Gravitational Waves from Electroweak Phase Transition

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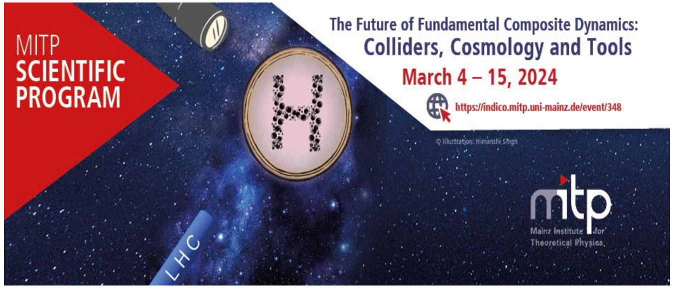

**MITP - Mainz Institute for Theoretical Physics,
Johannes Gutenberg University Mainz
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Motivation

- Gravitational waves (GW) first detected in 2016.

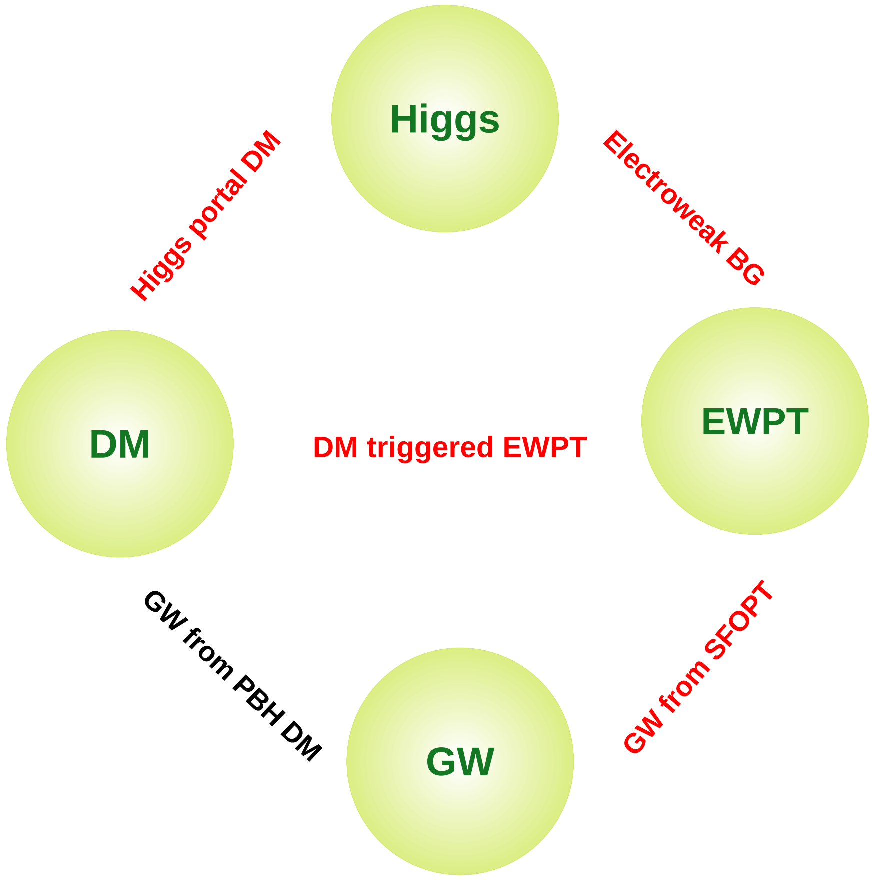
  -- A new window into the early Universe

- We have data from NANOGrav.

- New probes of Particle Physics phenomenology beyond the TeV scale (LHC)

  Complementarity!

Any connection among them ?



- **Why complementarity ?**

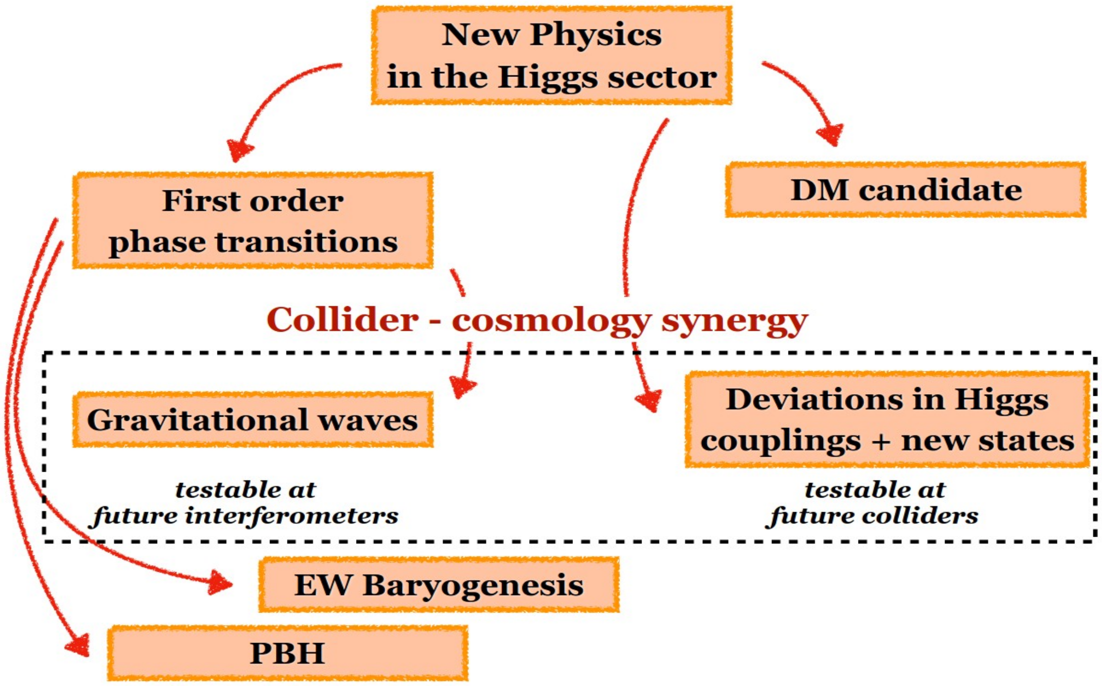


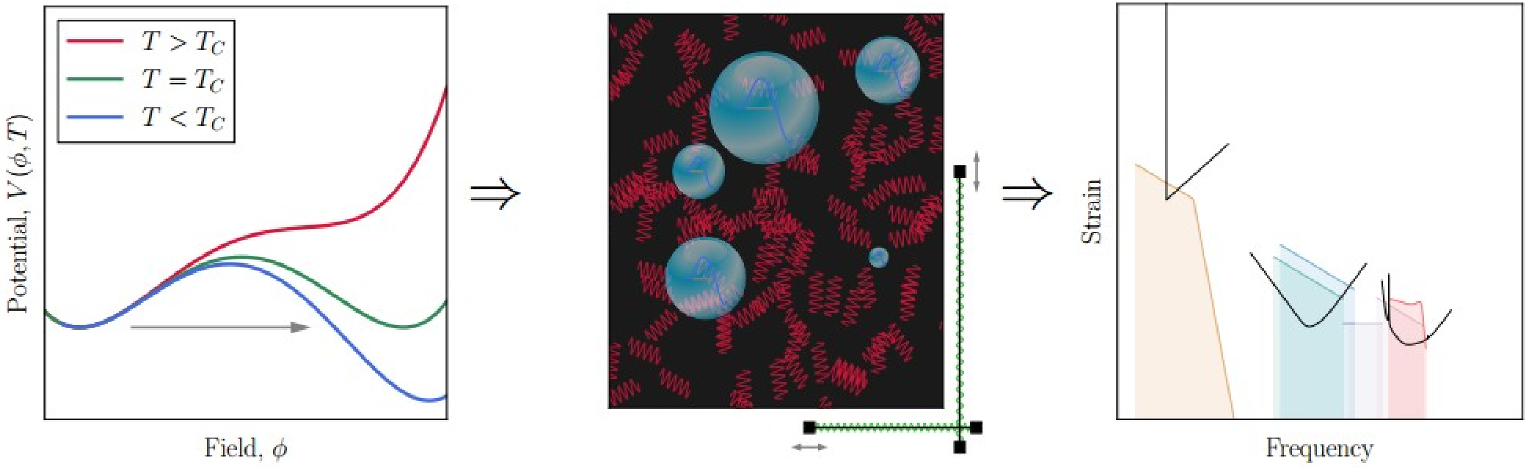
*Image: ***Luigi Delle Rose***

Gravitational Wave Observation → **A complementary way to explore or discard new physics model**

First-order EW Phase Transitions

Why

- Provides a better Understanding of EWSB in Early Universe.
- A possible answer to the open mysteries at Interface of Particle Physics & Cosmology. ***Electroweak Baryogenesis*.**
- **(Possible) Cosmological Relics from the EW Epoch.**



P. Athron et al., arXiv:2305.02357 [hep-ph]

First-order EW Phase Transitions

- The main ingredient to investigate the phase transition is the effective potential, in general,

$$V_{\rm eff}^{T} = V_{\rm tree} + \Delta V + V'^{\rm 1-loop}_{\rm CW} + V^{\rm 1-loop}_{T\neq 0} + V_{\rm ct}$$

$V_{\rm tree}$ is the tree level potential of the underlying theory.

$$V'^{\rm 1-loop}_{\rm CW} = \frac{1}{64\pi^2}\sum_{i=B,F}(-1)^{F_i} n_i m_i^4(\phi_\alpha, T)\left[\log\left(\frac{m_i^2(\phi_\alpha,T)}{\Lambda^2}\right) - \mathcal{C}_i\right]$$

$$m_i^2(\phi_\alpha, T) = m_i^2(\phi_\alpha) + c_i T^2,\ c_i \to \text{Daisy coefficients.}$$

c_i can be calculated using the high-temperature limit with $\frac{1}{T^2}\frac{\partial^2 V^{\rm 1-loop}_{T\neq 0}}{\partial\phi_i\partial\phi_j}$

$$V^{\rm 1-loop}_{T\neq 0} = \frac{T^4}{2\pi^2}\sum_{i=B,F}(-1)^{F_i} n_i J_{B/F}\left(\frac{m_i^2(\phi_\alpha,T)}{T^2}\right)$$

$$J_{B/F}\left(x^2 \equiv \frac{m_i^2(\phi_\alpha,T)}{T^2}\right) = \pm\int_0^\infty dy\ y^2 \log\left(1 \mp e^{-\sqrt{x^2+y^2}}\right)$$

V_{ct} = counter term potential

$$\frac{\partial}{\partial\phi_i}\left(V_{\rm eff} + V_{\rm ct}\right)\Big|_{\phi_i=\langle\phi_i\rangle} = 0 \quad \text{and} \quad \frac{\partial^2}{\partial\phi_i\partial\phi_j}\left(V_{\rm eff} + V_{\rm ct}\right)\Big|_{\phi_i=\langle\phi_j\rangle} = 0,$$

Theoretical Challenges

Gauge dependence of the effective potential

This gauge dependence is not surprising since the effective potential is not directly a physical quantity. However, gauge dependence has nonetheless presented challenges in phase transition calculations.

The order parameter (v_c/T_c) that defines the strength of the PT is inherently gauge dependent.

$$\text{For a } \varphi^4 \text{ theory, } V_{\rm eff}(\varphi, T) = m^2(T)\varphi^2 - ET\varphi^3 + \lambda\varphi^4$$

$$E \simeq 2m_W^3 + m_Z^3$$

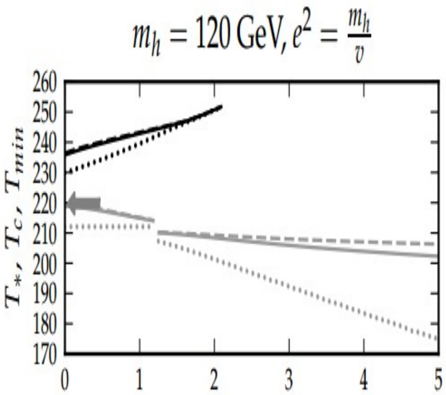
Gauge dependence first appears at *cubic* term, important for barrier formation.

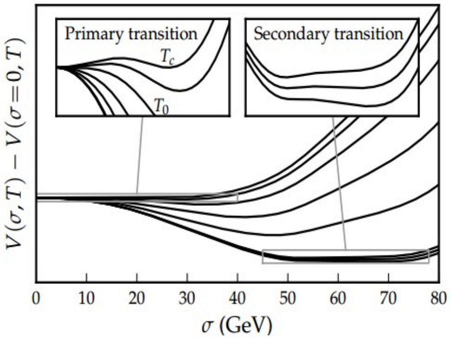
$$\frac{v_c}{T_c} \sim \frac{E}{\lambda_{\rm eff}} \geq 1.0$$

Recall, $J(y^2) \simeq -\frac{\pi^4}{45} + \frac{\pi^2}{12}y^2 - \frac{\pi}{6}y^3 - O(y^4)$

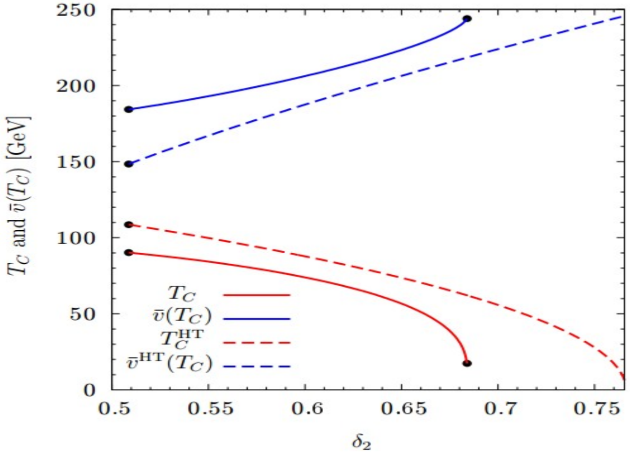
Challenges: Gauge Dependency

▶ Is gauge dependency that bad?





C. Wainwright, S. Prufumo, MJR Musolf., arXiv:1104.5487 [hep-ph]



T [GeV]	$T_C = 84.3$	$T_C^{HT} = 99.8$	$T_N = 96.6$
$\bar{v}(T)$ [GeV]	193.0	167.0	173.5
$E_{sph}(T)/T$	84.36	61.67	66.02
$\mathcal{E}(T)$	1.92	1.92	1.92

TABLE I: VEV's and sphaleron energies at different temperatures, T_C, T_C^{HT} and T_N, for $\delta_2 = 0.55$ in S2. The last two columns are calculated by use of the high-T effective potential

C. Chiang, MJR Musolf, E. Senaha, Phys.Rev.D 97 (2018) 1, 015005

solid, dashed and dotted lines denote the transition temperature, the critical temperature, and the minimum temperature at which the hot phase exists.

A secondary transition, pure gauge artifacts!

Gauge dependency not only impacts PT dynamics but also the GW spectrum!

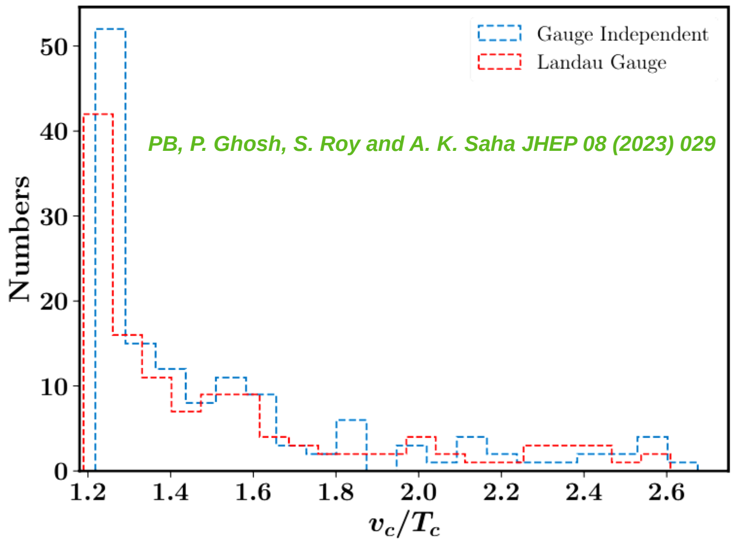
Any way-out?

▶ Gauge independent treatments are available

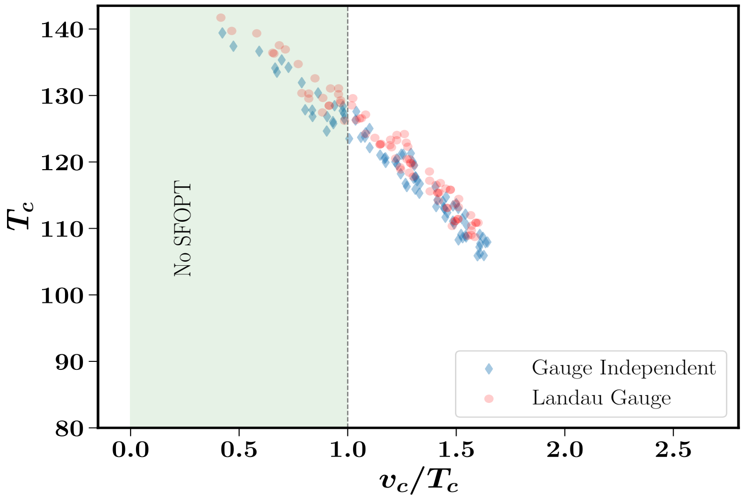
D. Croon et. al., JHEP 04 (2021) 055; P. M. Schicho et. al., JHEP 06 (2021) 130; L. Niemi et. al., Phys. Rev. D 103 115035; A. Katz et. al., Phys. Rev. D 92 (2015) 095019, and many more...

For eg., if the tree-level potential contains a cubic term, then one can, in principle, truncate the $\mathcal{O}(g^3)$ terms appearing due to the thermal contribution to minimize the gauge dependency.

However, for a complicated model, implementing the details to make the theory gauge invariant is very tedious.



NMSSM + RHN



Landau gauge is close to gauge invariant results.

Any way-out?

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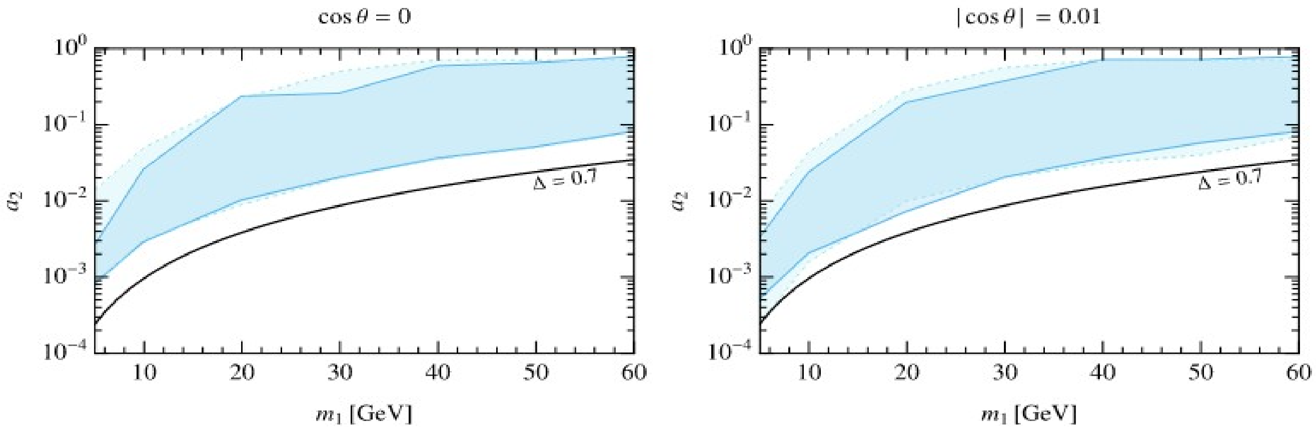


Dark shaded upto $\mathcal{O}(g^2)$

Light shaded, including $\mathcal{O}(g^3)$

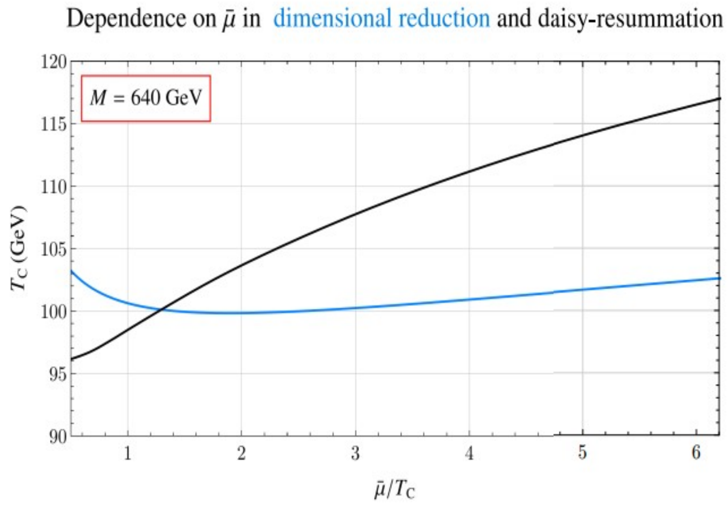
K. Kozaczuk et. al., Phys.Rev.D 101 (2020) 11, 115035

$$V = -\mu^2 |H|^2 + \lambda |H|^4 + \frac{1}{2}a_1 |H|^2 S + \frac{1}{2}a_2 |H|^2 S^2 + b_1 S + \frac{1}{2}b_2 S^2 + \frac{1}{3}b_3 S^3 + \frac{1}{4}b_4 S^4$$

Landau gauge is close to gauge invariant results.

Challenges: Reno. Scale Dependency

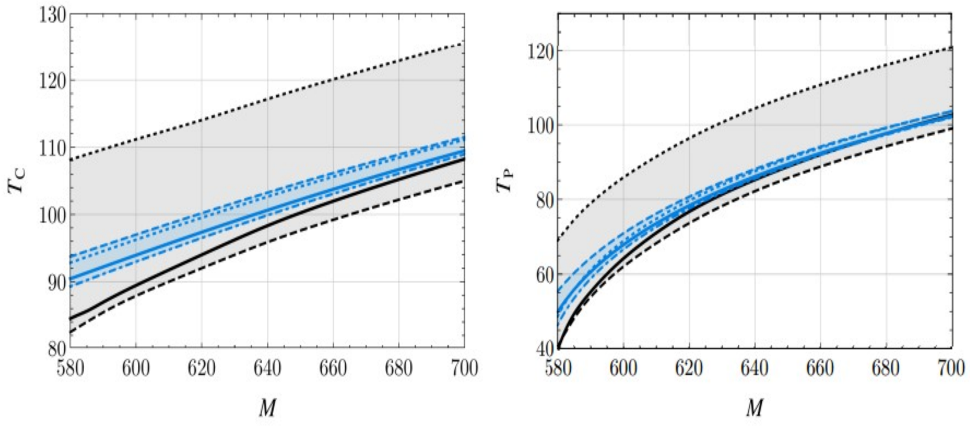
▶ Renormalization scale dependence



On shell prescriptions ? D. Curtin, P. Meade and C.-T. Yu., JHEP 11 (2014) 127

$$V'^{1-\text{loop}}_{\text{CW}} = \frac{1}{64\pi^2}\sum(-1)^{F_i} n_i \left[m_i^4(\phi_\alpha, T)\left(\log\frac{m_i^2(\phi_\alpha, T)}{m_{0i}^2} - \mathcal{C}_i\right) + 2m_i^2 m_{0i}^2 \right]$$

This ensures that the zero temperature vacuum conditions are completely determined by the tree-level contribution.



D. Croon, O. Gould et. al., arXiv:2009.10080 [hep-ph]

GWs from First-order Phase Transitions

- **WHY?**

GW – Collider – (DM) complementarity.

After LHC, LISA is next step in exploration of EW scale physics.

And, also, we have data from NANOGrav.

We further have,

BBO, DECIGO,

U-DECIGO, etc.

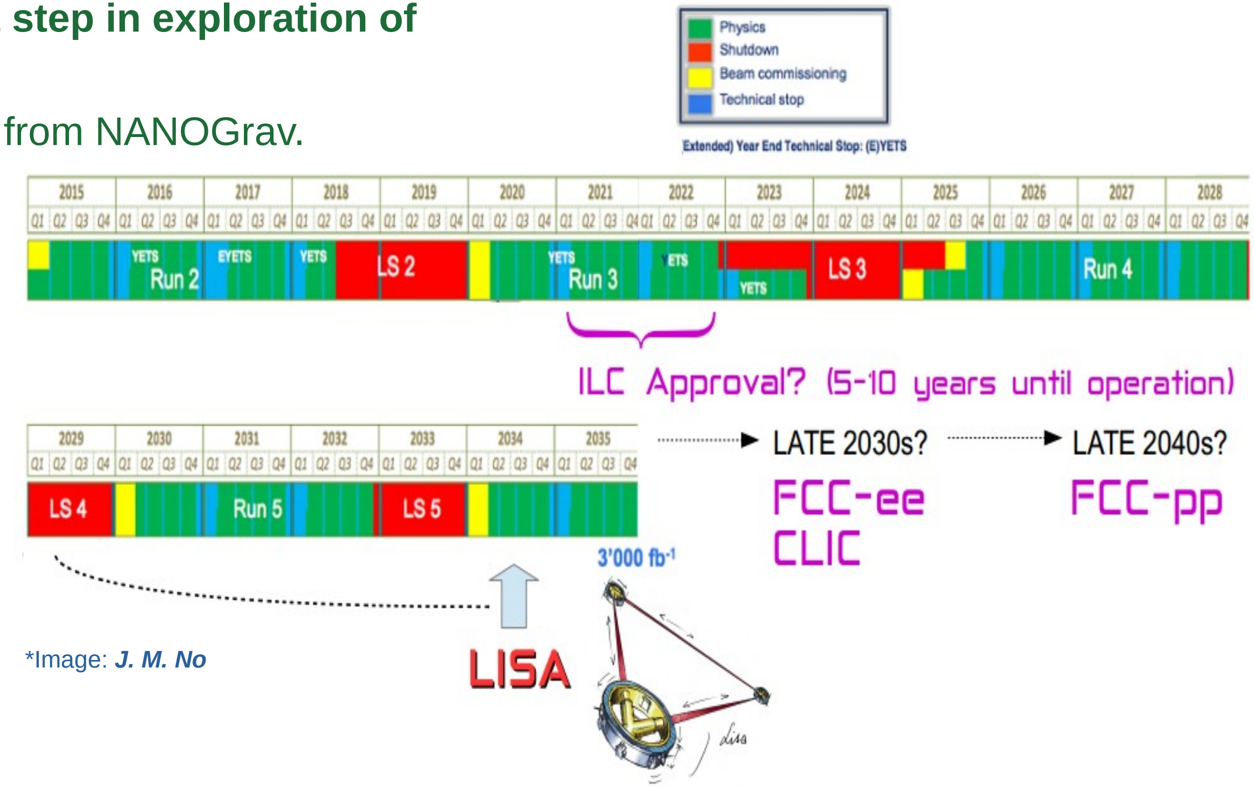
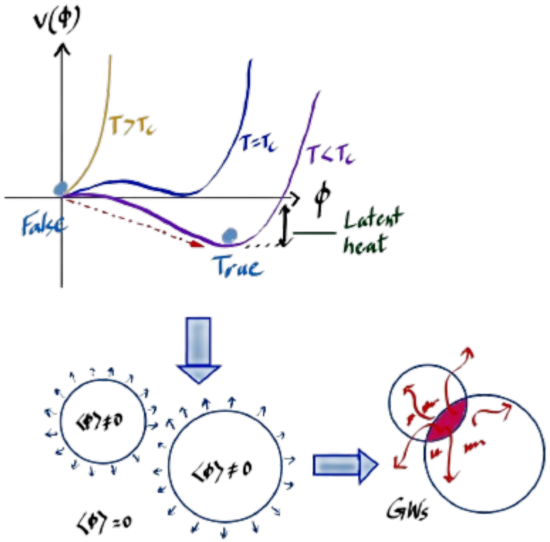


*Image: ***J. M. No***

GWs from First-order Phase Transitions

- The stochastic GWs are generated by a strong first-order phase transitions (SFOPT) in the early Universe. The main production processess are,
  - **Bubble collisions**
  - **Sound waves left behind in thermal plasma**
  - **Turbulance**



- The total GW energy budget can be obtained from 3 sources,

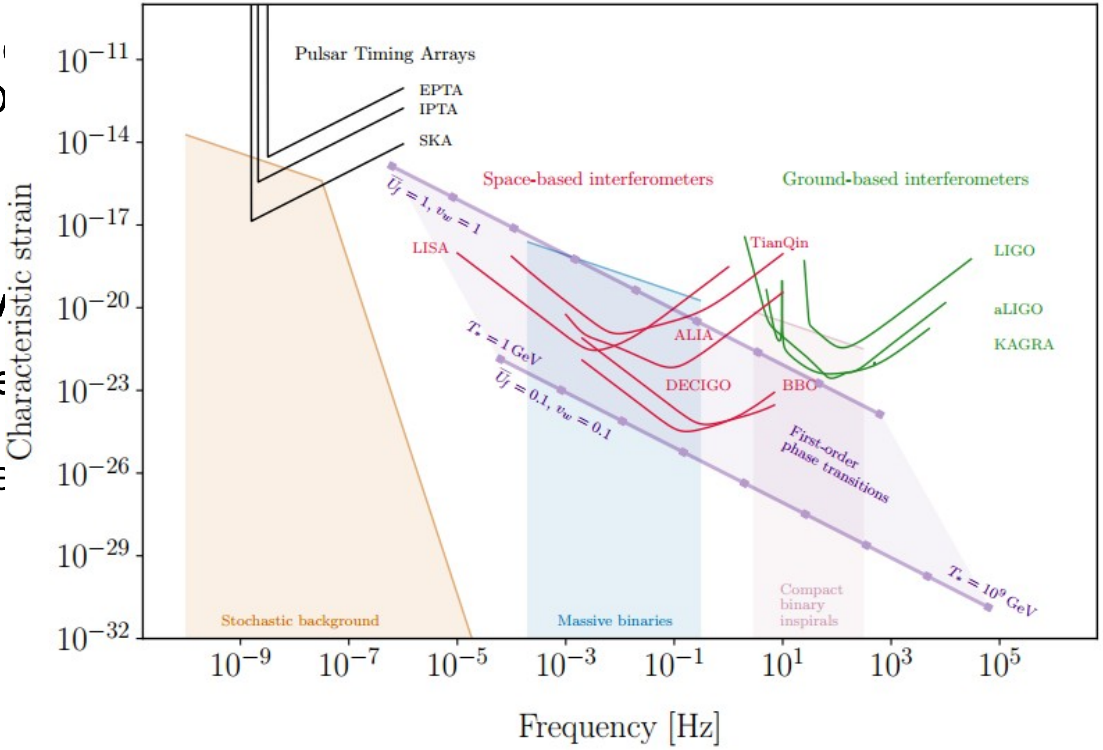
$$\Omega_{\rm GW}h^2(f) \approx \Omega_{\rm col}h^2 + \Omega_{\rm sw}h^2 + \Omega_{\rm tur}h^2$$

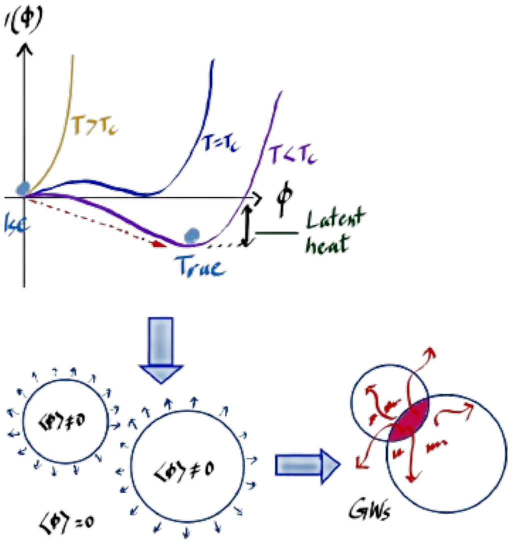
$$\Omega_{\rm col}h^2 \propto \left(\frac{\beta}{H_*}\right)^{-2},\ \Omega_{\rm sw}h^2 \propto \left(\frac{\beta}{H_*}\right)^{-1},\ \Omega_{\rm tur}h^2 \propto \left(\frac{\beta}{H_*}\right)^{-1}$$

C. Caprini et al., JCAP 04 (2016) 001; J. Ellis et al., JCAP 07 (2020) 050

GWs from First-order Phase Transitions

- The stochastic transitions (SFO processess are,
    - **Bubble col**
    - **Sound wav**
    - **Turbulance**
- The total GW ene





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C. Caprini et al., JCAP 04 (2016) 001; J. Ellis et al., JCAP 07 (2020) 050

GWs: approximations, productions

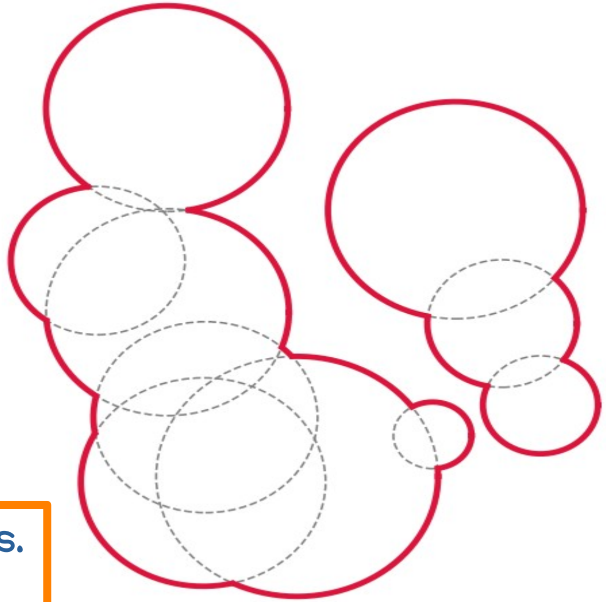
- The energy flow from a scalar field to the plasma and spacetime during a FOPT can be deduced from the coupled equations of motion of the scalar and gravitational fields and that of the plasma.

- **It is challenging to solve the coupled equations even numerically.**

Envelope approximation !

- Bubble walls are infinitesimally thin.
- The energy-momentum tensor immediately vanishes in sections of the bubble wall contained inside other bubbles.

  (radiation domination, adiabatic expansion...)



P. Athron et al., arXiv:2305.02357 [hep-ph]

There are recent developments that relaxes this approximations.

M. Hindmarsh et al., Phys. Rev. Lett. 125 (2020) 021302, Phys. Rev. D 92 (2015) 12300; S.J. Huber and T. Konstandin, JCAP 09 (2008) 022, etc.

GWs: approximations, productions

Bubble collisions: $\Omega_{\rm col} h^2$

Kosowsky, Turner, Watkins, PRL 69 (1992) 2026; PRD 45 (1992) 4514; Weir, PRD 93 (2016) 124037; Huber, Konstandin, JCAP 0809 (2008) 022; Cutting, Hindmarsh, Weir, PRD 97 (2018) 123513, etc.

In general negligible, except for very strong super cooling.

In most cases, such amount of supercooling incompatible with PT completion... Ellis, Lewicki, No, JCAP 1904 (2019) 003

A few exception, e.g., conformal scalar potentials

Sound waves: $\Omega_{\rm sw} h^2$

Hindmarsh, Huber, Rummukainen, Weir, PRL 112 (2014) 041301; PRD 92 (2015) 123009; PRD 96 (2017) 103520; Konstandin, JCAP 1803 (2018) 047; Hindmarsh, Hijazi, arXiv:1909.10040

Typically dominant signal.

Works for low bubble velocity!

$$h^2\Omega_{\rm sw}(f) = 2.59\times 10^{-6} \underbrace{\left(\frac{g_*}{100}\right)^{-\frac{1}{3}}}_{\text{Redshift}} \underbrace{\left(\frac{\kappa_{\rm sw}\alpha}{1+\alpha}\right)^2 \left(\frac{\max(v_w, c_{s,f})}{c}\right)\left(\frac{\beta}{H_*}\right)^{-1}\Upsilon(\tau_{\rm sw})}_{\text{Scaling}} \underbrace{S_{\rm sw}(f)}_{\text{Shape}},$$

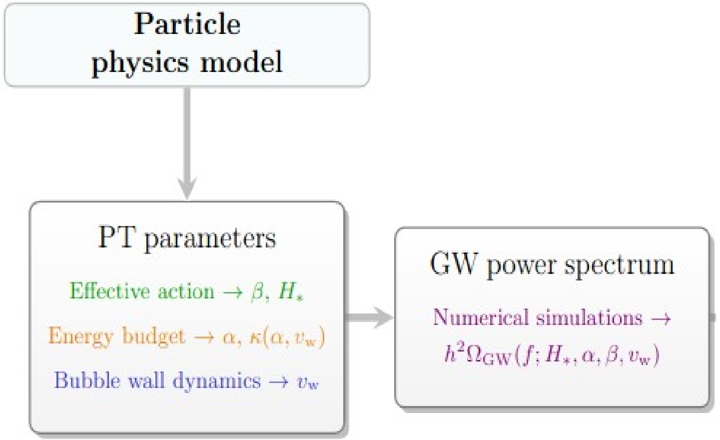
$$f_{\rm sw} = 8.9\times 10^{-6}\,{\rm Hz} \underbrace{\left(\frac{g_*}{100}\right)^{\frac{1}{6}}\left(\frac{T_*}{100\,{\rm GeV}}\right)}_{\text{Redshift}} \underbrace{\left(\frac{c}{\max(v_w, c_{s,f})}\right)\left(\frac{\beta}{H_*}\right)\left(\frac{z_p}{10}\right)}_{\text{Scaling}},$$

$$\Upsilon(\tau_{\rm sw}) = 1 - \frac{1}{\sqrt{1+2\tau_{\rm sw}H_*}}, \quad S_{\rm w}(f) = \left(\frac{f}{f_{\rm sw}}\right)^3\left[\frac{7}{4+3(f/f_{\rm sw})^2}\right]^{7/2}$$

Turbulence: $\Omega_{\rm turb} h^2$

Numerical simulations are going on! Semi-analytic approximations exists so far.

How the PT parameters are connected to GW spectra ?



- α and β/H_* depends on your particle physics model,

$\alpha = \frac{\Delta\rho}{\rho_{rad}}$, latent heat released by the PT process

$$\rho_{rad} = \frac{g^* \pi^2}{30} T_n^2$$

$$\Delta\rho = \left[V_{\text{eff}}^T(\phi_0, T) - T\frac{dV_{\text{eff}}^T(\phi_0,T)}{dT}\right]_{T=T_n} - \left[V_{\text{eff}}^T(\phi_n, T) - T\frac{dV_{\text{eff}}^T(\phi_n,T)}{dT}\right]_{T=T_n}$$

$$\frac{\beta}{H_*} = \left[T.\frac{d(S_E/T)}{dT}\right]_{T=T_n}, \quad v_w \longrightarrow 1 \text{ (a conservative choice)}$$

How to obtain those PT parameters ?

The probability of tunneling $\Gamma(T)$ from the false vacuum to the true one:

$$\Gamma(T) \approx T^4 \left(\frac{S_E}{2\pi T}\right)^{3/2} e^{-\frac{S_E}{T}}$$

The bounce action corresponding to the critical bubble:

$$S_E = \int_0^\infty 4\pi r^2 dr \left(V_T(\phi, T) + \frac{1}{2} \left(\frac{d\phi(r)}{dr}\right)^2 \right)$$

The bubble profile: $\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} = \frac{dV_T(\phi,T)}{dr}$,

boundary conditions: $\frac{d\phi}{dr} = 0$ when $r \to 0$ and $\phi(r) \to \phi_{\text{false}}$ when $r \to \infty$

Challanges in solving them for complicated physics models.

Numerical tools are necessary!

CosmoTransitions, PhaseTracer, BSMPT, FindBounce, DRalgo,...

Important Remarks

1. Although it is customary to calculate all the GW observables at nucleation temperature, however, care must be taken to check whether the transition completes.

2. Assumption is that once the bubble nucleate they will grow and fill the entire space. But this may be far from reality.

3. Better to calculate the false vacuum fraction.

$$P_f(t) = exp\left[-\frac{4\pi}{3}v_w^3\int_T^{T_c}\frac{\Gamma(T')dT'}{T'^4H(T')}\left(\int_T^{T'}\frac{dT''}{H(T'')}\right)^3\right]$$

Percolation temperature

4. Moreover, the nucleation temperature is not really connceted to bubble collisions.

Percolation is directly defined in terms of contact between bubbes.

5. Good choice for a temperature at which to evaluate thermal parameters determining the GWs spectrum.

However, it is still not established, which is the “right” reference temperature or what is the uncertainty using any single reference temperature.

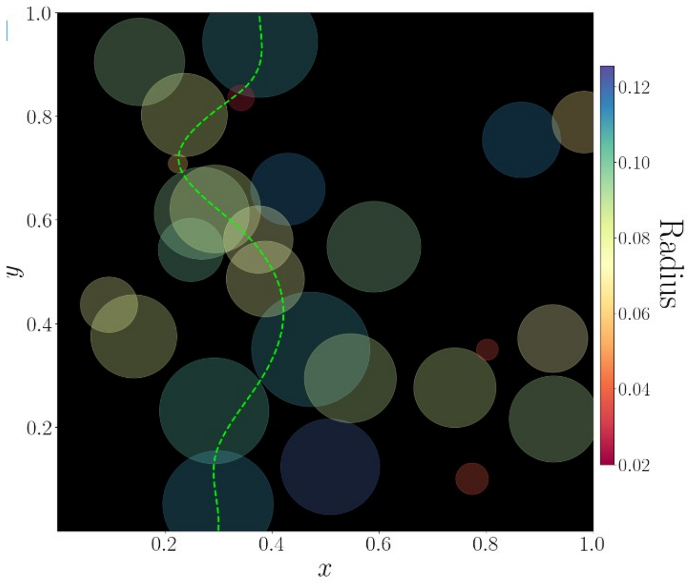
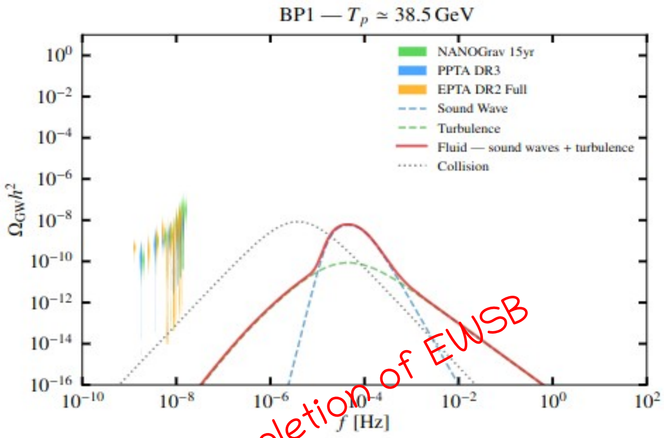


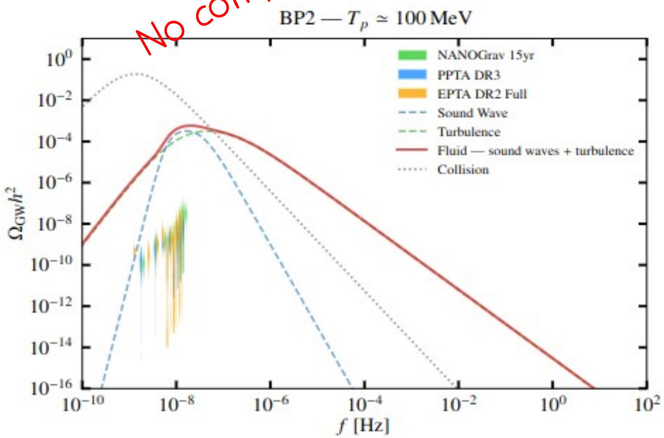
*Image: *P. Athron*

Important Remarks (Supercool transitions)

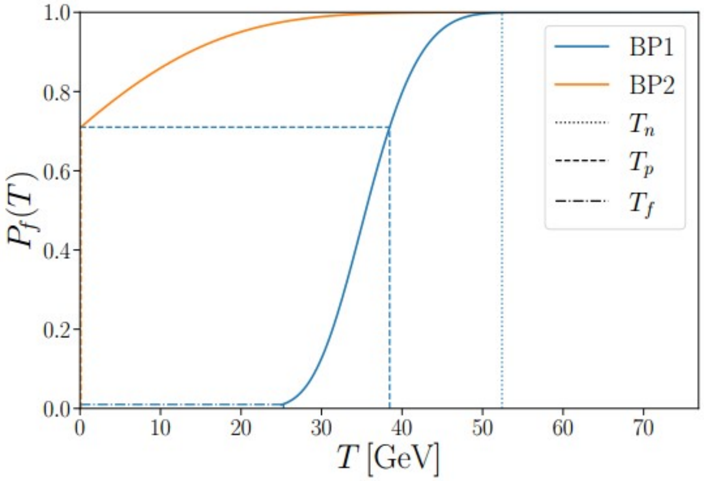
P. Athron, L. Morris, Z. Xu arXiv:2306.17239

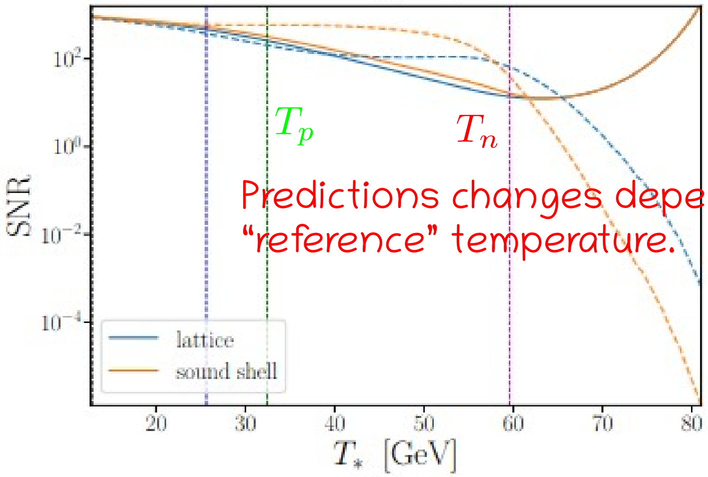


No completion of EWSB



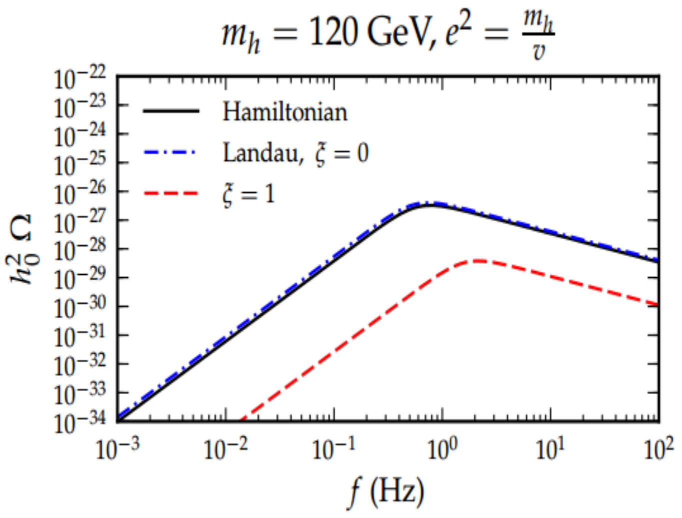
P. Athron et al., arXiv:2306.17239



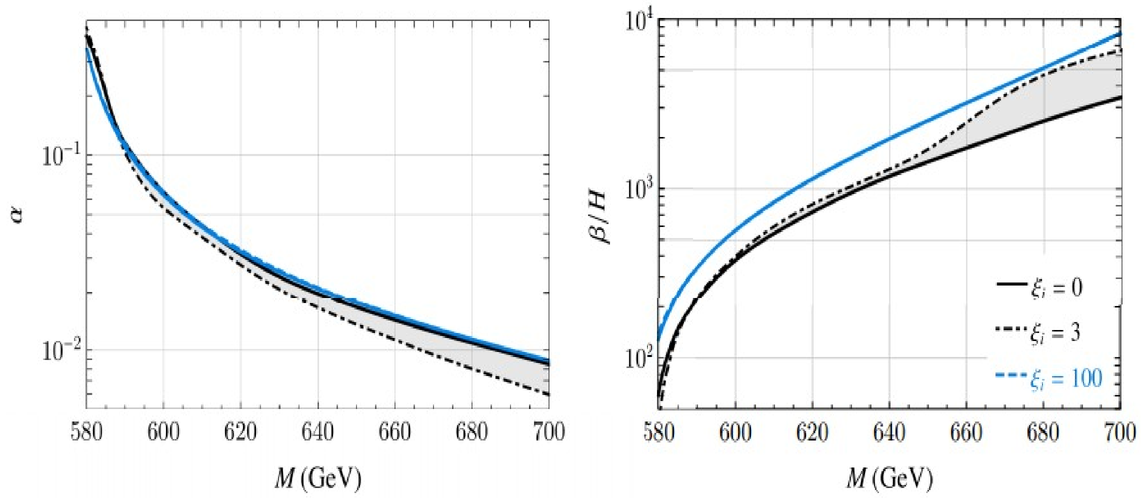


Predictions changes depending on "reference" temperature.

How the gauge/scale dependency in PT can impact GW spectra ?



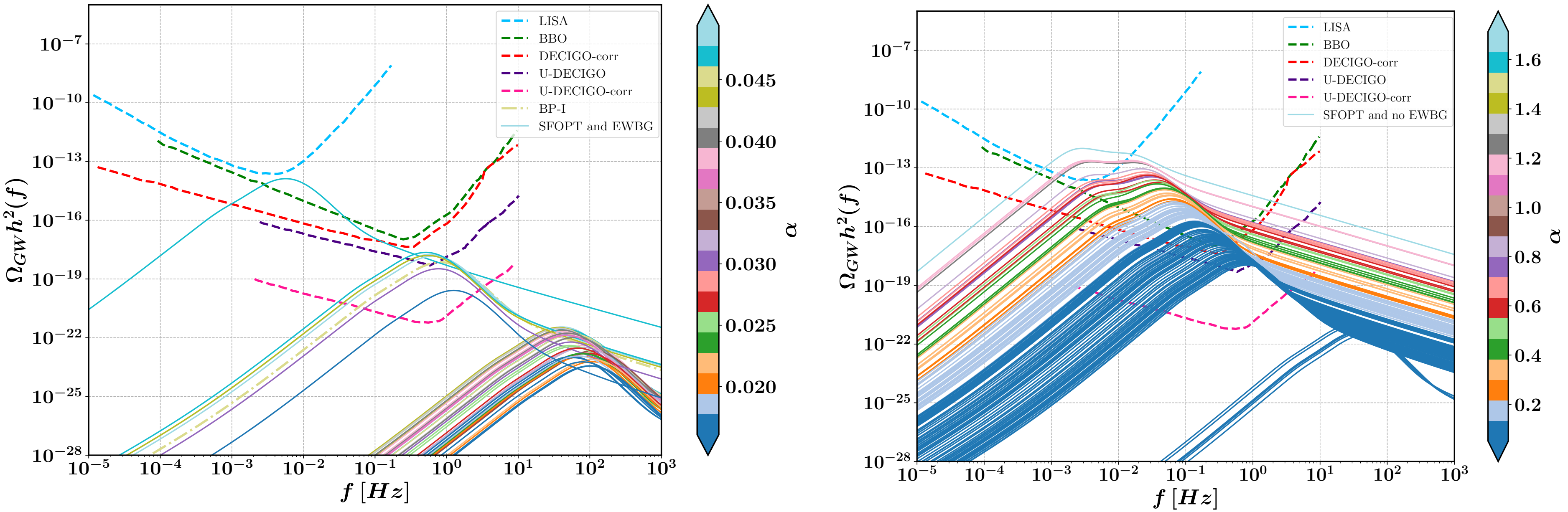
C. Wainwright, S. Prufumo, MJR Musolf., arXiv:1104.5487 [hep-ph]



D. Croon, O. Gould et. al., arXiv:2009.10080 [hep-ph]

Again, Landau gauge works better.

GW spectrum in NMSSM + an RHN



PB, P. Ghosh, S. Roy and A. K. Saha JHEP 08 (2023) 029

- Parameter points that favours EWBG mostly falls within the U-DECIGO and U-DECIGO-corr sensitivity.
- Majority of the points are within the reach of LISA, BBO, DECIGO-corr, U-DECIGO and U-DECIGO-corr.
- **NMSSM + RHN with lighter RH-sneutrino states, below 125 GeV SM-like Higgs, can accomodate SFOPT, hence promising for EWBG scenario, and offeres detactable GW spectrum at future space-based GW detectors.**

Conclusion

1. Estimation of Gravitational waves from first order electroweak phase transition is interesting and well motivated.

2. Depending on the effective potential, any BSM theory, SUSY, composite Higgs, or any model can accomodate FOPT and Gws, also can be promising for EWBG.

3. **Careful calculation is needed to avoid gauge dependency and scale dependency.**

4. **Choice of the best “reference” temperature is still ongoing. It may vary from problem to problem. A careful check of the completion of PT is well recommended in all the cases.**

Thank You

Back Up

$$
\begin{aligned}
V_{1,T=0}^{R_\xi} = \frac{1}{4(4\pi)^2}\Bigg[&\sum_{\phi} n_\phi m_\phi^4(\{\phi_j\},\xi)\left(\ln\left(\frac{m_\phi^2(\{\phi_j\},\xi)}{Q^2}\right) - k_s\right) \\
&+ \sum_V n_V m_V^4(\{\phi_j\})\left(\ln\left(\frac{m_V^2(\{\phi_j\})}{Q^2}\right) - k_V\right) \\
&- \sum_V (\xi m_V^2(\{\phi_j\}))^2\left(\ln\left(\frac{\xi m_V^2(\{\phi_j\})}{Q^2}\right) - k_V\right) \\
&- \sum_f n_f m_f^4(\{\phi_j\})\left(\ln\left(\frac{m_f(\{\phi_j\})^2}{Q^2}\right) - k_f\right)\Bigg],
\end{aligned}
$$

GWs: approximations, productions

- **Bubble collisions:**

$$h^2\Omega_{\rm coll}(f) = \underbrace{1.67\times10^{-5}\left(\frac{g_*}{100}\right)^{-\frac{1}{3}}}_{\it Redshift}\underbrace{\left(\frac{\kappa_{\rm coll}\alpha}{1+\alpha}\right)^2\left(\frac{\beta}{H_*}\right)^{-2}\Delta(f_{\rm coll},v_w)}_{\it Scaling}\underbrace{S_{\rm coll}(f,v_w)}_{\it Shape},$$

$$f_{\rm coll} = \underbrace{1.65\times10^{-5}\,{\rm Hz}\left(\frac{g_*}{100}\right)^{\frac{1}{6}}\left(\frac{T_*}{100\,{\rm GeV}}\right)\left(\frac{f_*}{\beta}\right)\left(\frac{\beta}{H_*}\right)}_{\it Redshift},$$

- **Sound waves:**

$$h^2\Omega_{\rm sw}(f) = 2.59\times10^{-6}\underbrace{\left(\frac{g_*}{100}\right)^{-\frac{1}{3}}}_{\it Redshift}\underbrace{\left(\frac{\kappa_{\rm sw}\alpha}{1+\alpha}\right)^2\left(\frac{\max(v_w,c_{s,f})}{c}\right)\left(\frac{\beta}{H_*}\right)^{-1}\Upsilon(\tau_{\rm sw})}_{\it Scaling}\underbrace{S_{\rm sw}(f)}_{\it Shape},$$

$$f_{\rm sw} = 8.9\times10^{-6}\,{\rm Hz}\underbrace{\left(\frac{g_*}{100}\right)^{\frac{1}{6}}\left(\frac{T_*}{100\,{\rm GeV}}\right)}_{\it Redshift}\underbrace{\left(\frac{c}{\max(v_w,c_{s,f})}\right)\left(\frac{\beta}{H_*}\right)\left(\frac{z_p}{10}\right)}_{\it Scaling},$$

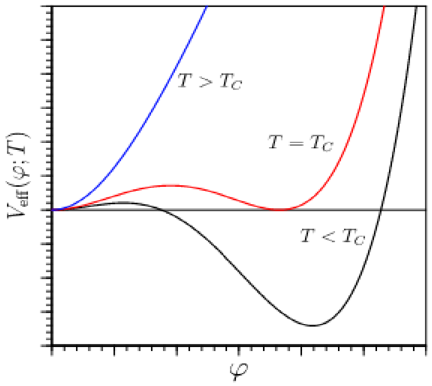
- **Turbulance:**

$$h^2\Omega_{\rm turb}(f) = 3.35\times10^{-4}\underbrace{\left(\frac{g_*}{100}\right)^{-\frac{1}{3}}}_{\it Redshift}\underbrace{\left(\frac{\beta}{H_*}\right)^{-1}\left(\frac{\kappa_{\rm turb}\alpha}{1+\alpha}\right)^{\frac{3}{2}}\left(\frac{\max(v_w,c_{s,f})}{c}\right)}_{\it Scaling}\underbrace{S_{\rm turb}(f)}_{\it Shape},$$

$$f_{\rm turb} = 2.7\times10^{-5}\,{\rm Hz}\underbrace{\left(\frac{g_*}{100}\right)^{\frac{1}{6}}\left(\frac{T_*}{100\,{\rm GeV}}\right)}_{\it Redshift}\underbrace{\left(\frac{c}{\max(v_w,c_{s,f})}\right)\left(\frac{\beta}{H_*}\right)}_{\it Scaling},$$

Strong FOPT in Scalar Extensions

- Thermal barrier in effective potential, $V_{\rm eff}(\varphi, T) = m^2(T)\varphi^2 - ET\varphi^3 + \lambda\varphi^4$



Strongly first order phase transition is realized when, $\frac{v_c}{T_c} \sim \frac{E}{\lambda_{\rm eff}} \geq 1.0$

$$E \simeq 2m_W^3 + m_Z^3 + \text{new scalar mass}$$

$$E \simeq \left(2m_W^3 + m_Z^3 + n\lambda_{\Phi S}^{3/2} v^2\right),\ V \supset \lambda_{\Phi S}\Phi^\dagger\Phi S_n^\dagger S_n$$

- New scalars need to be light $\sim \mathcal{O}(EW\, scale)$ to be in thermal plasma during PT
- Large scalars d.o.f enhances size of PT

- However, $\sigma_{\rm SI} \sim \frac{\lambda_{\Phi S}^2}{4\pi} \to \lambda_{\Phi S} \sim$ small!

Lessons:

- ***Strong FOPT is related to direct detection (DD) rate.***
- ***We require low scalar mass for EWPT, and thermal relic constraints demand heavy scalar masses away from the Higgs resonances.***

Need to avoid the general realtions !

I. A Supersymmetric Extended Model

- **We extend the Next-to-MSSM with a Right Handed Neutrino (RHN) superfield.**

  The *superpotential* is given as,

$$W = W'_{\rm MSSM}(\mu=0) + \lambda\, \widehat{S}\, \widehat{H}_u \cdot \widehat{H}_d + \frac{\kappa}{3}\, \widehat{S}^3 + Y_N^i\, \widehat{N}\, \widehat{L}_i \cdot \widehat{H}_u + \frac{\lambda_N}{2}\, \widehat{S}\, \widehat{N}\, \widehat{N}$$

$$-\mathcal{L}_{\rm soft} = -\mathcal{L}'_{\rm soft} + m_S^2\, S^* S + M_N^2 \widetilde{N}^* \widetilde{N} + \Big(\lambda A_\lambda S\, H_u \cdot H_d + h.c.\Big)$$
$$+\Big(\frac{\kappa A_\kappa}{3} S^3 + (A_N Y_N)^i\, \widetilde{L}_i \cdot H_u\, \widetilde{N} + \frac{A_{\lambda_N}\lambda_N}{2} S\widetilde{N}\widetilde{N}\Big)$$

- Along with the 3 CP-even Higgses in NMSSM, the scalar field of the RHN supermultiplet, $\widetilde{N}$ also develops a non-zero VEV, v_N. Hence, *R-parity* is spontenously broken. *Kitano and Oda, Phys.Rev.D 61 (2000)*

- The relevant part of the neutral scalar potential for an SFOPT is,

$$V^0_{\rm scalar} \supset \Big| -\lambda H_u^0 H_d^0 + \kappa S^2 + \frac{\lambda_N}{2}\widetilde{N}^2\Big|^2 + |\lambda|^2|S|^2(|H_u^0|^2 + |H_d^0|^2) + \Big|\lambda_N S \widetilde{N}\Big|^2 + \frac{\kappa A_\kappa}{3} S^3 + \frac{\lambda_N A_{\lambda_N}}{2} S\widetilde{N}\widetilde{N}$$

- The neutrino Yukawa couplings, Y_N^i, are ignored considering a TeV scale seesaw mechanism, leading to small left-sneutrino VEV and tiny R_p-violation.