QCD with an IR Fixed Point and a Dilaton

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mostly based on (other refs later)

Del Debbio, RZ	JHEP'22 2112.1364	Dilaton new phase?
RZ	PRD, 2306.06752	broken χ-sym.@IRFP - pions
RZ	2306.12914	Dilaton improves Goldstones
Shifman RZ	PRD, 2310.16449	β'_* in N=1 confomal window
RZ	2312.13761	broken χ-sym.@IRFP - pions & dilaton

Future of Fundamental Composite Higgs Dynamics - 14 March 2024

Overview

- **Prologue:**
  Overview of results & open questions
- **Introduction:**
  Dilaton terminology & phases of gauge theories
- **Main part QCD@IRFP**
  Equating QCD@IRFP & (dilaton)χPT
- **Selected topics**
  Double-soft dilaton theorem(s)
  Outlook: dilaton as σ-meson and Higgs boson
- **Conclusions**

Results & open questions of IRFP interpretation of QCD

results

- Mass anomalous dimension: $\gamma_* = \gamma_m|_{\mu=0} = 1$
- Slope of β-fct. $\beta'_* = \beta'|_{\mu=0} = 0$

solid & consistent

good evidence & attractive (realistic)

gauge-theory

- If **dilaton mass** non-zero & generated by $T^\rho_\rho \supset \mathcal{O}_m$ $\Rightarrow \Delta_{\mathcal{O}_m} = 2$

model-independent

↕

open

- **Dilaton mass**: zero or not? (presumably small at least)
- Is $r|_{N_f=2} = \frac{F_\pi}{F_D} \approx 1$? **If yes**, **dilaton** ⇒ **Higgs** boson

(I) Introduction

What is meant by **dilaton** in this talk (briefly)

RG-flows and **conformal window**

A collection of refs

more refs in my papers and surely more as an old and interesting topic

- **Pre-QCD work 69-70**

  PhD thesis: John Ellis & Rod Crewther'70 and paper resulting
  Imperial group; Isham, Salam, Strathdee also Mack

- **PDG banned σ-meson (dilaton candidate 20 years)**

  Pelaez et al'96 good values Caprini, Colangelo, Leutwyler'06 good value, small error, Roy eq. with LHC

- **Pre&post LHC model building - (not attached to gauge theories)**

  Rattazi et al 1306.4601 Grinstein et al 0708.1463 Terning et al 1406.5192} ….
  Some pragmatic, some negative conclusions

- **Conformal Window lattice efforts '07 - (walking TC & composite Higgs)**

  Del Debbio, Lucini, Patella, Kuti, Holland, Sannino, Pica, Appelquist, LSD, LatKMI, Feretti, Caccapaglia…….
  Some results established, finding light scalars more and more

- **σ-meson in χPT? Crewther'Tunstall, 12'-15'**
- **Dilaton (gravity explaining origin of Planck Mass)** **Wetterich, Zee, Shaposhnikov, Karananas..**
- **Crawling TC (connecting Pre-QCD work)** **Cata, Crewther'Tunstall, 18'**
- **Dilaton-EFT'15 (understand lattic results)** **Appelquist, Piai, Ingolby .. Golterman & Shamir .**

What is a **dilaton**?

- Always: particle vacuum quantum numbers $J^{PC} = 0^{++}$
  Otherwise: few different meanings

1. **Goldstone boson*** of spontaneously **broken scale invariance** of strong interactions 1968-1970 then largely forgotten
   (resurrected as Higgs as dilaton pre-LHC)

   this talk

2. **Scalar component of gravity (gravi-scalar)**
   Brans-Dicke, supergravity (string theory)

3. A **name** for a **light** $J^P = 0^+$ **scalar** in context of approximate scale inv.

   However, it is not a Goldstone (no limit when it's massless…)

Types of Renormalisation Group (RG)-flow

- assume UV fixed point (e.g. asymptotic freedom) g^*_{UV} , IR flow?

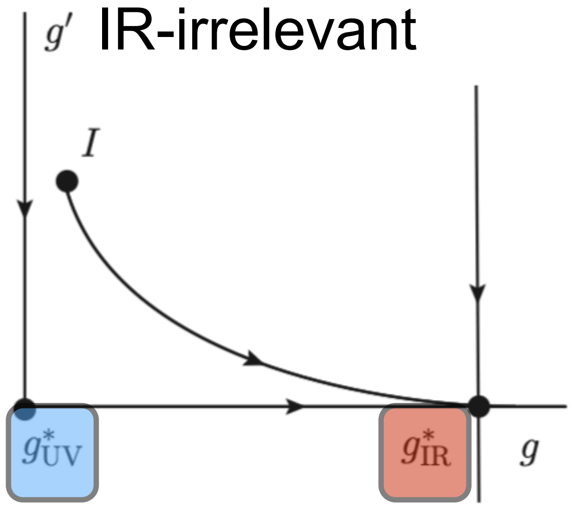
IR fixed point g^*_{IR}

conformal window

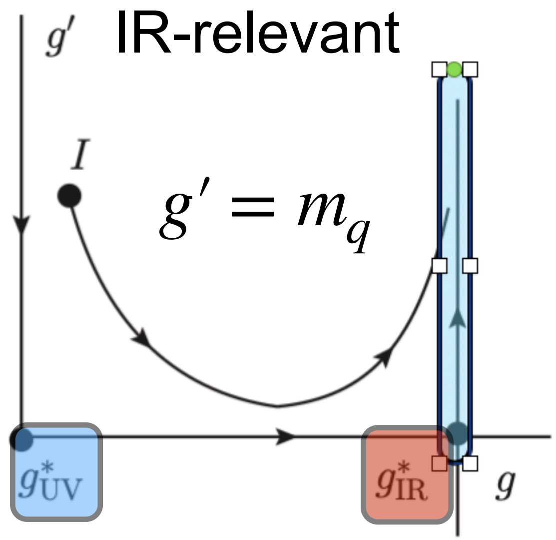
no IR fixed point

QCD-picture

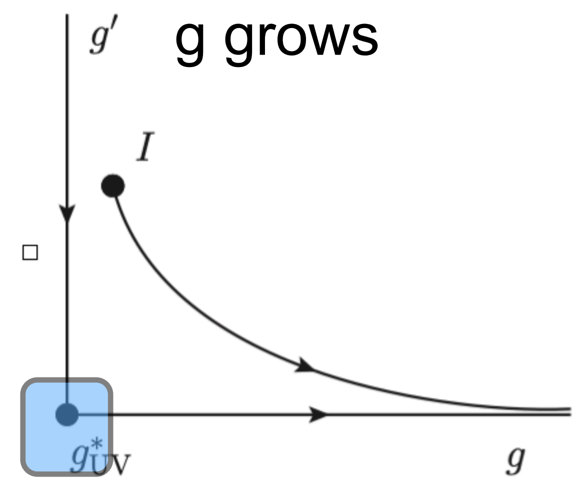
IR-irrelevant



IR-relevant

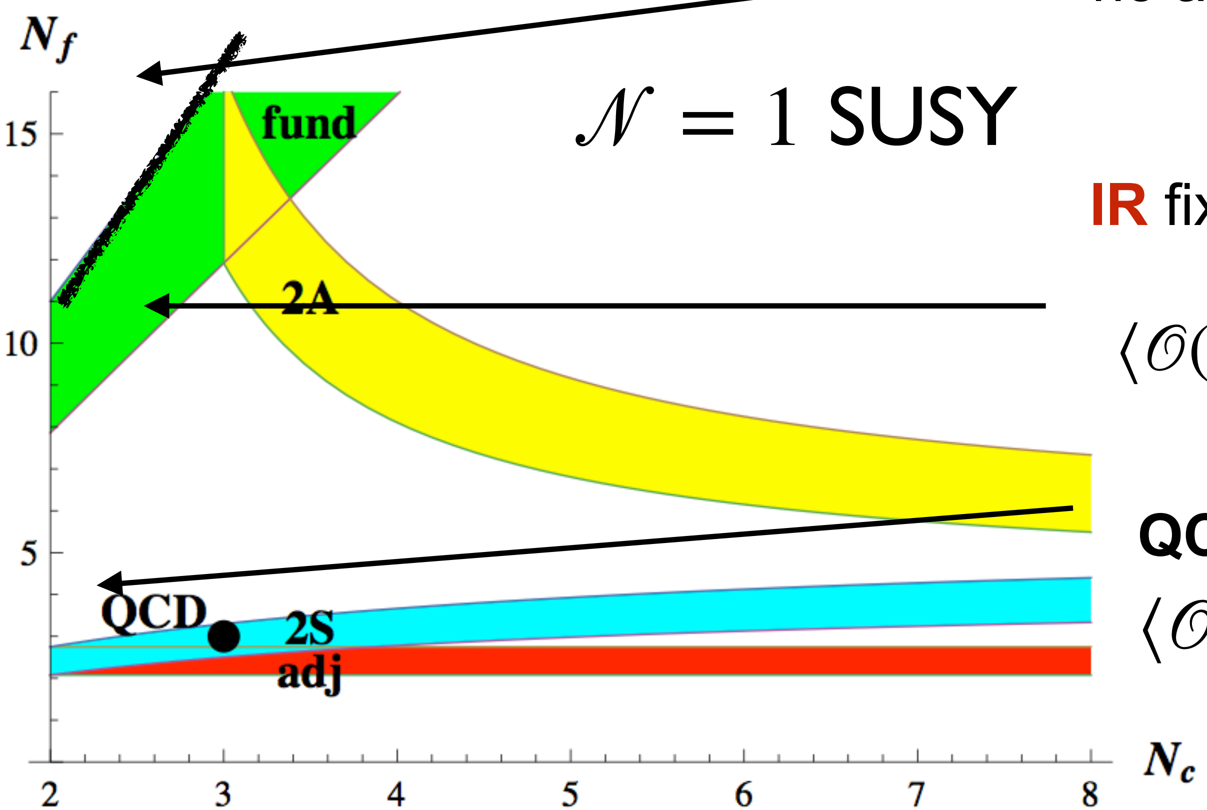


IR-irrrelevant &
g grows



Phases of gauge theories - **Conformal Window**

- gauge theory **massless quarks** in some **irrep** (e.g. fund. of say $SU(N_c)$)
- Focus on green = fund irrep



$\mathcal{N} = 1$ SUSY

no asymptotic freedom (ignore)

IR fixed point = **conformal window**

$$\langle \mathcal{O}(x)\mathcal{O}(0)\rangle_{CFT} \propto \frac{1}{(x^2)^{\Delta_{\mathcal{O}}}} \quad x^2 \to \infty$$

QCD: *chiral SSB* & *confinement*

$\langle \mathcal{O}(x)\mathcal{O}(0)\rangle_{QCD} \propto$ complicated

QCD@low energy: pion EFT = χPT

isospin

- QCD $\langle \bar{q} q \rangle \neq 0$ **breaks** chiral $SU_L(N_f) \times SU_L(N_f) \to SU_V(N_f)$ **spontaneously,** $N_f^2 - 1$ **Goldstones = pions** $[\, m_\pi^2 = \mathcal{O}(m_q) \,]$

- CCWZ construction $U = e^{i\pi^a T^a / F_\pi}$

$$\mathcal{M} \equiv \mathrm{diag}(m_{q_1}, \ldots, m_{q_{N_f}})$$

$$\mathscr{L}^{\chi PT}_{LO} = \frac{F_\pi^2}{4} \mathrm{Tr}[\partial^\mu U \partial_\mu U^\dagger] + \frac{B_0 F_\pi^2}{2} \mathrm{Tr}[\mathcal{M} U^\dagger + U \mathcal{M}^\dagger]$$

kinetic *m_q-term (spurion technique)* *GMOR* $m_\pi^2 F_\pi^2 = -2 m_q \langle \bar{q} q \rangle$

PCAC GMOR, Goldberger-Treiman
LO: Weinberg '67
NLO: Weinberg '79
Gasser Leutweyler '84,'85
NNLO: Bijnes, Colangelo, Gasser ...

- QCD $\langle \bar{q} q \rangle \neq 0$ also **breaks scale symmetry,** possibly spontaneously? If yes, **1** (pseudo) **Goldstones = dilaton**

$$\mathscr{L}^{d\chi PT}_{LO} = \text{later}$$

$$m_D^2 = \mathcal{O}(m_q, \beta'_*)$$

parametric expectation
does Goldstone mass remember the flow?
(Not settled - If CFT SSB then massless)

How the QCD phase is viewed

1. **Standard:** not related to an IR fixed point - **✗PT**

2. **Sometimes:** close to IRFP below boundary of CW - **d✗PT** (techni-dilaton)

   SSB of scale invariance? Dilaton mass $\bar{m}_D^2 \propto \beta \langle G^2 \rangle$ by "PCDC soft-thms"

3. **This talk**: **explore** IR fixed point **interpretation** in all of QCD phase

$\bar{m}_D \neq 0,$
deep-IR ⇒ standard **✗PT**

"QCD with an IRFP - pion sector"

$\bar{m}_D = 0,$
deep-IR ⇒ **dilaton-✗PT**

"QCD with an IRFP and a dilaton"

guiding principle: **consistency of results**

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* $\bar{m}_D$ is **chiral dilaton mass** ($m_q \to 0$), i.e. no explicit scale symmetry breaking

(2) QCD@IRFP (main part)

determination of **scaling dimensions** with **pion physics**

IRFP-interpretation - assumptions

- scaling @IRFP with SSB: $\langle \bar{q} q \rangle \neq 0$

$$\langle \mathcal{O}(x)\mathcal{O}(0)\rangle \propto \frac{1}{(x^2)^{\Delta_{\mathcal{O}}}} + \text{ GB-corrections}$$

$x^2 \to \infty$

$\Delta_{\mathcal{O}} = d_{\mathcal{O}} + \gamma_{\mathcal{O}}$

- **assume** exists a scheme: $\beta_* = \beta|_{\mu=0} = 0$

$$\beta = \beta'_* \delta g + \beta''_* \frac{(\delta g)^2}{2} + \mathcal{O}((\delta g)^3) \, , \quad \delta g \equiv g - g_*$$

$$T^\rho{}_\rho|_{\text{phys}} = \frac{\beta}{2g} G^2 + \sum_q m_q (1+\gamma_m) \bar{q} q$$

- idea: QCD@IRFP $\leftrightarrow$ EFT (**dilaton)-χPT** for $x^2 \to \infty$

determine anomalous dimension: e.g* $\gamma_{m_q} = -\gamma_{\bar{q}q}|_{\mu=0} \equiv \gamma_*$

*** main quantity in CW-hunt. and Walking technicolor** $-1 \leq \gamma_* \leq 2$ **allowed range**

irrelevant(PCAC) unitarity

Matching scalar adjoint correlator

$m_q = 0$

$S^a = \bar{q} T^a q$

$$\langle S^a(x) S^a(0) \rangle_{\text{CD-QCD}} = \langle S^a(x) S^a(0) \rangle_{\chi\text{PT}} \,, \quad \text{for } x^2 \to \infty$$

deep-IR

source theory

Gasser & Leutwyler'84

$$\langle \mathcal{O}(x) \mathcal{O}^\dagger(0) \rangle_{\text{CFT}} \propto (x^2)^{-\Delta_{\mathcal{O}}}$$

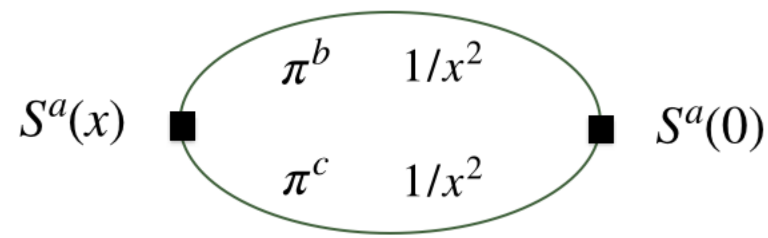
$$\Delta_{\mathcal{O}} = d_{\mathcal{O}} + \gamma_{\mathcal{O}}$$

$$\langle S^a(x) S^a(0) \rangle_{\text{CD-QCD}} \propto (x^2)^{-(3-\gamma_*)}$$

$$\Delta_{S^a} = d_{S^a} - \gamma_*$$

$$S^a|_{\text{LO}} = -\frac{F_\pi^2 B_0}{2} \text{Tr}[T^a U^\dagger + U T^a] \propto B_0 d^{abc} \pi^b \pi^c + \ldots$$

$$\langle S^a(x) S^a(0) \rangle_{\chi\text{PT}} \propto B_0^2 d^{abc} d^{abc} \langle \pi^a(x) \pi^a(0) \rangle^2 \propto \frac{1}{x^4}$$



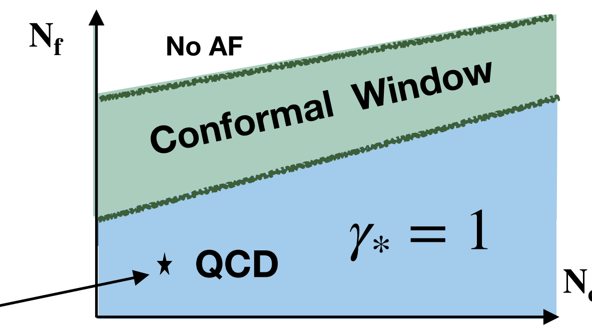
CFT-scaling matches Goldstone EFT

$$\gamma_* = 1$$

1st main result

Is $\gamma_* = 1$ accidental? Probably not …

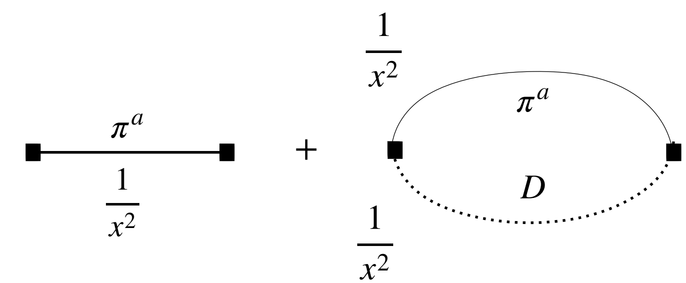
1. It's **consistent** with **end** of **conformal window** in
   a) $\mathcal{N} = 1$supersymmetric gauge theories
   b) many models & lattice



2. Go beyond: **might make sense** in **QCD phase**
   $\mathcal{N} = 1$: IR-free meson field $M^a_{\bar{b}} \leftrightarrow \tilde{Q}_{\bar{b}} Q^a \ \Rightarrow 1 = \Delta_M = \Delta_{\tilde{Q}Q} = 2 - \gamma_*$

3. Was the S^a-correlator a **coincidence?**
   Probably **not**, since same result follows from

   i) $P^a = \bar{q}\gamma_5 T^a q$-correlator [1]
   (amusing interplay with single π-channel)



   ii) hyperscaling $m_\pi^2 \propto m_q^{\frac{2}{1+\gamma_*}} \propto m_q$ (need to argue) [2]

   iii) matching trace anomaly & Feynman Hellman thm [2]

   iv) soft theorems [1]

[1] 2312.13761
[2] 2306.06752

Trace anomaly & Feynman-Hellmann thm $m_q \neq 0$

Dilatons only hides Λ-scale*
may use $2m_\pi^2 = \langle \pi^a | T^\rho{}_\rho | \pi^a \rangle$

$T^\rho_\rho |_{phys} = \beta/(2g)\, G^2 + N_f m_q (1+\gamma_m) \bar{q} q$

Ellis, Chanowist, Crewther, Minkowski
Adler, Duncan, Nielsen, Collins, Jogelekar '72-75 '

$$2m_\pi^2 = \langle \pi | \beta/(2g) G^2 + N_f m_q (1+\gamma_m) \bar{q} q | \pi \rangle$$

$\partial_{\ln m_q} E_\pi = N_f m_q \langle \hat{\pi} | \bar{q} q | \hat{\pi} \rangle + \mathcal{O}(m_q^2)$

main ingredient: $\partial_{m_q} \langle \hat{\pi}(p') | \hat{\pi}(p) \rangle = 0$

rewrite using GMOR $m_\pi^2 \propto m_q$ (QCD):

$$2m_\pi^2 |_{m_q} = 2 N_f m_q \langle \pi | \bar{q} q | \pi \rangle$$

reduces to GMOR double soft-pion thm

1. Note that these two **must equate at** $\mathcal{O}(m_q)$, also in standard QCD

2. Note that $\beta \to \beta_* = 0$, $\gamma_m \to \gamma_* = 1$ seems a simple $\mathcal{O}(m_q)$-solution

$\Rightarrow \gamma_* = 1$ follows once more

*residue $\mathcal{O}(q^2, m_\pi^2) \Rightarrow$ pole no "dramatic" effect

An emerging picture

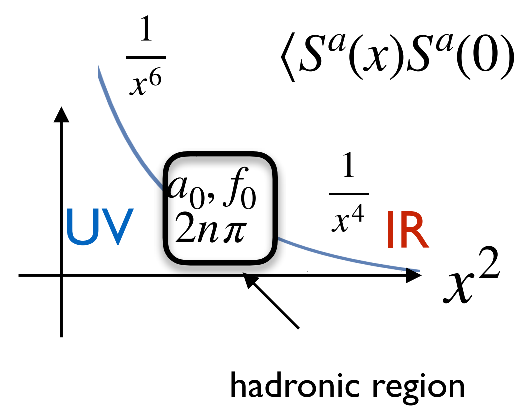
- Message seems to be: integer γ_* is special

$\gamma_* = 2$ unitarity bound (Mack'77) = 1 free scalar

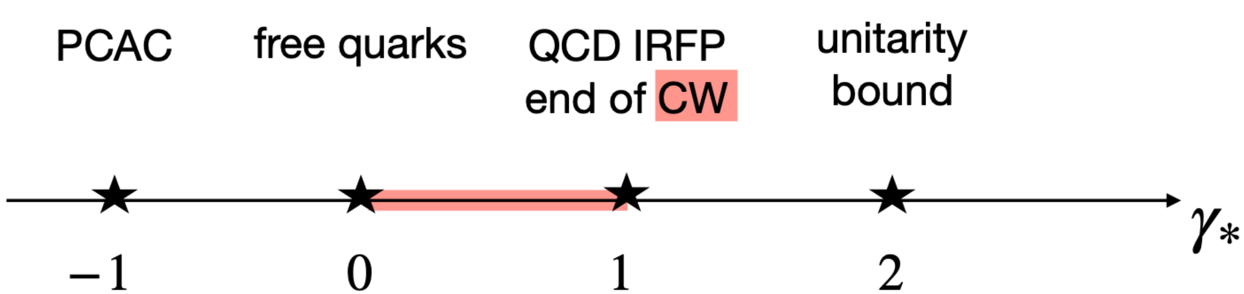
$\gamma_* = 1$ lower end of CW = 2 free scalars $\Delta^{UV}_{S^a} = 2$ } degenerate $\mathcal{N} = 1$ SUSY

$\gamma_* = 0$ upper end of CW = 2 free quarks $\Delta^{UV}_{S^a} = 3$

$\gamma_* = -1$ PCAC bound (Wilson'69)



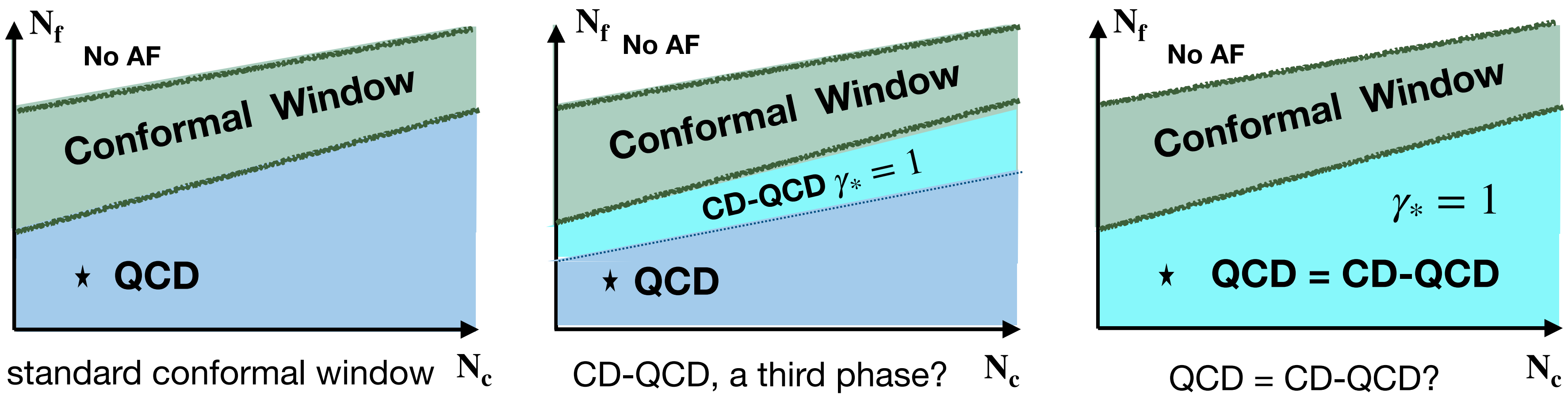
QCD-like theories (no scalars) $\gamma_m = -\gamma_{\bar{q}q}|_{\mu=0} = \gamma_*$



- Conformal window only uses 1/3 of allowed γ_*-range

End of main part and …

- At least any of these three possibilities is logically possible. Option 1 is what is what is taken for granted in standard view.



- Hope, convinced you that option 2 & 3 are not as absurd as .. I thought as well.

- Important: under assumptions got back consistent results.

Before going to T^ρ_ρ-correlator …

…. pause and introduce EFT: **dilaton-χPT**

chiral | **dilatation**

$$J^a_{5\mu} = \bar{q} T^a \gamma_\mu \gamma_5 q$$

$$J^D_\mu(x) = x^\nu T_{\mu\nu}(x)$$

sym. currents

$$\langle \pi^b(q) | J^a_{5\mu} | 0 \rangle = i F_\pi q_\mu \delta^{ab}$$

$$\langle D(q) | J^D_\mu | 0 \rangle = i F_D q_\mu$$

decay constants= order parameters

$$U = e^{i\pi^a T^a / F_\pi}$$

$$\chi \equiv F_D e^{-D/F_D}$$

coset rep.

$$U \to L U R^\dagger$$

$$\chi \to \chi e^{\alpha(x)}$$

transformation

$$(L, R) \in SU(N_f)_L \otimes SU(N_f)_R$$

$$\alpha(x) \in \mathbb{R}$$

Isham, Salam, Strathdee,
Mack, Zumino ca ‘70

Leading order dilaton-χPT

- Building principle: enforce Weyl invariance

$$g_{\mu\nu} \to e^{-2\alpha} g_{\mu\nu} \qquad \chi \to \chi e^{\alpha} \qquad U \to U$$

$$\Delta_{\bar{q}q} = 3 - \gamma_* = 2$$

quark mass = expl. sym-breaking

$$\mathcal{L}_{\rm LO}^{\rm d\chi PT} = \frac{F_\pi^2}{4} \hat{\chi}^2 {\rm Tr}[\partial^\mu U \partial_\mu U^\dagger] + \frac{B_0 F_\pi^2}{2} \hat{\chi}^{\Delta_{\bar{q}q}} ({\rm Tr}[\mathcal{M} U^\dagger + U \mathcal{M}^\dagger] + \frac{1}{2} (\partial \chi)^2$$

standard-extend χPT + dilaton **global Weyl inv.**

$$- \frac{\Delta_{\bar{q}q}}{4} {\rm Tr}[\mathcal{M} + \mathcal{M}^\dagger] \hat{\chi}^4) + \frac{1}{12} \chi^2 R + \mathcal{L}_{\rm anom}(\beta'_*) + V(\chi) \,,$$

removes tadpole (**Zumino-term**)

local Weyl inv. - solves Goldstone improvement problem (another talk RZ 2306.12914)

matches trace anomaly (talk later)

potential (big unknown) last part …

Ready for T^ρ_ρ-correlator …

- Trace of EMT: $T^\rho{}_\rho|_{\rm phys} = \dfrac{\beta}{2g} G^2$

- Formally (& RG)

$$(\gamma_{G^2})_* = \beta'_* \qquad \Rightarrow \Delta_{T^\rho{}_\rho} = \Delta_{G^2} = 4 + \beta'_*$$

$$\beta = \beta'_* \delta g + \beta''_* \frac{(\delta g)^2}{2} + \mathcal{O}((\delta g)^3)\,, \quad \delta g \equiv g - g_*$$

$$\langle T^\rho{}_\rho(x) T^\rho{}_\rho(0) \rangle \propto (\beta'_* \delta g + \beta''_* \frac{(\delta g)^2}{2})^2 \frac{1}{(x^2)^{4+\beta'_*}}$$

- EFT **difference between χPT and dilaton-χPT** (with improvement RZ 2306.12914)

$$T^\rho{}_\rho|^{\rm LO}_{\chi\rm PT} = -\frac{1}{2}\partial^2 \pi^a \pi^a\,, \quad T^\rho{}_\rho|^{\rm LO}_{d\chi\rm PT} = 0$$

$$\langle T^\rho{}_\rho(x) T^\rho{}_\rho(0) \rangle^{\rm LO}_{\chi\rm PT} \propto \frac{1}{x^8}\,, \quad \langle T^\rho{}_\rho(x) T^\rho{}_\rho(0) \rangle^{\rm LO}_{\rm d\chi PT} \propto 0$$

- χPT implies $\beta'_* = 0$ (dχPT non-conclusive)

  Second method by **RG equations** reveals $\beta_* = 0$ **equally!**

2nd main result

$\beta'_* = 0$ seems important for consistency

$$\delta g \propto \frac{1}{|\beta''_*| \ln(\mu/\lambda_{IR})}$$

- Power-running $\delta g \propto \mu^{\beta'_*}$ $\Rightarrow$ **log-running**

  $\Rightarrow$ seems can **drop** $\mathscr{L}_{\textbf{anom}}(\beta'_*)$ from LO Lagrangian

  as anomaly reproduced in extending "EMT in ✗PT" Donoghue & Leutwyler 90'

  $\Rightarrow$ **log-running**, sign of **mass-gap**. QCD asymptotes into Goldstone-EFT

- Makes light (or massless) dilaton more probable since: $m_D = \mathcal{O}(\beta'_*) \to \mathcal{O}(\beta''_*)$

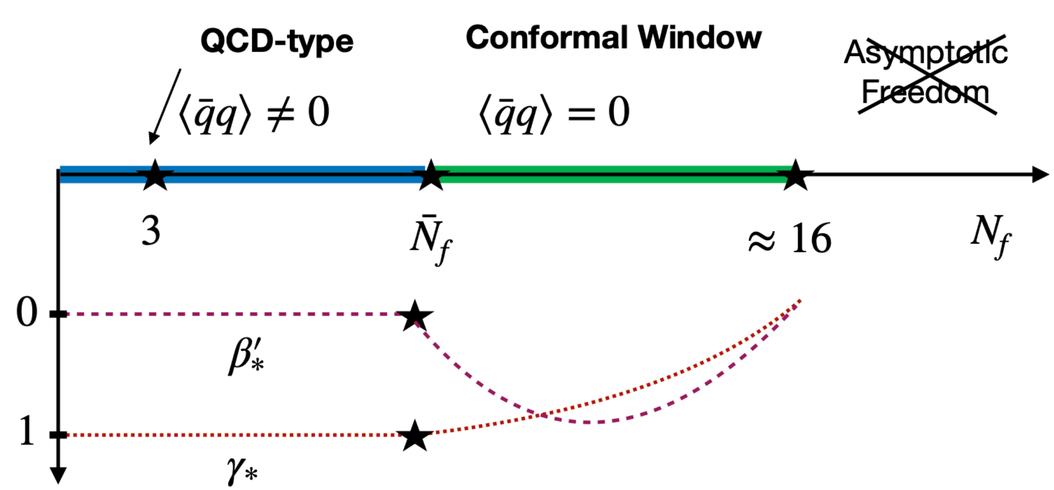
- Continuous **matching** to **N=1 SUSY** conformal window $\beta'_* \to 0$ @boundary

  Anselmi, Grisaru, Johanson 97' Shifman RZ '23

$$\beta'_*|_{\rm el} = \beta'_*|_{\rm mag} \quad \Leftrightarrow \quad \langle T^\rho{}_\rho(x) T^\alpha{}_\alpha(0)\rangle_{\rm mag} \overset{\rm IR}{\longleftrightarrow} \langle T^\rho{}_\rho(x) T^\alpha{}_\alpha(0)\rangle_{\rm el}$$

- Summary figure:

  $SU(N_c)\ , N_c = 3$



Applications & further topics …

….cannot cover all of them

1) **consistency** with **N=1** supersymmetric gauge theories CW and below

2) massless dilaton: $\langle N | T^{\rho}_{\rho} | N \rangle = 0$ and non-zero nucleon mass $m_N \neq 0$

→ *dilaton analogue of Goldberger-Treiman relation*

Del Debbio, RZ JHEP'22 2112.1364

3) Goldstone **improvement problem**

4) **soft-dilaton theorems**

5) **massive or massless** dilaton in chiral limit?

6) **σ-meson as dilaton** in (chiral limit of) QCD

7) **Higgs as a dilaton** BSM

present 4) and brief remarks on 6) & 7) if time permits

(Double) soft theorems for Goldstones

2312.13761

- Idea: light-particle X (mass gap above): $q_X \to 0$ (soft) matrix element simplifies!
- **Pion soft thm** as warm up

$$\langle \pi^a(q)\beta|\mathcal{O}(0)|\alpha\rangle = -\frac{i}{F_\pi}\langle\beta|[Q_5^a,\mathcal{O}(0)]|\alpha\rangle \;+\; \lim_{q\to 0} iq\cdot R^a$$

remainder, almost always zero $\quad R^a_\mu = -\frac{i}{F_\pi}\int d^dx e^{iq\cdot x}\langle\beta|TJ^a_{5\mu}(x)\mathcal{O}(0)|\alpha\rangle$

- GMOR formula: $\quad T^\rho{}_\rho = \mathcal{O}_{\bar{q}q} + \mathcal{O}(\delta g)$ $\quad \mathcal{O}_{\bar{q}q} = (1+\gamma_*)\sum_q m_q\bar{q}q$

$$2m_\pi^2 = \langle\pi^a|T^\rho{}_\rho|\pi^a\rangle = \langle\pi^a|\mathcal{O}_{\bar{q}q}|\pi^a\rangle = \frac{-(1+\gamma_*)m_q}{F_\pi}\langle 0|i[Q_5^a,\bar{q}\mathbb{1}_{N_f}q]|\pi^a\rangle$$

$$= \frac{2m_q(1+\gamma_*)}{F_\pi}\langle 0|P^a|\pi^a\rangle = \frac{-2m_q(1+\gamma_*)}{F_\pi^2}\langle\bar{q}q\rangle\ ,$$

$$m_\pi^2F_\pi^2 = -(1+\gamma_*)m_q\langle\bar{q}q\rangle = -2m_q\langle\bar{q}q\rangle$$

GMOR formula
(if $\gamma_* = 1$ once more)!

(Double) soft theorem for Dilaton (too quick)

$$\langle D(q)\beta|\mathcal{O}(0)|\alpha\rangle = -\frac{1}{F_D}\langle\beta|i[Q_D,\mathcal{O}(0)]|\alpha\rangle + \lim_{q\to 0} iq\cdot R$$

$$i[Q_D,\mathcal{O}(x)] = (\Delta_{\mathcal{O}} + x\cdot\partial)\mathcal{O}(x)$$

$$R_\mu = -\frac{i}{F_D}\int d^dx e^{iq\cdot x}\langle\beta|TJ^D_\mu(x)\mathcal{O}(0)|\alpha\rangle$$

- Dilaton-GMOR formula?

$$m_D^2F_D^2 = \frac{1}{2}\Delta_{\mathcal{O}}^2\langle\mathcal{O}\rangle < 0$$

Since vacuum energy shift needs to be negative: $\langle\delta T^\rho{}_\rho\rangle = \langle\mathcal{O}\rangle < 0$

- That surely went wrong …. negative mass! Let’s be more careful.

(Double) soft theorem for Dilaton (more careful)

$$2m_D^2 = \langle D|\mathcal{O}(x)|D\rangle = -\frac{1}{F_D}\langle 0|i[Q_D, \mathcal{O}(x)]|D\rangle = -(\Delta_{\mathcal{O}} + x\cdot\partial)\langle 0|\mathcal{O}(x)|D\rangle$$

- There is x-dependence in matrix element: $\langle 0|\mathcal{O}(x)|D(p)\rangle = F_{\mathcal{O}} e^{-ipx}$
- Interpret as distribution to be smeared out (form wave packet)

$$\mathbb{1}_V[x\cdot\partial\langle 0|\mathcal{O}(x)|D\rangle] = -d\frac{1}{V}\int_V d^dx\langle 0|\mathcal{O}(x)|D\rangle$$

$$\mathbb{1}_V = \frac{1}{V}\int_V d^dx$$

- Get a better formula with positive mass if $d > \Delta_{\mathcal{O}}$.

$$m_D^2 F_D^2 = \frac{1}{2}(\Delta_{\mathcal{O}} - d)\Delta_{\mathcal{O}}\langle\mathcal{O}\rangle > 0$$

Mass operator ought to be $\Delta_{\mathcal{O}} = 2$

- Intermediate result from before:

$$m_D^2 F_D = \frac{1}{2}(d - \Delta_{\mathcal{O}})\langle 0|\mathcal{O}|D\rangle$$

- Definition of decay constant:

$$\langle D(q)|T_{\mu\nu}|0\rangle = \frac{F_D}{d-1}(m_D^2 \eta_{\mu\nu} - q_\mu q_\nu)$$

$$m_D^2 F_D = \langle 0|T^\rho{}_\rho|D\rangle = \langle 0|\mathcal{O}|D\rangle$$

- Comparing, one concludes (d=4):

$$\Delta_{mass} = \Delta_{\mathcal{O}} = 2$$

3rd main result

Note: - valid beyond gauge theory
- reproducible by Lagrangian (a bit tricky …)
- $x \cdot \partial$**-term** is the **analogue** of **Zumino-term** in EFT!
- $\Delta_{mass} = 2$ is consistent with EFT mass operator (free field)

Consequences $\Delta_{mass} = 2$

- $m_q = 0,$ only $T^\rho_\rho \supset G^2$ with $\Delta_{G^2} = 4 + \beta'_* = 4 \neq 2$ cannot provide mass

sets **question mark** on PCDC-type formula $\bar{m}_D^2 \propto \beta \langle G^2 \rangle$ (widely assumed)

$\bar{m}_D = 0$
massless dilaton?

or assumptions break down
(does **dilaton-χPT** makes sense?)

- In literature: a) Δ_{mass} open parameter or b) Δ_{mass} = 4 (PCDC)

$$V_\Delta(\hat{\chi}) = \frac{m_D^2 F_D^2}{\Delta - d}\left(\frac{1}{\Delta}\hat{\chi}^\Delta - \frac{1}{d}\hat{\chi}^d\right) = m_D^2 F_D^2 \left(\frac{1}{2}\hat{D}^2 - \frac{d+\Delta}{3!}\hat{D}^3 + \frac{(d+\Delta)^2}{4!}\hat{D}^4 + \mathcal{O}(D^5)\right)$$

↑ Zumino-term

- $m_q \neq 0,$ only $T^\rho_\rho \supset m_q \bar{q} q$ with $\Delta_{\bar{q}q} = 3 - \gamma_* = 2$. exactly as it should!

The essence of QCD and the dilaton

- **A dilaton in QCD?** Who? Consensus it would be the $\sigma \equiv f_0(500)$-meson

$$\sqrt{s_\sigma} = m_\sigma - \frac{i}{2}\Gamma_\sigma = (441^{+16}_{-8} - i272^{+9}_{-12.5})\,\mathrm{MeV}\ ,$$

Caprini, Colangelo, Leutwyler'06
Roy-equations+input

- **Question**: does m_σ become massless or nearly massless in chiral limit?
  Fact: *nobody knows*, some indication it becomes lighter.

- using **dilaton-χPT:**
  1) can reproduce width ($SU(3)_F$-analysis): $\Gamma_\sigma = 616\,^{-108}_{+146} \pm \mathrm{syst}^{*}$ MeV
  2) soft-mass even too large (EFT-convergence broken)

- **Concluding**: 1) success (already 1970's) 2) inconclusive
  Hence, not bad but there could be more to it …

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* notion of σ decay constant F_σ not well-defined, took model-value needs further thought

The higgs boson as a dilaton

universal part

- If **v = 0, SM conformal** (up to log-running), Higgs like a dilaton

$$(1+\frac{h}{v}) \to \chi = e^{-\frac{D}{F_D}} \to (1+\frac{h}{F_D})$$

If number of **doublets = 1** $\Rightarrow v = F_\pi$

- $r = \frac{F_\pi}{F_D}$ = 1 is the Standard Model limit

- One can deduce indirectly: $r_{QCD} = 1.0(2) \pm$ syst, **intriguing!**
  a) **no symmetry reason** for this to happen (however, systematics…)
  b) closeness to unity, **LO-invisible @ LHC**

Why does the dilaton couple like the Higgs?

non-universal part

1. popular just before LHC $G_{CFT} = G_{SM} \times G'$ + $\delta\mathscr{L}_{\not{CFT}} = c\mathcal{O}$

Golfberger et al, Terning et al etc

new-sector

in trouble: $\delta_{SM}(gg \to h) \propto \delta_{SM}(h \to \gamma\gamma) \propto \Delta\beta_{decoupled}$ = too large

when it is said that *"the dilaton as a Higgs has been excluded by the LHC"*. then that's what people mean.

2. another idea (Cata, Crewther'Tunstall, 18') $G_{SM}^{\text{no Higgs}} \xleftrightarrow{\text{Yukawa}} G'$

$$\mathscr{L} \supset \frac{1}{4} v^2 tr[D^\mu U D_\mu U^\dagger] - v\bar{q}_L Y_d U \mathscr{D}_R + \ldots$$

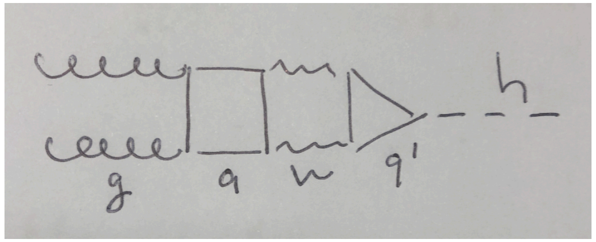
$$U = \exp(i2T^a\pi^a/F_\pi) \qquad U \to V_L U V_Y\,, \quad V_Y = e^{iyT_3}$$

how to bring back the higgs/dilaton is not addressed in that paper, one cannot use the compensator argument as only G' IR-CFT.

In 2312.13761 it is argued that if there is a symmetry reason for $r_2 \approx 1$, then same reason might enforce the right coupling aka

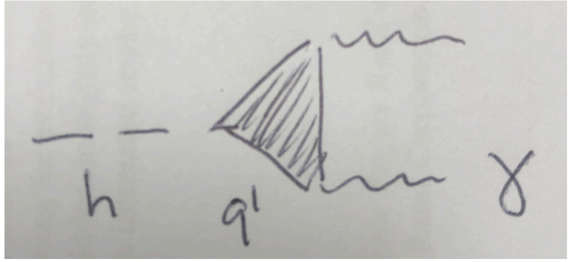
$$\mathcal{L} \supset \frac{1}{4} v^2 e^{-2D/F_D} \mathrm{Tr}[D^\mu U D_\mu U^\dagger] - v e^{-D/F_D} \bar{q}_L Y_d U \mathcal{D}_R + \dots$$

- **Constraints?**



$\delta_{SM}(gg \to h) =$ NNLO

$\delta_{SM}(h \to \gamma\gamma) =$ non-perturatbive



EWPO: e.g. S-parameter $\delta S = \mathcal{O}(2\%)$ if $r_2 = 1$

most “dangerous one” looks like $h \to \gamma\gamma$
… to be continued & discussed or other idea

- **Higgs-dilaton potential?**

radiatively induced aka composite Higgs with $\Lambda_{G'} = \mathcal{O}(1)\, \Lambda_{EW}$

Summary and Conclusions

- Aspects of **pions physics are consistent with IRFP** with $\gamma_* = 1, \beta'_* = 0$
- **Mass operator** $T^\rho_\rho \supset \mathcal{O}$ has $\Delta_{\mathcal{O}} = 2$ by double soft dilaton theorem
- **integer scaling dimension** (characteristic of EFT)
- **Consistency** with **N=1** SUSY gauge theories and more …
- **Open problems**:
  - **Dilaton mass** in **chiral limit**? (lattice, analyticity methods …)

    If $m_D \neq 0 \ll 4\pi F_\pi \approx m_\rho$ then still interesting (technically inconvenient)
  - Is it **accidental** that $r_{CDQCD} = F_\pi / F_D = 1.0(2)$ is close to SM-Higgs limit?
  - Investigate realistic models assuming F_π / F_D close to one

Thank you

BACKUP

Massive Hadrons in Conformal Phase

Chiral limit $m_q \to 0$ resolve the contradiction below

$$\langle \phi(p) \,|\, T^{\mu}_{\mu} \,|\, \phi(p) \rangle \begin{cases} = 2m_\phi^2 & \text{standard formula} \\ = 0 & \text{with (massless) dilaton} \end{cases}$$

"The dilaton can hide the nucleon mass"

Del Debbio, RZ JHEP'22 2112.1364

Gravitational Form Factors

focus scalar instead of nucleon

- parameterise using Lorentz & translation invariance ($\partial^\mu T_{\mu\nu} = 0$)

$$\langle \varphi(p') \,|\, T_{\mu\nu} \,|\, \varphi(p) \rangle = 2\mathscr{P}_\mu \mathscr{P}_\nu G_1(q^2) + (q_\mu q_\nu - q^2 \eta_{\mu\nu}) G_2(q^2)$$

$\mathscr{P} = \frac{1}{2}(p + p')\ , \quad q = p - p'$ momentum transfer

- consider soft limit $q \to 0$ then G_2 drops and using $P_\mu = \int d^3x T^0_\mu$

$$\langle \phi(p) \,|\, T^\mu_\mu \,|\, \phi(p) \rangle = 2m_\phi^2$$

$$G_1(0) = 1$$

… seems the end of the road (for massive hadrons and conformality)

- Let's have another look at*

$$\langle \varphi(p') \,|\, T_{\mu\nu} \,|\, \varphi(p) \rangle = 2P_\mu P_\nu G_1(q^2) + (q_\mu q_\nu - q^2 \eta_{\mu\nu}) G_2(q^2)$$

$\langle \phi(p) \,|\, T^\mu_\mu \,|\, \phi(p) \rangle = 2m_\phi^2$ does **not** need to **hold if**

$$G_2(q^2) = \frac{r}{q^2} + \ldots$$

Goldstone pole (the **dilaton)**

- That is already a bit of a shock - can we make this quantitative?

Yes in soft limit, as then can use $G_1(0) = 1$ and vanishing trace imposes

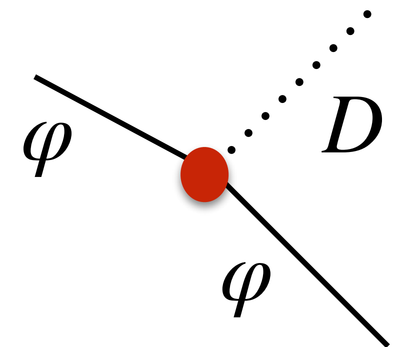
$$r = \frac{2m_\phi^2}{(d-1)}$$

*e.g lecture notes Gell-Mann '69 (pre-QCD), no details worked out

Computation of Residue (new)

$$r = \frac{2m_\phi^2}{(d-1)}$$

- need to know $\langle D\varphi \,|\, \varphi\rangle = i(2\pi)^d \delta\left(\sum p_i\right)$ $g_{\varphi\varphi D}$
- can get it via **compensator trick** **(Weyl scaling)**



$$g_{\mu\nu} \to e^{-2\alpha} g_{\mu\nu}\,, \quad \varphi \to e^{\alpha}\varphi \quad \Rightarrow \quad D \to D - \alpha F_D$$

compensates m_φ^2 by dilaton, regain``conformal inv'': $\delta_\alpha \sqrt{-g}\,\mathscr{L}^{eff} = 0$

$$\mathscr{L}^{eff} \supset -e^{-2D/F_D}\frac{1}{2}m_\varphi^2\varphi^2 \quad \Rightarrow \quad g_{D\varphi\varphi} = \frac{2m_\varphi^2}{F_D}$$

$$r = \frac{2m_\phi^2}{(d-1)}$$

- now apply the LSZ formula (or dispersion theory)

$$\langle D\varphi|\varphi\rangle = \lim_{q^2\to 0}(-i)\frac{q^2}{Z_D}\int d^dx e^{iq\cdot x}P_2^{\mu\nu}T_{\mu\nu}^{(\varphi)}(p,p',x)$$

$$= \lim_{q^2\to 0}(-i)\frac{q^2}{Z_D}G_2(q^2)(2\pi)^d\delta\left(\sum p_i\right)$$

use EMT as dilaton interpolator

$Z_D = -F_D/(d-1)$

- from where we get exactly the right residue

$$r = \lim_{q^2\to 0} q^2 G_2(q^2) = -g_{\varphi\varphi D}Z_D = \frac{2m_\varphi^2}{d-1}$$

- Rather encouraging. The **approach** is **self-consistent**!

The dilaton improves Goldstones

based on

2306.12914 RZ

The standard improved scalar field

- Two terms curved space, no dim. couplings* $\mathcal{L} = \frac{1}{2}\left((\partial\varphi)^2 - \xi R \varphi^2\right)$

$$T^{\rho}{}_{\rho} = -d_{\varphi}(\partial\varphi)^2 + \xi(d-1)\partial^2\varphi^2 = (d-1)(\xi - \xi_d)\partial^2\varphi^2$$

↑ eom

- Conformal $T^{\rho}_{\rho} = 0$, only for $\xi = \xi_d \equiv \frac{(d-2)}{4(d-1)} \to \frac{1}{6}$ (d=4)
  - improved EMT Callan, Coleman, Jackiw'70, finite EMT (necessary as observable)
  - earlier in GR: Penrose'64 required by weak equivalence principle Chernikov&Tagirov'68
  - finite integrated Casimir-effect deWitt'75

- Heuristically, $\mathscr{L} \propto R\phi^2$, not possible to write with coset field $U = e^{i\frac{\pi^a T^a}{F_\pi}}$

Dolgov & Voloshin'82 Leutwyler-Shifman '89, Donoghue-Leutwyler' 91

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* may also work in flat space from start, but less elegant

Intermezzo on relevance for flow theorems

- Focus d=2 for simplicity, Weyl anomaly $T^\rho_\rho = cR$ reveals central charge of CFT.

  c-theorem (Zamalodchikov'86).: $\Delta c = c_{UV} - c_{IR} \geq 0$

  Cardy'88.: $\Delta c \propto \int d^2x\, x^2 \langle T^\rho_\rho(x) T^\rho_\rho(0)\rangle \quad \Rightarrow \quad T^\rho_\rho \to 0$ in UV and IR fast enough

  d=2 ok, Goldstone special anyway

- d=4, if **Goldstones not improvable** $T^\rho_\rho = -\frac{1}{2}\partial^2\pi^2$, then **log-IR divergence**

  a-thm* & $\Box R$-flow analogue formula IR-divergent

$\Rightarrow$ Goldstone improvement desirable

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*for a-thm, Luty, Polchiniski, Rattazzi'12 ' provide argument formula is IR-onvergent as inclusive enough

The Goldstone improvement proposal

- dilaton-pion system improvement

$$\mathcal{L}_{\rm LO} = \mathcal{L}_{\rm kin,4} + \mathcal{L}_4^R - V_4(\chi)$$

$$\mathcal{L}_{\rm kin,d} = \frac{F_\pi^2}{4}\hat{\chi}^{d-2}\mathrm{Tr}[\partial^\mu U \partial_\mu U^\dagger] + \frac{1}{2}\chi^{d-4}(\partial\chi)^2$$

standard Lag.

$$\mathcal{L}_d^R = \frac{\kappa}{4} R \chi^{d-2}$$

0, no mass (later..)

improvement term, κ to be **determined**

- **locally Weyl invariant** $\Rightarrow$ conformal invariance.

$$\kappa = \kappa_d \equiv \frac{2}{(d-1)(d-2)} \stackrel{d\to 4}{\to} \frac{1}{3}$$

Compared to $\xi_4 = 1/6$ like a ``double improvement'' (more to say)

- realises decay constant in EFT

$$\langle 0|T_{\mu\nu}|D(q)\rangle \stackrel{\rm def}{=} \frac{F_D}{d-1}(m_D^2\eta_{\mu\nu} - q_\mu q_\nu) \qquad = \qquad \langle 0|T^R_{\mu\nu}|D(q)\rangle = \langle 0|\frac{1}{6}(\eta_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu)\chi^2|D(q)\rangle$$

3a. Improvement $T^{\rho}_{\rho} = 0$ use of equation of motion

- dilaton eom: $\chi \partial^2 \chi = 2\mathcal{L}^{\pi}_{\text{kin},4} - \partial_{\ln\chi} V_4$

$$T_{\mu\nu} = \frac{F_\pi^2}{2}\hat{\chi}^2 \text{Tr}[\partial_\mu U \partial_\nu U^\dagger] + \partial_\mu \chi \partial_\nu \chi - \eta_{\mu\nu}(\mathcal{L}_{\text{kin},4} - V_4) + T^R_{\mu\nu}$$

$$T^R_{\mu\nu} = \frac{\kappa}{2}(g_{\mu\nu}\partial^2 - \partial_\mu \partial_\nu)\chi^2$$

$$\begin{aligned} T^{\rho}{}_{\rho}|_{V=0} &= \frac{3}{2}\kappa \partial^2 \chi^2 - 2\mathcal{L}^{\pi}_{\text{kin},4} - 2\mathcal{L}^{D}_{\text{kin},4} \\ &\overset{eom}{=} \frac{3}{2}\kappa \partial^2 \chi^2 - (\partial\chi)^2 - \chi \partial^2 \chi \\ &= (3\kappa - 1)\{\chi \partial^2 \chi + (\partial \chi)^2\} = 0 \end{aligned}$$

$$\kappa = \kappa_4 = \frac{1}{3}$$

- **works** as expected from **local Weyl invariance**, also works d-dim curved space