Charmless inclusive B decays

Gil Paz

Department of Physics and Astronomy,

Wayne State University,

Detroit, Michigan, USA

Outline

- Prehistory (< 2001)
- History ($2001 - 2011$)
- Recent Past ($2011 - 2019$)
- Future
- Pictures
- Conclusions

Prehistory (< 2001)

Neubert-Rosner bound

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- Neubert-Rosner bound:
  A bound on CKM angle γ using $B^\pm \to \pi^\pm K^0$ and $B^\pm \to \pi^0 K^\pm$ using $SU(3)$ flavor symmetry
  [Neubert, Rosner, PLB **441** 403 (1998), hep-ph/9808493]

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- M.Sc. project: Generalize Neubert-Rosner bound to $B \to VP$ decays
  Supervisor: Michael Gronau

Generalizing Neubert-Rosner bound

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- Wigner Eckart theorem: 10 reduced matrix elements
- "Graphical method": 12 amplitudes
  [GP, hep-ph/0206312]

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- Results summarized in [GP, hep-ph/0206312]
  Not published, but cited in the literature, most recently in 2021

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- The relation between graphical amplitudes and reduced matrix elements was recently corrected in
  [He, Wang, Chin. Phys. C **42** 103108 (2018), arXiv:1803.04227]

History (2001 – 2011)

Papers with Matthias

- Bosch, Lange, Neubert, GP
  Nucl. Phys. B **699**, 335 (2004) [hep-ph/0402094]
- Bosch, Lange, Neubert, GP
  Phys. Rev. Lett. **93**, 221801 (2004) [hep-ph/0403223]
- Bosch, Neubert, GP
  JHEP **0411**, 073 (2004) [hep-ph/0409115]
- Lange, Neubert, GP
  Phys. Rev. D **72**, 073006 (2005) [hep-ph/0504071]
- Lange, Neubert, GP
  JHEP **0510**, 084 (2005) [hep-ph/0508178]
- Lee, Neubert, GP
  Phys. Rev. D **75**, 114005 (2007) [hep-ph/0609224]
- Benzke, Lee, Neubert, GP
  JHEP **1008**, 099 (2010) [arXiv:1003.5012 (hep-ph)]
- Benzke, Lee, Neubert, GP
  PRL **106**, 141801 (2011) [arXiv:1012.3167 (hep-ph)]

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$M_X^2 \sim m_b^2$	local OPE	("OPE region")
$M_X^2 \sim m_b \Lambda_{\rm QCD}$	Non local OPE	("end point region")
$M_X^2 \sim \Lambda_{\rm QCD}^2$	No inclusive description	("resonance region")

Non local OPE

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- Γ_s^{ij}: [Lee, Neubert, GP '06], [Benzke, Lee, Neubert, GP Mar. '10], [Benzke, Lee, Neubert, GP Dec. '10]

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- J and partonic S calculated at $\mathcal{O}(\alpha_s^3)$ [Brüser, Liu, Stahlhofen '18,'19]

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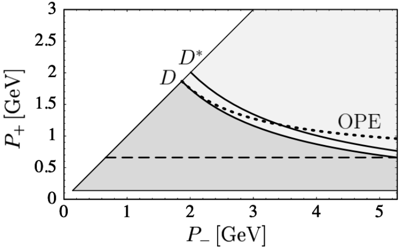

- P_+ spectrum used for $|V_{ub}|$ extraction
  [BaBar, Phys. Rev. D **86**, 032004 (2012) arXiv:1112.0702 (hep-ex)]

Bosch, Neubert, GP
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  [K. Lee, Stewart '04] [Beneke, Campanario, Mannel, Pecjak '04]

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  [K. Lee, Stewart '04] [Beneke, Campanario, Mannel, Pecjak '04]
- See also [Bauer, Luke, Mannel '01 & '02]

Lange, Neubert, GP
Phys. Rev. D **72**, 073006 (2005) [hep-ph/0504071]

- Putting it all together

$$d\Gamma \sim H \cdot J \otimes S + \frac{1}{m_b} \sum_i H \cdot J \otimes s_i + \ldots$$

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- Precision determination of $|V_{ub}|$ ("NLO")
  [Lange, Neubert, GP '05]
  Error on $|V_{ub}|$: **18%** (PDG 2004) $\Rightarrow$ **8%** (PDG 2006)

Jump to 2021: Inclusive $|V_{ub}|$ from Belle data

- Current extractions used
  - BLNP [Lange, Neubert, GP, PRD **72**, 073006, (2005)]
  - DGE [Andersen, Gardi, JHEP **01**, 097, (2006)]
  - GGOU [Gambino, Giordano, Ossola, Uraltsev, JHEP **10**, 058, (2007)]
  - ADFR [Aglietti, Di Lodovico, Ferrera, Ricciardi, EPJC **59**, 831, (2009)]

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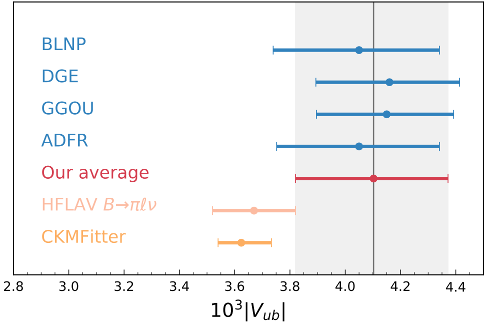
- Recent work: Inclusive $|V_{ub}|$ from Belle data
  [L. Cao *et al.* [Belle], PRD **104**, 012008 (2021)]



Lange, Neubert, GP
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- Later Björn generalized this to arbitrary spectra of $\bar{B} \to X_u \, \ell \, \bar{\nu}$
  B. O. Lange, JHEP **01**, 104 (2006) [hep-ph/0511098]

Lee, Neubert, GP
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- Surprising result: Unlike total rate $\Gamma(\bar{B} \to X_u \, l \, \bar{\nu})$
  Non perturbative effects in $\Gamma(\bar{B} \to X_s \, \gamma)$ arise at $\Lambda_{\rm QCD}/m_b$

Lee, Neubert, GP
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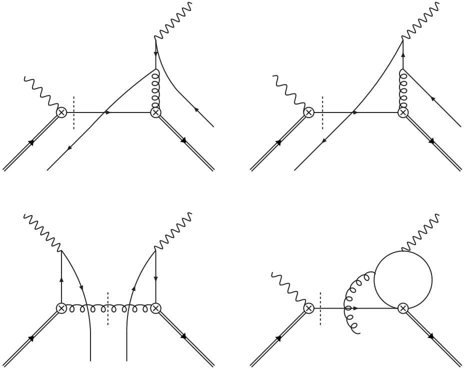

- "Resolved photon" contributions at $\Lambda_{\mathrm{QCD}}/m_b$
- Top line: $Q_{7\gamma} - Q_{8g}$
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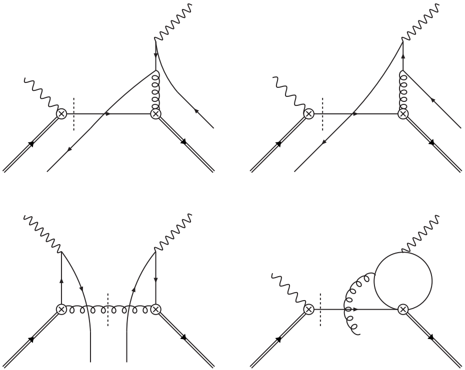

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- What do we find from a systematic analysis?

Benzke, Lee, Neubert, GP
JHEP **1008**, 099 (2010) [arXiv:1003.5012 (hep-ph)]

- Considering only $Q_{7\gamma} - Q_{7\gamma}$: factorization formula for $d\Gamma = H \cdot J \otimes S$
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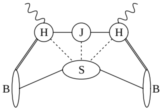

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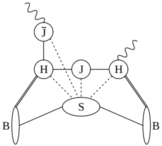

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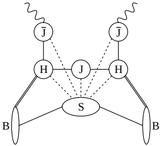

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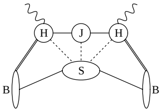

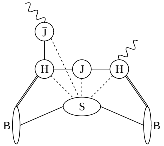

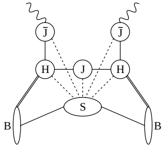

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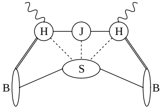

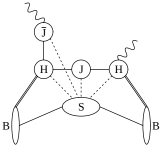

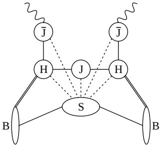

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$$h_{78}(\omega_1, \omega_2) \quad \text{F.T. of} \quad \langle \bar{B} | \bar{b}(0) \cdots b(0) \sum e_q \, \bar{q}(r\bar{n}) \cdots q(s\bar{n}) | \bar{B} \rangle$$

- These gave the largest uncertainty $\sim 5\%$ on $\Gamma(\bar{B} \to X_s \gamma)$

Benzke, Lee, Neubert, GP
PRL **106**, 141801 (2011) [arXiv:1012.3167 (hep-ph)]

- Resolved photon contributions are $\Lambda_{\rm QCD}/m_b$ effect

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- PDG 2022: $\mathcal{A}_{X_s\gamma} = 1.5\% \pm 1.1\%$
- **New** test of physics beyond the SM

$$\mathcal{A}_{X_s^-\gamma} - \mathcal{A}_{X_s^0\gamma} \approx 4\pi^2\alpha_s\,\frac{\tilde{\Lambda}_{78}}{m_b}\,\mathrm{Im}\,\frac{C_{8g}}{C_{7\gamma}} \approx 12\% \times \frac{\tilde{\Lambda}_{78}}{100\,\mathrm{MeV}}\,\mathrm{Im}\,\frac{C_{8g}}{C_{7\gamma}}$$

- BaBar $\Delta\mathcal{A}_{X_s\gamma} = (5.0 \pm 3.9 \pm 1.5)\%$ [BaBar '14]
- Belle $\Delta\mathcal{A}_{X_s\gamma} = (3.69 \pm 2.65 \pm 0.76)\%$ [Belle '18]
- PDG average $\Delta\mathcal{A}_{X_s\gamma} = (4.1 \pm 2.3)\%$ statistically limited

Recent Past (2011 – 2019)

Subleading jet functions (2009)

$$d\Gamma_u, d\Gamma_s^{77} \sim H \cdot J \otimes S + \frac{1}{m_b} \sum_i H \cdot J \otimes s_i + \frac{1}{m_b} \sum_i H \cdot j_i \otimes S + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{m_b^2}\right)$$

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- Subleading jet functions, j_i, calculated at $\mathcal{O}(\alpha_s)$ [GP '09]:
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- do *not* introduce new hadronic uncertainties

Subleading jet functions (2009)

$$d\Gamma_u, d\Gamma_s^{77} \sim H \cdot J \otimes S + \frac{1}{m_b} \sum_i H \cdot J \otimes s_i + \frac{1}{m_b} \sum_i H \cdot j_i \otimes S + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{m_b^2}\right)$$

- Subleading jet functions, j_i, calculated at $\mathcal{O}(\alpha_s)$ [GP '09]:
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by

$$W^{\rm SJF} = \frac{1}{m_b}\,\frac{C_F\alpha_s(\mu)}{4\pi}\int d\omega\;\theta(\omega+n\cdot p)\left[32\,\ln\frac{\mu^2}{m_b(\omega+n\cdot p)}-18\right]S(\omega)+\mathcal{O}(\alpha_s^2)$$

- *Factorization of the log suggested by Matthias*

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- Although α_s and $1/m_b$ suppressed, effect can be non-negligible e.g. constant change from $+30$ to -18

MITP Topical Workshops

- 2014

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Proton Radius Puzzle

Organizers: Carl Carlson (College of William and Mary), Richard Hill (Chicago Univ.), Savely Karshenboim (MPI for Quantum Optics Munich and Pulkovo Observatory St. Petersburg), Marc Vanderhaeghen (JGU Mainz)

June 2 - 6, 2014, Waldthausen Castle near Mainz

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  Challenges in Semileptonic B decays

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  April 20 - 24, 2015, JGU Campus Mainz

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April 9 - 13, 2018, JGU Campus Mainz

- July 2018

Precision Measurements and Fundamental Physics: The Proton Radius Puzzle and Beyond

Organizers: Richard Hill (University of Kentucky / Fermilab), Gil Paz (Wayne State University) and Randolf Pohl (JGU Mainz)

July 23 - 27, 2018, JGU Campus Mainz

Two topics coming together

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- Construct them by tensor decomposition of HQET matrix elements
- The same operators appear for NRQCD/NRQED

New Result: Dimension 9 HQET operators

- Using this method: SI Dimension 9 HQET operators

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$$
\begin{aligned}
&\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1}\, iD^{\mu_2}\, iD^{\mu_3}\, iD^{\mu_4}\, iD^{\mu_5}\, iD^{\mu_6}\, h|H\rangle = a^{(9)}_{12,34}\, \Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_4}\Pi^{\mu_5\mu_6} + \\
&+a^{(9)}_{12,35}\, \left(\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_5}\Pi^{\mu_4\mu_6} + \Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_4}\Pi^{\mu_5\mu_6}\right) + a^{(9)}_{12,36}\, \left(\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_6}\Pi^{\mu_4\mu_5} + \Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_3}\Pi^{\mu_5\mu_6}\right) + \\
&+a^{(9)}_{13,25}\, \Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_5}\Pi^{\mu_4\mu_6} + a^{(9)}_{13,26}\, \left(\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_6}\Pi^{\mu_4\mu_5} + \Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_3}\Pi^{\mu_4\mu_6}\right) + a^{(9)}_{14,25}\, \Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_5}\Pi^{\mu_3\mu_6} + \\
&+a^{(9)}_{14,26}\, \left(\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_6}\Pi^{\mu_3\mu_5} + \Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_4}\Pi^{\mu_3\mu_6}\right) + a^{(9)}_{15,26}\, \Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_6}\Pi^{\mu_3\mu_4} + a^{(9)}_{16,23}\, \Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_3}\Pi^{\mu_4\mu_5} + \\
&+a^{(9)}_{16,24}\, \Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_4}\Pi^{\mu_3\mu_5} + a^{(9)}_{16,25}\, \Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_5}\Pi^{\mu_3\mu_4} + b^{(9)}_{12,36}\, \left(\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_6}v^{\mu_4}v^{\mu_5} + \Pi^{\mu_1\mu_4}\Pi^{\mu_5\mu_6}v^{\mu_2}v^{\mu_3}\right) + \\
&+b^{(9)}_{12,46}\, \left(\Pi^{\mu_1\mu_2}\Pi^{\mu_4\mu_6}v^{\mu_3}v^{\mu_5} + \Pi^{\mu_1\mu_3}\Pi^{\mu_5\mu_6}v^{\mu_2}v^{\mu_4}\right) + b^{(9)}_{12,56}\, \Pi^{\mu_1\mu_2}\Pi^{\mu_5\mu_6}v^{\mu_3}v^{\mu_4} + \\
&+b^{(9)}_{13,26}\, \left(\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_6}v^{\mu_4}v^{\mu_5} + \Pi^{\mu_1\mu_5}\Pi^{\mu_4\mu_6}v^{\mu_2}v^{\mu_3}\right) + b^{(9)}_{13,46}\, \Pi^{\mu_1\mu_3}\Pi^{\mu_4\mu_6}v^{\mu_2}v^{\mu_5} + \\
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\end{aligned}
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New Result: Dimension 9 HQET operators

- Using this method: SI Dimension 9 HQET operators

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- There are also multiple color structures

  At the current level of precision only the ones above are needed

New Result: Moments of the leading power shape function

- Shape function moments related to HQET parameters
  The matrix elements decomposition makes their calculation easy

$$2M_B \int d\omega \, \omega^k \, S(\omega) = n_{\mu_1} ... n_{\mu_k} \langle \bar{B}(v) | \bar{h} \, iD^{\mu_1} ... iD^{\mu_k} \, h | \bar{B}(v) \rangle$$

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$$\int d\omega\, S(\omega) = 1, \qquad \int d\omega\, \omega\, S(\omega) = 0, \qquad \int d\omega\, \omega^2\, S(\omega) = -a^{(5)} = -\lambda_1/3,$$

$$\int d\omega\, \omega^3\, S(\omega) = -a^{(6)} = -\rho_1/3,$$

$$\int d\omega\, \omega^4\, S(\omega) = a^{(7)}_{12} + a^{(7)}_{13} + a^{(7)}_{14} - b^{(7)} = m_1/5 - m_2/3,$$

$$\int d\omega\, \omega^5\, S(\omega) = 2a^{(8)}_{12} + 2a^{(8)}_{13} + 2a^{(8)}_{15} + b^{(8)}_{12} + b^{(8)}_{14} + b^{(8)}_{15} - c^{(8)} =$$
$$= \left(-8r_1 + 2r_2 + 2r_3 + 2r_4 + r_5 + r_6 + r_7\right)/15,$$

$$\begin{aligned}\int d\omega\, \omega^6\, S(\omega) = &-a^{(9)}_{12,34} - 2a^{(9)}_{12,35} - 2a^{(9)}_{12,36} - a^{(9)}_{13,25} - 2a^{(9)}_{13,26} - a^{(9)}_{14,25} - 2a^{(9)}_{14,26} - a^{(9)}_{15,26} \\ &- a^{(9)}_{16,23} - a^{(9)}_{16,24} - a^{(9)}_{16,25} + 2b^{(9)}_{12,36} + 2b^{(9)}_{12,46} + b^{(9)}_{12,56} + 2b^{(9)}_{13,26} + b^{(9)}_{13,46} \\ &+ 2b^{(9)}_{14,26} + b^{(9)}_{14,36} + b^{(9)}_{15,26} + 2b^{(9)}_{16,23} + 2b^{(9)}_{16,24} + b^{(9)}_{16,25} + b^{(9)}_{16,34} - c^{(9)}\end{aligned}$$

- m_i and r_i notation from [Mannel, Turczyk, Uraltsev '10]
- $a^{(9)}, b^{(9)}, c^{(9)}$ notation from [Gunawardna, GP '17]

New Result: Dimension 8 NRQCD Lagrangian

- We can now list the dimension 8 NRQCD Lagrangian
  [Gunawardna, GP JHEP **1707** 137 (2017), Kobach, Pal PLB **772** 225 (2017)]

$$
\begin{aligned}
\mathcal{L}^{\text{dim}=8}_{\text{NRQCD}} = \psi^\dagger \Big\{ & \ldots c_{X1} g \frac{[\boldsymbol{D}^2, \{\boldsymbol{D}^i, \boldsymbol{E}^i\}]}{m_p^4} + c_{X2} g \frac{\{\boldsymbol{D}^2, [\boldsymbol{D}^i, \boldsymbol{E}^i]\}}{m_p^4} + c_{X3} g \frac{[\boldsymbol{D}^i, [\boldsymbol{D}^i, [\boldsymbol{D}^j, \boldsymbol{E}^j]]]}{m_p^4} \\
& + i c_{X4a}\, g^2 \frac{\{\boldsymbol{D}^i, \epsilon^{ijk} \boldsymbol{E}^j_a \boldsymbol{B}^k_b \{T^a, T^b\}\}}{2M^4} + i c_{X4b}\, g^2 \frac{\{\boldsymbol{D}^i, \epsilon^{ijk} \boldsymbol{E}^j_a \boldsymbol{B}^k_b \, \delta^{ab}\}}{m_p^4} + i c_{X5} g \frac{\boldsymbol{D}^i \boldsymbol{\sigma} \cdot (\boldsymbol{D} \times \boldsymbol{E} - \boldsymbol{E} \times \boldsymbol{D}) \boldsymbol{D}^i}{m_p^4} \\
& + i c_{X6} g \frac{\epsilon^{ijk} \sigma^i \boldsymbol{D}^j [\boldsymbol{D}^l, \boldsymbol{E}^l] \boldsymbol{D}^k}{m_p^4} + c_{X7a}\, g^2 \frac{\{\boldsymbol{\sigma} \cdot \boldsymbol{B}_a T^a, [\boldsymbol{D}^i, \boldsymbol{E}^i]_b T^b\}}{2M^4} + c_{X7b}\, g^2 \frac{\boldsymbol{\sigma} \cdot \boldsymbol{B}_a [\boldsymbol{D}^i, \boldsymbol{E}^i]_a}{m_p^4} \\
& + c_{X8a}\, g^2 \frac{\{\boldsymbol{E}^i_a T^a, [\boldsymbol{D}^i, \boldsymbol{\sigma} \cdot \boldsymbol{B}]_b T^b\}}{2M^4} + c_{X8b}\, g^2 \frac{\boldsymbol{E}^i_a [\boldsymbol{D}^i, \boldsymbol{\sigma} \cdot \boldsymbol{B}]_a}{m_p^4} + c_{X9a}\, g^2 \frac{\{\boldsymbol{B}^i_a T^a, [\boldsymbol{D}^i, \boldsymbol{\sigma} \cdot \boldsymbol{E}]_b T^b\}}{2M^4} \\
& + c_{X9b}\, g^2 \frac{\boldsymbol{B}^i_a [\boldsymbol{D}^i, \boldsymbol{\sigma} \cdot \boldsymbol{E}]_a}{m_p^4} + c_{X10a}\, g^2 \frac{\{\boldsymbol{E}^i_a T^a, [\boldsymbol{\sigma} \cdot \boldsymbol{D}, \boldsymbol{B}^i]_b T^b\}}{2M^4} + c_{X10b}\, g^2 \frac{\boldsymbol{E}^i_a [\boldsymbol{\sigma} \cdot \boldsymbol{D}, \boldsymbol{B}^i]_a}{m_p^4} \\
& + c_{X11a}\, g^2 \frac{\{\boldsymbol{B}^i_a T^a, [\boldsymbol{\sigma} \cdot \boldsymbol{D}, \boldsymbol{E}^i]_b T^b\}}{2M^4} + c_{X11b}\, g^2 \frac{\boldsymbol{B}^i_a [\boldsymbol{\sigma} \cdot \boldsymbol{D}, \boldsymbol{E}^i]_a}{m_p^4} + \tilde{c}_{X12a}\, g^2 \frac{\epsilon^{ijk} \boldsymbol{\sigma}^i \boldsymbol{E}^j_a [D_t, \boldsymbol{E}^k]_b \{T^a, T^b\}}{2M^4} \\
& + \tilde{c}_{X12b}\, g^2 \frac{\epsilon^{ijk} \boldsymbol{\sigma}^i \boldsymbol{E}^j_a [D_t, \boldsymbol{E}^k]_a}{m_p^4} + i c_{X13} g^2 \frac{[\boldsymbol{E}^i, [D_t, \boldsymbol{E}^i]]}{m_p^4} + i c_{X14} g^2 \frac{[\boldsymbol{B}^i, (\boldsymbol{D} \times \boldsymbol{E} + \boldsymbol{E} \times \boldsymbol{D})^i]}{m_p^4} \\
& + i c_{X15} g^2 \frac{[\boldsymbol{E}^i, (\boldsymbol{D} \times \boldsymbol{B} + \boldsymbol{B} \times \boldsymbol{D})^i]}{m_p^4} + c_{X16} g^2 \frac{[\boldsymbol{\sigma} \cdot \boldsymbol{B}, \{\boldsymbol{D}^i, \boldsymbol{E}^i\}]}{m_p^4} + c_{X17} g^2 \frac{[\boldsymbol{B}^i, \{\boldsymbol{D}^i, \boldsymbol{\sigma} \cdot \boldsymbol{E}\}]}{m_p^4} + c_{X18} g^2 \frac{[\boldsymbol{E}^i, \{\boldsymbol{\sigma} \cdot \boldsymbol{D}, \boldsymbol{B}^i\}]}{m_p^4} \Big\} \psi
\end{aligned}
$$

- 25 operators
- c_{Xib} start at $\mathcal{O}(\alpha_s)$

Improving the uncertainty of $\bar{B} \to X_s\gamma$

- Recall that resolved photon contributions give
- largest uncertainty on $\Gamma(\bar{B} \to X_s\gamma)$
- dominant effect on $\mathcal{A}^{\text{SM}}_{X_s\gamma}$

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- Use
- Better control of higher dim. operators [Gunawardna, GP '17]
- HQET parameters extraction from [Gambino, Healey, Turczyk '16]

- To improve the estimates of uncertainties

  [Ayesh Gunawardna, GP JHEP **11** 141 (2019)]

Results: Reducing non-perturbative uncertainties

- $\bar{B} \to X_s \gamma$ uncertainty depends on a soft function $g_{17}(\omega, \omega_1, \mu)$
- Moments in ω and ω_1 are related to HQET parameters

$$\langle \omega^l \, \omega_1^k \, g_{17} \rangle \equiv \int_{-\infty}^{\bar{\Lambda}} d\omega \, \omega^l \int_{-\infty}^{\infty} d\omega_1 \, \omega^k \, g_{17}(\omega, \omega_1, \mu) = \left(i v^\rho \epsilon_{\rho\mu\alpha_\perp\lambda} \bar{n}^\mu - g_{\alpha_\perp\lambda} \right) (-1)^k$$

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- New estimate of uncertainty: Total rate $\downarrow$ 50%, CP asymmetry $\uparrow$ 33%
- Using different models, *some* $\Lambda_{\text{QCD}}^2/m_b^2$ corrections, and larger m_c range, a smaller reduction was found in [Benzke, Hurth '20]

Future

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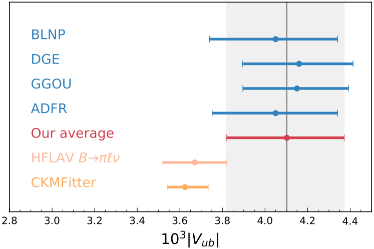
  [L. Cao *et al.* [Belle], PRD **104**, 012008 (2021)]



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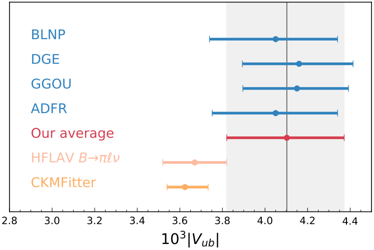

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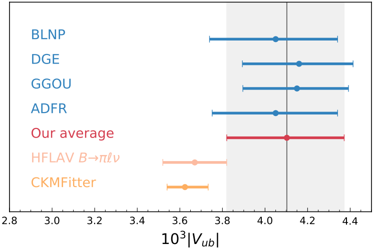

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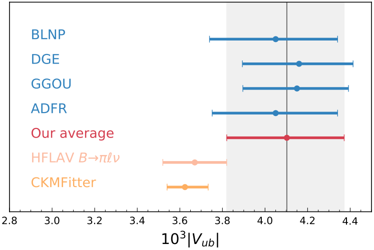

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  [Gunawardana, Lange, Mannel, Olschewsky, Vos, GP, *in progress*]

Pictures

2002

(Picture taken by Stefan Bosch)

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BLNP 2002

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Outside the Grotta Azzurra Capri 2022

(Picture taken by an unknown photographer)

Acknowledgements from my Ph.D Dissertation

ACKNOWLEDGEMENTS

I would like to thank my advisor Matthias Neubert for his help, encouragement and for teaching me to always try to improve the work that we do.

(hep-ph/0607217)

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Happy Birthday Matthias!