# EW corrections in SMEFT

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# SMEFT PERSPECTIVES (AT NLO)

$$\mathcal{L}^{\mathrm{SMEFT}} = \mathcal{L}^{\mathrm{SM}} + \sum_{i=1}^{59} C_i(\mu) Q_i(\mu) + \ldots$$

Two perspectives:

- bottom up (the present): NP if any $C_i \neq 0 \Rightarrow$ many efforts to see this in data through global fits
- top down (the future): if NP is known, SMEFT is tool for calculating RG-improved cross sections at scales $\mu_{\mathrm{EW}} \ll \Lambda_{\mathrm{NP}}$

NLO calculations:

- reduce dependence on renormalization scheme (for instance $\mu$-dependence in $\overline{\mathrm{MS}}$)
- more robust fits (bottom up), better agreement with data (top-down)

# The NLO SMEFT landscape

A rapidly expanding field:

- Current calculations done on case-by-case basis: [Giardino, Dawson, Maltoni, Zhang, Trott, Petriello, Duhr, Schulze, Passarino, Signer, Pruna, Shepherd, Hartmann, Baglio, Lewis, Zhang, Boughezal, Degrande, BP, Vryonidou, Mimasu, Deutschmann, Scott, Dedes, Suxho, Trifyllis, Gomez-Ambrosio, Durieux, Cullen, Gauld, Haisch, Zanderighi, . . . ]
- Future is NLO automation as in SM
  (already available for QCD corrections [Degrande et al. arXiv:2008.11743])

This talk: issues common to all NLO EW calculations in SMEFT

- general procedure with $h \to b\bar{b}$ as an example
  [Cullen, BP, Scott, Gauld: '15,'19]
- EW input schemes and universal corrections in SMEFT
  [Biekötter, BP, Scott, Smith arXiv:2305.03763]

# Motivation for $h \to b\bar{b}$ at NLO in SMEFT

1) **Phenomenology**:

- can measure *hbb* coupling at (sub)percent level at Higgs factory
- $\Rightarrow$ NLO SMEFT calculation sets long-term baseline for analysis in EFT

2) **SMEFT development**:

- reveals many non-trivial features of SMEFT at NLO in (relatively) simple setting
- analytic results useful for benchmarking automated codes for NLO SMEFT

3) **FUN WITH EFT**

- a main lesson from my PhD with Matthias: EFTs combine the conceptual and the calculational in a unique and fascinating way


DEUTSCHLAND

# Outline of an NLO calculation

Basic outline:

- specify input parameters and renormalization scheme
- write down LO and UV counterterm amplitudes
- calculate one-loop matrix elements and UV counterterms (2-point functions apart from operator mixing)
- calculate real emissions of photons, gluons, add together with UV-renormalized virtual corrections to get IR finite answer

In general, every piece of calculation gets dim-4 (SM) and dim-6 (SMEFT) contributions, dim-8 terms are dropped

Will illustrate the procedure with $h \to b\bar{b}$

# Input parameters

In the "$\alpha$ scheme", input parameters are:

$$\{M_W, M_Z, \alpha\}, m_f, m_H, V_{ij},\ C_i(\mu), \alpha_s(\mu)$$

- $C_i$ and $\alpha_s$ are renormalized in the $\overline{\text{MS}}$ scheme
- $M_W$, $M_Z$, $m_H$, $m_t$ renormalized in on-shell scheme
- renormalization scheme for $\alpha = e^2/(4\pi)$ and $m_b \neq 0$ kept flexible
- all other $m_f = 0$
- we approximate $V_{ij} = \delta_{ij}$

Will come back to other EW input schemes later:

- $\{M_W, M_Z, G_F\} =$ "$\alpha_\mu$ scheme"
- $\{\alpha, M_Z, G_F\} =$ "LEP scheme"

# The LO amplitude for $h \to b\bar{b}$

- LO decay amplitude

$$i\mathcal{M}^{(0)}(h \to b\bar{b}) = -i\bar{u}(p_b)\left(\mathcal{M}_L^{(0)} P_L + \mathcal{M}_L^{(0)*} P_R\right) v(p_{\bar{b}})$$

- split into dim-4 and dim-6 contributions

$$\mathcal{M}_L^{(0)} = \mathcal{M}_L^{(4,0)} + \mathcal{M}_L^{(6,0)}$$

- Explicit results ($v_\alpha \equiv 2M_w s_w/e, \quad c_w = M_W/M_Z = \sqrt{1-s_w^2}$)

$$\begin{aligned}
\mathcal{M}_L^{(4,0)} &= \frac{m_b}{v_\alpha}\,, \\
\mathcal{M}_L^{(6,0)} &= m_b v_\alpha \left[ C_{H\Box} - \frac{C_{HD}}{4}\left(1 - \frac{c_w^2}{s_w^2}\right) + \frac{c_w}{s_w} C_{HWB} - \frac{v_\alpha}{m_b}\frac{C_{bH}^*}{\sqrt{2}} \right]
\end{aligned}$$

- $Q_{bH} \sim (H^\dagger H)(\bar{b}_L b_R H)$+h.c. contributes to LO Feynman diagram
- other $Q_i$, where $Q_i \xrightarrow{\text{SSB}} \langle H^\dagger H \rangle Q_i^{(4)} = \frac{1}{2} v_T^2 Q_i^{(4)}$ appear from rotation to mass basis

$$Q_{H\Box} = (H^\dagger H)\Box(H^\dagger H), \quad Q_{HWB} = H^\dagger \sigma^I H W_{\mu\nu}^I B^{\mu\nu}, \quad Q_{HD} = \left(H^\dagger D_\mu H\right)^* \left(H^\dagger D_\mu H\right)$$

# The counterterm amplitude

dimension-4 counterterm is

$$\delta\mathcal{M}_L^{(4)} = \frac{m_b}{v_\alpha}\left(\frac{\delta m_b^{(4)}}{m_b} - \frac{\delta v_\alpha^{(4)}}{v_\alpha} + \frac{1}{2}\delta Z_h^{(4)} + \frac{1}{2}\delta Z_b^{(4),L} + \frac{1}{2}\delta Z_b^{(4),R*}\right)$$

dimension-6 counterterm is

$$\begin{aligned}\delta\mathcal{M}_L^{(6)} =& \frac{m_b}{v_\alpha}\left(\frac{\delta m_b^{(6)}}{m_b} - \frac{\delta v_\alpha^{(6)}}{v_\alpha} + \frac{1}{2}\delta Z_h^{(6)} + \frac{1}{2}\delta Z_b^{(6),L} + \frac{1}{2}\delta Z_b^{(6),R*}\right)\\ &+ \mathcal{M}_L^{(6,0)}\left(\frac{\delta m_b^{(4)}}{m_b} + \frac{\delta v_\alpha^{(4)}}{v_\alpha} + \frac{1}{2}\delta Z_h^{(4)} + \frac{1}{2}\delta Z_b^{(4),L} + \frac{1}{2}\delta Z_b^{(4),R*}\right)\\ &- \frac{v_\alpha^2}{\sqrt{2}}C_{bH}^*\left(\frac{\delta v_\alpha^{(4)}}{v_\alpha} - \frac{\delta m_b^{(4)}}{m_b}\right) + m_b v_\alpha\left[C_{HWB} + \frac{c_w}{2s_w}C_{HD}\right]\delta\left(\frac{c_w}{s_w}\right)^{(4)}\\ &+ m_b v_\alpha\left(\delta C_{H\Box} - \frac{\delta C_{HD}}{4}\left(1 - \frac{c_w^2}{s_w^2}\right) + \frac{c_w}{s_w}\delta C_{HWB} - \frac{v_\alpha}{m_b}\frac{\delta C_{bH}^*}{\sqrt{2}}\right)\end{aligned}$$

where

$$\frac{\delta v_\alpha}{v_\alpha} \equiv \frac{\delta M_W}{M_W} + \frac{\delta s_w}{s_w} - \frac{\delta e}{e}$$

and

$$\frac{\delta s_w}{s_w} = -\frac{c_w^2}{s_w^2}\left(\frac{\delta M_W}{M_W} - \frac{\delta M_Z}{M_Z}\right), \quad \delta\left(\frac{c_w}{s_w}\right)^{(4)} = -\frac{1}{c_w s_w}\left(\frac{\delta s_w^{(4)}}{s_w}\right)$$

# Calculational procedure

One-loop $h \to bb$ matrix elements and two-point functions for counterterms involve many Feynman diagrams and dim-6 operators

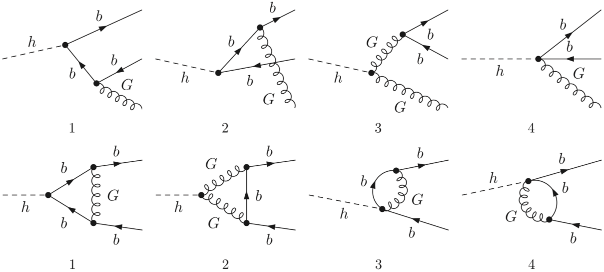
- **automation**: Feynrules (in-house model file, including gauge fixing and ghosts), Feynarts, FormCalc, Package X
- loop integrals obtained analytically in terms of Passarino-Veltmann integrals and standard functions

Decay rate also requires real emission corrections $h \to bb(g, \gamma)$

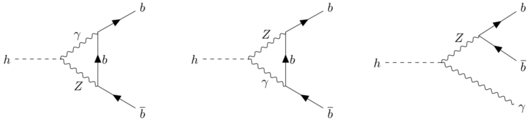
- squared matrix elements generated with automated tools
- 3-body phase space integrals done by hand

# Example: QCD-QED corrections

- QCD corrections by far simplest to calculate [Gauld, B.P., Scott '16]
- UV-renormalized one-loop amplitudes have IR divergences canceled by real emissions




- most corrections involving photons can obtained analogously, exception is graphs involving $h\gamma Z$ vertex




(a) (b) (c)

# Analytic structure of $h\gamma Z$ corrections

$$\Gamma_{h\gamma Z} \propto v_b \left[2(C_{HB} - C_{HW})c_w s_w + C_{HWB}(c_w^2 - s_w^2)\right] F_{h\gamma Z}\left(\frac{M_Z^2}{m_H^2}, \frac{\mu^2}{m_H^2}, \frac{m_b^2}{m_H^2}\right)$$

$$\begin{aligned}
F_{h\gamma Z}\left(z, \hat{\mu}^2, b\right) = {} & \frac{3}{4}\beta(8z-5) - \beta^3\left(\frac{39}{4} + \frac{z}{b}\right) - \frac{4}{3}\beta^2\pi^2\bar{z} + \frac{4}{3}\pi^2 z\bar{z} + 6\beta\left(\beta^2 - \frac{2}{3}z\right. \\
& \left. + \frac{(2b-\beta^2)z^2}{12b^2}\right)\ln(b) + 2(\beta^2 - z)\bar{z}\ln(x_z)^2 - 4\beta_z z\bar{z}\ln(x_{\beta z}) \\
& + \ln(x)\left(-\frac{1}{8}\left(15 + 7\beta^4 + 8z(4z-7) + \beta^2(2+8z)\right) + 2(z-\beta^2)\bar{z}\ln(x_z)\right. \\
& \left. + 4(\beta^2 - z)\bar{z}\ln(1 - xx_z) + 2(\beta^2 - z)\bar{z}\ln(x_{\beta z})\right) \\
& + \ln(x_z)\left(\frac{\beta\beta_z z\left(\beta^2(2b+z) - 2bz\right)}{2b^2} + 2(z-\beta^2)\bar{z}\ln(x_{\beta z})\right) \\
& + 4\beta z\bar{z}\ln(\bar{z}) + \frac{\beta^3(\beta^2+2b)z^2\ln(z)}{2b^2} - 6\beta^3\ln\left(\hat{\mu}^2\right) \\
& + 4(\beta^2 - z)\bar{z}\left(\mathrm{Li}_2\left(\frac{x}{x_z}\right) + \mathrm{Li}_2\left(xx_z\right)\right)
\end{aligned}$$

where

$$\beta = \sqrt{1-4b}, \quad \beta_z = \sqrt{1 - \frac{4b}{z}}, \quad x = \frac{1-\beta}{1+\beta}, \quad x_z = \frac{1-\beta_z}{1+\beta_z}, \quad x_{\beta z} = \frac{\beta - \beta_z}{\beta + \beta_z}, \quad \bar{z} = 1 - z$$

# Cross-checks and features

Results involve 45 different Wilson coefficients (generally complex). Cross-checks:

- all UV and IR poles cancel (and $\mu$-dependence consistent with RG eqns)
- SM results reproduced from dim. 4 terms
- all results calculated in unitary and Feynman gauge with full agreement

Interesting features:

- structure of wave-function renormalization of $b$-quark field
- Higgs-$Z$ mixing
- EW Ward identities and electric charge renormalization
- structure of tadpole contributions
- decoupling relations and hybrid renormalization schemes (mix of $\overline{\text{MS}}$ and on-shell)

# Enhanced NLO corrections I: QCD

- QCD/QED corrections generate $\ln m_b/m_H$ terms when $\mu = m_H$:

$$\frac{\Gamma^{(1)}_{g,\gamma}}{\Gamma^{(4,0)}} \approx \ln^2\left(\frac{m_b^2}{m_H^2}\right)\frac{v_\alpha^2}{\pi}\left(C_F\alpha_s c_{hgg} + Q_b^2\alpha c_{h\gamma\gamma}\right)$$
$$+ \; c_{m_b}\ln\left(\frac{m_b^2}{m_H^2}\right)\frac{3}{2}\left(\frac{C_F\alpha_s + Q_b^2\alpha}{\pi}\right)\left[1 + 2v_\alpha^2\left(C_{H\Box} - \frac{C_{HD}}{4}\left(1 - \frac{c_w^2}{s_w^2}\right)\right.\right.$$
$$\left.\left. + \frac{c_w}{s_w}C_{HWB} - \frac{v_\alpha}{m_b}\frac{C_{bH}}{2\sqrt{2}}\right)\right]$$

- double logs of IR origin are largest NLO correction
- $c_{m_b} = 1$ in on-shell scheme, $c_{m_b} = 0$ in $\overline{\text{MS}}$ scheme for $m_b$

$\Rightarrow$ **QCD/QED prefers $\overline{\text{MS}}$ scheme for** $m_b$ (running mass resums single UV logs)

# Enhanced NLO corrections II: tadpoles

- QCD and QED corrections prefer $\overline{\text{MS}}$ scheme for $m_b$ and $e$
- however, in $\overline{\text{MS}}$ scheme tadpoles give enhanced corrections $\propto m_t^4$
  Example: SM in $m_t \to \infty$ limit

$$\overline{\text{MS}}\text{ scheme:} \qquad \frac{\overline{\Gamma}_t^{(4,1)}}{\Gamma^{(4,0)}} \approx -\frac{N_c}{2\pi^2}\frac{m_t^4}{v_\alpha^2 m_H^2} \approx -15\%$$

$$\text{on-shell scheme:} \qquad \frac{[\Gamma_t]^{\mathrm{O.S.}(4,1)}}{\Gamma^{(4,0)}} = \frac{m_t^2}{16\pi^2 v_\alpha^2}\left(-6 + N_c\frac{7-10c_w^2}{3s_w^2}\right) \approx -3\%$$

- similar behaviour in SMEFT contributions
- $\Rightarrow$ **EW corrections better behaved in on-shell scheme to eliminate tadpoles**

Combining EW and QCD corrections is a non-trivial problem. Would like to calculate QCD in $\overline{\text{MS}}$ scheme, but EW in on-shell scheme...

# Decoupling relations I

- decoupling relations connect $\overline{\mathrm{MS}}$ parameters in SM, with those in low-energy theory where top and heavy bosons integrated out:

$$\overline{m}_b(\mu) = \zeta_b(\mu, m_t, m_H, M_W, M_Z)\overline{m}_b^{(\ell)}(\mu)$$

- $\zeta_b$ calculated by relating on-shell with $\overline{\mathrm{MS}}$ mass in SM and low-energy theory:

$$m_b = z_b^{-1}(\mu, m_b, m_t, m_H, M_W, M_Z)\overline{m}_b(\mu) = \left[z_b^{(\ell)}(\mu, m_b)\right]^{-1}\overline{m}_b^{(\ell)}(\mu)$$

$$\Rightarrow \zeta_b(\mu, m_t, m_H, M_W, M_Z) = \left.\frac{z_b(\mu, m_b, m_t, m_H, M_W, M_Z)}{z_b^{(\ell)}(\mu, m_b)}\right|_{m_b \to 0}$$

- works analogously for electric charge. The connection between low energy parameters and experiment are:

from $B$-physics: $\quad \overline{m}_b^{(\ell)}(\overline{m}_b^{(\ell)}) \approx 4.2\ \mathrm{GeV}$

from LEP: $\quad \overline{\alpha}^{(\ell)}(M_Z) = \alpha(M_Z)\left(1 + \frac{100\alpha}{27\pi}\right), \quad \alpha(M_Z) \approx 1/129,\ \alpha \approx 1/137$

# Decoupling relations II

- dim.4 contributions to $\zeta_i$ well known, we calculated dim.6 corrs. Example:

$$\zeta_e^{(4,1)} = \frac{\alpha}{\pi}\left[-\frac{1}{12} - \frac{7}{8}\ln\left(\frac{\mu^2}{M_W^2}\right) + \frac{N_c}{6}Q_t^2\ln\left(\frac{\mu^2}{m_t^2}\right)\right]$$

$$\zeta_e^{(6,1)} = \frac{\alpha}{\pi}\left[\sqrt{2}v_\alpha m_t N_c Q_t\left(c_w\frac{\mathrm{Re}(C_{tB})}{e} + s_w\frac{\mathrm{Re}(C_{tW})}{e}\right)\ln\left(\frac{\mu^2}{m_t^2}\right) + 9\frac{C_W}{e}s_w M_W^2\ln\left(\frac{\mu^2}{M_W^2}\right)\right] + \left.\frac{\delta e^{\mathrm{cl.}4(6)}}{e}\right|_{\mathrm{fin.},\, m_b\to 0}$$

- relation between NLO decay rate using low-energy parameters vs. SM params:

$$\overline{\Gamma}_\ell^{(4,1)} = \overline{\Gamma}^{(4,1)} + 2\overline{\Gamma}^{(4,0)}\left(\zeta_b^{(4,1)} + \zeta_e^{(4,1)}\right),$$

$$\overline{\Gamma}_\ell^{(6,1)} = \overline{\Gamma}^{(6,1)} + 2\overline{\Gamma}^{(4,0)}\left(\zeta_b^{(6,1)} + \zeta_e^{(6,1)}\right) + 2\overline{\Gamma}^{(6,0)}\zeta_b^{(4,1)} + \sqrt{2}C_{bH}\frac{(\overline{v}^{(\ell)})^3}{\overline{m}_b^{(\ell)}}\overline{\Gamma}^{(4,0)}\left(\zeta_b^{(4,1)} + \zeta_e^{(4,1)}\right)$$

- illustrative results: QCD-QED corrections and EW corrections in $m_t \to \infty$ limit:

$$\overline{\Gamma}_{\ell,g,\gamma} = \overline{\Gamma}_{g,\gamma}\,, \qquad \overline{\Gamma}_{\ell,t} = [\Gamma_t]^{\mathrm{O.S.}}$$

  - interpretation: QCD-QED corrections in $\overline{\mathrm{MS}}$ scheme (UV logs resummed), heavy-particle EW corrections in on-shell (tadpoles cancel)

# Corrections to LO results

| | SM | $C_{HWB}$ | $C_{H\Box}$ | $C_{bH}$ | $C_{HD}$ |
|---|---|---|---|---|---|
| NLO QCD-QED | 18.2% | 17.9% | 18.2% | 18.2% | 18.2% |
| NLO large-$m_t$ | -3.1% | -4.6% | 3.2% | 3.5% | -9.0% |
| NLO remainder | -2.2% | -1.9% | -1.2 % | 0.6% | -2.0% |
| NLO correction | 12.9% | 11.3% | 20.2% | 22.3% | 7.1% |

Table: Size of NLO corrections to different terms in LO decay rate, split into QCD-QED, large $m_t$, and remaining components.

- applying SM $K$-factor to dim.6 coefficients bad approximation for EW corrections
- this is generally the case, also for other decays such as $W \to \ell\nu$ and $Z \to \ell^+\ell^-$
- nonetheless, possible to decipher patterns across the $C_i$, input schemes, and decays [Biekötter, BP, Scott, Smith arXiv:2305.03763]

# 3 decays in 3 schemes

EW corrections ($\alpha_s = 0$ in $h \to bb$)

| $h \to b\bar{b}$ | SM | $C_{H\Box}$ | $C_{HD}$ | $C_{\substack{dH\\33}}$ | $C_{HWB}$ | $C^{(3)}_{\substack{Hl\\jj}}$ | $C_{\substack{ll\\1221}}$ |
|---|---|---|---|---|---|---|---|
| $\alpha$-scheme: $\{M_W, M_Z, \alpha\}$ | -5.2 % | 2.1% | -11.0% | 4.2% | -6.7% | - | - |
| $\alpha_\mu$-scheme: $\{M_W, M_Z, G_F\}$ | -0.8 % | 2.1% | 2.0% | 1.9% | - | 0.9% | -0.8% |
| LEP scheme: $\{\alpha, M_Z, G_F\}$ | -0.7 % | 2.1% | 1.6% | 1.9% | - | 0.7% | -0.9% |

| $W \to \tau\nu$ | SM | $C_{HD}$ | $C_{HWB}$ | $C^{(3)}_{\substack{Hl\\jj}}$ | $C_{\substack{ll\\1221}}$ | $C^{(3)}_{\substack{Hl\\33}}$ |
|---|---|---|---|---|---|---|
| $\alpha$ | $-4.2\%$ | $-1.7\%$ | $-3.0\%$ | — | — | 2.2% |
| $\alpha_\mu$ | $-0.3\%$ | — | — | 2.5% | $-0.2\%$ | 2.2% |
| LEP | 2.0% | 8.1% | 3.2% | 5.1% | 2.5% | 4.6% |

| $Z \to \tau\tau$ | SM | $C_{HD}$ | $C_{HWB}$ | $C_{\substack{He\\33}}$ | $C^{(1)}_{\substack{Hl\\33}}$ | $C^{(3)}_{\substack{Hl\\33}}$ | $C^{(3)}_{\substack{Hl\\jj}}$ | $C_{\substack{ll\\1221}}$ |
|---|---|---|---|---|---|---|---|---|
| $\alpha$ | $-4.0\%$ | $-10.6\%$ | $-5.4\%$ | 7.7% | 0.3% | $-0.5\%$ | — | — |
| $\alpha_\mu$ | $< 0.1\%$ | 71.1% | $-27.2\%$ | 7.6% | 0.1% | $-0.4\%$ | 2.9% | 0.6% |
| LEP | 1.0% | 7.8% | 17.4% | 2.0% | 4.7% | 4.2% | 6.9% | 4.5% |

Is there any rhyme or reason to the pattern across $C_i$?

# Connecting schemes

Start with $\mathcal{L}_{\text{bare}}(M_W, M_Z, v_T, \dots)$, and renormalise $v_T$ as

$$\frac{1}{v_{T,0}^2} = \frac{1}{v_\sigma^2}\left[1 - v_\sigma^2 \Delta v_\sigma^{(6,0,\sigma)} - \frac{1}{v_\sigma^2}\Delta v_\sigma^{(4,1,\sigma)} - \Delta v_\sigma^{(6,1,\sigma)}\right]; \quad \sigma \in \{\alpha, \mu\}$$

$$v_\alpha \equiv \frac{2M_W s_w}{\sqrt{4\pi\alpha}}, \qquad v_\mu \equiv \left(\sqrt{2}G_F\right)^{-\frac{1}{2}} \equiv \frac{2M_W s_w}{\sqrt{4\pi\alpha_\mu}}$$

- for $\alpha$ scheme $\{M_W, M_Z, \alpha\}$: use $\sigma = \alpha$ and determine $\Delta v_\alpha$ from charge ren.
- for $\alpha_\mu$ scheme $\{M_W, M_Z, G_F\}$: use $\sigma = \mu$ and determine $\Delta v_\mu$ from muon decay
- for LEP scheme $\{\alpha, M_Z, G_F\}$: start with $\alpha_\mu$ scheme, and then eliminate $M_W$ using

$$\frac{v_\alpha^2}{v_\mu^2} - 1 \equiv \Delta r = v_\mu^2 \Delta r^{(6,0)} + \frac{1}{v_\mu^2}\Delta r^{(4,1)} + \Delta r^{(6,1)}$$

where $\boldsymbol{\Delta r^{(i,j)}}$ **are finite** and related to $\Delta v_{\mu\alpha} = \Delta v_\mu - \Delta v_\alpha$

$$\Delta r^{(6,0)} = \Delta v_{\mu\alpha}^{(6,0)}, \quad \Delta r^{(4,1)} = \Delta v_{\mu\alpha}^{(4,1)}, \quad \Delta r^{(6,1)} = \Delta v_{\mu\alpha}^{(6,1)} + 2\Delta v_\mu^{(4,1,\mu)}\Delta v_{\mu\alpha}^{(6,0)}$$

# Top loops and universal corrections

- $\Delta r$ is physical, $\Delta v_\sigma$ is not. However, in large-$m_t$ limit in SM:

$$\frac{1}{v_{T,0}^2}\bigg|_{m_t\to\infty} = \frac{1}{v_\sigma^2}\left[1+\frac{1}{v_\sigma^2}\left(\Delta r_t^{(4,1)}\delta_{\alpha\sigma} - 2\Delta M_{W,t}^{(4,1)}\right)\right]\,; \quad \sigma\in\{\alpha,\mu\}$$

$$\frac{\Delta r_t^{(4,1)}}{v_\alpha^2} = -\frac{c_w^2}{s_w^2}\frac{\Delta\rho_t^{(4,1)}}{v_\alpha^2} \approx -3.5\%\,, \qquad \frac{\Delta\rho_t^{(4,1)}}{v_\alpha^2} \equiv \frac{3}{16\pi^2}\frac{m_t^2}{v_\alpha^2} \approx 1\%$$

- universal correction $\Delta r_t^{(4,1)}$ in $\alpha$-scheme comes along with LO (can resum!)
- we generalised this to include universal scheme-dependent corrections in SMEFT through a substitution procedure on LO [arXiv:2305.03763], for example

$$\frac{1}{v_T^2} \to \frac{1}{v_\sigma^2}\left[\underbrace{1+v_\sigma^2 K_t^{(6,0,\sigma)} + \frac{K_t^{(4,1,\sigma)}}{v_\sigma^2} + K_t^{(6,1,\sigma)}}_{\mathrm{LO}_K} + (\text{divergent and unphysical stuff})\right]$$

- the $K_t$ are physical top-loop corrections that always come along with LO
- $\Rightarrow$ re-organise pert. theory. to include them already in "$\mathrm{LO}_K$" approximation

NLO corrections to $\mathrm{LO}_K$ results

| $W \to \tau\nu$ | SM | $C_{HD}$ | $C_{HWB}$ | $C^{(3)}_{\underset{jj}{Hl}}$ | $C_{\underset{1221}{ll}}$ | $C^{(3)}_{\underset{33}{Hl}}$ |
|---|---|---|---|---|---|---|
| $\alpha$ | −0.9% | 1.1% | 0.6% | — | — | 2.2% |
| $\alpha_\mu$ | −0.3% | — | — | 0.6% | −0.2% | 2.2% |
| LEP | 0.0 % | 1.9% | 0.9 % | 0.1% | 0.2% | 2.5% |

| $Z \to \tau\tau$ | SM | $C_{HD}$ | $C_{HWB}$ | $C_{\underset{33}{He}}$ | $C^{(1)}_{\underset{33}{Hl}}$ | $C^{(3)}_{\underset{33}{Hl}}$ | $C^{(3)}_{\underset{jj}{Hl}}$ | $C_{\underset{1221}{ll}}$ |
|---|---|---|---|---|---|---|---|---|
| $\alpha$ | −0.9% | −1.4% | −0.1% | 3.3% | 2.0% | 1.3% | — | — |
| $\alpha_\mu$ | 0.0% | 11.2% | −3.4% | 3.2% | 1.8% | 1.3% | 0.8% | 0.0% |
| LEP | 0.0% | 2.3% | −3.0% | 2.5% | 2.5% | 2.0% | 0.8% | 0.0% |

| $h \to b\bar{b}$ | SM | $C_{H\Box}$ | $C_{HD}$ | $C_{\underset{33}{dH}}$ | $C_{HWB}$ | $C^{(3)}_{\underset{jj}{Hl}}$ | $C_{\underset{1221}{ll}}$ |
|---|---|---|---|---|---|---|---|
| $\alpha$ | -1.9 % | 2.1% | 2.5% | 2.5% | -1.5% | - | - |
| $\alpha_\mu$ | -0.8 % | 2.1% | 2.0% | 1.9% | - | 0.9% | -0.8% |
| LEP | -0.8 % | 2.1% | 1.6% | 1.9% | - | 0.7% | -0.9% |

Corrections smaller and less scheme dependent compared to pure fixed order

# Summary

- NLO in SMEFT is an active field whose future is automation
- My interest has been in sorting out general EFT aspects of these calculations before they transition to button pushing
- My thanks to Matthias for helping me develop the skillset for appreciating and doing this and many other EFT-based projects!