Top-down approach to flavor I: The heterotic origin

Saúl Ramos-Sánchez

MITP Topical Wokshop on Modular Flavor Physics

May 15, 2024

Based on collaborations with

A. Baur, M. Kade, H.P. Nilles & P. Vaudrevange: 2001.01736, 2008.07534, 2010.13798, 2104.03981, 2107.10677,...

A. Baur, H.P. Nilles, A. Trautner & P. Vaudrevange: 2112.06940 & 2207.10677

Flavor puzzle

Despite the great success of the SM

- Need to explain
  - three flavors of SM particles
  - observed mass hierarchies
  - observed quark and lepton mixing *textures*
  - CP violation in CKM and PMNS
  - neutrino physics
  - ...

$$\begin{pmatrix} 0.974 & 0.224 & 0.0039 \\ 0.218 & 0.997 & 0.042 \\ 0.008 & 0.039 & 1.019 \end{pmatrix}_{CKM} , \qquad \begin{pmatrix} 0.829 & 0.539 & 0.147 \\ 0.493 & 0.584 & 0.645 \\ 0.262 & 0.607 & 0.75 \end{pmatrix}_{PMNS}$$

$m_{u_i} \sim 2.16, 1270, 172900$ MeV

$m_{d_i} \sim 4.67, 93, 4180$ MeV

$\Delta m^2_{21} = 7.4 \cdot 10^{-5}, \Delta m^2_{31(23)} \approx 2.5 \cdot 10^{-3}$ eV^2

$m_{e_i} \sim 0.511, 105.7, 1776.9$ MeV

normal ordering

[talks by Steve,Myriam,Ye-Ling,Omar,Matteo,Xueqi,Gui-Jun,Morimitsu,João,...]

Bottom-up approaches towards solving the flavor puzzle

Traditional: discrete non-Abelian flavor symmetries $G_{traditional}$ lead to models for quarks and leptons with great fits, $\theta_{13} \neq 0$,...

see reviews by Ishimori, Kobayashi, Ohki, Okada, Shimizu, Tanimoto (2010); Feruglio, Romanino (2019)

Modular: Yukawa couplings are modular forms $Y = Y(T)$ Feruglio (2017)

$$Y(T) \to Y(\gamma T) = (cT+d)^{n_Y} \rho_Y(\gamma) Y(T), \quad \gamma \in \Gamma = \mathrm{SL}(2,\mathbb{Z}), \rho_Y \in \Gamma_N$$

Matter fields transform similarly: $\phi \to (cT+d)^{n_\phi} \rho_\phi(\gamma)\phi$

$\Rightarrow$ finite modular groups Γ_N = modular flavor symmetries $G_{modular}$

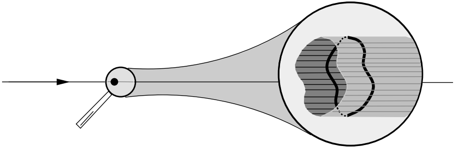
- $\Gamma_N \cong S_3, A_4, S_4, A_5$ for $N = 2,3,4,5$
  $\Rightarrow$ less parameters (despite some Kähler issues [see e.g. Chen,Ratz,SRS(2019) & Hajime's talk])
- double cover $\Gamma'_N \cong S_3, T', \mathrm{SL}(2,4), \mathrm{SL}(2,5)$ for $N = 2,3,4,5$
- 4-fold cover $\widetilde{\Gamma}_4 \cong [96,67], \widetilde{\Gamma}_8 \cong [768,1085324], \widetilde{\Gamma}_{12} \cong [2304,\ldots]$
- multiple moduli, e.g. Siegel modular groups $\Gamma_{g,N} \cong \mathrm{Sp}(2g,\mathbb{Z})/K_N$
- $\Gamma/\ker(\varrho)$ with vector-valued modular forms

Chen,Ding,Feruglio,King,Knapp-Pérez,Li,Liu,Mondragón,Parriciatu,Qu,Ratz,Yau,Zhou,Penedo,Petcov,Titov...


String
Theory

Stringy ingredients

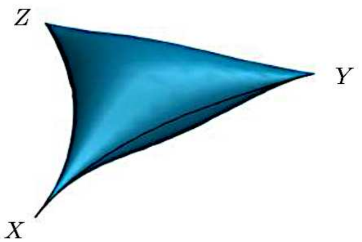
particles $\longleftrightarrow$ strings



Focus on heterotic strings

- closed strings with SUGRA & 10D space-time
  $\rightarrow$ compactify 6D on spaces with shapes and sizes set by moduli
- matter fields get all their properties from string features
  $\rightarrow$ all field charges (reps, weights,...) are computable
- field couplings arise from string interactions
  $\rightarrow$ coupling strengths are determined by CFT
  $\rightarrow$ couplings are modular forms with fixed properties
- gauge group: either $\mathrm{E}_8\times\mathrm{E}_8$ or $\mathrm{SO}(32)$ $\rightarrow$ SM by compactification

Heterotic Orbifolds



Dixon, Harvey, Vafa, Witten (1985-86)

Ibáñez, Nilles, Quevedo (1987)

Font, Ibáñez, Quevedo, Sierra (1990)
Katsuki, Kawamura, Kobayashi, Ohtsubo, Ono, Tanioka (1990)

Kobayashi, Raby, Zhang (2004)

Buchmüller, Hamaguchi, Lebedev, Ratz (2004-06)

Kobayashi, Nilles, Plöger, Raby, Ratz (2006)
Lebedev, Nilles, Ratz, SRS, Vaudrevange, Wingerter (2006-08)

⋮

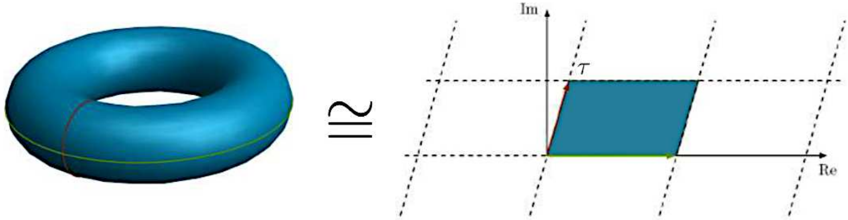
Mütter, Parr, Vaudrevange + Biermann, Ratz (2018-19)

Baur, Nilles, Trautner, Vaudrevange (2018-19)

...

Example: 2D $\mathbb{T}^2/\mathbb{Z}_3$ orbifolds

* Start with a $\mathbb{T}^2$

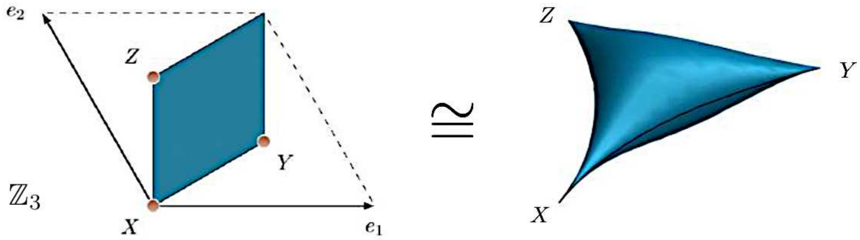


$$Y = \begin{pmatrix} y_{\rm R} \\ y_{\rm L} \end{pmatrix} \sim Y + E\,\hat{N} = \begin{pmatrix} y_{\rm R} \\ y_{\rm L} \end{pmatrix} + E \begin{pmatrix} n \\ m \end{pmatrix}, \quad y_{\rm L}, y_{\rm R} \text{ are } 2D$$

vielbein $E = E(T, U)$, $n \in \mathbb{Z}^2$: winding, $m \in \mathbb{Z}^2$: KK momentum

Example: 2D $\mathbb{T}^2/\mathbb{Z}_3$ orbifolds

* Mod out a discrete $\mathbb{Z}_3$ symmetry generated by twist Θ



$$\begin{pmatrix} y_\mathrm{R} \\ y_\mathrm{L} \end{pmatrix} \sim \begin{pmatrix} \vartheta & 0 \\ 0 & \vartheta \end{pmatrix} \begin{pmatrix} y_\mathrm{R} \\ y_\mathrm{L} \end{pmatrix} + E \begin{pmatrix} n \\ m \end{pmatrix}, \quad \vartheta \in \mathrm{SO}(2), \quad \vartheta^3 = \mathbb{1}_2$$

$$S_\mathrm{Narain} := \langle (\Theta^k, 0),\ (\mathbb{1}, E_i) \rangle, \qquad k = 1, 2, \quad i = 1, \ldots, 4$$

Outer automorphisms of S_Narain are symmetries of orbifold

e.g. $T_1 := (1/3, 2/3, 0, 0)^T \ \Rightarrow\ X \stackrel{T_1}{\to} Z, Y \stackrel{T_1}{\to} X, Z \stackrel{T_1}{\to} Y$

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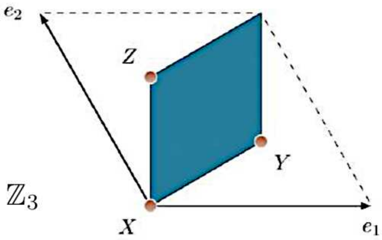
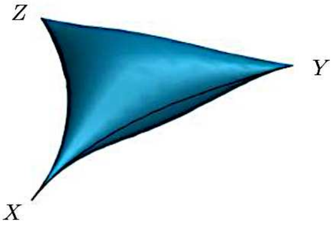


$\cong$



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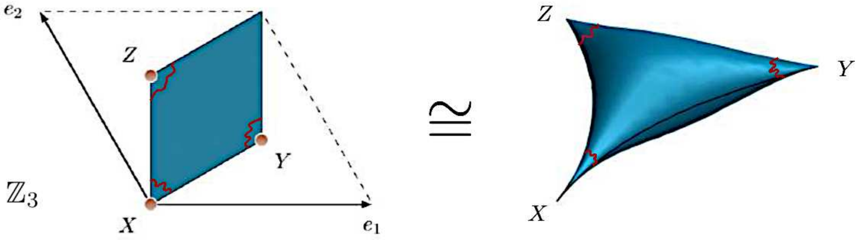
Types: rotational $\Longleftrightarrow$ change T, U, translational $\Longleftrightarrow$ don't change T, U

Example: 2D $\mathbb{T}^2/\mathbb{Z}_3$ orbifolds

* Consider twisted string states localized at singularities of Θ^k sector



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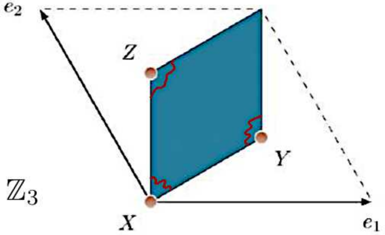

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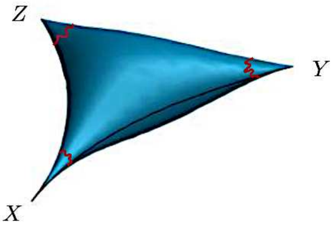

$\Rightarrow$ $G_{modular}$ = rotational outer automorphisms
$G_{traditional}$ = translational outer automorphisms

We can compute the flavor group: $G_{modular} \cup G_{traditional}$ ☺

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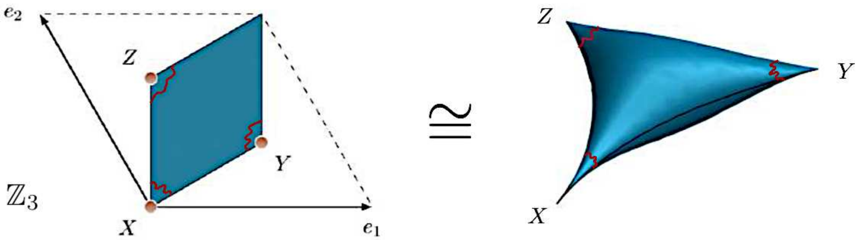

$$\Rightarrow \quad G_{modular} = \text{rotational outer automorphisms}$$
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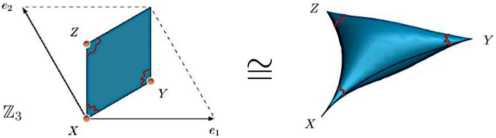
In $\mathbb{T}^2/\mathbb{Z}_3$: $\quad T' \cup \Delta(54) \cong [658, 533]$


modular
WEIGHT
MANAGEMENT
Stringy
Nutrition

More on matter fields in $\mathbb{T}^2/\mathbb{Z}_3$



Define $\Phi_n = (X, Y, Z)$ with n: a computable modular weight

[Ibáñez,Lüst(1992), Olguín-Trejo,SRS(2017)]

n depends on: twisted sector, winding, momentum in compact space.

- In $\mathbb{T}^2/\mathbb{Z}_3$: $n = -2/3$ for $\Theta^{k=1}$, $n = -1/3$ for $\Theta^{k=2}$
  Strings that are (untwisted) not localized, get only $n = 0, -1$

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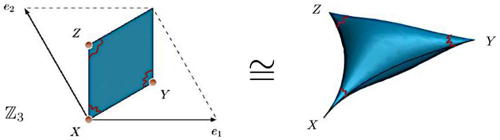

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  Strings that are (untwisted) not localized, get only $n = 0, -1$
- The transformations under $G_{modular}$ and $G_{traditional}$ can be computed

[Lauer,Mas,Nilles(1989)]

	Φ_0	Φ_{-1}	$\Phi_{-2/3}$	$\Phi_{-1/3}$
$\Delta(54)$	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{3}_2$	$\bar{\mathbf{3}}_2$
T'	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{2}'' \oplus \mathbf{1}$

Baur, Nilles, Trautner, Vaudrevange (2019); Nilles, SRS, Vaudrevange (2020)

More symmetries in $\mathbb{T}^2/\mathbb{Z}_3$?

- Always also a $\mathbb{Z}_2^{\mathcal{CP}}$ $\mathcal{CP}$-like trafo: $T \to -\bar{T}$ and $X, Y, Z \to \bar{X}, \bar{Y}, \bar{Z}$

  [Dent(2001); Baur,Nilles,Trautner,Vaudrevange(2019)] also in bottom-up:[Novichkov,Penedo,Petcov,Titov(2019)]

- For the torus to be consistent with Θ, $\langle U \rangle = \omega := \mathrm{e}^{2\pi\mathrm{i}/3}$

  $\to$ some discrete modular symmetry is unbroken $\to \mathbb{Z}_9^R$ in $\mathbb{T}^2/\mathbb{Z}_3$

$$q_R = (-\langle U \rangle - 1)^n = \omega^{2n}, \qquad n = 0, -1, -{}^2\!/3, -{}^1\!/3$$

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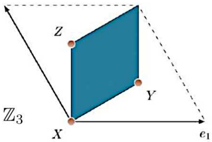
In summary



$\mathbb{T}^2/\mathbb{Z}_3$	bulk matter		θ sector		θ^2 sector		
	Φ_0	Φ_{-1}	$\Phi_{-2/3}$	$\Phi_{-5/3}$	$\Phi_{-1/3}$	$\Phi_{+2/3}$	$\mathcal{W}$
traditional $\Delta(54)$	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{3}_2$	$\mathbf{3}_1$	$\bar{\mathbf{3}}_2$	$\bar{\mathbf{3}}_1$	$\mathbf{1}'$
modular T'	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{2}'' \oplus \mathbf{1}$	$\mathbf{2}'' \oplus \mathbf{1}$	$\mathbf{1}$
modular weight n_T	0	-1	$-2/3$	$-5/3$	$-1/3$	$+2/3$	-1
R-charge of $\mathbb{Z}_9^R$	0	3	1	-2	2	5	3

Example: 2D $\mathbb{T}^2/\mathbb{Z}_3$ orbifolds

Yukawa coupling coefficients $\hat{Y}$ are modular forms

modular forms $\hat{Y}_{\boldsymbol{s}}^{(n_Y)}$	eclectic flavor group $\Omega(1)$: modular T' subgroup: irrep $\boldsymbol{s}$	$\rho_{\boldsymbol{s}}(\mathrm{S})$	$\rho_{\boldsymbol{s}}(\mathrm{T})$	n_Y	traditional $\Delta(54)$ subgroup: irrep $\boldsymbol{r}$	$\rho_{\boldsymbol{r}}(\mathrm{A})$	$\rho_{\boldsymbol{r}}(\mathrm{B})$	$\rho_{\boldsymbol{r}}(\mathrm{C})$
$\hat{Y}_{\mathbf{2}''}^{(1)}$	$\mathbf{2}''$	$\rho_{\mathbf{2}''}(\mathrm{S})$	$\rho_{\mathbf{2}''}(\mathrm{T})$	1	$\mathbf{1}$	1	1	1
$\hat{Y}_{\mathbf{1}}^{(4)}$	$\mathbf{1}$	1	1	4	$\mathbf{1}$	1	1	1
$\hat{Y}_{\mathbf{1}'}^{(4)}$	$\mathbf{1}'$	1	ω	4	$\mathbf{1}$	1	1	1
$\hat{Y}_{\mathbf{3}}^{(4)}$	$\mathbf{3}$	$\rho_{\mathbf{3}}(\mathrm{S})$	$\rho_{\mathbf{3}}(\mathrm{T})$	4	$\mathbf{1}$	1	1	1

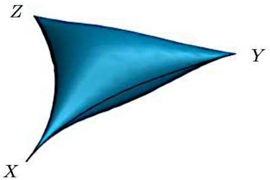
$$\hat{Y}_{\mathbf{2}''}^{(1)} := \begin{pmatrix} \hat{Y}_1(T) \\ \hat{Y}_2(T) \end{pmatrix} = \begin{pmatrix} -3\sqrt{2} & 0 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} \eta(3T)^3/\eta(T) \\ \eta(T/3)^3/\eta(T) \end{pmatrix}$$

No arbitrary modular weights n_Y nor representations $\boldsymbol{s}$! ☺

Superpotential and Kähler in $\mathbb{T}^2/\mathbb{Z}_3$

Restricted superpotential

Baur,Nilles,Trautner,SRS,Vaudrevange(2021-22)



$$\Rightarrow \mathcal{W} \quad \supset \quad c\Big[\hat{Y}_2(T)\big(X_1\,X_2\,X_3 + Y_1\,Y_2\,Y_3 + Z_1\,Z_2\,Z_3\big) - \frac{\hat{Y}_1(T)}{\sqrt{2}}\big(X_1\,Y_2\,Z_3 + X_1\,Y_3\,Z_2 + X_2\,Y_1\,Z_3 + X_3\,Y_1\,Z_2 + X_2\,Y_3\,Z_1 + X_3\,Y_2\,Z_1\big)\Big]\,,$$

with $\quad \Phi^{i}_{-2/3} := (X_i, Y_i, Z_i)^T, \quad c \in \mathbb{R}$

Restricted superpotential

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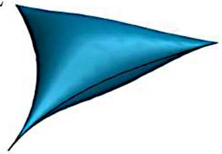

$$\Rightarrow \mathcal{W} \quad \supset \quad c\Big[\hat{Y}_2(T)\big(X_1\,X_2\,X_3+Y_1\,Y_2\,Y_3+Z_1\,Z_2\,Z_3\big) - \frac{\hat{Y}_1(T)}{\sqrt{2}}\big(X_1\,Y_2\,Z_3+X_1\,Y_3\,Z_2+X_2\,Y_1\,Z_3 + X_3\,Y_1\,Z_2+X_2\,Y_3\,Z_1+X_3\,Y_2\,Z_1\big)\Big]\,,$$

with $\quad \Phi^i_{-2/3} := (X_i, Y_i, Z_i)^T, \quad c\in\mathbb{R}$

More interestingly

$$\mathcal{K} \quad = \quad -\log(-\mathrm{i}T+\mathrm{i}\bar{T}) + \textstyle\sum_i \Big[(-\mathrm{i}T+\mathrm{i}\bar{T})^{-2/3} + (-\mathrm{i}T+\mathrm{i}\bar{T})^{1/3}|\hat{Y}^{(1)}_{\mathbf{2''}}|^2 + \ldots\Big]\,|\Phi^i_{-2/3}|^2$$
$$+ \quad \text{suppressed corrections with flavon fields}$$

Only canonical terms are allowed

$\rightarrow$ predictivity of bottom-up models with Γ'_N recovered! ☺

Chen,SRS,Ratz(2019); Nilles,SRS,Vaudrevange (2004.05200)

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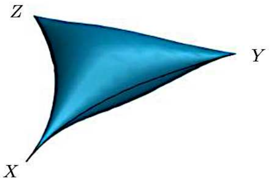

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$+$ suppressed corrections with flavon fields

Only canonical terms are allowed (due to traditional symmetry)

$\rightarrow$ predictivity of bottom-up models with Γ'_N recovered! ☺

Chen,SRS,Ratz(2019); Nilles,SRS,Vaudrevange (2004.05200)

Lessons from 2D $\mathbb{T}^2/\mathbb{Z}_3$ orbifolds

- Common origin of all kinds of flavor symmetries
- $G_{modular}$ and $G_{traditional}$ appear together → eclectic picture with

$$G_{modular} \subset \mathrm{Out}(G_{traditional})$$

Also in bottom-up [Nilles,SRS,Vaudrevange(2020); Ding,King,Li,Liu,Lu(2023)]

- $G_{modular}$ is a double cover (e.g. T') → $n_Y \in \mathbb{Z}$
- Discrete $\mathbb{Z}_M^R$ symmetry
- Generalized $\mathbb{Z}_2^{\mathcal{CP}}$ $\mathcal{CP}$ trafo
- Fractional modular weights (with both signs) - only $0, -1$ for untwisted matter
- $\mathcal{K}$ constrained to canonic form by $G_{traditional}$
- $\mathcal{W}$ constrained by all
- Flavons are needed ☹

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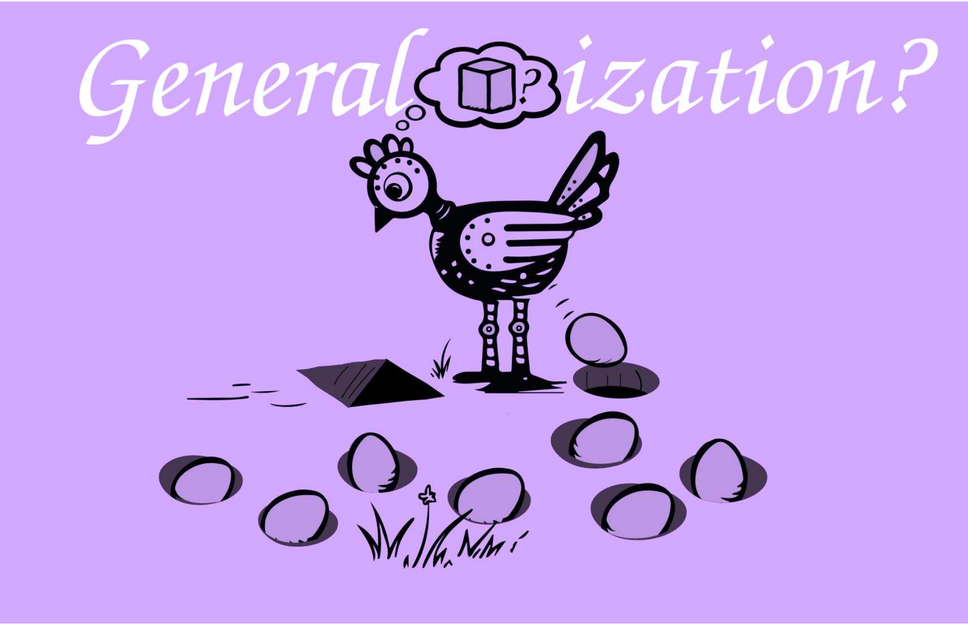
- $\mathcal{W}$ constrained by all
- Flavons are needed ☹
- Since this means two Gold medals, it wins!! 


General
?
ization?

The heterotic origin of flavor in $\mathbb{T}^2/\mathbb{Z}_k$

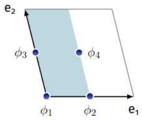
Basic idea:

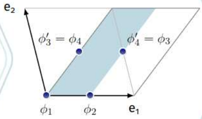
The heterotic origin of flavor in $\mathbb{T}^2/\mathbb{Z}_k$

Basic idea:

Momenta lie on a lattice → Symmetries of the lattice → States transform under symmetries → Flavor transformations

In a bit more detail:





$$\begin{pmatrix}\phi_1\\ \phi_2\\ \phi_3\\ \phi_4\end{pmatrix} \xrightarrow{T} \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}\begin{pmatrix}\phi_1\\ \phi_2\\ \phi_3\\ \phi_4\end{pmatrix}$$

$\mathbb{Z}_2$

The heterotic origin of flavor in $\mathbb{T}^2/\mathbb{Z}_k$

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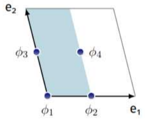
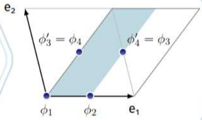
In a bit more detail:





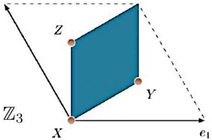
$$\begin{pmatrix}\phi_1\\ \phi_2\\ \phi_3\\ \phi_4\end{pmatrix} \xrightarrow{T} \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}\begin{pmatrix}\phi_1\\ \phi_2\\ \phi_3\\ \phi_4\end{pmatrix}$$

$\mathbb{Z}_2$

In even further detail:

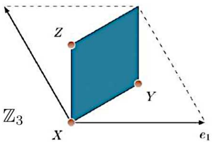
Narain lattice of signature (d,d) → $\mathrm{O}_{\hat\eta}(d,d,\mathbb{Z})$ + translations → $\Phi \xrightarrow{g} j(\tau,k)\,\rho(g)\,\Phi$ → $(S_3\times S_3)\rtimes\mathbb{Z}_4 \cup \dfrac{D_8\times D_8}{\mathbb{Z}_2}$

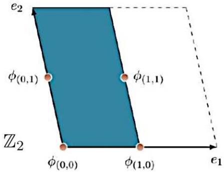
The heterotic origin of flavor in $\mathbb{T}^2/\mathbb{Z}_k$



$\mathbb{T}^2/\mathbb{Z}_3$	bulk matter		θ sector		θ^2 sector		
	Φ_0	Φ_{-1}	$\Phi_{-2/3}$	$\Phi_{-5/3}$	$\Phi_{-1/3}$	$\Phi_{+2/3}$	$\mathcal{W}$
traditional $\Delta(54)$	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{3}_2$	$\mathbf{3}_1$	$\bar{\mathbf{3}}_2$	$\bar{\mathbf{3}}_1$	$\mathbf{1}'$
modular T'	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{2}'' \oplus \mathbf{1}$	$\mathbf{2}'' \oplus \mathbf{1}$	$\mathbf{1}$
modular weight n_T	0	-1	$-2/3$	$-5/3$	$-1/3$	$+2/3$	-1
R-charge of $\mathbb{Z}_9^R$	0	3	1	-2	2	5	3

The heterotic origin of flavor in $\mathbb{T}^2/\mathbb{Z}_k$





$\mathbb{T}^2/\mathbb{Z}_3$	bulk matter		θ sector		θ^2 sector		
	Φ_0	Φ_{-1}	$\Phi_{-2/3}$	$\Phi_{-5/3}$	$\Phi_{-1/3}$	$\Phi_{+2/3}$	$\mathcal{W}$
traditional $\Delta(54)$	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{3}_2$	$\mathbf{3}_1$	$\bar{\mathbf{3}}_2$	$\bar{\mathbf{3}}_1$	$\mathbf{1}'$
modular T'	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}'\oplus\mathbf{1}$	$\mathbf{2}'\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{1}$
modular weight n_T	0	-1	$-2/3$	$-5/3$	$-1/3$	$+2/3$	-1
R-charge of $\mathbb{Z}_9^R$	0	3	1	-2	2	5	3

$\mathbb{T}^2/\mathbb{Z}_2$	bulk matter		twisted matter				
	$\Phi_{(0,0)}$	$\Phi_{(-1,-1)}$	$\Phi_{(-1/2,-1/2)}$	$\Phi_{(-3/2,\,1/2)}$	$\Phi_{(1/2,-3/2)}$	$Y^{(2)}_{\mathbf{4}_3}$	$\mathcal{W}$
traditional	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{4}$	$\mathbf{4}$	$\mathbf{4}$	$\mathbf{1}_0$	$\mathbf{1}_0$
modular	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{4}_1$	$(\mathbf{4}_1\oplus\mathbf{4}_1)$		$\mathbf{4}_3$	$\mathbf{1}_0$
n_T	0	-1	$-1/2$	$-3/2$	$1/2$	2	-1
n_U	0	-1	$-1/2$	$1/2$	$-3/2$	2	-1
R-charge	0	2	3	1	1	0	2 mod 4

$$G_{traditional} = \frac{D_8\times D_8}{\mathbb{Z}_2}\cup\mathbb{Z}_4^R \cong [64,266], \quad G_{modular} = \left[(S_3^U\times S_3^T)\rtimes\mathbb{Z}_4^M\right]\cup\mathbb{Z}_2^{\mathcal{CP}} = [288,880]$$

$$\hat{Y}^{(2)}_{\mathbf{4}_3} := \left(\hat{Y}_1(T)\hat{Y}_1(U),\hat{Y}_2(T)\hat{Y}_1(U),\hat{Y}_1(T)\hat{Y}_2(U),\hat{Y}_2(T)\hat{Y}_2(U)\right)^T$$

$$\mathcal{K}\supset(-\mathrm{i}T+\mathrm{i}\bar{T})^{n_T}(-\mathrm{i}U+\mathrm{i}\bar{U})^{n_U}|\Phi_{(n_U,n_T)}|^2$$

$$\mathcal{W}\supset\hat{Y}^{(0)}(T,U)\Phi_{(0,0)}\Phi_{(-1/2,-1/2)}\Phi_{(-1/2,-1/2)}+\hat{Y}^{(2)}(T,U)\Phi_{(-1,-1)}\Phi_{(-1/2,-1/2)}\Phi_{(-1/2,-1/2)}\Phi_{(-1/2,-1/2)}\Phi_{(-1/2,-1/2)}$$

Baur,Kade,Nilles,SRS,Vaudrevange(2021)

Modular transformations in $\mathbb{T}^2/\mathbb{Z}_2$

Observation: if T and U are included in a modulus matrix

$$\Omega := \begin{pmatrix} T & 0 \\ 0 & U \end{pmatrix} \qquad \text{subject to} \quad \operatorname{Im}\Omega > 0$$

all modular transformations (no $\mathcal{CP}$) are 4×4 matrices

$$\mathcal{M} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \in \mathrm{Sp}(4,\mathbb{Z}) \qquad \text{e.g.} \quad \mathcal{M}_{(\gamma_T,\gamma_U)} = \begin{pmatrix} a_U & 0 & b_U & 0 \\ 0 & a_T & 0 & b_T \\ c_U & 0 & d_U & 0 \\ 0 & c_T & 0 & d_T \end{pmatrix}$$

with

$$\mathrm{Sp}(4,\mathbb{Z}) = \{\mathcal{M} \in \mathbb{Z}^{4\times 4} | \mathcal{M}^T J \mathcal{M} = J\} \quad \text{and} \quad J = \begin{pmatrix} 0 & \mathbb{1}_2 \\ -\mathbb{1}_2 & 0 \end{pmatrix}$$

But we need one extra transformation $\mathcal{M}$ to generate $\mathrm{Sp}(4,\mathbb{Z})$

Modular transformations of $\mathbb{T}^2/\mathbb{Z}_2$ vs $\mathrm{Sp}(4,\mathbb{Z})$

symmetry	$\mathrm{Sp}(4,\mathbb{Z})$	$\mathrm{O}_{\hat{\eta}}(2,2,\mathbb{Z})$	transformation of moduli
$\mathrm{SL}(2,\mathbb{Z})_T$	$\mathcal{M}_{(\mathrm{S},\mathbb{1}_2)}$	S_T	$T \to -\frac{1}{T}$ $U \to U$
	$\mathcal{M}_{(\mathrm{T},\mathbb{1}_2)}$	T_T	$T \to T+1$ $U \to U$
$\mathrm{SL}(2,\mathbb{Z})_U$	$\mathcal{M}_{(\mathbb{1}_2,\mathrm{S})}$	S_U	$T \to T$ $U \to -\frac{1}{U}$
	$\mathcal{M}_{(\mathbb{1}_2,\mathrm{T})}$	T_U	$T \to T$ $U \to U+1$
Mirror	$\mathcal{M}_\times$	M	$T \to U$ $U \to T$
?	$\mathcal{M}\binom{\ell}{m}$	?	
$\mathcal{CP}$-like	$\mathcal{M}_* \in$ $\mathrm{GSp}(4,\mathbb{Z})$	$\mathbb{Z}_2^{\mathcal{CP}}$	$T \to -\bar{T}$ $U \to -\bar{U}$

Modular transformations of $\mathbb{T}^2/\mathbb{Z}_2$ vs $\mathrm{Sp}(4,\mathbb{Z})$

symmetry	$\mathrm{Sp}(4,\mathbb{Z})$	$\mathrm{O}_{\hat\eta}(2,2,\mathbb{Z})$	transformation of moduli
$\mathrm{SL}(2,\mathbb{Z})_T$	$\mathcal{M}_{(\mathrm{S},\mathbb{1}_2)}$	S_T	$T \to -\frac{1}{T}$ $U \to U$
	$\mathcal{M}_{(\mathrm{T},\mathbb{1}_2)}$	T_T	$T \to T+1$ $U \to U$
$\mathrm{SL}(2,\mathbb{Z}$	$\mathcal{M}_{(\mathbb{1}_2,\mathrm{S})}$	S_U	$T \to T$ $U \to -\frac{1}{U}$
Mirro			
?	$\mathcal{M}\binom{\ell}{m}$	?	
$\mathcal{CP}$-like	$\mathcal{M}_* \in$ $\mathrm{GSp}(4,\mathbb{Z})$	$\mathbb{Z}_2^{\mathcal{CP}}$	$T \to -\bar{T}$ $U \to -\bar{U}$

Include continuous Wilson-line modulus Z

$$\Omega = \begin{pmatrix} T & Z \\ Z & U \end{pmatrix}, \quad Z = -a_2 + U a_1, a_i \in \mathbb{R}$$

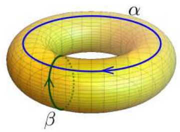
Demand Wilson-line shift $a_1 \to a_1 + \ell, a_2 \to a_2 + m$

$$\ell, m \in \mathbb{Z}$$

Modular transformations of $\mathbb{T}^2/\mathbb{Z}_2$ vs $\mathrm{Sp}(4,\mathbb{Z})$

symmetry	$\mathrm{Sp}(4,\mathbb{Z})$	$\mathrm{O}_{\hat\eta}(2,3,\mathbb{Z})$	transformation of moduli
$\mathrm{SL}(2,\mathbb{Z})_T$	$\mathcal{M}_{(\mathrm{S},\mathbb{1}_2)}$	S_T	$T\to-\frac{1}{T}$ $U\to U-\frac{Z^2}{T}$ $Z\to-\frac{Z}{T}$
	$\mathcal{M}_{(\mathrm{T},\mathbb{1}_2)}$	T_T	$T\to T+1$ $U\to U$ $Z\to Z$
$\mathrm{SL}(2,\mathbb{Z})_U$	$\mathcal{M}_{(\mathbb{1}_2,\mathrm{S})}$	S_U	$T\to T-\frac{Z^2}{U}$ $U\to-\frac{1}{U}$ $Z\to-\frac{Z}{U}$
	$\mathcal{M}_{(\mathbb{1}_2,\mathrm{T})}$	T_U	$T\to T$ $U\to U+1$ $Z\to Z$
Mirror	$\mathcal{M}_\times$	M	$T\to U$ $U\to T$ $Z\to Z$
Wilson line shift	$\mathcal{M}\binom{\ell}{m}$	$\mathrm{W}\binom{\ell}{m}$	$T\to T+m\,(m\,U+2\,Z-\ell)$ $U\to U$ $Z\to Z+m\,U-\ell$
$\mathcal{CP}$-like	$\mathcal{M}_*\in$ $\mathrm{GSp}(4,\mathbb{Z})$	$\mathbb{Z}_2^{\mathcal{CP}}$	$T\to-\bar T$ $U\to-\bar U$ $Z\to-\bar Z$

Origin of the Siegel modular flavor group



$\mathrm{Sp}(2,\mathbb{Z}) \cong \mathrm{SL}(2,\mathbb{Z})$

$\mathrm{GSp}(2,\mathbb{Z}) \cong \mathrm{GL}(2,\mathbb{Z})$

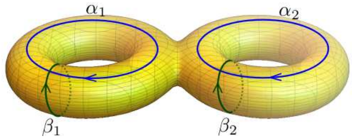
γ

T

Γ_N

Φ_n

modular forms



$\mathrm{Sp}(4,\mathbb{Z})$

$\mathrm{GSp}(4,\mathbb{Z})$

$\mathcal{M}$

$\Omega(T,U,Z)$

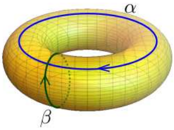
$\Gamma_{2,N}$

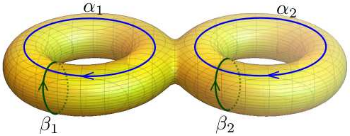
Φ_{n_T,n_U}

Siegel modular forms

in bottom-up: Ding,Feruglio,Liu(2021)

Origin of the Siegel modular flavor group





$\mathrm{Sp}(2,\mathbb{Z}) \cong \mathrm{SL}(2,\mathbb{Z})$	$\mathrm{Sp}(4,\mathbb{Z})$
$\mathrm{GSp}(2,\mathbb{Z}) \cong \mathrm{GL}(2,\mathbb{Z})$	$\mathrm{GSp}(4,\mathbb{Z})$
γ	$\mathcal{M}$
T	$\Omega(T,U,Z)$
Γ_N	$\Gamma_{2,N}$
Φ_n	Φ_{n_T,n_U}
modular forms	Siegel modular forms

in bottom-up: Ding,Feruglio,Liu(2021)

Extend to $n_T \neq n_U$, new modular weight associated with Z?

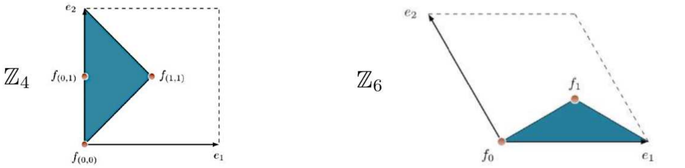
Find out the exact form of all transformations

Compare with other approaches

Ishiguro, Kobayashi, Otsuka (2021)

More flavors for $\mathbb{T}^2/\mathbb{Z}_k$

Not every $\mathbb{Z}_k$ is possible: $k = 2, 3, 4, 6$ only!



orbifold	$\mathbb{Z}_2$	$\mathbb{Z}_3$	$\mathbb{Z}_4$	$\mathbb{Z}_6$
$G_{modular}$	$(S_3 \times S_3) \rtimes \mathbb{Z}_4$	T'	$2D_3$	$S_3 \times T'$
$G_{traditional}$	$(D_8 \times D_8)/\mathbb{Z}_2$	$\Delta(54)$	$(D_8 \times \mathbb{Z}_4)/\mathbb{Z}_2$	$\mathbb{Z}_6$
$\mathbb{Z}_M^R$	$\mathbb{Z}_4^R$	$\mathbb{Z}_9^R$	$\mathbb{Z}_{16}^R$	$\mathbb{Z}_{36}^R$

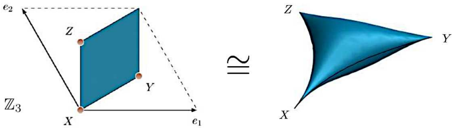
Baur,Nilles,SRS,Trautner,Vaudrevange (to appear)

Flavor in

semi-realistic orbifold models

Explicit string model $\mathbb{T}^6/\mathbb{Z}_3 \times \mathbb{Z}_3$

- Contains a sector $\mathbb{T}^2/\mathbb{Z}_3$



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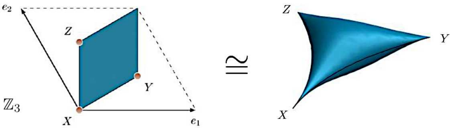

- $G_{traditional} = \Delta(54)$ & $G_{modular} = T' \cong \Gamma_3'$

Lauer, Mas, Nilles (89-90)

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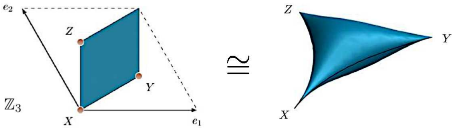

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use only the few representations for quarks and leptons:

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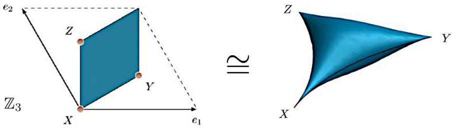

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Lauer, Mas, Nilles (89-90)

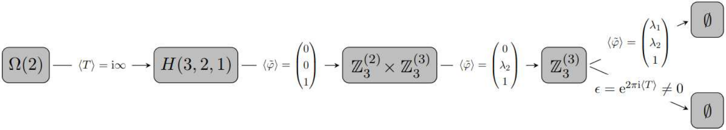
use only the few representations for quarks and leptons:

	quarks and leptons						Higgs fields		flavons							
label	q	$\bar{u}$	$\bar{d}$	ℓ	$\bar{e}$	$\bar{\nu}$	H_u	H_d	φ_e	φ_u	φ_ν	ϕ^0	ϕ^0_M	ϕ^0_e	ϕ^0_u	ϕ^0_d
$SU(3)_c$	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$SU(2)_L$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$U(1)_Y$	$1/6$	$-2/3$	$1/3$	$-1/2$	1	0	$1/2$	$-1/2$	0	0	0	0	0	0	0	0
$\Delta(54)$	$\mathbf{3}_2$	$\mathbf{3}_2$	$\mathbf{3}_2$	$\mathbf{3}_2$	$\mathbf{3}_2$	$\mathbf{3}_2$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}_2$	$\mathbf{3}_2$	$\mathbf{3}_2$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
T'	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$\mathbb{Z}_9^R$	1	1	1	1	1	1	0	0	1	1	1	0	0	0	0	0
n	$-2/3$	$-2/3$	$-2/3$	$-2/3$	$-2/3$	$-2/3$	0	0	$-2/3$	$-2/3$	$-2/3$	0	0	0	0	0

Baur, Nilles, SRS, Trautner, Vaudrevange (2207.10677)

Explicit string model $\mathbb{T}^6/\mathbb{Z}_3 \times \mathbb{Z}_3$

- write the corresponding action,
- fit the value of the modulus ($\langle T \rangle \sim 3\mathrm{i}$), and
- compute effective particle interactions (with 20 params)
- breake the eclectic flavor symmetry



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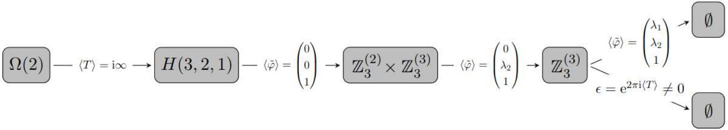
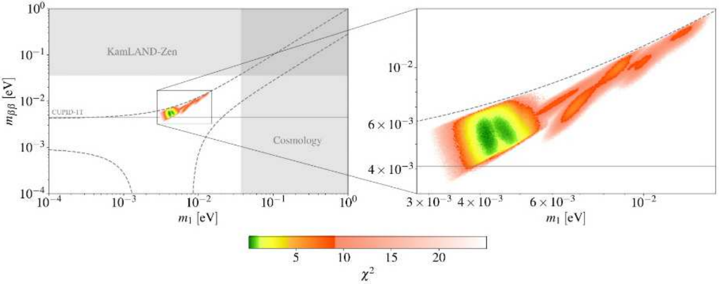


Resultados:



Explicit string model $\mathbb{T}^6/\mathbb{Z}_3 \times \mathbb{Z}_3$

Resultados:

	parameter	best-fit value
superpotential	$\mathrm{Im}\langle T\rangle$	3.195
	$\mathrm{Re}\langle T\rangle$	0.02279
	$\langle\tilde\varphi_{u,1}\rangle$	$2.0332\cdot10^{-4}$
	$\langle\vartheta_{u,1}\rangle$	1.6481
	$\langle\tilde\varphi_{u,2}\rangle$	$6.3011\cdot10^{-2}$
	$\langle\vartheta_{u,2}\rangle$	-1.5983
	$\langle\tilde\varphi_{e,1}\rangle$	$-4.069\cdot10^{-5}$
	$\langle\tilde\varphi_{e,2}\rangle$	$5.833\cdot10^{-2}$
	$\langle\tilde\varphi_{\nu,1}\rangle$	$1.224\cdot10^{-3}$
	$\langle\tilde\varphi_{\nu,2}\rangle$	-0.9857
	Λ_ν [eV]	0.05629
Kähler potential	α_1^u	-0.94917
	α_2^u	0.0016906
	α_3^u	0.31472
	α_1^d	0.95067
	α_2^d	0.0077533
	α_3^d	0.30283
	α_1^q	-0.96952
	α_2^q	-0.20501
	α_3^q	0.041643

(a)

	observable	model best fit	exp. best fit	exp. 1σ interval
quark sector	m_u/m_c	0.00193	0.00193	$0.00133\to0.00253$
	m_c/m_t	0.00280	0.00282	$0.00270\to0.00294$
	m_d/m_s	0.0505	0.0505	$0.0443\to0.0567$
	m_s/m_b	0.0182	0.0182	$0.0172\to0.0192$
	ϑ_{12} [deg]	13.03	13.03	$12.98\to13.07$
	ϑ_{13} [deg]	0.200	0.200	$0.193\to0.207$
	ϑ_{23} [deg]	2.30	2.30	$2.26\to2.34$
	$\delta^q_{\mathcal{CP}}$ [deg]	69.2	69.2	$66.1\to72.3$
lepton sector	m_e/m_μ	0.00473	0.00474	$0.00470\to0.00478$
	m_μ/m_τ	0.0586	0.0586	$0.0581\to0.0590$
	$\sin^2\theta_{12}$	0.303	0.304	$0.292\to0.316$
	$\sin^2\theta_{13}$	0.0225	0.0225	$0.0218\to0.0231$
	$\sin^2\theta_{23}$	0.449	0.450	$0.434\to0.469$
	$\delta^\ell_{\mathcal{CP}}/\pi$	1.28	1.28	$1.14\to1.48$
	η_1/π	0.029	-	-
	η_2/π	0.994	-	-
	$J_{\mathcal{CP}}$	-0.026	-0.026	$-0.033\to-0.016$
	$J^{\max}_{\mathcal{CP}}$	0.0335	0.0336	$0.0329\to0.0341$
	$\Delta m^2_{21}/10^{-5}$ [eV^2]	7.39	7.42	$7.22\to7.63$
	$\Delta m^2_{31}/10^{-3}$ [eV^2]	2.521	2.510	$2.483\to2.537$
	m_1 [eV]	0.0042	<0.037	-
	m_2 [eV]	0.0095	-	-
	m_3 [eV]	0.0504	-	-
	$\sum_i m_i$ [eV]	0.0641	<0.120	-
	$m_{\beta\beta}$ [eV]	0.0055	<0.036	-
	m_β [eV]	0.0099	<0.8	-
	χ^2	0.11		

Baur, Nilles, SRS, Trautner, Vaudrevange (2207.10677)

In summary

Concluding remarks

- Flavor puzzle: open questions about flavor (number and mixings of particles)

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- String theory: candidate theory for quantum gravity and all other quantum interactions

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  Caveat: some free parameters, less than the number of predictions

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To work on

- flavor with $\mathbb{T}^2/\mathbb{Z}_4$ & $\mathbb{T}^2/\mathbb{Z}_6$?
  Baur, Nilles, SRS, Trautner, Vaudrevange (2024)
- $\mathcal{CP}$ and $\mathcal{CP}$ violation ?
  Nilles, Ratz, Trautner, Vaudrevange (2018)
- bottom-up understanding of these fetures?
  We all?
- dynamic moduli stabilization & de Sitter ?
  see Michael's talk
- more pheno in these models ?
  should we join forces?
- already testable predictions ?


THANK
YOU FOR
SUPPORTING
MY SMALL
BUSINESS

Backup slides

Modular symmetries as flavor symmetries

Congruence modular subgroups: $\Gamma(N) \subset \mathrm{SL}(2,\mathbb{Z})$

$$\Gamma(N) = \{\gamma \in \mathrm{SL}(2,\mathbb{Z}) \,|\, \gamma = \mathbb{1} \mod N\}$$

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(Double-cover) finite modular subgroups: $\Gamma'_N \cong \mathrm{SL}(2,\mathbb{Z})/\Gamma(N)$

$$\Gamma'_N = \langle \mathrm{S}, \mathrm{T} \,|\, \mathrm{S}^4 = (\mathrm{ST})^3 = T^N = \mathbb{1}, \quad \mathrm{S}^2\mathrm{T} = \mathrm{TS}^2, \qquad N = 2,3,4,5 \rangle$$

$$\Gamma'_2 \cong S_3,\ \Gamma'_3 \cong T',\ \Gamma_4 \cong \mathrm{SL}(2,4),\ \Gamma_5 \cong \mathrm{SL}(2,5), \ldots$$

e.g. Liu, Ding (2019)

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Finite modular subgroups: $\Gamma_N \cong \mathrm{PSL}(2,\mathbb{Z})/\bar{\Gamma}(N)$ $(\mathrm{PSL}(2,\mathbb{Z}) \cong \mathrm{SL}(2,\mathbb{Z})/\{\pm\mathbb{1}\})$

$$\Gamma_N = \langle \mathrm{S},\mathrm{T} \,|\, \mathrm{S}^2 = (\mathrm{ST})^3 = T^N = \mathbb{1}, \quad N = 2,3,4,5 \rangle$$

$$\Gamma_2 \cong S_3,\ \Gamma_3 \cong A_4,\ \Gamma_4 \cong S_4,\ \Gamma_5 \cong A_5, \ldots, \Gamma_7 \cong \Sigma(168), \ldots$$

e.g. de Adelhaart, Feruglio, Hagedorn (2011)

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Couplings $\hat{Y}^{(n_Y)}(T)$ are $modular\ forms$

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Admissible iff

$$W(\Phi_{n_1},\ldots) \xrightarrow{\gamma} (cT+d)^{-1}\,\mathbb{1}\,W(\Phi_{n_1},\ldots), \qquad \text{i.e. } n_Y+\sum n_i=-1, \quad \prod\rho(\gamma)=\mathbb{1}$$

Note the nontrivial *automorphy factor* $(cT+d)^{-1} \to W$ covariant

How to proceed with *modular* flavor symmetries

- Take your favorite symmetry: $G_{mod} = \Gamma_N \in \{S_3, A_4, S_4, A_5, \ldots\}$
- Choose your favorite representations $\rho(\gamma)$ for quark and lepton fields

  e.g. quark doublets Q as $\mathbf{3}$ or $\mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}''$ of $\Gamma_3 \cong A_4, \ldots$

- Pick your favorite modular weights n_i and n_Y
- Write your G_{mod}-covariant superpotential W

  e.g. $W \supset \hat{Y}^u H_u Q \bar{u} + \hat{Y}^d H_d Q \bar{d} + \hat{Y}^e H_d L \bar{e} + \dfrac{\hat{Y}}{\Lambda} L H_u \, L H_u$

- Take your favorite inv. Kähler potential K; typical choice $K = \sum |\Phi_{n_i}|^2$

  MANY other modular invariant K possible! - Chen, SR-S, Ratz (1909.06910)

- Choose a $\langle T \rangle \neq 0$ $\rightarrow$ nontrivial rep. of $\hat{Y}(\langle T \rangle)$ breaks G_{mod}
- EW breakdown with $\langle H_u \rangle, \langle H_d \rangle \neq 0$
- Diagonalize quark and lepton matrices to compute V_{CKM} and U_{PMNS} and adjust only $\langle T \rangle$ to data

From top-down to bottom-up

eclectic flavor symmetries

Eclectic flavor groups

Key observation: T' is an outer automorphism group of $\Delta(54)$ ☺

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- $G_{eclectic} \cong$ multiplicative closure of G_{flavor} and $G_{modular}$
- Verify whether there is a third (class-inverting) outer automorphism that act as a $\mathbb{Z}_2$ CP-like transformation to further enhance the eclectic flavor symmetry

Eclectic flavor groups

flavor group $\mathcal{G}_{\text{fl}}$	GAP ID	$\text{Aut}(\mathcal{G}_{\text{fl}})$	finite modular groups		eclectic flavor group
Q_8	[8, 4]	S_4	without $\mathcal{CP}$	S_3	$\text{GL}(2,3)$
			with $\mathcal{CP}$	–	–
$\mathbb{Z}_3 \times \mathbb{Z}_3$	[9, 2]	$\text{GL}(2,3)$	without $\mathcal{CP}$	S_3	$\Delta(54)$
			with $\mathcal{CP}$	$S_3 \times \mathbb{Z}_2$	[108, 17]
A_4	[12, 3]	S_4	without $\mathcal{CP}$	S_3 S_4	S_4 S_4
			with $\mathcal{CP}$	–	–
T'	[24, 3]	S_4	without $\mathcal{CP}$	S_3	$\text{GL}(2,3)$
			with $\mathcal{CP}$	–	–
$\Delta(27)$	[27, 3]	[432, 734]	without $\mathcal{CP}$	S_3 T'	$\Delta(54)$ $\Omega(1)$
			with $\mathcal{CP}$	$S_3 \times \mathbb{Z}_2$ $\text{GL}(2,3)$	[108, 17] [1296, 2891]
$\Delta(54)$	[54, 8]	[432, 734]	without $\mathcal{CP}$	T'	$\Omega(1)$
			with $\mathcal{CP}$	$\text{GL}(2,3)$	[1296, 2891]

Nilles, SR-S, Vaudrevange (2001.01736)

More details on top-down

eclectic flavor symmetries

Towards the *eclectic* flavor picture

Use Narain formalism: split string in independent components

$$X(\tau,\sigma) = X_R(\sigma - \tau) + X_L(\sigma + \tau)$$

Groot-Nibbelink, Vaudrevange (2017)

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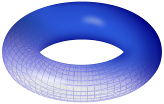
What are the outer automorphisms of $S_{Narain} = \{g\}$?

$$Out(S_{Narain}) = \left\{h = (\Sigma, t) \notin S_{Narain} \mid hgh^{-1} \in S_{Narain}\right\}$$

Rotations: $h_\Sigma = (\Sigma, 0) \to O(2,2;\mathbb{Z})$, Translations: $h_t = (\mathbb{1}_4, t)$

Towards the *eclectic* picture: what $Out(S_{Narain})$ is

String 2D toroidal compactifications have two moduli: T, U



$$G = \frac{\mathrm{Im}\,T}{\mathrm{Im}\,U}\begin{pmatrix} 1 & \mathrm{Re}\,U \\ \mathrm{Re}\,U & |U|^2 \end{pmatrix}, \qquad B = \mathrm{Re}\,T\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

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$h_\Sigma =$	S_U	T_U	S_T	T_T	M	$\mathbb{Z}_2^{\mathcal{CP}}$
$U \xrightarrow{h_\Sigma}$	$-1/U$	$U+1$	U	U	T	$-\bar{U}$
$T \xrightarrow{h_\Sigma}$	T	T	$-1/T$	$T+1$	U	$-\bar{T}$

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Recall: in $\mathrm{SL}(2,\mathbb{Z})$ $\qquad T \xrightarrow{\mathrm{S}} -\frac{1}{T}, \qquad T \xrightarrow{\mathrm{T}} T+1$

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$$\mathrm{SL}(2,Z)_T = \langle \mathrm{S}_T, \mathrm{T}_T \rangle, \quad \mathrm{SL}(2,Z)_U = \langle \mathrm{S}_U, \mathrm{T}_U \rangle \qquad ☺$$

M: mirror symmetry, K$_*$: $\mathcal{CP}$-like transformation ☺

Nilles, Ratz, Trautner, Vaudrevange (2018); Novichkov, Penedo, Petcov, Titov (2019)

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Further, $\{h_t\}$ don't change T, U, but do transform fields
$\rightarrow$ traditional flavor symmetry ☺

Common origin of modular and traditional flavor

Modular weights n_i, representations and couplings of Φ_{n_i} not *ad hoc*! ☺

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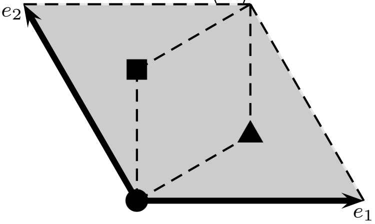
Example $\mathbb{T}^2/\mathbb{Z}_3$: must fix U to $\langle U \rangle = \omega = e^{2\pi \mathrm{i}/3} \rightarrow$ broken $\mathrm{SL}(2,\mathbb{Z})_U$



Lauer, Mas, Nilles (1989)

By using CFT formalism, inspect $\mathrm{SL}(2,\mathbb{Z})_T$ on the triplet of matter fields:

$$h_\Sigma : \rho(\mathrm{S}_T) = \frac{\mathrm{i}}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}, \quad \rho(\mathrm{T}_T) = \begin{pmatrix} \omega^2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\rho(\mathrm{S}_T)$ and $\rho(\mathrm{S}_T)$ build the reps. $\mathbf{2}' \oplus \mathbf{1}$ of modular group $\Gamma'_3 = T'$ ☺

$$\Phi_{n=-2/3,-5/3} \xrightarrow{\mathrm{S}_T} (-T)^n \rho(\mathrm{S}_T)\Phi_n, \qquad \Phi_n \xrightarrow{\mathrm{T}_T} \rho(\mathrm{T}_T)\Phi_n$$

Ibáñez, Lüst (1992)

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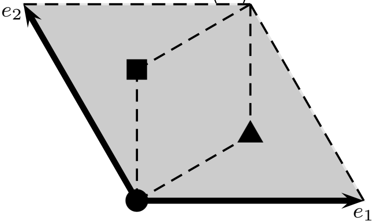

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$$h_t : \rho(\mathrm{A}) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \ \rho(\mathrm{B}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \ \rho(\mathrm{C}) = \rho(\mathrm{S}_T^2)$$

$\rho(\mathrm{A})$, $\rho(\mathrm{B})$ and $\rho(\mathrm{C})$ build the reps $\mathbf{3}_{2(1)}$ and $\mathbf{3}_{1(1)}$ of traditional flavor group $\Delta(54)$ for $\Phi_{-2/3}$ and $\Phi_{-5/3}$

cf. also in Kobayashi, Plöger, Nilles, Raby, Ratz (2006)

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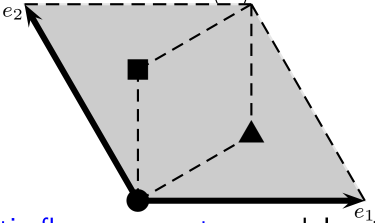

first eclectic flavor symmetry: modular + traditional flavor

$$\Delta(54) \cup T' \cong \Omega(1) = SG[648, 533]$$

with $\mathcal{CP}$: $\Delta(54) \cup T' \cup \mathbb{Z}_2^{\mathcal{CP}} \cong SG[1296, 2891]$

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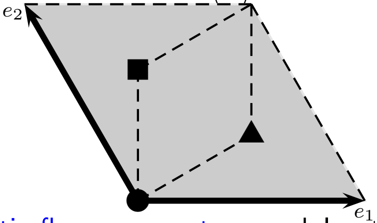

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Can we generalize this in a bottom-up fashion ?