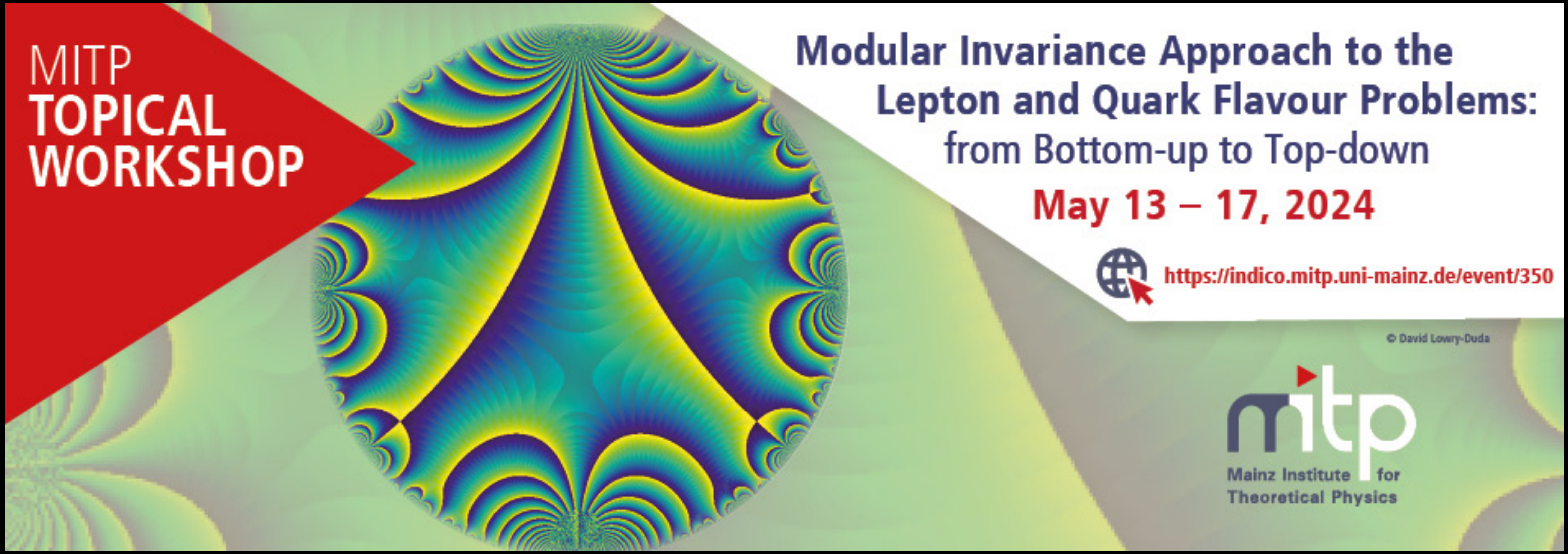
UNIVERSITY OF Southampton
School of Physics and Astronomy

HIDDeν
Hunting Invisibles: Dark sectors, Dark matter and Neutrinos

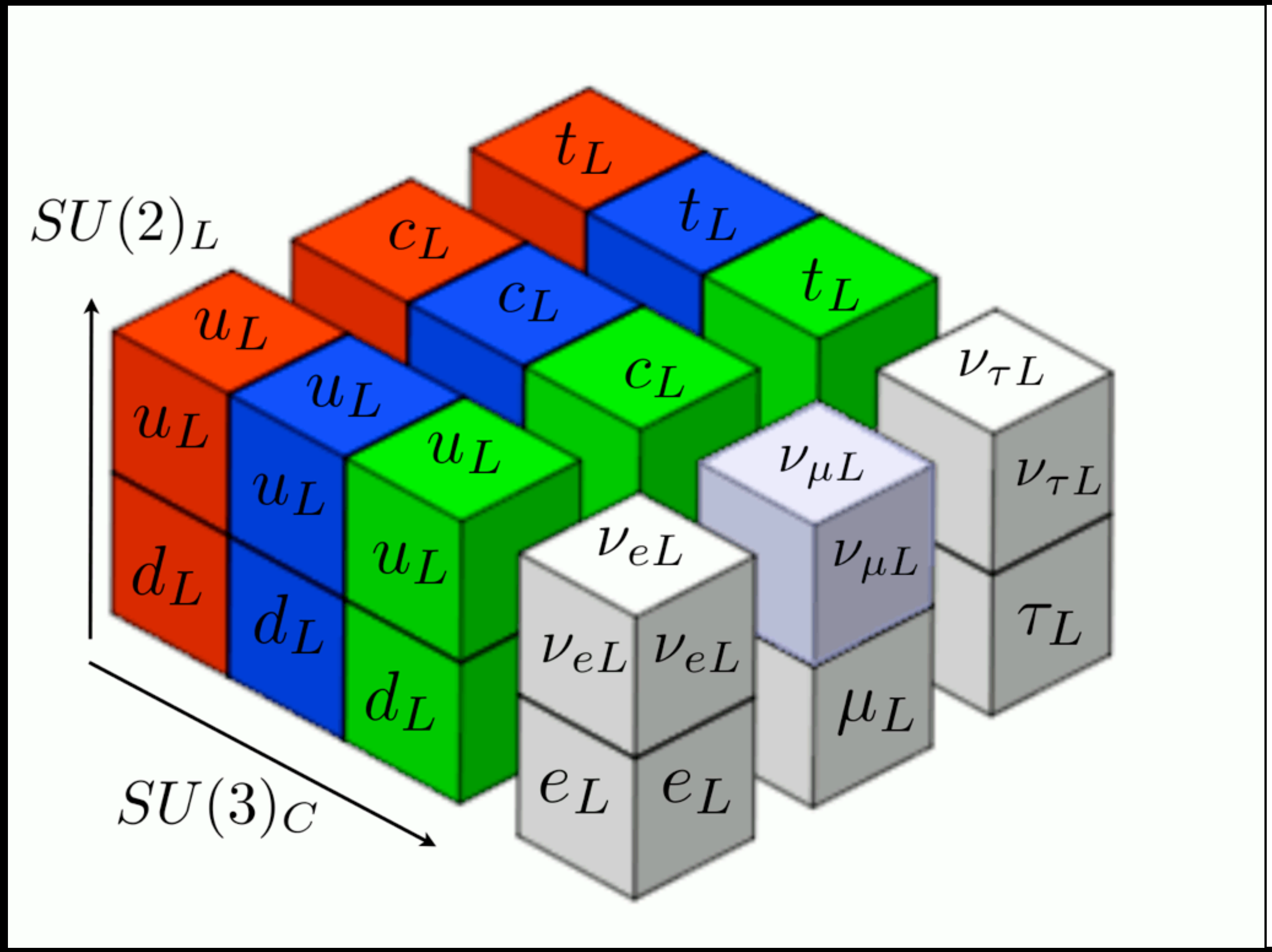
Flavour Model Building I

Steve King, 13th May 2024, Mainz

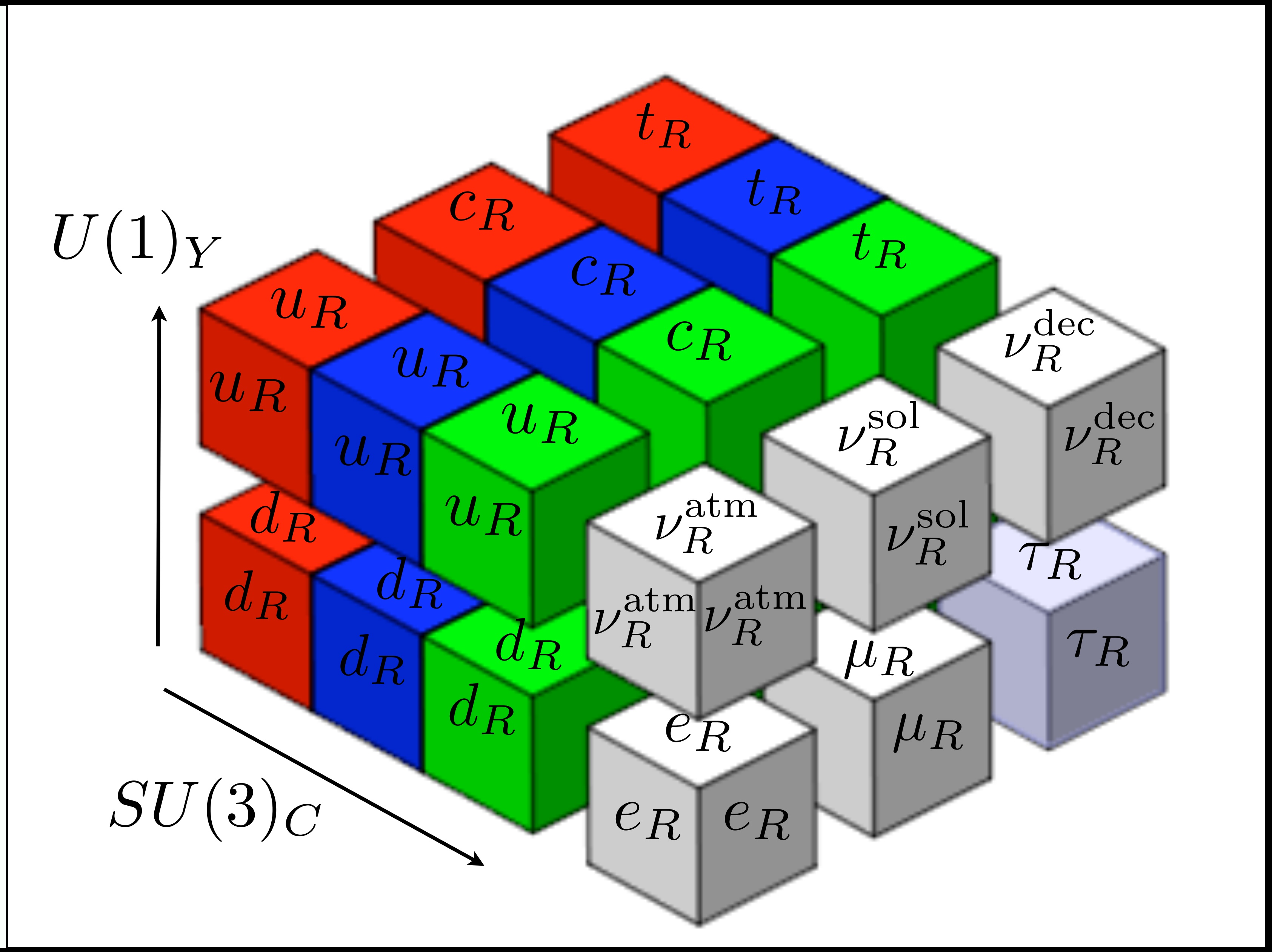


The Standard Model

Left-handed

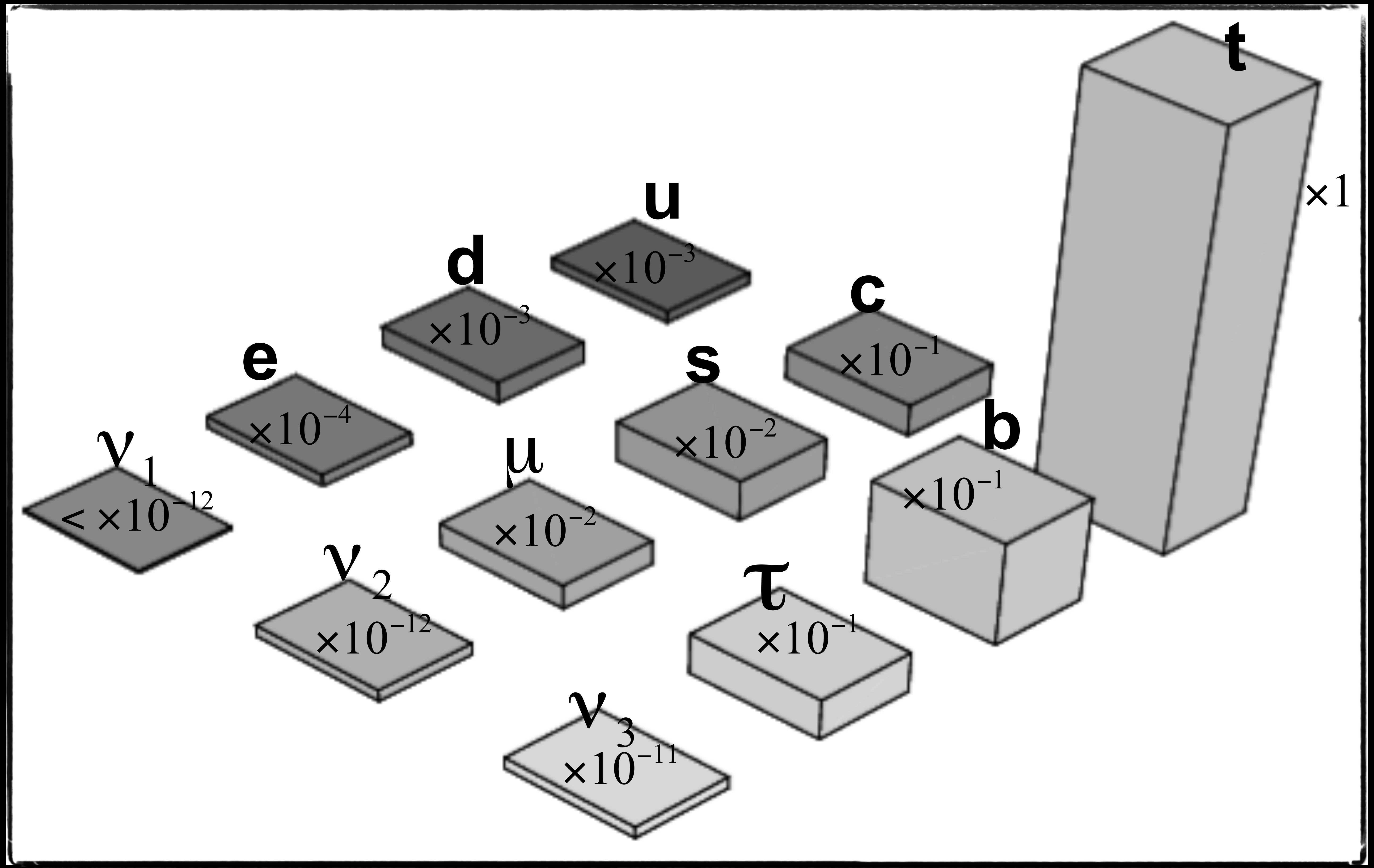


Right-handed

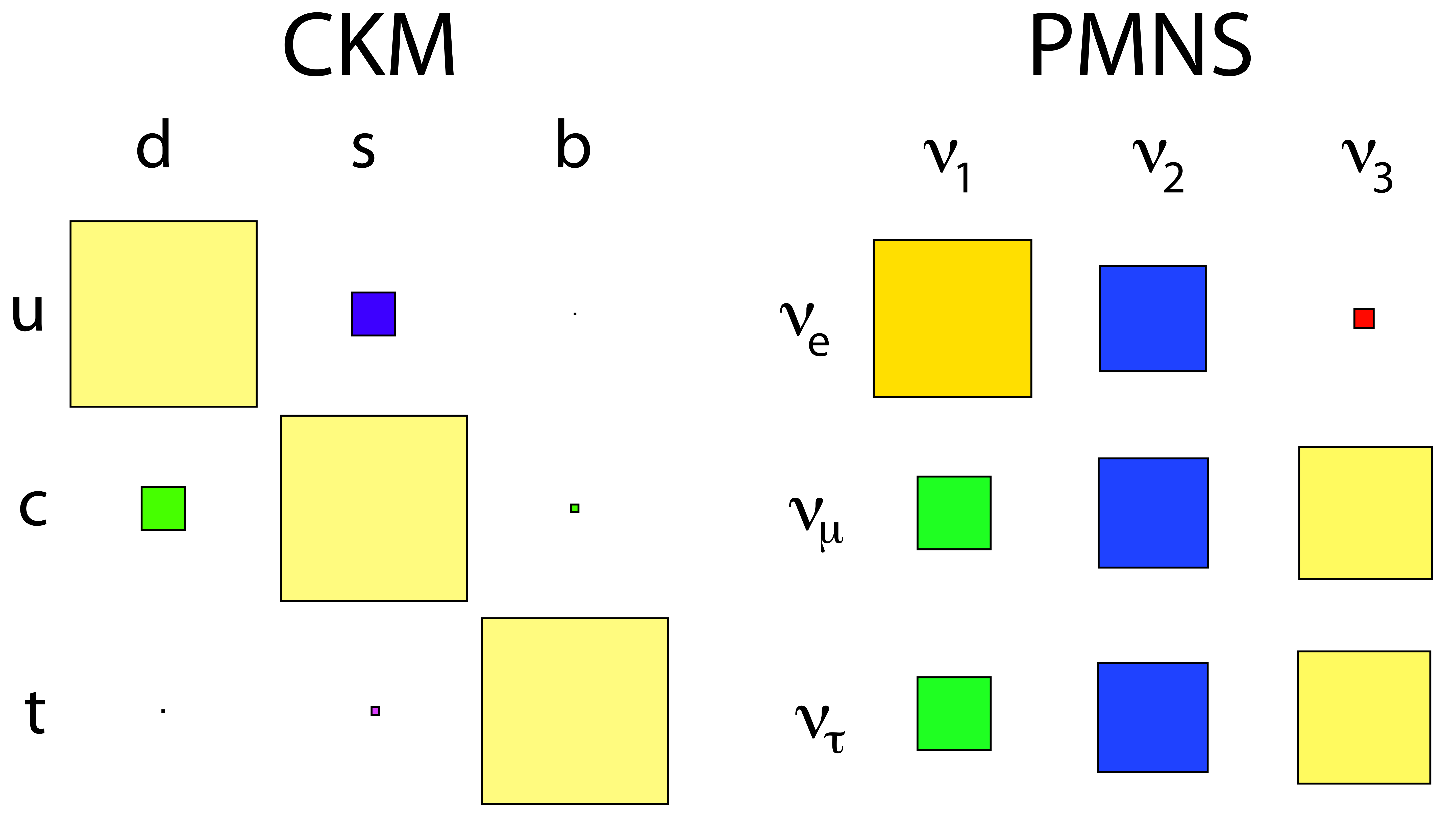


(Including three right-handed neutrinos)

The Flavour Problem

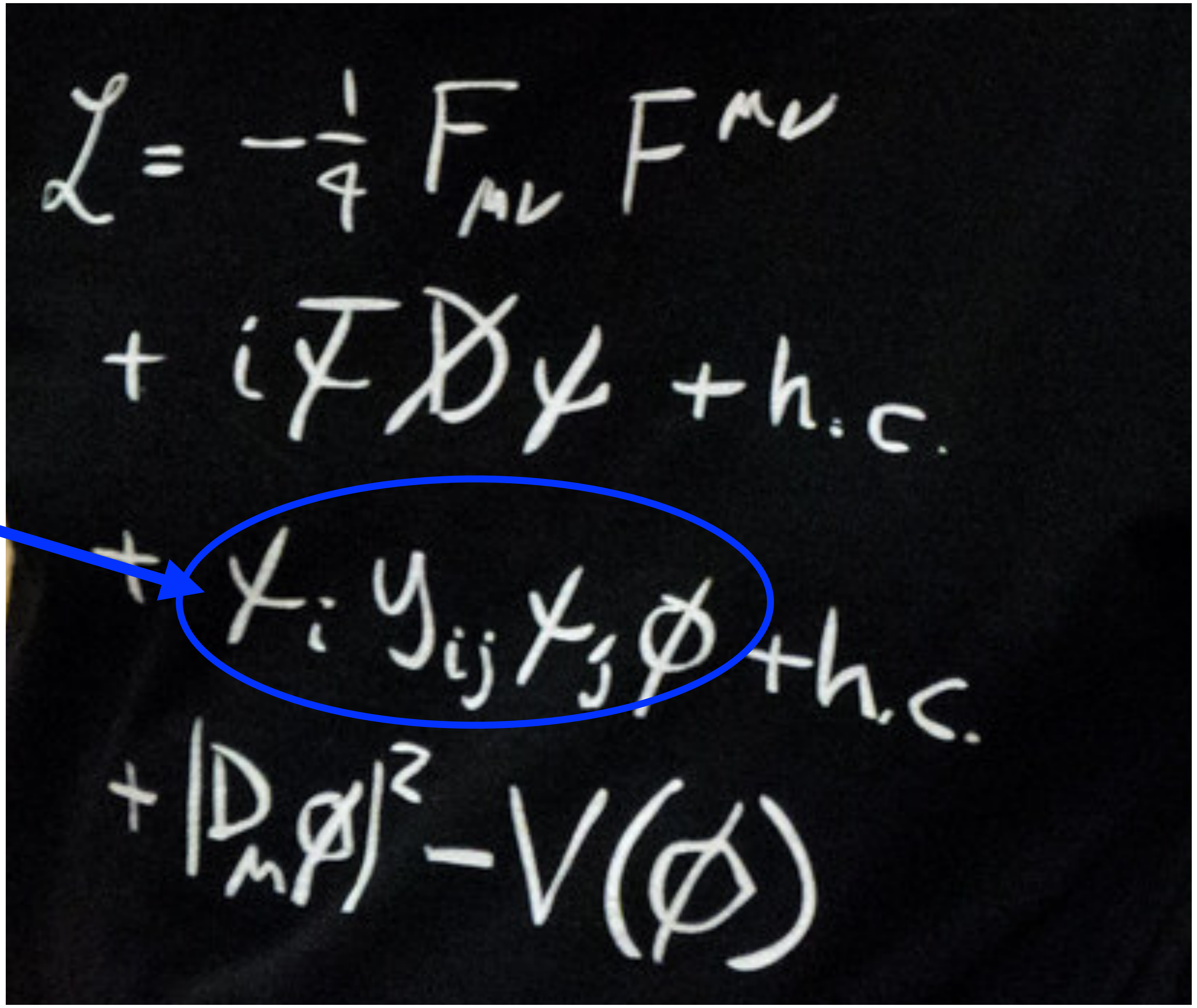


Mixing



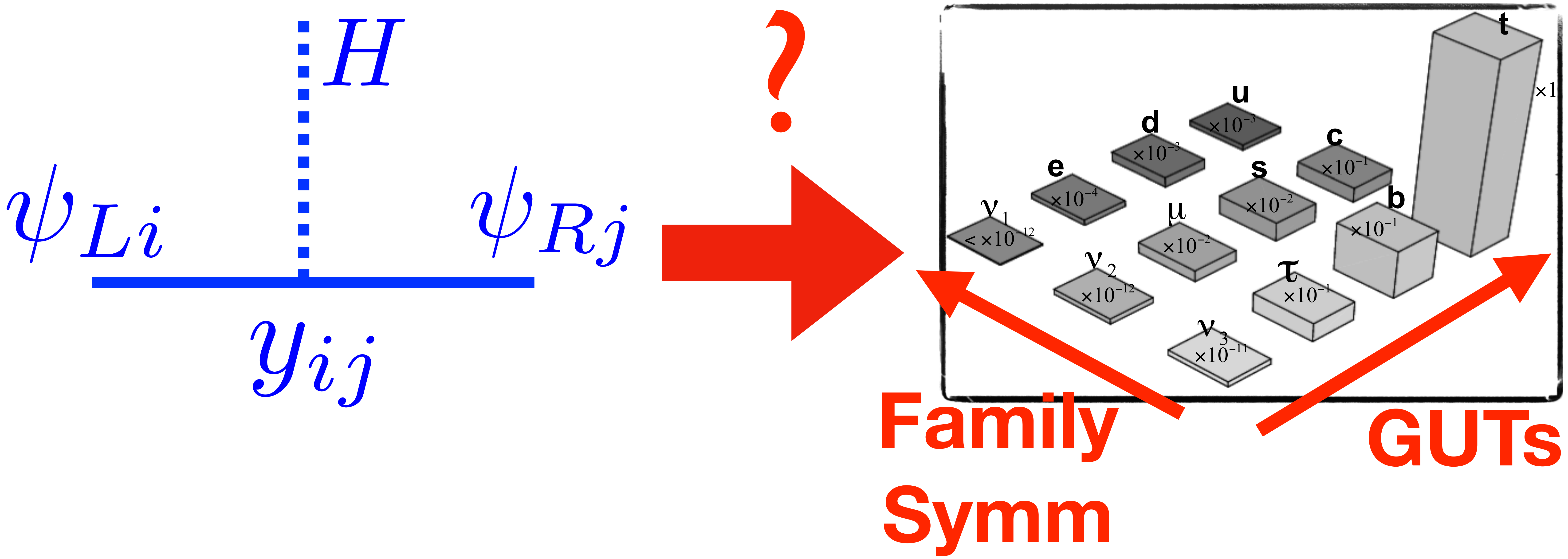
SM Yukawa couplings

Many undetermined
free parameters



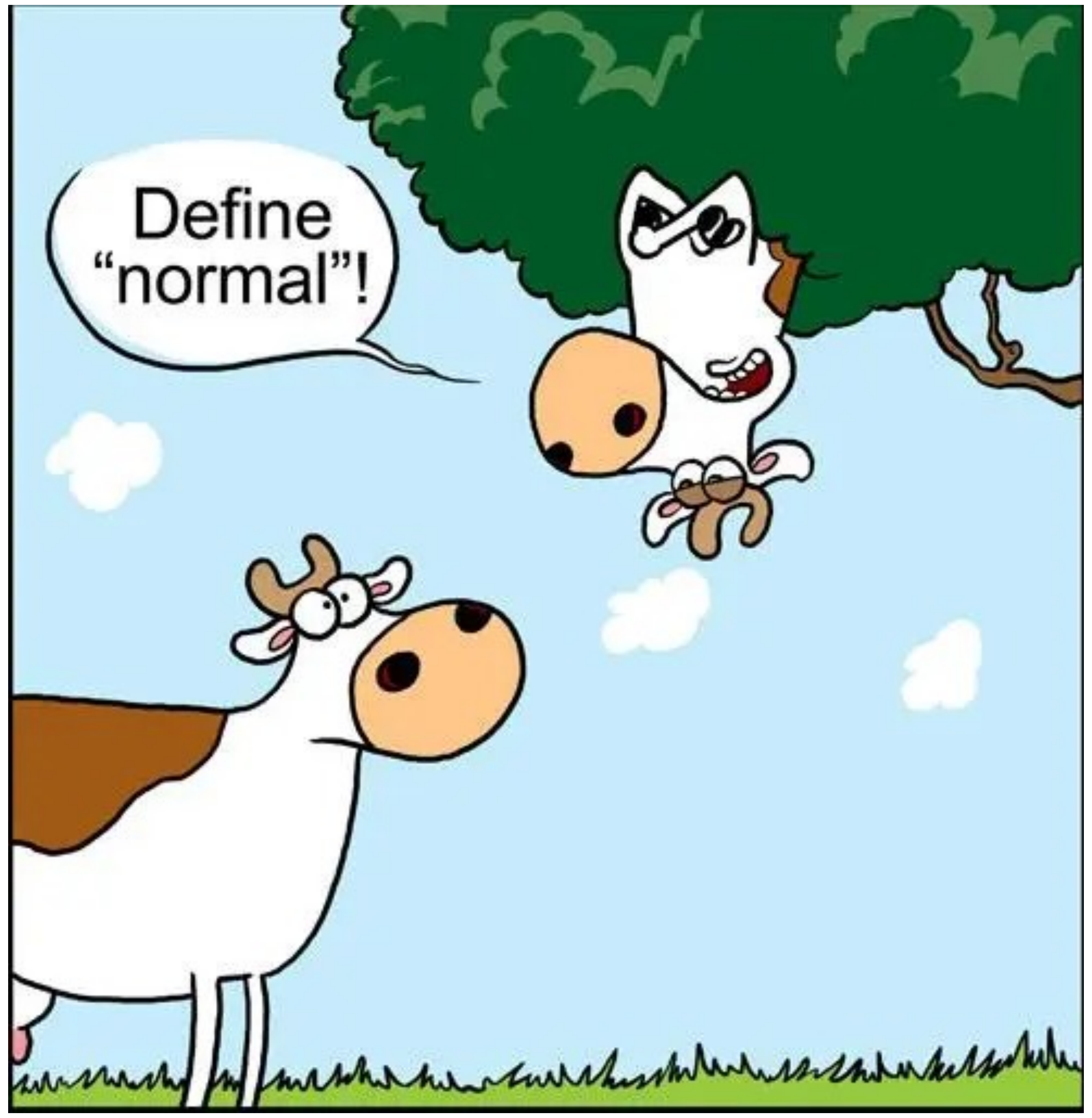
SM Yukawa couplings

$$y_{ij} H \overline{\psi}_{Li} \psi_{Rj}$$

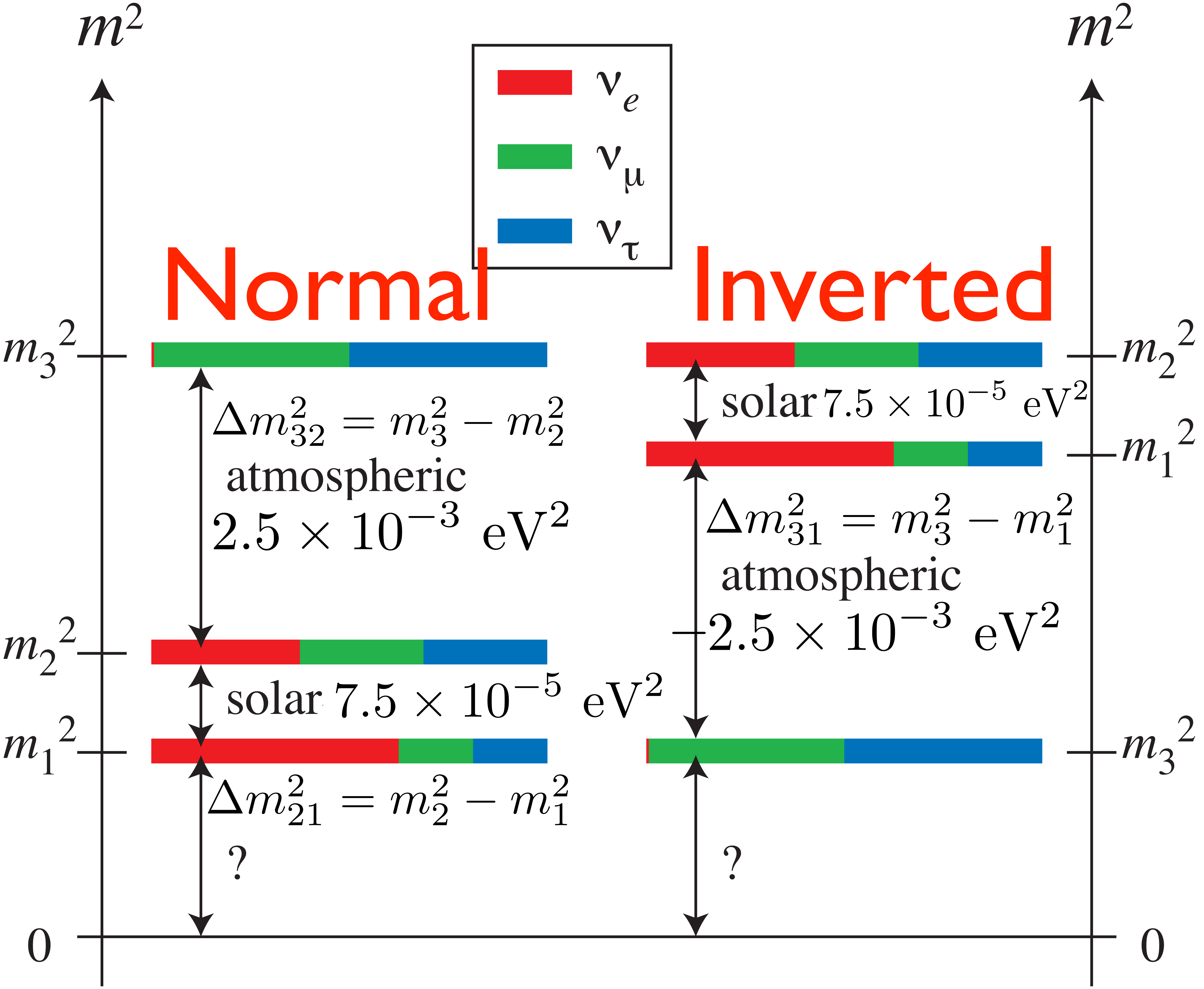


Neutrino mass and mixing

- Neutrinos have tiny masses (much less than electron)
- Neutrinos mix a lot (unlike the quarks)
- Up to 9 new params: 3 masses, 3 angles, 3 phases
- Origin of mass and mixing is unknown



Normal or Inverted?

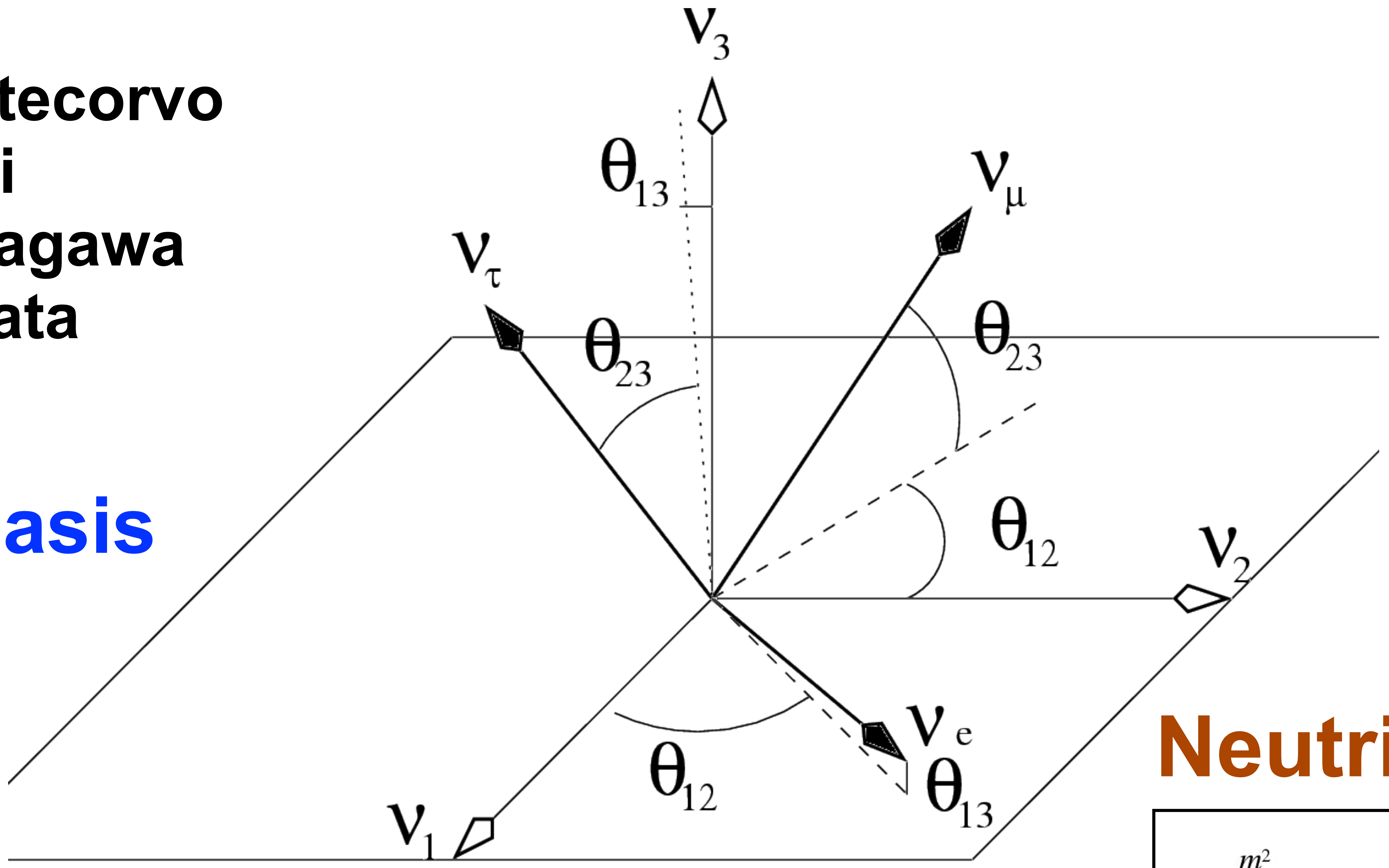


PMNS mixing matrix

B. Pontecorvo

S. Sakata Z. Maki M. Nakagawa

Pontecorvo
Maki
Nakagawa
Sakata

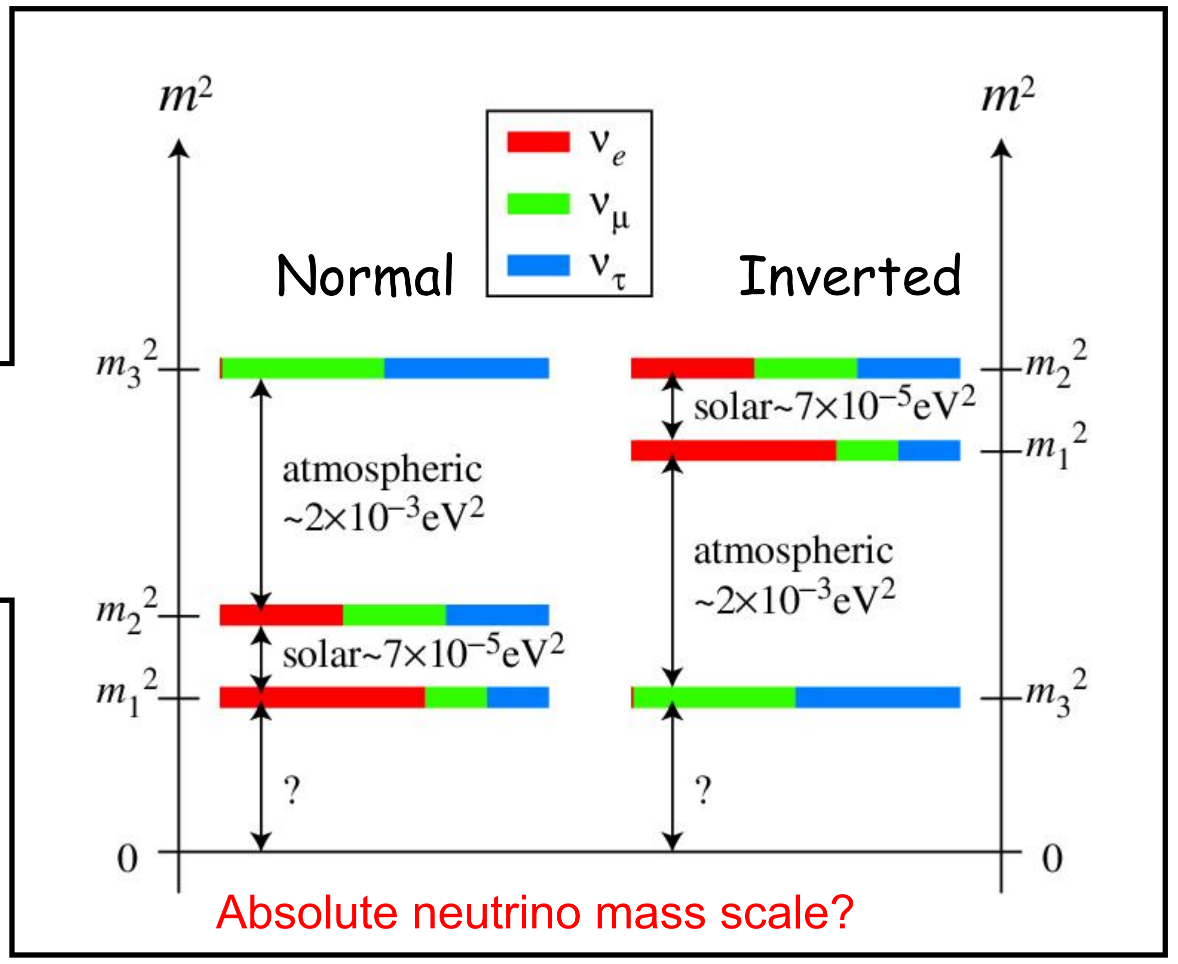


Charged lepton mass basis

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Neutrino mass basis



PMNS mixing matrix

$$U_{PMNS}=\begin{pmatrix}1&0&0\\0&c_{23}&s_{23}\\0&-s_{23}&c_{23}\end{pmatrix}\begin{pmatrix}c_{13}&0&s_{13}e^{-i\delta}\\0&1&0\\-s_{13}e^{i\delta}&0&c_{13}\end{pmatrix}\begin{pmatrix}c_{12}&s_{12}&0\\-s_{12}&c_{12}&0\\0&0&1\end{pmatrix}\begin{pmatrix}1&0&0\\0&e^{i\frac{\alpha_{21}}{2}}&0\\0&0&e^{i\frac{\alpha_{31}}{2}}\end{pmatrix}$$

Atmospheric **Reactor** **Solar** **Majorana**

CP violating phase

$$=\begin{pmatrix}c_{12}c_{13}&s_{12}c_{13}&s_{13}e^{-i\delta}\\-s_{12}c_{23}-c_{12}s_{13}s_{23}e^{i\delta}&c_{12}c_{23}-s_{12}s_{13}s_{23}e^{i\delta}&c_{13}s_{23}\\s_{12}s_{23}-c_{12}s_{13}c_{23}e^{i\delta}&-c_{12}s_{23}-s_{12}s_{13}c_{23}e^{i\delta}&c_{13}c_{23}\end{pmatrix}$$

CP violating Majorana phases

$$\times\,\mathrm{diag}(1,e^{i\alpha_{21}/2},e^{i\alpha_{31}/2})$$

NuFIT 5.2 (2022)

Global Fits

3σ ranges

$\theta_{23} = [39.6^\circ, 51.9^\circ]$ **Octant?**

$\sin^2\theta_{23} = \frac{1}{2}$? 45°? **Max Mix?**

$\theta_{12} = [31.31^\circ, 35.74^\circ]$

$\sin^2\theta_{12} = \frac{1}{3}$? 35.26°? **TBM?**

$\delta = [0^\circ, 44^\circ]$ & $[108^\circ, 360^\circ]$

0°? 180°? 270°?

CPC? **Max CPV?**

without SK atmospheric data

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.3$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.303^{+0.012}_{-0.011}$	$0.270 \to 0.341$	$0.303^{+0.012}_{-0.011}$	$0.270 \to 0.341$
$\theta_{12}/^\circ$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$
$\sin^2\theta_{23}$	$0.572^{+0.018}_{-0.023}$	$0.406 \to 0.620$	$0.578^{+0.016}_{-0.021}$	$0.412 \to 0.623$
$\theta_{23}/^\circ$	$49.1^{+1.0}_{-1.3}$	$39.6 \to 51.9$	$49.5^{+0.9}_{-1.2}$	$39.9 \to 52.1$
$\sin^2\theta_{13}$	$0.02203^{+0.00056}_{-0.00059}$	$0.02029 \to 0.02391$	$0.02219^{+0.00060}_{-0.00057}$	$0.02047 \to 0.02396$
$\theta_{13}/^\circ$	$8.54^{+0.11}_{-0.12}$	$8.19 \to 8.89$	$8.57^{+0.12}_{-0.11}$	$8.23 \to 8.90$
$\delta_{CP}/^\circ$	197^{+42}_{-25}	$108 \to 404$	286^{+27}_{-32}	$192 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$	$+2.511^{+0.028}_{-0.027}$	$+2.428 \to +2.597$	$-2.498^{+0.032}_{-0.025}$	$-2.581 \to -2.408$

with SK atmospheric data

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 6.4$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.303^{+0.012}_{-0.012}$	$0.270 \to 0.341$	$0.303^{+0.012}_{-0.011}$	$0.270 \to 0.341$
$\theta_{12}/^\circ$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$
$\sin^2\theta_{23}$	$0.451^{+0.019}_{-0.016}$	$0.408 \to 0.603$	$0.569^{+0.016}_{-0.021}$	$0.412 \to 0.613$
$\theta_{23}/^\circ$	$42.2^{+1.1}_{-0.9}$	$39.7 \to 51.0$	$49.0^{+1.0}_{-1.2}$	$39.9 \to 51.5$
$\sin^2\theta_{13}$	$0.02225^{+0.00056}_{-0.00059}$	$0.02052 \to 0.02398$	$0.02223^{+0.00058}_{-0.00058}$	$0.02048 \to 0.02416$
$\theta_{13}/^\circ$	$8.58^{+0.11}_{-0.11}$	$8.23 \to 8.91$	$8.57^{+0.11}_{-0.11}$	$8.23 \to 8.94$
$\delta_{CP}/^\circ$	232^{+36}_{-26}	$144 \to 350$	276^{+22}_{-29}	$194 \to 344$
$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$	$+2.507^{+0.026}_{-0.027}$	$+2.427 \to +2.590$	$-2.486^{+0.025}_{-0.028}$	$-2.570 \to -2.406$

Is there a pattern in the matrix?

Small

$$|U|_{3\sigma}^{\text{w/o SK-atm}} = \begin{pmatrix} 0.803 \to 0.845 & 0.514 \to 0.578 & 0.142 \to 0.155 \\ 0.233 \to 0.505 & 0.460 \to 0.693 & 0.630 \to 0.779 \\ 0.262 \to 0.525 & 0.473 \to 0.702 & 0.610 \to 0.762 \end{pmatrix}$$

NuFit 5.2

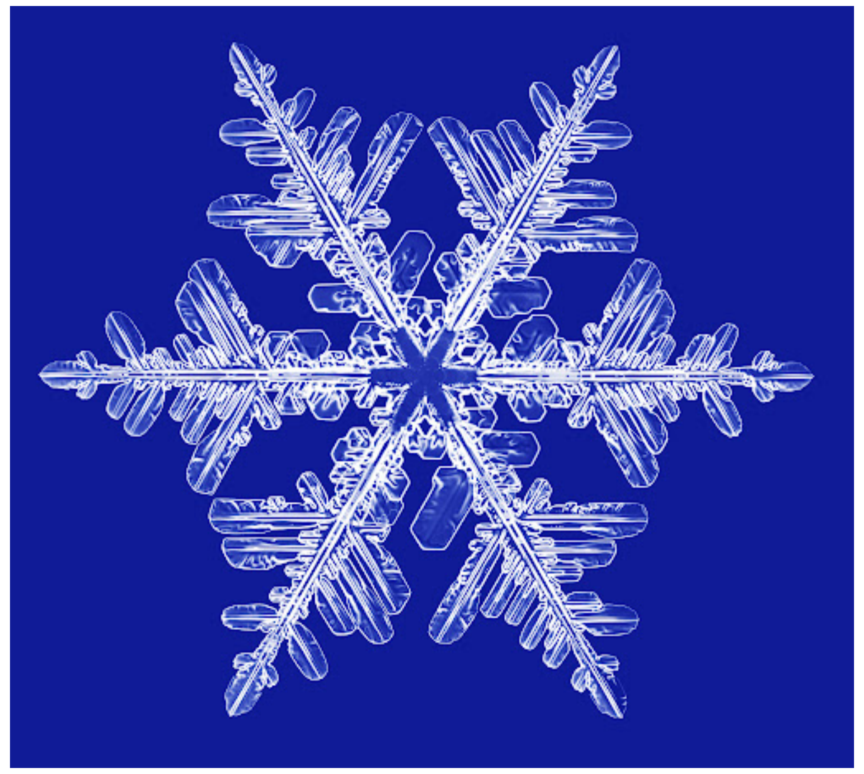
Large   Large

Symmetry can enforce

$$\sin\theta_{23} = \frac{1}{\sqrt{2}} \qquad \sin\theta_{13} = 0$$

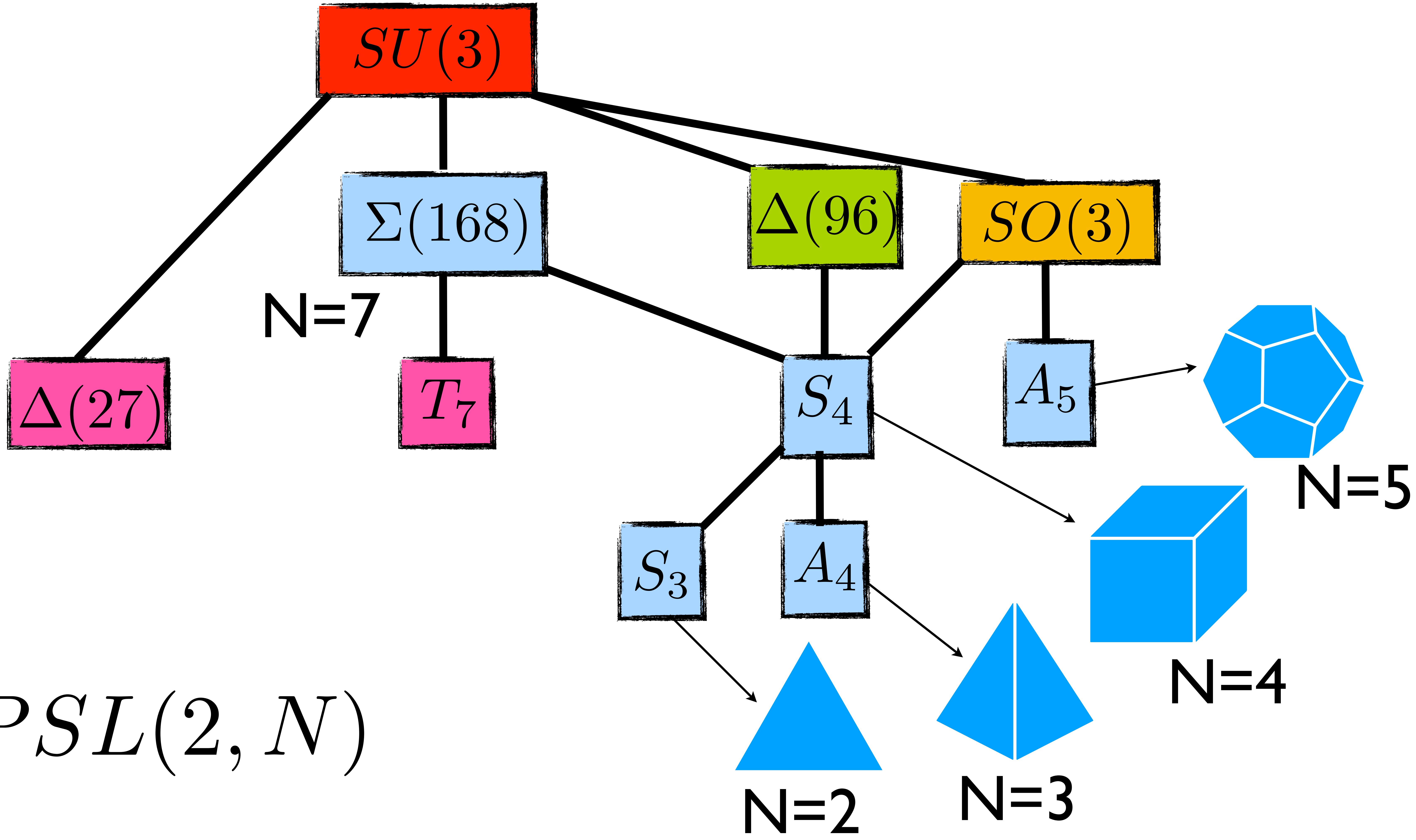
$$U_0 = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{s_{12}}{\sqrt{2}} & -\frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Where large $\sin\theta_{12}$ **can come from the same symmetry**



- H. Ishimori, T. Kobayashi, H. Ohki, Y. Shimizu, H. Okada, and M. Tanimoto, 1003.3552
- S. F. K., A. Merle, S. Morisi, Y. Shimizu, and M. Tanimoto, 1402.4271

Non-Abelian Family Symmetry



$PSL(2, N)$

A_4 and S_4 Group Theory

Ma-Rajarsakaran A_4 basis

A_4 $\quad S^2 = T^3 = (ST)^3 = \mathbb{I}$

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} ; \quad T = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$3 \times 3 = 1 + 1' + 1'' + 3 + 3$$

$a = (a_1, a_2, a_3)$ and $b = (b_1, b_2, b_3)$

$$\begin{aligned}
(ab)_1 &= a_1 b_1 + a_2 b_2 + a_3 b_3 \, ; \\
(ab)_{1'} &= a_1 b_1 + \omega a_2 b_2 + \omega^2 a_3 b_3 \, ; \\
(ab)_{1''} &= a_1 b_1 + \omega^2 a_2 b_2 + \omega a_3 b_3 \, ; \\
(ab)_{3_1} &= (a_2 b_3, a_3 b_1, a_1 b_2) \, ; \; \omega^3 = 1 \\
(ab)_{3_2} &= (a_3 b_2, a_1 b_3, a_2 b_1) \, ,
\end{aligned}$$

Altarelli-Feruglio A_4 basis

$$S^2 = T^3 = U^2 = (ST)^3 = (SU)^2 = (TU)^2 = (STU)^4 = 1$$

S_4	A_4	S	T	U
$\mathbf{1}, \mathbf{1}'$	$\mathbf{1}$	1	1	± 1
$\mathbf{2}$	$\begin{pmatrix} \mathbf{1}'' \\ \mathbf{1}' \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$\mathbf{3}, \mathbf{3}'$	$\mathbf{3}$	$\frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$	$\mp \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

VEV $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ preserves T in Ma basis
preserves S,U in AF basis

VEV $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ preserves S in Ma basis
preserves T in AF basis

Some Simple Symmetrical Examples

$$Z_3 \;\; \text{Tri} \qquad\qquad\qquad Z_2 \;\; \text{Bi} \qquad\qquad\qquad Z_5 \;\; \text{GR}$$

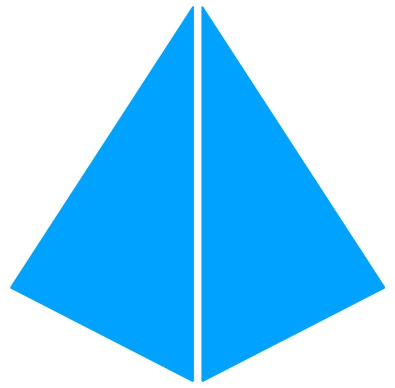
$$\begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \text{Bi} \; Z_2 \qquad \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \text{Bi} \; Z_2 \qquad \begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{s_{12}}{\sqrt{2}} & -\frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \text{Bi} \; Z_2$$

Tri-bimaximal

$$\sin\theta_{12} = \frac{1}{\sqrt{3}}$$

- F. Harrison, D. H. Perkins, and W. G. Scott, hep-ph/0202074.
- Z.-z. Xing, hep-ph/0204049. P

A_4

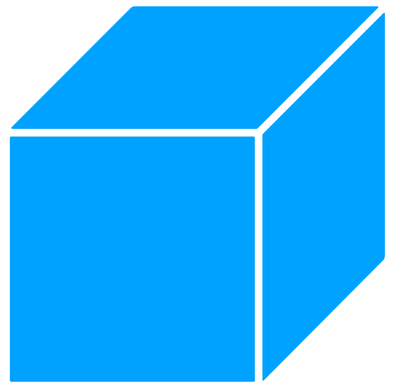


Bimaximal

$$\sin\theta_{12} = \frac{1}{\sqrt{2}}$$

- V. D. Barger, S. Pakvasa, T. J. Weiler, and K. Whisnant, hep-ph/9806387.
- S. Davidson and S. F. K. hep-ph/9808296.

S_4



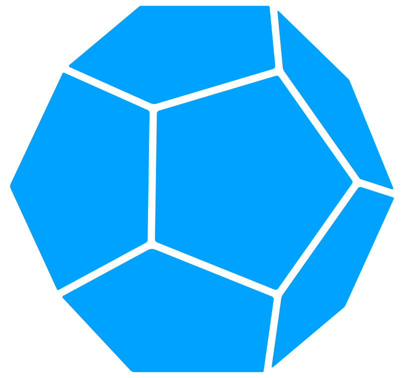
Golden Ratio

a) $\tan\theta_{12} = \frac{2}{1+\sqrt{5}} = \frac{1}{\phi}$

b) $\cos\theta_{12} = \phi/2$

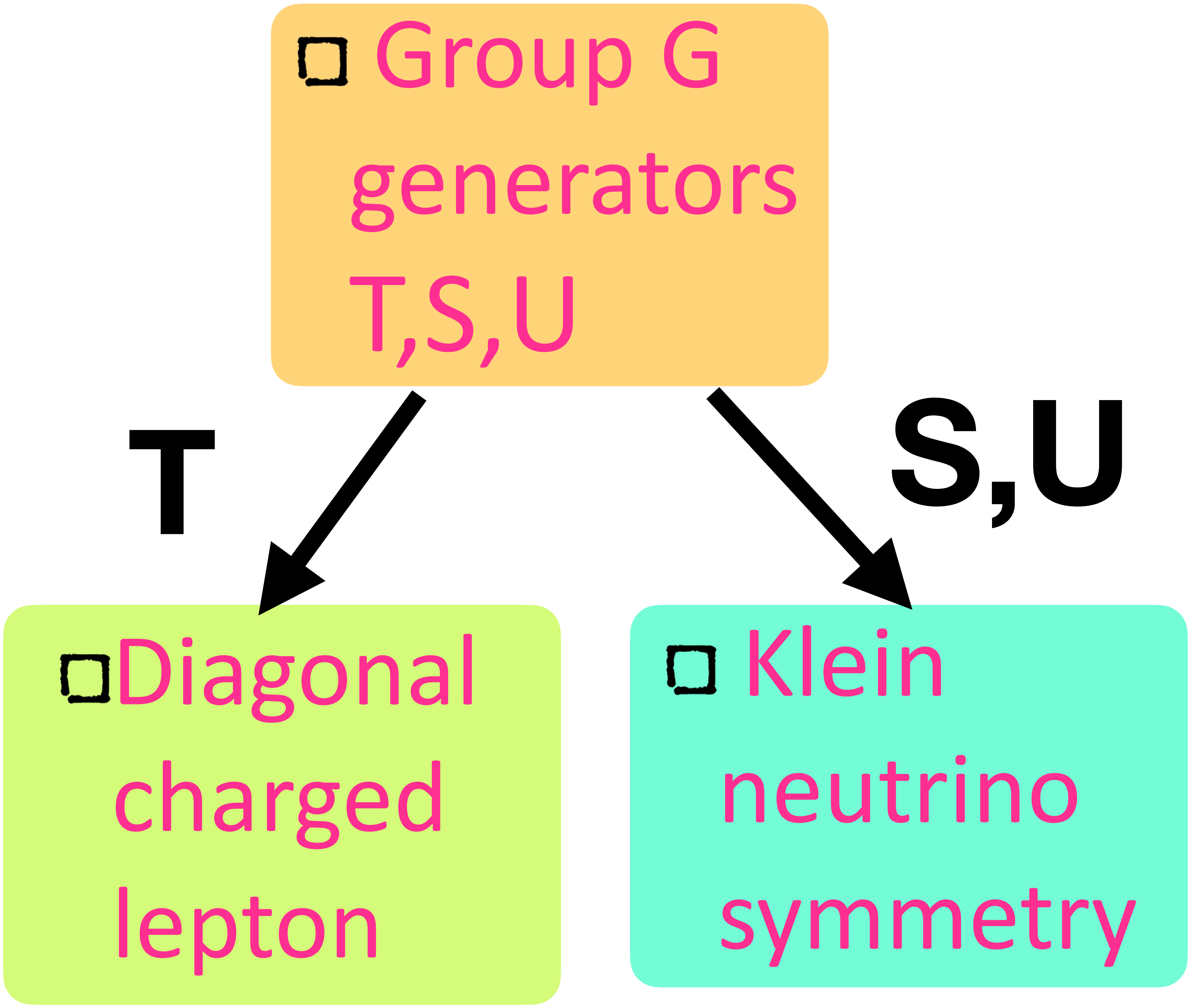
- A. Datta, F.-S. Ling, and P. Ramond, hep-ph/0306002.
- Y. Kajiyama, M. Raidal, and A. Strumia, 0705.4559
- L. L. Everett and A. J. Stuart, 0812.1057
- W. Rodejohann, 0810.5239

A_5



All these patterns involve $\sin\theta_{13} = 0$ **so they need to be corrected**

Why is θ_{13} predicted to be zero?



Altarelli-Feruglio basis

Why is θ_{13} predicted to be zero?

S.F.K. and C.Luhn, 0908.1897, 1301.1340

- Diagonal charged lepton

$$T^\dagger (M_e M_e^\dagger) T = M_e M_e^\dagger$$

$$T = \mathrm{diag}(1, \omega^2, \omega)$$

$$\omega = e^{i2\pi/N}$$

- Group G generators T,S,U

T → Diagonal charged lepton

S,U → Klein neutrino symmetry

- Klein neutrino symmetry

$$M^\nu = S^\dagger M^\nu S^* \quad M^\nu = U^\dagger M^\nu U^*$$

$$S = U_{\mathrm{PMNS}}\, \mathrm{diag}(-1,+1,-1)\, U_{\mathrm{PMNS}}^\dagger$$

$$U = U_{\mathrm{PMNS}}\, \mathrm{diag}(-1,-1,+1)\, U_{\mathrm{PMNS}}^\dagger$$

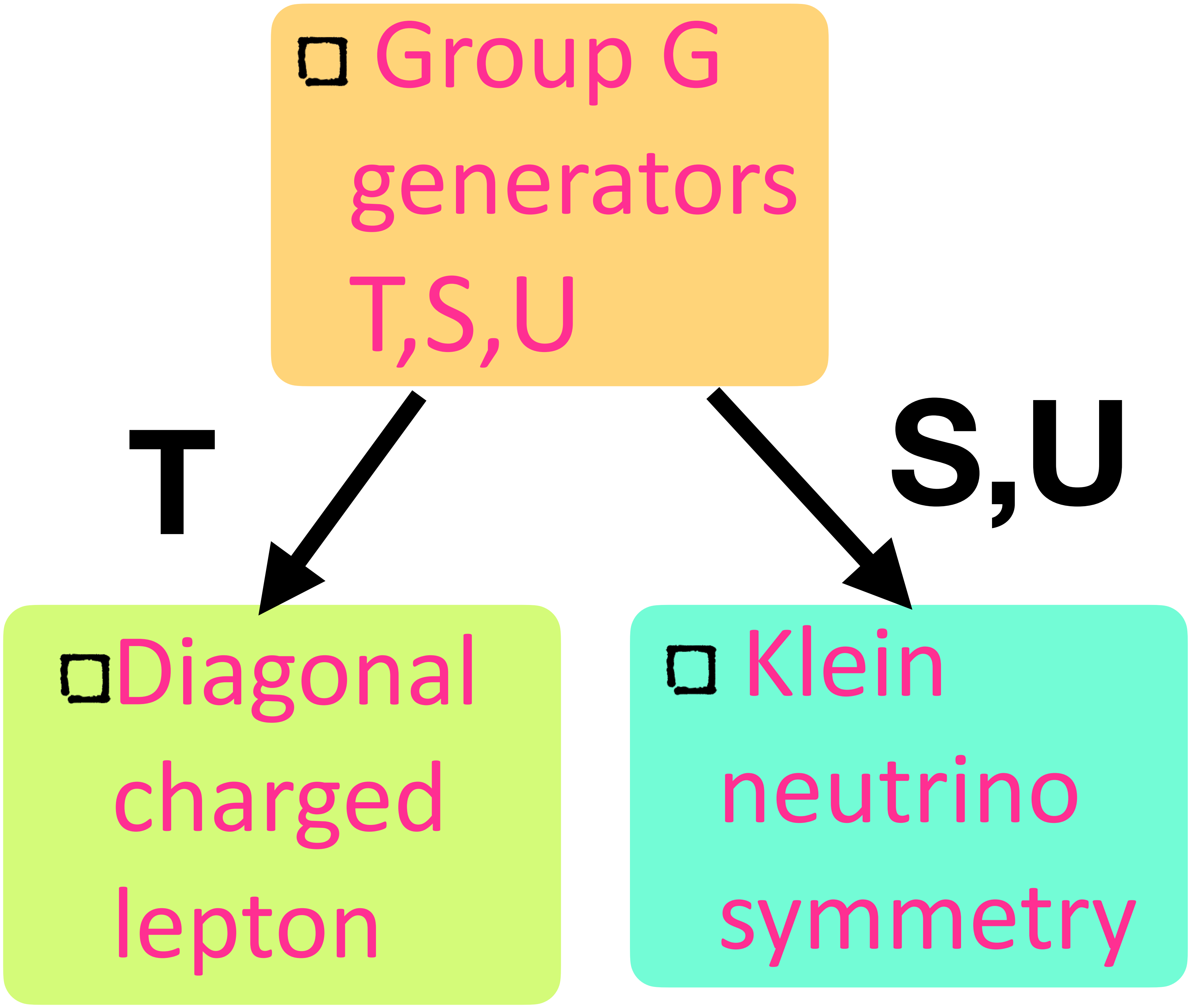
- Diagonal charged lepton

- Klein neutrino symmetry

Altarelli-Feruglio basis

S_4 generators S,U enforce TB mixing

How to switch on θ_{13}?



How to switch on θ_{13}?

1. Break T

Charged Lepton Corrections

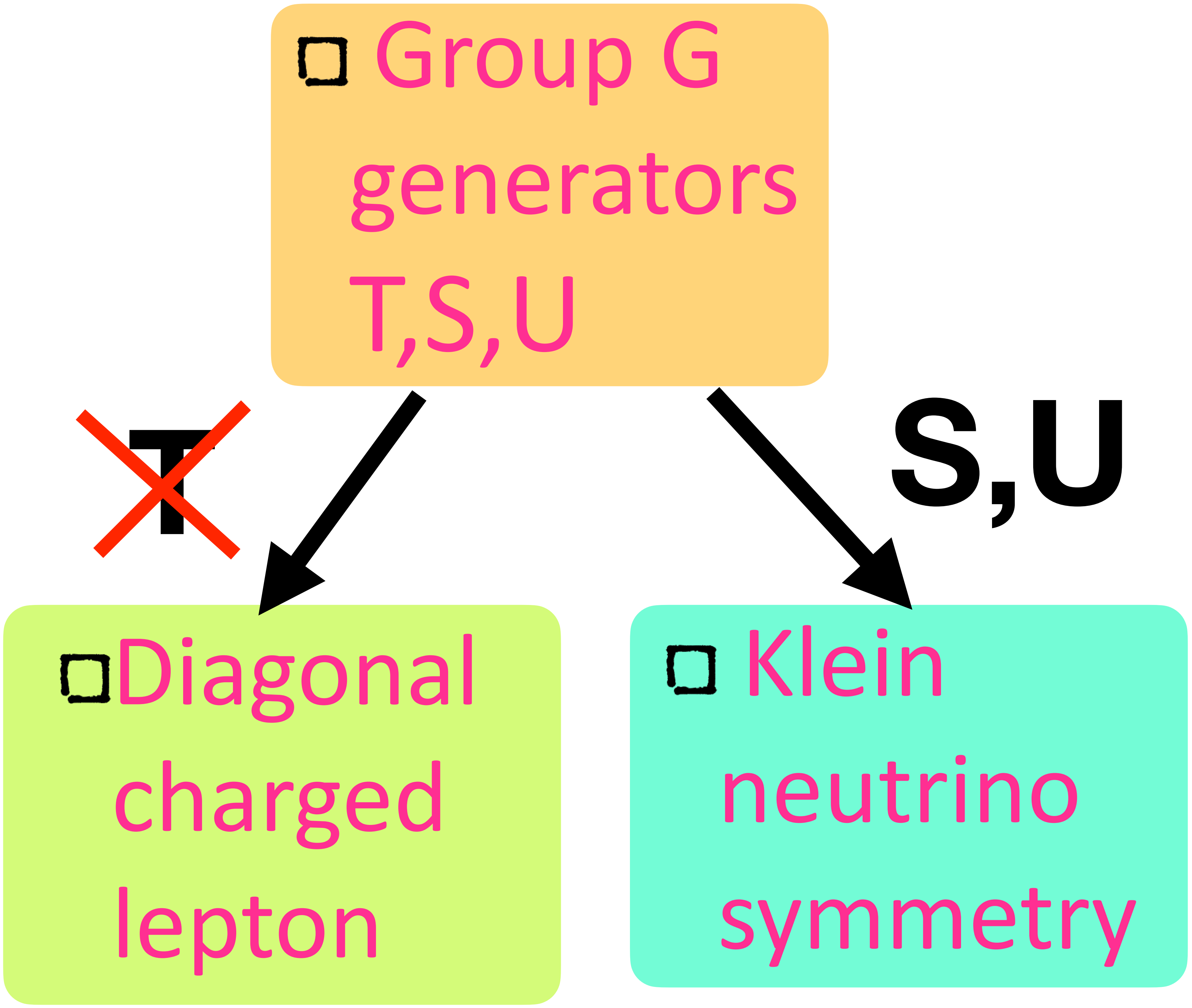
$\theta^e_{12} \neq 0$

Assume

$\theta^e_{23} = 0 \quad \theta^e_{13} = 0$

$U_{\text{PMNS}} = U_e U_\nu$

$s_{13} = \frac{s^e_{12}}{\sqrt{2}}$



Leading order solar sum rule

$$\theta_{12} + \theta_{13}\cos(\delta - \pi) \approx \theta^\nu_{12}$$

- S. F. K., hep-ph/0506297; I. Masina, hep-ph/0508031
- S. Antusch and S. F. K., hep-ph/0508044
- S. Antusch, P. Huber, S. F. K. and T. Schwetz, hep-ph/0702286

How to switch on θ_{13}?

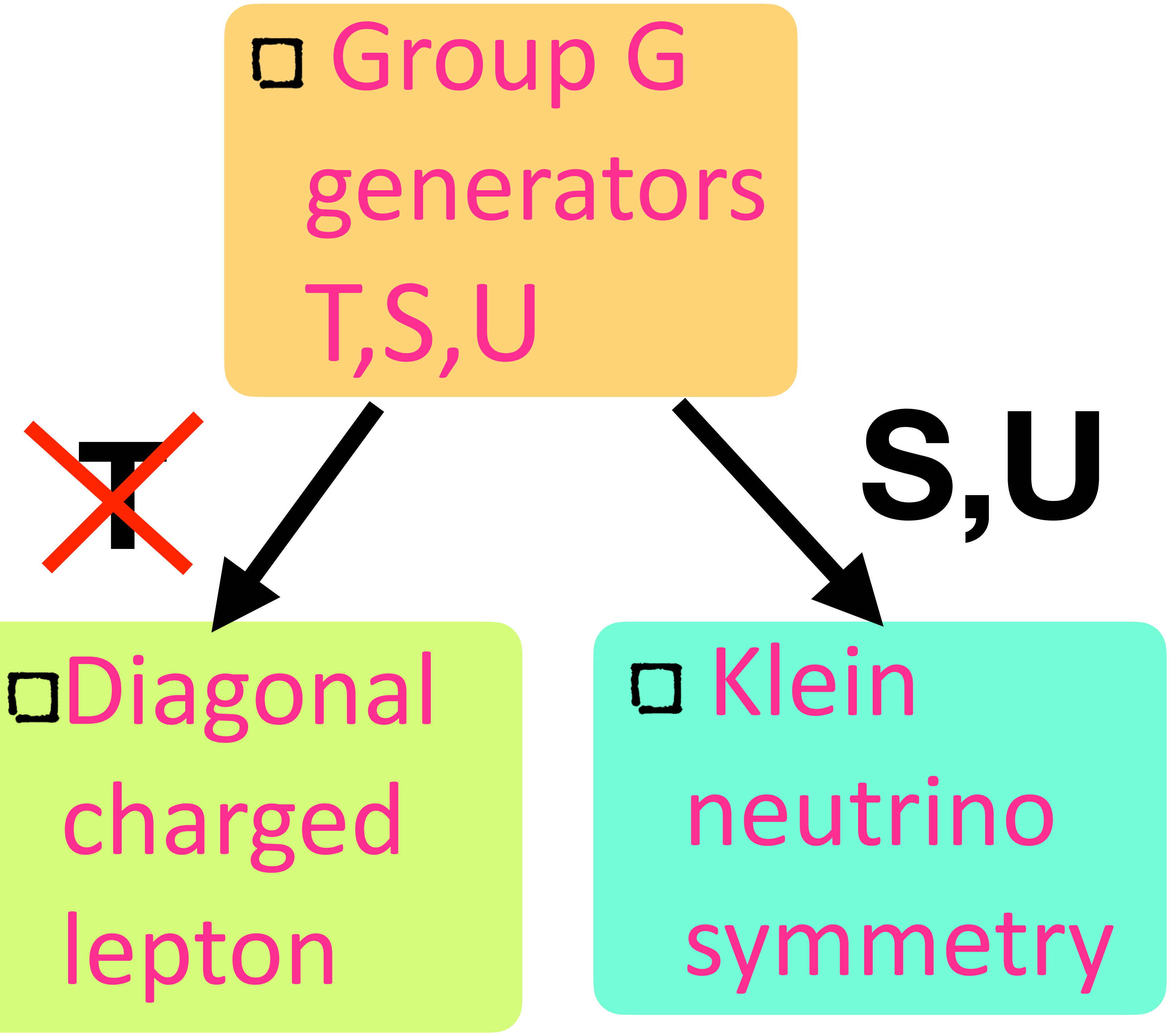
1. Break T

Charged Lepton Corrections

$\theta^e_{12} \neq 0$

Assume

$\theta^e_{23} \neq 0 \quad \theta^e_{13} = 0$



$$U_{\mathrm{PMNS}} = U_e U_\nu$$

$$s_{13} = \frac{s^e_{12}}{\sqrt{2}}$$

More precise Solar Sum Rule

- D. Marzocca, S. T. Petcov, A. Romanino, M. C. Sevilla, 1302.0423];
- I. Girardi, S. T. Petcov, A. V. Titov, 1410.8056

$$\cos\delta = \frac{\tan\theta_{23}\sin\theta_{12}^2 + \sin\theta_{13}^2\cos\theta_{12}^2/\tan\theta_{23} - (\sin\theta_{12}^{\nu})^2\left(\tan\theta_{23} + \sin\theta_{13}^2/\tan\theta_{23}\right)}{\sin 2\theta_{12}\sin\theta_{13}}$$

How to switch on θ_{13}?

P. Ballett, S. F. K., C. Luhn, S. Pascoli and M. A. Schmidt, 1410.7573

Simple derivation

1. Break T

Charged Lepton Corrections

$\theta^e_{12} \neq 0$

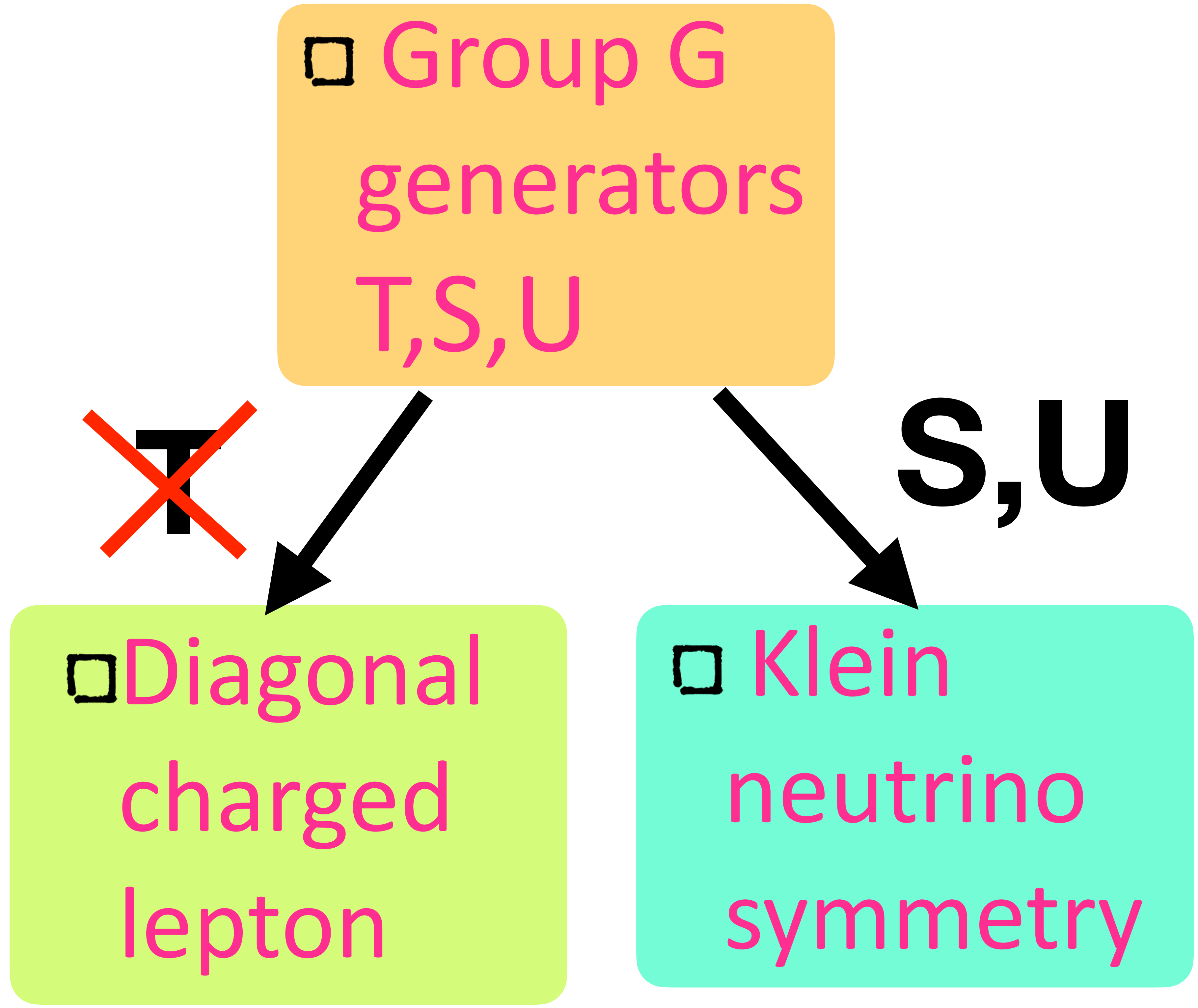
$\theta^e_{23} \neq 0$

Assume

$\theta^e_{13} = 0$

$U_{\rm PMNS} = U_e U_\nu$

$s_{13} = \frac{s^e_{12}}{\sqrt{2}}$



- Charged lepton corrections (not s^e_{13})

$$U = U^{e\dagger}_{12} U^{e\dagger}_{23} R^\nu_{23} R^\nu_{12} P^\nu$$

$$U_{\tau 1} = s^\nu_{12}(s^\nu_{23} c^e_{23} - c^\nu_{23} s^e_{23} e^{i\delta^e_{23}}),$$

$$U_{\tau 2} = -c^\nu_{12}(s^\nu_{23} c^e_{23} - c^\nu_{23} s^e_{23} e^{i\delta^e_{23}})$$

$$\frac{|U_{\tau 1}|}{|U_{\tau 2}|} = \frac{|s_{23}s_{12} - s_{13}c_{23}c_{12}e^{i\delta}|}{|s_{23}c_{12} + s_{13}c_{23}s_{12}e^{i\delta}|} = t^\nu_{12}$$

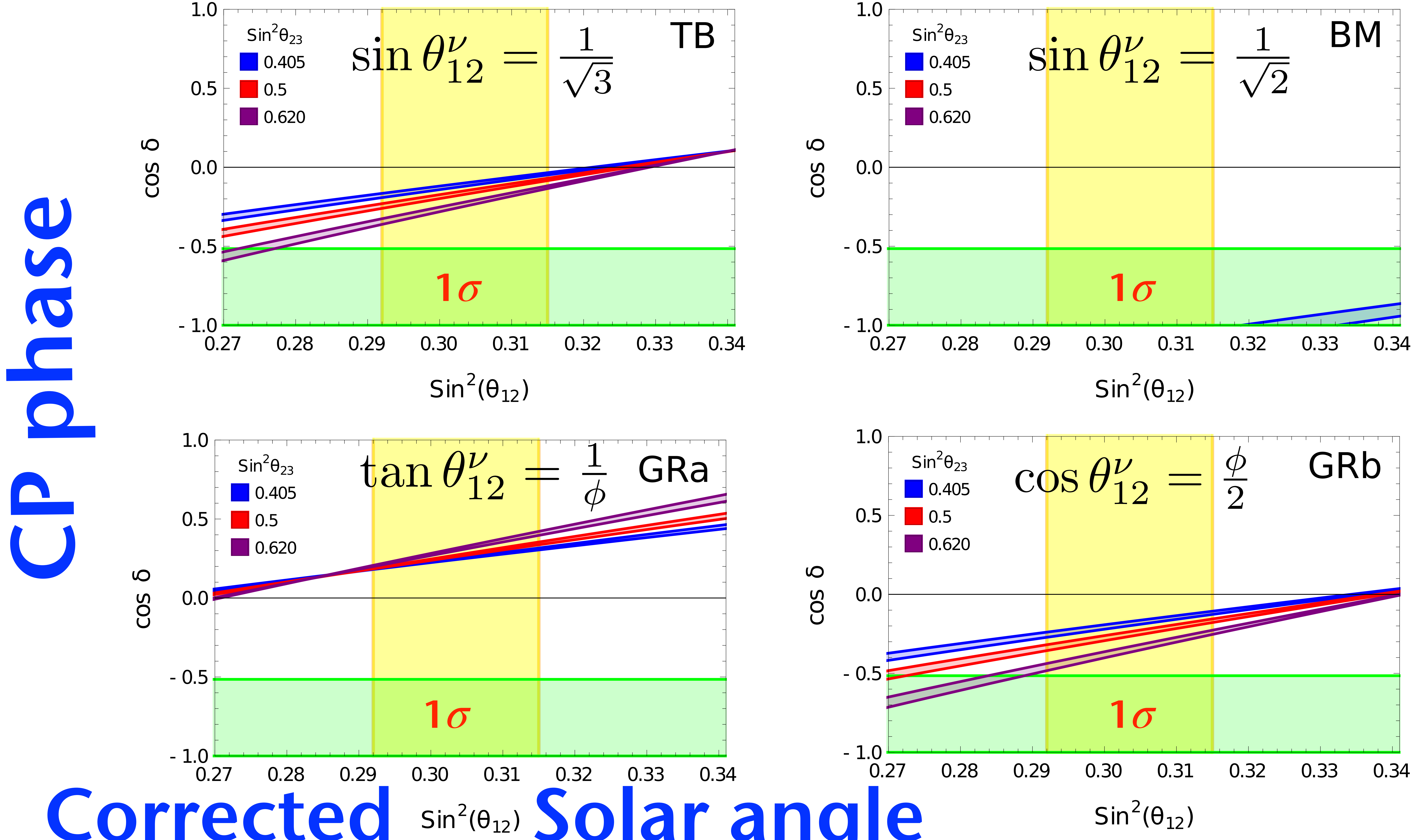
- D. Marzocca, S. T. Petcov, A. Romanino, M. C. Sevilla, 1302.0423];
- I. Girardi, S. T. Petcov, A. V. Titov, 1410.8056

More precise Solar Sum Rule

$$\cos\delta = \frac{\tan\theta_{23}\sin\theta^2_{12} + \sin\theta^2_{13}\cos\theta^2_{12}/\tan\theta_{23} - (\sin\theta^\nu_{12})^2\left(\tan\theta_{23} + \sin\theta^2_{13}/\tan\theta_{23}\right)}{\sin 2\theta_{12}\sin\theta_{13}}$$

Solar Sum Rule Predictions

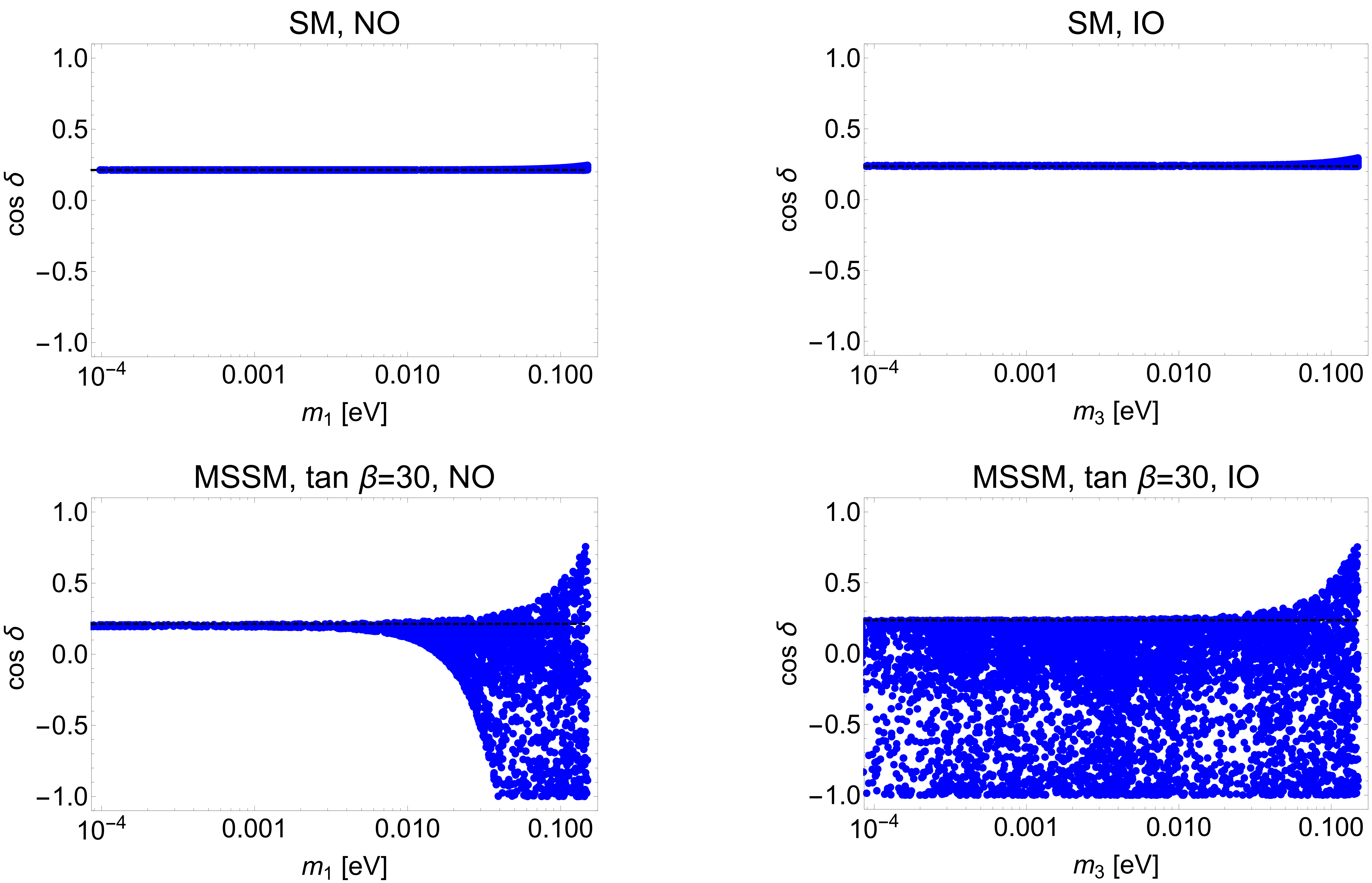
F.Costa and S.F.K., 2307.13895



Corrected Solar angle

RG corrections to GRa solar sum rule

J.Gehrlein, S.T.Petcov, M.Spinrath and A.V.Titov, 1608.08409



$$10^{13}\ \text{GeV} \rightarrow M_Z$$

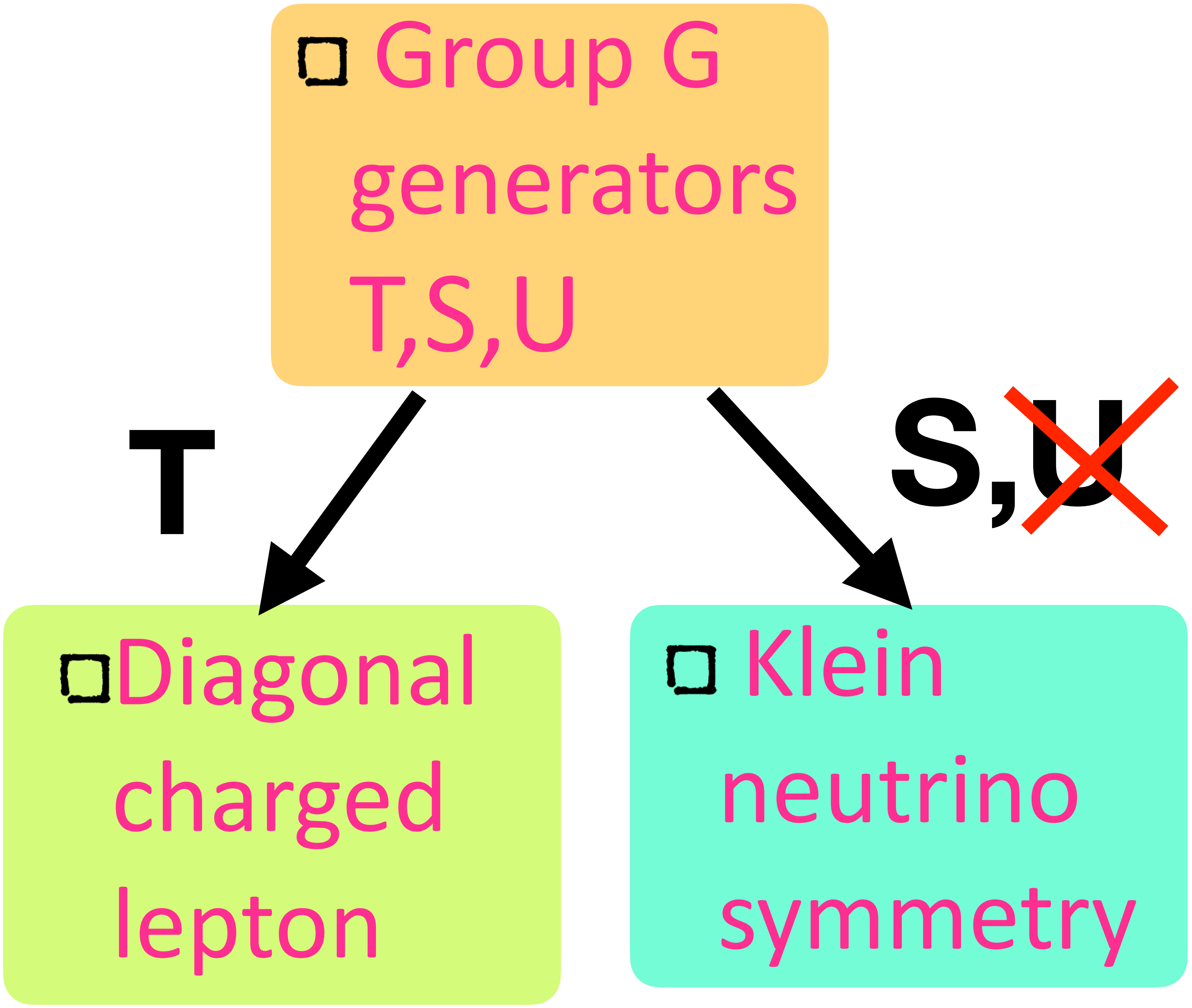
$$\theta^e_{12} \neq 0$$

$$\theta^e_{23} \neq 0$$

$$\theta^e_{13} = 0$$

How to switch on θ_{13}?

- C. H. Albright and W. Rodejohann, 0812.0436
- C. Luhn, 1306.2358
- S. F. King and C. Luhn, 1107.5332
- P. Ballett, S. F. King, C. Luhn, S. Pascoli and M. A. Schmidt, 1308.4314



2. Break U

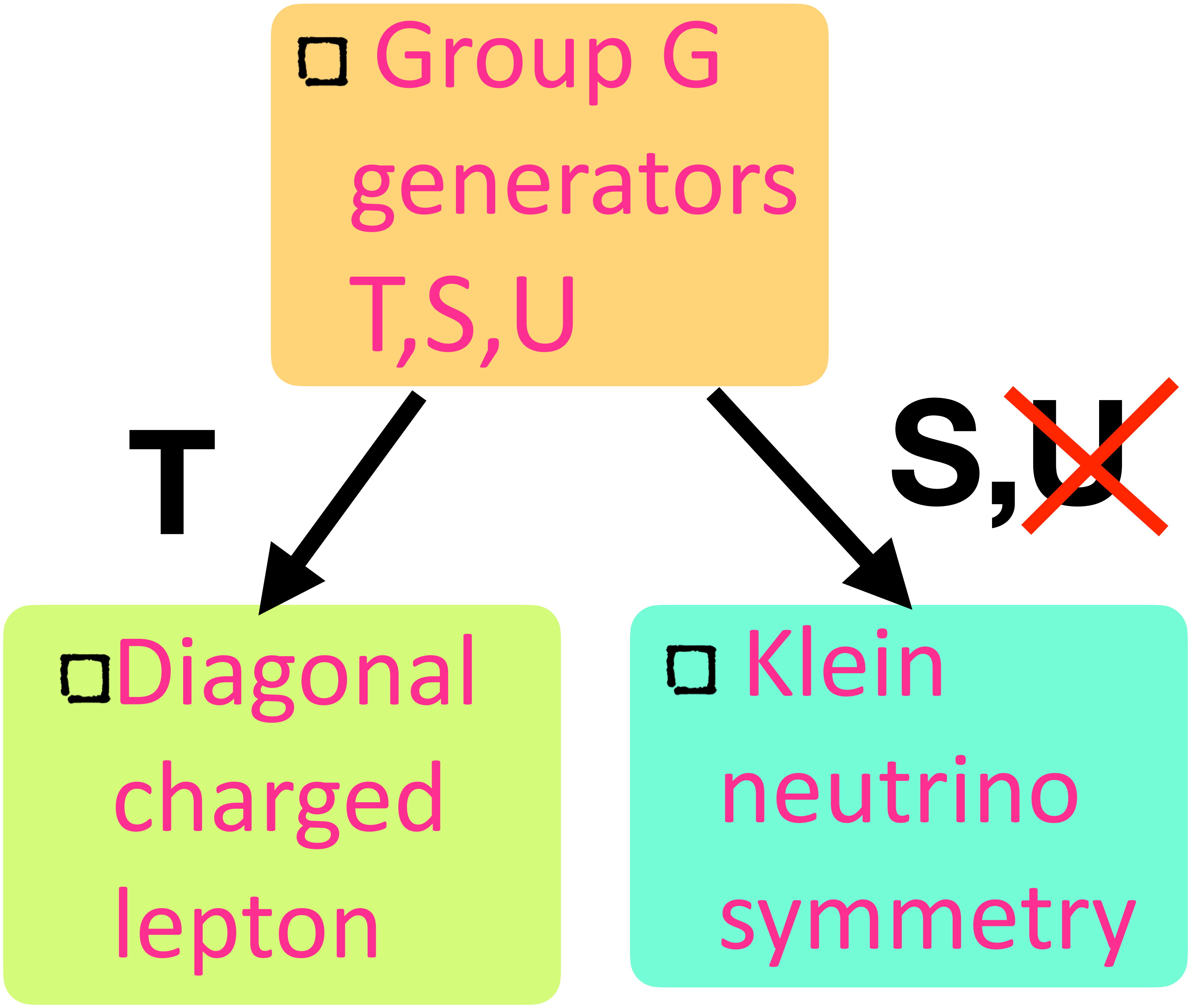
First or second PMNS column preserved

$$\begin{pmatrix} \sqrt{\frac{2}{3}} & - & - \\ -\frac{1}{\sqrt{6}} & - & - \\ \frac{1}{\sqrt{6}} & - & - \end{pmatrix} \quad \begin{pmatrix} - & \sqrt{\frac{1}{3}} & - \\ - & \sqrt{\frac{1}{3}} & - \\ - & -\sqrt{\frac{1}{3}} & - \end{pmatrix}$$

s_{13} **free parameter**

How to switch on θ_{13}?

- C. H. Albright and W. Rodejohann, 0812.0436
- C. Luhn, 1306.2358
- S. F. King and C. Luhn, 1107.5332
- P. Ballett, S. F. King, C. Luhn, S. Pascoli and M. A. Schmidt, 1308.4314



2. Break U

First or second PMNS column preserved

$$\begin{pmatrix} \sqrt{\frac{2}{3}} & - & - \\ -\frac{1}{\sqrt{6}} & - & - \\ \frac{1}{\sqrt{6}} & - & - \end{pmatrix} \quad \begin{pmatrix} - & \sqrt{\frac{1}{3}} & - \\ - & \sqrt{\frac{1}{3}} & - \\ - & -\sqrt{\frac{1}{3}} & - \end{pmatrix}$$

SU preserved

Atmospheric Sum Rules

s_{13} free parameter

$$\begin{pmatrix} \sqrt{\frac{2}{3}} & - & - \\ -\frac{1}{\sqrt{6}} & - & - \\ \frac{1}{\sqrt{6}} & - & - \end{pmatrix}$$

$$s_{12}^2 = \frac{(1-3s_{13}^2)}{3(1-s_{13}^2)} \qquad \cos\delta = -\frac{\cot 2\theta_{23}(1-5s_{13}^2)}{2\sqrt{2}s_{13}\sqrt{1-3s_{13}^2}}$$

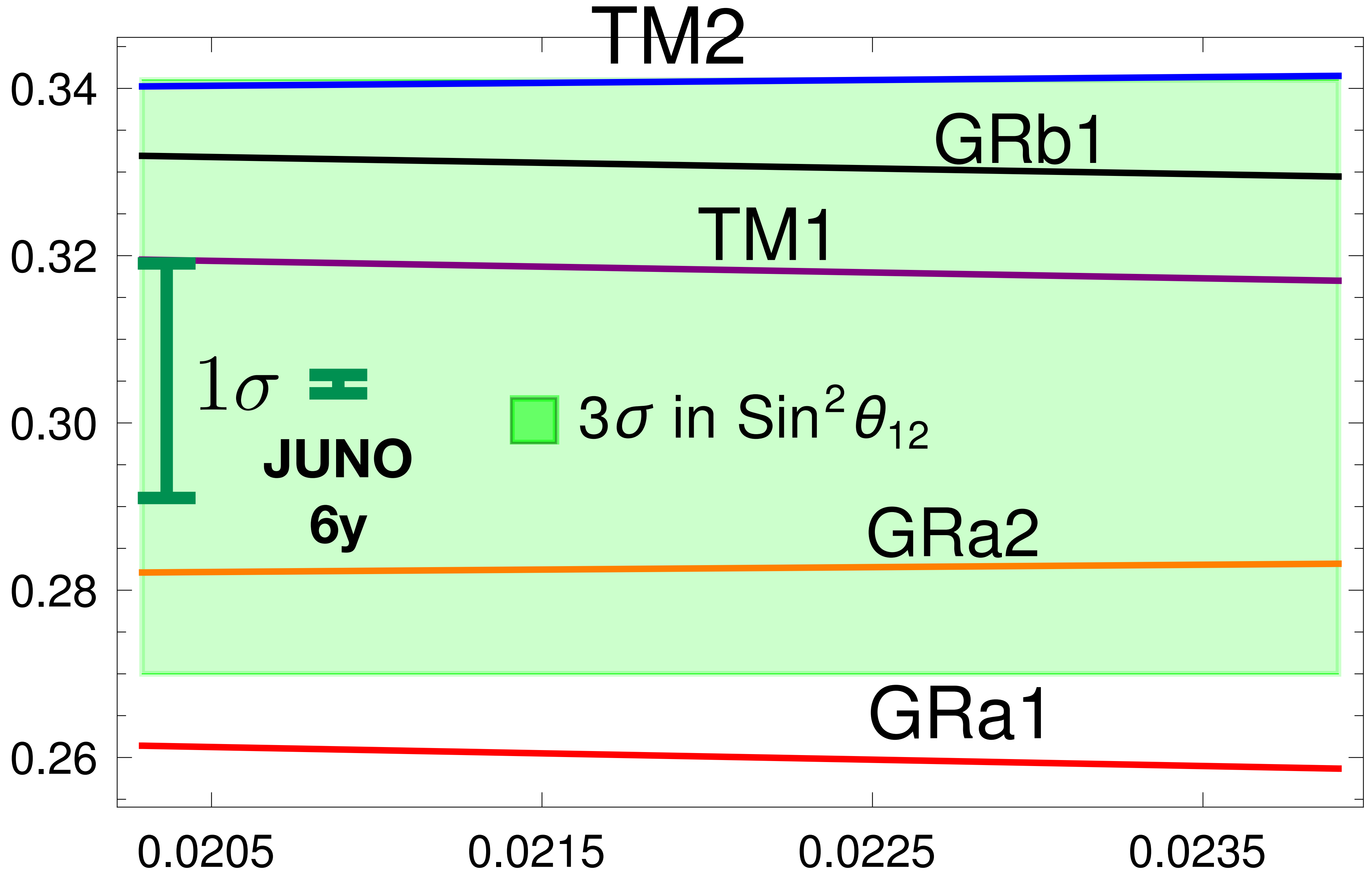
$$s_{12}^2 = \frac{1}{3(1-s_{13}^2)} \qquad \cos\delta = \frac{2c_{13}\cot 2\theta_{23}\cot 2\theta_{13}}{\sqrt{2-3s_{13}^2}}$$

$$\begin{pmatrix} - & \sqrt{\frac{1}{3}} & - \\ - & \sqrt{\frac{1}{3}} & - \\ - & -\sqrt{\frac{1}{3}} & - \end{pmatrix}$$

F.Costa and S.F.K., 2307.13895

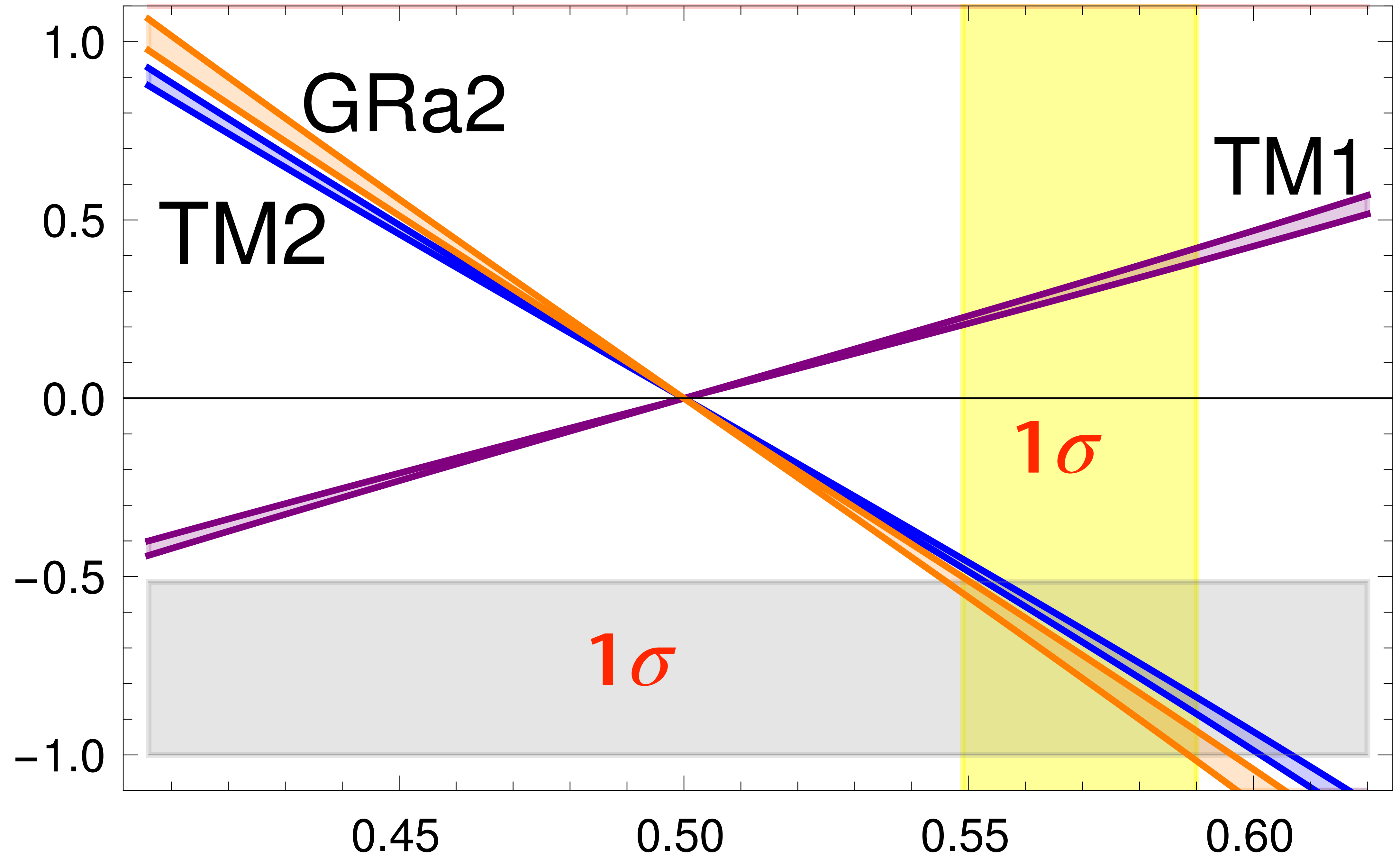
Atmospheric Sum Rule Predictions

Solar angle



Reactor angle $\sin^2(\theta_{13})$

CP phase



Atmospheric angle $\sin^2(\theta_{23})$

Only viable patterns

TM1 $\begin{pmatrix} \sqrt{\frac{2}{3}} & - & - \\ -\frac{1}{\sqrt{6}} & - & - \\ \frac{1}{\sqrt{6}} & - & - \end{pmatrix}$

TM2 $\begin{pmatrix} - & \sqrt{\frac{1}{3}} & - \\ - & \sqrt{\frac{1}{3}} & - \\ - & -\sqrt{\frac{1}{3}} & - \end{pmatrix}$

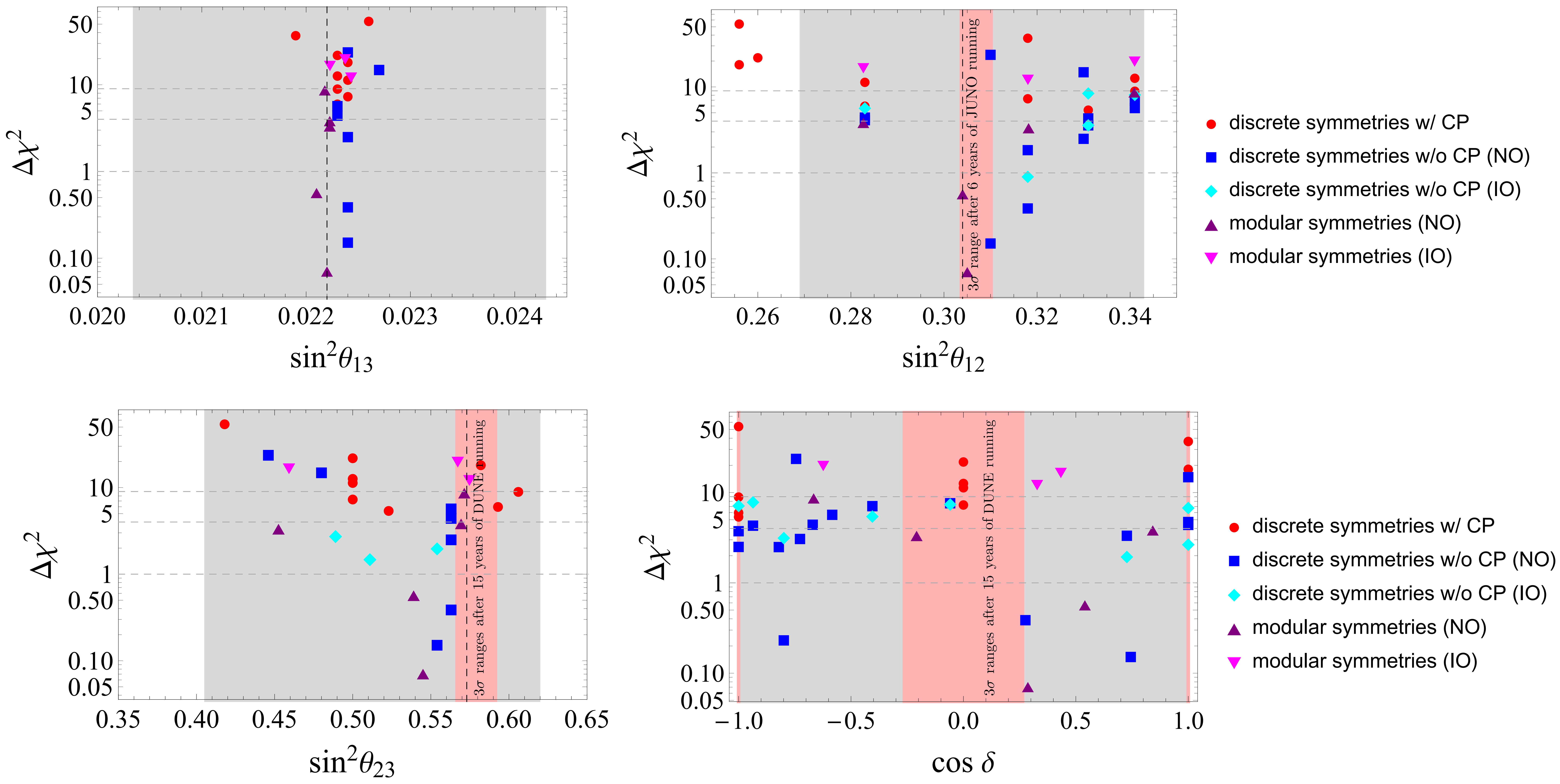
disfavoured

GRa2 $\begin{pmatrix} - & s^\nu_{12} & - \\ - & \frac{c^\nu_{12}}{\sqrt{2}} & - \\ - & -\frac{c^\nu_{12}}{\sqrt{2}} & - \end{pmatrix}$

$\tan\theta^\nu_{12} = \frac{1}{\phi}$

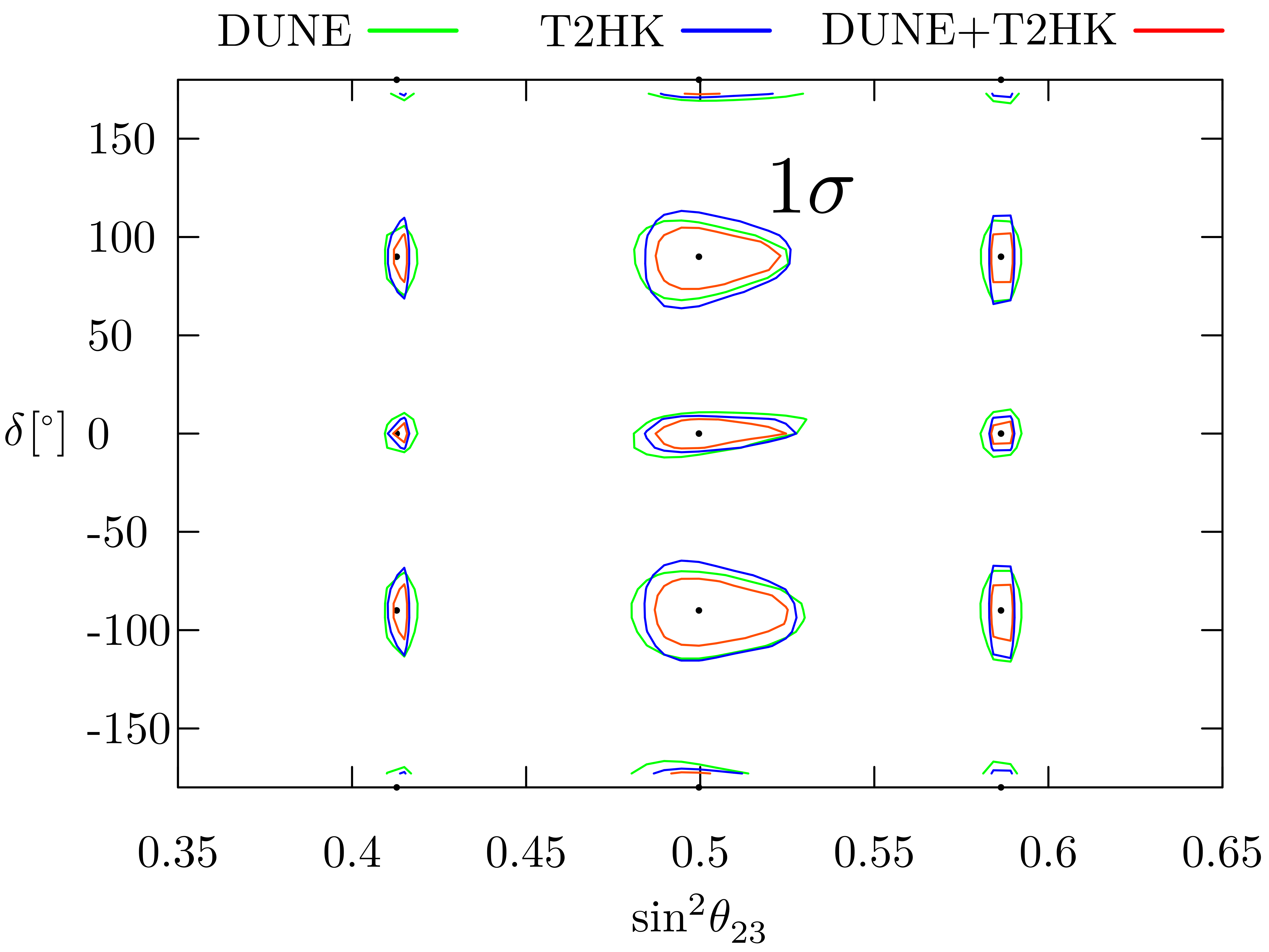
Gehrlein, S.Petcov, M.Spinrath and A.Titov 2203.06219

Survey of symmetry predictions



Future Prospects

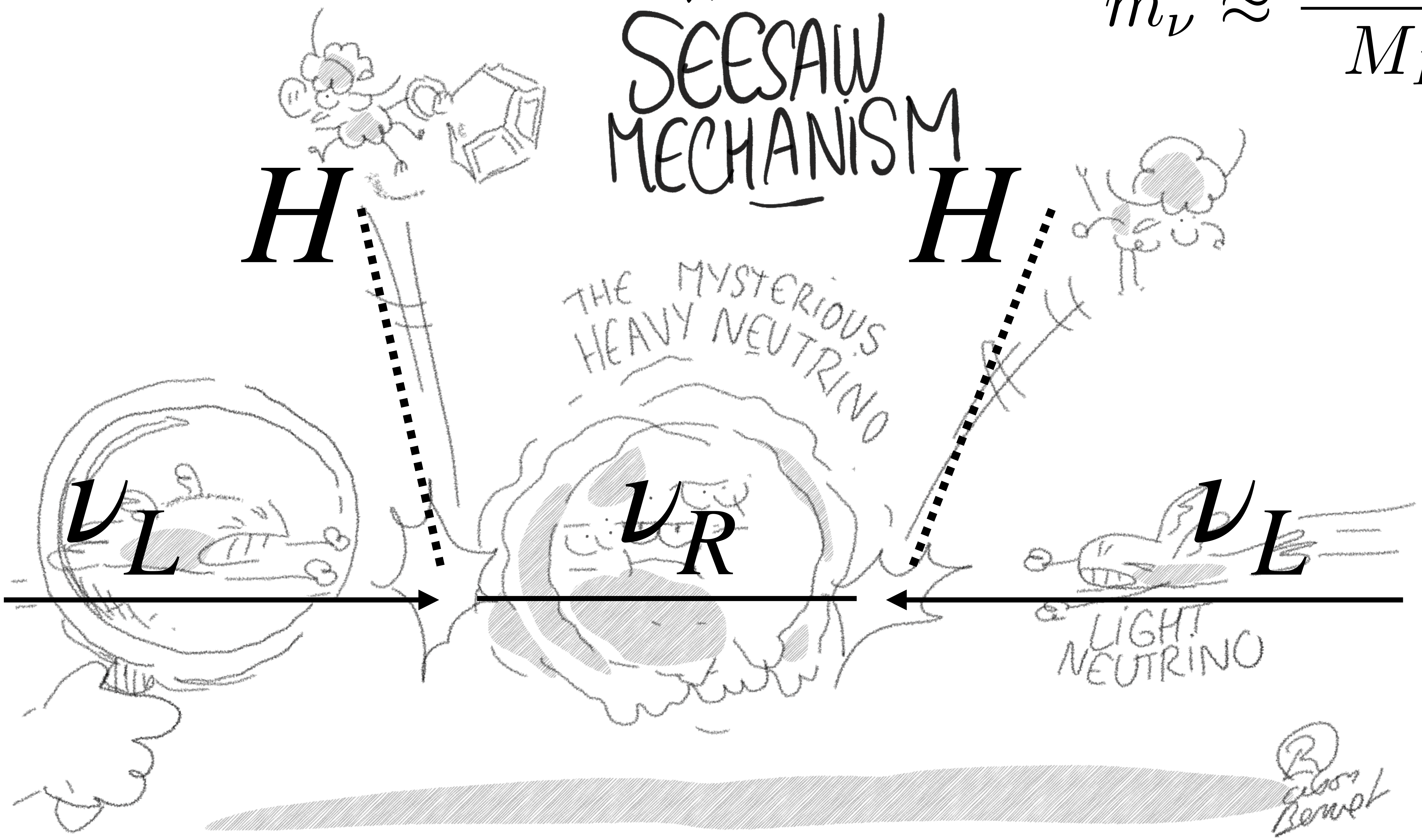
P.Ballett, S.F.K., S.Pascoli, N.W.Prouse and T.Wang,1612.07275



Will put flavour symmetry models to the test!

Type Ia SEESAW MECHANISM

$$m_\nu \approx \frac{m^2_{\text{Dirac}}}{M_R}$$



Consider type Ia seesaw models with a natural neutrino mass hierarchy $m_3 \gg m_2 \gg m_1 \approx 0$

hep-ph/9806440

Single RHN model (1998)

Just add a single RHN to the SM

$$(H_u/v_u)(d\overline{L}_e + e\overline{L}_\mu + f\overline{L}_\tau)\nu_R^{\rm atm} + M_{\rm atm}\overline{\nu_R^{\rm atm}}(\nu_R^{\rm atm})^c$$

To explain atmospheric neutrino oscillations assume

$$d \ll e \sim f$$

Assume charged lepton mass matrix is approximately diagonal (like the quarks)

So that

$$\tan\theta_{23} \sim e/f \sim 1$$

Maximal atmospheric mixing

$$\tan\theta_{13} \sim d/\sqrt{e^2+f^2} \ll 1$$

Small reactor mixing

Two RHN Model (1999)

hep-ph/9904210

hep-ph/9912492

Add a second RHN to the SM to account for solar neutrino oscillations as well

Solar $$(H_u/v_u)(a\overline{L}_e + b\overline{L}_\mu + c\overline{L}_\tau)\nu_R^{\rm sol} + (H_u/v_u)(d\overline{L}_e + e\overline{L}_\mu + f\overline{L}_\tau)\nu_R^{\rm atm}$$ **Atmospheric**

$$+ M_{\rm sol}\overline{\nu_R^{\rm sol}}(\nu_R^{\rm sol})^c + M_{\rm atm}\overline{\nu_R^{\rm atm}}(\nu_R^{\rm atm})^c$$

Simpler matrix notation

$$m^D = \begin{pmatrix} a & d \\ b & e \\ c & f \end{pmatrix} \qquad M_R = \begin{pmatrix} M_{\rm sol} & 0 \\ 0 & M_{\rm atm} \end{pmatrix}$$

Assume diagonal M_R

Assume charged lepton mass matrix is approx diagonal

Seesaw matrix

$$m^\nu = m^D M_R^{-1}(m^D)^T = \begin{pmatrix} \frac{a^2}{M_{\rm sol}} + \frac{d^2}{M_{\rm atm}} & \frac{ab}{M_{\rm sol}} + \frac{de}{M_{\rm atm}} & \frac{ac}{M_{\rm sol}} + \frac{df}{M_{\rm atm}} \\ \frac{ab}{M_{\rm sol}} + \frac{de}{M_{\rm atm}} & \frac{b^2}{M_{\rm sol}} + \frac{e^2}{M_{\rm atm}} & \frac{bc}{M_{\rm sol}} + \frac{ef}{M_{\rm atm}} \\ \frac{ac}{M_{\rm sol}} + \frac{df}{M_{\rm atm}} & \frac{bc}{M_{\rm sol}} + \frac{ef}{M_{\rm atm}} & \frac{c^2}{M_{\rm sol}} + \frac{f^2}{M_{\rm atm}} \end{pmatrix}$$

Single RHN Dominance

$$\frac{(e,f)^2}{M_{\rm atm}} \gg \frac{(a,b,c)^2}{M_{\rm sol}} \qquad d = 0$$

Leads to natural hierarchy

Atmospheric mixing from dominant RHN

$$\tan\theta_{23} \sim \frac{e}{f},$$

$$\tan\theta_{12} \sim \frac{\sqrt{2}a}{b-c}$$

Solar mixing from subdominant RHN

Reactor angle

$$\theta_{13} \lesssim m_2/m_3$$

hep-ph/0506297

Constrained Sequential Dominance (2005)

Recall $m^D = \begin{pmatrix} a & d \\ b & e \\ c & f \end{pmatrix}$

Assume charged lepton mass matrix is exactly diagonal

We now add further constraints to enhance predictivity

$$d = 0 \quad e = f \qquad \tan\theta_{23} \sim e/f \sim 1$$

$$a = b = -c \qquad \tan\theta_{12} \sim \sqrt{2}a/(b-c) \sim 1/\sqrt{2}$$

It turns out that this gives exact tri-bimaximal mixing with

$$\theta_{13} = 0$$

Tri-bimaximal

$$\begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Excluded in 2012

Accidentally occurs due to orthogonality of two columns as in Form Dominance

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \perp \begin{pmatrix} d \\ e \\ f \end{pmatrix}$$

M.C.Chen, S.F.K., 0903.0125

1304.6264

CSD(n) (n=real number) (2013)

More generally assume the two columns of the Dirac matrix are proportional to

$$\begin{pmatrix} d \\ e \\ f \end{pmatrix} \propto \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} a \\ b \\ c \end{pmatrix} \propto \begin{pmatrix} 1 \\ n \\ n-2 \end{pmatrix}$$

For $n \neq 1$ the two columns are no longer orthogonal (violating FD) so now expect $\theta_{13} \neq 0$

But still find approx TB mixing as before (since n cancels)

$$\tan\theta_{23} \sim e/f \sim 1$$

$$\tan\theta_{12} \sim \sqrt{2}a/(b-c) \sim 1/\sqrt{2}$$

$$\theta_{13} \sim (n-1)\frac{\sqrt{2}}{3}\frac{m_2}{m_3}$$

The case $n = 1$ corresponds to the exact TBM case previously with FD but for values of $n \neq 1$ find only approximate TBM

S.F.K., S.Molina Sedgwick and S.J.Rowley, 1808.01005

Flipped CSD(n)

Normal | Flipped | (n= real number)

$$\begin{pmatrix} d \\ e \\ f \end{pmatrix} \propto \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \qquad \begin{pmatrix} a \\ b \\ c \end{pmatrix} \propto \begin{pmatrix} 1 \\ n \\ n-2 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} a \\ b \\ c \end{pmatrix} \propto \begin{pmatrix} 1 \\ n-2 \\ n \end{pmatrix}$$

The two predictions only differ in atmospheric angle and CP phase (solar angle, reactor angle and neutrino mass unchanged)

Octant flipped $\tan\theta_{23} \to \cot\theta_{23}$ $\delta \to \delta + \pi$

Alternatively we could use the following (only differs by unphysical phases):

$$\begin{pmatrix} d \\ e \\ f \end{pmatrix} \propto \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \qquad \begin{pmatrix} a \\ b \\ c \end{pmatrix} \propto \begin{pmatrix} 1 \\ n \\ 2-n \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} a \\ b \\ c \end{pmatrix} \propto \begin{pmatrix} 1 \\ 2-n \\ n \end{pmatrix}$$

Bjorkeroth, SFK 1412.6996

Results for CSD(n) (2014)

Seesaw formula

$$m^\nu = m^D M_R^{-1} (m^D)^T \qquad m^D = \begin{pmatrix} a & d \\ b & e \\ c & f \end{pmatrix} \qquad \begin{pmatrix} d \\ e \\ f \end{pmatrix} \propto \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \text{ and } \begin{pmatrix} a \\ b \\ c \end{pmatrix} \propto \begin{pmatrix} 1 \\ n \\ n-2 \end{pmatrix}$$

$$m^\nu_{(n)} = m_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + m_b e^{i\eta} \begin{pmatrix} 1 & n & n-2 \\ n & n^2 & n(n-2) \\ n-2 & n(n-2) & (n-2)^2 \end{pmatrix}$$

Three effective input parameters (for given n)

N.B. TM1 mixing ∀ n

SRHND $m_a \sim \frac{(e,f)^2}{M_{\rm atm}} \gg \frac{(a,b,c)^2}{M_{\rm sol}} \sim m_b$

Charged leptons diagonal

$m_1 = 0$ $\qquad \theta_{13} \sim (n-1)\frac{\sqrt{2}}{3}\frac{m_2}{m_3}$

n	m_a (meV)	m_b (meV)	η (rad)	θ_{12} (°)	θ_{13} (°)	θ_{23} (°)	$\lvert\delta_{\rm CP}\rvert$ (°)	m_2 (meV)	m_3 (meV)	χ^2	
1	24.8	2.89	3.14	35.3	0	45.0	0	8.66	49.6	485	CSD(1)=TBM
2	19.7	3.66	0	34.5	7.65	56.0	0	8.85	48.8	95.1	CSD(2) Antusch et al 1108.4278
3	27.3	2.62	2.17	34.4	8.39	44.5	92.2	8.69	49.5	3.98	CSD(3) 1304.6264
4	36.6	1.95	2.63	34.3	8.72	38.4	120	8.61	49.8	8.82	CSD(4) 1305.4846
5	45.9	1.55	2.88	34.2	9.03	34.4	142	8.53	50.0	33.8	

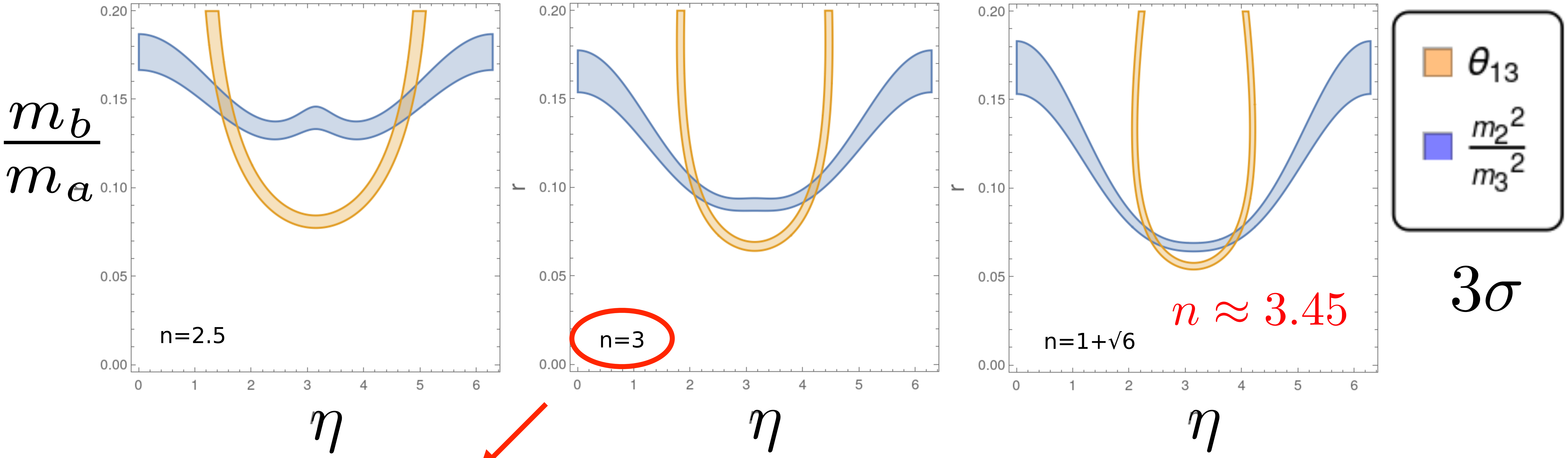
Find best fit for n~3

Highly predictive - 3 inputs for 9 observables (6 so far measured)

1512.07531

Littlest Seesaw CSD(~3) (2015)

F.Costa, SFK 2307.13895



Fit these accurately measured parameters

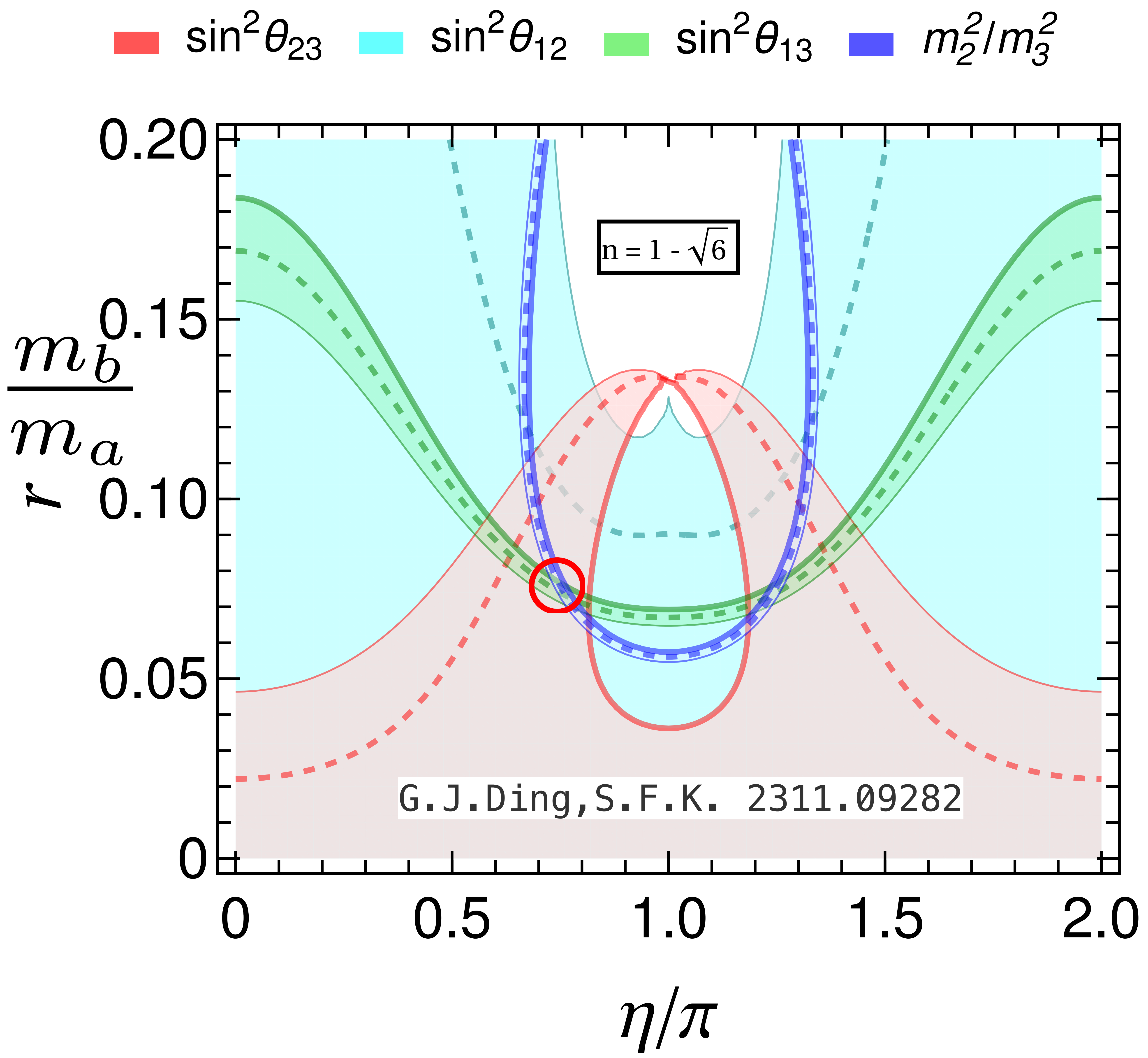
m_b/m_a	$n = 3$	$\eta = 2.11 \pm 0.15$	$\eta = 4.17 \pm 0.15$	Exp. range
0.072 ± 0.004	θ_{12} [°]	$34.32^{+0.20}_{-0.24}$	$34.32^{+0.20}_{-0.25}$	$31.31 - 35.74$
normal	θ_{23} [°]	$45.5^{+2.3}_{-2.4}$	$45.5^{+2.3}_{-2.4}$	$39.6 - 51.9$
normal	δ [°]	$272.2^{+9.6}_{-11.0}$	$87.9^{+11.0}_{-9.6}$	$0 - 44$ & $108 - 360$
flipped	θ_{23} [°]	$44.5^{+2.3}_{-2.4}$	$44.5^{+2.3}_{-2.4}$	$39.6 - 51.9$
flipped	δ [°]	$92.2^{+9.6}_{-11.0}$	$267.9^{+11.0}_{-9.6}$	$0 - 44$ & $108 - 360$

Predict the less well measured solar, and atmospheric angles and CP phase δ

N.B. not just $\cos\delta$

G.J.Ding, S.F.K., X.G.Liu and J.N.Lu, 1910.03460

Littlest Modular Seesaw (2019)



CSD(n) $n = 1 + \sqrt{6} \approx 3.45$

Flipped modular Littlest seesaw		
	bf	allowed ranges
η/π	0.742	[0.725, 0.806]
r	0.0758	[0.0683, 0.0786]
$\sin^2\theta_{13}$	0.0231	[0.0205, 0.0240]
$\sin^2\theta_{12}$	0.318	[0.317, 0.319]
$\sin^2\theta_{23}$	0.535	[0.517, 0.595]
δ_{CP}/π	−0.452	[−0.478, −0.354]
β/π	−0.441	[−0.562, −0.409]
m_2^2/m_3^2	0.0283	[0.0270, 0.0321]

How does $n = 1 + \sqrt{6}$ originate?

De Anda, SFK 2304.05958

Littlest Modular Seesaw from Orbifold

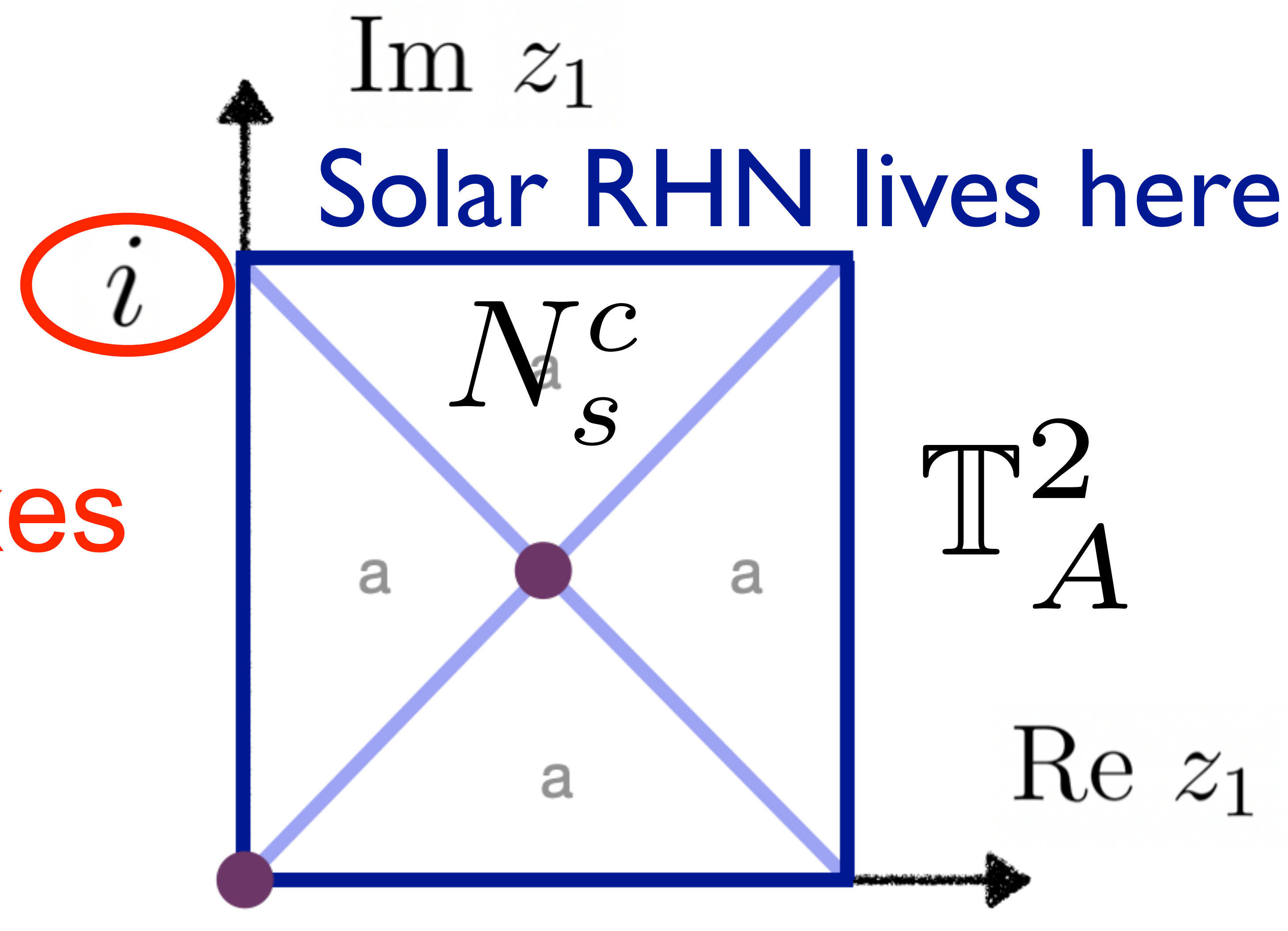
6d SUSY orbifold $(\mathbb{T}^2)^3/(\mathbb{Z}_4 \times \mathbb{Z}_2)$

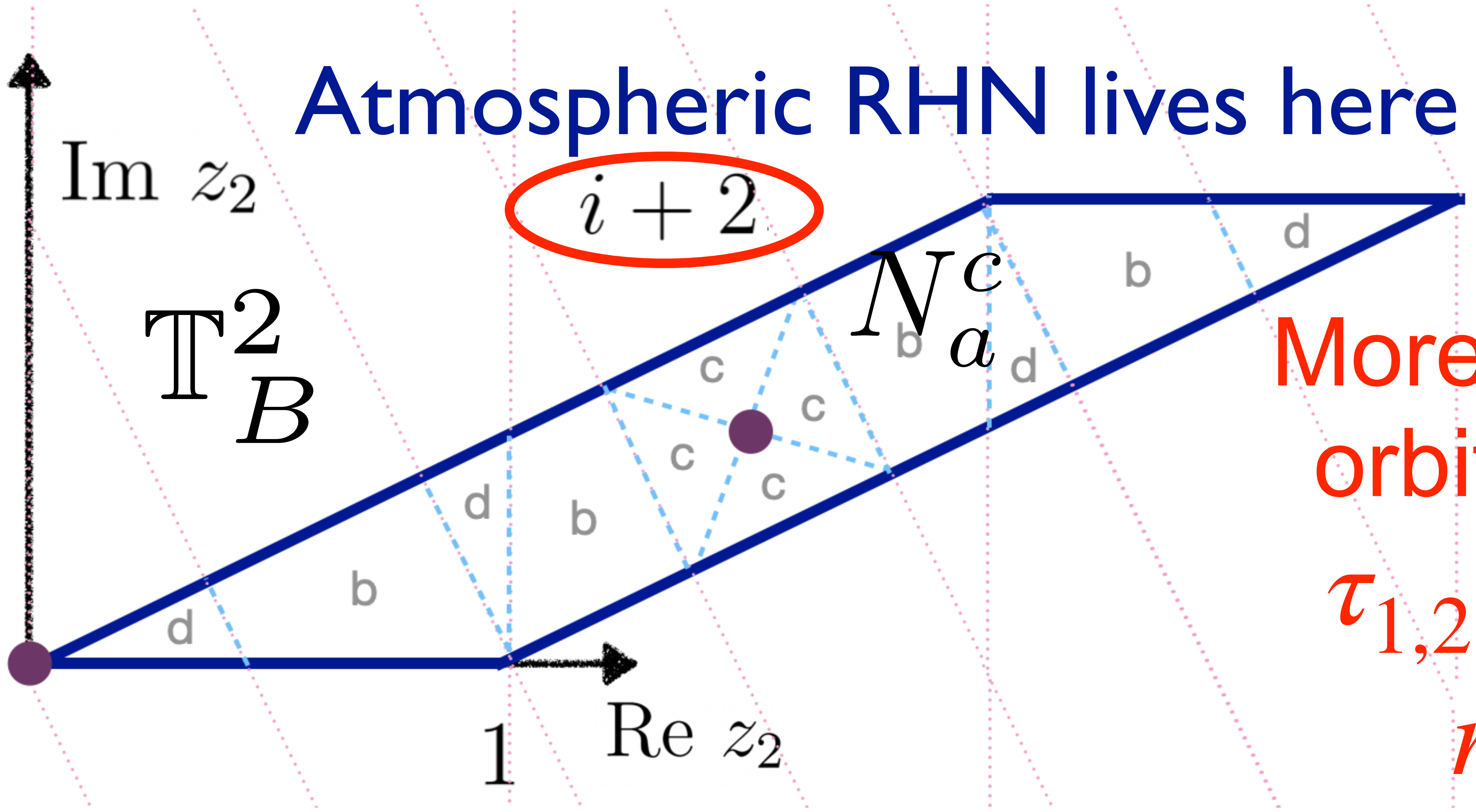
S_4 in each 2d space $S_4^A \times S_4^B \times S_4^C$

10d model with 3 factorisable tori

S_4^A

Orbifold fixes $\tau_1 = i$





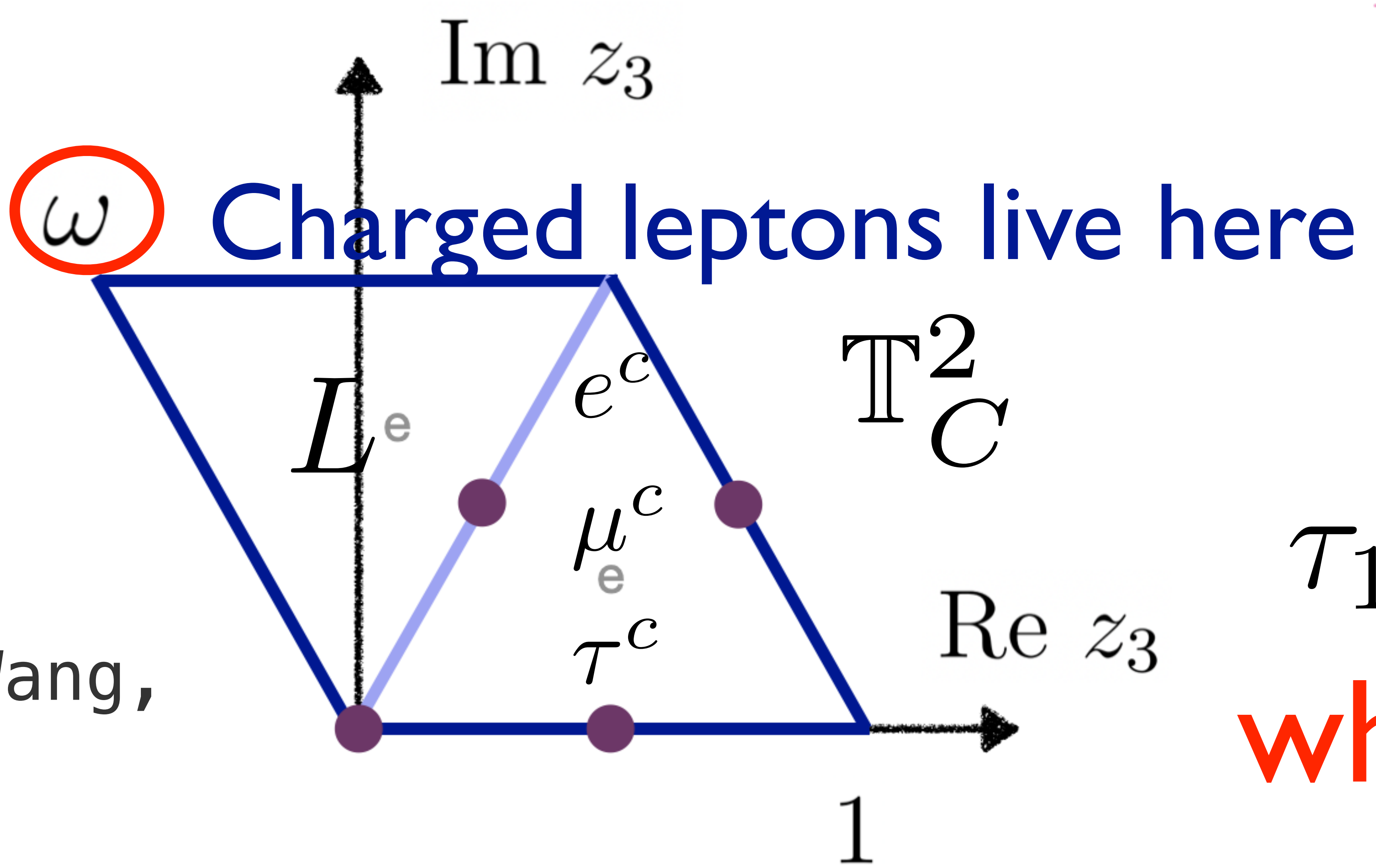
S_4^B

More precisely orbifold fixes $\tau_{1,2} = i + n$

$n \leq 3$

Fix τ_3 by stability

S_4^C



S.F.K., X.Wang, 2310.10369

Lattice vectors for each torus are $(1, \tau_i)$

$$\tau_1 = i,\ \tau_2 = i+2,\ \tau_3 = \omega = e^{\frac{2\pi i}{3}}$$

which define 3 fixed moduli

De Anda, SFK 2304.05958

Littlest Modular Seesaw from Orbifold

de Medeiros Varzielas,S.F.K.,M.Levy 2211.00654

Also see Multiple moduli talk by Zhou

Field	S_4^A	S_4^B	S_4^C	$2k_A$	$2k_B$	$2k_C$	Loc
L	**1**	**1**	**3**	0	0	0	$\mathbb{T}^2_C$
e^c	**1**	**1**	**1**	0	0	-6	$\mathbb{T}^2_C$
μ^c	**1**	**1**	**1**	0	0	-4	$\mathbb{T}^2_C$
τ^c	**1**	**1**	**1**	0	0	-2	$\mathbb{T}^2_C$
N_a^c	**1**	**1**	**1**	0	-4	0	$\mathbb{T}^2_B$
N_s^c	**1**	**1**	**1**	-2	0	0	$\mathbb{T}^2_A$
Φ_{BC}	**1**	**3**	**3**	0	0	0	Bulk
Φ_{AC}	**3**	**1**	**3**	0	0	0	Bulk

Yuk/Mass	S_4^A	S_4^B	S_4^C	$2k_A$	$2k_B$	$2k_C$
$Y_e(\tau_3)$	**1**	**1**	**3**	0	0	6
$Y_\mu(\tau_3)$	**1**	**1**	**3**	0	0	4
$Y_\tau(\tau_3)$	**1**	**1**	**3**	0	0	2
$Y_a(\tau_2)$	**1**	**3**	**1**	0	4	0
$Y_s(\tau_1)$	**3**	**1**	**1**	2	0	0
$M_a(\tau_2)$	**1**	**1**	**1**	0	8	0
$M_s(\tau_1)$	**1**	**1**	**1**	4	0	0

Yukawa couplings are modular forms evaluated at the fixed points of the moduli fields (the lattice vectors)

Fixed points of S_4

	τ	$Y_{\mathbf{3}}^{(2)}(\tau),\ Y_{\mathbf{3,I}}^{(6)}(\tau)$	$Y_{\mathbf{3}}^{(4)}(\tau),\ Y_{\mathbf{3'}}^{(6)}(\tau)$
τ_1	i	$(1, 1+\sqrt{6}, 1-\sqrt{6})$	$(1, -\frac{1}{2}, -\frac{1}{2})$
	$i+1$	$(1, -\frac{\omega}{3}(1+i\sqrt{2}), -\frac{\omega^2}{3}(1+i\sqrt{2}))$	$(0, 1, -\omega)$
τ_2	$i+2$	$(1, \frac{1}{3}(-1+i\sqrt{2}), \frac{1}{3}(-1+i\sqrt{2}))$	$(0, 1, -1)$
	$i+3$	$(1, \omega(1+\sqrt{6}), \omega(1-\sqrt{6}))$	$(1, -\frac{\omega}{2}, -\frac{\omega^2}{2})$

	τ	$Y_{\mathbf{3}}^{(2)}(\tau)$	$Y_{\mathbf{3}}^{(4)}(\tau), Y_{\mathbf{3'}}^{(4)}(\tau)$	$Y_{\mathbf{3,II}}^{(6)}(\tau), Y_{\mathbf{3'}}^{(6)}(\tau)$
τ_3	ω	$(0,1,0)$	$(0,0,1)$	$(1,0,0)$
	$\omega+1$	$(1,1,-\frac{1}{2})$	$(1,-\frac{1}{2},1)$	$(1,-2,-2)$
	$\omega+2$	$(1,-\frac{\omega^2}{2},\omega)$	$(1,\omega^2,-\frac{\omega}{2})$	$(1,-2\omega^2,-2\omega)$
	$\omega+3$	$(1,\omega,-\frac{\omega^2}{2})$	$(1,-\frac{\omega}{2},\omega^2)$	$(1,-2\omega,-2\omega^2)$

$$\frac{1}{\Lambda}\left[L\Phi_{BC}Y_aN_a^c + L\Phi_{AC}Y_sN_s^c\right]H_u + \left[LY_ee^c + LY_\mu\mu^c + LY_\tau\tau^c\right]H_d + \frac{1}{2}M_aN_a^cN_a^c + \frac{1}{2}M_sN_s^cN_s^c\,.$$

$$\begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} \qquad \begin{pmatrix} 0 & b \\ a & b\left(1-\sqrt{6}\right) \\ -a & b\left(1+\sqrt{6}\right) \end{pmatrix}$$

Diagonal Charged leptons

Dirac neutrino matrix

Flipped CSD(n)

$n = 1+\sqrt{6}$

$n \approx 3.45$

de Anda, SFK 2312.09010

Littlest Modular Seesaw from Orbifold GUTs

Field	$SU(5)$	S_4^A	S_4^B	S_4^C	$2k_A$	$2k_B$	$2k_C$	Loc
F	$\mathbf{5}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	0	0	0	$\mathbb{T}^2_C$
T_1	$\mathbf{10}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	0	1	$\mathbb{T}^2_C$
T_2	$\mathbf{10}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	0	1/2	$\mathbb{T}^2_C$
T_3	$\mathbf{10}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	0	0	$\mathbb{T}^2_C$
N_a^c	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	-4	0	$\mathbb{T}^2_B$
N_s^c	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	-2	0	0	$\mathbb{T}^2_A$
H_u	$\mathbf{5}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	0	0	Bulk
H_d	$\overline{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	0	1/2	Bulk
H_{45}	$\mathbf{45}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	0	1/2	Bulk
$H_{\overline{45}}$	$\overline{\mathbf{45}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	0	0	Bulk
Φ_{BC}	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	0	0	0	Bulk
Φ_{AC}	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{3}$	0	0	0	Bulk
ξ_F	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	0	$-5/2$	$\mathbb{T}^2_C$
ξ_T	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	0	$-1/2$	$\mathbb{T}^2_C$

Yuk/Mass	S_4^A	S_4^B	S_4^C	$2k_A$	$2k_B$	$2k_C$
$Y_e(\tau_3)$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	0	0	6
$Y_\mu(\tau_3)$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	0	0	4
$Y_\tau(\tau_3)$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	0	0	2
$Y_a(\tau_2)$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	0	4	0
$Y_s(\tau_1)$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	2	0	0
$M_a(\tau_2)$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	8	0
$M_s(\tau_1)$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	4	0	0

- 10d orbifold B.C.s break SU(5) with DT splitting
- Triangular form of M_d, M_e yields CKM mixing plus very suppressed charged lepton corrections
- Two weightons ξ_F, ξ_T control the hierarchies

$$M_u = \begin{pmatrix} 0 & y^u_{12}\tilde{\xi}^3_{T,F}e^{i\phi_{u1}} & y^u_{13}\tilde{\xi}^2_T \\ y^u_{12}\tilde{\xi}^3_{T,F}e^{i\phi_{u1}} & y^u_{22}\tilde{\xi}^2_T & y^u_{23}\tilde{\xi}_T e^{i\phi_{u2}} \\ y^u_{13}\tilde{\xi}^2_T & y^u_{23}\tilde{\xi}_T e^{i\phi_{u2}} & y^u_{33} \end{pmatrix} v_u$$

de Medeiros Varzielas, S.F.K., M.Levy 2309.15901

$$M_d = \begin{pmatrix} y_{d11}\tilde{\xi}^3_F & y_{d12}\tilde{\xi}^2_F\tilde{\xi}_T & y_{d13}\tilde{\xi}_F\tilde{\xi}^2_T \\ 0 & y_{d22}\tilde{\xi}^2_F & y_{d23}\tilde{\xi}_F\tilde{\xi}_T e^{i\phi_{d2}} \\ 0 & 0 & y_{d33}\tilde{\xi}_F \end{pmatrix} v_d$$

Upper/lower LR triangular form

$$M_e = \begin{pmatrix} y_{e11}\tilde{\xi}^3_F & 0 & 0 \\ y_{e21}\tilde{\xi}^2_F\tilde{\xi}_T & y_{e22}\tilde{\xi}^2_F & 0 \\ y_{e31}\tilde{\xi}_F\tilde{\xi}^2_T & y_{e32}\tilde{\xi}_F\tilde{\xi}_T e^{i\phi_{d1}} & y_{e33}\tilde{\xi}_F \end{pmatrix} v_d,$$

Flipped CSD(n)

$n = 1 + \sqrt{6}$

Dirac neutrino matrix

$$M_D = \begin{pmatrix} 0 & y_s\tilde{\Phi}_{AC} \\ y_a\tilde{\Phi}_{BC} & y_s\tilde{\Phi}_{AC}(1-\sqrt{6}) \\ -y_a\tilde{\Phi}_{BC} & y_s\tilde{\Phi}_{AC}(1+\sqrt{6}) \end{pmatrix} v_u, \quad M_N = \begin{pmatrix} M_a & 0 \\ 0 & M_s \end{pmatrix}$$

Summary

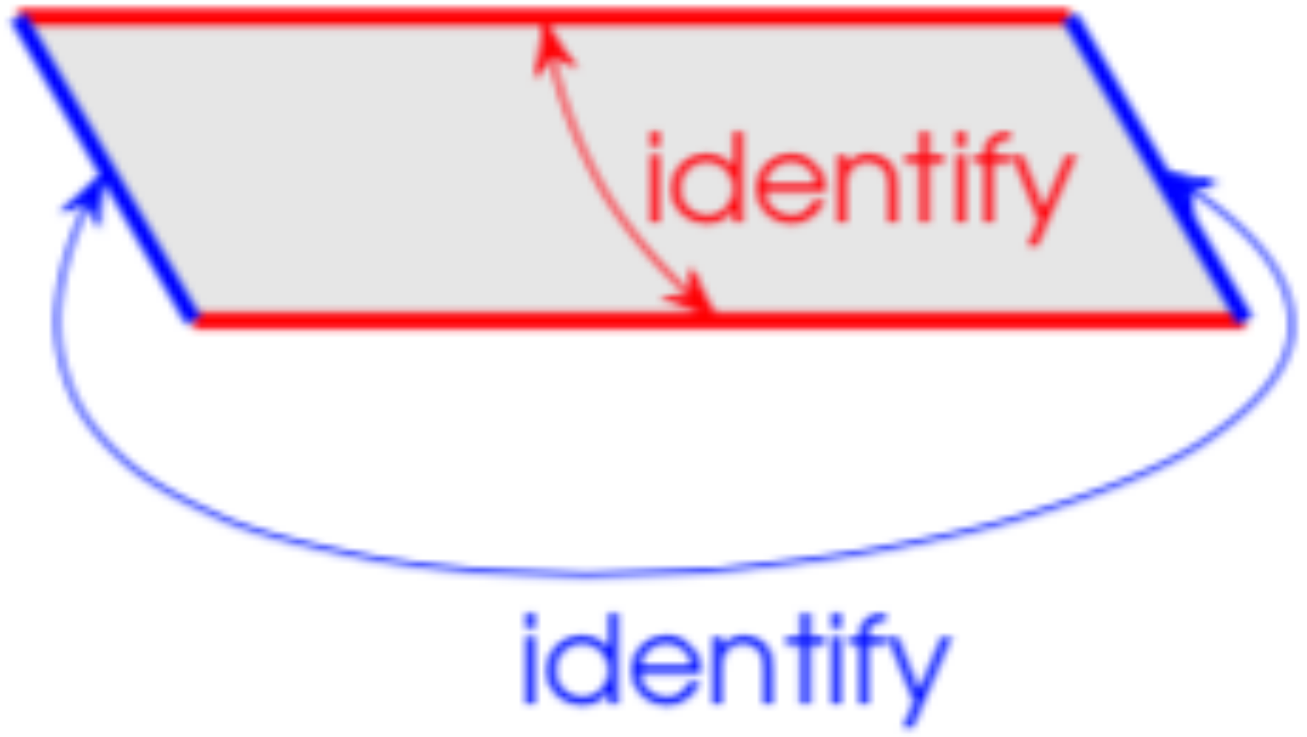
- Flavour problem motivates family/flavour symmetry
- Neutrino mass and mixing motivates non-Abelian
- A4, S4, A5 can enforce TBM, BM, GR patterns via Z^T_N and $Z^S_2 x Z^U_2$
- Reactor angle can be non-zero if only a subgroup is preserved
- Breaking Z^T_N leads to charged lepton corrections and solar sum rules
- Breaking Z^U_2 preserves 1st or 2nd columns, atmospheric sum rules
- Such symmetry predictions will be tested in coming years
- Type 1a seesaw: 2RHN + SRHND for natural hierarchy
  $m_3 \gg m_2 \gg m_1 \approx 0$ (large mixing with no tuning)
- Predictivity motivates CSD(n) with n~3 a.k.a. Littlest Seesaw
- Littlest Modular Seesaw yields excellent predictions
- Can arise from 10d orbifold and may be combined with SU(5) GUTs

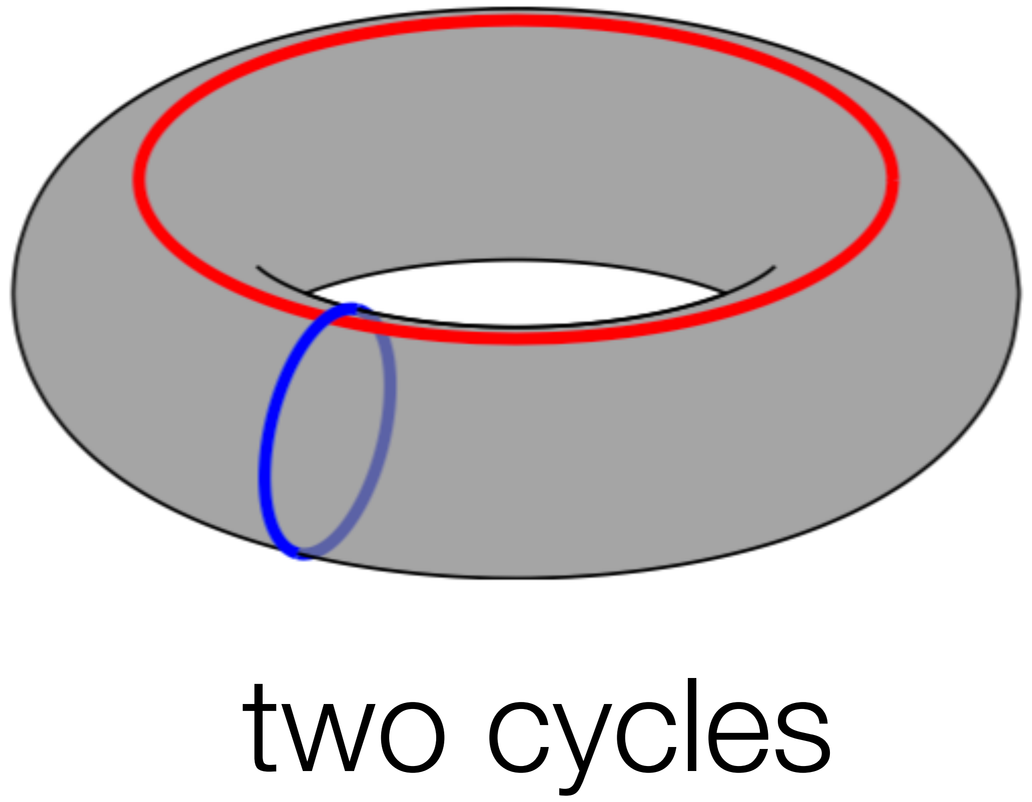
Back-up: our bottom-up Orbifold

For more details see:

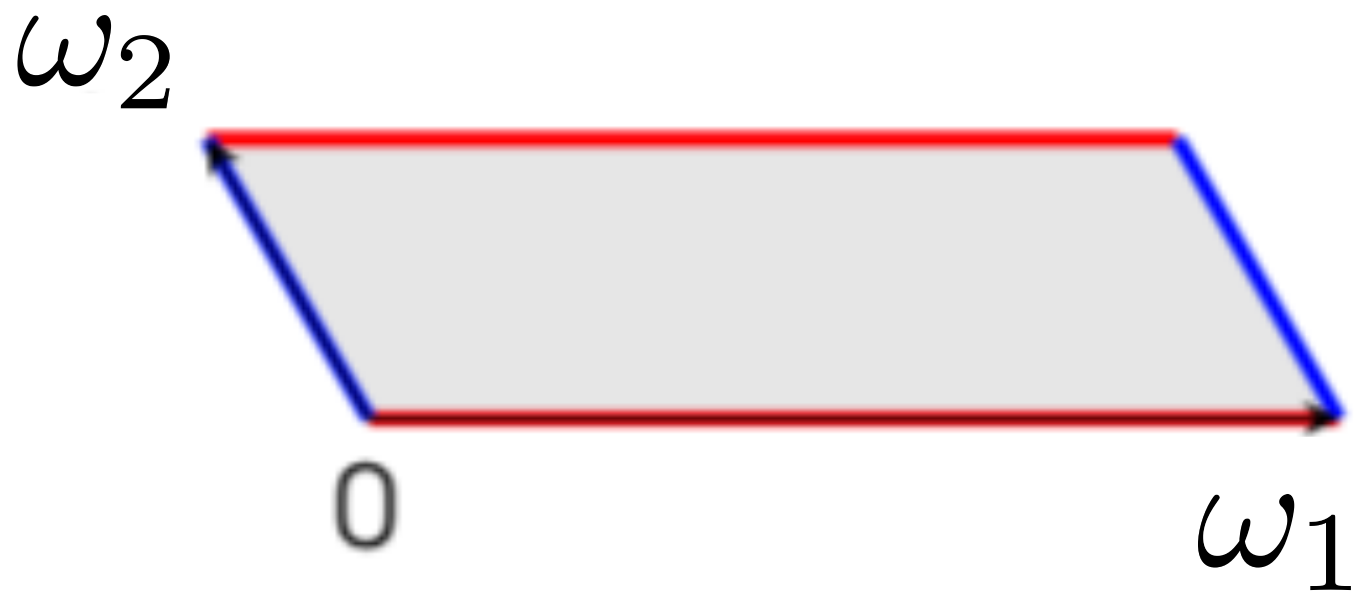
`De Anda, SFK 2304.05958`

Consider Two Extra Dimensions compactified on a torus, equivalent to a parallelogram

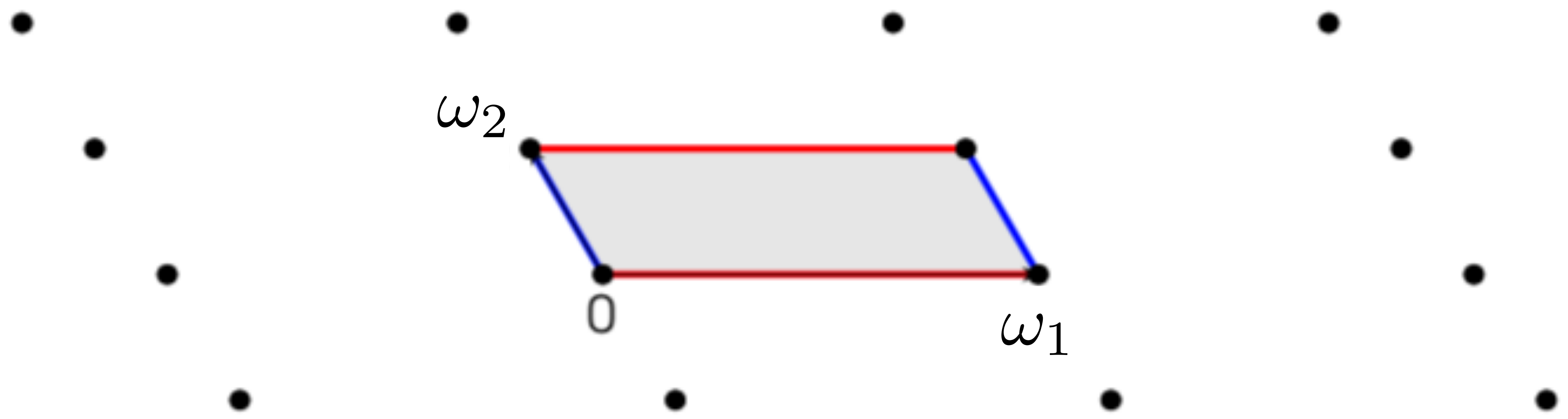




Surface area of torus = area of parallelogram



Parallelogram is defined by two vectors



Adding the vectors together with arbitrary integers generates a lattice of points.
Any two lattice points give a new torus.

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Bottom-up Orbifolds

Two dimensional twisted torus $\mathbb{T}^2$ $\quad z = x_5 + ix_6$

Identify $\quad z = z + \omega_1, \quad z = z + \omega_2$

Define **modulus field** $\quad \tau \equiv \omega_1/\omega_2$

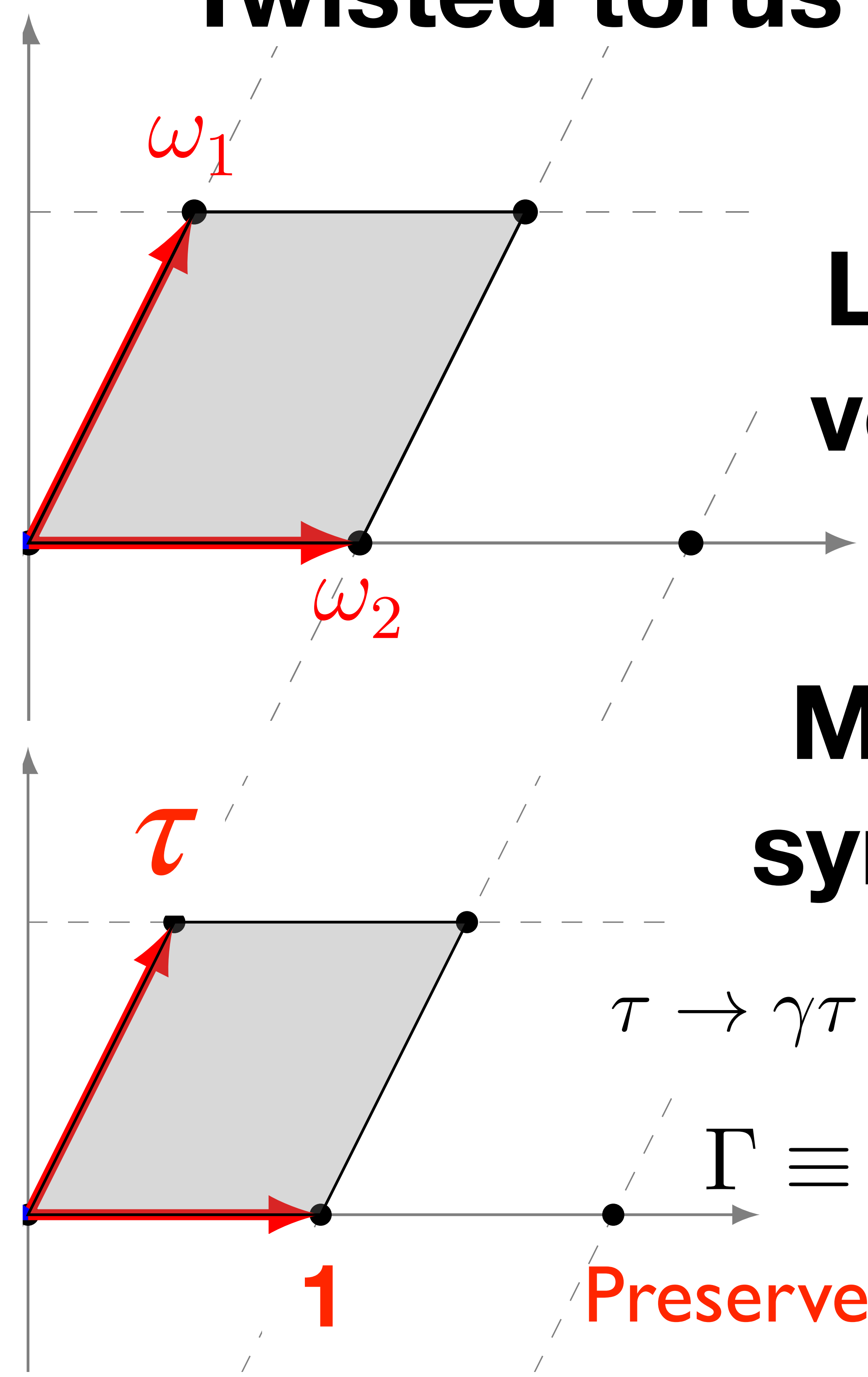
Basis vectors become $\quad \{1, \tau\}$

Then identify $\quad z \sim z + 1, \quad z \sim z + \tau$

$\mathbb{T}^2/\mathbb{Z}_N$ **Orbifold condition** $\quad z \sim e^{2i\pi/N} z$ **Fixes** τ

$$\begin{cases} N = 2, & \tau = z \in \mathbb{C}, \\ N = 3, & \tau = \omega, \qquad \omega = e^{2i\pi/3} \\ N = 4, & \tau = i, \end{cases}$$

Twisted torus



Lattice vectors

Modular symmetry

$$\tau \to \gamma\tau = \frac{a\tau + b}{c\tau + d}$$

$$\Gamma \equiv SL(2, \mathbb{Z})$$

Preserves torus area

Consider 6d space with 3 factorisable tori

Three twisted tori $z_i \sim z_i + 1, \quad z_i \sim z_i + \tau_i$ with three **moduli fields**

These are the complex structure moduli in string theory

SUSY preserving orbifolds $(\mathbb{T}^2)^3/(\mathbb{Z}_N \times \mathbb{Z}_M)$

$$\theta_N : \ (x, z_1, z_2, z_3) \sim (x, \alpha_N z_1, \beta_N z_2, \gamma_N z_3),$$
$$\theta_M : \ (x, z_1, z_2, z_3) \sim (x, \alpha_M z_1, \beta_M z_2, \gamma_M z_3).$$

$\alpha_{N,M}, \beta_{N,M}, \gamma_{N,M}$ are Nth, Mth roots of unity.

Allowed SUSY preserving orbifolds $(\mathbb{T}^2)^3/(\mathbb{Z}_N \times \mathbb{Z}_M)$

M.Fischer, M.Ratz, J.Torrado, P.K.S.Vaudrevange 1209.3906

(N, M)	$(\alpha_N, \beta_N, \gamma_N)$	$(\alpha_M, \beta_M, \gamma_M)$	(τ_1, τ_2, τ_3)
			Fixes
$(3,1)$	(ω,ω,ω)	$(1,1,1)$	(ω,ω,ω)
$(4,1)$	$(i,i,-1)$	$(1,1,1)$	(i,i,z)
$(6,1)_I$	$(-\omega^2,-\omega^2,\omega^2)$	$(1,1,1)$	$(\{\omega,\rho/\sqrt{3}\},\{\omega,\rho/\sqrt{3}\},\omega)$
$(6,1)_{II}$	$(-\omega^2,\omega,-1)$	$(1,1,1)$	$(\{\omega,\rho/\sqrt{3}\},\omega,z)$
$(2,2)$	$(1,-1,-1)$	$(-1,1,-1)$	(z,z,z)
$(4,2)$	$(i,-i,1)$ $\mathbb{Z}_4$	$(1,-1,-1)$ $\mathbb{Z}_2$	(i,i,z)
$(6,2)_I$	$(-\omega^2,1,-\omega)$	$(1,-1,-1)$	$(\{\omega,\rho/\sqrt{3}\},z,\{\omega,\rho/\sqrt{3}\})$
$(6,2)_{II}$	$(\omega^2,-\omega^2,-\omega^2)$	$(1,-1,-1)$	$(\omega,\{\omega,\rho/\sqrt{3}\},\{\omega,\rho/\sqrt{3}\})$
$(3,3)$	$(1,\omega,\omega^2)$	$(\omega,1,\omega^2)$	(ω,ω,ω)
$(6,3)$	$(-\omega^2,1,-\omega)$	$(1,\omega,\omega^2)$	$(\{\omega,\rho/\sqrt{3}\},\{\omega,\rho/\sqrt{3}\},\omega)$
$(4,4)$	$(1,i,-i)$	$(i,1,-i)$	(i,i,i)
$(6,6)$	$(1,-\omega^2,-\omega)$	$(-\omega^2,1,-\omega)$	$(\{\omega,\rho/\sqrt{3}\},\{\omega,\rho/\sqrt{3}\},\{\omega,\rho/\sqrt{3}\})$

$\omega = e^{2i\pi/3}$

$\rho = e^{i\pi/6}$

$z \in \mathbb{C}$

$(\mathbb{T}^2)^3/(\mathbb{Z}_4 \times \mathbb{Z}_2)$

Now consider this example

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Consider (N,M)= (4,2) example from table

$$(\mathbb{T}^2)^3/(\mathbb{Z}_4 \times \mathbb{Z}_2)$$

6d SUSY orbifold

$$\theta_4 : (x, z_1, z_2, z_3) \sim (x, iz_1, -iz_2, z_3)$$
$$\theta_2 : (x, z_1, z_2, z_3) \sim (x, z_1, -z_2, -z_3)$$

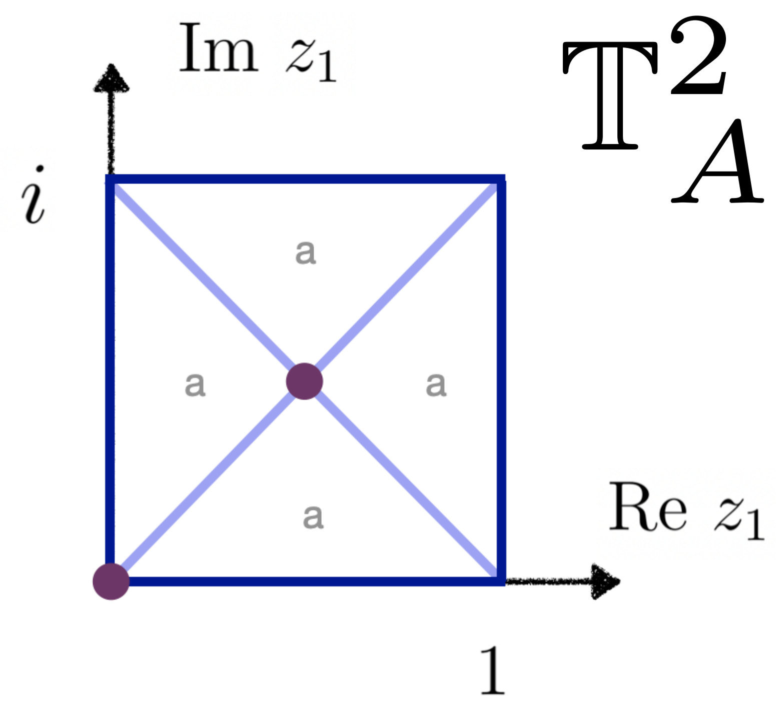
Fixes $\tau_{1,2} = i + n_{1,2}, \quad | \quad n_{1,2} \in \mathbb{Z},$

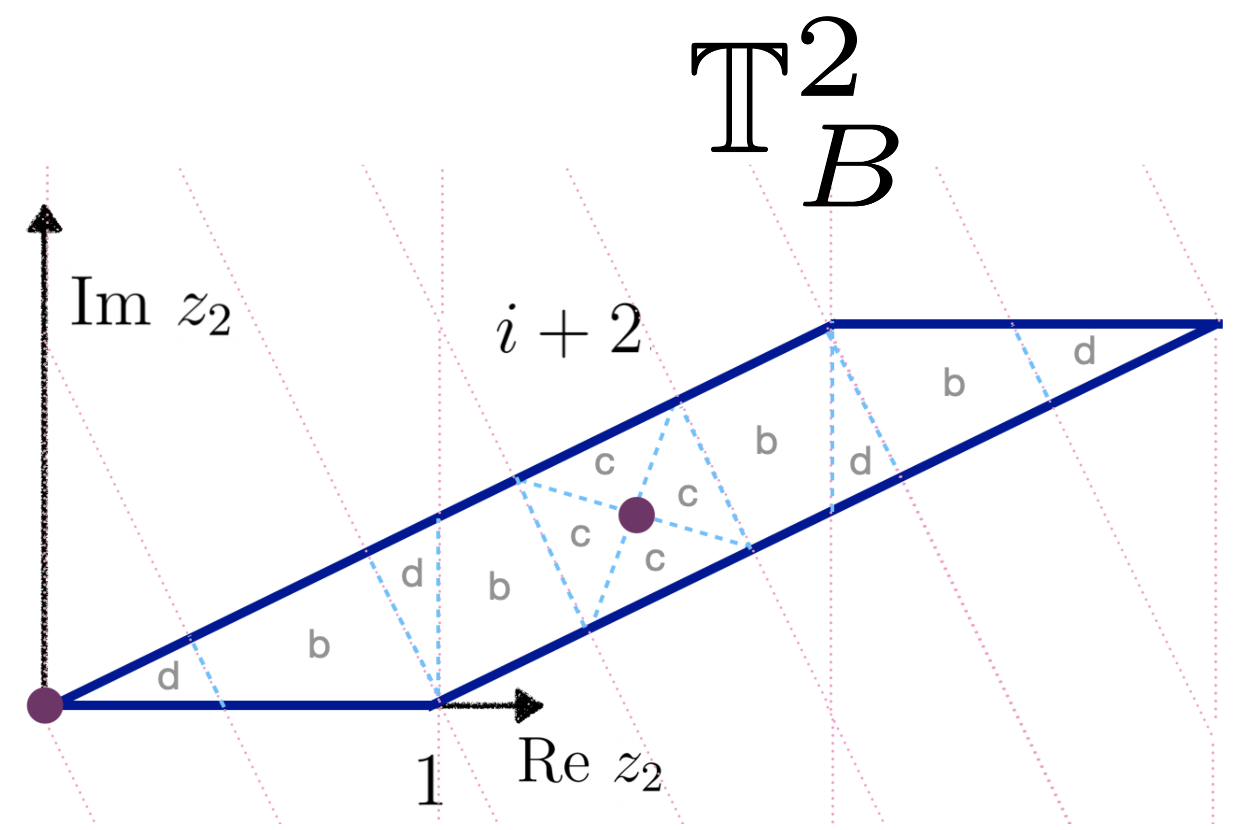
Assume $\tau_3 = \omega$ **(stability)**

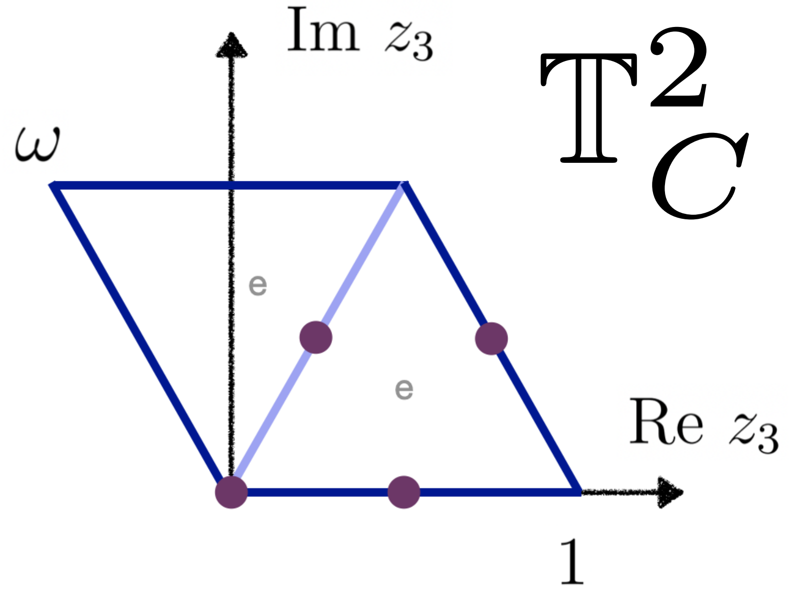
Choose 6d invariant fixed branes

$$\mathbb{T}^2_A = (x, z_1, 0, 0), \qquad \mathbb{Z}_4$$
$$\mathbb{T}^2_B = (x, 0, z_2, 0), \qquad \mathbb{Z}_4$$
$$\mathbb{T}^2_C = (x, 0, 0, z_3), \qquad \mathbb{Z}_2$$

which all overlap the 6d origin







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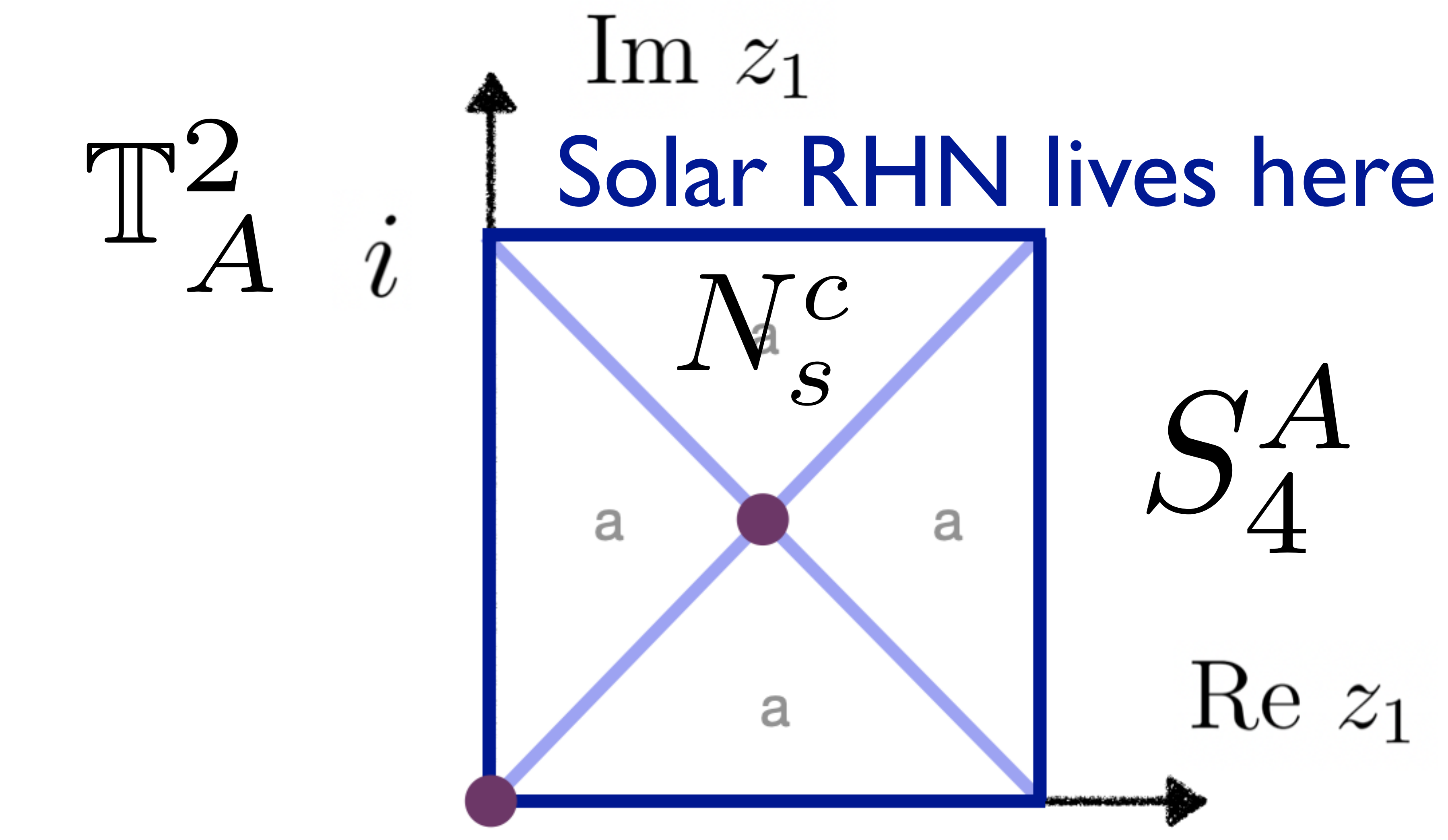
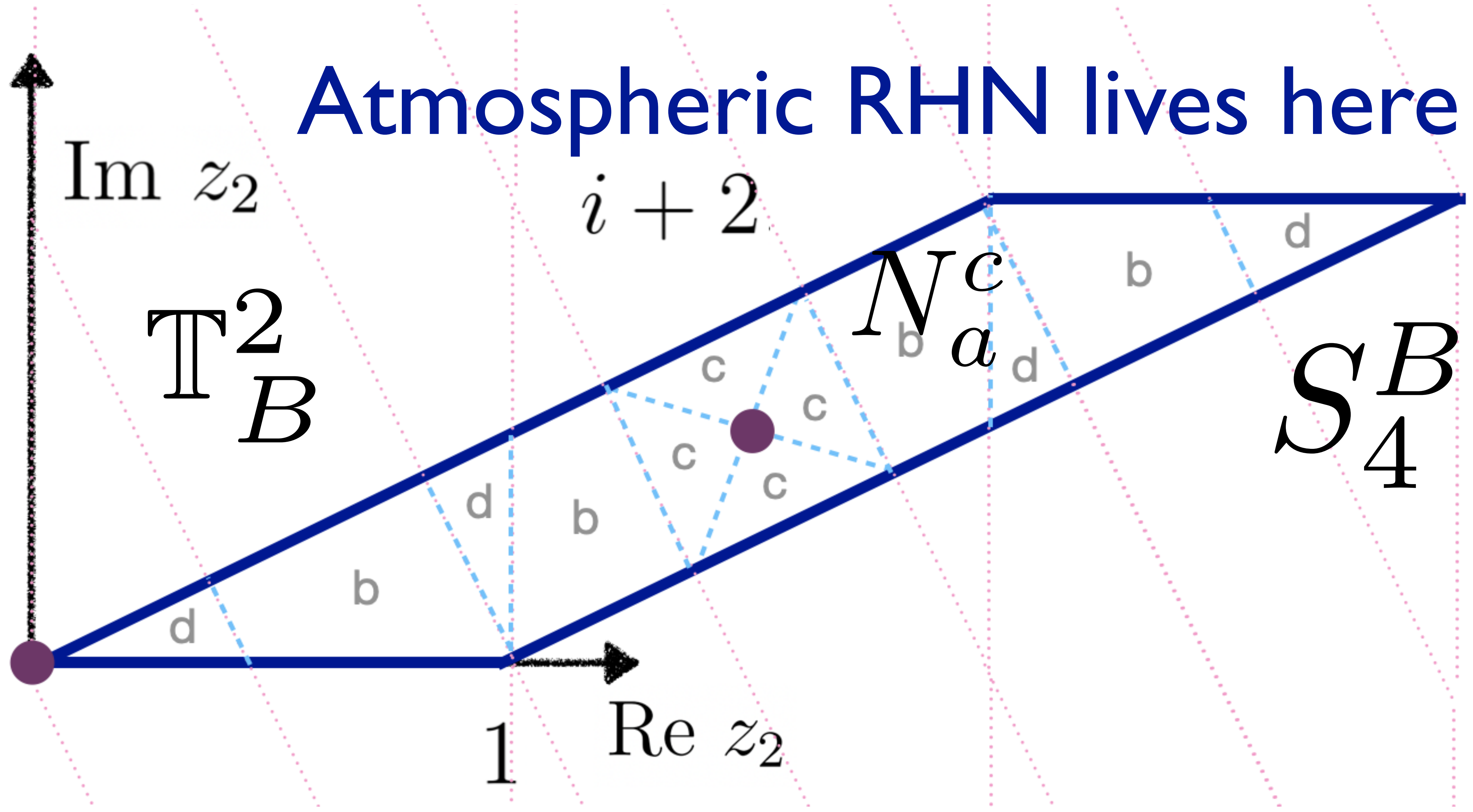
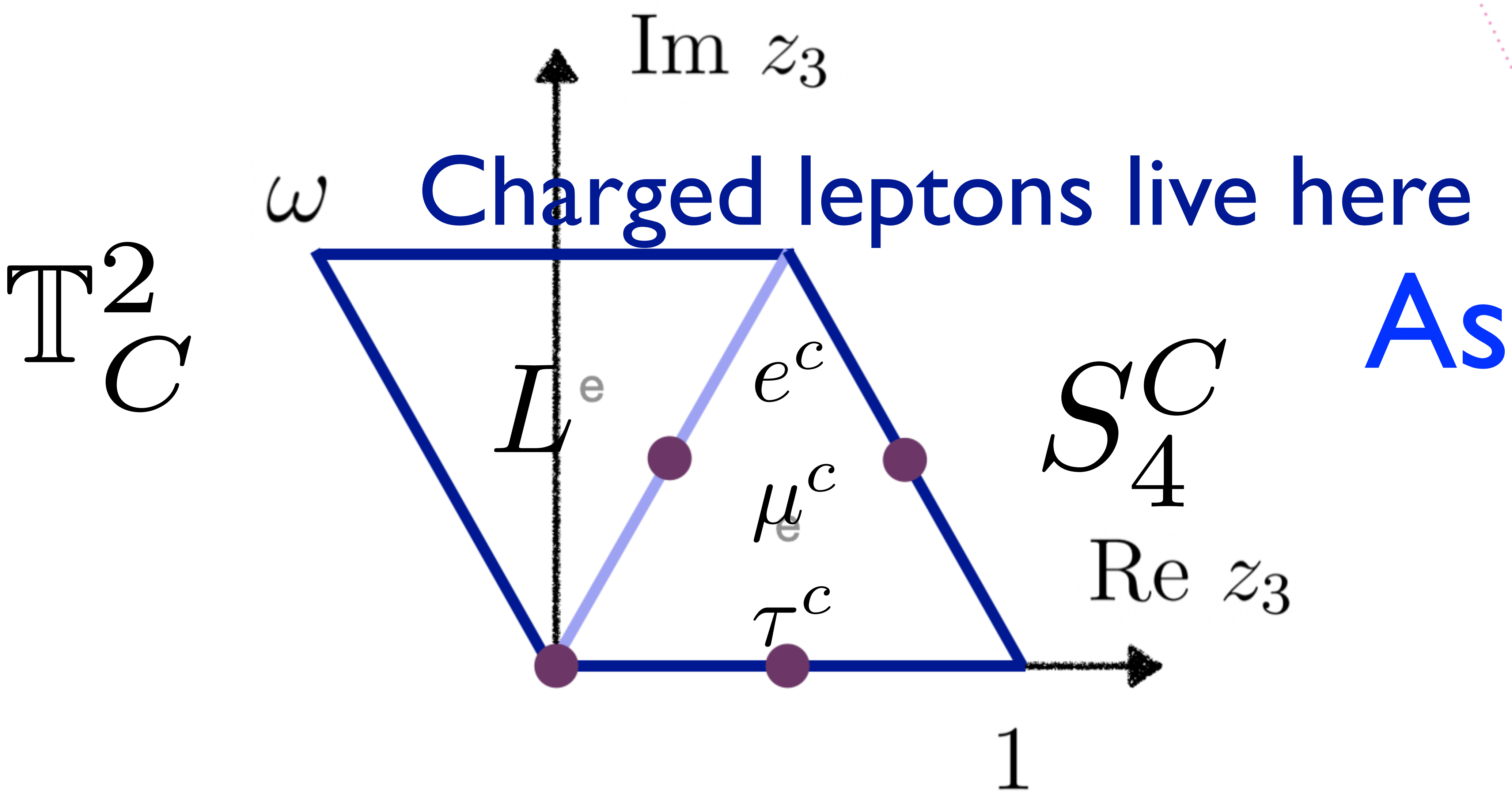
Littlest Modular Seesaw from Orbifold







Assume modular S4 in each 2d space

$$S_4^A \times S_4^B \times S_4^C$$