Beyond SL(2,Z) in the top-down approach

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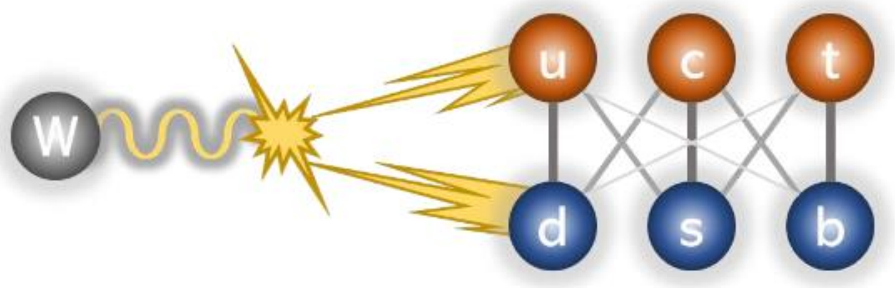
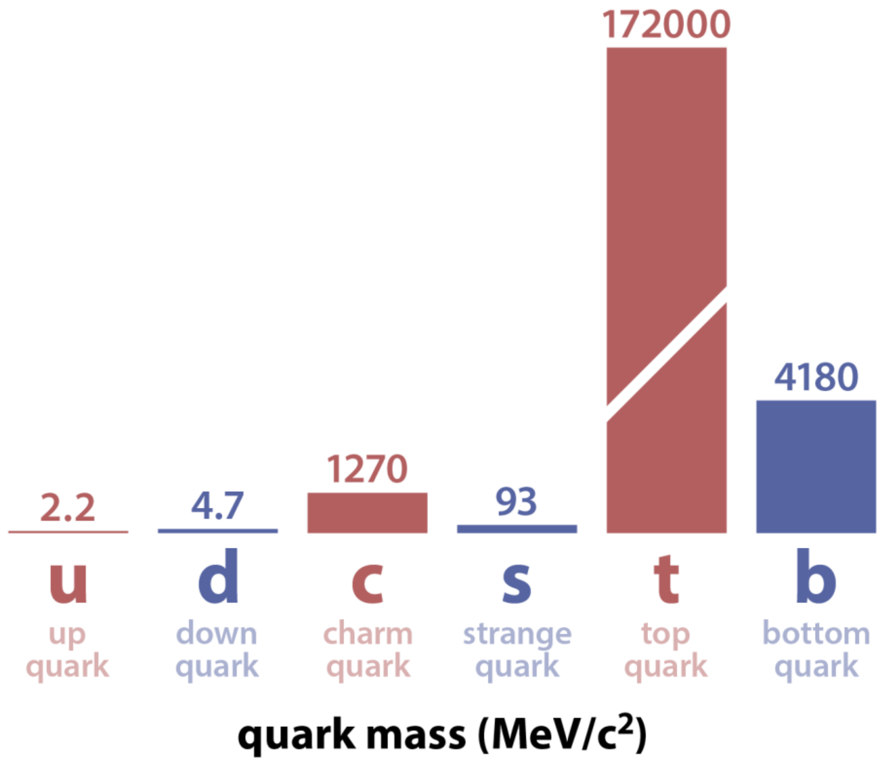
Contents:

Flavour structure

- Flavour puzzles of the Standard Model(SM):
  - e.g. quark sector:
    - ① Mass hierarchy of quarks
    - ② CKM mixing matrix
    - ③ CP violation



$$|V_{CKM}| = \begin{pmatrix} 0.974 & 0.225 & 0.00347 \\ 0.225 & 0.973 & 0.0410 \\ 0.00862 & 0.0403 & 0.999 \end{pmatrix}$$



$$J_{CP} = \mathrm{Im}\left((V_{\mathrm{CKM}})_{12}(V_{\mathrm{CKM}})_{23}(V_{\mathrm{CKM}})^*_{13}(V_{\mathrm{CKM}})^*_{22}\right) \sim 2.8 \times 10^{-5}$$

Flavour structure

- The SM cannot explain why the parameters show such patterns.
  → Need to study the underlying fundamental theory
  (Beyond the SM)

Energy scale

Superstring theory (10D)

M_{GUT} 4D Effective field theory

M_{EW} SM

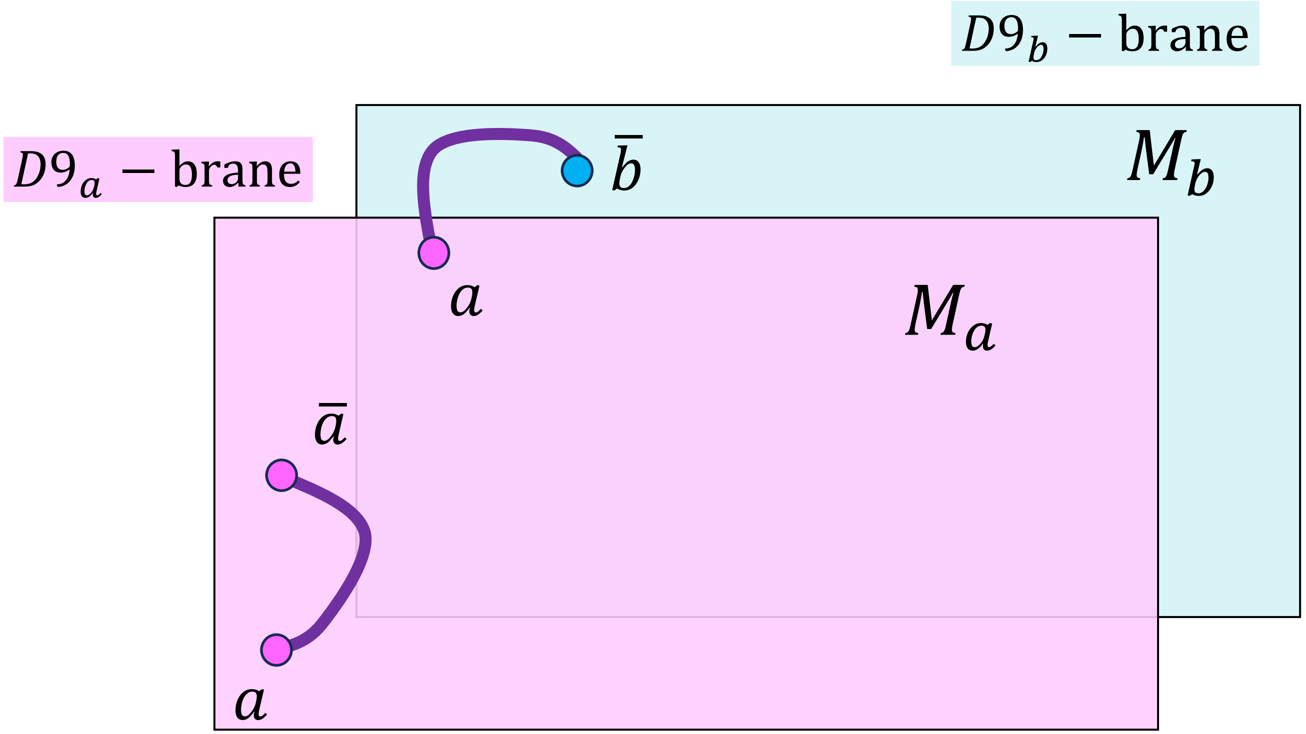
e.g. Magnetized torus compactification

The Lagrangian respects the modular symmetry
(geometrical discrete symmetry of the torus)

Contents:

Magnetized D-brane model

◆Magnetized D9-branes in Type IIB string theory



- **Background magnetic flux:**

$$F_{45} = \begin{pmatrix} M_a \mathbf{1}_{N_a \times N_a} & \\ & M_b \mathbf{1}_{N_b \times N_b} \end{pmatrix}$$

Gauge space

$$U(N_a + N_b) \to U(N_a) \times U(N_b)$$

- **Gaugino:**

$$\lambda = \begin{pmatrix} \lambda^{a\bar{a}} & \lambda^{a\bar{b}} \\ \lambda^{b\bar{a}} & \lambda^{b\bar{b}} \end{pmatrix}$$

Gauge space

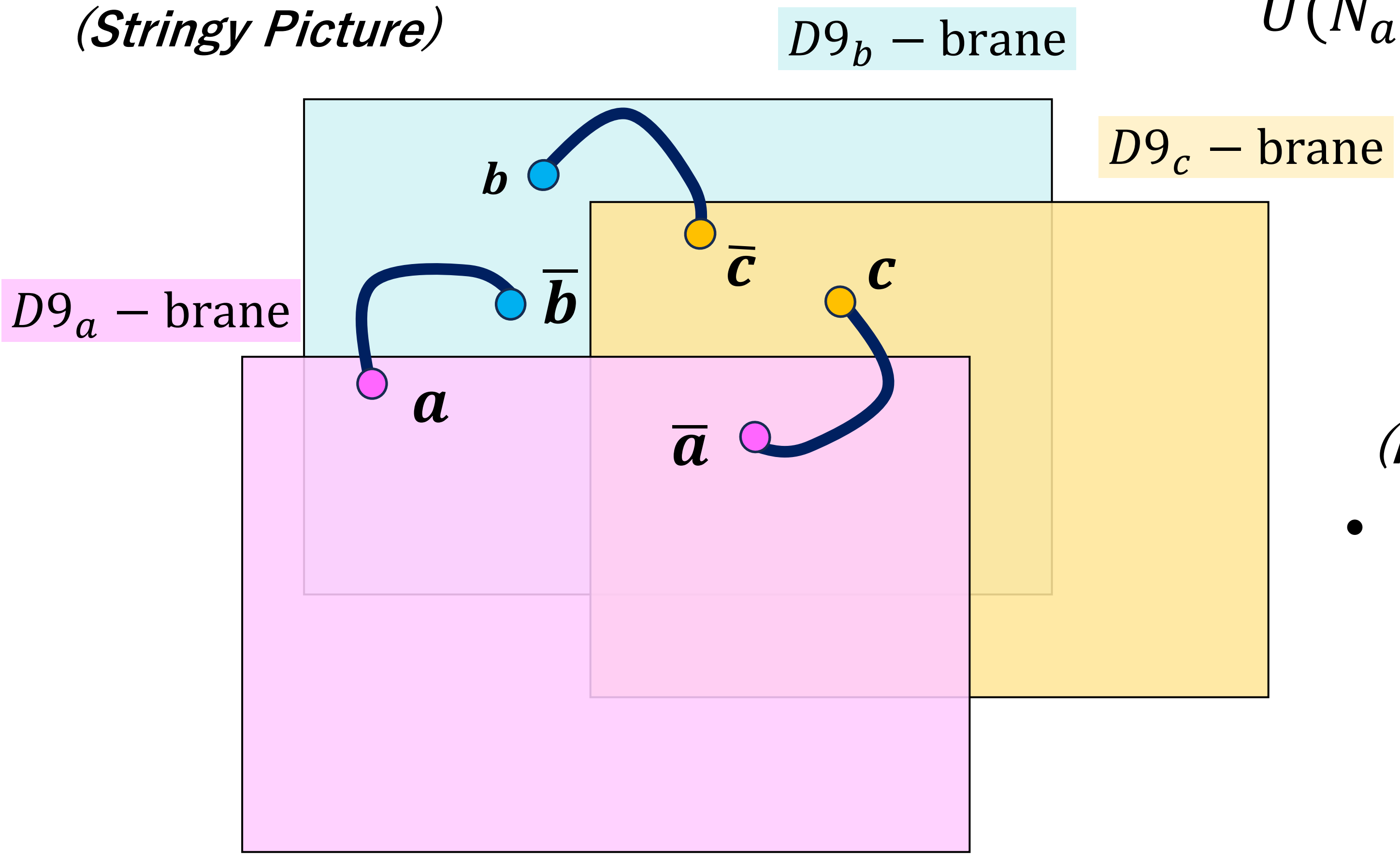
- **Same structure in Gauge boson**

Magnetized D-brane model

◆Magnetized D9-branes in Type IIB string theory

(Stringy Picture)

$$U(N_a + N_b + N_c) \to U(N_a) \times U(N_b) \times U(N_c)$$



- Yukawa coupling corresponds to the 3-point coupling.

(Field Theory Picture)

- Overlap integral of zero-modes

$$Y \propto \int d^6y\, \lambda_{6D}^{a\bar{b}} \lambda_{6D}^{c\bar{a}} A_{6D}^{b\bar{c}}$$

Top-down approach

High Energy

$M_{planck} \sim 10^{19}$GeV

Type II-B superstring

10D $\mathcal{N} = 1$ super Yang-Mills

Compactification

$M_{EW} \sim 10^{2}$GeV

Standard Model

Low Energy

$$\mathcal{L}_{SYM} = -\frac{1}{4g^2}\mathrm{Tr}\left\{F^{MN}F_{MN}\right\} + \frac{i}{2g^2}\mathrm{Tr}\left\{\bar{\lambda}\Gamma^{M}D_{M}\lambda\right\}$$

Gauge Boson

Higgs

4D gauge boson

λ: Gauginos

Matter fermions

$$S_{4D} = \int d^4x\ \mathcal{L}_{SM} + \cdots$$

Top-down approach

Higher dimensional Lagrangian (e.g. 10D)

$$L_{10} = g\int d^4x d^6y\, \bar{\lambda}(x,y) A(x,y) \lambda(x,y)$$

Kaluza-Klein decomposition + integrate the compact space
⇒ 4D theory

$$L_4 = Y\int d^4x \bar{\chi}(x)\varphi(x)\chi(x)$$

$$Y = g\int d^6y \bar{\psi}(y)\phi(y)\psi(y)$$

Yukawa couplings

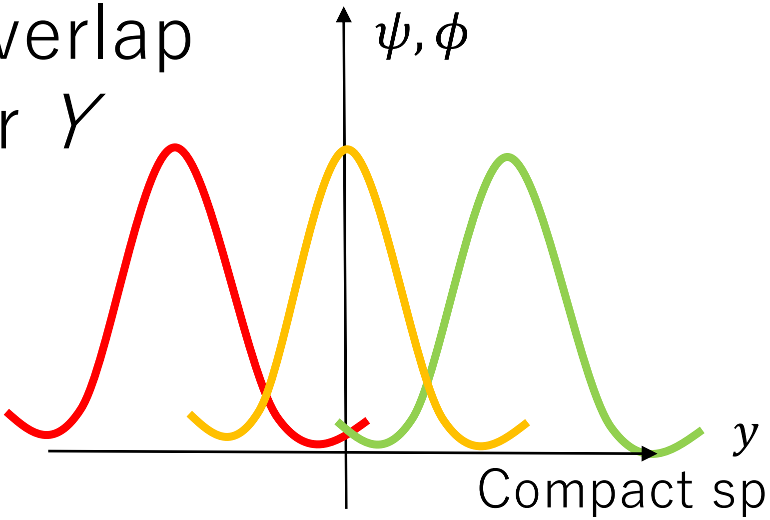
◆Yukawa coupling is computed by the overlap integrals of zero-modes over the extra dimension.

Fermion zero-modes in 6D space $\mathcal{M}$

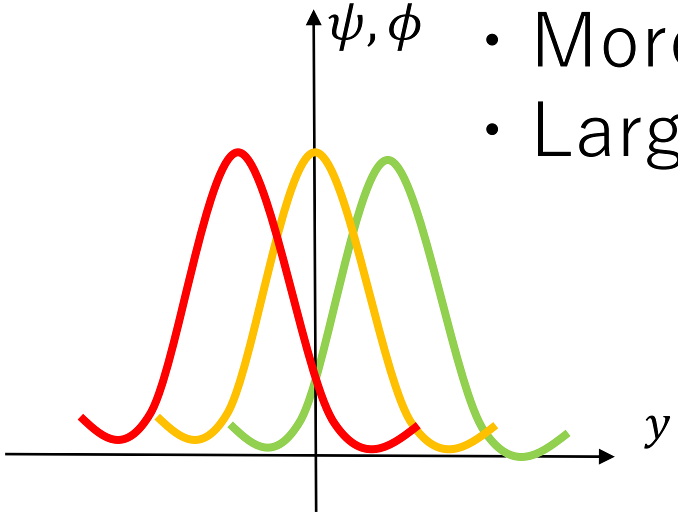
$$Y_{IJK} = \frac{1}{2g^2}\int_{\mathcal{M}} d^6y\, \psi_I^{ab\dagger}\phi_{J,i}^{bc}\Gamma^i\psi_K^{ca}$$

6D part of Higgs

- Less overlap
- Smaller Y



Compact space

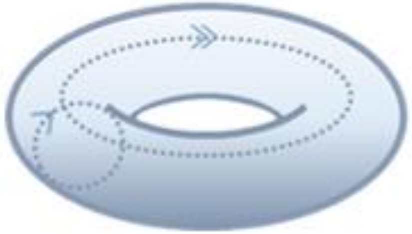


- More overlap
- Larger Y

◆Explicit computation of zero-modes is possible on toroidal compact space.

Magnetized T^2 model

Magnetized T^2 model



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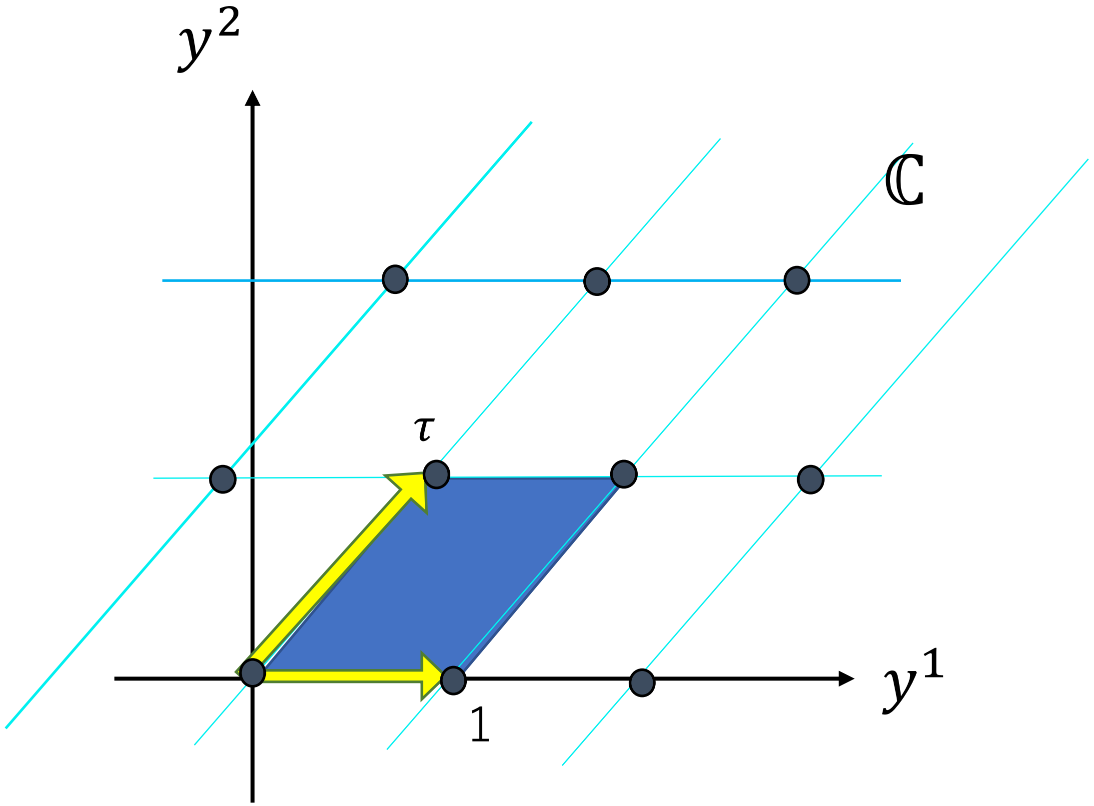


Magnetic flux is quantized to integer

$F = 2\pi M_a \cdot dy^1 \wedge dy^2 \quad (M_a \in \mathbb{Z})$

$$z \sim z + 1 \sim z + \tau$$

τ: complex structure modulus $(\mathrm{Im}\tau > 0)$

Magnetized T^2 model

◆Zero-mode wavefunctions on magnetized T^2:

[D. Cremades, L. E. Ibanez, and F. Marchesano (2004)]

- Let us consider fermion zero-modes in the bi-fundamental representation of $U(N_a) \times U(N_b)$:

$\psi^{a\bar{b}}$ feels the difference of magnetic flux, i.e. $M_{ab} = M_a - M_b$

- Dirac equation for zero-modes on magnetized T^2:

$$i(\partial_z + iA)\psi^{a\bar{b}} = 0$$

where

$$\psi^{a\bar{b}} = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

ψ_+ ← Positive chirality

ψ_- ← Negative chirality

$$A = -i\frac{\pi M_{ab}}{2Im\tau}z$$

Magnetized T^2 model

◆Zero-mode wavefunctions on magnetized T^2:

[D. Cremades, L. E. Ibanez, and F. Marchesano (2004)]

- $|M_{ab}| \in \mathbb{Z}$ degenerated zero-modes appear.
- If $M_{ab} > 0$, we obtain

Jacobi theta function

Solution: $\psi_+^j \propto e^{\frac{\pi i M_{ab} z}{Im\tau}} \vartheta \begin{bmatrix} \frac{j}{M_{ab}} \\ 0 \end{bmatrix} (M_{ab}z, M_{ab}\tau),$ $\quad j = 0, 1, \ldots, |M_{ab}| - 1$

$\psi_- = 0$

Useful property to realize the **4D chiral theory**

- If $M_{ab} < 0$, we obtain $\psi_+ = 0$, $\psi_- \neq 0$.

Magnetized T^2 model

◆Transformation of zero-mode wavefunctions under S and T:

- Transforms like modular forms of weight 1/2

$$S : \psi^{j,|M|}(z,\tau) \to \psi^{j,|M|}\left(-\frac{z}{\tau}, -\frac{1}{\tau}\right) = (-\tau)^{1/2} \sum_{k=0}^{|M|-1} e^{i\pi/4} \frac{1}{\sqrt{|M|}} e^{2\pi i \frac{jk}{|M|}} \psi^{k,|M|}(z,\tau),$$

$$T : \psi^{j,|M|}(z,\tau) \to \psi^{j,|M|}(z,\tau+1) = e^{i\pi \frac{j^2}{|M|}} \psi^{j,|M|}(z,\tau),$$

- Weight 1/2 modular forms are relevant to the double covering of $\Gamma = SL(2,Z)$

$$\tilde{\Gamma} = S\widetilde{L(2,Z)}$$

[S. Kikuchi, T. Kobayashi, S. Takada, T. Tatsuishi, H. Uchida (2020)]

Magnetized T^2 model

- The double-covering group $\tilde{\Gamma}$ is defined by

$$\widetilde{\Gamma} \equiv \left\{[\gamma, \epsilon] \middle| \gamma \in \Gamma,\ \epsilon \in \{\pm 1\}\right\}.$$

$\Gamma = SL(2, Z)$

The multiplication of arbitrary two elements, $[\gamma_1, \epsilon_1], [\gamma_2, \epsilon_2] \in \widetilde{\Gamma}$, is defined by

$$[\gamma_1, \epsilon_1][\gamma_2, \epsilon_2] = [\gamma_1\gamma_2, A(\gamma_1, \gamma_2)\epsilon_1\epsilon_2],$$

Cocycle condition $\quad A(\gamma_1, \gamma_2)A(\gamma_1\gamma_2, \gamma_3) = A(\gamma_1, \gamma_2\gamma_3)A(\gamma_2, \gamma_3).$

- The zero-mode wavefunctions behave like modular forms of ½ for $\tilde{\Gamma}(2|M|)$.

$$\widetilde{\Gamma}(2|M|) \equiv \{[h, \epsilon] \in \widetilde{\Gamma} | h \in \Gamma(2|M|), \epsilon = 1\}.$$

[S. Kikuchi, T. Kobayashi, S. Takada, T. Tatsuishi, H. Uchida (2020)]

Magnetized T^2 model

- The zero-mode wavefunctions behave like modular forms of ½ for $\tilde{\Gamma}(2|M|)$.
- Unitary representation of $\tilde{\Gamma}_{2|M|} = \tilde{\Gamma}/\tilde{\Gamma}(2|M|)$.

[S. Kikuchi, T. Kobayashi, S. Takada, T. Tatsuishi, H. Uchida (2020)]

When $M = 2$,

$$\rho(\widetilde{S}) = \frac{e^{i\pi/4}}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad \rho(\widetilde{T}) = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}.$$

$$\widetilde{S_4} \simeq T' \rtimes Z_4.$$ Order: 96

When $M = 4$,

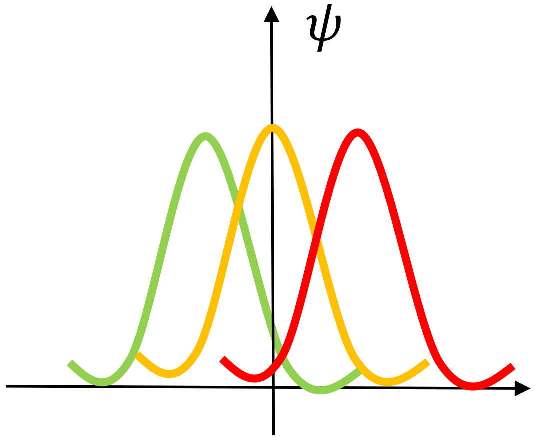
$$\rho(\widetilde{S}) = \frac{e^{i\pi/4}}{2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{pmatrix}, \quad \rho(\widetilde{T}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{i\pi/4} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & e^{i\pi/4} \end{pmatrix}.$$

$$\Delta(48) \rtimes Z_8.$$ Order: 384

Magnetized T^2 model

◆Modular transformation of Yukawa couplings

$$\tilde{Y}^{ijk} \propto \int_{T^2} dz d\bar{z}\, \psi_{T^2}^{i\,M_{ab}} \psi_{T^2}^{j,M_{ca}} \left(\psi_{T^2}^{k,M_{cb}}\right)^*$$



$$\tilde{Y}^{ijk} = (2\mathrm{Im}\tau)^{-1/2} \mathcal{A}^{-1/2} \left|\frac{M_{ab}M_{ca}}{M_{cb}}\right|^{1/4} \sum_{n=1}^{g} \vartheta \begin{bmatrix} \frac{|M_{ca}|k - |M_{cb}|j + |M_{ca}M_{cb}|\ell_0}{|M_{ab}M_{ca}M_{cb}|} + \frac{n}{g} \\ 0 \end{bmatrix} (0, |M_{ab}M_{ca}M_{cb}|\tau)$$

Where $g = \mathrm{gcd}(|M_{ab}|, |M_{cb}|)$

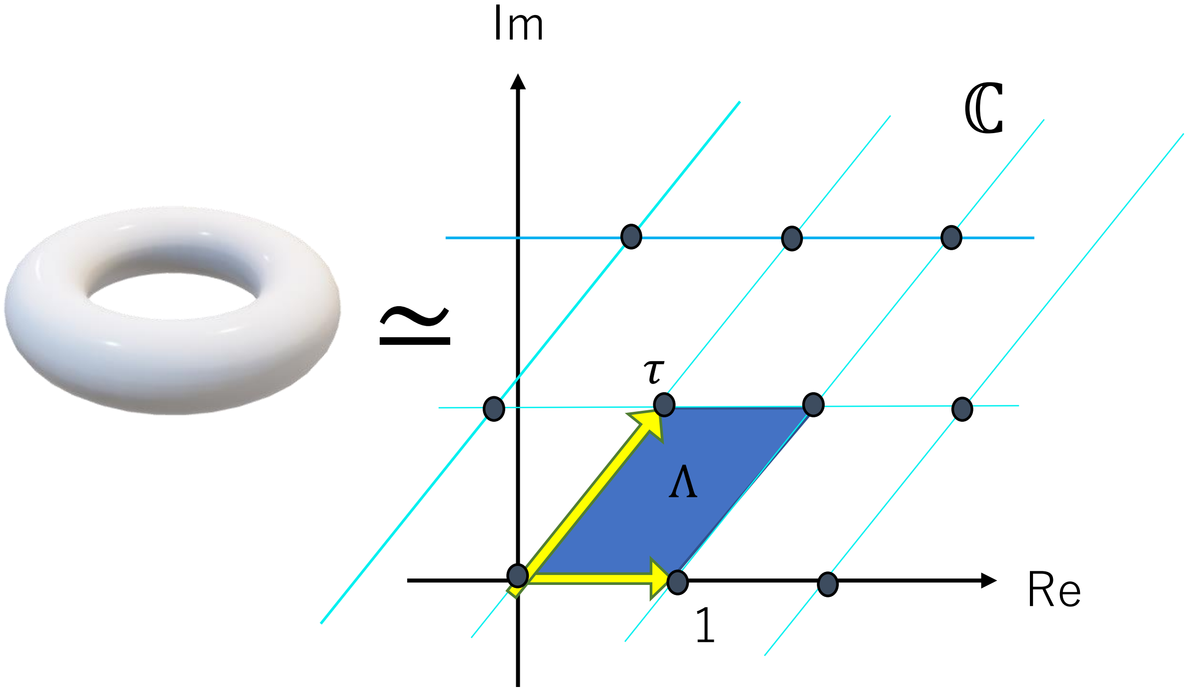
◆$Y^{ijk}(\tau)$ transforms as a modular form of weight 1/2 for $\tilde{\Gamma}(2N)$

where $N = \mathrm{lcm}(M_{ab}M_{ca}M_{cb})$

[K. Hoshiya, S. Kikuchi, T. Kobayashi, Y. Ogawa, H. Uchida (2021)]

Higher dimensional torus model

$T^2 \cong \mathbb{C}/\Lambda$



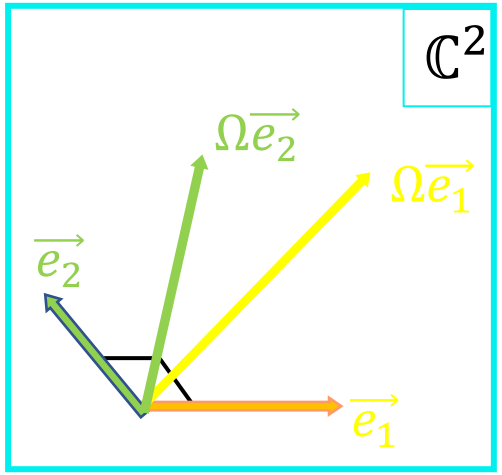
$$z \sim z + 1 \sim z + \tau$$

$(\mathrm{Im}\tau > 0)$

$T^4 \cong \mathbb{C}^2/\Lambda$

Λ: 4つの独立なベクトルからなる格子

$\vec{e_1} = (1,0)$
$\vec{e_2} = (0,1)$
$\Omega\vec{e_1} = (\Omega_{11}, \Omega_{12})$
$\Omega\vec{e_2} = (\Omega_{21}, \Omega_{22})$



$$\vec{z} = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \sim \vec{z} + \vec{e_i} \sim \vec{z} + \Omega\vec{e_i}$$

Magnetized T^4 model

◆Backgroud magnetic flux:

[I. Antoniadis, A. Kumar, and B. Panda (2009)]

$$F = \pi[M^T \cdot (\mathrm{Im}\Omega)^{-1}]_{ij}\left(idz^i \wedge d\bar{z}^j\right) \qquad i,j \in \{1,2\}$$

- M is an integer-valued matrix:

$$M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \qquad M_{ij} \in \mathbb{Z}$$

- F-term SUSY condition:

$$(M\Omega)^T = M\Omega$$

Magnetized T^4 model

◆Zero-mode wavefunctions on magnetized T^4:

[I. Antoniadis, A. Kumar, and B. Panda (2009)]

Vector potential:

$$A = \pi\, \mathrm{Im}\{[M_{ab}\bar{\vec{z}}] \cdot (Im\Omega)^{-1} d\vec{z}\}$$

Spinors:

$$\Psi(\vec{z}, \bar{\vec{z}}) = \begin{pmatrix} \psi_+^1 \\ \psi_-^2 \\ \psi_-^1 \\ \psi_+^2 \end{pmatrix}$$

If $\det[\mathrm{Im}(M\Omega)] > 0$, we obtain:

Riemann theta function

$$\psi_+^1 \propto e^{\pi i [M\vec{z}]\cdot(Im\Omega)^{-1}\mathrm{Im}\vec{z}}\, \vartheta \begin{bmatrix} (J_1, J_2) M_{ab}{}^{-1} \\ \vec{0} \end{bmatrix} (M_{ab}\vec{z}, M_{ab}\Omega) \equiv |J_1, J_2\rangle$$

$$\psi_-^2 = \psi_-^1 = \psi_+^2 = 0$$

$(J_1, J_2) \in \mathbb{Z}^2$ inside the cell spanned by $Ne_{k(=1,2)}$.

$$e_1 = \binom{1}{0}, e_2 = \binom{0}{1}$$

(degeneracies of zero-modes) $= |\det M_{ab}|$

Magnetized T^4 model

- Modular Transformations $Sp(4,\mathbb{Z})$

$$(\vec{z},\boldsymbol{\Omega}) \to \left({}^{t}(C\boldsymbol{\Omega}+D)^{-1}\vec{z}, (A\boldsymbol{\Omega}+B)(C\boldsymbol{\Omega}+D)^{-1} \right)$$

Sets of 4×4 real matrices with integer entries,

$$\gamma = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

satisfying ${}^{t}\gamma J \gamma = J$, where J is the symplectic form:

$$J = \begin{pmatrix} 0 & \mathbb{I}_2 \\ -\mathbb{I}_2 & 0 \end{pmatrix}$$

Behaviors of zero-mode wave functions?

Magnetized T^4 model

◆$Sp(4,Z)$ modular transformation:

Generators in $Sp(4,Z)$:

$$S = \begin{pmatrix} 0 & I_2 \\ -I_2 & 0 \end{pmatrix}, \quad T_{ab} = \begin{pmatrix} I_2 & B_{ab} \\ 0 & I_2 \end{pmatrix}$$

$$B_{11} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix},$$

$$B_{22} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix},$$

$$B_{12} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

- Magnetic flux M is invariant under S if $M^T = M$.
- Magnetic flux M is invariant under T_{ab} if $B_{ab}M^T = MB_{ab}$.

Magnetized T^4 model

◆Example

$$M = \begin{pmatrix} 4 & -2 \\ -2 & 4 \end{pmatrix} \quad \Omega = \begin{pmatrix} \tau & \tau_{12} \\ \tau_{12} & \tau \end{pmatrix}$$

General form of Ω satisfying the F-term SUSY condition: $(M\Omega)^T = M\Omega$

12 degeneracies → 3-dim irreducible rep.

$$|J_1, J_2\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}}(|-1,3\rangle - |3,-1\rangle) \\ \frac{1}{2}(|1,0\rangle - |0,1\rangle + |2,1\rangle - |1,2\rangle) \\ \frac{1}{\sqrt{2}}(|2,0\rangle - |0,2\rangle) \end{pmatrix},$$

$$\rho_{T^4/(\mathbb{Z}_2^{1t}\times\mathbb{Z}_2^{1p})}(\widetilde{S}) = -\frac{1}{2}\begin{pmatrix} 1 & \sqrt{2} & 1 \\ \sqrt{2} & 0 & -\sqrt{2} \\ 1 & -\sqrt{2} & 1 \end{pmatrix}$$

$$\rho_{T^4/(\mathbb{Z}_2^{1t}\times\mathbb{Z}_2^{1p})}(\widetilde{T_{11}}\widetilde{T_{22}}) = \begin{pmatrix} e^{\pi i/3} & & \\ & e^{\pi i/3} & \\ & & e^{4\pi i/3} \end{pmatrix}$$

$$\rho_{T^4/(\mathbb{Z}_2^{1t}\times\mathbb{Z}_2^{1p})}(\widetilde{T}_{12}) = \begin{pmatrix} e^{-\pi i/3} & & \\ & e^{\pi i/6} & \\ & & e^{2\pi i/3} \end{pmatrix}$$

Siegel modular form of weight 1/2

Automorphy factor:

$$\tilde{J}_{1/2}(\tilde{S},\tau) = (\det(-\Omega))^{1/2}$$

$$\Delta(96) \times Z_4 \times Z_3$$

[S. Kikuchi, T. Kobayashi, K.N, S. Takada, H. Uchida (2023)]

Magnetized T^4 model

◆If we restrict the complex structure moduli as

$$M = \begin{pmatrix} 4 & -2 \\ -2 & 4 \end{pmatrix} \qquad \Omega = \begin{pmatrix} \tau & \tau_{12} \\ \tau_{12} & \tau \end{pmatrix} \Rightarrow \begin{pmatrix} \tau & 0 \\ 0 & \tau \end{pmatrix}$$

$$\rho_{T^4/(\mathbb{Z}_2^{1t}\times\mathbb{Z}_2^{1p})}(\widetilde{S}) = -\frac{1}{2}\begin{pmatrix} 1 & \sqrt{2} & 1 \\ \sqrt{2} & 0 & -\sqrt{2} \\ 1 & -\sqrt{2} & 1 \end{pmatrix}$$

$$\Delta(96) \times Z_4 \times Z_3 \xrightarrow{\tau_{12}=0} S_3 \times Z_3.$$

$$\rho_{T^4/(\mathbb{Z}_2^{1t}\times\mathbb{Z}_2^{1p})}(\widetilde{T_{11}}\widetilde{T_{22}}) = \begin{pmatrix} e^{\pi i/3} & & \\ & e^{\pi i/3} & \\ & & e^{4\pi i/3} \end{pmatrix}$$

$$\rho_{T^4/(\mathbb{Z}_2^{1t}\times\mathbb{Z}_2^{1p})}(\widetilde{T}_{12}) = \begin{pmatrix} e^{-\pi i/3} & & \\ & e^{\pi i/6} & \\ & & e^{2\pi i/3} \end{pmatrix}$$

Weight 1

Automorphy factor:

$\tilde{J}_1\left(\tilde{S},\tau\right) = (-\tau)^1$

[S. Kikuchi, T. Kobayashi, K.N, S. Takada, H. Uchida (2023)]

Magnetized T^4 model

◆Transformation of zero-mode wavefunctions under S and T:

- Transforms like modular forms of weight 1/2

$$\tilde{\gamma}: |J_1, J_2\rangle(z,\tau) \to |J_1, J_2\rangle(\tilde{\gamma}(z,\tau)) = \tilde{J}_{1/2}(\tilde{\gamma},\tau)\rho(\tilde{\gamma})_{JK}|K_1, K_2\rangle(z,\tau)$$

$$\tilde{S} \equiv [S,1] \qquad \tilde{J}_{1/2}(\tilde{S},\tau) = (\det(-\Omega))^{1/2} \qquad \rho(\tilde{S})_{JK} = \frac{i}{\sqrt{\det N}} e^{2\pi i J^T N^{-1} K}$$

$$\tilde{T} \equiv [T,1] \qquad \tilde{J}_{1/2}(\tilde{T},\tau) = 1 \qquad \rho(\tilde{T})_{JK} = e^{\pi i J^T B N^{-1} J}\delta_{J,K}$$

[S. Kikuchi, T. Kobayashi, K.N, S. Takada, H. Uchida (2023)]

Contents:

Spontaneous CP violation (Introduction)

◆4D CP is embedded to 10D proper Lorentz transformation.

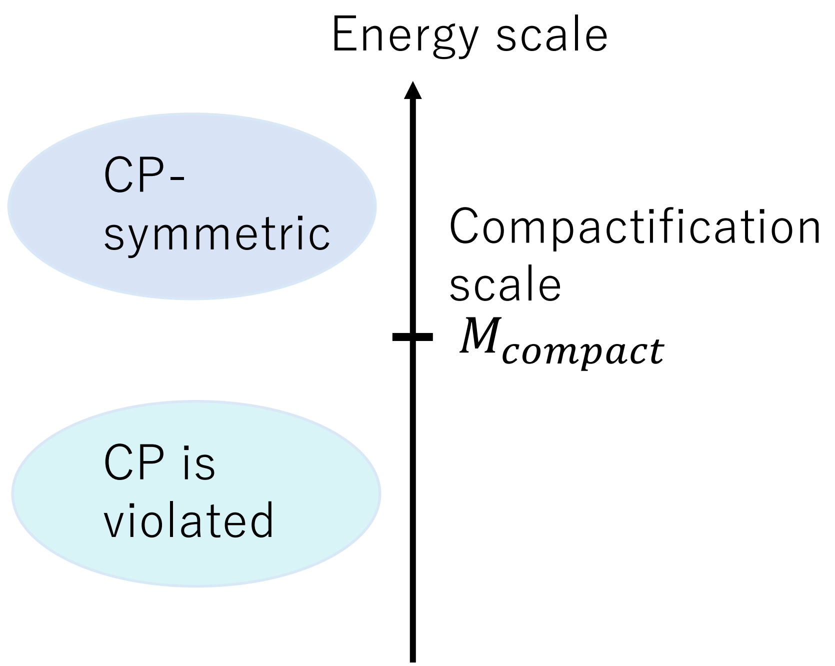
[M.B. Green, J.H. Schwarz, E. Witten (1987)]

4D space-time × 6D compact space

$$(t, x, y, z) \xrightarrow{CP} (t, -x, -y, -z) \times Z_i \xrightarrow{CP} -\bar{Z}_i, \ (i = 1,2,3)$$

∈ 10D proper Lorentz (Symmetry of string theory)

- Compactification (fixing the vacuum state) is the source of CP violation.
- CP symmetry is expected to restore at very high energy, $E \gg M_{compact}$.
- Spontaneous CP violation is natural.



Spontaneous CP violation (Introduction)

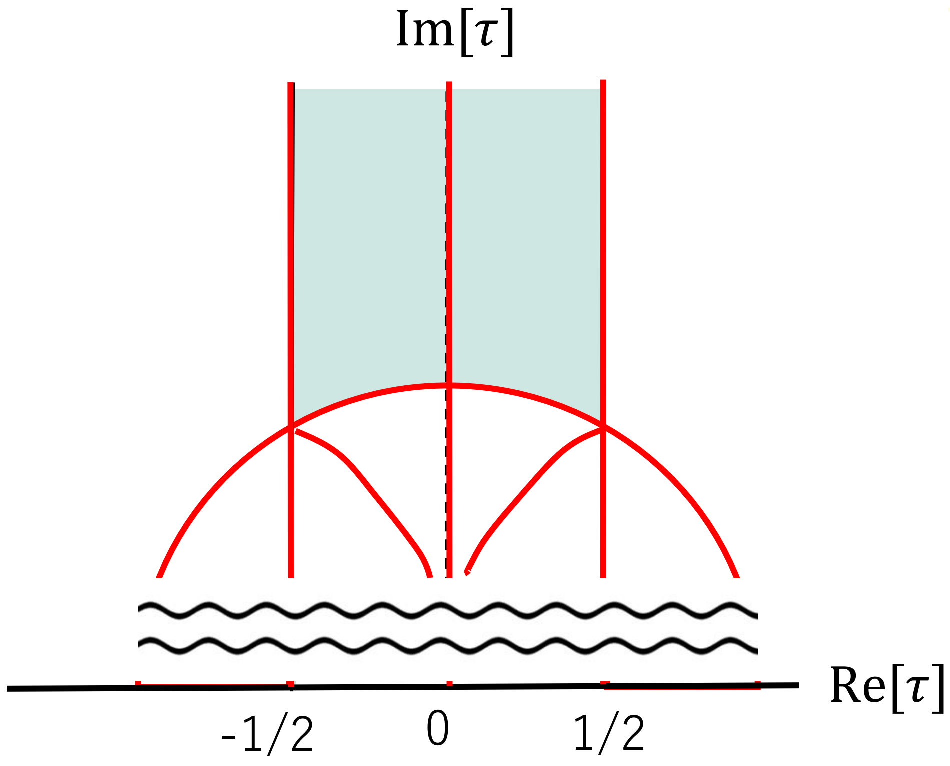
◆CP is violated by the VEV of complex structure modulus.

- In the case of toroidal compact space,

$$\tau \overset{CP}{\rightarrow} -\bar{\tau}. \quad Z \overset{CP}{\rightarrow} -\bar{Z}$$

- In modular flavour models, CP is conserved if and only if there exists $\gamma \in SL(2,Z)$ s.t.

$$-\bar{\tau} = \gamma\tau.$$



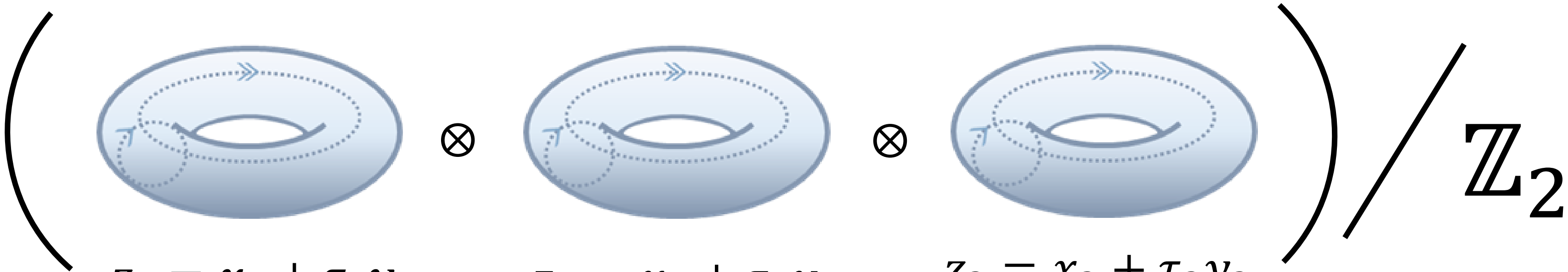
[P. P. Novichkov, J. T. Penedo, S. T. Petcov, A. V. Titov (2019)]

Red curves denote the CP-conserving region.

Purpose and set-up

◆**Purpose: Realization of spontaneous CP violation from the top-down**

◆Type IIB string based on factorizable $T^6/\mathbb{Z}_2$ toroidal orientifold:



$z_1 = x_1 + \tau_1 y_1$ $\quad$ $z_2 = x_2 + \tau_2 y_2$ $\quad$ $z_3 = x_3 + \tau_3 y_3$

Identification

$z_i \sim -z_i, (i = 1,2,3)$

- 3 complex structure moduli: τ_1, τ_2, τ_3
- Overall Kahler (volume) modulus: $\mathcal{V}$
- Axio-dilaton : S

$SL(2,Z)_1 \otimes SL(2,Z)_2 \otimes SL(2,Z)_3 \otimes SL(2,Z)_S$

Type II-B flux compactification

◆Compactification with 3-form background flux→ Scalar potential of moduli

- Scalar potential　(4D $\mathcal{N} = 1$ supergravity)

$$V = e^K \left[K^{A\bar{B}} D_A W \overline{D_B W} - 3|W|^2 \right].$$

$$K_{A\bar{B}} = \partial_A \partial_B K$$

$$K^{A\bar{B}} K_{C\bar{B}} = \delta_C^A$$

- Gukov-Vafa-Witten superpotential:

$$W_{\rm flux} = \frac{1}{l_s^2} \int G_3 \wedge \Omega,$$

Holomorphic form: $\Omega = dz_1 \wedge dz_2 \wedge dz_3,$

$z_i = x_i + \tau_i y_i$ **Moduli dependence**

3-form flux: $G_3 = F_3 - S H_3$

R-R

NS-NS　Axio-dilaton

- Kähler potential:

$$K_{\rm moduli} = -\ln\left[-i(S - \bar{S})\right] - \ln\left[i(\tau_1 - \bar{\tau}_1)(\tau_2 - \bar{\tau}_2)(\tau_3 - \bar{\tau}_3)\right] - 2\ln \mathcal{V},$$

Type II-B flux compactification

◆The 3-form fluxes are expanded by the dual-basis of 3-forms, i.e. $H^3(T^6, Z)$

$$\frac{1}{l_s^2}F_3 = a^0\alpha_0 + a^i\alpha_i + b_i\beta^i + b_0\beta^0,$$
$$\frac{1}{l_s^2}H_3 = c^0\alpha_0 + c^i\alpha_i + d_i\beta^i + d_0\beta^0,$$

- Dual basis:

$$\begin{aligned} \alpha_0 &= dx^1 \wedge dx^2 \wedge dx^3, & \alpha_1 &= dy^1 \wedge dx^2 \wedge dx^3, \\ \alpha_2 &= dy^2 \wedge dx^3 \wedge dx^1, & \alpha_3 &= dy^3 \wedge dx^1 \wedge dx^2, \\ \beta_0 &= dy^1 \wedge dy^2 \wedge dy^3, & \beta_1 &= -dx^1 \wedge dy^2 \wedge dy^3, \\ \beta_2 &= -dx^2 \wedge dy^3 \wedge dy^1, & \beta_3 &= -dx^3 \wedge dy^1 \wedge dy^2. \end{aligned}$$

$$\int_{T^6} \alpha_I \wedge \beta^J = \delta_I^J$$

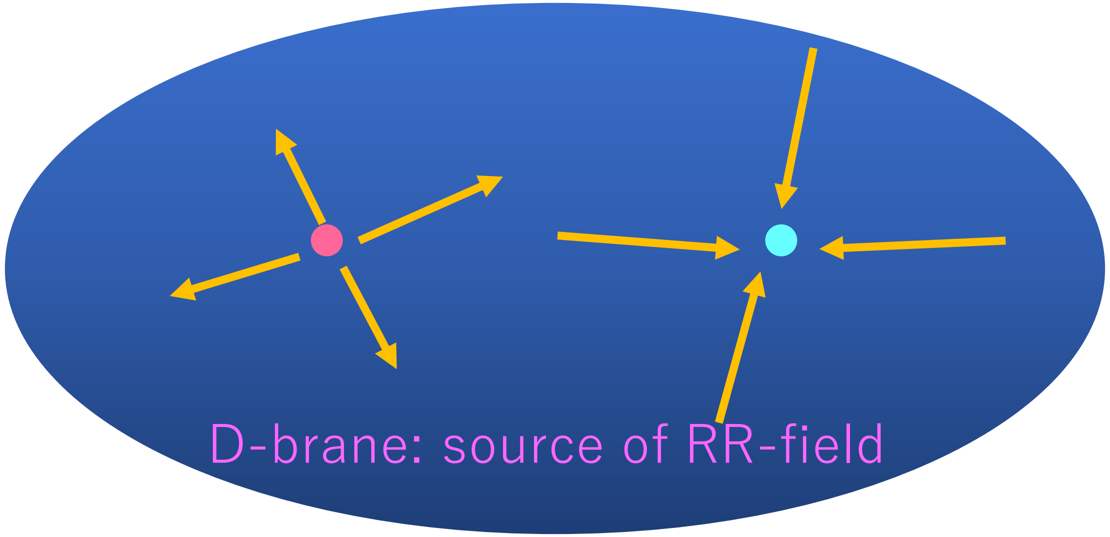
- Flux quanta $\{a^0, a^i, b_0, b_i, c^0, c^i, d_0, d_i\}$ are quantized to integers:

Type II-B flux compactification

◆Tadpole cancellation condition:

- Flux quanta cannot be arbitrarily large

$$
\begin{aligned}
n_{\text{flux}} &= \frac{1}{l_s^2}\int H_3 \wedge F_3 \\
&= c^0 b_0 - d_0 a^0 + \sum_i (c^i b_i - d_i a^i) \\
&= 32 - 2n_{\text{D3}} - n_{\text{O3}'} \leq 32,
\end{aligned}
$$



n_{D3}: number of D3 branes

$n_{O3'}$: number of exotic O3' planes

Type II-B flux compactification

◆Requiring CP-invariance to the scalar potential:

$$V = e^K K^{i\bar{j}} D_i W_{\rm flux} \overline{D_j W_{\rm flux}}.$$

- We require $W_{\rm flux}$ to transform under CP as

$$W_{\rm flux} \xrightarrow{\rm CP} -\bar{W}_{\rm flux},$$

where

$$W_{\rm flux} = \frac{1}{l_s^2} \int \Omega \wedge G_3,$$

$$\Omega \xrightarrow{\rm CP} -\bar{\Omega}.$$

$$G_3 \rightarrow -\overline{G_3}$$

- Restrictions to the flux quanta [T. Kobayashi, H. Otsuka (2020)]

$$\frac{1}{l_s^2} F_3 = a^0 \alpha_0 + \cancel{a^i \alpha_i} + b_i \beta^i + \cancel{b_0 \beta^0},$$

$$\frac{1}{l_s^2} H_3 = \cancel{c^0 \alpha_0} + c^i \alpha_i + \cancel{d_i \beta^i} + d_0 \beta^0,$$

Spontaneous CP violation

◆CP invariant flux compactification:

[T. Kobayashi, H. Otsuka (2020)]

$$K_{\rm moduli} = -\ln\left[-i(S-\bar{S})\right] - \ln\left[i(\tau_1-\bar{\tau}_1)(\tau_2-\bar{\tau}_2)(\tau_3-\bar{\tau}_3)\right] - 2\ln\mathcal{V},$$

$$W_{\rm flux} = a^0\tau_1\tau_2\tau_3 + c^1 S\tau_2\tau_3 + c^2 S\tau_1\tau_3 + c^3 S\tau_1\tau_2 - \sum_{i=1}^{3} b_i\tau_i + d_0 S.$$

◆Solving the supersymmetric conditions:

$$D_I W_{\rm flux} = 0$$

- One generally finds flat directions, where CP-breaking and conserving vacua are degenerate.

Spontaneous CP violation

◆Following the model in [A. Hebecker, P. Henkenjohann, L. T. Witkowski (2017)] [T. Kobayashi, H. Otsuka (2020)]

$$W_{\rm flux} = a^0\tau_1\tau_2\tau_3 + c^1 S\tau_2\tau_3 + c^2 S\tau_1\tau_3 + c^3 S\tau_1\tau_2 - \sum_{i=1}^{3} b_i\tau_i + d_0 S.$$

$$c^1 = c^2 = 0, \quad b_1 = b_2 = 0, \quad c^3 = -fa^0, \quad d_0 = fb_3.$$

$$W_{\rm flux} = (\tau_3 - fS)\left[a^0\tau_1\tau_2 - b_3\right].$$

◆Supersymmetric vacuum (SUSY) is obtained by solving

$$\partial_{\tau_i} W_{\rm flux} = 0, \quad \partial_S W_{\rm flux} = 0, \quad W_{\rm flux} = 0,$$

- One finds flat directions:

$$\tau_1\tau_2 = \frac{b_3}{a^0}, \quad \tau_3 = fS.$$

Spontaneous CP violation (Model I)

◆Resolving the degeneracy between CP-violating and CP-conserving vacua?

- Coupling between the moduli and a matter field X:

$$W = W_{\rm flux} + W_{\rm matter},$$

e.g.

$$W_{\rm matter} = \Lambda^2 Y(\tau_1) X,$$

$Y(\tau_1)$: weight k_Y, trivial singlet modular form

Λ :mass parameter

$$K_{\rm matter} = Z|X|^2, \qquad Z = (-i\tau_1 + i\bar{\tau}_1)^k (-i\tau_2 + i\bar{\tau}_2)^{-1} (-i\tau_3 + i\bar{\tau}_3)^{-1} (-iS + i\bar{S})^{-1},$$

- $SL(2,Z)_1 \otimes SL(2,Z)_2 \otimes SL(2,Z)_3 \otimes SL(2,Z)_S$ symmetry if

$$X \to (c_1\tau_1 + d_1)^k (c_2\tau_2 + d_2)^{-1} (c_3\tau_3 + d_3)^{-1} (c_S S + d_S)^{-1} X \qquad \text{where} \qquad -1 = k + k_Y.$$

- Matter contributions are CP symmetric.

$$W_{\rm matter} \xrightarrow{\rm CP} -\bar{W}_{\rm matter},$$

Minus sign is possible
(arbitrariness in the normalization of modular forms)

because $\tau \xrightarrow{\rm CP} -\bar{\tau}, \quad X \xrightarrow{\rm CP} \bar{X}. \quad Y(\tau_1) \xrightarrow{\rm CP} Y(-\bar{\tau}_1) = -\bar{Y}(\tau_1)$

[P. P. Novichkov, J. T. Penedo, S. T. Petcov, A. V. Titov (2019)]

Spontaneous CP violation (Model I)

◆Supersymmetric vacuum (SUSY) is obtained by solving

$$\partial_{\tau_i} W = 0, \quad \partial_S W = 0, \quad \partial_X W = 0, \quad W = 0.$$

◆ SUSY Minkowski vacuum:

$$\tau_1 \tau_2 = \frac{b_3}{a^0}, \quad \tau_3 = fS, \quad Y(\tau_1) = 0, \quad X = 0 \ (\text{if } \partial_{\tau_1} Y \neq 0).$$

- τ_1 is stabilized at a zero-point of $Y(\tau_1)$.
- τ_2 is stabilized at $\tau_2 = \frac{b_3}{a^0}\frac{1}{\tau_1}$
- There remains one flat direction, $\tau_3 = fS$.

Lifting of flat direction!

◆If we choose certain flux quanta a^0, b_3, spontaneous CP violation is realized quite easily.

Spontaneous CP violation (Model I)

◆ Let us consider A_4 trivial singlet modular form with weight $k_Y = 4$:

$$Y(\tau_1) = Y_{\mathbf{1}}^{(4)}(\tau_1),$$

- τ_1 is stabilized at the Z_3 fixed-point, i.e. $\tau_1 = \omega$

$$Y_{\mathbf{1}}^{(4)}(\omega) = 0, \quad \partial_\tau Y_{\mathbf{1}}^{(4)}|_{\tau=\omega} \simeq -6.04i.$$

- We take the following flux quanta:

$$a^0 = 2, \quad b_3 = -3, \quad f = 1,$$

Tadpole cancellation is satisfied.
$n_{flux} = 12 \leq 32$

- **CP violation by the VEV of τ_2**

$$\langle \tau_2 \rangle = -\frac{3}{2\omega} = \frac{3}{4} + \frac{3\sqrt{3}}{4} i$$

c.f. $\tau_1 \tau_2 = \frac{b_3}{a^0},$

Spontaneous CP violation (Model I)

◆ Modular symmetry and CP violation

- No CP violation by τ_1

Any good reason?

$SL(2,Z)_1$ symmetry in the matter SUGRA scalar potential

$$\tau_1 \to \frac{a\tau_1 + b}{c\tau_1 + d}$$

- CP violation by τ_2

Any good reason?

Modification of $SL(2,Z)_2$ due to the flux quanta

Spontaneous CP violation (Model I)

◆ SUGRA scalar potential:

$$V = V_{flux} + V_{matter}$$

- On the flat directions, $\tau_1\tau_2 = \frac{b_3}{a^0}, \tau_3 = fS$:

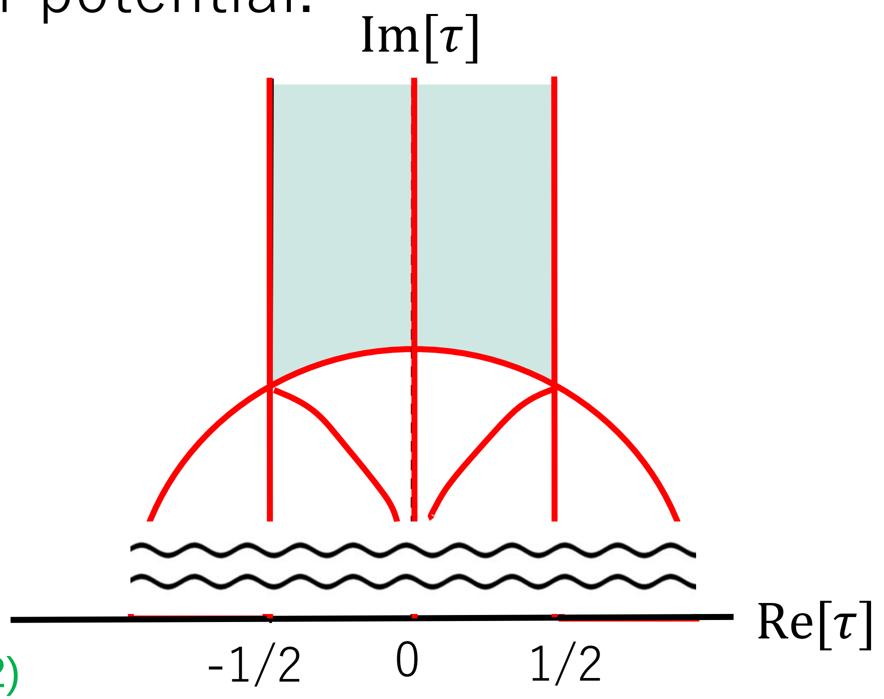
$$V \to V_{\text{matter}} = \frac{\Lambda^4}{\mathcal{V}^2}[2\text{Im}(\tau_1)]^{k_Y}|Y_1^{(k_Y)}(\tau_1)|^2.$$

Manifestly invariant under $SL(2,Z)_1 \rtimes$CP

◆ General properties of SL(2,Z) ⋊CP invariant scalar potential:

$$\frac{\partial V}{\partial \text{Re}[\tau]} = 0,\ \text{for}\ \forall \text{Im}[\tau]\ \text{if}\ \text{Re}[\tau] \equiv 0 \mod(1/2)$$

$$\frac{\partial V}{\partial r} = 0,\ \text{for}\ \forall\theta\ \text{if}\ r = 1 \qquad (\tau = re^{i\theta})$$



Tendency to preserve CP

[M. Cvetic, A. Font, L. E. Ibanez, D. Lust, and F. Quevedo (1991)]

[S. F. King and X. Wang (2023)]

P. P. Novichkov, J. T. Penedo and S. T. Petcov(2022)

Spontaneous CP violation (Model I)

◆ V_{matter} as a function of $\tau_2, \bar{\tau_2}$

$$V \to V_{\rm matter} = \frac{\Lambda^4}{\mathcal{V}^2}[2{\rm Im}(\tau_1)]^{k_Y}|Y_{\bf 1}^{(k_Y)}(\tau_1)|^2.$$

$$= \frac{\Lambda^4}{\mathcal{V}^2}\left[\frac{a^0}{b_3}2{\rm Im}(\tau_2)\right]^{k_Y}\left|Y_{\bf 1}^{(k_Y)}\left(\frac{|a^0|}{|b_3|}\tau_2\right)\right|^2,$$

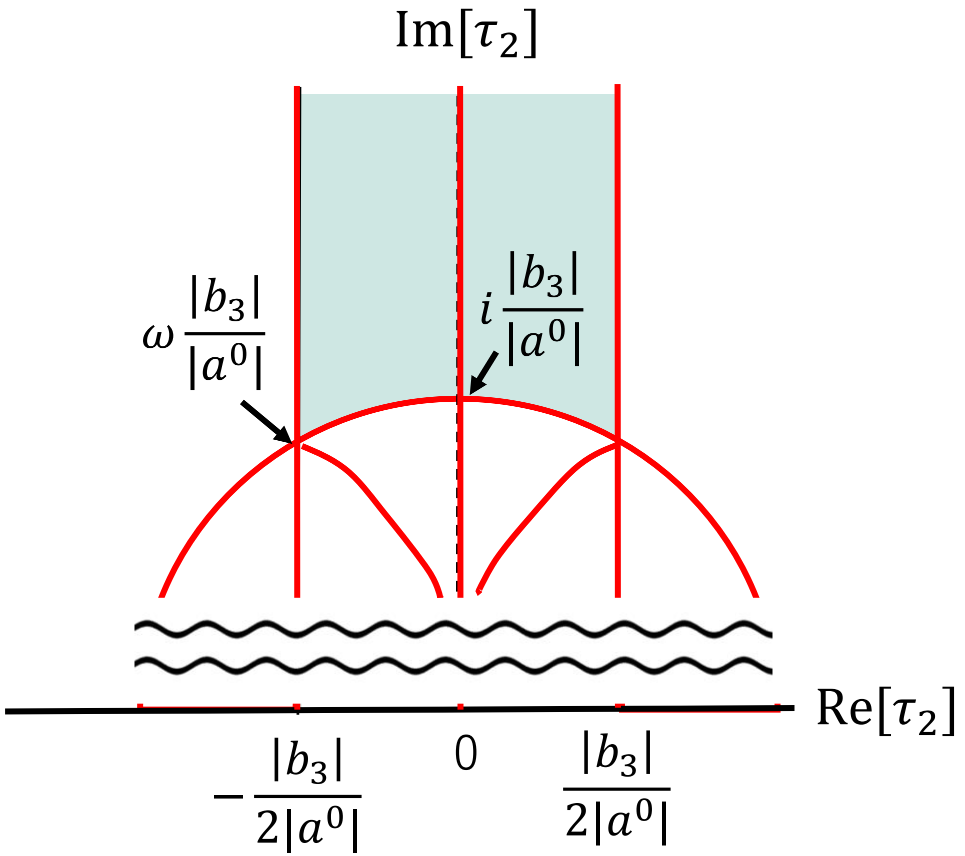
- V_{matter} is symmetric under

$$\tau_2 \to \frac{a\tau_2 + b}{c\tau_2 + d}$$

$$\gamma \in \left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{R}) \middle| \begin{pmatrix} a & b \\ c & d \end{pmatrix} \equiv \begin{pmatrix} 0 \pmod 1 & 0 \pmod{|b_3/a^0|} \\ 0 \pmod{|a^0/b_3|} & 0 \pmod 1 \end{pmatrix}\right\},$$

$$\frac{\partial V}{\partial {\rm Re}[\tau]} = 0, \text{ for } \forall {\rm Im}[\tau] \text{ if } {\rm Re}[\tau] \equiv 0 \mod(1/2)$$

$$\frac{\partial V}{\partial r} = 0, \text{ for } \forall \theta \text{ if } r = |b_3|/|a^0|.$$



Spontaneous CP violation (Model II)

◆ Let us consider the following Lagrangian:

$$W = W_{\rm flux} + W_{\rm matter},$$

where

$$W_{\rm matter} = \Lambda^2 (Y(\tau_1) X + \alpha\, y(\tau_3) x), \quad \alpha \in \mathbb{R},$$

$Y(\tau_1)$, $y(\tau_3)$: trivial singlet modular forms
X, x: matter fields

◆ Supersymmetric vacuum (SUSY) is obtained:

- τ_1 at a zero-point of $Y(\tau_1)$
- τ_3 at a zero-point of $y(\tau_3)$
- $\langle X \rangle = 0$

- $\langle \tau_2 \rangle = \frac{b_3}{a^0} \frac{1}{\langle \tau_1 \rangle}$
- $\langle S \rangle = f \langle \tau_3 \rangle$
- $\langle x \rangle = 0$

All flat directions are lifted.

Spontaneous CP violation (Model II)

◆ A_4 trivial singlet modular form with weight $k_Y = 4, 6$:

$$Y(\tau_1) = Y_{\mathbf{1}}^{(4)}(\tau_1), \quad y(\tau_3) = Y_{\mathbf{1}}^{(6)}(\tau_3),$$

- τ_1, τ_3 are stabilized at the fixed points,

$$\langle \tau_1 \rangle = \omega, \ \langle \tau_3 \rangle = i$$

c.f. $Y_{\mathbf{1}}^{(4)}(\omega) = 0, \quad Y_{\mathbf{1}}^{(6)}(i) = 0,$

- If take the following flux quanta:

$$a^0 = 4, \quad b_3 = -6, \quad f = \frac{1}{2},$$

Tadpole cancellation is satisfied.

$$n_{\rm flux} = 24 \leq 32.$$

- τ_2 and S are stabilized at

$$\langle \tau_2 \rangle = -\frac{3}{2\omega} = \frac{3}{4} + \frac{3\sqrt{3}}{4} i$$

$$\langle S \rangle = 2i$$

c.f. $\tau_1 \tau_2 = \frac{b_3}{a^0},$

$\tau_3 = fS,$

CP violation under the weak coupling regime, ($\mathrm{Im}[S] > 1$)

Spontaneous CP violation (Model II)

◆ Phenomenological implication of $\langle S \rangle = 2i$:

- Suppose that the gauge kinetic function f_h of the standard gauge group is given by

$$f_h = -iS$$

$$-\frac{1}{4}(\mathrm{Re}f_h)_{ab}\, F^a_{\mu\nu}F^{b\,\mu\nu} - \frac{1}{4}(\mathrm{Im}f_h)_{ab}\, F^a_{\mu\nu}\widetilde{F^{b\,\mu\nu}},$$

$$\frac{1}{4\pi\alpha}$$

Theta term

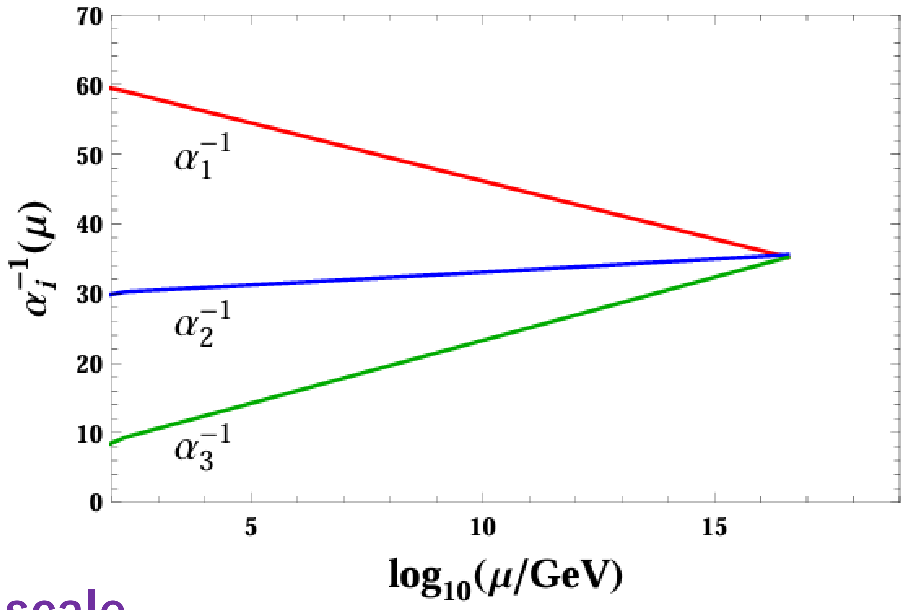


Figure from B.Dutta, et.al (2015)

- We realize **Unified gauge coupling of MSSM at the GUT scale**

$$\frac{1}{\alpha} = 4\pi(\mathrm{Re}f) = 4\pi(\mathrm{Im}S) \approx 25$$

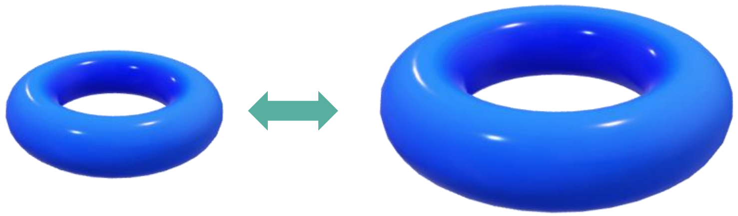
$$\langle S \rangle = 2i$$

- **Vanishing strong QCD phase**

Spontaneous CP violation (Model II')

◆Kahler modulus stabilization

- We fix $\mathcal{V} \simeq [2\mathrm{Im}(T)]^{3/2}$



◆KKLT-like scenario [S. Kachru, R. Kallosh, A. D. Linde, S. P. Trivedi (2003)]

- **Non-perturbative effects** such as D-brane instanton effect $W = W_{\mathrm{flux}} + W_{\mathrm{matter}} + W_{\mathrm{np}},$ [K. Ishiguro, H. Okada, H. Otsuka (2022)]

$$W_{np} = \Lambda'^3 e^{ibS} + Ce^{iaT}$$ e.g. $a = b = 2\pi$

- We have $W_{flux} + W_{matter} = 0$, effective superpotential becomes

$$W_{eff} = \underline{\underline{\Lambda'^3 e^{ib\langle S\rangle}}} + Ce^{iaT}$$

Constant term.

- By solving the SUSY condition, $D_T W_{eff} = 0$

$$a\,\mathrm{Im}\langle T\rangle \simeq \ln\left(C/w_0\right),$$ with $w_0 = \langle \Lambda'^3 e^{ibS}\rangle \ll 1\ (= M_{\mathrm{Pl}}^3).$

Spontaneous CP violation (Model II')

◆Note that the true vacua are solutions of the following equation:

$$D_I W = 0,$$

where W denotes the full superpotential.

$$W = W_{\rm flux} + W_{\rm matter} + W_{\rm np},$$

◆Some deviations to the moduli as $\phi = \langle\phi\rangle + \delta\phi$

$$\delta\tau_1 = \mathcal{O}(\varepsilon^2),$$

$$\delta\tau_2 = -\frac{W_{\rm eff}}{W_{\tau_2 S}} G_S\Big|_{\rm VEV} + \mathcal{O}(\varepsilon^2),$$

$$\delta\tau_3 = \mathcal{O}(\varepsilon^2),$$

$$\delta S = -\frac{W_{\rm eff}}{W_{\tau_2 S}} G_{\tau_2}\Big|_{\rm VEV} + \mathcal{O}(\varepsilon^2),$$

$$\cdots$$

Where $\varepsilon = |W_{eff}|/(\Lambda'^2 M_{Pl})$

In the regime $\mathrm{Im}\langle T\rangle > 1, \mathrm{Im}\langle S\rangle > 1$,
deviations are exponentially suppressed.

- If $\mathrm{Re}\langle\delta S\rangle < 10^{-10}$ → consistent with experiments of electric dipole moment of neutrons

Spontaneous CP violation (Model II')

◆The vacuum energy is negative.

$$V = -3\langle e^K |W_{\rm eff}|^2\rangle < 0.$$

◆We need up-lifting by SUSY breaking.

◆Required F-terms and D-terms are small enough compared with the masses of $\tau_{i=1,2,3}, S, X, x$.

Summary

◆Magnetized D-brane models

- In magnetized T^2 models, zero-mode wavefunctions and Yukawa couplings behave like modular forms of weight 1/2.

Non-Abelian discrete symmetry depends on the magnetic flux $M \in \mathbb{Z}$

- In magnetized T^4 models, zero-mode wavefunctions behave like Siegel modular forms of weight 1/2.

◆Spontaneous CP violation

- Flux compactification + moduli-matter couplings ➞ spontaneous CP violation by the VEV of τ_2.

- CP violation under the weak coupling regime $Im(S) > 0$ is possible.

Supplementary slides

◆Origin of matter contributions:

$$Y(\tau_1)Q\bar{Q}X$$

Condensation $\langle Q\bar{Q}\rangle = \Lambda^2$

$$\Lambda^2 Y(\tau_1)X$$

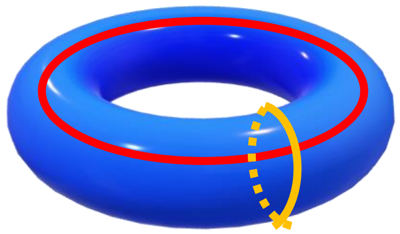
Q: hidden matter fields

◆ SUGRA scalar potential at $\langle X\rangle = 0$:

$$V|_{X=0} = \frac{\Lambda^4}{\mathcal{V}^2}[2\mathrm{Im}(\tau_1)]^{k_Y}|Y_1^{(k_Y)}(\tau_1)|^2 + \frac{2|\tau_3 - fS|^2\ |a^0\bar{\tau}_1\tau_2 - b_3|^2 + 2|\tau_3 - f\bar{S}|^2\ |a^0\tau_1\tau_2 - b_3|^2}{\mathcal{V}^2[2\mathrm{Im}(\tau_1)][2\mathrm{Im}(\tau_2)][2\mathrm{Im}(\tau_3)][2\mathrm{Im}(S)]}.$$

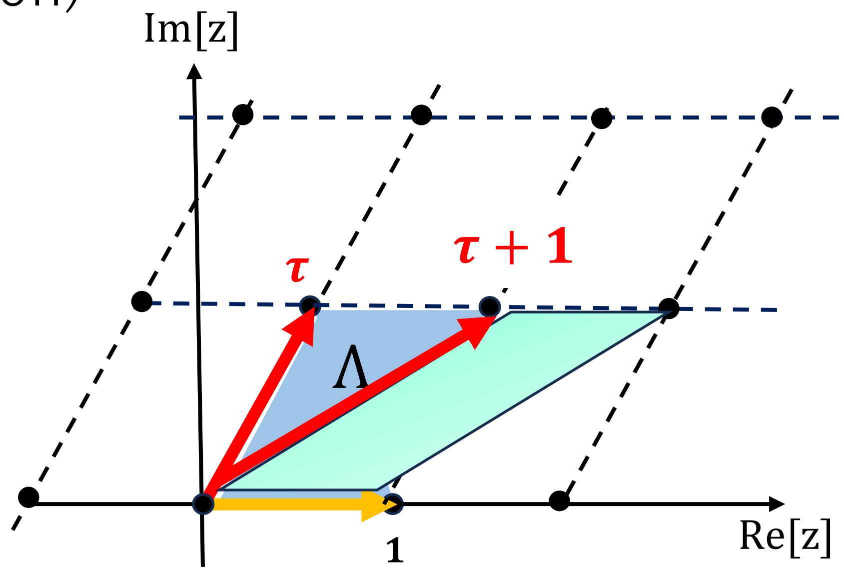
Modular symmetry

- Modular symmetry approach to the flavour puzzles:
  - ➢modular symmetry (brief explanation)



$T^2 \simeq \mathbb{C}/\Lambda$

$\simeq$



Transformation of τ:

$$\tau \to \gamma\tau = \frac{a\tau + b}{c\tau + d}, \qquad \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma \equiv SL(2, \mathbb{Z})$$

τ: complex structure modulus

Generators of SL(2,Z):

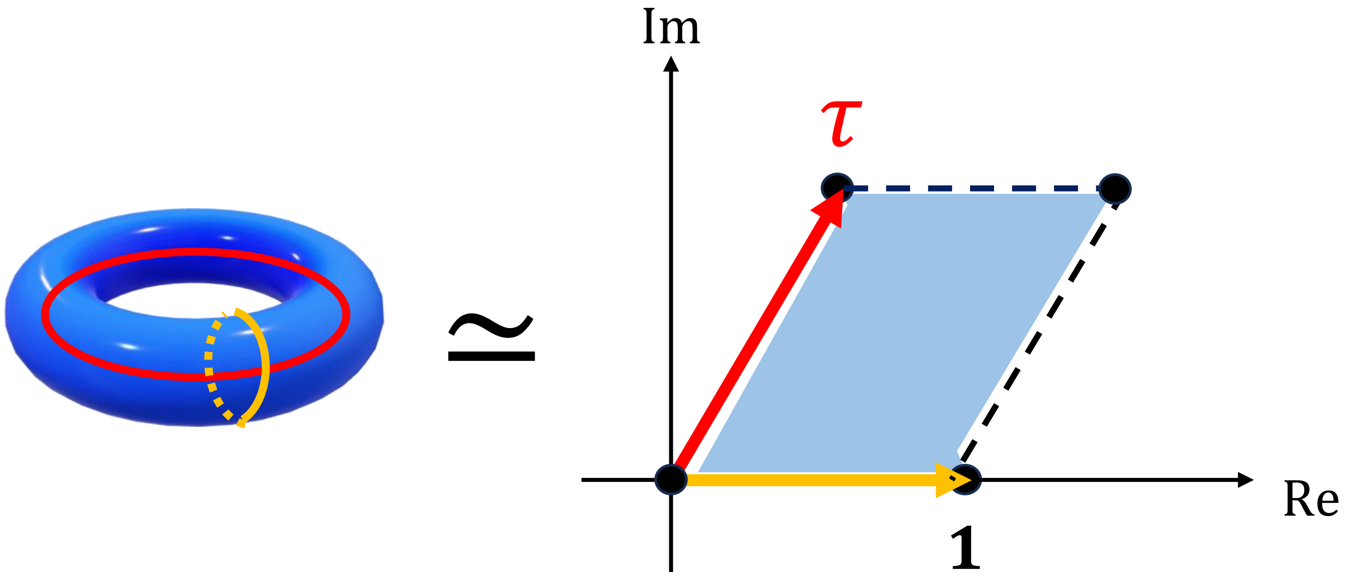
$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$S\tau \to -\frac{1}{\tau}, \quad T\tau \to \tau + 1$$

Modular symmetry

- **Yukawa coupling constants are holomorphic functions of τ**

$Y_{ij}(\tau)$: Computed Yukawa couplings



We know how τ is transformed under SL(2,Z):

$$\tau \rightarrow \gamma\tau = \frac{a\tau + b}{c\tau + d}, \qquad \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma \equiv SL(2, \mathbb{Z})$$

How does $Y_{ij}(\tau)$ transform?

[D. Cremades, L. E. Ibanez, and F. Marchesano (2004)]