# FLAVOUR MODEL BUILDING: CONSEQUENCES IN THE QUARK AND HIGGS SECTORS

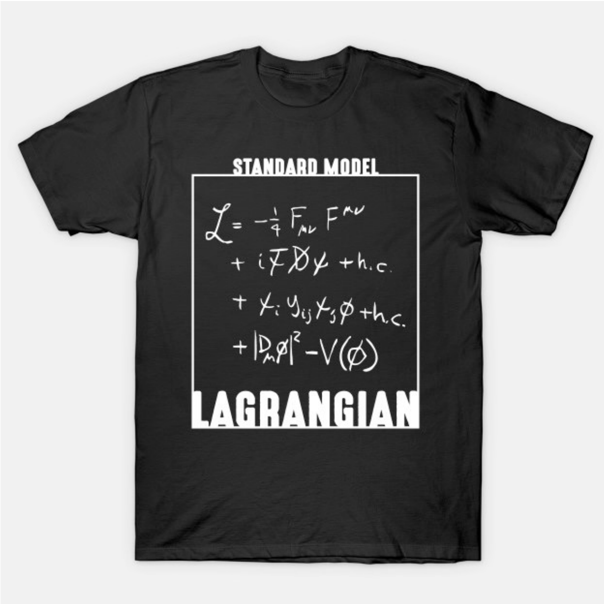
*Myriam Mondragón*

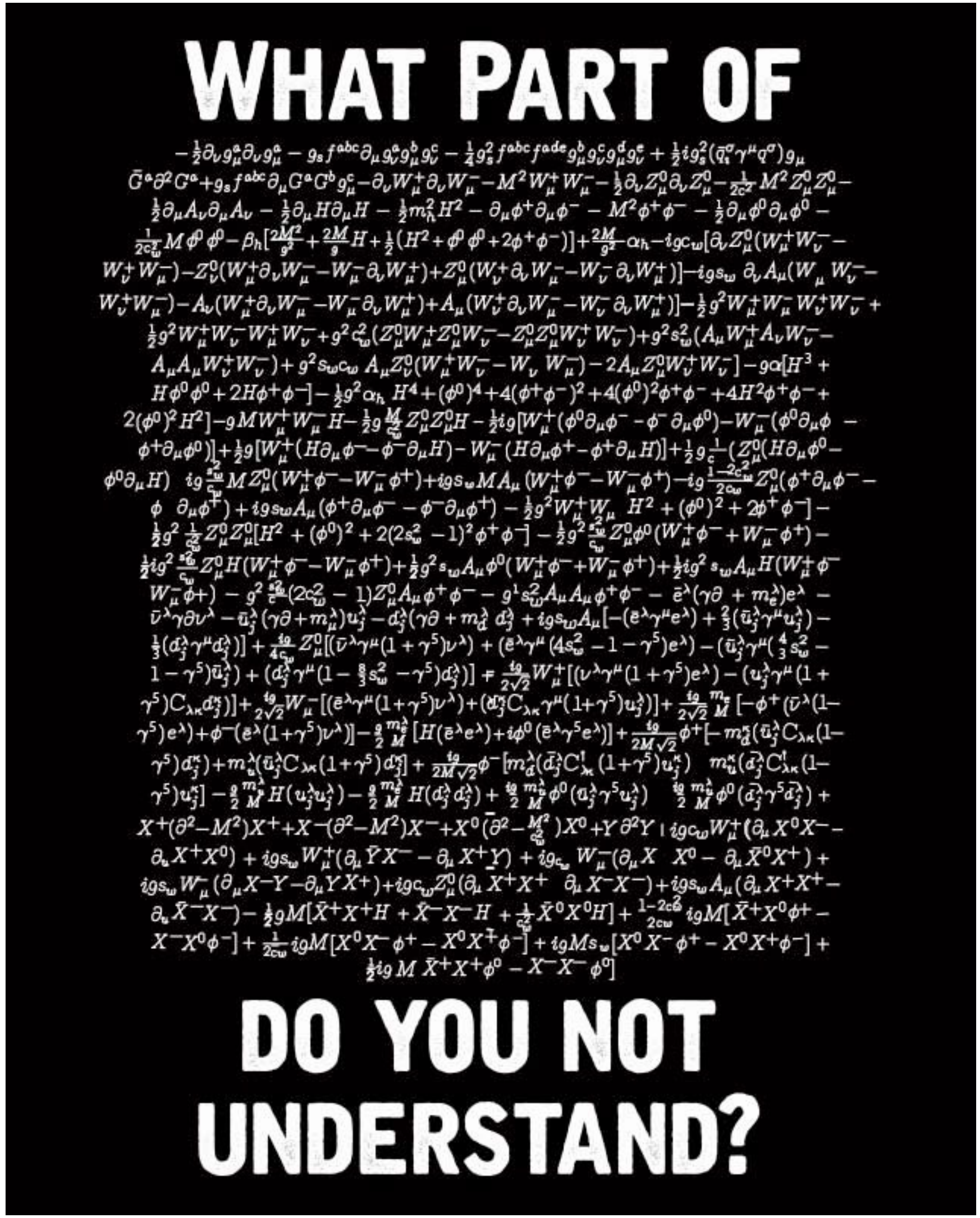
*Instituto de Física, UNAM*

*Modular Invariance Approach to the Lepton and Quark Flavour Problems*

*May 14, 2024*

*MITP, Mainz*


STANDARD MODEL
$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$
$+ i\bar{\psi} \not{D} \psi + h.c.$
$+ \psi_i y_{ij} \psi_j \phi + h.c.$
$+ |D_\mu \phi|^2 - V(\phi)$
LAGRANGIAN


WHAT PART OF
DO YOU NOT UNDERSTAND?


WHAT PART OF
DO YOU NOT UNDERSTAND?
more at hereticwear.com

# THE FLAVOUR PROBLEM

- Steve gave the motivation
    - Examples for neutrinos
- What happens in the quark sector?
    - Textures
- What happens in the scalar sector?
- An S3 example multi-Higgs example
    - quarks and Higgs sectors
        - problems and an unusual solution

# FLAVOUR

- Interactions that distinguish between flavours

- why 3 generations?
- why those masses?
- why the gap between neutral and charged fermions
- why the difference between mixing matrices?
- why that amount of CP violation?
- …

- *Fermion masses*
- *Mixing*
- *CP violation*

### Lead to discoveries

- $\Gamma(K_L \to \mu^+ \mu^-) / \Gamma(K^+ \to \mu^+ \nu) \to$ *charm quark*
- $\Delta m_K \to$ *charm mass*
- $\Delta m_B \to$ *top mass*
- $\varepsilon_K \to$ *third generation*
- $\nu$ *oscillation* $\to$ $\nu$ *mass*

### Connections to new/unknown physics

- *Dark matter*
- *Baryogenesis*
- *Leptogenesis*
- *EW phase transition*
- *??*

Nir, CERN–LATAM School HEP (2015)

# SOME ASPECTS OF THE FLAVOUR PROBLEM

- Quark and charged lepton masses very different, very hierarchical

$$m_u : m_c : m_t \sim 10^{-6} : 10^{-3} : 1$$

$$m_d : m_s : m_b \sim 10^{-4} : 10^{-2} : 1$$

$$m_e : m_\mu : m_\tau \sim 10^{-5} : 10^{-2} : 1$$

- Neutrino masses unknown, only difference of squared masses.
- Type of hierarchy (normal or inverted) also unknown
- Higgs sector under study

- Quark mixing angles

$$\theta_{12} \approx 13.0^o$$
$$\theta_{23} \approx 2.4^o$$
$$\theta_{13} \approx 0.2^o$$

- Neutrino mixing angles

$$\Theta_{12} \approx 33.8^\circ$$
$$\Theta_{23} \approx 48.6^\circ$$
$$\Theta_{13} \approx 8.6^\circ$$

- Small mixing in quarks, large mixing in neutrinos. Very different
- Is there an underlying symmetry?

?

*The matter particles*

$$Q_{Li}(3,2)_{+1/6},\quad U_{Ri}(3,1)_{+2/3},\quad D_{Ri}(3,1)_{-1/3},\quad L_{Li}(1,2)_{-1/2},\quad E_{Ri}(1,1)_{-1}\quad (i=1,2,3)$$

$$\phi(1,2)_{+1/2}.$$

*The scalar*

*The Lagrangian*

$$\mathcal{L} = \mathcal{L}_{\text{cin}} + \mathcal{L}_{\psi} + \mathcal{L}_{\text{Yuk}} + \mathcal{L}_{\phi}$$

*The fields strengths*

$$G_a^{\mu\nu} = \partial^\mu G_a^\nu - \partial^\nu G_a^\mu - g_s f_{abc} G_b^\mu G_c^\nu$$
$$W_a^{\mu\nu} = \partial^\mu W_a^\nu - \partial^\nu W_a^\mu - g \epsilon_{abc} W_b^\mu W_c^\nu$$
$$B^{\mu\nu} = \partial^\mu B^\nu - \partial^\nu B^\mu$$

*The covariant derivative*

*U(1) charges*

$$D^\mu = \partial^\mu + i g_s G_a^\mu L_a + i g W_b^\mu T_b + i g' B^\mu Y$$

*group generators*

*The Yukawa interactions*

$$\mathcal{L}_{\text{Y}}^{\text{ME}} = Y_{ij}^d \overline{Q_{Li}} \phi D_{Rj} + Y_{ij}^u \overline{Q_{Li}} \tilde{\phi} U_{Rj} + Y_{ij}^e \overline{L_{Li}} \phi E_{Rj} + \text{h.c.}.$$

$$\tilde{\phi} = i\tau_2 \phi^\dagger$$

# The electroweak sector of the SM

# HIGGS POTENTIAL

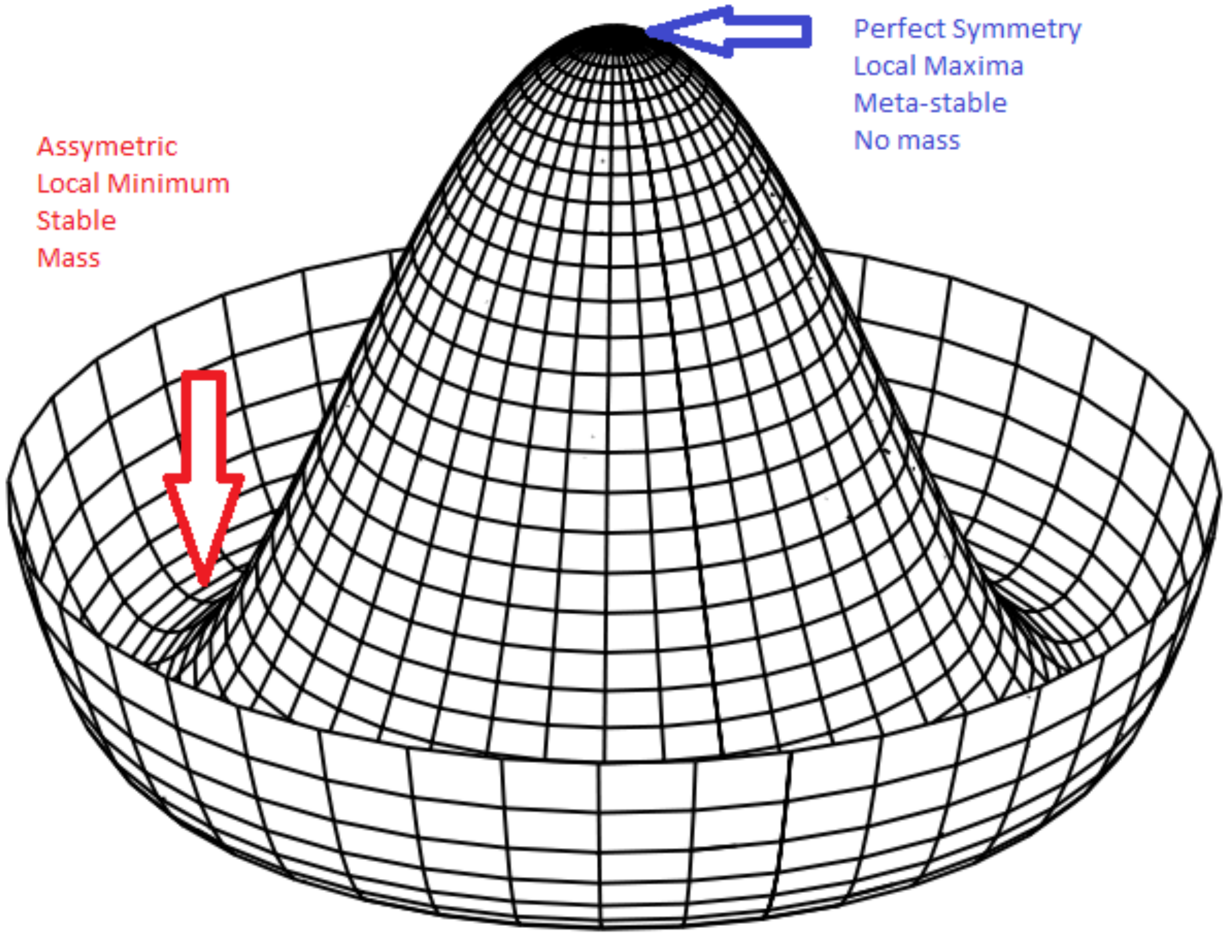
$$\mathcal{L}_{\phi}^{\mathrm{ME}} = -\mu^2 \phi^\dagger \phi - \lambda \left(\phi^\dagger \phi\right)^2$$

$$\mu^2 < 0 \,,\ \lambda > 0$$

$$v^2 = -\frac{\mu^2}{\lambda}$$

$$\langle \phi \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

$$SU(2)xU(1) \rightarrow U(1)_{EM}$$




# QUARKS AND HIGGS INTERRELATED

- Yukawa couplings: several orders of magnitude of difference, strong hierarchy

$$\mathcal{L}_{\rm Y}^{\rm ME} = Y_{ij}^d \overline{Q_{Li}} \phi D_{Rj} + Y_{ij}^u \overline{Q_{Li}} \tilde{\phi} U_{Rj} + Y_{ij}^e \overline{L_{Li}} \phi E_{Rj} + \text{h.c.}.$$

Also neutrinos, but they could acquire mass other ways.

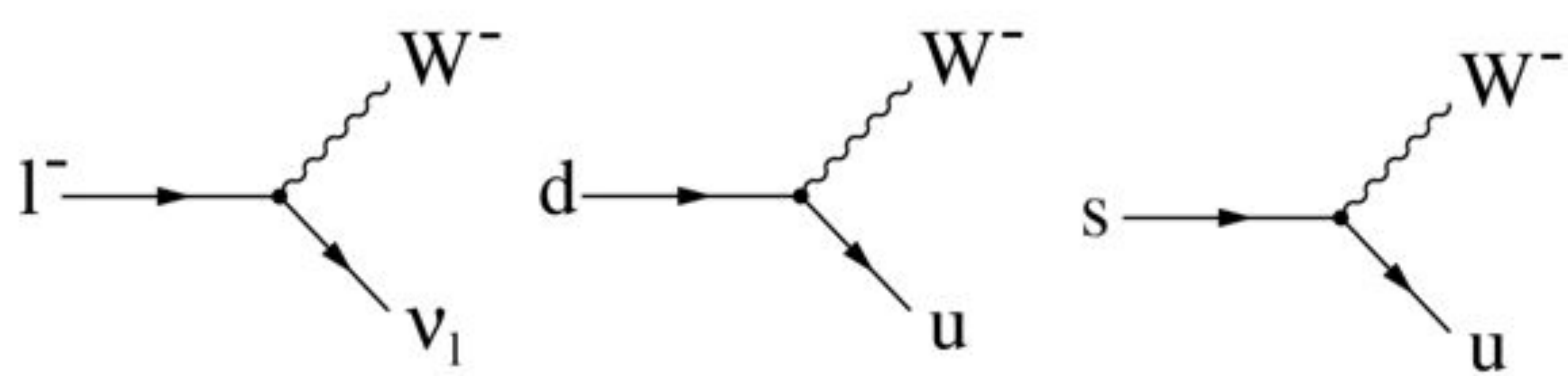
- Higgs sector:

$$\mathcal{L}_{\phi}^{\rm ME} = -\mu^2 \phi^\dagger \phi - \lambda \left(\phi^\dagger \phi\right)^2 \qquad v^2 = -\frac{\mu^2}{\lambda}$$

  - hierarchy problem (quadratic radiative corrections)
  - limits to perturbative unitarity
  - Why $M_{\rm Higgs} \sim 125$ GeV?

# CHARGED CURRENT INTERACTIONS

- Quarks change flavour through charged current interactions
- CP violation in the weak interactions
- Coupling is complex
- On




- Flavour changing neutral currents greatly suppressed

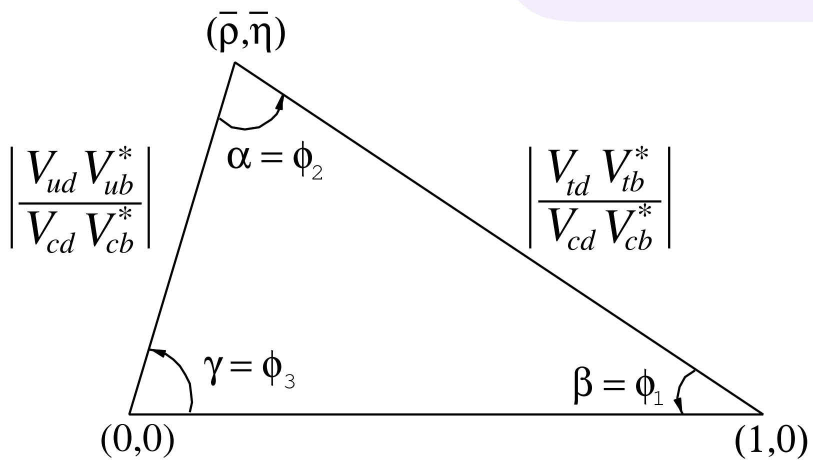
$$V_{\rm CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

*$V_{CKM}$ very well determined*

*PDG 2023*

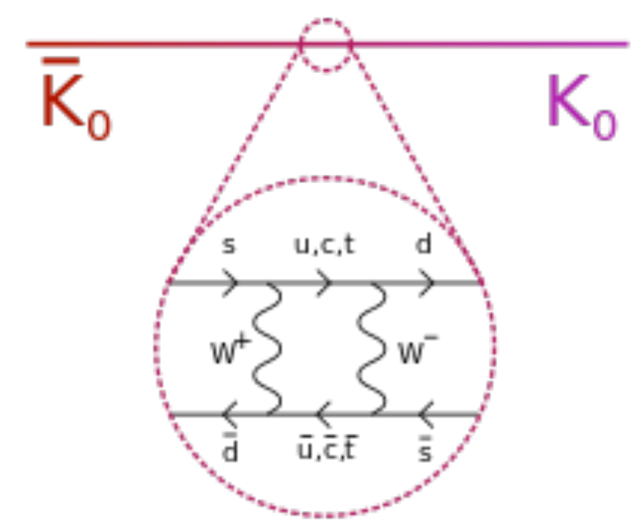
$$|V_{\rm CKM}| = \begin{pmatrix} 0.97435 \pm 0.00016 & 0.22500 \pm 0.00067 & 0.00369 \pm 0.00011 \\ 0.22486 \pm 0.00067 & 0.97349 \pm 0.00016 & 0.04182^{+0.00085}_{-0.00074} \\ 0.00857^{+0.00020}_{-0.00018} & 0.04110^{+0.00083}_{-0.00072} & 0.999118^{+0.000031}_{-0.000036} \end{pmatrix}$$




$$J = (3.08^{+0.15}_{-0.13}) \times 10^{-5}$$

$$\sin\theta_{12} = 0.22500 \pm 0.00067\,, \qquad \sin\theta_{13} = 0.00369 \pm 0.00011\,,$$

$$\sin\theta_{23} = 0.04182^{+0.00085}_{-0.00074}\,, \qquad \delta = 1.144 \pm 0.027\,.$$




*K, B, $B_S$, D processes can be used to study new physics*

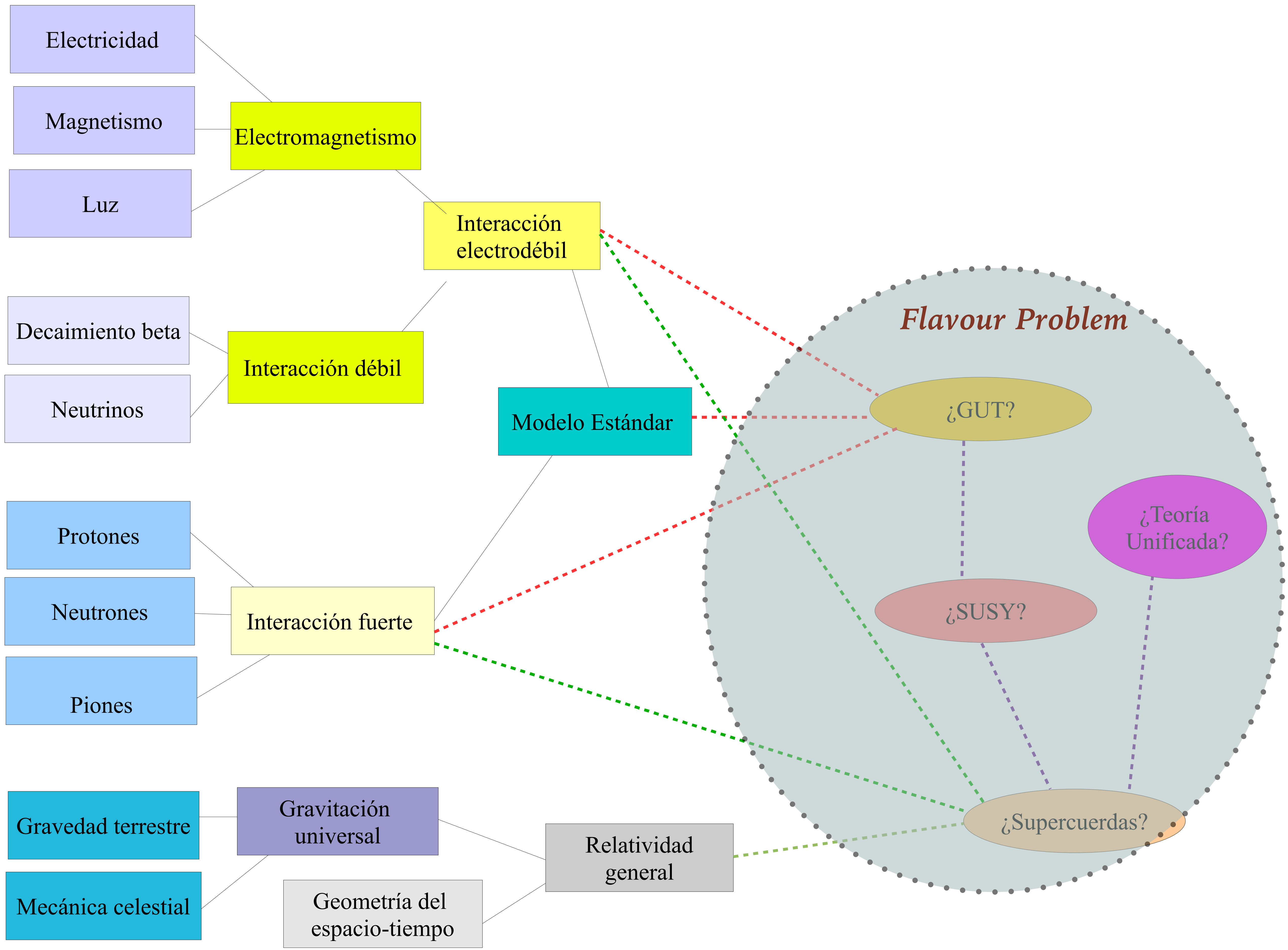
***FCNCs very sensitive to BSM***

# FERMION AND SCALAR SECTORS

- Free parameters in quarks:
  6 masses ->Yukawa couplings
  3 mixing angles
  CP violating phase
- Unitarity —> Jarlskog invariants

- Free parameters in neutrinos:
  6 masses
  3 mixing angles
  CP violating phase
  2 Majorana phases
- Unitarity? —> Also Jarlskog invariants

*Plus Higgs vev*



Electricidad
Magnetismo
Luz
Electromagnetismo
Interacción electrodébil
Decaimiento beta
Neutrinos
Interacción débil
Modelo Estándar
Flavour Problem
¿GUT?
¿Teoría Unificada?
¿SUSY?
Protones
Neutrones
Piones
Interacción fuerte
Gravedad terrestre
Mecánica celestial
Gravitación universal
Relatividad general
Geometría del espacio-tiempo
¿Supercuerdas?

# FLAVOUR SYMMETRIES

- Flavour symmetries: continuous or discrete?

discrete
could lead to domain walls

continuous
breaking may give massless
Goldstone bosons

- At low energies now discrete preferred. Could be:
    - Residual symmetry from breaking from continuous one
    - From the breaking of a larger discrete group
    - Discrete from the "beginning"

# MASS MATRICES TEXTURES — TEXTURE ZEROES

- Zeroes in the mass matrices —> less parameters, underlying symmetries: Fritzsch

$$M_q = \begin{pmatrix} 0 & C_q & 0 \\ C_q^* & 0 & B_q \\ 0 & B_q^* & A_q \end{pmatrix}$$

*This version excluded already*

hierarchical A≫|B|≫|C|

- In SM and extensions (no FC right-handed currents) is always possible to simultaneously the Mu and Md to Hermitian or NNI textures

- NNI

$$M'_q = \begin{pmatrix} 0 & C_q & 0 \\ C'_q & 0 & B_q \\ 0 & B'_q & A_q \end{pmatrix}$$

B' ≠ B, C' ≠ C

- For any Hermitian 3x3 Mu, Md always possible to change basis to (1,3)=(3,1)=0

*Xing Phys.Rept. 854 (2020)*

# MORE ON TEXTURES

- Add zeroes? Use $Z_N$, arbitrary but effective
- Better, theoretical motivation
- Use invariants, calculate mass ratios —> $V_{CKM}$
- What works? up and down sector same structure, coming from same dynamics
- Best type of texture with current data

$$M_q = \begin{pmatrix} 0 & C_q & 0 \\ C_q^* & B'_q & B_q \\ 0 & B_q^* & A_q \end{pmatrix}$$

$A \gg |B| \gg |B'| \gg |C|$

$A > 0$, $B'$ real

# ALLOWED TEXTURES

Table 14: The five phenomenologically viable five-zero textures of Hermitian quark mass matrices.

| | I | II | III | IV | V |
|---|---|---|---|---|---|
| $M_u =$ | $\begin{pmatrix} 0 & C_u & 0 \\ C_u^* & B'_u & 0 \\ 0 & 0 & A_u \end{pmatrix}$ | $\begin{pmatrix} 0 & C_u & 0 \\ C_u^* & 0 & B_u \\ 0 & B_u^* & A_u \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & D_u \\ 0 & B'_u & 0 \\ D_u^* & 0 & A_u \end{pmatrix}$ | $\begin{pmatrix} 0 & C_u & 0 \\ C_u^* & B'_u & B_u \\ 0 & B_u^* & A_u \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & D_u \\ 0 & B'_u & B_u \\ D_u^* & B_u^* & A_u \end{pmatrix}$ |
| $M_d =$ | $\begin{pmatrix} 0 & C_d & 0 \\ C_d^* & B'_d & B_d \\ 0 & B_d^* & A_d \end{pmatrix}$ | $\begin{pmatrix} 0 & C_d & 0 \\ C_d^* & B'_d & B_d \\ 0 & B_d^* & A_d \end{pmatrix}$ | $\begin{pmatrix} 0 & C_d & 0 \\ C_d^* & B'_d & B_d \\ 0 & B_d^* & A_d \end{pmatrix}$ | $\begin{pmatrix} 0 & C_d & 0 \\ C_d^* & B'_d & 0 \\ 0 & 0 & A_d \end{pmatrix}$ | $\begin{pmatrix} 0 & C_d & 0 \\ C_d^* & B'_d & 0 \\ 0 & 0 & A_d \end{pmatrix}$ |

Above textures first found by Ramond et al (1993), work today if not strongly hierarchical.

- But so far the best one is:

$$M_q = \begin{pmatrix} 0 & C_q & 0 \\ C_q^* & B'_q & B_q \\ 0 & B_q^* & A_q \end{pmatrix}$$

*From Z. Xing, Phys.Rept. 854 (2020)*

# TEXTURES AT HIGH ENERGIES

- Usually express mass matrices as mass ratios →
they remain stable below eW scale, but renormalize above it, depending on model

- From high to low energies they get renormalized as,

$$M_{\rm u}(\Lambda_{\rm EW}) \simeq \gamma_{\rm u} \left[ \begin{pmatrix} 0 & C_{\rm u} & 0 \\ C_{\rm u}^* & B'_{\rm u} & B_{\rm u} I_t^{C_{\rm u}^{\rm u}} \\ 0 & B_{\rm u}^* I_t^{C_{\rm u}^{\rm u}} & A_{\rm u} I_t^{C_{\rm u}^{\rm u}} \end{pmatrix} + \frac{I_t^{C_{\rm u}^{\rm u}} - 1}{A_{\rm u}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & |B_{\rm u}|^2 & B_{\rm u} B'_{\rm u} \\ 0 & B_{\rm u}^* B'_{\rm u} & 0 \end{pmatrix} \right]$$

$$M_{\rm d}(\Lambda_{\rm EW}) \simeq \gamma_{\rm d} \left[ \begin{pmatrix} 0 & C_{\rm d} & 0 \\ C_{\rm d}^* & B'_{\rm d} & B_{\rm d} \\ 0 & B_{\rm d}^* I_t^{C_{\rm d}^{\rm u}} & A_{\rm d} I_t^{C_{\rm d}^{\rm u}} \end{pmatrix} + \frac{I_t^{C_{\rm d}^{\rm u}} - 1}{A_{\rm u}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & B_{\rm u} B_{\rm d}^* & A_{\rm d} B_{\rm u} \\ 0 & B_{\rm u}^* B'_{\rm d} & B_{\rm u}^* B_{\rm d} \end{pmatrix} \right]$$

*I's are the one-loop corrections,γ anomalous dimensions, C's coefficients in the running*

- Textures remain, coefficients change, for MSSM there is dependence on soft breaking terms

# WHAT ABOUT THE HIGGS SECTOR? ORIGIN OF FLAVOUR PROBLEM(S)?

- One Higgs field: "takes care" of all masses, might be too much
- More Higgs fields:
  more doublets, absolutely necessary in SUSY models, always in pairs
  2HDM without SUSY
  3HDM also studied
- More scalars: potential more complicated breaking of flavour symmetry at low energies… either by "hand" or spontaneously
- Where does the flavour symmetry breaking come from?

# N-HIGGS DOUBLET MODELS — NHDM

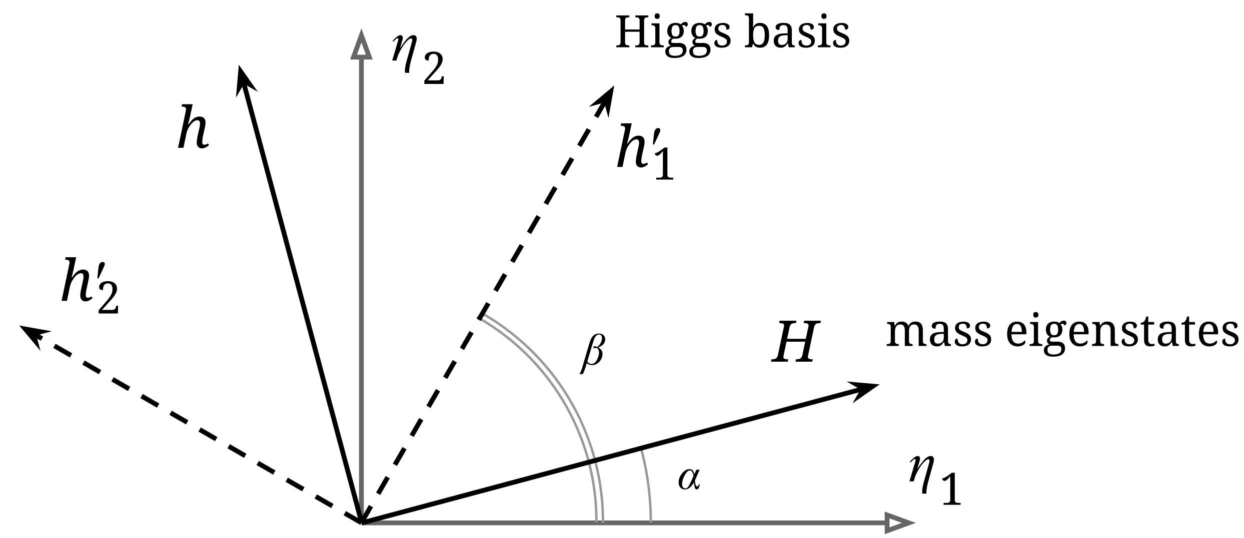
- Add more complex electroweak doublets
  All with same hyper charge Y=1

$$V(\phi) = Y_{ij}\phi_i^\dagger\phi_j + Z_{ijkl}(\phi_i^\dagger\phi_j)(\phi_k^\dagger\phi_l)\,.$$

- $N^2 + N^2(N^2+1)/2$ real parameters:
  12 for 2HDM, 54 for 3HDM…
- Potential must be bounded by below, no charge or colour breaking minima
- Must respect unitarity bounds
- Can have CP breaking minima → baryogenesis (or disaster)

# BASIS, FLAVOUR BASIS

- Convenient to rotate to Higgs basis, vev all in first doublet
- Goldstone bosons in first one, physical Higgses in the rest




*Ivanov, Prog.Part.Nucl.Phys. 95 (2017)*

- N-1 pairs of charged Higgses, 2N-1 neutral scalars (odd and even)
- Suitable basis for studying phenomenology, e.g. FCNCs

# MULTI-HIGHS MODEL AND FLAVOUR SYMMETRIES

- 2HDM widely studied, several studies on 3HDM (Branco et al,; King et al, *JHEP* 01 (2014) 052 al, 2014)
- Minimization of scalar potential must be performed. Sometimes vev alignments are chosen by hand, e.g. v1»v2» v3 → maybe only local minima
- Extra Higgs doublets and discrete symmetries → continuous symmetries
- Also usually after minimization of the potential there are residual symmetries → unphysical quark sector, either degenerate masses, zero masses or zeroes in $V_{CKM}$
    - $S_3$, $S_4$, $A_4$, $\Delta(54)$ all have residual symmetries in 3HDM
- If soft breaking performed, stability and unitarity conditions must be recalculated
- Connection with dark matter, inert scalars vev=0

*Ivanov, Prog.Part.Nucl.Phys.* 95 (2017)

# MORE SCALARS

- Add singlets, same considerations as before
- Flavons: responsible for family symmetry breaking at high energies, Froggatt-Nielsen mechanism
- Scalars can be used for a number of other purposes: inflation, dark matter, dark energy, phase transitions

  - Is there evidence for new scalars?
    95 GeV? CMS ~ 2.9 sigma
    150 GeV? multilepton anomalies
    650 GeV? CMS ~ 3.8 sigma
    All of them???

- Not significative, but persistent…

# INTERPLAY BETWEEN FLAVOUR AND ASTROPARTICLE PHYSICS

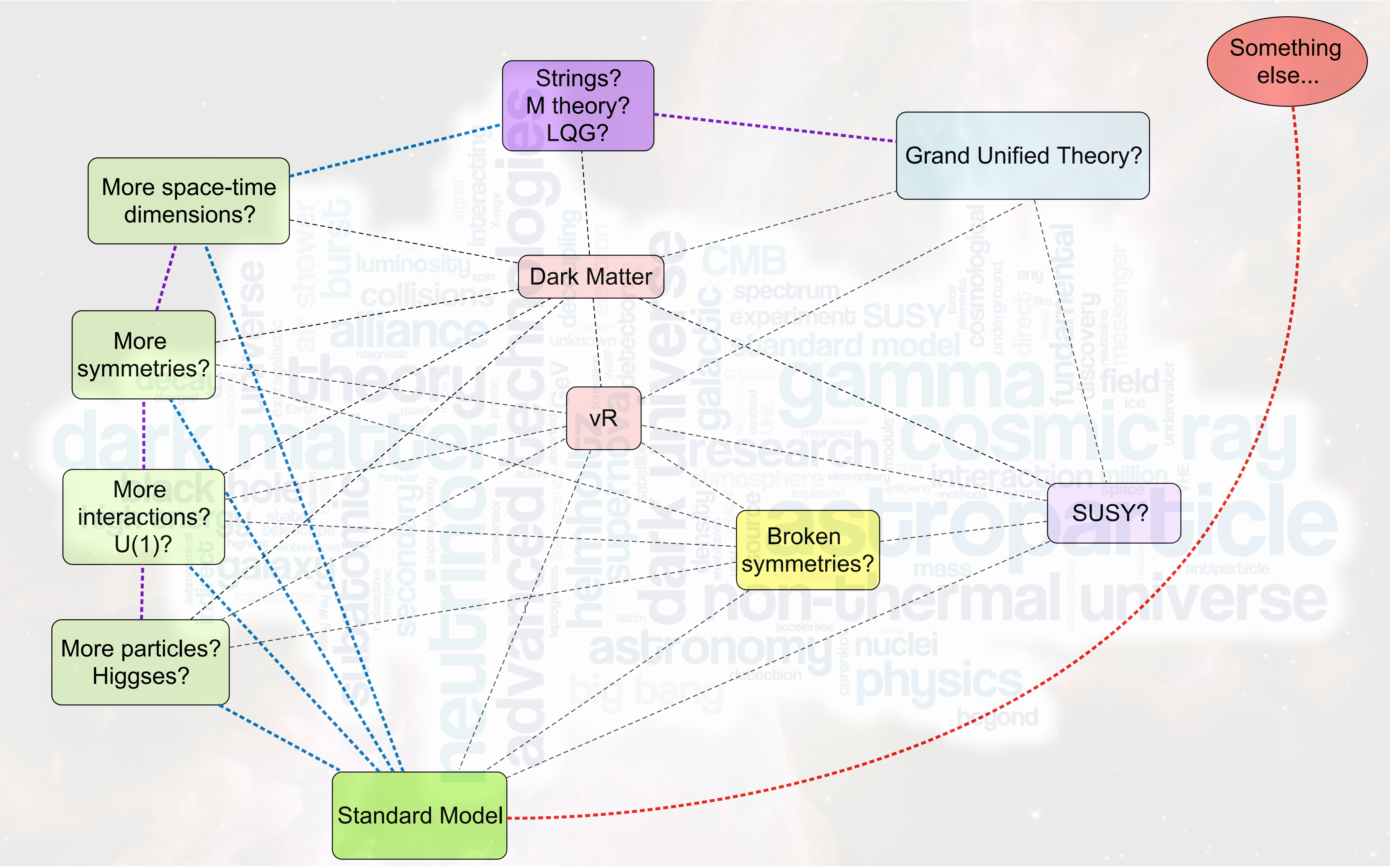
- Dark matter candidates:

scalars:
exotic Higgses, axion-like particles,
KK particles,

fermions:
right-handed neutrinos,
neutralinos, KK particles…

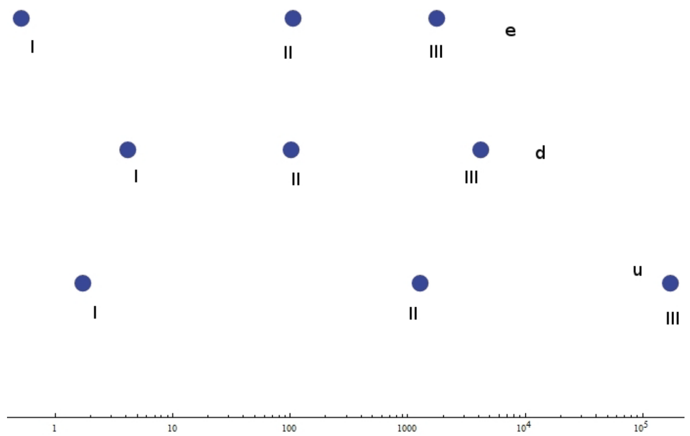
**Related to flavour,**
**Constrained by symmetries**

- CP violation: baryogenesis, leptogenesis
- g-2: many extensions attempt explanation. LHC and DM experiments constrain it
- Effective field theory approach ($\kappa$ formalism) helps constrain new processes

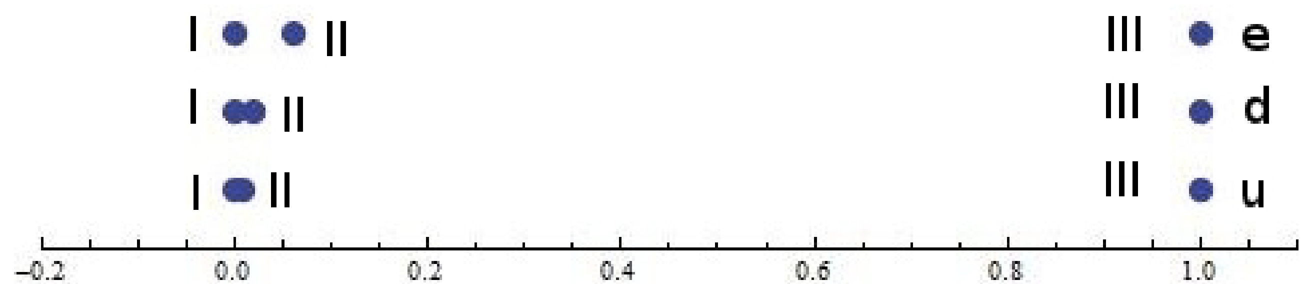

Strings?
M theory?
LQG?
Something else...
Grand Unified Theory?
More space-time dimensions?
Dark Matter
More symmetries?
νR
More interactions? U(1)?
SUSY?
Broken symmetries?
More particles? Higgses?
Standard Model

# HOW DO WE CHOOSE A FLAVOUR SYMMETRY?

- Several ways:
- Look for inspiration in a high energy extension of SM, i.e. strings or GUTs
- Look at low energy phenomenology
- At some point they should intersect…
- In here:
  - Find the smallest flavour symmetry suggested by data
  - Explore how generally it can be applied (universally)
  - Follow it to the end
  - Compare it with the data



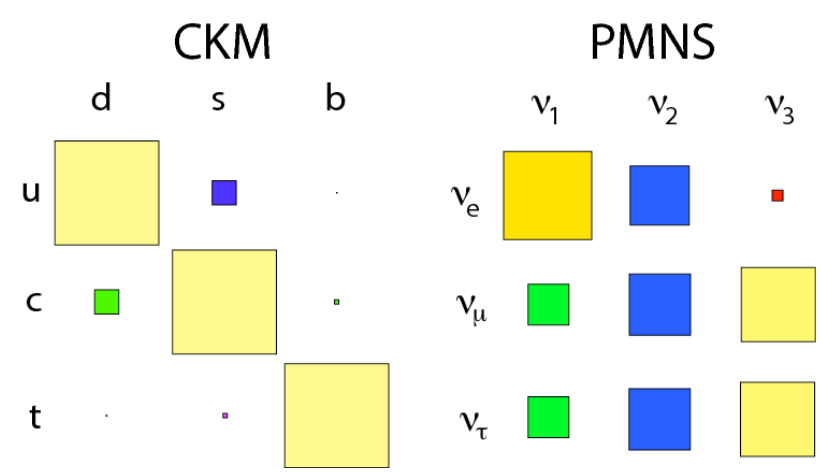

*Plot of mass ratios*




*Logarithmic plot of quark masses*

$$\begin{bmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{bmatrix} \approx \begin{bmatrix} 0.974 & 0.225 & 0.003 \\ 0.225 & 0.973 & 0.041 \\ 0.009 & 0.040 & 0.999 \end{bmatrix},$$

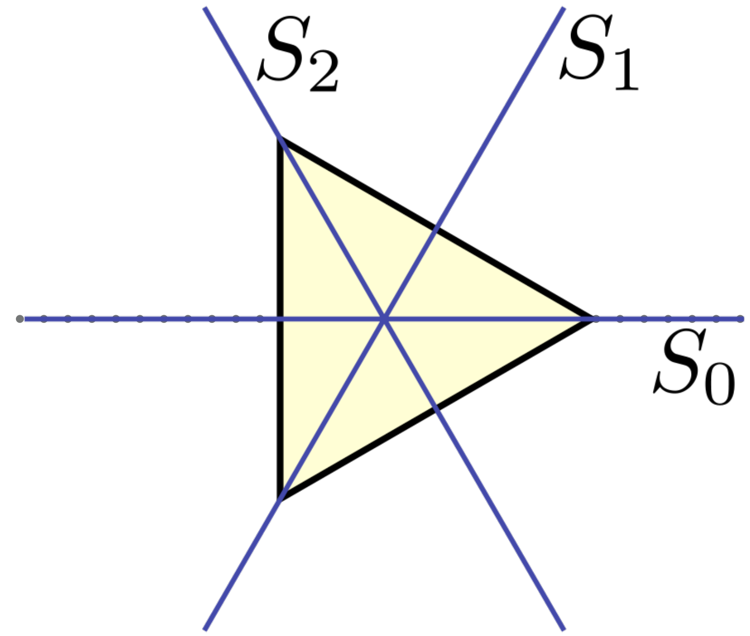
Suggests a 2⊕1 structure



# 3HDM

- Without symmetry $\Longrightarrow$ 54 real parameters in potential
- Complemented with additional symmetry(ies)
- Studies started in the 70's, hope to find global symmetry that explains the mass and mixing patterns
- The first symmetries to be added were the permutational groups S3 and S4
- Different modern versions of these models exist

# S3




- Smallest non-Abelian discrete group
- Permutation symmetry of three objects; reflections and rotations that leave an equilateral triangle invariant
- Has irreducible representations, 2, $1_S$ d $1_A$
- 3 right handed neutrinos
- 3 Higgs doublets

- We apply the symmetry "universally" to quarks, leptons and Higgs-es
  - First two families in the doublet
  - Third family in symmetric singlet
- Treat scalars and fermions simultaneously

# *A sample of S3 models*

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S Pramanick, Phys Rev D100 (2019)

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M. Gómez-Bock, A. Pérez, MM, EPJC81 (2021)

Kobayashi et al , PTEP 2020 (2020)

Petcov and Tenedo, 2024

*Just a sample, there are many more…*
*I apologize for those not included*

# PREDICTIONS, ADVANTAGES?

- Possible to reparametrize mixing matrices in terms of mass ratios, successfully
- CKM has NNI and Fritzsch textures
- PMNS → fix one mixing angle, predictions for the other two within experimental range
- Reactor mixing angle $\theta_{13} \neq 0$
- Some FCNCs suppressed by symmetry
- Higgs potential has 8 couplings
- Underlying symmetry in quark, leptons and Higgs → residual symmetry of a more fundamental one?
- Lots of Higgses: 3 neutral, 4 charged, 2 pseudoscalars
- Further predictions will come from Higgs sector: decays, branching ratios

*A. Mondragón, M. M., F. González, E. Peinado, U. Saldaña, O. Félix, E. Rodríguez, A. Pérez, H. Reyes...; Das, Dey et al; Teshima et al;*

# FERMION MASSES

- The Lagrangian of the model

$$\mathcal{L}_Y = \mathcal{L}_{Y_D} + \mathcal{L}_{Y_U} + \mathcal{L}_{Y_E} + \mathcal{L}_{Y_\nu},$$

- The general form of the fermion mass matrices in the symmetry adapted basis is

$$\mathbf{M} = \begin{pmatrix} m_1 + m_2 & m_2 & m_5 \\ m_2 & m_1 - m_2 & m_5 \\ m_4 & m_4 & m_3 \end{pmatrix}.$$

where $m_{1,3} = Y_{1,3}v_3$ and $m_{1,2,4,5} = Y_{1,2,4,5}$ ($v_1$ or $v_2$)

# QUARKS

*without taking into account minimization conditions*

3HDM: $G_{SM} \otimes S_3$

| | $\psi_L^f$ | $\psi_R^f$ | Mass matrix | Possible mass textures | |
|---|---|---|---|---|---|
| $A$ | $\mathbf{2}, 1_{\mathbf{S}}$ | $\mathbf{2}, 1_{\mathbf{S}}$ | $\begin{pmatrix} \mu_1^f + \mu_2^f & \mu_4^f & \mu_6^f \\ \mu_4^f & \mu_1^f - \mu_2^f & \mu_7^f \\ \mu_8^f & \mu_9^f & \mu_3^f \end{pmatrix}$ | $\begin{pmatrix} 0 & \mu_2^f sc\left(3 - t^2\right) & 0 \\ \mu_2^f sc\left(3 - t^2\right) & -2\mu_2^f c^2\left(1 - 3t^2\right) & \mu_7^f/c \\ 0 & \mu_7^{f*}/c & \mu_3^f - \mu_1^f - \mu_2^f c^2(1 - 3t^2) \end{pmatrix}$ | |
| $A^{'}$ | | | | $\begin{pmatrix} 0 & \frac{2}{\sqrt{3}}\mu_2^f & 0 \\ \frac{2}{\sqrt{3}}\mu_2^f & 0 & \frac{2}{\sqrt{3}}\mu_7^f \\ 0 & \frac{2}{\sqrt{3}}\mu_9^f & \mu_3^f - \mu_1^f \end{pmatrix}$ | *NNI* |
| $B$ | $\mathbf{2}, 1_{\mathbf{A}}$ | $\mathbf{2}, 1_{\mathbf{A}}$ | $\begin{pmatrix} \mu_1^f + \mu_2^f & \mu_4^f & \mu_7^f \\ \mu_4^f & \mu_1^f - \mu_2^f & -\mu_6^f \\ -\mu_9^f & \mu_8^f & \mu_3^f \end{pmatrix}$ | $\begin{pmatrix} 0 & -\mu_4^f c^2\left(1 - 3t^2\right) & 0 \\ -\mu_4^f c^2\left(1 - 3t^2\right) & 2\mu_4^f sc\left(3 - t^2\right) & -\mu_6^f/c \\ 0 & -\mu_6^{f*}/c & \mu_3^f - \mu_1^f + \vert\mu_4^f sc(3 - t^2) \end{pmatrix}$ | |
| $B^{'}$ | | | | $\begin{pmatrix} 0 & -2\mu_4^f & 0 \\ -2\mu_4^f & 0 & -2\mu_6^f \\ 0 & 2\mu_8^f & \mu_3^f - \mu_1^f \end{pmatrix}$ | *NNI* |

Table 2: Mass matrices in $S_3$ family models with three Higgs $SU(2)_L$ doublets: $H_1$ and $H_2$, which occupy the $S_3$ irreducible representation $\mathbf{2}$, and $H_S$, which transforms as $1_{\mathbf{S}}$ for the cases when both the left- and right-handed fermion fields are in the same assignment. The mass matrices shown here follow a normal ordering of their mass eigenvalues $(m_1^f, m_2^f, m_3^f)$. We have denoted $s = \sin\theta$, $c = \cos\theta$ and $t = \tan\theta$. The third column of this table corresponds to the general case, while the fourth column to a case where we have rotated the matrix to a basis where the elements $(1,1)$, $(1,3)$ and $(3,1)$ vanish. The primed cases, A' or B', are particular cases of the unprimed ones, A or B, with $\theta = \pi/6$ or $\theta = \pi/3$, respectively.

*Mass matrices reproduce the NNI or the Fritzsch forms (rotation + shift)*

*F. González, AM, MM et al, Phys.Rev. D88 (2013) 096004*

# HIGGS SECTOR – TESTS FOR THE MODEL

*General Potential:*

$$
\begin{aligned}
V \;=\;& \mu_1^2\left(H_1^\dagger H_1 + H_2^\dagger H_2\right) + \mu_0^2\left(H_s^\dagger H_s\right) + a\left(H_s^\dagger H_s\right)^2 + b\left(H_s^\dagger H_s\right)\left(H_1^\dagger H_1 + H_2^\dagger H_2\right) \\
&+c\left(H_1^\dagger H_1 + H_2^\dagger H_2\right)^2 + d\left(H_1^\dagger H_2 - H_2^\dagger H_1\right)^2 + e f_{ijk}\left(\left(H_s^\dagger H_i\right)\left(H_j^\dagger H_k\right) + h.c.\right) \\
&+f\left\{\left(H_s^\dagger H_1\right)\left(H_1^\dagger H_s\right) + \left(H_s^\dagger H_2\right)\left(H_2^\dagger H_s\right)\right\} + g\left\{\left(H_1^\dagger H_1 - H_2^\dagger H_2\right)^2 + \left(H_1^\dagger H_2 + H_2^\dagger H_1\right)^2\right\} \\
&+h\left\{\left(H_s^\dagger H_1\right)\left(H_s^\dagger H_1\right) + \left(H_s^\dagger H_2\right)\left(H_s^\dagger H_2\right) + \left(H_1^\dagger H_s\right)\left(H_1^\dagger H_s\right) + \left(H_2^\dagger H_s\right)\left(H_2^\dagger H_s\right)\right\} \qquad (1)
\end{aligned}
$$

*Derman and Tsao (1979); Sugawara and Pakwasa (1978); Kubo et al (2004); Felix-Beltrán, Rodríguez-Jáuregui, M.M (2009); Das and Dey (2014), Barradas et al (2014); Costa, Ogreid, Osland and Rebelo (2016), etc*

- *The minimum of potential can be parameterised in spherical coordinates, two angles and v*
- *Minimisation fixes* $v_1^2 = 3v_2^2$
- *e = 0 massless scalar, residual continuous S2 symmetry*
- *Conditions for normal vacuum already studied, also for CP breaking ones*
  *Felix-Beltrán, Rodríguez-Jáuregui, M.M (2007); Barradas et al (2015); Costa et al (2016)*

$$v_1 = v\cos\varphi\sin\theta, \quad v_2 = v\sin\varphi\sin\theta \quad v_3 = v\cos\theta.$$

$$\tan\varphi = 1/\sqrt{3} \;\Rightarrow\; \sin\varphi = \frac{1}{2} \;\&\; \cos\varphi = \frac{\sqrt{3}}{2}$$

$$\tan\theta = \frac{2v_2}{v_3} \;\Rightarrow\; \sin\theta = \frac{2v_2}{v} \;\&\; \cos\theta = \frac{v_3}{v}$$

# STABILITY CONDITIONS

$$\lambda_8 > 0$$
$$\lambda_1 + \lambda_3 > 0$$
$$\lambda_5 > -2\sqrt{(\lambda_1+\lambda_3)\lambda_8}$$
$$\lambda_5 + \lambda_6 - 2|\lambda_7| > \sqrt{(\lambda_1+\lambda_3)\lambda_8}$$
$$\lambda_1 - \lambda_2 > 0$$
$$\lambda_1 + \lambda_3 + \mid 2\lambda_4 \mid + \lambda_5 + 2\lambda_7 + \lambda_8 > 0$$
$$\lambda_{13} > 0$$
$$\lambda_{10} > -2\sqrt{(\lambda_1+\lambda_3)\lambda_{13}}$$
$$\lambda_{10} + \lambda_{11} - 2|\lambda_{12}| > \sqrt{(\lambda_1+\lambda_3)\lambda_{13}}$$
$$\lambda_{14} > -2\sqrt{\lambda_8\lambda_{13}}.$$

Das and Dey (2014)

# UNITARITY CONDITIONS

$$a_1^{\pm} = (\lambda_1 - \lambda_2 + \frac{\lambda_5+\lambda_6}{2}) \pm \sqrt{(\lambda_1 - \lambda_2 + \frac{\lambda_5+\lambda_6}{2})^2 - 4[(\lambda_1-\lambda_2)(\frac{\lambda_5+\lambda_6}{2}) - \lambda_4^2]}$$

$$a_2^{\pm} = (\lambda_1 + \lambda_2 + 2\lambda_3 + \lambda_8) \pm \sqrt{(\lambda_1 + \lambda_2 + 2\lambda_3 + \lambda_8)^2 - 4[\lambda_8(\lambda_1+\lambda_2+2\lambda_3) - 2\lambda_7^2]}$$

$$a_3^{\pm} = (\lambda_1 - \lambda_2 + 2\lambda_3 + \lambda_8) \pm \sqrt{(\lambda_1 - \lambda_2 + 2\lambda_3 + \lambda_8)^2 - 4[\lambda_8(\lambda_1+\lambda_2+2\lambda_3) - \frac{\lambda_6^2}{2}]}$$

$$a_4^{\pm} = (\lambda_1 + \lambda_2 + \frac{\lambda_5}{2} + \lambda_7) \pm \sqrt{(\lambda_1 + \lambda_2 + \frac{\lambda_5}{2} + \lambda_7)^2 - 4[(\lambda_1-\lambda_2)(\frac{\lambda_5}{2} + \lambda_7) - \lambda_4^2]}$$

$$a_5^{\pm} = (5\lambda_1 - \lambda_2 + 2\lambda_3 + 3\lambda_8) \pm \sqrt{(5\lambda_1 - \lambda_2 + 2\lambda_3 + 3\lambda_8)^2 - 4[3\lambda_8(5\lambda_1-\lambda_2+2\lambda_3) - \frac{1}{2}(2\lambda_5+\lambda_6)^2]}$$

$$a_6^{\pm} = (\lambda_1 + \lambda_2 + 4\lambda_3 + \frac{\lambda_5}{2} + \lambda_6 + 3\lambda_7) \pm ((\lambda_1 + \lambda_2 + 4\lambda_3 + \frac{\lambda_5}{2} + \lambda_6 + 3\lambda_7)^2 - 4[(\lambda_1+\lambda_2+4\lambda_3)(\frac{\lambda_5}{2} + \lambda_6 + 3\lambda_7) - 9\lambda_4^2])^{1/2}$$

$$\begin{aligned}
b_1 &= \lambda_5 + 2\lambda_6 - \lambda_7 \\
b_2 &= \lambda_5 - 2\lambda_7 \\
b_3 &= 2(\lambda_1 - 5\lambda_1 - 2\lambda_3) \\
b_4 &= 2(\lambda_1 - \lambda_1 - 2\lambda_3) \\
b_5 &= 2(\lambda_1 + \lambda_1 - 2\lambda_3) \\
b_6 &= \lambda_5 - \lambda_6.
\end{aligned}$$

# HIGGS MASSES

➤ After electroweak symmetry breaking (Higgs mechanism) we are left with 9 massive particles

*doesn't couple to gauge bosons: Z2 symmetry*
*massless when e=0, S2 symmetry*

$$m^2_{h_0} = -9ev^2 \sin\theta\cos\theta$$

$$m^2_{H_1,H_2} = (M_a^2 + M_c^2) \pm \sqrt{(M_a^2 - M_c^2)^2 + (M_b^2)^2}$$

$$\begin{aligned} M_a^2 &= \left[2(c+g)v^2\sin^2\theta + \frac{3}{2}ev^2\sin\theta\cos\theta\right] \\ M_b^2 &= \left[3ev^2\sin^2\theta + 2(b+f+2h)v^2\sin\theta\cos\theta\right] \\ M_c^2 &= 2av^2\cos^2\theta - \frac{ev^2\tan\theta\sin^2\theta}{2} \end{aligned}$$

*H1 or H2 can be the SM Higgs boson*

$$\begin{aligned} m^2_{A_1} &= -v^2\left[2(d+g)\sin^2\theta + 5e\cos\theta\sin\theta + 2h\cos^2\theta\right] \\ m^2_{A_2} &= -v^2(e\tan\theta + 2h) \end{aligned}$$

*Das and Dey (2014)*
*Barradas,Félix, González (2014)*
*Gómez-Bock, MM, Perez-Martínez (2022)*

$$\begin{aligned} m^2_{H_1^\pm} &= -v^2\left[5e\sin\theta\cos\theta + (f+h)\cos^2\theta + 2g\sin^2\theta\right] \\ m^2_{H_2^\pm} &= -v^2\left[e\tan\theta + (f+h)\right] \end{aligned}$$

# RESIDUAL Z2 SYMMETRY

- After eW symmetry breaking, S3 breaks -> residual Z2 symmetry
  Das and Dey (2014), Ivanov (2017)
- h0 decoupled from gauge bosons
- There are 2 "alignment" limits 👀
  - H2 is the SM Higgs → H1 decoupled from gauge bosons
  - H1 is the SM Higgs → H2 decoupled from gauge bosons
    mH2 < mH1
- Z2 parity:

  $h_0$, $A_1$, $H_1^{\pm}$ parity -1,
  $H_1$, $H_2$ parity +1
  $H_2^{\pm}$, $A_2$ parity +1

  Das and Dey (2014)
- This forbids certain couplings

# MASSES — TREE LEVEL — ALIGNMENT LIMITS

- Scenario A, H2 SM Higgs
  - Upper bound for masses
    mh0 ≲ 900 GeV , mH1 ≲ 3 TeV
    mA1 ≲ 1 TeV, mA2 ≲ 3 TeV
    mH1 ≲ 1 TeV, mH2 ≲ 3 TeV
  - Taking $(\alpha\text{-}\theta)$ 1% lowers mH1, mA2, MH2 ≲ 1 TeV
- **Allows for a neutral scalar lighter than SM Higgs**
  **h0 in this case**
- Some of scalar masses are almost degenerate → good for oblique parameters

*Gómez-Bock, MM, Perez-Martínez (2022)*

# EXACT ALIGNMENT LIMIT A

- In the exact alignment limit A (SM Higgs the lightest scalar)

$$\sin(\alpha - \theta) = 1,\ \cos(\alpha - \theta) = 0,$$

- "Our" SM Higgs trilinear and quartic couplings reduce exactly to SM ones

$$g_{H_2H_2H_2} = \frac{1}{v\, s_{2\theta}}\left[m_{H_2}^2 s_\alpha s_\theta\right] = \frac{1}{2v}\frac{s_\alpha}{c_\theta} m_{H_2}^2 = \frac{m_{H_2}^2}{2v} \equiv \lambda_{SM}.$$

$$g_{H_1H_1H_1} = \frac{1}{v\, s_{2\theta}}\left[\frac{1}{9c_\theta^2} m_{h_0}^2 - s_\theta^2 m_{H_1}^2\right] = \frac{1}{v\, s_{2\theta} c_\theta^2}\left[\frac{1}{9} m_{h_0}^2 - \frac{1}{2} s_{2\theta} m_{H_1}^2\right].$$

$$g_{H_2H_2H_2H_2} = \frac{1}{2v^2 s_{2\theta}^2} m_{H_2}^2 (-s_\theta^3 c_\theta - c_\theta^3 s_\theta)^2 = \frac{m_{H_2}^2}{8v^2}.$$

$$g_{H_2H_2h_0h_0} = \frac{1}{v^2 s_{2\theta}}\left(\frac{1}{6} m_{h_0}^2 3 s_{2\theta} + \frac{1}{4} m_{H_2}^2 s_{2\theta}\right) = \frac{1}{4v^2}(2m_{h_0}^2 + m_{H_2}^2)\ .$$

# CONSTRAINTS ON SCALARS

- Constraints are imposed over the parameter space:
  - Vacuum stability and unitarity conditions
  - SM Higgs boson mass within 125 ± 3 GeV
- We recover SM Higgs boson properties, trilinear and quartic couplings are the same, extra heavier scalars, bounded from above and below
- BUT residual Z2 symmetry: 😵

$$M_q = \begin{pmatrix} x & 0 & 0 \\ 0 & x & x \\ 0 & x & x \end{pmatrix}$$

# ALIGNMENT NOT EXACT — LIMITS ON PARAMETERS

- Higgs-gauge couplings have been determined with 5% precision → $\kappa_\lambda$ scaling factor
- -1.8 < $\kappa_\lambda$ < 9.2

Degrassi, Di Micco, Giardino, Rossi (2021)

- If the alignment limit is not exact we can parameterize deviations from SM

$$g_{H_2H_2H_2} \equiv \lambda_{SM}\kappa_\lambda \; = \; \frac{m_{H_2}^2}{2v}\left[(1+2\delta^2)\sqrt{1-\delta^2}+\delta^3(\tan\theta-\cot\theta)-\frac{m_{h_0}^2}{m_{H_2}^2}\frac{\delta^3}{9s_\theta c_\theta^3}\right]$$

$$\cos(\alpha-\theta)=\cos(\frac{\pi}{2}-\epsilon)=\sin\epsilon\equiv\delta,$$

- The max value for $m_{h0}$ sets constraints on tan$\theta$

  e.g. for $\delta \sim 0.1$ → tan$\theta \leq 15$

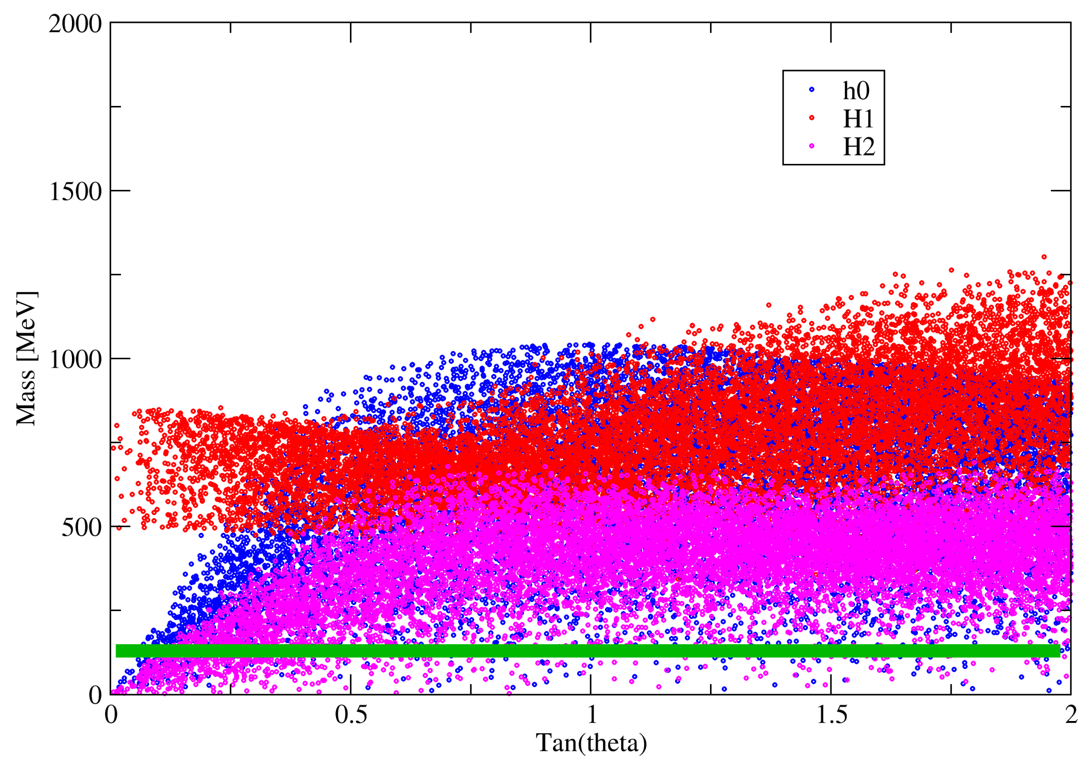
*Gómez-Bock, MM, Perez-Martínez (2022)*

# 4HDM –S3 WITH DM

- We add another doublet, inert, to have a DM candidate. We assign it to the $1^A$, and thus "saturate" the irreps
- First two generations in a flavour doublet, third in a singlet, extra anti-symmetric singlet is inert → DM candidates
- A lot of Higgses (13), but the good features of 3H-S3 remain
  Quark and lepton sectors remain unchanged
  DM candidate in inert sector
- Add a Z2 symmetry to prevent the DM candidate to decay
- S3 symmetry constrains strongly the allowed couplings

C. Espinoza, E. Garcés, M.M., H. Reyes (2019)

# NEUTRAL SCALAR MASSES




*S3-3H Neutral scalar masses with stability and unitarity bounds only*

*Pink will be constrained to be SM Higgs*

*Red neutral H1*

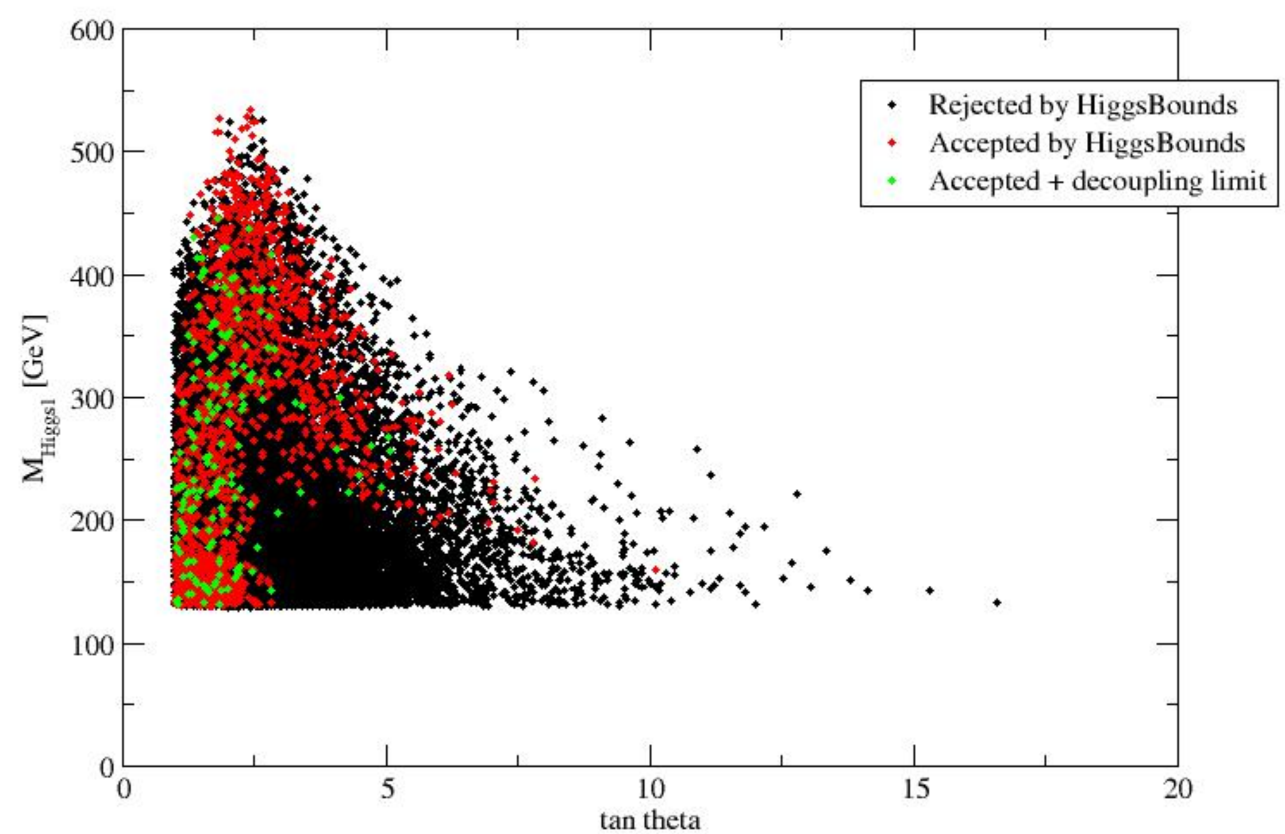
*Blue h0 decoupled from gauge bosons*

*S3-4H*

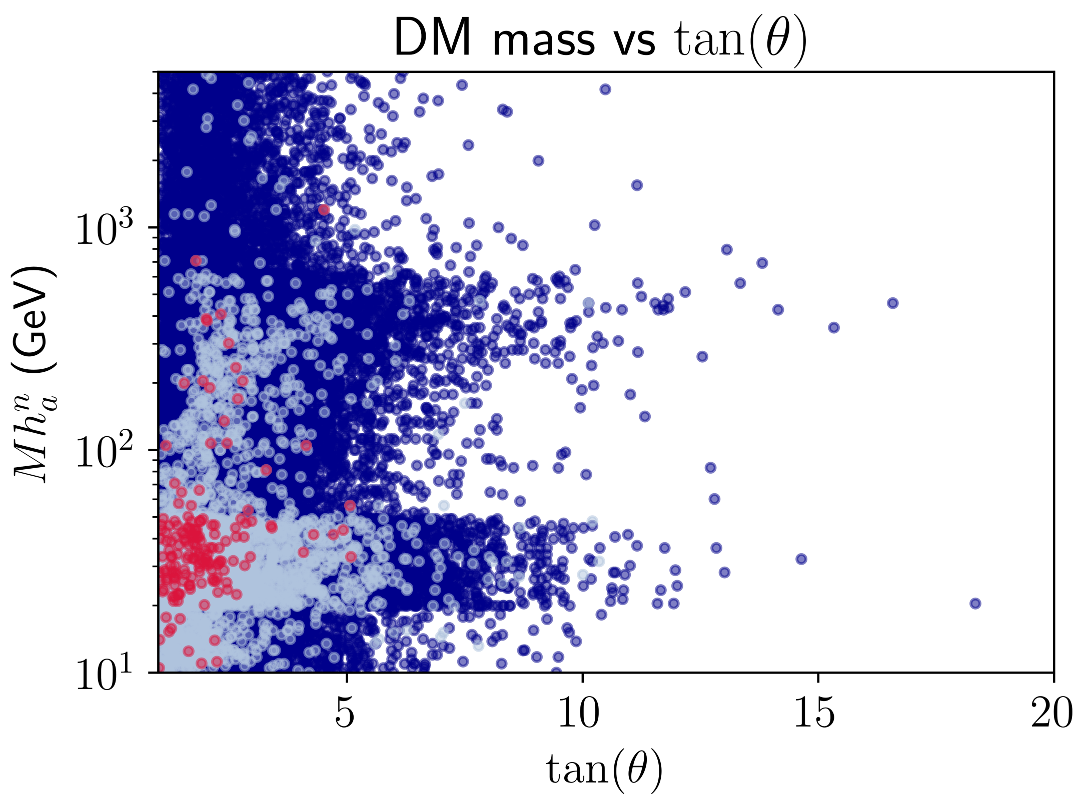
*H2 constrained to be SM-H*

*Shown H1 vs tan $\theta$*

*Green passes unitarity, stability and* HiggsBounds + *decoupling limit* $\Longrightarrow$ *small tan $\theta$*



# DM MASS AND RELIC DENSITY — S3-4H

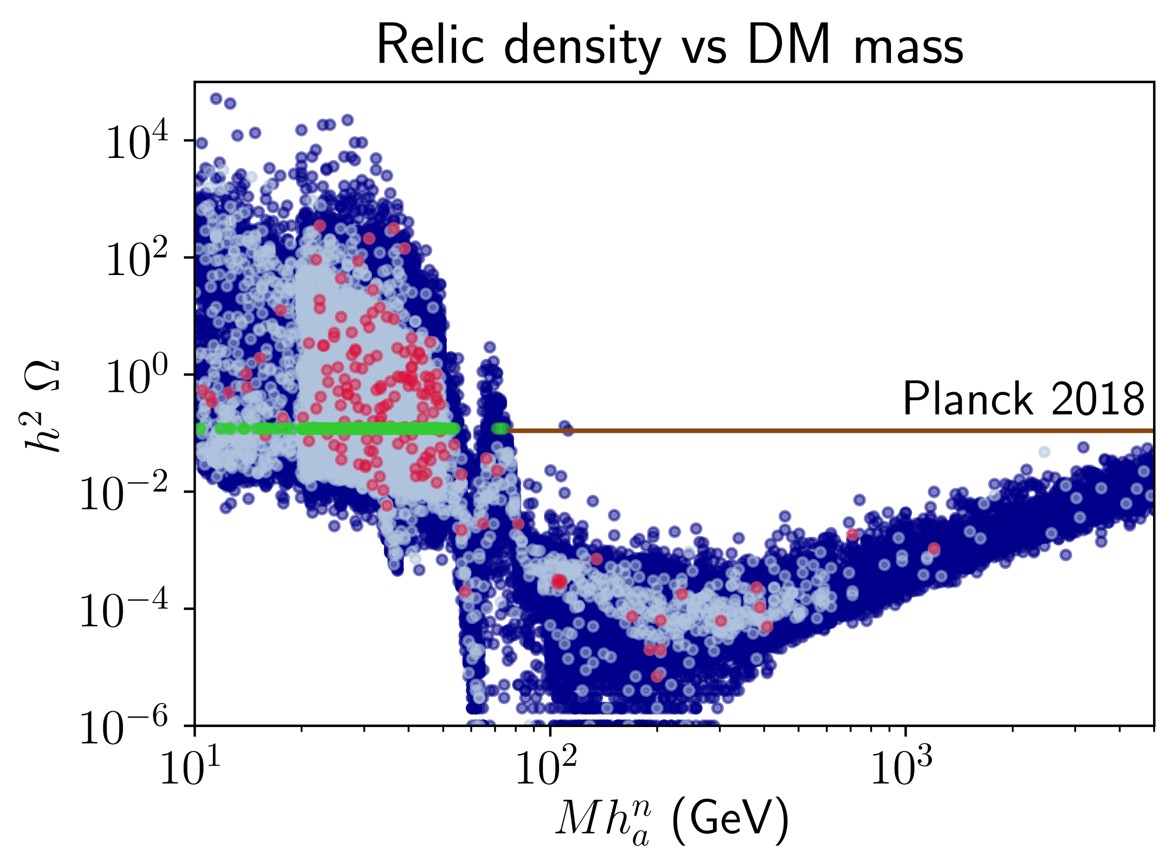



Blue points → stability and unitarity

Light blue → also Higgs bounds

Red points → also alignment limit

*The bounds apply to S3-3H too*




Green points → DM Planck limits

Small values of tanθ preferred

# IN YUKAWA SECTOR

- The Higgs Z2 symmetry will lead to zeroes in the CKM and PMNS matrices 😱 Das, Dey, Pal (2015), Ivanov (2017)
- To recover the good features of the symmetry:
  - Add S3 singlet Brown, Deshpande,Sugawara, Pakwasa (1984)
  - Break very softly the S3 symmetry with mass terms, recover original structure e.g., Kubo, Okada, Sakamaki (2004), Das, Dey, Pal (2015)
  - Consider CP violation Costa, Ogreid, Osland, Rebelo(2014,2021)
  - Higher order interactions
  - Second B-L sector at high scale with small interaction

    Gómez-Izquierdo, MM (2018)
- Combinations of the above: all introduce more parameters

*Perform a little experiment…*

# MAKE IT MODULAR

*will it help?*

# MODULAR SYMMETRIES

- Using modular symmetries as flavour symmetries:
  Inspiration from supersymmetric theories, initially with extra dimensions

  Feruglio, Altarelli (2006-2022); Petcov et al (2019, 2021, 2022); …

  Magnetized branes, magnetized tori, superstring theories

  Cremades et al (2004); Kobayashi et al (2018); Almumin et al (2022);…

  Superstring compactifications, especially from orbifold compactifications

  e.g. Kobayashi et al (2018, 2019); Chen, Ramos-Sánchez, Ratz (2022); …

- Usually applied in supersymmetric models, but also possible in non-susy settings

  Nomura, Okada et al, (2019,2020); Review M. Ratz (2024)

# MODULAR GROUPS AS FLAVOUR GROUPS

- Isomorphism between some finite modular groups and some groups associated to polygons (invariance under rotations and reflections)

$$\begin{aligned} \Gamma_2 &\simeq S_3 \\ \Gamma_3 &\simeq A_4 \\ \Gamma_4 &\simeq S_4 \\ \Gamma_5 &\simeq A_5 \end{aligned}$$

- Yukawa couplings expressed in terms of modular forms, i.e. functions of a complex scalar field

$$Y(\alpha, \beta, \gamma|\tau) = \frac{d}{d\tau}\left(\alpha \log \eta\left(\frac{\tau}{2}\right) + \beta \log \eta\left(\frac{\tau+1}{2}\right) + \gamma \log \eta\,(2\tau)\right)$$

  with $\tau$ acquiring a vev on the upper half of complex plane

- Fermions and scalar fields transform with a weight

$$\phi \to (c\tau + d)^{k_\phi}\phi,$$

# S3 MODULAR SYMMETRY

- We will impose a modular S3 or $\Gamma_2$ to a non-supersymmetric Lagrangian

$$SU(3)_C \times SU_L(2) \times U_y(1) \times \Gamma_2$$

3HDM, 3 $\nu_R$, quarks and leptons:

first two generations in a doublet
third generation in a singlet

same for 3 Higgses: 2 of them in a doublet, third in a singlet

- We assign specific modular weights (again, some liberty there…) to get a good texture

- Weight of matter fields, together with modular forms (couplings) has to be zero

# THE ASSIGNMENTS FOR THE MODEL

- We assign the fields the following weights

| | $(Q_1, Q_2)$ | $(q_1, q_2)$ | $Q_3$ | $q_3$ | $(H_1, H_2)$ | $H_s$ | $(Y_1^{(2,4)}(\tau), Y_2^{(2,4)}(\tau))$ | $Y_s^{(4)}(\tau)$ |
|---|---|---|---|---|---|---|---|---|
| $SU(2)$ | 2 | 1 | 2 | 1 | 2 | 2 | 1 | 1 |
| $S_3$ | 2 | 2 | 1 | 1 | 2 | 1 | 2 | 1 |
| $k$ | $-2$ | $-2$ | 0 | 0 | 0 | 0 | $(2,4)$ | 4 |

Table 2: charges, assignments, and modular weights of $SU(2)$ and $S_3$. The superscript $(2,4)$ on the modular forms indicates that they are of modular weight 2 or 4. The subscript $s$ indicates the symmetric singlet of the modular form of weight 4.

- The Yukawa part of the Lagrangian is

$$\begin{aligned}\mathcal{L}_y^{(u)} &= C_1\overline{Q}\otimes u\otimes\tilde{H}\otimes Y^{(4)} + C_2\overline{Q}\otimes u\otimes\tilde{H}\otimes Y_s^{(4)} + C_3\overline{Q}\otimes u\otimes\tilde{H}_s\otimes Y^{(4)} \\ &+ C_4\overline{Q}\otimes u\otimes\tilde{H}_s\otimes Y_s^{(4)} + C_5\overline{Q}\otimes u_{3R}\otimes\tilde{H}\otimes Y^{(2)} + C_6\overline{Q}\otimes u_{3R}\otimes\tilde{H}_s\otimes Y^{(2)} \\ &+ C_7\overline{Q}_3\otimes u\otimes\tilde{H}\otimes Y^{(2)} + C_8\overline{Q}_3\otimes u\otimes\tilde{H}_s\otimes Y^{(2)} + C_9\overline{Q}_3\otimes u_{3R}\otimes\tilde{H}_s + \text{h.c.}\end{aligned}$$

# ELEMENTS OF MASS MATRIX

- The elements of the quark mass matrix are now

$$
\begin{aligned}
M_{11}^{(u)} &= (\alpha+\gamma)v_1 Y_1^{(4)} + (\alpha-\gamma)v_2 Y_2^{(4)} + C_2 v_2 Y_s^{(4)} + C_3 v_s Y_2^{(4)} + C_4 v_s Y_s^{(4)} \\
M_{12}^{(u)} &= (\beta+\gamma)v_2 Y_1^{(4)} + (\gamma-\beta)v_1 Y_2^{(4)} + C_2 v_1 Y_s^{(4)} + C_3 v_s Y_1^{(4)} \\
M_{13}^{(u)} &= C_5(v_2 Y_1^{(2)} + v_1 Y_2^{(2)}) + C_6 v_s Y_1^{(2)} \\
M_{21}^{(u)} &= (\beta+\gamma)v_1 Y_2^{(4)} + (\gamma-\beta)v_2 Y_1^{(4)} + C_2 v_1 Y_s^{(4)} + C_3 v_s Y_1^{(4)} \\
M_{22}^{(u)} &= (\alpha+\gamma)v_2 Y_2^{(4)} + (\alpha-\gamma)v_1 Y_1^{(4)} - C_2 v_2 Y_s^{(4)} - C_3 v_s Y_2^{(4)} + C_4 v_s Y_s^{(4)} \\
M_{23}^{(u)} &= C_5(v_1 Y_1^{(2)} - v_2 Y_2^{(2)}) + C_6 v_s Y_2^{(2)} \\
M_{31}^{(u)} &= C_7(v_2 Y_1^{(2)} + v_1 Y_2^{(2)}) + C_8 v_s Y_1^{(2)} \\
M_{32}^{(u)} &= C_7(v_1 Y_1^{(2)} - v_2 Y_2^{(2)}) + C_8 v_s Y_2^{(2)} \\
M_{33}^{(u)} &= C_9 v_s,
\end{aligned}
$$

*Lots of free parameters!!* $\alpha,\ \beta,\ \gamma,\ v_2,\quad C_2,\ C_3,\ C_4,\ C_5,\ C_6,\ C_7,\ C_8,\ C_9$ y $\tau$

# WHAT CAN WE DO?

- A lot of freedom!  too many parameters…
- Can we do something about it?
- But, look at the symmetries — geometry, of the problem
- In the symmetry points parameters are identified or related: only few parameters remain
- This way: possible to explain mixings, S4 and A5 studied

  Novichkov, Penedo, Petcov, Titov;(2019-2022, 2024)
- S3 studied too, but so far without exploiting these symmetric points    Kobayashi et al (2019,2020)
- In our analysis, interplay between minimization of scalar potential and symmetric modular points crucial

# MODULAR SYMMETRIC POINTS

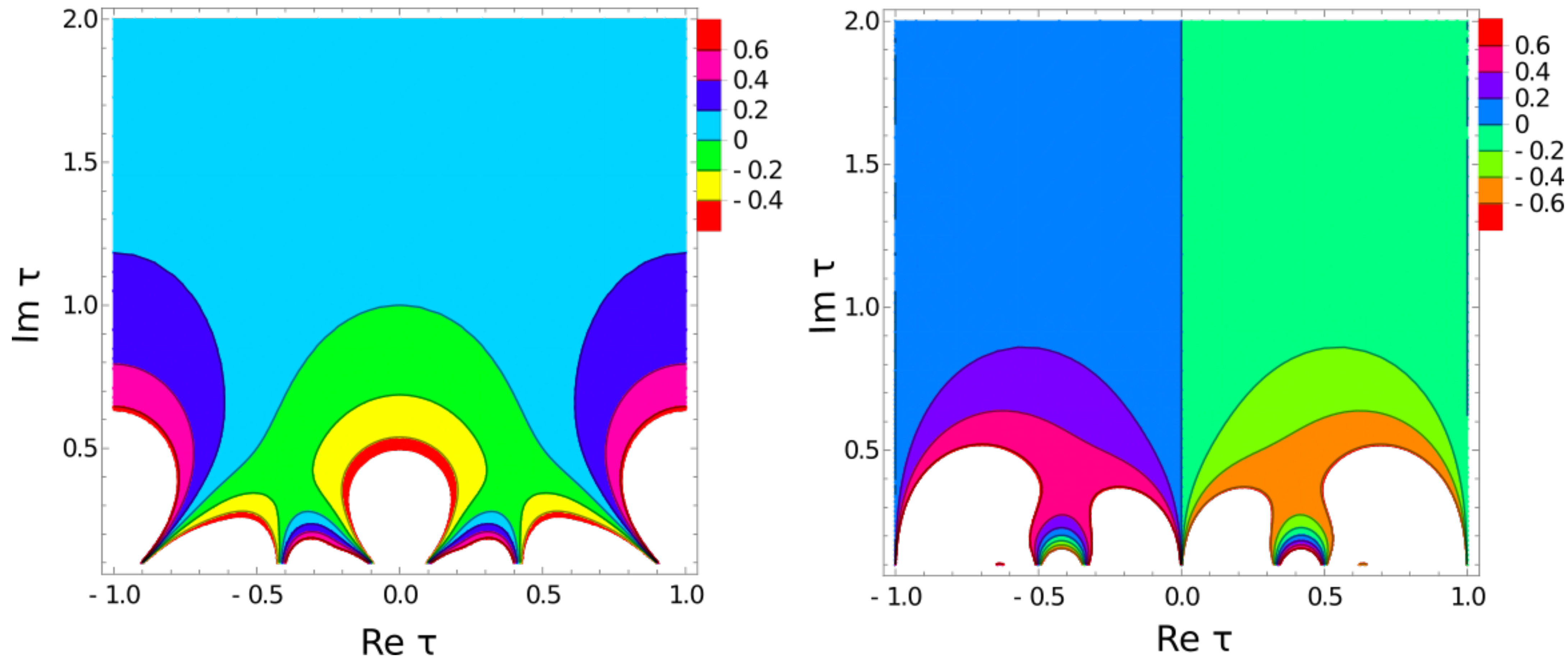



Figure 3: Real (left) and imaginary (right) part of the given expression in $M_{13}$ y $M_{31}$, that is, $Y_2^{(2)}(\tau) - \sqrt{3}Y_1^{(2)}(\tau)$. It is observed that $Y_2^{(2)}(\tau) - \sqrt{3}Y_1^{(2)}(\tau) = 0$, for both its real and imaginary parts, at the point $\tau = i$, which guarantees that $M_{13} = M_{31} = 0$.

# LAGRANGIAN AND FREE PARAMETERS SO FAR

- We want a matrix of the form, which is known to reproduce the VCKM (not every symmetry leads to this form)

$$\begin{pmatrix} 0 & a & 0 \\ a^* & b & c \\ 0 & c^* & d \end{pmatrix}$$

- Conditions on parameters:
- Minimisation condition $v_1^2 = 3v_2^2$
- Evaluate $\tau$ in the modular symmetric points

$$Y_2^{(2)}(\tau) - \sqrt{3}Y_1^{(2)}(\tau) = 0, \qquad \tau = i$$

# REPARAMETERIZATION

- Rewrite mass matrices in polar form, real matrix multiplied by phase matrix
- Use three matrix invariants: trace, determinant, and the trace of the square matrix

$$\bar{M}^{(u)} = \begin{pmatrix} 0 & |C| & 0 \\ |C| & C'_4 & |C'_5| \\ 0 & |C'_5| & C'_9 \end{pmatrix}$$

$$|C| = \sqrt{\frac{\widetilde{\sigma}_1 \widetilde{\sigma}_2}{C'_9}}$$

$$\widetilde{\sigma}_i = m_i / m_3$$

$$C'_4 = (\widetilde{\sigma}_1 - \widetilde{\sigma}_2 + 1 - C'_9)$$

$$P_f = diag(1, e^{i\phi_1}, e^{i(\phi_1 - \phi_2)})$$

$$|C'_5| = \sqrt{\frac{(1 - C'_9)(C'_9 - \widetilde{\sigma}_1)(C'_9 + \widetilde{\sigma}_2)}{C'_9}}$$

$C'_{9u}$, $C'_{9d}$ , $\phi_{1u}$,$\phi_{2u}$, $\phi_{1d}$ and $\phi_{2d}$.

# $V_{CKM}$ MATRIX

- Assuming the NNI form and a hierarchical structure for the mass matrices u and d, we can reparameterize them in terms of mass ratios $\widetilde{\sigma}_i = m_i/m_3$

F. González, A. Mondragón, M. Mondragón et al, (2013;) J. Barranco, F. González, A. Mondragón (2010)

- Exact analytical expression for the $V_{CKM}$ corresponding to the symmetry S3 with the NNI structure
- Without loss of generality we can fix the values of 2 phases

$$\phi_{1d} = \phi_{2d} = 0$$

- Now only 4 free parameters to fit the $V_{CKM}$
- We perform a $\chi^2$ analysis to find the numerical values of our parameters

$$V_{ud}^{th} = \sqrt{\frac{\widetilde{\sigma}_c\widetilde{\sigma}_s\xi_1^u\xi_1^d}{\mathcal{D}_{1u}\mathcal{D}_{1d}}} + \sqrt{\frac{\widetilde{\sigma}_u\widetilde{\sigma}_d}{\mathcal{D}_{1u}\mathcal{D}_{1d}}}\left(\sqrt{(1-\delta_u)\,(1-\delta_d)\,\xi_1^u\xi_1^d} + \sqrt{\delta_u\delta_d\xi_2^u\xi_2^d}e^{i\phi_2}\right)e^{i\phi_1},$$

$$V_{us}^{th} = -\sqrt{\frac{\widetilde{\sigma}_c\widetilde{\sigma}_d\xi_1^u\xi_2^d}{\mathcal{D}_{1u}\mathcal{D}_{2d}}} + \sqrt{\frac{\widetilde{\sigma}_u\widetilde{\sigma}_s}{\mathcal{D}_{1u}\mathcal{D}_{2d}}}\left(\sqrt{(1-\delta_u)\,(1-\delta_d)\,\xi_1^u\xi_2^d} + \sqrt{\delta_u\delta_d\xi_2^u\xi_1^d}e^{i\phi_2}\right)e^{i\phi_1},$$

$$V_{ub}^{th} = \sqrt{\frac{\widetilde{\sigma}_c\widetilde{\sigma}_d\widetilde{\sigma}_s\delta_d\xi_1^u}{\mathcal{D}_{1u}\mathcal{D}_{3d}}} + \sqrt{\frac{\widetilde{\sigma}_u}{\mathcal{D}_{1u}\mathcal{D}_{3d}}}\left(\sqrt{(1-\delta_u)\,(1-\delta_d)\,\delta_d\xi_1^u} - \sqrt{\delta_u\xi_2^u\xi_1^d\xi_2^d}e^{i\phi_2}\right)e^{i\phi_1},$$

$$V_{cd}^{th} = -\sqrt{\frac{\widetilde{\sigma}_u\widetilde{\sigma}_s\xi_2^u\xi_1^d}{\mathcal{D}_{2u}\mathcal{D}_{1d}}} + \sqrt{\frac{\widetilde{\sigma}_c\widetilde{\sigma}_d}{\mathcal{D}_{2u}\mathcal{D}_{1d}}}\left(\sqrt{(1-\delta_u)\,(1-\delta_d)\,\xi_2^u\xi_1^d} + \sqrt{\delta_u\delta_d\xi_1^u\xi_2^d}e^{i\phi_2}\right)e^{i\phi_1},$$

$$V_{cs}^{th} = \sqrt{\frac{\widetilde{\sigma}_u\widetilde{\sigma}_d\xi_2^u\xi_2^d}{\mathcal{D}_{2u}\mathcal{D}_{2d}}} + \sqrt{\frac{\widetilde{\sigma}_c\widetilde{\sigma}_s}{\mathcal{D}_{2u}\mathcal{D}_{2d}}}\left(\sqrt{(1-\delta_u)\,(1-\delta_d)\,\xi_2^u\xi_2^d} + \sqrt{\delta_u\delta_d\xi_1^u\xi_1^d}e^{i\phi_2}\right)e^{i\phi_1},$$

$$V_{cb}^{th} = -\sqrt{\frac{\widetilde{\sigma}_u\widetilde{\sigma}_d\widetilde{\sigma}_s\delta_d\xi_2^u}{\mathcal{D}_{2u}\mathcal{D}_{3d}}} + \sqrt{\frac{\widetilde{\sigma}_c}{\mathcal{D}_{2u}\mathcal{D}_{3d}}}\left(\sqrt{(1-\delta_u)\,(1-\delta_d)\,\delta_d\xi_2^u} - \sqrt{\delta_u\xi_1^u\xi_1^d\xi_2^d}e^{i\phi_2}\right)e^{i\phi_1},$$

$$V_{td}^{th} = \sqrt{\frac{\widetilde{\sigma}_u\widetilde{\sigma}_c\widetilde{\sigma}_s\delta_u\xi_1^d}{\mathcal{D}_{3u}\mathcal{D}_{1d}}} + \sqrt{\frac{\widetilde{\sigma}_d}{\mathcal{D}_{3u}\mathcal{D}_{1d}}}\left(\sqrt{\delta_u\,(1-\delta_u)\,(1-\delta_d)\,\xi_1^d} - \sqrt{\delta_d\xi_1^u\xi_2^u\xi_2^d}e^{i\phi_2}\right)e^{i\phi_1},$$

$$V_{ts}^{th} = -\sqrt{\frac{\widetilde{\sigma}_u\widetilde{\sigma}_c\widetilde{\sigma}_d\delta_u\xi_2^d}{\mathcal{D}_{3u}\mathcal{D}_{2d}}} + \sqrt{\frac{\widetilde{\sigma}_s}{\mathcal{D}_{3u}\mathcal{D}_{2d}}}\left(\sqrt{\delta_u\,(1-\delta_u)\,(1-\delta_d)\,\xi_2^d} - \sqrt{\delta_d\xi_1^u\xi_2^u\xi_1^d}e^{i\phi_2}\right)e^{i\phi_1},$$

$$V_{tb}^{th} = \sqrt{\frac{\widetilde{\sigma}_u\widetilde{\sigma}_c\widetilde{\sigma}_d\widetilde{\sigma}_s\delta_u\delta_d}{\mathcal{D}_{3u}\mathcal{D}_{3d}}} + \left(\sqrt{\frac{\xi_1^u\xi_2^u\xi_1^d\xi_2^d}{\mathcal{D}_{3u}\mathcal{D}_{3d}}} + \sqrt{\frac{\delta_u\delta_d(1-\delta_u)(1-\delta_d)}{\mathcal{D}_{3u}D_{3d}}}e^{i\phi_2}\right)e^{i\phi_1}\ .$$

$$\begin{aligned}
\delta_{u,d} &= 1 - C'_{9u,d} \\
\xi_1^{u,d} &= 1 - \widetilde{\sigma}_{u,d} - \delta_{u,d}, \\
\xi_2^{u,d} &= 1 + \widetilde{\sigma}_{c,s} - \delta_{u,d}, \\
\mathcal{D}_{1(u,d)} &= (1-\delta_{u,d})(\widetilde{\sigma}_{u,d} + \widetilde{\sigma}_{c,s})(1-\widetilde{\sigma}_{u,d}), \\
\mathcal{D}_{2(u,d)} &= (1-\delta_{u,d})(\widetilde{\sigma}_{u,d} + \widetilde{\sigma}_{c,s})(1+\widetilde{\sigma}_{c,s}), \\
\mathcal{D}_{3(u,d)} &= (1-\delta_{u,d})(1-\widetilde{\sigma}_{u,d})(1+\widetilde{\sigma}_{c,s}).
\end{aligned}$$

# VCKM FIT

- We have 4 parameters to fit a 3x3 unitary matrix, constructed to fit
- Analytical expression successful, comes from **symmetry**

| | Center value and error |
|---|---|
| $\widetilde{\sigma}_u$ | $7.032 \times 10^{-6}$ |
| $\widetilde{\sigma}_d$ | $9.44 \times 10^{-4}$ |
| $\widetilde{\sigma}_s$ | $0.0190 \pm 0.00046$ |
| $\widetilde{\sigma}_c$ | $0.00375 \pm 0.00023$ |

| | Values in the fit |
|---|---|
| $C'_{9u}$ | 0.816393 |
| $C'_{9d}$ | 0.828604 |
| $\phi_{1u}$ | 1.63797 |
| $\phi_{1d}$ | 0 |
| $\phi_{2u}$ | 0.0981477 |
| $\phi_{2d}$ | 0 |
| $\chi^2$ | 0.00070 |

👍👍

$$V_{CKM}^{th} = \begin{pmatrix} 0.97435 & 0.2250 & 0.00369 \\ 0.22486 & 0.97349 & 0.04182 \\ 0.00857 & 0.04110 & 0.999118 \end{pmatrix}$$

$$\mathcal{J}^{th} = 3.07 \times 10^{-5}.$$

# GOING UP?

- Possible to have a modular S3 with SU(5) SUSY GUT, 3 pairs of Higgs doublets

  Antonio C. Samaniego, M.Sc. Thesis (2022), work in progress

- You can embed the model (or a version of it, not modular) in a SUSY model with Q6 symmetry

- Grand Unified SU(5) x Q6 model already studied, preserves the nice features of S3 in quarks and leptons. Mixing angles in good agreement with experiment, both hierarchies allowed. J.C. Gómez-Izquierdo, F. González-Canales, M.M. (2014)

  Neutrino masses: add singlets or non-renormalizable interactions or radiatively

- Possible to have different assignments of Q6 in leptonic sector

  $\Longrightarrow$ breaking of mu-tau symmetry J.C. Gómez-Izquierdo, M.M. (2017)

- Flavour structure in trilinear soft SUSY breaking terms $\rightarrow$

  LFV $\tau\mu \rightarrow \gamma$, g-2 contributions through LFV in leptonic sector

  F. Flores-Báez, M. Gómez-Bock, M.M. (2018)

- Non-SUSY B-L model with S3, also breaking of mu-tau symmetry and DM

  J.C. Gómez-Izquierdo, M.M. (2019), Lucía Gutiérrez, Ph.D. Thesis

- Q4-2HDM with lots of singlets connecting with DM, leptogenesis and g-2

  A. Cárcamo, C. Espinoza, J.C. Gómez-Izquierdo, MM (2023)

# RECAP

- Flavour problem: one of the most important open problems in HEP
- Has served as guidance for discoveries
- Far reaching consequences in particles and astroparticle physics
  work them out!
- Flavour symmetries:
  - Might give insight into what lies ahead,
    either top-down or bottom up
  - Important to look both at **fermionic** and **scalar sector simultaneously** (surprises, pleasant and not, might appear)
- Where do the Yukawa couplings come from? Why those?

# THANKS!