"Workshop on Modular Flavour Symmetries "

Mainz May 2024

On the origin of discrete flavour symmetries
in String Theory

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GREECE

Outline of the Talk

- ▲ Motivation/Remarks
- ▲ Modular symmetries in string theories
- ▲ *Some ideas how to implement them in an* $\mathcal{F}$-GUT

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based on

JHEP 04 (2012) 094, 12 (2013) 037, 09 (2014) 107. 1 (2020) 149)

and work ... *in progress* with

V. Basiouris, Crispim-Romão, S.F. King

Introductory Remarks

▲ Recent activity on Modular symmetries:
⇒ inspired by modular properties of String Theory.

Fundamental constituents in this approach of flavor symmetries are the moduli fields, weights, and modular functions (→ $Yukawas$.)

▲ **Bottom-up approach**: mostly working with moduli, endowed with some "generic" properties, **However:**

▲ Each String -Theory version (Heterotic, IIA, IIB, M, F) involves different types of moduli with different properties.

▲ Moduli Stabilisation, and other issues must be taken care of.

▲ This makes top-bottom approach a necessity.

▲ Most appealing framework ⇒ IIB & $\mathcal{F}$-theory...→ calculability

▲ Effective Field Theory: no scale Supergravity

Modular Symmetries in IIB/F-theory

★ Basics ★

⇓

II-B: *closed string spectrum obtained by combining left and right moving open strings with NS and R-boundary conditions:*

$$(NS_+, NS_+),\ (R_-, R_-),\ (NS_+, R_-),\ (R_-, NS_+)$$

Bosonic spectrum:

(NS_+, NS_+): graviton, dilaton and 2-form KB-field:

$$g_{\mu\nu},\ \phi,\ B_{\mu\nu} \to B_2$$

(R_-, R_-): scalar, 2- and 4-index fields (*p-form potentials*)

$$C_0, C_{\mu\nu}, C_{\kappa\lambda\mu\nu} \to C_p,\ p = 0, 2, 4$$

Definitions (*bosonic part*)

1. *String coupling:* $g_s = e^{\phi}$

2. *Combining the two scalars* C_0, ϕ *to one* *modulus:*

$$\tau = C_0 + i\, e^{-\phi} \to C_0 + \frac{i}{g_s}$$

3. *Field strengths etc*

$$H_3 = d\, B_2 \quad (1)$$

$$F_p = d\, C_{p-1}, \; p = 1, 3, 5 \quad (2)$$

$$\mathbf{G_3} = F_3 - \tau H_3 \quad (3)$$

$$\tilde{F}_5 = F_5 - \frac{1}{2} C_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3 \quad (4)$$

IIB - action leading to equs of motion: (*see 0803.1194*)

$$\mathbf{S_{IIB}} \quad \propto \quad \int d^{10}x\sqrt{-g}\, R - \frac{1}{2}\int \frac{1}{(\mathrm{Im}\tau)^2} d\tau \wedge *d\bar{\tau}$$

$$+ \quad \frac{1}{\mathrm{Im}\tau} G_3 \wedge *\overline{G}_3 + \frac{1}{2}\tilde{F}_5 \wedge *\tilde{F}_5 + C_4 \wedge H_3 \wedge F_3$$

Properties:

1. Invariant under $SL(2,Z)$ S-duality:

   $$\tau \to \frac{p\tau+q}{s\tau+t} \quad \text{and} \quad \begin{pmatrix} F \\ H \end{pmatrix} \to \begin{pmatrix} p & q \\ s & t \end{pmatrix} \begin{pmatrix} F \\ H \end{pmatrix}$$

2. This is the analogue of a 12-d. theory compactified on torus with modulus τ with F_3, H_3 *components of some* 12-d. $\hat{F}_4$ reduced along the 1-cycles of torus τ.

Types of moduli fields in IIB (and F)- theory

IIB	symbol	#
Kähler	T_k	$2\,h_+^{1,1}$
Complex Structure	τ_i	$2\,h_+^{2,1}$
Axio-dilaton	τ	1
C_4 axions	a_i	$h_+^{1,1}$
B_2 axions	b_i	$h_-^{1,1}$
C_2 axions	c_i	$h_-^{1,1}$
D_3 positions		N_{D3}
D_7 deformations		$2h_-^{2,0}$

▲ **Framework** ▲

↓

After these preliminaries, next we will discuss the framework where the analysis takes place

Type II-B effective Supergravity

▲ ▲

▲ *Basic 'ingredients':* ▲

↓

$\mathcal{A}$) *Superpotential* $\mathcal{W}$

$\mathcal{B}$) *Kähler potential* $\mathcal{K}$

$\mathcal{C}$) $\mathcal{K}$*inetic function*

$\mathcal{A}$

▲ The **Superpotential** $\mathcal{W}$ ▲

▲ *Flux-induced (Gukov-Vafa-Witten) superpotential* with $G_3 = F_3 - \tau H_3$ and holomorphic $(3,0)$-form $\Omega(z_a)$:

$$\mathcal{W}_0 = \int \mathbf{G_3} \wedge \Omega(z_a) \;\Rightarrow\; \mathcal{W}_0 = \mathcal{W}_0(z_a, \tau)$$

$\Rightarrow$ $\mathcal{W}_0$ **does not** depend on Kähler moduli T_i.

Thus:

▲ **Flatness conditions only w.r.t. CS and axiodilaton** ▲

$$\mathcal{D}_{z_a}\mathcal{W}_0 = 0, \quad \mathcal{D}_S\mathcal{W}_0 = 0 \;:$$

$\Rightarrow$ z_a *and* τ **stabilised** $\Leftarrow$

but!

▲ Kähler moduli $\notin \mathcal{W}_0$ $\Rightarrow$ remain unfixed! ▲

$\mathcal{B}$

▲ The Kähler potential ▲

$$\mathcal{K}_0 = -\ln(-i(\tau - \bar{\tau})) - 2\ln(\hat{\mathcal{V}}) - 2\ln\left(\int \Omega \wedge \overline{\Omega}\right)$$

where $\hat{\mathcal{V}}$ is the 6d volume in Einstein frame :

$$\hat{\mathcal{V}} = e^{-\frac{3}{2}\phi}\mathcal{V} = \frac{\mathcal{V}}{{g_s}^{3/2}} = \frac{1}{3!}\kappa_{ijk}\hat{t}^i\hat{t}^j\hat{t}^k$$

given in terms of $\hat{t^i}$ defined through the Kähler moduli:

$$t^k = -\mathrm{Im}(T^k) = \hat{t}^k\left(\frac{\tau - \bar{\tau}}{2i}\right)^{-1/2} \equiv \hat{t}^k\ {g_s}^{1/2}$$

▲ The classical scalar potential computed from $\mathcal{K}_0$: ▲

$$V = e^{\mathcal{K}}\Big(\sum_{I,J} \mathcal{D}_I \mathcal{W}_0 \mathcal{K}^{I\bar{J}} \mathcal{D}_{\bar{J}} \mathcal{W}_0 - 3|\mathcal{W}_0|^2\Big) \equiv 0,$$

is identically zero *due to flatness conditions and the no-scale structure.*

$\Downarrow$

Kähler moduli completely **undetermined**!

Quantum Corrections

▲ Non renormalisation theorems do not allow for any perturbative corrections to the superpotential

Kähler moduli can be fixed by:

▲ NP-contributions (hep-th/0301240) to $\mathcal{W}_0 \to \mathcal{W} = \mathcal{W}_0 + \mathcal{W}_{NP}$:

$$\mathcal{W} = \mathcal{W}_0 + \sum_k A_k e^{-\alpha T_k}, \quad D_{T_k}\mathcal{W} = 0$$

and α'^3 (hep-th/0204254) corrections $\in$ Kähler potential.

However

Presence of Kähler moduli in W, neither necessary, nor always possible.

Loop corrections $\in K$ suffice for stabilisation!

Perturbative Quantum Corrections

▲ Loop corrections to the Kähler potential exhibit logarithmic dependence on Kähler moduli (*see 1803.08941, 1909.10525*)

They are included by a shift to $\mathcal{V}$:

$$\hat{\mathcal{V}} \to \mathcal{U} \equiv \hat{\mathcal{V}} + \frac{\xi}{2}\frac{1}{{g_s}^{3/2}} + \eta {g_s}^{1/2} \log(\hat{\mathcal{V}})$$

—

Perturbative $\mathcal{W}$ and the final form of the corresponding Kähler potential:

$$\mathcal{K} = -\log(-i(\tau - \bar{\tau})) - 2\log \mathcal{U} + \mathcal{K}_{cs}$$

fix the values of the Kähler moduli.

—

Based on above arguments, we can exclude Kähler moduli from $\mathcal{W}$

Allow me for a small deviation here to explain that, based only on F-terms in scalar potential, the minimum is always AdS.
In this construction, however, this problem has a natural solution.
By virtue of D7 brane stacks, there are universal $U(1)$'s which contribute a D-term. Hence, in the Large Volume Limit,
$V_{\rm eff}$ acquires the following (simple) form

$$V_{\rm eff} = V_F + V_D \approx C\frac{\hat{\xi} - 4\hat{\eta} + 2\hat{\eta}\log(\mathcal{V})}{\mathcal{V}^3} + \frac{d}{\mathcal{V}^2}$$

Minimising $V_{\rm eff}$ we find a minimum and a maximum defined by the:
double-valued **Lambert** W-function (i.e., solution of $\mathbf{We^W} = z$):

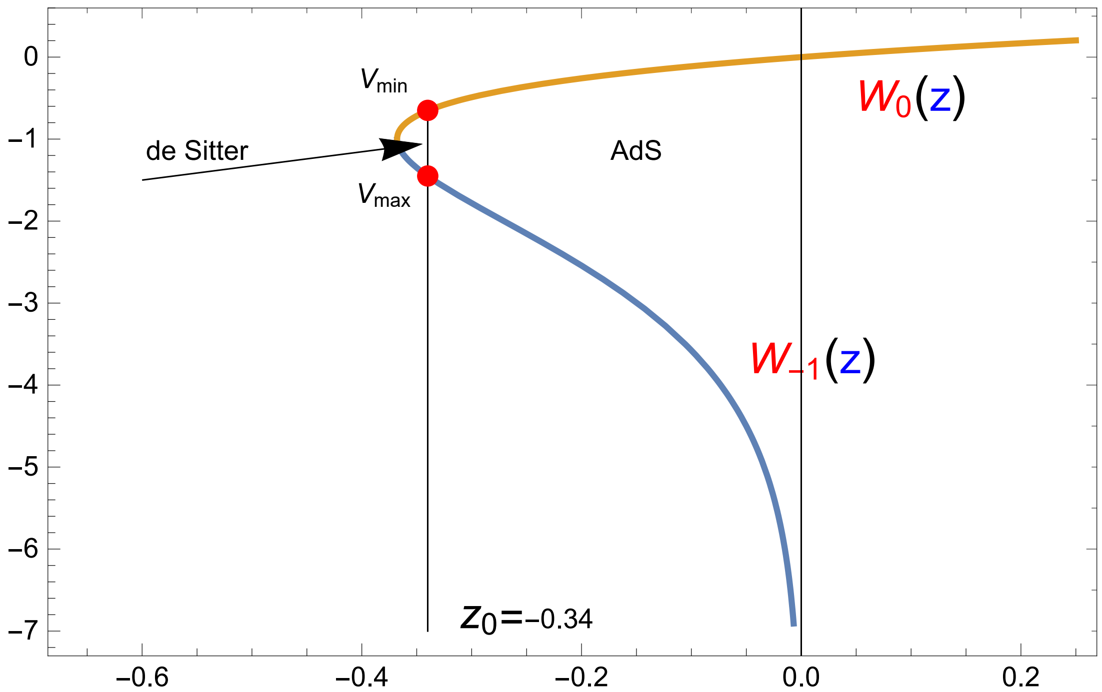
$$\boxed{\mathcal{V}_0 = \frac{\hat{n}}{d}\mathbf{W_{0/-1}}\left(\frac{d}{\hat{n}}e^{\frac{7}{3}-\frac{\hat{\xi}}{2\hat{n}}}\right)}$$

(for details see JHEP 07 (2022) 047, e-print: 2203.03362)

▲ de Sitter vacua ▲

minimum $V_{\rm eff} = V_F + V_{\mathcal{D}}$ *at* $\mathcal{V}_0$ *must be positive:*

$$V_{\rm eff}^{\rm min} = \frac{c}{\mathcal{V}_0^3} + \frac{d}{\mathcal{V}_0^2} > 0$$



This dramatically constrains acceptable string vacua (fluxes etc)

Inflation

Hybrid scenario can be realised with **open string states** χ *at D7-brane intersections playing the role of waterfall fields (2109.03243, JHEP 2022, 2405.06738)*

Shape of V_{total} in the presence of χ at large $\mathcal{V}$ regime:

$$V_{total} = C\frac{\hat{\xi} - 4\hat{\eta} + 2\hat{\eta}\log(\mathcal{V})}{\mathcal{V}^3} + \frac{d}{\mathcal{V}^2} + V_\chi$$

with V_χ the waterfall field potential:

$$V_\chi = \sim m^2(\mathcal{V})\chi^2 + \lambda(\mathcal{V})\chi^4$$

▲ The volume modulus can play the role of Inflaton field

$$\phi \propto \log \mathcal{V}$$

▲ *We find that inflation can be realised with most of the 60 efolds collected near the metastable local mimimum* $V_{total}(\mathcal{V}_{min})$.

▲ Inflation ends and false vacuum decays to *Global minimum* through a

$$\textbf{waterfall field } \chi\text{:} \;\; V_\chi \sim m^2(\mathcal{V})\chi^2 + \lambda(\mathcal{V})\chi^4 \; .$$

For $m^2 > 0$ minimum in the χ-field direction is at the origin

$$m^2 > 0 \; \rightarrow \; \langle\chi\rangle = 0$$

When the mass of χ becomes tachyonic, a phase transition occurs and the new vacuum is obtained at a non-vanishing $\langle\chi\rangle$:

$$m^2 < 0 \; \rightarrow \; \langle\chi\rangle \neq 0$$

A configuration to realise the hybrid scenario in our D7 set-up

	$\mathcal{T}^2_{(45)}$	$\mathcal{T}^2_{(67)}$	$\mathcal{T}^2_{(89)}$
$D7_1$	$\cdot$	$\otimes$	$\times_{\mathcal{A}_1}$
$D7_2$	$\times$	$\cdot_{\pm\mathbf{x_2}}$	$\otimes$
$D7_3$	$\otimes$	$\times_{\mathcal{A}_3}$	$\cdot$

- A circled cross shows magnetic field on specific $D7$ and $\mathcal{T}^2$.
- $\mathcal{A}_{1,3}$ denote Wilson lines
- $\pm\mathbf{x_2}$ brane separations (uplifting tachyons)

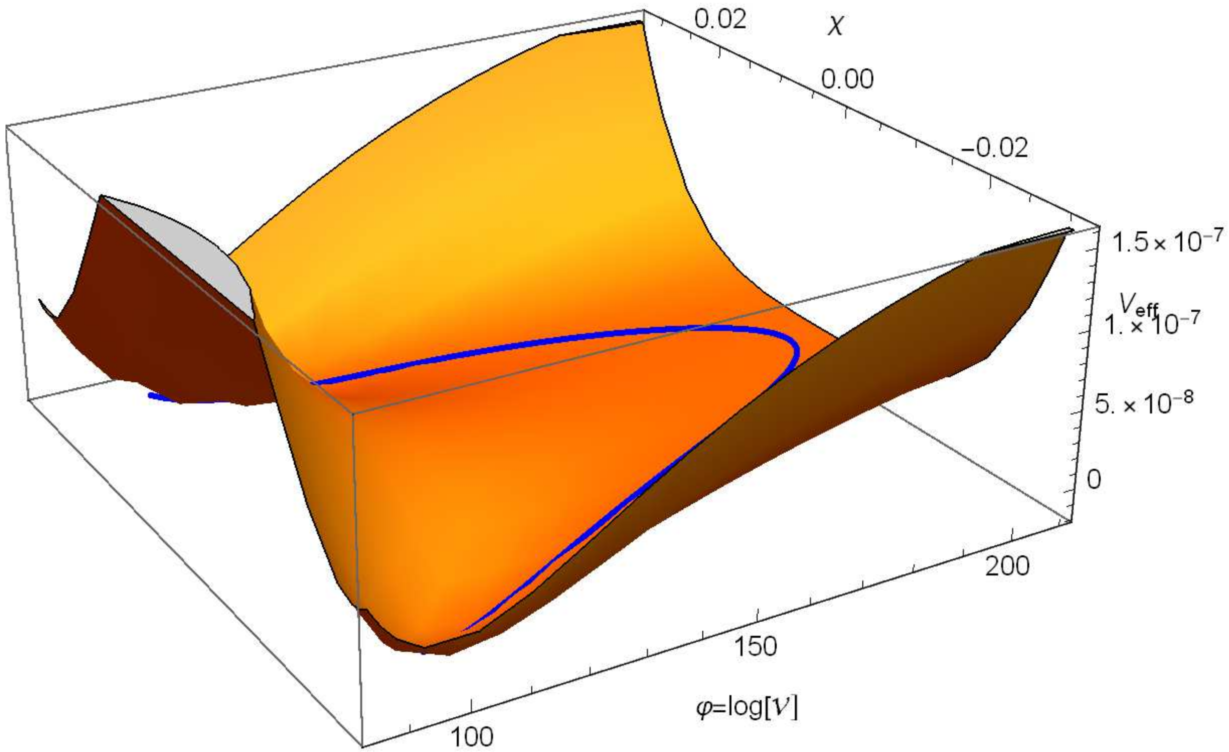
$\Rightarrow$ only one tachyonic state playing the role of waterfall field:

$$\alpha' {m_{22}}^2 \approx -\frac{A}{\mathcal{V}^{1/3}} + B\mathcal{V}^{1/3}, \quad (A, B) \to \textit{positive constants}$$

$\langle\chi\rangle \neq 0$ for

$$\mathcal{V} \; < \; \mathcal{V}_{\text{critical}} = \left(\frac{A}{B}\right)^{\frac{3}{2}}$$

- Hybrid scenario: Inflaton: $\phi \propto \log \mathcal{V}$
- Blue curve: waterfall field χ trajectory



... back to moduli ...

Excluding T_k from W in the present framework, we focus on τ and CS τ_i in the context of

Toroidal Compactifications

$$T^6/\mathbb{Z}_2 = (T_1^2 \times T_2^2 \times T_3^2)/\mathbb{Z}_2$$

Theory will also manifest the modular invariance w.r.t. τ and CS moduli of each torus

$$SL(2,\mathbb{Z})_\tau \otimes (\otimes_{i=1}^3 SL(2,\mathbb{Z})_i)$$

where the associated transformation matrices read

$$R = \begin{pmatrix} p & q \\ s & t \end{pmatrix}, \; R_i = \begin{pmatrix} p_i & q_i \\ s_i & t_i \end{pmatrix}, \; R, R_i \in SL(2, \mathbf{Z}), \; i = 1, 2, 3$$

A 'generic' form of the superpotential for the moduli fields is

$$W_{\rm IIB} \;=\; \int \mathbf{G_3} \wedge \Omega \;\Rightarrow \tag{5}$$

$$(a^0 - \tau c^0)(\Pi_j \tau_j) - (a^i - \tau c^i)(\Pi_{j\neq i}\tau_j) - (b_i - \tau d_i)\tau^i - (b_0 - \tau d_0)$$

Minimisation conditions

$$D_\tau W_{\rm IIB} = D_{\tau_k} W_{\rm IIB} = 0$$

imply the relations among the various moduli

$$\tau_3 \propto \tau \tag{6}$$

$$\tau_1 = \frac{a^1\tau_2 + b_3}{a^0\tau_2 - a^2}$$

Additional requirements on $R_2(\tau_2) \in SL(2,Z)_2$ (integer entries, tadpole conditions etc) further restrict $R_2(\tau_2) \in S_4$.

In IIB a generic Yukawa coupling depends on g_s:

$$W \supset \lambda_{ij}(g_s) f_i f_j h$$

Intimately related to S-duality invariance of IIB. Recall that

$$K = -\log(-i(\tau - \bar{\tau})) + \cdots \Rightarrow$$

$$K \to K + |c\tau + d|^2$$

$$\mathcal{L} \supset -e^{K/2} \bar{W} \psi_\mu \sigma^{\mu\nu} \psi_\nu \to \mathit{Gravitino\,mass} : m_{3/2}^2 = e^K |W|^2.$$

Invariance implies

$$W \to \frac{W}{c\tau + d} \;\Rightarrow\; |W|^2 \to \frac{|W|^2}{|c\tau + d|^2} \equiv \frac{|W|^2}{C_0^2 + {g_2}^{-2}}$$

The Yukawa coupling is:

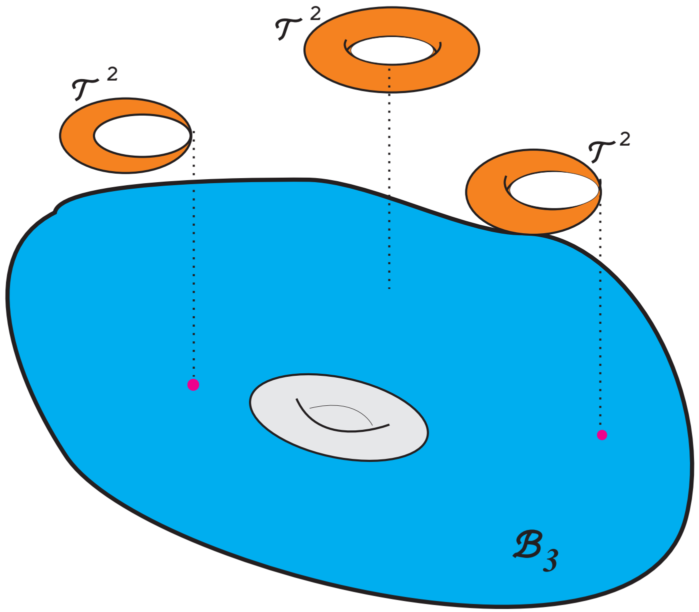
$$\lambda_{ij}^2(g_s) \propto \frac{1}{g_s} \equiv \mathrm{Im}\tau \to \mathrm{Im}\tau' = \frac{\mathrm{Im}\tau}{|c\tau + d|^2}$$

which matches the $SL(2,Z)_\tau$ transformation property of W.

The Yukawa sector in F-theory

▲ ▲ **F-theory** is compactified on an **elliptically fibered complex manifold** where
the threefold $\mathcal{B}_3$ is the base of the fibration.

Fibration *is implemented by the axio-dilaton modulus*
$\tau = C_0 + \imath\, e^{-\phi}$ *which can be thought as describing a torus*



Geometry and Gauge Symmetries

The elliptic fibration is represented by the Weierstraß equation:

$$y^2 = x^3 + f(z)\,x + g(z)$$

- At the points where the discriminant $\Delta = 27g^2 + 4f^3$ vanishes, the elliptic fiber **degenerates**.
- The type of Manifold **singularity** is specified by the vanishing order of Δ and the polynomials $f(z)$, $g(z)$ of Weierstraß eqn
- These **geometric singularities** are classified in terms of $\mathcal{A}\,\mathcal{D}\,\mathcal{E}$ Lie groups.

In F-theory these singularities are interpreted as:

$$\Downarrow$$

$\mathcal{CY}_4$-**Singularities** $\rightleftarrows$ **GAUGE SYMMETRIES**

▲▼ $\mathcal{A}n\ \mathcal{E}_6 \times \mathcal{S}_4\ \mathcal{MODEL}$ ▲▼

▲ The candidate GUT is embedded in $\mathcal{E}_8$ which is the maximal exceptional group in elliptic fibration.

We consider a CY with a divisor accommodating our model $\mathcal{E}_6$ while the rest is the symmetry commutant to it.

$$\mathcal{E}_8 \supset \mathcal{E}_6 \times SU(3)_\perp$$

The spectrum descends from the $\mathcal{E}_8$-Adjoint which decomposes as:

$$248 \to (78, 1) + (1, 8) + (27, \bar{3}) + (\overline{27}, 3)$$

Observe that the $\mathcal{E}_6$ matter content transforms non-trivially under the perpendicular $SU(3)_\perp$.

It suffices to proceed with the $SU(3)_\perp$ Cartan subalgebra where the representations are labeled by the weights t_i satisfying

$$\sum_{i=1}^{3} t_i = 0$$

Matter consists of $\mathbf{27}$'s and singlets $\in (1,8)$ residing on matter curves

$$\mathbf{27} \in \Sigma_{t_i}, \quad \& \quad \mathbf{1}_{t_i - t_j} \in \Sigma_{t_i - t_j}, \; i \neq j$$

The perpendicular symmetry conveys the geometry and topological properties of the internal manifold to matter fields through the coefficients $\mathbf{b_k}$ *of the spectral cover equation*

$$\mathcal{C}_3 = \sum_{k=0}^{3} b_k t^{3-k} \equiv b_0 t^3 + b_1 t^2 + b_2 t + b_3 = 0$$

The solution for t_i's may involve branchcuts which might imply identifications of the roots (weights)

Three possibilities:

1. $\mathcal{C}_3$ completely reducible, then all three roots distinct
   $t_i \neq t_j, \ \ \forall i \neq j$

$$\mathcal{C}_3 = \alpha(t - t_1)(t - t_2)(t - t_3)$$

2. Partial split of $\mathcal{C}_3$:

$$\mathcal{C}_3 = (a_1 t^2 + a_2 t + a_3)(a_4 t + a_5)$$

   Two roots connected by Z_2 monodromy,

$$t_1 \leftrightarrow t_2$$

3. $\mathcal{C}_3$ non-"splitable" $\Rightarrow \exists\, Z_3$ monodromy

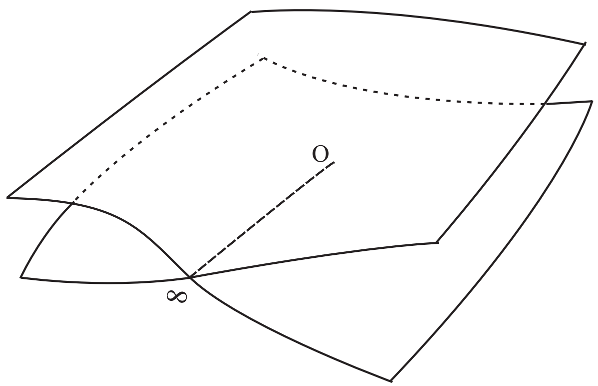
$$t_1 \leftrightarrow t_2 \leftrightarrow t_3$$

EXAMPLE ..Simplest monodromy Z_2 : :

$$a_1 + a_2 t + a_3 t^2 = 0 \to t_{1,2} = \frac{-a_2 \pm \sqrt{\Delta}}{2a_3}$$

Under $\theta \to \theta + 2\pi$: $\sqrt{\Delta} \to -\sqrt{\Delta}$ branes interchange locations

$$t_1 \leftrightarrow t_2$$



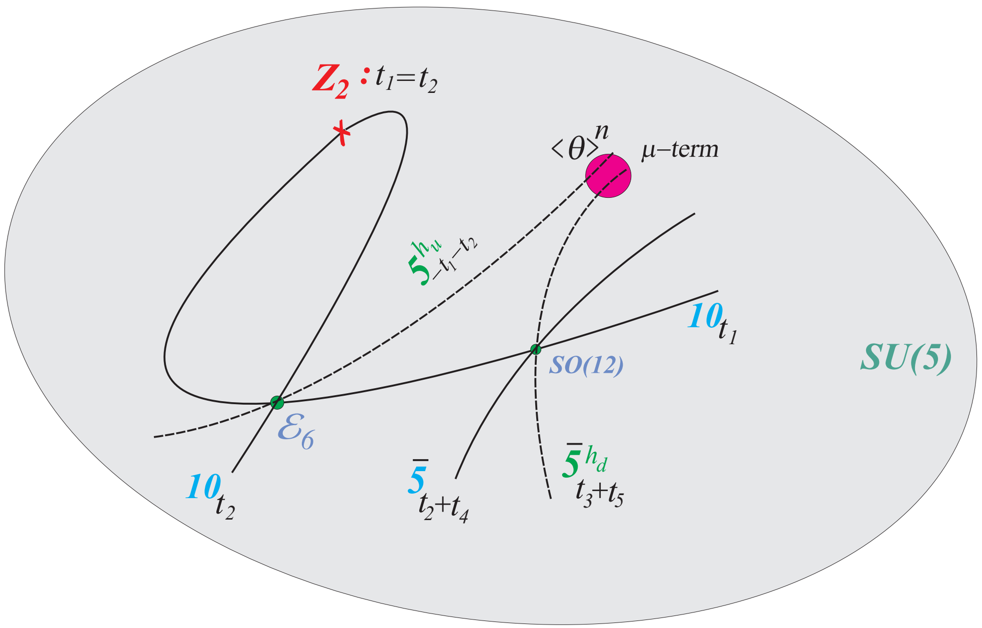
Two $\mathbf{U(1)}$'s related by monodromies, gauge symmetry reduces to:

$$\mathcal{E}_6 \times U(1)^2 \to \mathcal{E}_6 \times \mathbf{U(1)}$$

Case 2, i.e., $Z_2 : t_1 \leftrightarrow t_2$ *is the most appealing, since it identifies*

$$27_{t_1} \equiv 27_{t_2}$$

thus allows a tree-level Yukawa coupling



The matter content is distributed in the following representations

$\mathcal{E}_6$	Section	Homology	#
$\mathbf{\Sigma}_{27_{t_{1,2}}}$	$[a_1]$	$\eta - 2c_1 - \chi$	$n_1 = \mathcal{F}_{U(1)} \cdot (\eta - 2c_1 - \chi)$
$\mathbf{\Sigma}_{27_{t_3}}$	$[a_4]$	$\chi - c_1$	$n_3 = \mathcal{F}_{U(1)} \cdot (c_1 - \chi)$
$\mathbf{\Sigma}_{1_{t_i - t_j}}$	$[a_1 a_5]$	$\eta - 2c_1$	$n_{singlets} = \mathcal{F}_{U(1)} \cdot (\eta - c_1)$

Their multiplicities are determined from the topological properties of the matter curves accommodating the representations and the restrictions of the $U(1)_\perp \mathcal{F}$lux $\in SU(3)_\perp$ on them.

The final model is achieved by a consecutive symmetry breaking with $U(1)_\perp$ fluxes, through the following chain

$$\mathcal{E}_6 \supset \mathcal{SO}(10) \supset \mathcal{SU}(5) \supset G_{SM}$$

At the final step we generate SM chirality and break $SU(5)$ by turning on

Hypercharge Flux

$U(1)_Y-$**Flux**-splitting of **10**'s:

$$\begin{aligned} n_{(3,2)_{\frac{1}{6}}} - n_{(\bar{3},2)_{-\frac{1}{6}}} &= M_{10} \\ n_{(\bar{3},1)_{-\frac{2}{3}}} - n_{(3,1)_{\frac{2}{3}}} &= M_{10} - N_{Y_{10}} \\ n_{(1,1)_1} - n_{(1,1)_{-1}} &= M_{10} + N_{Y_{10}} \end{aligned}$$

$U(1)_Y-$ **Flux**-splitting of **5**'s:

$$\begin{aligned} n_{(3,1)_{-\frac{1}{3}}} - n_{(\bar{3},1)_{\frac{1}{3}}} &= M_5 \\ n_{(1,2)_{\frac{1}{2}}} - n_{(1,2)_{-\frac{1}{2}}} &= M_5 + N_{Y_5} \end{aligned}$$

E_6	$SU(5)$	weight vector	particle content	SM spectrum
$\mathbf{\Sigma}_{27_{t'_1}}$	$\overline{5}_3$	t_1+t_5	$4d^c+5L$	$3d^c+3L$
$\mathbf{\Sigma}_{27_{t'_1}}$	10_M	t_1	$4Q+5u^c+3e^c$	$3(Q+u^c+e^c)$
$\mathbf{\Sigma}_{27_{t'_1}}$	θ_{15}	t_1-t_5	$3\nu^c$	-
$\mathbf{\Sigma}_{27_{t'_1}}$	5_1	$-t_1-t_3$	$3D+2H_u$	-
$\mathbf{\Sigma}_{27_{t'_1}}$	$\overline{5}_2$	t_1+t_4	$3\overline{D}+4H_d$	H_d
$\mathbf{\Sigma}_{27_{t'_3}}$	$\overline{5}_5$	t_3+t_5	$\overline{d^c}+2\overline{L}$	-
$\mathbf{\Sigma}_{27_{t'_3}}$	10_2	t_3	$\overline{Q}+2\bar{u}^c$	-
$\mathbf{\Sigma}_{27_{t'_3}}$	5_{H_u}	$-2t_1$	H_u	H_u
$\mathbf{\Sigma}_{27_{t'_3}}$	$\overline{5}_4$	t_3+t_4	$\overline{H_d}$	-
$\mathbf{\Sigma}_{t_{ij}}$	θ_{ij}	t_i-t_j	θ_{ij}	-

▲ The Yukawa Sector ▲

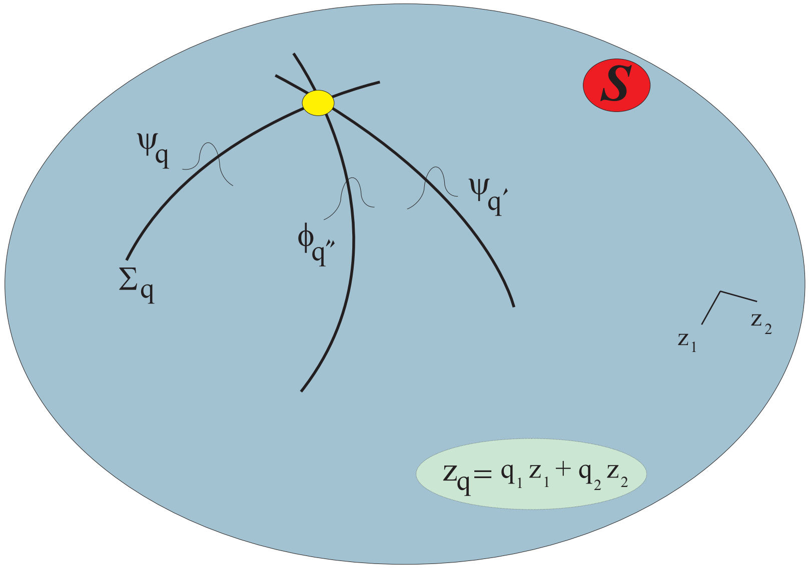
In F-theory, we start with the superpotential W_{8d} of the 8-d adjoint fields A, Φ and the D-term

$$W_{8d} = m_*^4 \int_S \mathrm{Tr}(F \wedge \Phi), \qquad D = \int_S \omega \wedge F + \frac{1}{2}[\Phi, \bar{\Phi}] \ ,$$

$$\text{with} : F = dA - iA \wedge A \qquad \omega = i\frac{g}{2}(dz_1 \wedge \bar{d}z_1 + dz_2 \wedge \bar{d}z_2)$$

A and Φ satisfy the EoM which can be solved by expanding the fields assuming linear fluctuations around their background values. (*Heckmann et al 0802.3391, 0806.0102*)
Solutions exhibit a Gaussian profile peaked along the matter curve and waning out along the transverse direction

Matter fields are represented by wavefunctions ψ_i, ϕ on the intersections of 7-branes with **S**.



$$\psi \propto R_a f(z_{\|}) \exp(-|z_{\perp}|^2)$$

(Font et al, 1211.6529, Camara et al, 1110.2206, GKL, GG Ross, 1009.6000, ...)

$f(z_{\|}) \to g(\tau_i)$ is a holomorphic function unspecified by the EoM. Since this is associated with a specific matter curve determined by $z_2(\tau_2)$ it would be natural to assume that it inherits the corresponding modular properties. We may then impose

$$g(\tau_2') = (c\tau_2 + d)^{k_2} g(\tau_2)$$

By the same token, R_a is some representation of the congruence subgroup of $SL(2, Z)$ left over after restrictions are imposed.

The Yukawa coupling emerges as an integral over overlapping wavefunctions

$$W_{Yuk} = -im_*^4 \int_S \mathrm{Tr}(\psi^i \wedge \psi^j \wedge \psi^k) \; = \; C_I \, \mathcal{R} \, G(\tau_k) \qquad (7)$$

where $G(\tau_k)$ modular function of CS moduli, C_I constant of integration and $\mathcal{R}$ irrep, all associated with the specific Yukawa coupling.

We may utilise the results of the previous analysis, where for our choice of fluxes, the residual symmetry is S_4

Consistency with the properties already discussed leads to the following transformation rules under S_4

Matter Fields	Charge	S_4	k
$10_M^A(Q_1, Q_2)$	t_1	**2**	2
$10_M^B(Q_3)$	t_1	**1**	4
$\bar{5}_3$	$t_1 + t_5$	**3**	2
5_{H_u}	$-2t_1$	**1**	0
5_2	$t_1 + t_4$	**1**	0

The superpotential is

$$
\begin{aligned}
\mathcal{W} &= \alpha\, Y_1^{(4)}\, 10_M^A\, 10_M^A\, 5_{H_u} \\
&+ \beta\, Y_2^{(4)} 10_M^A 10_M^A 5_{H_u} + \gamma\, Y_1^{(8)} 10_M^B 10_M^B 5_{H_u} + \\
&+ \delta\, Y_2^{(6)} 10_M^B 10_M^A 5_{H_u} \\
&+ \frac{\langle\theta_{31}\rangle}{M}\left(\alpha'\, 10_M^A \bar{5}_3 Y_3^{(4)} \bar{5}_2 + \beta'\, 10_M^A \bar{5}_3 Y_{3'}^{(4)} \bar{5}_2 + \gamma'\, 10_M^B \bar{5}_3 Y_3^{(6)} \bar{5}_2\right)
\end{aligned}
$$

As an example, let's see the mass matrix for the up quarks.
The superpotential terms consistent with all symmetries give rise to the following Yukawa matrix

$$\begin{pmatrix} (\alpha+\beta)Y_1^2+(\alpha-\beta)Y_2^2 & 2\beta Y_1 Y_2 & 0 \\ 2\beta Y_1 Y_2 & (\alpha-\beta)Y_1^2+(\alpha+\beta)Y_2^2 & 0 \\ \delta Y_1(Y_2^2+Y_1^2) & -\delta Y_2(Y_2^2+Y_1^2) & \gamma(Y_1^2+Y_2^2)^2 \end{pmatrix}$$

- Similar matrices are derived for down quarks and charged leptons.
- The coefficient are in principle calculable...
- There benchmark points where all masses and CKM mixing are in agreement with experimental data
- Neutrino sector includes 3 ν^c to realise the see-saw mechanism.

CONCLUSIONS

or

What Geometry and Topology of internal space can tell us about Low Energy Theory

1. Geometric Singularities of internal space determine the **Gauge Symmetry** of the Effective Field Theory
2. Euler Characteristic of the compactification manifold counts the number of **Massless Fields**
3. Hierarchy and Yukawa matrices: associated with Modular Symmetries of CS moduli underlying the shape of matter curves and CY manifold in general.
4. g_s $\rightarrow$ determines strength of Yukawa coupling
5. Geometric $U(1)$ Fluxes break **Gauge Symmetries** and produce Chiral Spectrum
6. Kähler moduli control the size of Internal Volume which, amogst other things, determines cosmological properties.

THANK YOU

for your attention

APPENDIX

Closed String Spectrum

Emerges from the combination of L-moving and R-moving open strings with NS and R boundary conditions

Closed String Sectors:

$$(NS, NS),\ (NS, R),\ (R, NS),\ (R, R)$$

bosons: (NS, NS) and (R, R)
fermions: (NS, R) and (R, NS)

Type **II-B**:
truncation of spectrum to preserve supersymmetry: $(-)^{F_{L,R}}$

$$Left = \left\{ \begin{array}{c} NS_+ \\ R_- \end{array} \right\}, \quad Right = \left\{ \begin{array}{c} NS_+ \\ R_- \end{array} \right\}$$

★ multiplicity of spectrum on GUT surface counted by topological invariant (Euler characteristic)

$$-n_j = \chi(S, \mathcal{L}_j) = 1 + \frac{1}{2} c_1(\mathcal{L}_j) \cdot c_1(\mathcal{L}_j) + \frac{1}{2} c_1(\mathcal{L}_j) \cdot c_1(S)$$

$$-n_j^* = \chi(S, \mathcal{L}^*) = 1 + \frac{1}{2} c_1(\mathcal{L}_j^*) \cdot c_1(\mathcal{L}_j^*) + \frac{1}{2} c_1(\mathcal{L}_j^*) \cdot c_1(S)$$

▲ $c_1(S), c_1(L)$ Chern classes (topological invariants counting independent sections)

At the level of $SU(5)$ *breaking consider* $(3,2)_{-\frac{5}{6}} + (\bar{3},2)_{\frac{5}{6}} \in 24$:

$$\chi(S, \mathcal{L}^*) + \chi(S, \mathcal{L}) = 2 + c_1(\mathcal{L}) \cdot c_1(L)$$

These states can be eliminated by choosing

$$c_1(\mathcal{L})^2 = -2$$

Hyper-Flux Doublet-Triplet splitting :

$U(1)_Y-$ **Flux**-splitting of $\mathbf{5_{H_u}}$:

$$n_{(3,1)_{-\frac{1}{3}}} - n_{(\bar{3},1)_{\frac{1}{3}}} = M_5 = 0$$

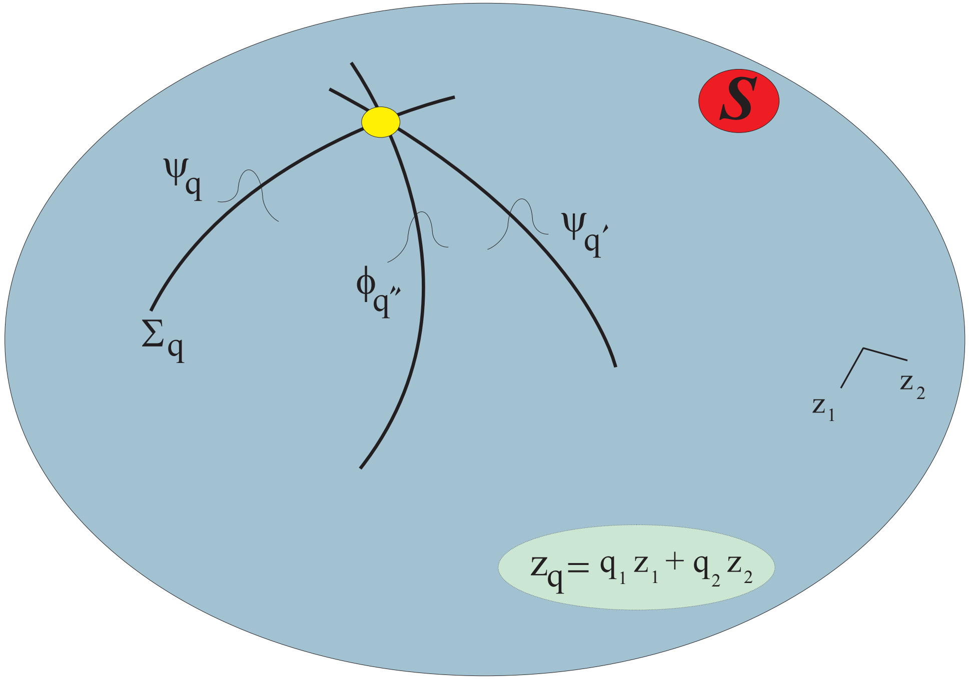
$$n_{(1,2)_{\frac{1}{2}}} - n_{(1,2)_{-\frac{1}{2}}} = M_5 + N_{Y_5} = 0 + 1 = 1\ (H_u)$$

$U(1)_Y-$ **Flux**-splitting of $\mathbf{\bar{5}_{H_d}}$ $\rightarrow$:

$$n_{(3,1)_{-\frac{1}{3}}} - n_{(\bar{3},1)_{\frac{1}{3}}} = M_5 = 0$$

$$n_{(1,2)_{\frac{1}{2}}} - n_{(1,2)_{-\frac{1}{2}}} = M_5 + N_{Y_5} = 0 - 1 = -1\ (H_d)$$

Particles' Wavefunctions: solving EoM $\rightarrow \psi \sim f(z_i)e^{-M|z_j|^2}$



Strength of Yukawa coupling $\propto$ integral of overlapping ψ's at 3-intersection:

$$\lambda_{ij} \propto \int \psi_i(z_1, z_2)\psi_j(z_1, z_2)\psi_H(z_1, z_2)dz_1 \wedge dz_2 \approx \lesssim 1$$

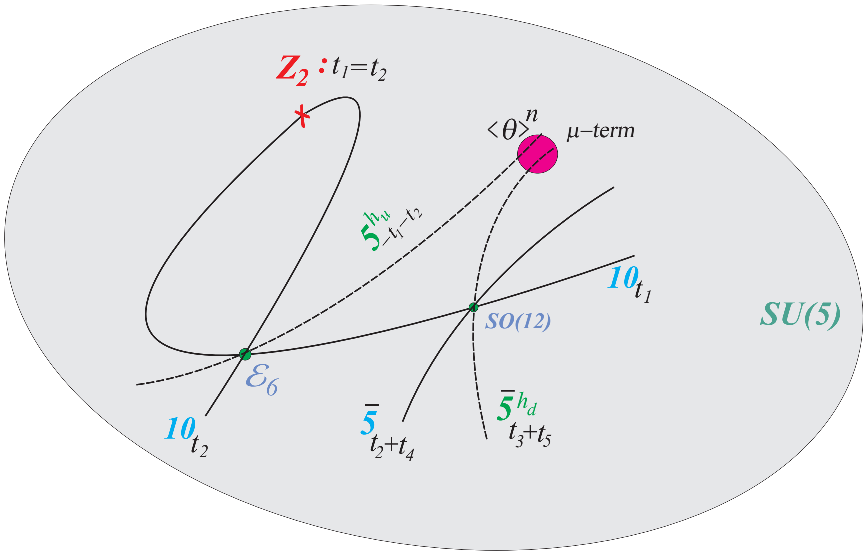
In a holomorphic basis of the wavefunctions the Yukawa Lagrangian is

$$\mathcal{L} = \sum_p \psi_i(p)\psi_j(p)\psi_k(p) \int d\theta^2 \chi_i\chi_j\chi_k \equiv \lambda_{ijk} \int d\theta^2 \chi_i\chi_j\chi_k$$

1. $\psi_i(p)$: component corresponds to the internal value of the wavefunction
2. χ_i: Corresponding chiral superfield, evaluated at the point p

Taking into account the normalisation of kinetic terms, while assuming comparable size of volumes for matter curves, $\mathcal{V}_\Sigma^2 \sim \mathcal{V}$,

$$\mathcal{L} \propto a_{GUT}^{3/4} \lambda_{ijk}^0 \int d\theta^2 \chi_i\chi_j\chi_k$$



Z2 : t1=t2
⟨θ⟩n
μ−term
5hu −t1−t2
10 t1
SO(12)
SU(5)
E6
10 t2
5̄ t2+t4
5̄hd t3+t5

Recall that type IIB string theory admits $SL(2,\mathbb{Z})$
This implies invariance of the resulting EFT under some subgroup
$\Gamma_s \subset SL(2,\mathbb{Z})$.

$\Downarrow$

This Motivates us to look for a $SL(2,\mathbb{Z})$ completion of
loop-corrections.

Consider the non-holomorphic Eisenstein series: $\mathcal{E}_{\frac{3}{2}} \equiv E_{\frac{3}{2}}(S,\overline{S})$:

$$\mathcal{E}_{\frac{3}{2}} = 2\zeta(3)\underbrace{\left(\frac{\tau-\bar{\tau}}{2i}\right)^{\frac{3}{2}}}_{\alpha'^3-\text{part}} + 4\zeta(2)\underbrace{\left(\frac{\tau-\bar{\tau}}{2i}\right)^{-\frac{1}{2}}}_{\text{loop}-\text{part}} + \underbrace{\left(\frac{\tau-\bar{\tau}}{2i}\right)^{\frac{1}{2}}}_{\text{non}-\text{pert.part}} \mathcal{O}(e^{-2\pi\mathrm{Re}\tau}) \tag{8}$$

Observation:

1^{st} and 2^{nd} terms are associated with α'^3 and loop corrections.

Higher derivative couplings in curvature

(generated by multigraviton scattering)

(see hep-th/9704145; 9707013; 9707018)

Leading correction term in type II-B action:

proportional to the fourth power of curvature:

$$\mathcal{S} \supset A \int_{M_{10}} e^{-2\phi_{10}} \mathcal{R}_{(10)} - B \int_{M_{10}} (-2\zeta(3) e^{-2\phi_{10}} - 4\zeta(2)) R^4 \wedge e^2$$

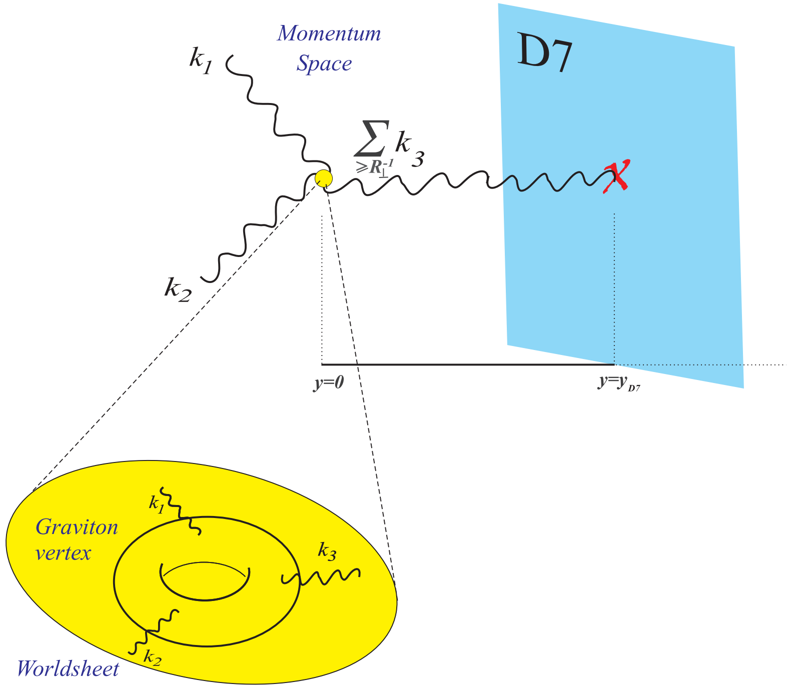
Upon compactification R^4-term induces a **novel** Einstein-Hilbert ($\mathcal{EH}$) term $\mathcal{R}_{(4)} \propto$ by the Euler characteristic χ:

$$\propto \underbrace{\chi \int_{M_4} (\zeta(2) - \zeta(3) e^{-2\phi}) \mathcal{R}_{(4)}}_{\textit{induced } \mathcal{EH} \textit{ term}}, \qquad \text{where:} \quad \chi \sim \int_{\mathcal{X}_6} R^3$$

▲▲ *this $\mathcal{EH}$ term is possible in 4-dimensions only*!

▲▲ New $\mathcal{EH}$-term is localised at points with $\chi \neq 0$▲▲

Consider graviton scattering with KK-exchange between graviton vertex and a $D7$-brane



$$\propto \zeta(2)\chi \int_{M_4} \left(1 + \sum_{i=1,2,3} e^{2\phi} \mathcal{T}_i \log(R_\perp^i) \right) \mathcal{R}_{(4)} \ ,$$

Solutions for massless fields residing on 7-branes.
Equations solved by expanding the fields A, Φ assuming linear fluctuations around the background:

$$A_{\bar{m}} \to \langle A_{\bar{m}} \rangle + a_{\bar{m}}, \qquad \Phi \to \langle \Phi \rangle + \varphi \tag{9}$$

with the definitions

$$a = a_{\bar{z}_1} d\bar{z}_1 + a_{\bar{z}_2} d\bar{z}_2, \qquad \varphi = \varphi_{\bar{z}_1 \bar{z}_2} d\bar{z}_1 \wedge d\bar{z}_2 \tag{10}$$

Then, keeping only linear terms regarding the fluctuations φ, a, in the holomorphic gauge the EoM take the form

$$\begin{aligned} \bar{\partial}_{\langle A \rangle} a &= 0 \\ \bar{\partial}_{\langle A \rangle} \varphi - i[a, \langle \bar{\Phi} \rangle] &= 0 \\ \omega \wedge \partial_{\langle A \rangle} a - \frac{1}{2}[\langle \bar{\Phi} \rangle, \varphi] &= 0 \end{aligned}$$

Master formula for F-term potential

(*for generic* $\mathcal{U}_1$ loop corrections)

$\Downarrow$

$$\boxed{V_{\alpha'+\text{generic}} = e^{\mathcal{K}}\left(\frac{3\mathcal{V}}{2\mathcal{U}^2}\left(1+\frac{\partial\mathcal{U}_1}{\partial\mathcal{V}}\right)^2\frac{4\mathcal{V}^2+\mathcal{V}\hat{\xi}+4\hat{\xi}^2}{\mathcal{V}-\hat{\xi}}-3\right)|W_0|^2}$$

▲▲ For α' and logarithmic corrections: $\mathcal{U}_1 = -\hat{\eta}+\hat{\eta}\log\mathcal{V}$:

$$V_{\alpha'+\log} \;=\; 12 g_s e^{K_{cs}}|W_0|^2\,\underbrace{\hat{\xi}\frac{\mathcal{V}^2+7\hat{\xi}\mathcal{V}+\hat{\xi}^2}{\left(\mathcal{V}-\hat{\xi}\right)\left(2\mathcal{V}+\hat{\xi}\right)^4}}_{\alpha'^3-corrections}$$

$$-\frac{3\kappa}{2}|W_0|^2\underbrace{\frac{2\hat{\eta}-\hat{\eta}\log\mathcal{V}}{2\mathcal{V}^3}}_{logarithmic}+\cdots$$