## AdS/Yang Mills From QCD to Composite Higgs

## Nick Evans

## University of Southampton

$$
\begin{array}{ll}
\text { Work with Johanna Erdmenger, Kostas Rigatos } \\
\text { and Werner Porod: } \\
& \begin{array}{l}
1907.09489 \text { [hep-th] } \\
2009.10737 \\
\text { 2010.10279 [hep-ph] } \\
2012.00032[\text { hep-ph] }
\end{array}
\end{array}
$$



Work with JE, WP and
Yang Liu: 2304.09190 [hep-th]
Work with Matt Ward: 2304.10816 [hep-ph]

## The Program...

- Dynamical mass generation in strongly coupled non-supersymmetric gauge theory
- A holographic description in terms of running anomalous dimensions and NJL operators
- Apply to QCD... the proton and sexaquark... composite higgs theories... including Sp(2Nc)...


## Introduction

One of the most remarkable aspects of the Standard Model is that the ground state symmetries are less than those of the bare Lagrangian...

- Higgs potential is adhoc and not yet understood
- QCD provides a DYNAMICAL symmetry breaking mechanism

$$
S U(2)_{L} \times S U(2)_{R} \rightarrow S U(2)_{V}
$$

$$
m \bar{\psi} \psi=m\left(\bar{\psi}_{L} \psi_{R}+\text { h.c. }\right)
$$

$$
\bar{u} \gamma^{\mu} u=\bar{u}_{L} \gamma^{\mu} u_{L}+\bar{u}_{R} \gamma^{\mu} u_{R}
$$

Evidence: lack of parity doubling, proton mass, Goldstone pions

$$
\left\langle\bar{u}_{L} u_{R}+\bar{d}_{L} d_{R}+h . c .\right\rangle \neq 0
$$



## How Does AdS/CFT Work 1

A weak strong duality that at least works for N=4 SYM and its deformations...


Dilatations

$$
\int d^{4} x \partial^{\mu} \phi \partial_{\mu} \phi, \quad x \rightarrow e^{-\alpha} x, \quad \phi \rightarrow e^{\alpha} \phi
$$

Become spacetime symmetry of AdS

$$
\rho \rightarrow e^{\alpha} \rho
$$

$\rho \quad$ is a continuous mass dimension $\rightarrow$ RG Scale

## How Does AdS/CFT Work 2



$$
\sqrt{-\operatorname{Detg}}=\operatorname{Det}\left[-\left(\begin{array}{ccccc}
-\rho^{2} & 0 & 0 & 0 & 0 \\
0 & \rho^{2} & 0 & 0 & 0 \\
0 & 0 & \rho^{2} & 0 & 0 \\
0 & 0 & 0 & \rho^{2} & 0 \\
0 & 0 & 0 & 0 & \frac{1}{\rho^{2}}
\end{array}\right)\right]^{1 / 2}=\rho^{3}
$$



Operators and sources appear as fields in the bulk

Eg

$$
\int d^{4} x m \bar{\psi} \psi
$$

$m$ is the quark mass
$c$ is the quark condensate

A field for the mass/condensate:

$$
S=\int d^{4} x \int d \rho \frac{1}{2} \rho^{3}\left(\partial_{\rho} L\right)^{2}
$$

$$
\partial_{\rho}\left[\rho^{3} \partial_{\rho} L\right]=0
$$

$$
L=m+\frac{c}{\rho^{2}}
$$

## Top/down

hep-th/0306018


Sakai-Sugimoto model (D4/D8)


Top down models that describe dynamical chiral symmetry breaking exist....

Magnetic catalysis is the most controlled case... (Johnson, Filev)

They all look a bit baroque...

Probe limit DBI Action captures key elements


$$
\mathcal{O}=\tilde{\mathcal{O}} \mu^{d} ; \quad \mathrm{Z}_{\mathrm{On}}
$$

$$
d=\frac{1}{\mathcal{O}} \mu \frac{d \mathcal{O}}{d \mu}
$$

$$
\gamma_{\mathcal{O}}=-\frac{1}{Z_{\mathcal{O}}} \mu \frac{d Z_{\mathcal{O}}}{d \mu}
$$

FIG. 1: Diagrams at one loop order contributing to the anomalous dimension of a gauge invariant scalar operator with $n$ quark legs.

Wave function renormalization of n-legs

Vertex factor (upto constant Cn)

$$
Z_{\psi}=1-C_{2}(R) \xi \frac{\alpha}{4 \pi} \frac{1}{\epsilon}
$$

$\xi$ is gauge
parameter

$$
Z_{\mathcal{O} n}=\left(1+C_{n}(3+\xi) \frac{\alpha}{4 \pi} \frac{1}{\epsilon}\right) Z_{\psi}^{n / 2}
$$

$$
C_{n}=n C_{2}(R) / 2
$$

For $\xi$ independence

$$
Z_{\mathcal{O} n}=1+\frac{3 n}{2} C_{2}(R) \frac{\alpha}{4 \pi} \frac{1}{\epsilon}
$$

$$
\gamma_{\mathcal{O} n}(\mu)=-n \frac{\alpha(\mu)}{\pi}
$$

# Running Dimensions in Holography 

Holographically we can change the dimension of our operator by adding a mass term

$$
\partial_{\rho}\left[\rho^{3} \partial_{\rho} L\right]-\rho \Delta m^{2} L=0
$$

$$
L=\frac{m_{F P}}{\rho^{\gamma}}+\frac{c_{F P}}{\rho^{2-\gamma}}, \quad \gamma(\gamma-2)=\Delta m^{2}
$$

$\Delta m^{2}=-1$ corresponds to $\gamma=1$ and is special - the Breitenlohner Freedman bound instability...

So we can include a running coupling by a $\rho$ dependent mass squared for the scalar.

Top down derivation: many string constructions eg probe D7 branes in D3 backgrounds are examples of this...

Very complex geometries describe the gauge theory glue-dynamics... a single quark in that background is described by a DBI field such as this with the running of the mass determined by the glue-dynamics...

## Dynamic AdS/YM

Timo Alho, NE, KimmoTuominen 1307.4896

$$
S=\int d^{4} x d \rho \operatorname{Tr} \rho^{3}\left[\frac{1}{\rho^{2}+|X|^{2}}|D X|^{2}+\frac{\Delta m^{2}}{\rho^{2}}|X|^{2}\right.
$$

$$
X=L(\rho) e^{2 i \pi^{a} T^{a}}
$$

$$
d s^{2}=\frac{d \rho^{2}}{\left(\rho^{2}+|X|^{2}\right)}+\left(\rho^{2}+|X|^{2}\right) d x^{2}
$$

$|X|=L$ is now the dynamical field whose solution will determine the condensate as a function of $m$ - the phase is the pion.

We use the top-down IR boundary condition on mass-shell: $\quad X^{\prime}(\rho=X)=0$

X enters into the AdS metric to cut off the radial scale at the value of $m$ or the condensate - no hard wall

The gauge DYNAMICS is input through a guess for $\Delta m$

$$
\Delta m^{2}=-2 \gamma=-\frac{3\left(N_{c}^{2}-1\right)}{2 N_{c} \pi} \alpha \text { The only free parameters are } \mathrm{Nc}, \mathrm{Nf}, \mathrm{~m}, \Lambda
$$

## The Meaning of $X$

Q.CD: we can treat the up quark as a probe in a background of glue + other quarks:

$$
X=\bar{u} u
$$

$\gamma$ Includes the running from the quarks so there is some by hand "backreaction".... We can study mesons made of u quarks which by symmetry are degenerate with $d$ states and mixed ud states..

Or we can promote $X$ to a $2 \times 2$ matrix, Tr over the action... now we have a nonabelian DBI which can in principle include mass splitting....

Real Representations: If quarks are in a real rep (eg adjoint) we form a Majorana spinor $\mathrm{u}=\left(\psi\right.$, -i $\left.\sigma \psi^{* *}\right)$ and again $X=\bar{u} u$

And bound states are $\mathbf{X}$ with $\gamma$-matrix structure inserted...
Assuming quarks are mass degenerate again we just look at one set of states, degenerate by symmetry to all the others....

## Formation of the Chiral Condensate

We solve for the vacuum configuration of $L$

$$
\partial_{\rho}\left[\rho^{3} \partial_{\rho} L\right]-\rho \Delta m^{2} L=0 .
$$



Read off $m$ and $q q$ in the UV...
$\Delta m^{2}$ from QCD
Shoot out with

$$
L^{\prime}(\rho=L)=0
$$

## Meson Fluctuations

$$
S=\int d^{4} x d \rho \operatorname{Tr} \rho^{3}\left[\frac{1}{\rho^{2}+|X|^{2}}|D X|^{2}+\frac{\Delta m^{2}}{\rho^{2}}|X|^{2}+\frac{1}{2 \kappa^{2}}\left(F_{V}^{2}+F_{A}^{2}\right)\right]
$$

$$
L=L_{0}+\delta(\rho) e^{i k x} \quad k^{2}=-M^{2}
$$

$$
\partial_{\rho}\left(\rho^{3} \delta^{\prime}\right)-\Delta m^{2} \rho \delta-\left.\rho L_{0} \delta \frac{\partial \Delta m^{2}}{\partial L}\right|_{L_{0}}
$$

$$
+M^{2} R^{4} \frac{\rho^{3}}{\left(L_{0}^{2}+\rho^{2}\right)^{2}} \delta=0
$$



The source free solutions pick out particular mass states... the $\sigma$ and its radial excited states...

The gauge fields let us also study the operators and states

## Decay Constants (a la. AdS/QCD - hep-ph/0501128 [hep-ph])

 Decay constants are determined by allowing a source to couple to a physical state

## $\mathrm{F}_{\mathrm{V}}{ }^{2}$

## Note not $F_{V} m_{V}$

Now we need to fix the normalizations of the holographic linear perturbations...

For the physical states we canonically normalize the kinetic terms...

For the source solutions we fix $\kappa$ and the norms so that we match perturbative results for eg $\Pi_{\mathrm{VV}}$ in the UV... $N_{V}^{2}=N_{A}^{2}=\frac{g_{5}^{2} d(R) N_{f}(R)}{48 \pi^{2}}$

## Baryons

cf Brodsky, de Teramond hep-th/0501022 [hep-th]

In D3/D7 system some quark-gaugino-quark tri-fermion states are described by world volume fermions on the D7 it does not seem unreasonable to include three quark states in this way therefore.

## Plus our

### 1907.09489 [hep-th]

$$
S_{1 / 2}=\int d^{5} x \rho^{3} \bar{\Psi}\left(\not D_{\mathrm{AAdS}}-m\right) \Psi
$$

The four component fermion satisfies the second order equation

$$
\left(\partial_{\rho}^{2}+\mathcal{P}_{1} \partial_{\rho}+\frac{M_{B}^{2}}{r^{4}}+\mathcal{P}_{2} \frac{1}{r^{4}}-\frac{m^{2}}{r^{2}}-\mathcal{P}_{3} \frac{m}{r^{3}} \gamma^{\rho}\right) \psi=0
$$

where $M_{B}$ is the baryon mass and the pre-factors are given by

$$
\begin{aligned}
& \mathcal{P}_{1}=\frac{6}{r^{2}}\left(\rho+L_{0} \partial_{\rho} L_{0}\right), \\
& \mathcal{P}_{2}=2\left(\left(\rho^{2}+L_{0}^{2}\right) L \partial_{\rho}^{2} L_{0}+\left(\rho^{2}+3 L_{0}^{2}\right)\left(\partial_{\rho} L_{0}\right)^{2}+4 \rho L_{0} \partial_{\rho} L_{0}+3 \rho^{2}+L_{0}^{2}\right), \\
& \mathcal{P}_{3}=\left(\rho+L_{0} \partial_{\rho} L_{0}\right) .
\end{aligned}
$$

$$
\begin{aligned}
\psi_{+} & \sim \mathcal{J} \sqrt{\rho}+\mathcal{O} \frac{M_{B}}{6} \rho^{-11 / 2}, \\
\psi_{-} & \sim \mathcal{J} \frac{M_{B}}{4} \frac{1}{\sqrt{\rho}}+\mathcal{O} \rho^{-9 / 2}
\end{aligned}
$$

$$
\begin{aligned}
& \text { IR boundary conditions } \\
& \qquad \begin{array}{ll}
\psi_{+}\left(\rho=L_{I R}\right)=1, & \partial_{\rho} \psi_{+}\left(\rho=L_{I R}\right)=0 \\
\psi_{-}\left(\rho=L_{I R}\right)=0, & \partial_{\rho} \psi_{-}\left(\rho=L_{I R}\right)=\frac{1}{L_{I R}}
\end{array}
\end{aligned}
$$

## NJL Operators

$$
\mathcal{L}=\bar{\psi}_{L} \not \psi_{L}+\bar{\psi}_{R} \not \psi_{R}+\frac{g^{2}}{\Lambda_{U V}^{2}} \bar{\psi}_{L} \psi_{R} \bar{\psi}_{R} \psi_{L}
$$

## $\square-\square$

$$
-i m=-\frac{g^{2}}{\Lambda_{U V}^{2}} \int \frac{k^{2} d k^{2}}{16 \pi^{2}} \frac{\operatorname{Tr}(k+m)}{k^{2}+m^{2}}
$$



$$
1=\frac{g^{2}}{4 \pi^{2}}\left(1-\frac{m^{2}}{\Lambda_{U V}^{2}} \log \left[\left(\Lambda_{U V}^{2}+m^{2}\right) / m^{2}\right]\right)
$$

## Witten's Holographic Prescription

$$
\begin{aligned}
\frac{g^{2}}{\Lambda_{U V}^{2}} \bar{\psi}_{L} \psi_{R} \bar{\psi}_{R} \psi_{L} & \rightarrow \frac{g^{2}}{\Lambda_{U V}^{2}}\left\langle\bar{\psi}_{L} \psi_{R}\right\rangle \bar{\psi}_{R} \psi_{L} \\
m & =\frac{g^{2}}{\Lambda_{U V}^{2}} \sigma
\end{aligned}
$$

## QCD Dynamics - Nc=3, $\mathrm{Nf}=2, \mathrm{~m}_{\mathrm{q}}=\mathbf{0}$

$$
\mu \frac{d \alpha}{d \mu}=-b_{0} \alpha^{2}, \quad b_{0}=\frac{1}{6 \pi}\left(11 N_{c}-2 N_{F}\right), \quad \gamma=\frac{3 C_{2}}{2 \pi} \alpha=\frac{3\left(N_{c}^{2}-1\right)}{4 N_{c} \pi} \alpha .
$$

| Observables <br> $(\mathrm{MeV})$ | QCD | AdS/SU $(3)$ <br> $2 \mathrm{~F} 2 \bar{F}$ | Deviation |
| :---: | :---: | :---: | :---: |
| $M_{\rho}$ | 775 | $775^{*}$ | fitted |
| $M_{A}$ | 1230 | 1183 | $-4 \%$ |
| $M_{S}$ | $500 / 990$ | 973 | $+64 \% /-2 \%$ |
| $M_{B}$ | 938 | 1451 | $+43 \%$ |
| $f_{\pi}$ | 93 | 55.6 | $-50 \%$ |
| $f_{\rho}$ | 345 | 321 | $-7 \%$ |
| $f_{A}$ | 433 | 368 | $-16 \%$ |
|  |  |  |  |
| $M_{\rho, n=1}$ | 1465 | 1678 | $+14 \%$ |
| $M_{A, n=1}$ | 1655 | 1922 | $+19 \%$ |
| $M_{S, n=1}$ | 990 | $/ 1200-1500$ | 2009 |
| $M_{B, n=1}$ | 1440 | 2406 | $+64 \% /+35 \%$ |

Table 1: The predictions for masses and decay constants (in MeV ) for $N_{f}=2$ massless QCD. The $\rho$-meson mass has been used to set the scale (indicated by the *).

Scale fixed by V-
meson

Pattern sensible

Pion decay constant needs a mass term

Baryon mass high

Radial excitations scale wrongly no string physics included

## Perfecting with HDOs



The weakly coupled gravity dual should only live between the red lines... probably we need HDOs at the UV scale to include matching effects... and stringy effects in the gravity model....
$\frac{g_{S}^{2}}{\Lambda_{U V}^{2}}|\overline{q q}|^{2}$,
$\frac{g_{V}^{2}}{\Lambda_{U V}^{2}}\left|\bar{q} \gamma^{\mu} q\right|^{2}$,
$\frac{g_{A}^{2}}{\Lambda_{U V}^{2}}\left|\bar{q} \gamma^{\mu} \gamma_{5} q\right|^{2}$,

$$
\frac{g_{\mathrm{B}}^{2}}{\Lambda_{U V}^{5}}|q q q|^{2}
$$

| Observables | QCD | Dynamic AdS/QCD | HDO coupling |
| :---: | :---: | :---: | :---: |
| $(\mathrm{MeV})$ |  |  |  |
| $M_{V}$ | 775 | 775 | sets scale |
| $M_{A}$ | 1230 | 1230 | fitted by $g_{A}^{2}=5.76149$ |
| $M_{S}$ | $500 / 990$ | 597 | prediction $+20 \% /-40 \%$ |
| $M_{B}$ | 938 | 938 | fitted by $g_{B}^{2}=25.1558$ |
| $f_{\pi}$ | 93 | 93 | fitted by $g_{S}^{2}=4.58981$ |
| $f_{V}$ | 345 | 345 | fitted by $g_{V}^{2}=4.64807$ |
| $f_{A}$ | 433 | 444 | prediction $+2.5 \%$ |
| $M_{V, n=1}$ | 1465 | 1532 | prediction $+4.5 \%$ |
| $M_{A, n=1}$ | 1655 | 1789 | prediction $+8 \%$ |
| $M_{S, n=1}$ | $990 / 1200-1500$ | 1449 | prediction $+46 \% / 0 \%$ |
| $M_{B, n=1}$ | 1440 | 1529 | prediction $+6 \%$ |

Pretty good... but we've lost some predictivity....

Table 2: The spectum and the decay constants for two-flavour QCD with HDOs from fig. 7 used to improve the spectrum.

## Proton/neutron mass still unconvincing 2304.10816 [hep-ph]

Add in the anomalous dimension for the qqq operator...

$$
\Delta m_{\psi}=\gamma=-\frac{3}{\pi} \alpha
$$

## $M_{B}=1.40 M_{\rho}=1.08 \mathrm{GeV}$

## Sexaquark

If anomalous dimensions have this big an effect how about elsewhere.. Eg sexaquark uuddss (Farrar)

$$
\partial_{r}\left[r^{3} \partial_{r} L\right]-r \Delta m^{2} L+\frac{r^{3} M^{2}}{\left(r^{2}+\frac{2}{3} L_{u d}^{2}+\frac{1}{3} L_{s}^{2}\right)^{2}} L=0
$$

$$
\Delta m^{2}=\left(3+m^{2}\right)+14 \gamma=48-\frac{84}{\pi} \alpha
$$

## $M_{S}=2.27 M_{o} \approx 1.75 \mathrm{GeV}$

Depends on IR assumptions about running (how does di-neutron differ?) but possible deeply bound stable state ( $m<2 m \_\Lambda$ ).... DM candidate or cosmological relic...

## Composite Higgs Models

Our current focus is an $\mathrm{Sp}(2 \mathrm{Nc})$ gauge theory with 2 Dirac fundamental fermions.

The pseudo-reality means the flavour symmetry is $U(4)$ on the 4 Weyl fermions (we neglect the anomaly)

$$
\psi_{i}=\left(\begin{array}{c}
U_{L}^{C} \\
D_{L}^{C} \\
D_{R} \\
U_{R}
\end{array}\right)
$$

The vacuum condensate is anti-symmetric in spin $\left(2 \times 2=1_{A}+3_{S}\right)$, anti-symmetric in colour, so anti-symmetric in flavour... possible vacua
$X=\left(\begin{array}{cccc}0 & L_{0} & 0 & 0 \\ -L_{1} & 0 & 0 & 0 \\ 0 & 0 & 0 & -L_{0} \\ 0 & 0 & L_{0} & 0\end{array}\right)$
$X=\left(\begin{array}{cccc}0 & 0 & 0 & -Q \\ 0 & 0 & Q & 0 \\ 0 & -Q & 0 & 0 \\ Q & 0 & 0 & 0\end{array}\right)$

Both break $U(4)->\operatorname{Sp}(4)^{\ldots}$. . The first is $S U(2)_{L}$ invariant the second breaks $S U(2)_{L}$

$$
X=\left(\begin{array}{cccc}
0 & L_{0} & 0 & 0 \\
-L_{\mathrm{C}} & 0 & 0 & 0 \\
0 & 0 & 0 & -L_{0} \\
0 & 0 & L_{0} & 0
\end{array}\right)
$$

$$
M=\left(\begin{array}{cccc}
0 & m_{1} & 0 & 0 \\
-m_{1} & 0 & 0 & 0 \\
0 & 0 & 0 & m_{2} \\
0 & 0 & -m_{2} & 0
\end{array}\right)
$$

$$
X_{f}=\left(\begin{array}{cccc}
0 & \sigma-Q_{5}+i S-i \pi_{5} & Q_{2}-\pi_{2}+i \pi_{1}-i Q_{1}-Q_{4}+\pi_{4}+i Q_{3}-i \pi_{3} \\
-\sigma+Q_{5}+i \pi_{5}-i S & 0 & Q_{4}+\pi_{4}+i Q_{3}+i \pi_{3} & Q_{2}+\pi_{2}+i Q_{1}+i \pi_{1} \\
\pi_{2}-Q_{2}+i Q_{1}-i \pi_{1}-Q_{4}-\pi_{4}-i Q_{3}-i \pi_{3} & 0 & \sigma+Q_{5}+i S+i \pi_{5} \\
Q_{4}-\pi_{4}+i \pi_{3}-i Q_{3}-Q_{2}-\pi_{2}-i Q_{1}-i \pi_{1}-\sigma-Q_{5}-i S-i \pi_{5} & 0
\end{array}\right)
$$

The 6 Goldstones (16 generators -> 10) are $\pi$ and $S$
$\pi 1-4$ are a 4-plet that transform as bi-doublet under SU(2)L x SU(2)R and can be made into a higgs boson... they obtain a mass from $m 1$ and $m 2$... and from loop corrections - W loops and top loops... here concentrate on the strong dynamics...

S (anomaly) and $\pi 5$ might be dark matter candidates.... Cacciapaglia/Sannino
$X=\left(\begin{array}{cccc}0 & L_{0} & 0 & 0 \\ -L_{0}(\rho) & 0 & 0 & 0 \\ 0 & 0 & 0 & -L_{0} \\ 0 & 0 & L_{0} & 0\end{array}\right)$

Biased with SU(2) L preserving masses

$$
M=\left(\begin{array}{cccc}
0 & m_{1} & 0 & 0 \\
-m_{1} & 0 & 0 & 0 \\
0 & 0 & 0 & m_{2} \\
0 & 0 & -m_{2} & 0
\end{array}\right)
$$

$$
X_{f}=\left(\begin{array}{cccc}
0 & \sigma-Q_{5}+i S-i \pi_{5} & Q_{2}-\pi_{2}+i \pi_{1}-i Q_{1} & -Q_{4}+\pi_{4}+i Q_{3}-i \pi_{3} \\
-\sigma+Q_{5}+i \pi_{5}-i S & 0 & Q_{4}+\pi_{4}+i Q_{3}+i \pi_{3} & Q_{2}+\pi_{2}+i Q_{1}+i \pi_{1} \\
\pi_{2}-Q_{2}+i Q_{1}-i \pi_{1}-Q_{4}-\pi_{4}-i Q_{3}-i \pi_{3} & 0 & \sigma+Q_{5}+i S+i \pi_{5} \\
Q_{4}-\pi_{4}+i \pi_{3}-i Q_{3}-Q_{2}-\pi_{2}-i Q_{1}-i \pi_{1}-\sigma-Q_{5}-i S-i \pi_{5} & 0
\end{array}\right)
$$

Holography: we move to $X$ being a non-abelian structure with the vevs and fluctuations shown...

The bulk must contain $16 \mathrm{U}(4)$ gauge field dual to the operators $\mathrm{q} \gamma^{\mu} \mathrm{q}$ - masses and vevs cause a higgs mechanism in the bulk...

We can have mixing between fields if we don't guess the mass eigenstates...
In 2304.09190 [hep-th] we worked through all this structure in detail... now applying it....

The vacuum for the SU(2) case's running...

$$
\partial_{\rho}\left(\rho^{3} \partial_{\rho} L_{0}\right)-\rho \Delta m^{2} L_{0}=0
$$



The Goldstone mechanism...

$$
A_{\mu}=A_{i, \mu \perp}+\partial_{\mu} \phi_{i}
$$

$$
\begin{aligned}
\partial_{\rho}\left(\rho^{3} \partial_{\rho} \phi_{j}\right)+g_{5}^{2} \frac{L_{0} \rho^{3}}{r^{4}}\left(\sqrt{2} \pi_{i}-L_{0} \phi_{j}\right) & =0, \\
\partial_{\rho}\left(\rho^{3} \partial_{\rho} \pi_{i}\right)-\rho \Delta m^{2}\left(\rho^{2}+L_{0}^{2}\right) \pi_{i}+M^{2} \frac{\rho^{3}}{r^{4}}\left(\pi_{i}-\frac{L_{0}}{\sqrt{2}} \phi_{j}\right) & =0,
\end{aligned}
$$

$\mathrm{M}^{2}=0$ solutions with $\pi=\mathrm{L}_{0}$. and $\phi=$ sqrt[2].... Higgs in the bulk but is only a physical state in the massless theory....

| Observables | $S U(2)$ | Lattice $(S U(2))$ | $S p(4)$ | $S p(6)$ | $S p(8)$ | $S p(10)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $m_{V}(10)$ | $1^{*}$ | $1.00(3)$ | $1^{*}$ | $1^{*}$ | $1^{*}$ | $1^{*}$ |
| $m_{A}(6)$ | 1.66 | $1.11(46)$ | 1.26 | 1.18 | 1.14 | 1.12 |
| $m_{\sigma}(1)$ | 1.26 | $1.5(1.1)$ | 1.20 | 1.22 | 1.23 | 1.23 |
| $m_{Q}(5)$ | 1.13 |  | 1.13 | 1.13 | 1.13 | 1.13 |
| $m_{\pi, S}(6)$ | 0.02 |  | 0.01 | 0.01 | 0.01 | 0.01 |
| $F_{V}$ | 0.38 |  | 0.53 | 0.59 | 0.64 | 0.67 |
| $F_{A}$ | 0.48 |  | 0.54 | 0.59 | 0.63 | 0.66 |
| $f_{\pi}$ | 0.06 |  | 0.10 | 0.12 | 0.12 | 0.13 |



(c) $M_{V}$

(d) $M_{\sigma}$

## NJL Competition

$$
\mathcal{L}=\frac{g^{2}}{\Lambda_{U V}^{2}}\left(\bar{\Psi}_{L} U_{R} \bar{U}_{R} \Psi_{L}+\bar{\Psi}_{L} D_{R} \bar{D}_{R} \Psi_{L}\right)
$$

$$
L=\left(\begin{array}{cccc}
0 & L_{0} & 0 & -Q \\
-L_{0} & 0 & Q & 0 \\
0 & -Q & 0 & L_{0} \\
Q & 0 & -L_{0} & 0
\end{array}\right)
$$

$$
U^{T} L U=\frac{1}{2}\left(\begin{array}{cccc}
0 & -L_{0}(\rho)-Q(\rho) & 0 & 0 \\
L_{0}(\rho)+Q(\rho) & 0 & 0 & 0 \\
0 & 0 & 0 & -L_{0}(\rho)+Q(\rho) \\
0 & 0 & L_{0}(\rho)-Q(\rho) & 0
\end{array}\right)
$$

We now have to change the boundary conditions on the left right states...

$$
X_{f}=\left(\begin{array}{cccc}
0 & \sigma-Q_{5}+i S-i \pi_{5} & Q_{2}-\pi_{2}+i \pi_{1}-i Q_{1} & -Q_{4}+\pi_{4}+i Q_{3}-i \pi_{3} \\
-\sigma+Q_{5}+i \pi_{5}-i S & 0 & Q_{4}+\pi_{4}+i Q_{3}+i \pi_{3} & Q_{2}+\pi_{2}+i Q_{1}+i \pi_{1} \\
\pi_{2}-Q_{2}+i Q_{1}-i \pi_{1}-Q_{4}-\pi_{4}-i Q_{3}-i \pi_{3} & 0 & \sigma+Q_{5}+i S+i \pi_{5} \\
Q_{4}-\pi_{4}+i \pi_{3}-i Q_{3}-Q_{2}-\pi_{2}-i Q_{1}-i \pi_{1}-\sigma-Q_{5}-i S-i \pi_{5} & 0
\end{array}\right)
$$

The Q1-4 are now exact Goldstones for all Q vevs because SU(2)L is broken explicitly by the vev... we can rotate from composite higgs to technicolour as the NJL g rises.... Paper of spectrum soon! The LHC phenomenology....

(a) $M_{S_{+}}^{2}$

(d) $M_{V_{1+13,2+7,3+12}}$

## 

(g) $M_{A_{8,10,14,15}}$

(b) $M_{S_{-}}^{2}$

(e) $M_{V_{1-13,2-7,3-12}}$

(h) $M_{A_{11+16}}$

(c) $M_{\pi_{5}}$

(f) $M_{A_{(4,5,6,9)}}$

(i) $M_{A_{11-16}}$

## Multi-Representation Theories

We got interested in lattice studies, inspired by composite higgs model building, of theories with two different representation quark matter.

We run the holographic model with two scalars - one for the F condensate and one for the HD condensate.. We input perturbative runnings of $\gamma$ at two loop in each case to fix $\Delta m^{2} \ldots$


How you decouple the quarks is important and unknown - I'll concentrate on when they are removed below their IR mass scale. Quench = pure glue running.

The gap between F and HD grows the less you decouple the quarks - the slower the running the more conformal the theory is around the chiral symmetry breaking point - this will lead to a lighter scalar meson...

## SU(4) $3 \mathrm{~F} 3 \overline{\mathrm{~F}} 5 \mathrm{~A}_{2}$

G. Ferretti, "UV Completions of Partial Compositeness: The Case for a SU(4) Gauge Group," JHEP 06 (2014) 142, arXiv:1404.7137 [hep-ph].

In this model the A2 symmetry breaking generates the SM higgs and the Fs are to give $F A_{2} F$ top partners

> V. Ayyar, T. DeGrand, M. Golterman, D. C. Hackett, W. I. Jay, E. T. Neil, Y. Shamir, and B. Svetitsky, "Spectroscopy of SU(4) composite Higgs theory with two distinct fermion representations," Phys. Rev. D 97 no. $7,(2018) 074505$, arXiv:1710.00806 [hep-lat]. The lattice has simulated (unquenched) SU(4) $2 \mathrm{~F} 2 \mathrm{~F} 4 \mathrm{~A}_{2}$

|  | Lattice [80] $4 A_{2}, 2 F, 2 \bar{F}$ unquench | $\begin{aligned} & \text { AdS } / S U(4) \\ & 4 A_{2}, 2 F, 2 \bar{F} \\ & \text { no decouple } \end{aligned}$ | $\begin{gathered} \text { AdS } / S U(4) \\ 4 A_{2}, 2 F, 2 \bar{F} \\ \text { decouple } \end{gathered}$ | $\begin{gathered} \text { AdS } / S U(4) \\ 5 A_{2}, 3 F, 3 \bar{F} \\ \text { no decouple } \end{gathered}$ | $\begin{gathered} \mathrm{AdS} / S U(4) \\ 5 A_{2}, 3 F, 3 \bar{F} \\ \text { decouple } \end{gathered}$ | $\begin{gathered} \mathrm{AdS} / S U(4) \\ 5 A_{2}, 3 F, 3 \bar{F} \\ \text { quench } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f_{\pi A_{2}}$ | 0.15(4) | 0.0997 | 0.0997 | 0.111 | 0.111 | 0.102 |
| $f_{\pi F}$ | 0.11(2) | 0.0949 | 0.0953 | 0.0844 | 0.109 | 0.892 |
| $M_{V A_{2}}$ | 1.00(4) | 1* | 1* | 1* | 1* | 1* |
| $f_{V A_{2}}$ | 0.68(5) | 0.489 | 0.489 | 0.516 | 0.516 | 0.517 |
| $M_{V F}$ | 0.93(7) | 0.933 | 0.939 | 0.890 | 0.904 | 0.976 |
| $f_{V F}$ | 0.49(7) | 0.458 | 0.461 | 0.437 | 0.491 | 0.479 |
| $M_{A A_{2}}$ |  | 1.37 | 1.37 | 1.32 | 1.32 | 1.28 |
| $f_{A A_{2}}$ |  | 0.505 | 0.505 | 0.521 | 0.521 | 0.522 |
| $M_{A F}$ |  | 1.37 | 1.37 | 1.21 | 1.23 | 1.28 |
| $f_{A F}$ |  | 0.501 | 0.504 | 0.453 | 0.509 | 0.492 |
| $M_{S A_{2}}$ |  | 0.873 | 0.873 | 0.684 | 0.684 | 1.18 |
| $M_{S F}$ |  | 1.03 | 1.02 | 0.811 | 0.798 | 1.25 |
| $M_{J A_{2}}$ | 3.9(3) | 2.21 | 2.21 | 2.21 | 2.21 | 2.22 |
| $M_{J F}$ | 2.0(2) | 2.07 | 2.08 | 1.97 | 2.00 | 2.17 |
| $M_{B A_{2}}$ | 1.4(1) | 1.85 | 1.85 | 1.85 | 1.85 | 1.86 |
| $M_{B F}$ | 1.4(1) | 1.74 | 1.75 | 1.65 | 1.68 | 1.81 |

The pattern is right...

The A2-F gap is very well described...

Adding extra flavours is not a huge change...

Scalar masses get lighter as add extra flavours

## Confinement vs Chiral Symmetry breaking.

We've seen multi-rep theories with gaps between chiral symmetry breaking scales.... How big can they be?

This would be a measure of the gap to confinement also... ???

Our models are driven by the perturbative running of $\gamma$ passing through 1..

$$
\mathcal{Q}(R)=\frac{\Lambda_{\chi S B R}}{\Lambda_{\chi S B} F}
$$



$$
\mathcal{Q}(R)=\frac{\Lambda_{\chi S B R}}{\Lambda_{\chi S B F}}
$$

## CONCLUSIONS

We have holog inge theories provided you allow us to inpur the perturbative beta function running...

- We aim to eventually catalogue all AF theories...but are pulling out interesting cases
- Mass splittings and NJL interactions are computable...
- The $\operatorname{Sp}(2 \mathrm{Nc})$ theories show off all this technology and the spectra are phenomenologically interesting for LHC searches....
- Theories with multiple representations large scale separations are interesting
- DO ASK US FOR THE SPECTRA OF THEORIES YOU ARE INTERESTED IN!

