainty from theory at the level of precision reached by experiment

- The missing part has to be provided by weak and strong interactions (or else, new physics...)

EW contribution

One-loop contributions $a_\mu^{\text{weak(1)}} = 194.79(1) \cdot 10^{-11}$

W.A. Bardeen, R. Gastmans and B.E. Lautrup, Nucl. Phys. B46, 315 (1972)
G. Altarelli, N. Cabbibo and L. Maiani, Phys. Lett. 40B, 415 (1972)
R. Jackiw and S. Weinberg, Phys. Rev. D 5, 2473 (1972)
I. Bars and M. Yoshimura, Phys. Rev. D 6, 374 (1972)
M. Fujikawa, B.W. Lee and A.I. Sanda, Phys. Rev. D 6, 2923 (1972)

EW contribution

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W.A. Bardeen, R. Gastmans and B.E. Lautrup, Nucl. Phys. B46, 315 (1972)
G. Altarelli, N. Cabbibo and L. Maiani, Phys. Lett. 40B, 415 (1972)
R. Jackiw and S. Weinberg, Phys. Rev. D 5, 2473 (1972)
I. Bars and M. Yoshimura, Phys. Rev. D 6, 374 (1972)
M. Fujikawa, B.W. Lee and A.I. Sanda, Phys. Rev. D 6, 2923 (1972)

Two-loop bosonic contributions

A. Czarnecki, B. Krause, W. J. Marciano, Phys. Rev. Lett. 76, 3267 (1996)
T. Gribouk, A. Czarnecki, Phys. Rev. D 72, 053016 (2005)

Two-loop fermionic contributions

A. Czarnecki, B. Krause, W. J. Marciano, Phys. Rev. D 52, R2619 (1995)
M. K., S. Peris, M. Perrottet, E. de Rafael, JHEP11, 003 (2002)
A. Czarnecki, W.J. Marciano, A. Vainshtein, Phys. Rev. D 67, 073006 (2003). Err.-ibid. D 73, 119901 (2006)

Complete three-loop short-distance leading logarithms

G. Degrassi and G. F. Giudice, Phys. Rev. D 58, 053007 (1998)

Updated after higgs discovery: $a_\mu^{\text{weak}} = 153.6(1.0) \cdot 10^{-11}$

C. Gnendiger, D. Stöckinger, H. Stöckinger-Kim, Phys. Rev. D 88, 053005 (2013)

Complete numerical evaluation: $a_\mu^{\text{weak}} = 152.9(1.0) \cdot 10^{-11}$

T. Ishikawa, N. Nakazawa and Y. Yasui, Phys. Rev. D 99, 073004 (2019)

EW contribution

- $$a_\mu^{\text{weak}} = 153.6(1.0) \cdot 10^{-11}$$
- $$a_\mu^{\mathrm{exp;WA}} - a_\mu^{\mathrm{QED}}(\mathrm{Cs18}) - a_\mu^{\text{weak}} = 7187(22) \cdot 10^{-11}$$
- Still no uncertainty from theory at the level of precision reached by experiment

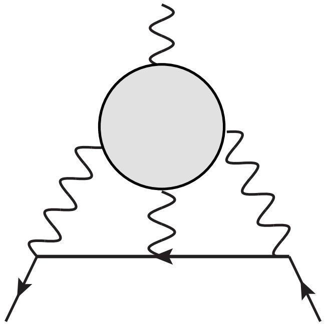
QCD contribution

QCD contribution

Hadronic light-by-light

Hadronic vacuum polarization

QCD contribution: HLxL



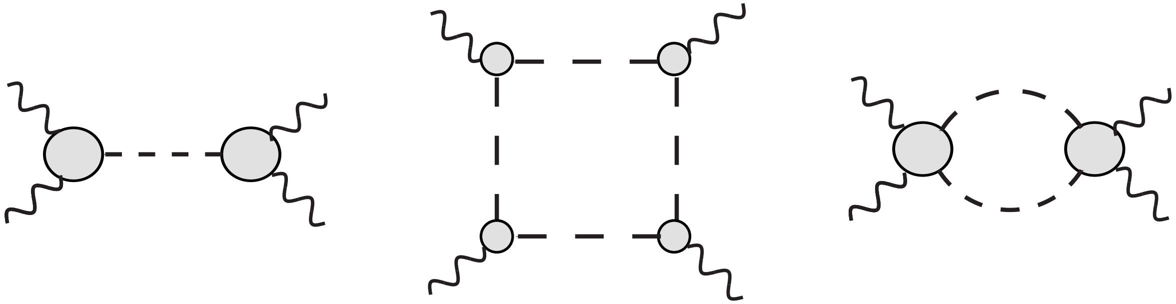
Two main approaches

Dispersion relations

Lattice simulations

QCD contribution: HLxL

Dispersion relations



$$\Pi = \Pi^{\pi^0,\eta,\eta' \text{ poles}} + \Pi^{\pi^\pm,K^\pm \text{ loops}} + \Pi^{\pi\pi} + \Pi^{\text{residual}}$$

G. Colangelo, M. Hoferichter, M. Procura, P. Stoffer, JHEP09, 091 (2014); JHEP09, 074 (2015);
Phys. Rev. Lett. 118, 232001 (2017); JHEP04, 161 (2017)
J. Lüdtke, M. Procura, P. Stoffer, JHEP04, 125 (2023)

Needs input (transition form factors,...)

G. Colangelo, M. Hoferichter, B. Kubis, M. Procura, P. Stoffer, Phys. Lett. B 738, 6 (2014)
A. Nyffeler, arXiv:1602.03398 [hep-ph]

...either from data or from lattice

Short-distance constraints have been worked out... K. Melnikov, A. Vainshtein, Phys.Rev.D 70, 113006 (2004)

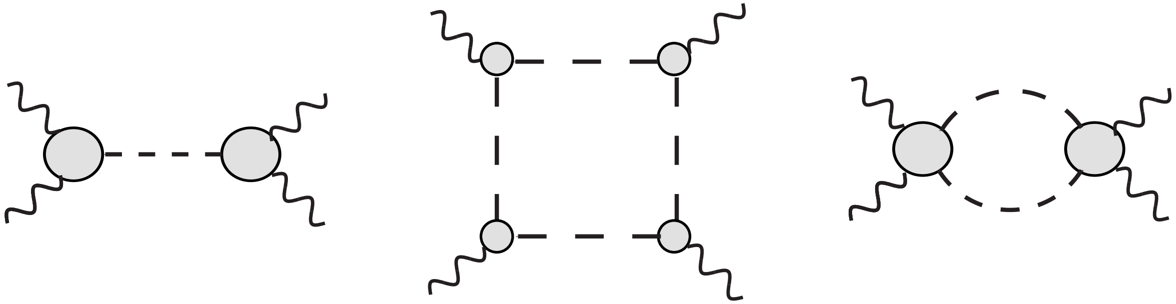
J. Bijnens, N. Hermansson-Truedsson, A. Rodríguez-Sánchez, Phys.Lett. B 798, 134994 (2019)
J. Bijnens, N. Hermansson-Truedsson, L. Laub and A. Rodríguez-Sánchez, JHEP 10, 203 (2020); JHEP 04, 240 (2021)
J. Bijnens, N. Hermansson-Truedsson and A. Rodríguez-Sánchez, JHEP 02, 167 (2023)

... and implemented G. Colangelo, F. Hagelstein, M. Hoferichter, L. Laub, P. Stoffer, JHEP03, 101 (2020)

J. Lüdtke, M. Procura, Eur. Phys. J. C 80, 1108 (2020)
J. G. Colangelo, F. Hagelstein, M. Hoferichter, L. Laub and P. Stoffer, Eur. Phys. J. C 81, 702 (2021)

QCD contribution: HLxL

Dispersion relations



$$\Pi = \Pi^{\pi^0,\eta,\eta' \text{ poles}} + \Pi^{\pi^\pm,K^\pm \text{ loops}} + \Pi^{\pi\pi} + \Pi^{\text{residual}}$$

Form factors: dispersive approach, e.g. $\eta^{(\prime)} \to \gamma^*\gamma^*$

S. Holz, J. Plenter, C. W. Xiao and T. Dato, Eur. Phys. J. C 81, 11 (2021)

S. Holz, C. Hanhart, M. Hoferichter and B. Kubis, Eur. Phys. J. C 82, 434 (2022); Eur. Phys. J. C 82, 1159 (2022)

or $f_1^{(\prime)} \to \gamma^*\gamma^*$ M. Hoferichter, B. Kubis and M. Zanke, JHEP07, 106 (2021); JHEP08, 209 (2023)

or e.m. FF of the kaon D. Stamen, D. Hariharan, M. Hoferichter, B. Kubis and P. Stoffer, Eur. Phys. J. C 82, 432 (2022)

or...

Form factors: lattice QCD, eg. π^0, η, η' TFFs

A. Gérardin, H. Meyer, A. Nyffeler, Phys. Rev. D 94 (2016); Phys. Rev. D 100, 034520 (2019)

J. Koponen, A. Gérardin, H. Meyer, K. Ottnad, G. von Hippel, arXiv:2311.07330

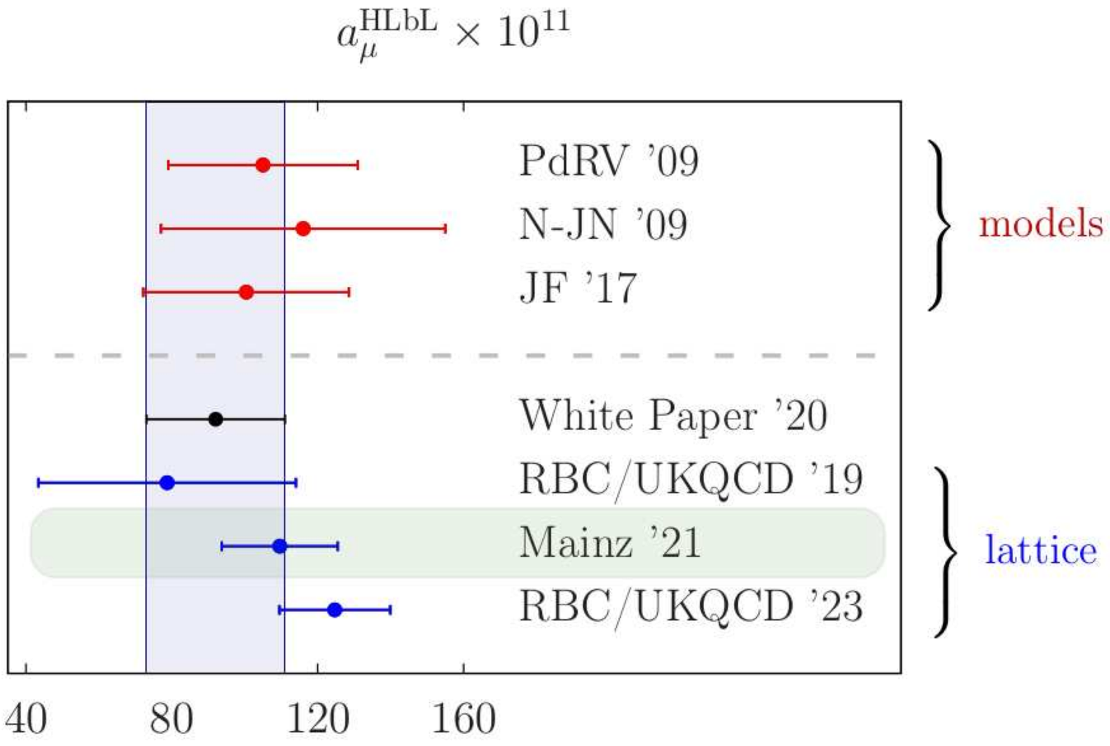
ETMc, Phys. Rev. D 108, 094514 (2023)

BMWc, arXiv:2305.04570 and work in progress

RBC-UKQCD, work in progress

QCD contribution: HLxL

Lattice QCD: full calculations



RBC-UKQCD 19: PRL 124, 132002 (2020)

Mainz 21: Eur. Phys. J. C 81, 651 (2021); C 82, 664 (2022)

RBC-UKQCD 23: arXiv:2304.04423

QCD contribution: HLxL

- $$a_\mu^{\text{HLxL}} = 92(19) \cdot 10^{-11} \quad \text{[WPsummary]}$$
- $$a_\mu^{\mathrm{exp;WA}} - a_\mu^{\mathrm{QED}}(\mathrm{Cs18}) - a_\mu^{\text{weak}} - a_\mu^{\text{HLxL;WP}} = 7095(29) \cdot 10^{-11}$$
- Uncertainty from theory starts to become significant
- WP update: expect (cf. lattice results) some upward shift of central value,

  precision $\lesssim 15\%$

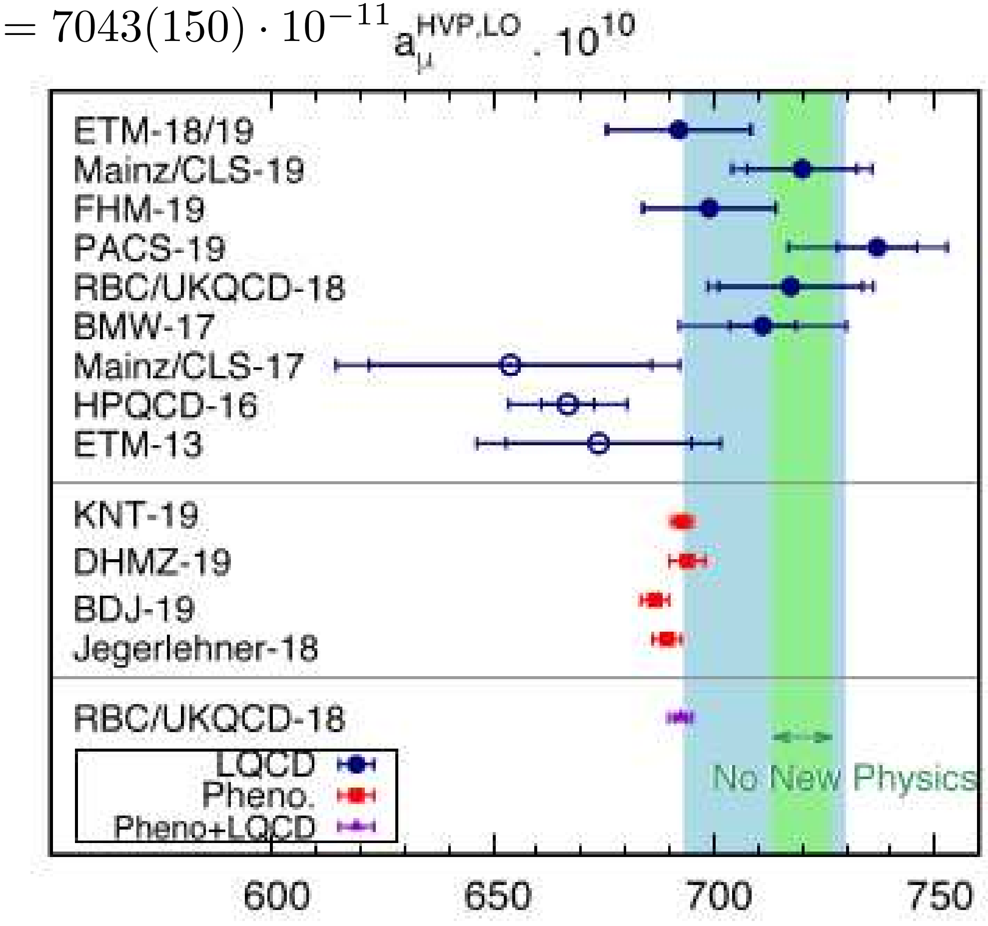
QCD contribution: HVP

White Paper summary

- Data evaluation:

$$a_\mu^{\mathrm{HVP;LO}} = 6931(40)\cdot 10^{-11} \quad a_\mu^{\mathrm{HVP;NLO}} = -98.3(7)\cdot 10^{-11} \quad a_\mu^{\mathrm{HVP;NNLO}} = 12.4(1)\cdot 10^{-11}$$

- Lattice WA: $a_\mu^{\mathrm{HVP;LO}} = 7043(150)\cdot 10^{-11}$

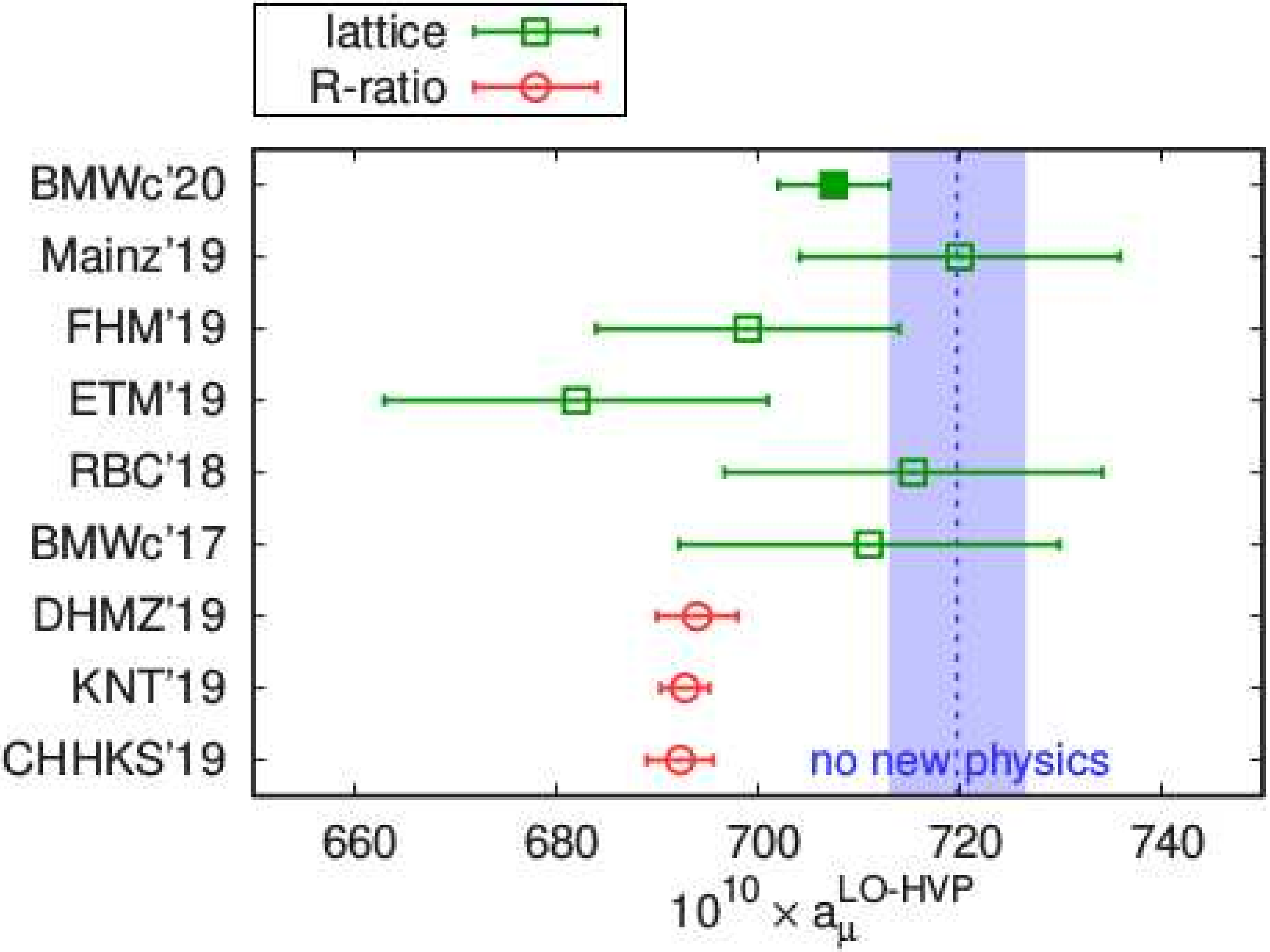


QCD contribution: HVP

- New lattice QCD result for HVP with 0.8% accuracy

$$a_\mu^{\mathrm{HVP;LO}} = 7075(55) \cdot 10^{-11}$$

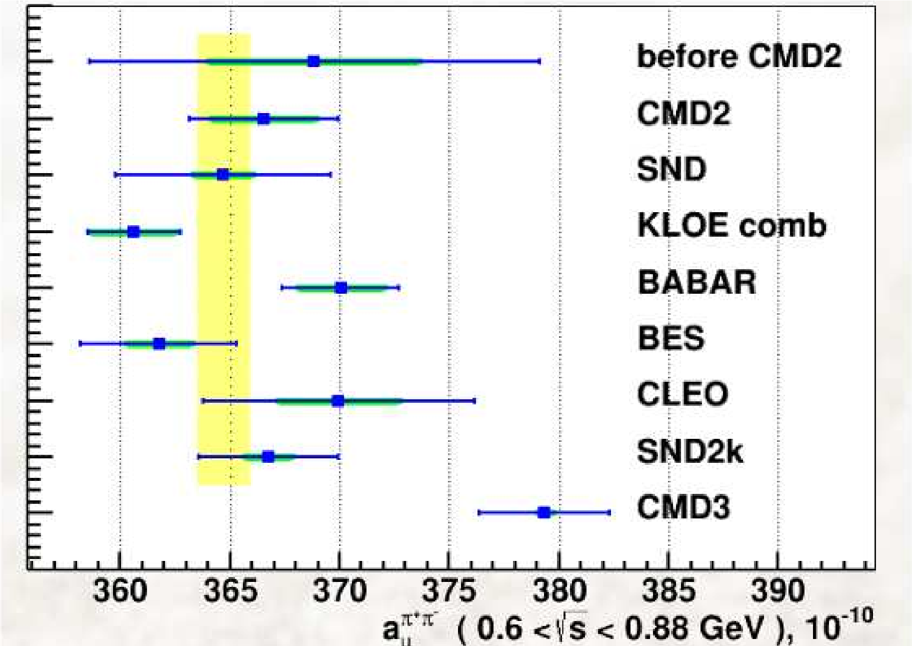
S. Borsanyi et al., Nature 593, 7857 (2021)



QCD contribution: HVP

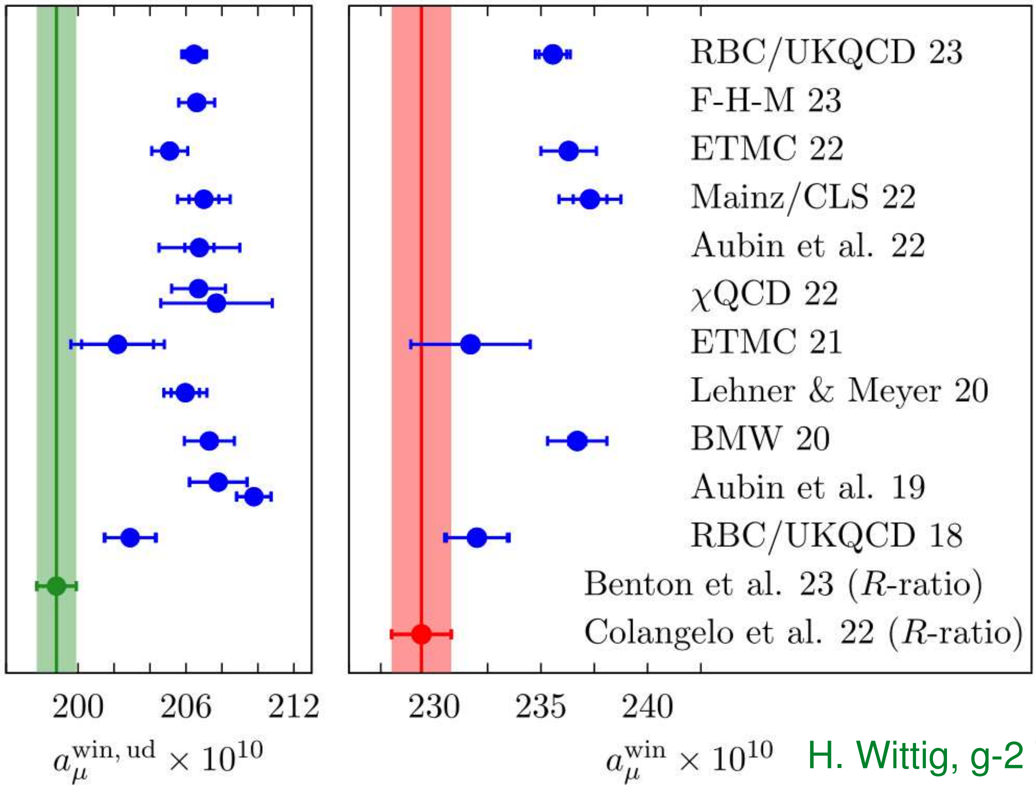
- New cross-section measurement in the $\pi\pi$ channel from CMD-3

Experiment	$a_\mu^{\mathrm{HVP-LO}\,2\pi}\cdot 10^{10}$	
CMD2	366.5(3.4)	
SND	364.7(4.9)	
KLOE	360.6(2.1)	KLOE vs. CMD3 5.1σ
BaBar	370.1(2.7)	BaBar vs. CMD3 2.5σ
BESIII	361.8(3.6)	
SND2k	366.7(3.2)	
CMD3	379.3 (3.0)	F. V. Ignatov *et al.* [CMD-3], arXiv:2302.08834



QCD contribution: HVP

- Several confirmations of BMWc result in the intermediate and short-distance windows



H. Wittig, g-2 TI Worshop, Bern, 4-8 Sept. 2023

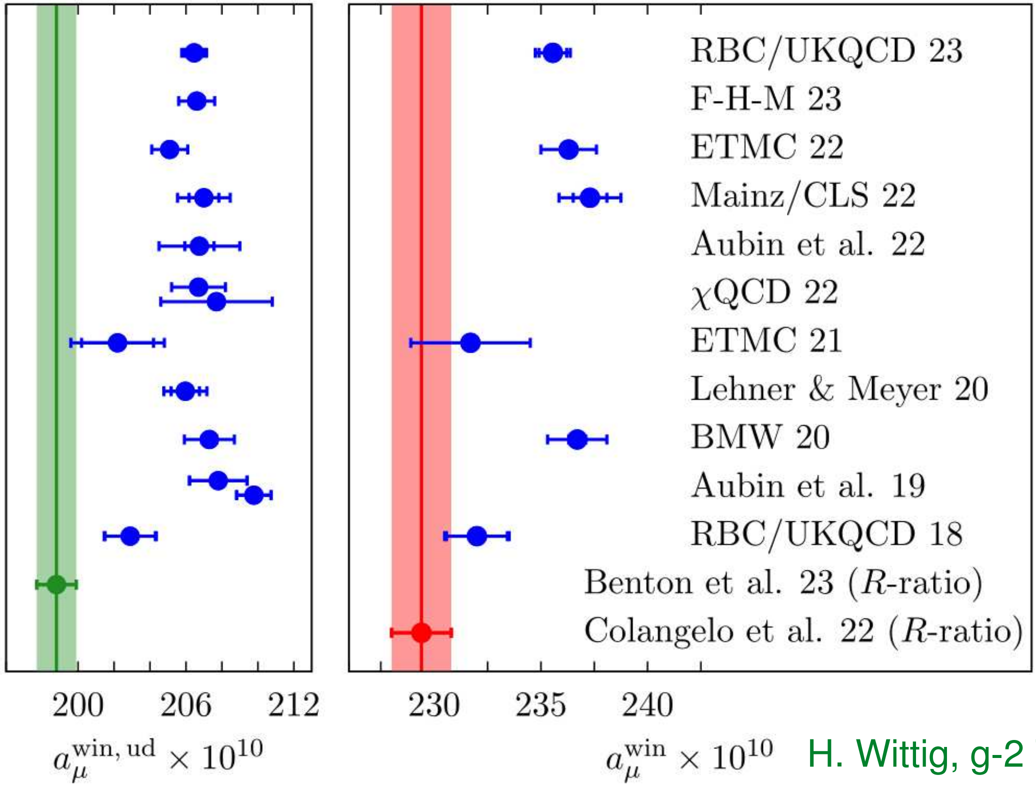
$$a_\mu^{\mathrm{SDW}} \sim 10\% \text{ of } a_\mu^{\mathrm{HVP;LO}}$$

$$a_\mu^{\mathrm{IW}} \sim 30\% \text{ of } a_\mu^{\mathrm{HVP;LO}}$$

$$a_\mu^{\mathrm{LDW}} \sim 60\% \text{ of } a_\mu^{\mathrm{HVP;LO}}$$

QCD contribution: HVP

- Several confirmations of BMWc result in the intermediate and short-distance windows



H. Wittig, g-2 TI Worshop, Bern, 4-8 Sept. 2023

To date, no other complete lattice evaluation of the HVP contribution at the same level of precision as BMWc...

Summary

- FNAL-E989 on the right course to achieve a measurement at 0.14ppm

- Unfortunately, the theory situation is not quite in such a good shape

  - QED and EW contributions under control at the required level of precision

  - results on HLxL at $\lesssim 10\%$ within reach (lattice QCD, dispersive approach with exp$^{\mathrm{al}}$ input, e.g. form factors,...)

  - important tensions at the level of HVP

    — between KLOE and BABAR

    — between CMD-3 and earlier experiments (incl. CMD-2!)

    — between BMWc and data-based evaluations (except CMD-3)

Summary

- More data are being analyzed (BaBar, KLOE)

or will become available in the future (BESIII, BelleII,...)

- Important to have independent confirmation of the full BMWc result

- Possibilities for inclusive measurements of HVP even more interesting
  - in the space-like region (MUonE)
  - or even directly in the time-like region

- Reappraisal of τ-data-based evaluation of HVP

M. Davier, A. Höcker, A. M. Lutz, B. Malaescu, Z. Zhang, arXiv:2312.02053

- Next milestone on the theory side: updated WP
  - to be ready before release of FNAL-E989 final analysis (Spring 2025?)