

10 years

Pushing the Limits of Theoretical Physics

08 – 12 May 2023

celebrating the 10th
anniversary of the

mtp
Mainz Institute for
Theoretical Physics

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How to discover axionlike particles



Martin Bauer

Mainz, May 12, 2023

Happy Birthday Matthias

3. Spezialvorlesungen

Grundlegende Experimente aus der Neutronenphysik

Vorlesung; 2 SWS; Mi, 8:30 - 10:00, Newton-Raum 01-122

Hydrodynamik und Elastizitätstheorie

Vorlesung; 2 SWS; Zeit und Raum n.V.

530

Statistische Physik kolloidaler Systeme (F)

Vorlesung; 2 SWS; Zeit und Raum n.V.

Theorie der Quantenflüssigkeiten (F)

Vorlesung; 2 SWS; Zeit und Raum n.V.

Übungen zur Theorie der Quantenflüssigkeiten (F)

Übung; 2 SWS; Zeit und Raum n.V.

Einführung in die Computersimulation (M,F)

Vorlesung; 2 SWS; Zeit und Raum n.V.

Physik auf dem Computer (M)

Vorlesung; 2 SWS; vorbereitende Veranstaltung für das Wahlpflichtfach

Computerphysik; Mo, 13:00 - 15:00, Lorentz-Raum 05-127

Übungen zur Physik auf dem Computer (M)

Übung; 3 SWS; Zeit n.V., CIP-Raum 05-422

Fraktionale Infinitesimalrechnung (M,F)

Vorlesung; 2 SWS; Zeit und Raum n.V.

Nanooptik (M,F)

Vorlesung; 2 SWS; Do, 15:00 - 17:00, Galilei-Raum 01-128

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Master Lectures about Theoretical Particle Physics

The basic lecture in the Master Program (Theoretical Physics 6a) is offered every term.

	Winter term	Summer term
Theoretical Physics 6a: Relativistic Quantum Field Theory (QFT) (4 hours lectures + 2 hours exercises per week)	x	x

All additional lectures can be chosen as Topical or as Advanced Lecture in the Master Program.

	Winter term	Summer term
Elementary Particle Physics (QFT II) (3+1)	x	x
Model and Electroweak Physics (QFT III) (3+1)	x	
String Theory (3+1)	x	
Classical General Relativity (3+1)		x

In addition there is at least one lecture in each term about Modern Methods in Particle Physics (3+1) typically from following list of topics:

- Amplitudes and Precision Physics at the LHC
- Effective Field Theories and Flavour Physics
- Introduction to String Theory
- Theoretical Astroparticle Physics
- Functional Methods and Exact Renormalization Group

There are also regular offerings in special topics in particle physics (2V). These include the following topics:

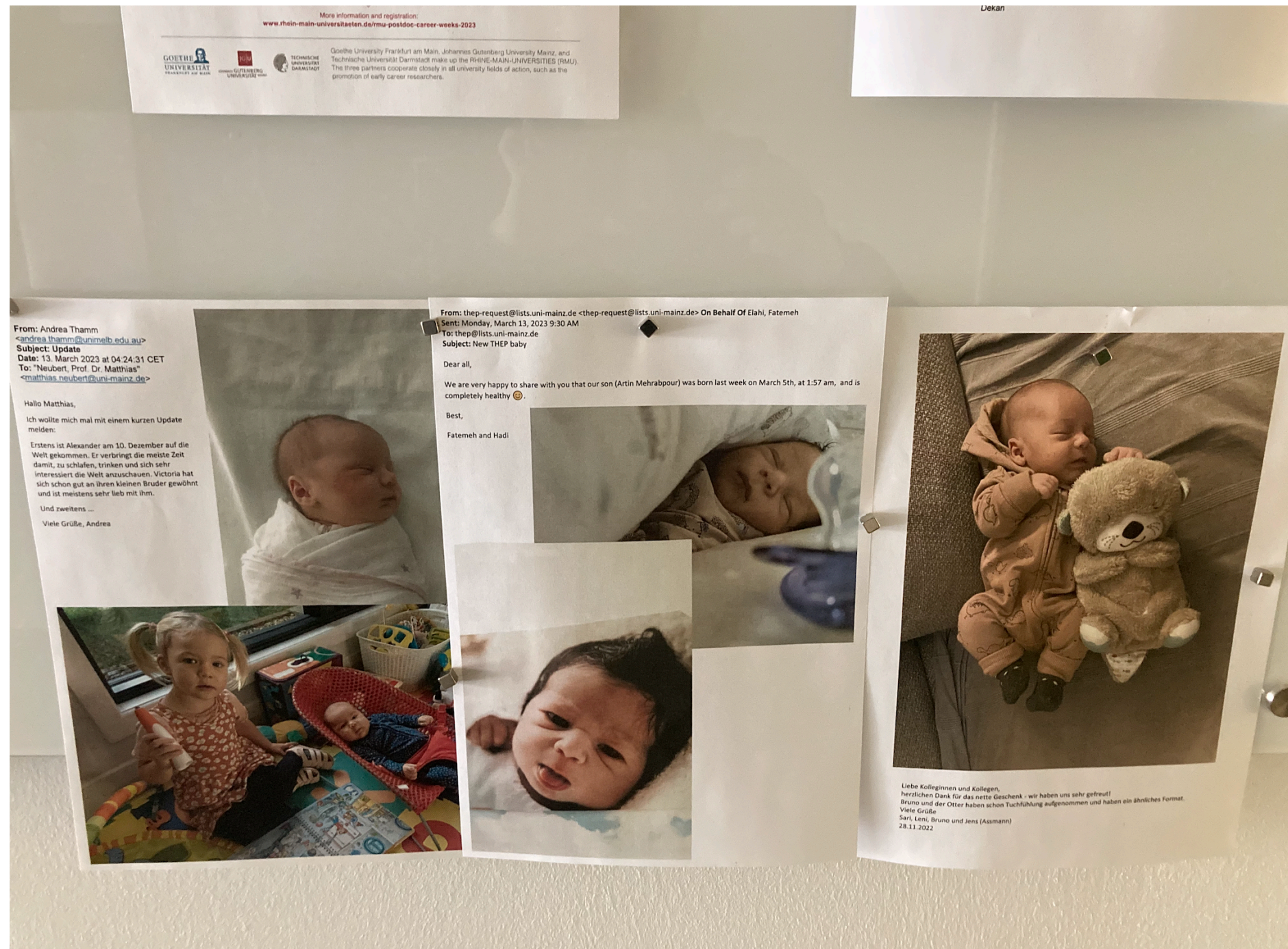
- Finite Temperature QFT and Phase Transitions
- Chirality and Gauge Theories
- Supersymmetry

20 years later

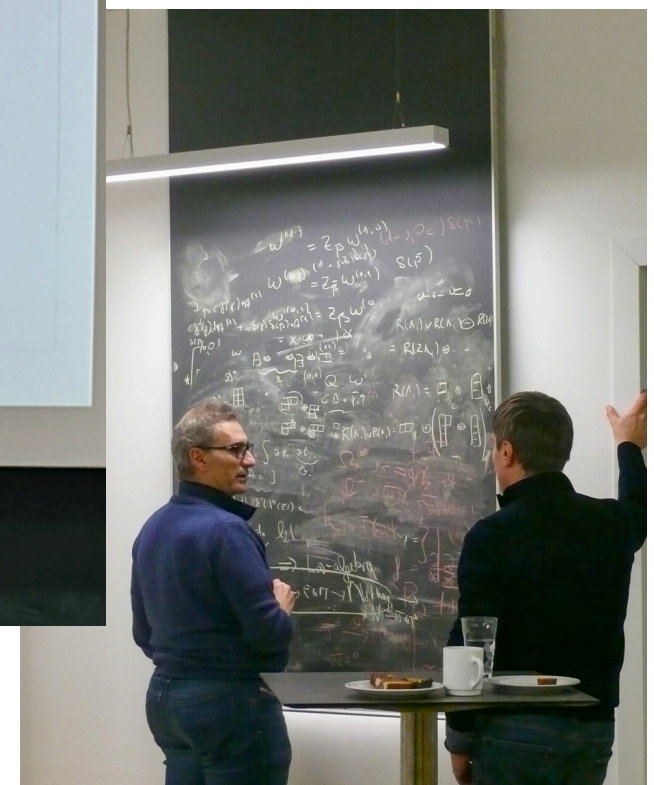
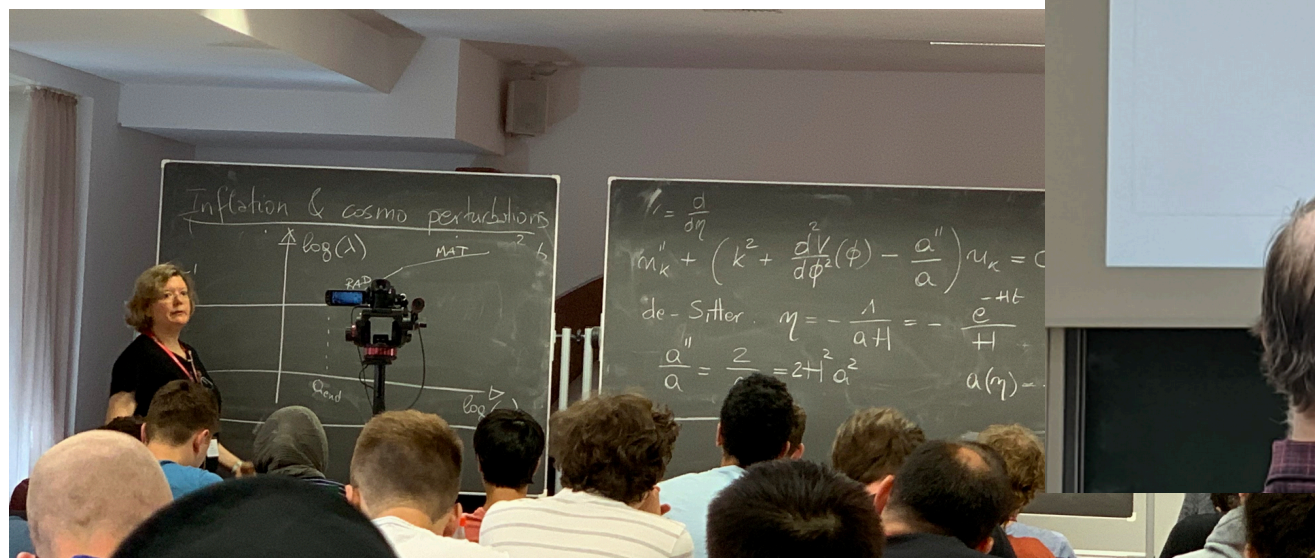
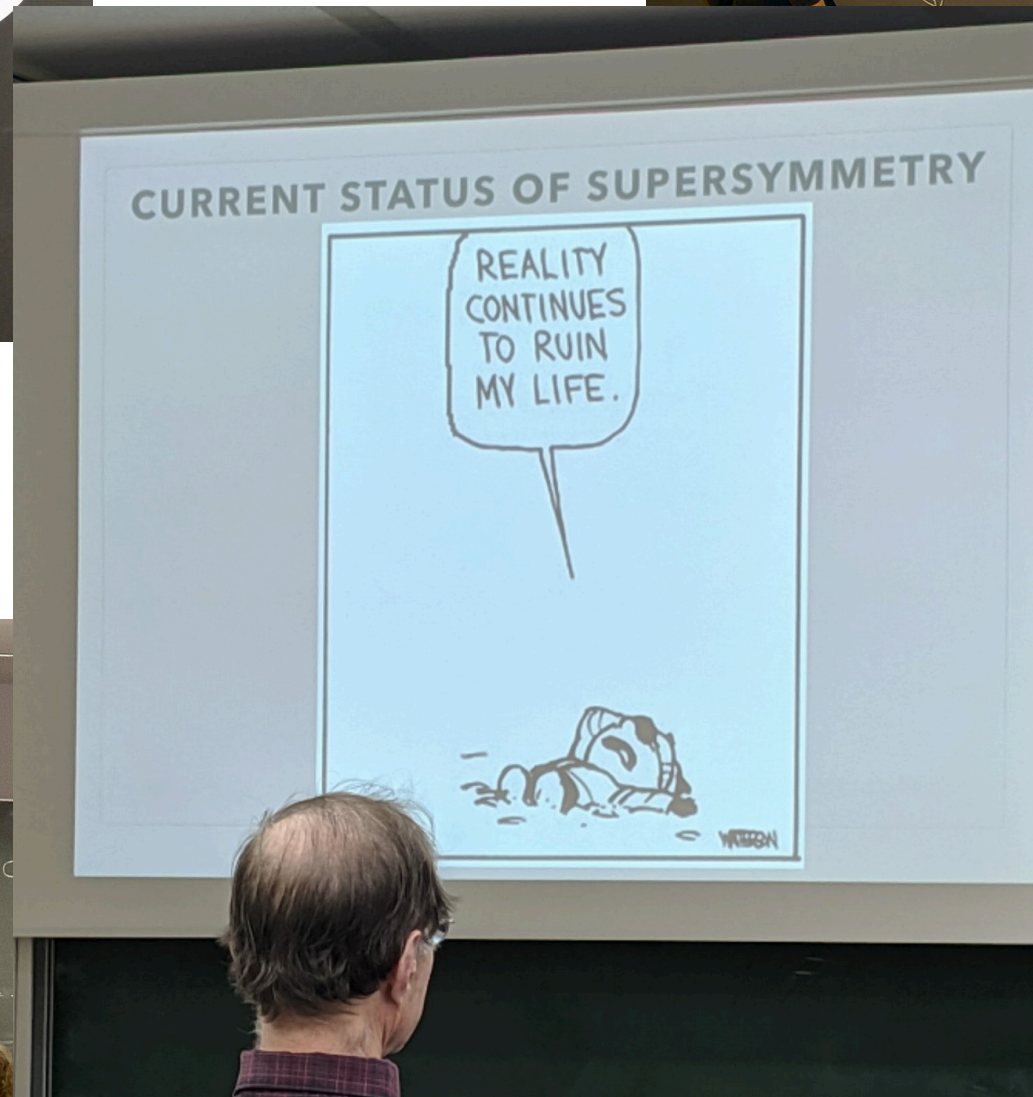
Johannes Gutenberg-Universität Mainz
Personen- und Vorlesungsverzeichnis
Sommersemester 2004



Happy Birthday Matthias

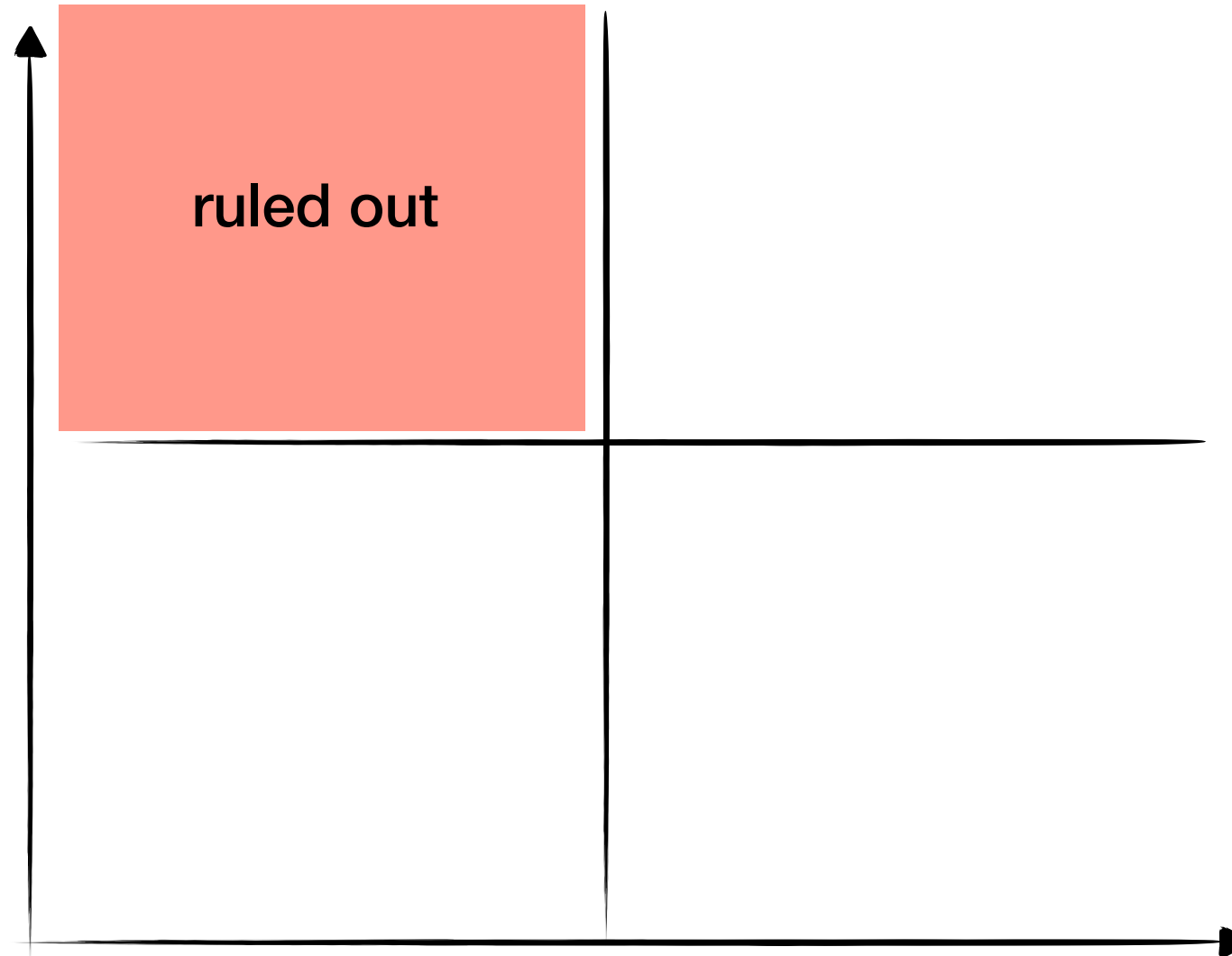


Happy Birthday MiTP



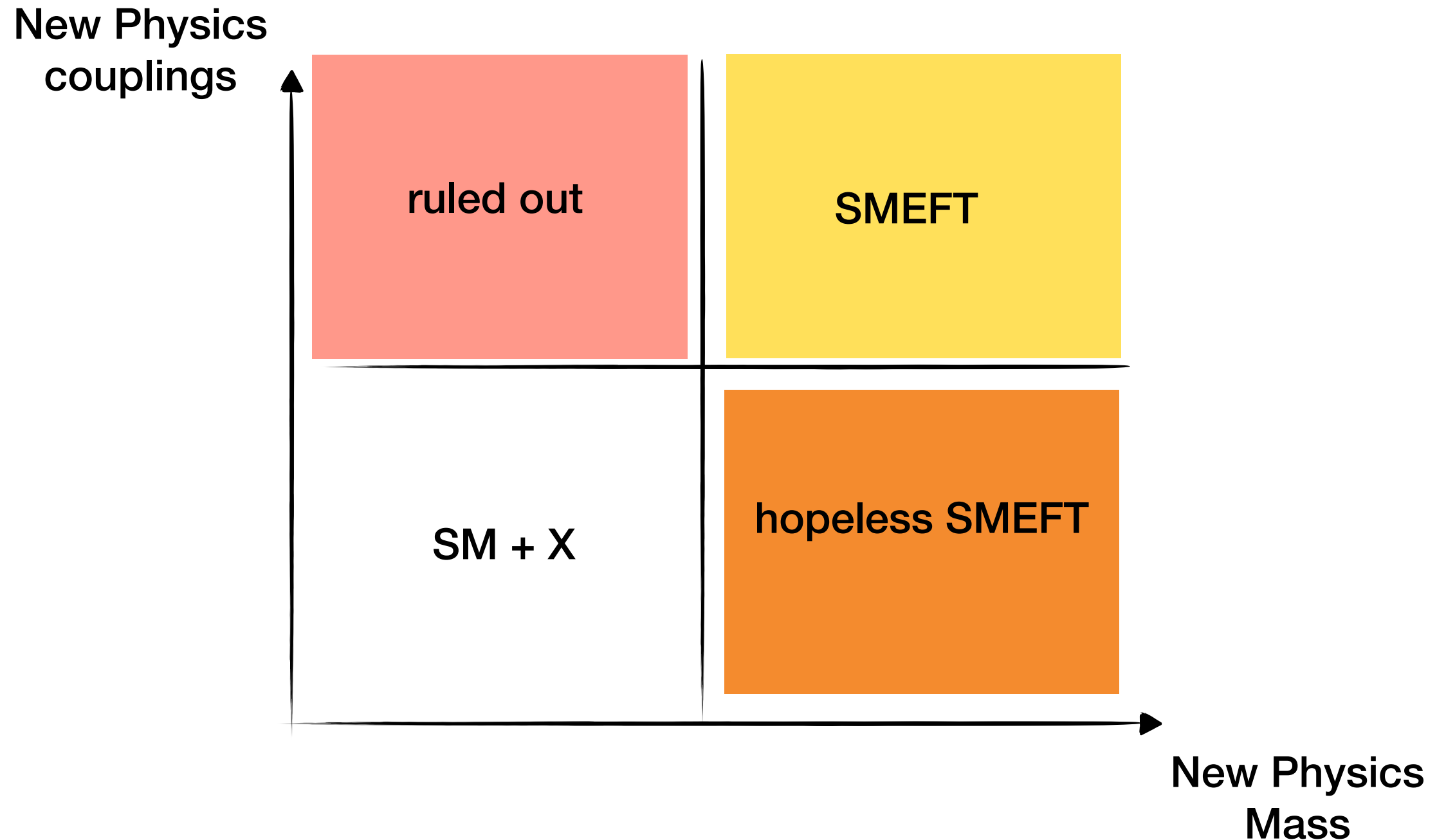
Landscape of new physics

New Physics
couplings



New Physics
Mass

Landscape of new physics



Why should there be *any* new physics that is light and weakly coupled?

Light new physics ?

Goldstone bosons

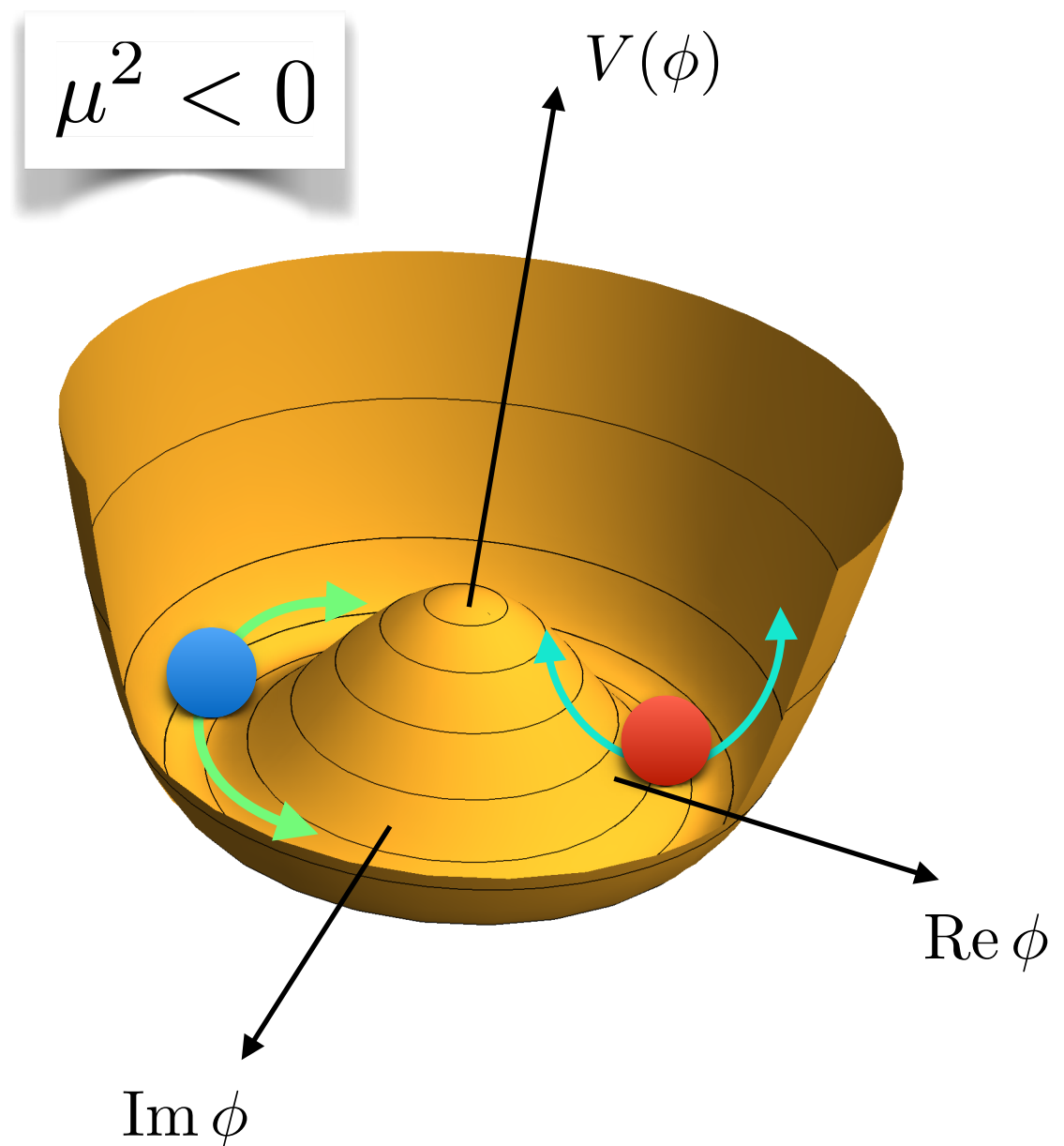
Every spontaneously broken continuous symmetry gives rise to massless spin-0 fields.

$$V(\phi) = \mu^2 \phi \phi^\dagger + \lambda (\phi \phi^\dagger)^2$$

$$\phi = (f + s)e^{ia/f}$$

$$m_s^2 = 4\lambda f^2 = |\mu^2|$$

$$m_a^2 = 0$$



Goldstone bosons

Since the GB corresponds to the phase of a complex field, it is protected by a shift symmetry

$$\phi = (\textcolor{blue}{f} + s)e^{ia/\textcolor{blue}{f}}$$

it is protected by a shift symmetry

$$e^{ia(x)/\textcolor{blue}{f}} \rightarrow e^{i(a(x)+c)/\textcolor{blue}{f}} = e^{ia(x)/\textcolor{blue}{f}} e^{ic/\textcolor{blue}{f}}$$

This symmetry forbids a mass term, and all couplings are suppressed by the UV scale

$$\mathcal{L} = \frac{1}{2} \partial_\mu a \partial^\mu a + c_\mu \frac{\partial^\nu a}{4\pi \textcolor{blue}{f}} \bar{\mu} \gamma_\nu \mu + \dots$$

Goldstone bosons

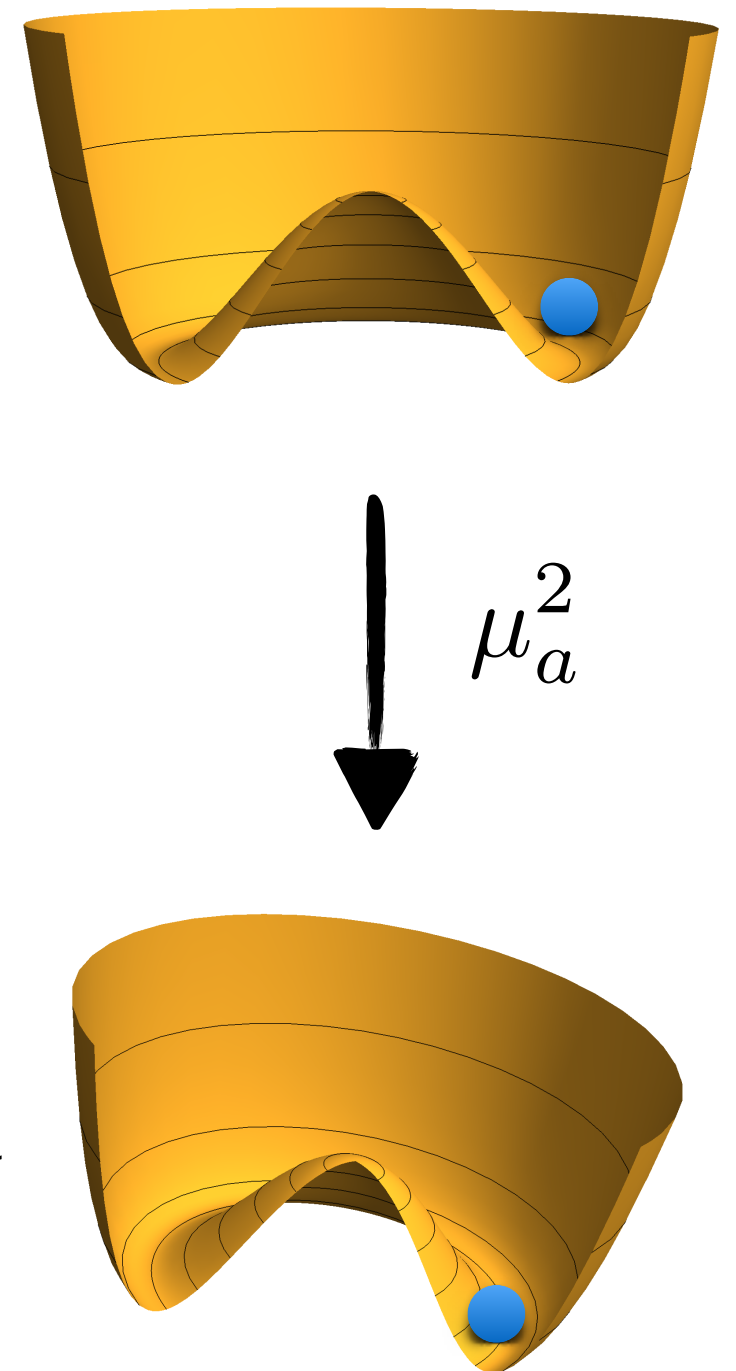
An exactly massless boson is very problematic.

The global symmetry can be broken by explicit masses or anomalous effects

$$\mathcal{L} = \frac{1}{2} \partial_\mu a \partial^\mu a + c_\mu \frac{\partial^\nu a}{4\pi f} \bar{\mu} \gamma_\nu \mu + \dots + \frac{1}{2} m_a^2 a^2$$

$$m_a = \frac{\mu_a^2}{f}$$

Small couplings correspond to small masses and a decoupled NP sector.



Goldstone bosons



The most famous example is the pion

ρ, P, N

$$\mathcal{L}_{\text{QCD}} = \bar{q}_L i \not{D} q_L + \bar{q}_R i \not{D} q_R + m_q \bar{q}_L q_R$$

$$\langle \bar{q}_L q_R \rangle = \Lambda_{\text{QCD}}^3 \approx \text{GeV}^3$$

The pion mass is controlled by the explicit breaking through light quark masses

$$m_\pi^2 = \frac{m_u + m_d}{f_\pi^2} \Lambda_{\text{QCD}}^3 \approx (140 \text{ MeV})^2$$

π



Goldstone bosons



The most famous example is the pion

NP at f

$$\mathcal{L}_{\text{QCD}} = \bar{q}_L i \not{D} q_L + \bar{q}_R i \not{D} q_R + m_q \bar{q}_L q_R$$

$$\langle \bar{q}_L q_R \rangle = \Lambda_{\text{QCD}}^3 \approx \text{GeV}^3$$

The pion mass is controlled by the explicit breaking through light quark masses

$$m_\pi^2 = \frac{m_u + m_d}{f_\pi^2} \Lambda_{\text{QCD}}^3 \approx (140 \text{ MeV})^2$$

axion



Axionlike particles

Most general dimension five Lagrangian at the UV scale

$$\begin{aligned}\mathcal{L}_{\text{eff}}^{D\leq 5} = & \frac{1}{2} (\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2} a^2 + \frac{\partial^\mu a}{f} \sum_F \bar{\psi}_F \mathbf{c}_F \gamma_\mu \psi_F + c_\phi \frac{\partial^\mu a}{f} (\phi^\dagger i D_\mu \phi + \text{h.c.}) \\ & + c_{GG} \frac{\alpha_s}{4\pi} \frac{a}{f} G_{\mu\nu}^a \tilde{G}^{\mu\nu,a} + c_{WW} \frac{\alpha_2}{4\pi} \frac{a}{f} W_{\mu\nu}^A \tilde{W}^{\mu\nu,A} + c_{BB} \frac{\alpha_1}{4\pi} \frac{a}{f} B_{\mu\nu} \tilde{B}^{\mu\nu} .\end{aligned}$$

All couplings are suppressed by the UV scale f

Axionlike particles

Most general dimension five Lagrangian at the UV scale

$$\begin{aligned}
 \mathcal{L}_{\text{eff}}^{D \leq 5} = & \frac{1}{2} (\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2} a^2 + \frac{\partial^\mu a}{f} \sum_F \bar{\psi}_F \mathbf{c}_F \gamma_\mu \psi_F + c_\phi \frac{\partial^\mu a}{f} (\phi^\dagger i D_\mu \phi + \text{h.c.}) \\
 & + c_{GG} \frac{\alpha_s}{4\pi} \frac{a}{f} G_{\mu\nu}^a \tilde{G}^{\mu\nu,a} + c_{WW} \frac{\alpha_2}{4\pi} \frac{a}{f} W_{\mu\nu}^A \tilde{W}^{\mu\nu,A} + c_{BB} \frac{\alpha_1}{4\pi} \frac{a}{f} B_{\mu\nu} \tilde{B}^{\mu\nu} .
 \end{aligned}$$

explicit mass term \rightarrow $\frac{m_{a,0}^2}{2} a^2$
 couplings to fermions $F=Q,u,d,L,e$ \rightarrow $\frac{\partial^\mu a}{f} \sum_F \bar{\psi}_F \mathbf{c}_F \gamma_\mu \psi_F$
 coupling to the Higgs current \rightarrow $c_\phi \frac{\partial^\mu a}{f} (\phi^\dagger i D_\mu \phi + \text{h.c.})$
 coupling to gluons \rightarrow $c_{GG} \frac{\alpha_s}{4\pi} \frac{a}{f} G_{\mu\nu}^a \tilde{G}^{\mu\nu,a}$
 coupling to $SU(2)_L$ gauge bosons \rightarrow $c_{WW} \frac{\alpha_2}{4\pi} \frac{a}{f} W_{\mu\nu}^A \tilde{W}^{\mu\nu,A}$
 coupling to hypercharge \rightarrow $c_{BB} \frac{\alpha_1}{4\pi} \frac{a}{f} B_{\mu\nu} \tilde{B}^{\mu\nu}$

All couplings are suppressed by the UV scale f

Axionlike particles

This Lagrangian captures all possible ALP coupling structures up to dimension 5.

It is easy to imagine scenarios in which a single coupling dominates:

For example: A UV theory in which the ALP couples only to $SU(2)_L$ gauge bosons

$$\mathcal{L}_{\text{eff}}^{D \leq 5} = \frac{1}{2} (\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2} a^2 + c_{WW} \frac{\alpha_2}{4\pi} \frac{a}{f} W_{\mu\nu} \tilde{W}^{\mu\nu}$$

Axionlike particles

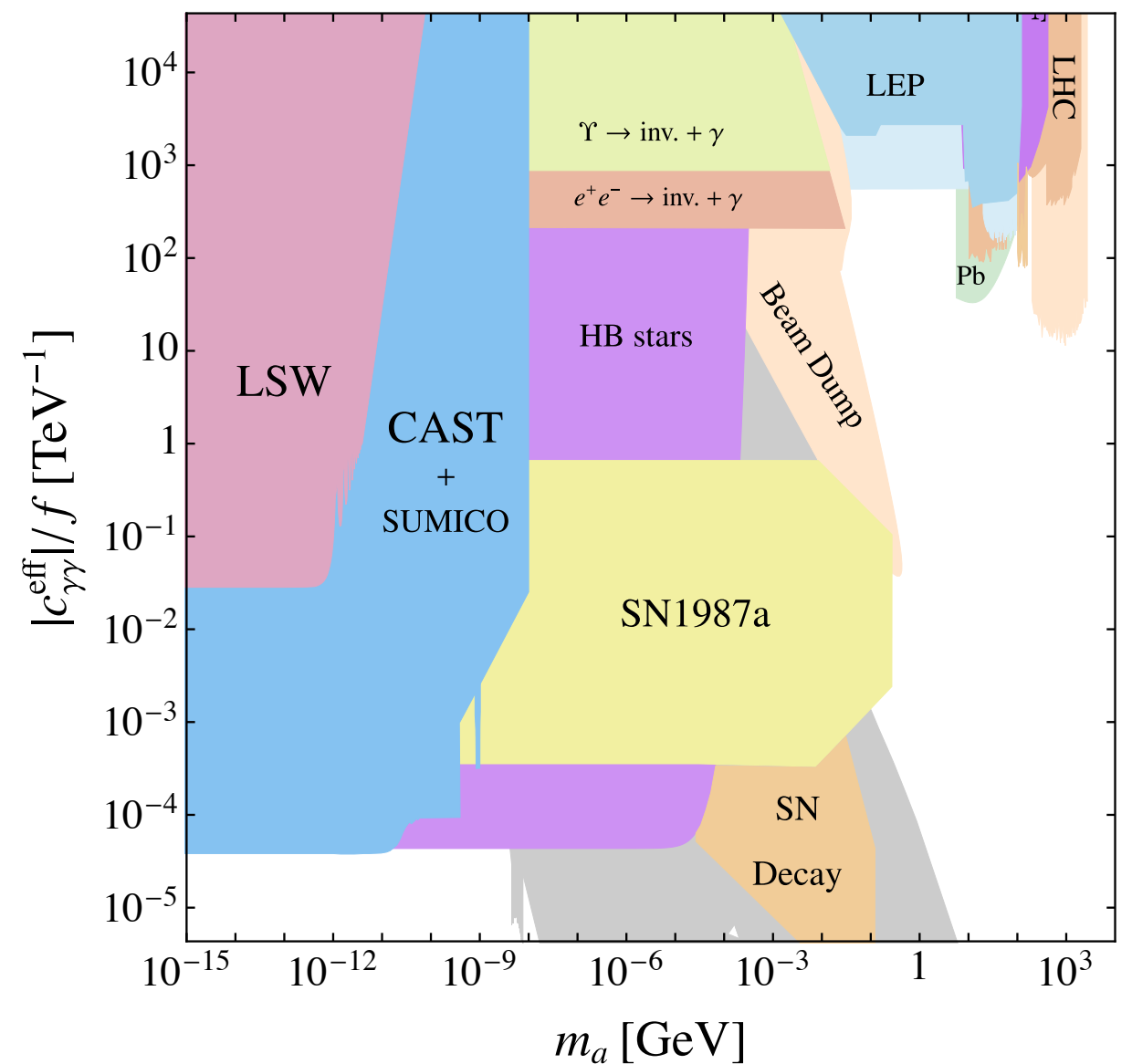
For example: A UV theory in which the ALP couples only to $SU(2)_L$ gauge bosons

$$\mathcal{L}_{\text{eff}}^{D \leq 5} = \frac{1}{2} (\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2} a^2 + c_{WW} \frac{\alpha_2}{4\pi} \frac{a}{f} W_{\mu\nu} \tilde{W}^{\mu\nu}$$

After EW symmetry breaking this ALP couples to photons.

$$W_\mu^3 = s_w A_\mu + c_w Z_\mu$$

But at higher loop order it couples to fermions

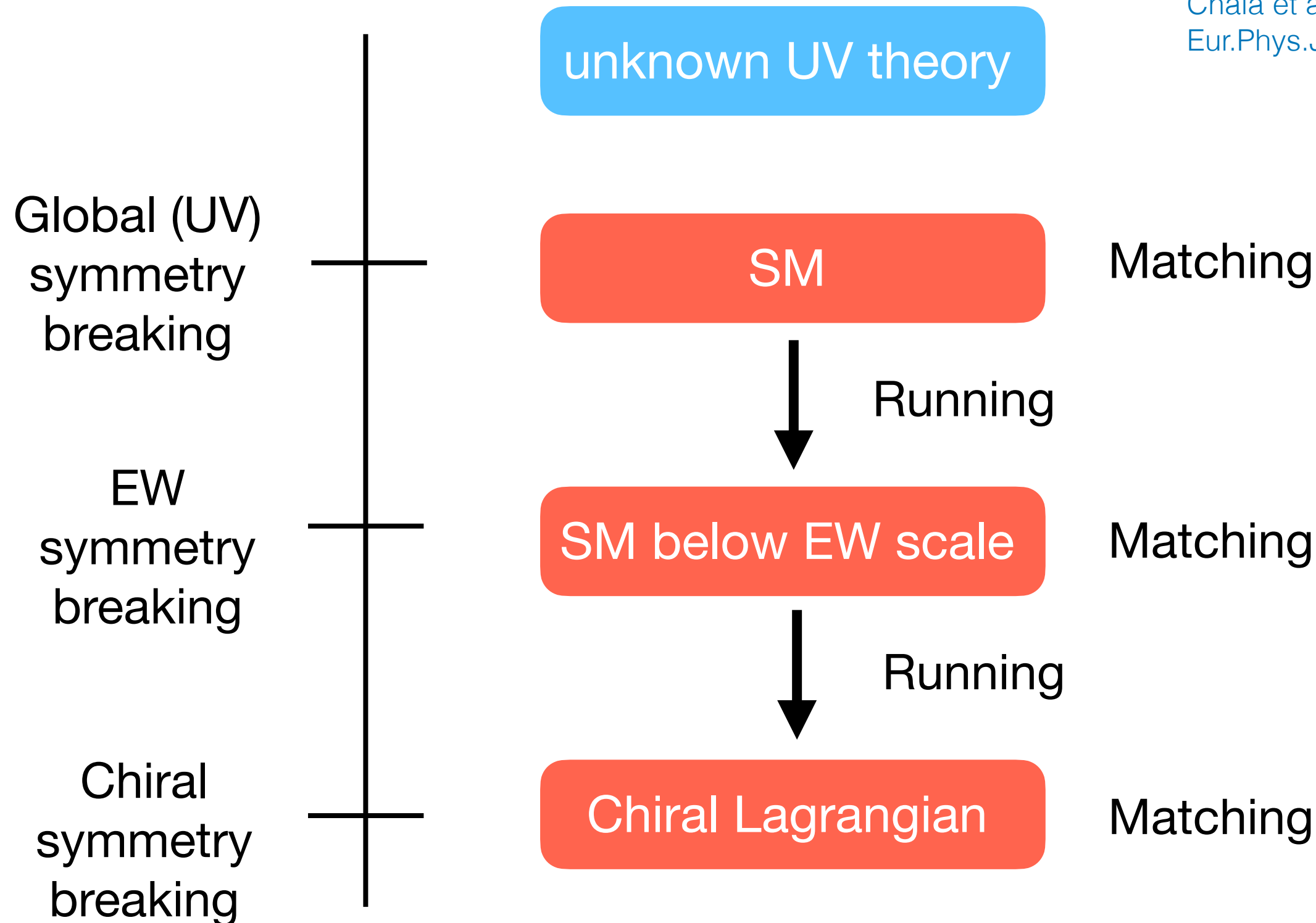


ALPs at different scales

MB, Neubert, Renner, Schnubel,
Thamm, *JHEP* 04 (2021) 063

MB, Neubert, Renner, Schnubel,
Thamm, [2102.13112](#), PRL. 127

Chala et al.,
Eur.Phys.J.C 81 (2021) 2

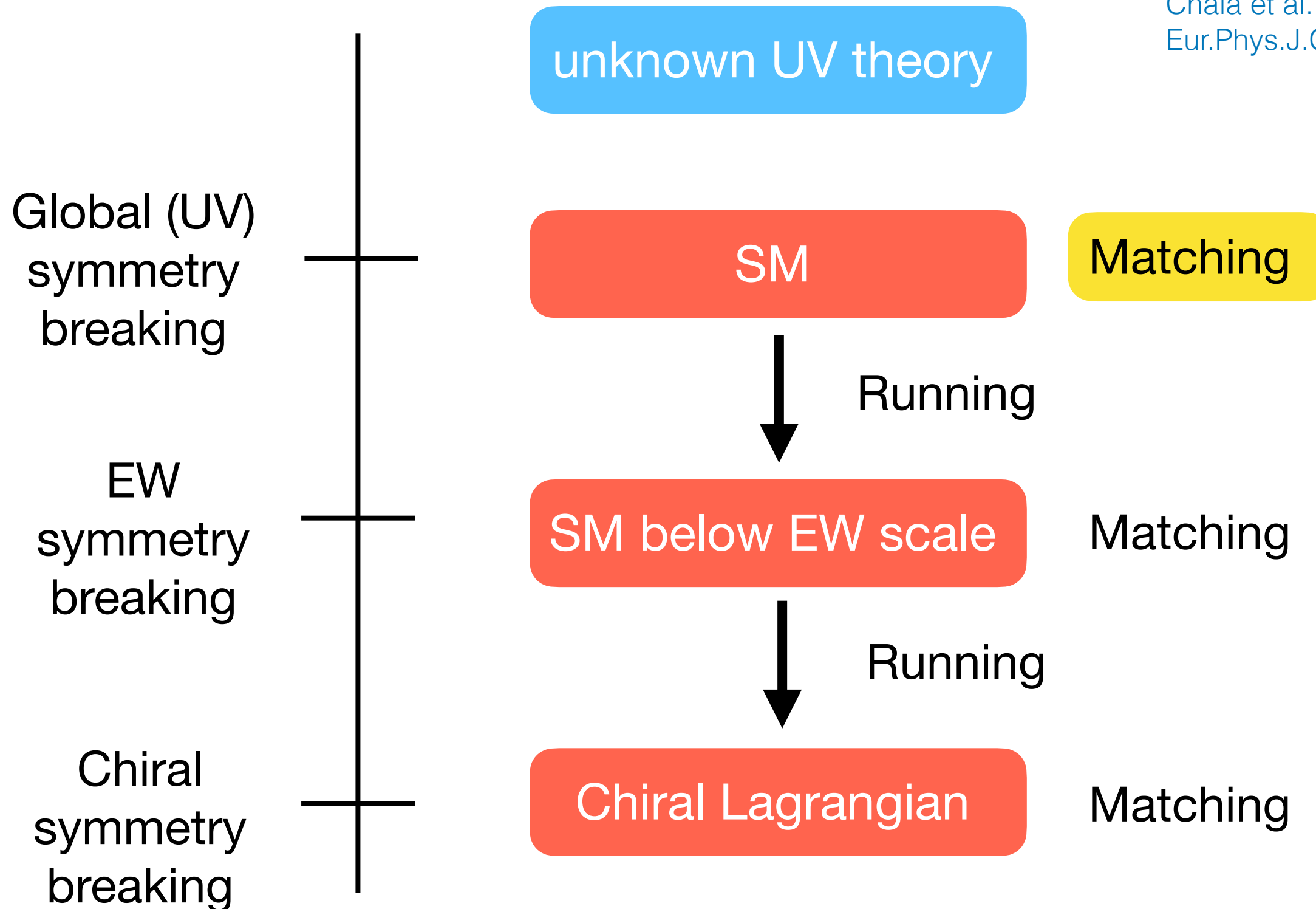


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MB, Neubert, Renner, Schnubel, Thamm, [2102.13112](#), PRL. 127

Chala et al.,
Eur.Phys.J.C 81 (2021) 2

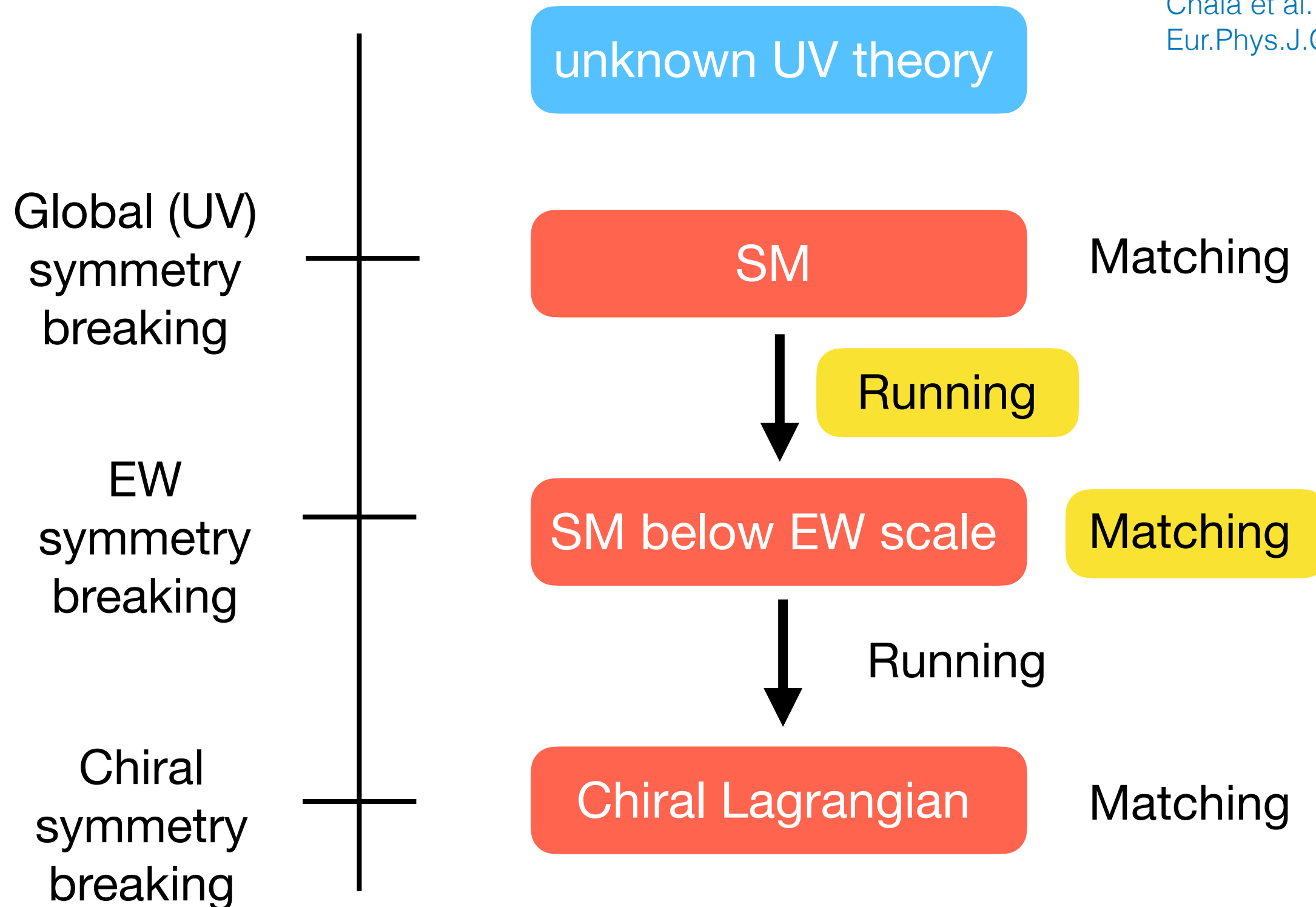


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Chala et al.,
Eur.Phys.J.C 81 (2021) 2

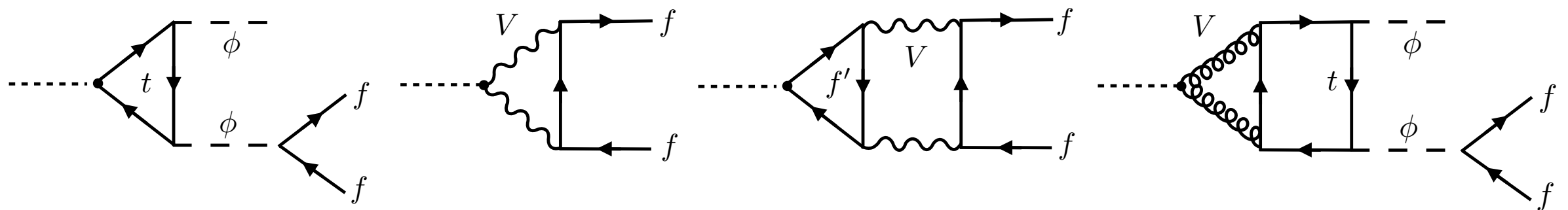


Running and matching at the weak scale

- The gauge boson couplings do not run

$$\frac{d}{d \ln \mu} c_{VV}(\mu) = 0; \quad V = G, W, B \quad \text{Bardeen et al. Nucl. Phys. B 535,(1998)}$$

- Neither are there matching contributions at 1-loop
- The running and matching of ALP fermion couplings receives various contributions



Running and matching at the weak scale

The ALP Lagrangian at the weak scale can be written as

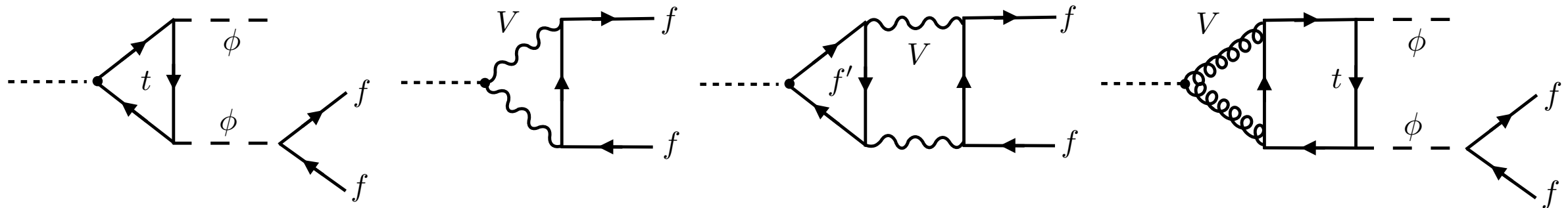
$$\begin{aligned}\mathcal{L}_{\text{eff}}(\mu_w) = & \frac{1}{2} (\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2} a^2 + \mathcal{L}_{\text{ferm}}(\mu_w) + c_{GG} \frac{\alpha_s}{4\pi} \frac{a}{f} G_{\mu\nu}^a \tilde{G}^{\mu\nu,a} + c_{\gamma\gamma} \frac{\alpha}{4\pi} \frac{a}{f} F_{\mu\nu} \tilde{F}^{\mu\nu} \\ & + c_{\gamma Z} \frac{\alpha}{2\pi s_w c_w} \frac{a}{f} F_{\mu\nu} \tilde{Z}^{\mu\nu} + c_{ZZ} \frac{\alpha}{4\pi s_w^2 c_w^2} \frac{a}{f} Z_{\mu\nu} \tilde{Z}^{\mu\nu} + c_{WW} \frac{\alpha}{2\pi s_w^2} \frac{a}{f} W_{\mu\nu}^+ \tilde{W}^{-\mu\nu},\end{aligned}$$

with $c_{\gamma\gamma} = c_{WW} + c_{BB}$, $c_{\gamma Z} = c_w^2 c_{WW} - s_w^2 c_{BB}$, $c_{ZZ} = c_w^4 c_{WW} + s_w^4 c_{BB}$.

with fermion couplings in the mass basis with $\mathbf{k}_U = \mathbf{U}_u^\dagger \mathbf{c}_Q \mathbf{U}_u$, \dots

$$\begin{aligned}\mathcal{L}_{\text{ferm}}(\mu_w) = & \frac{\partial^\mu a}{f} \left[\bar{u}_L \mathbf{k}_U \gamma_\mu u_L + \bar{u}_R \mathbf{k}_u \gamma_\mu u_R + \bar{d}_L \mathbf{k}_D \gamma_\mu d_L + \bar{d}_R \mathbf{k}_d \gamma_\mu d_R \right. \\ & \left. + \bar{\nu}_L \mathbf{k}_\nu \gamma_\mu \nu_L + \bar{e}_L \mathbf{k}_E \gamma_\mu e_L + \bar{e}_R \mathbf{k}_e \gamma_\mu e_R \right].\end{aligned}$$

Flavor diagonal ALP-fermion couplings



ALP fermion couplings at the weak scale for $f = 1 \text{ TeV}$

$$c_{uu,cc}(m_t) \simeq c_{uu,cc}(\Lambda) - 0.116 c_{tt}(\Lambda) - \left[6.35 \tilde{c}_{GG}(\Lambda) + 0.19 \tilde{c}_{WW}(\Lambda) + 0.02 \tilde{c}_{BB}(\Lambda) \right] \cdot 10^{-3},$$

$$c_{dd,ss}(m_t) \simeq c_{dd,ss}(\Lambda) + 0.116 c_{tt}(\Lambda) - \left[7.08 \tilde{c}_{GG}(\Lambda) + 0.22 \tilde{c}_{WW}(\Lambda) + 0.005 \tilde{c}_{BB}(\Lambda) \right] \cdot 10^{-3},$$

$$c_{bb}(m_t) \simeq c_{bb}(\Lambda) + 0.097 c_{tt}(\Lambda) - \left[7.02 \tilde{c}_{GG}(\Lambda) + 0.19 \tilde{c}_{WW}(\Lambda) + 0.005 \tilde{c}_{BB}(\Lambda) \right] \cdot 10^{-3},$$

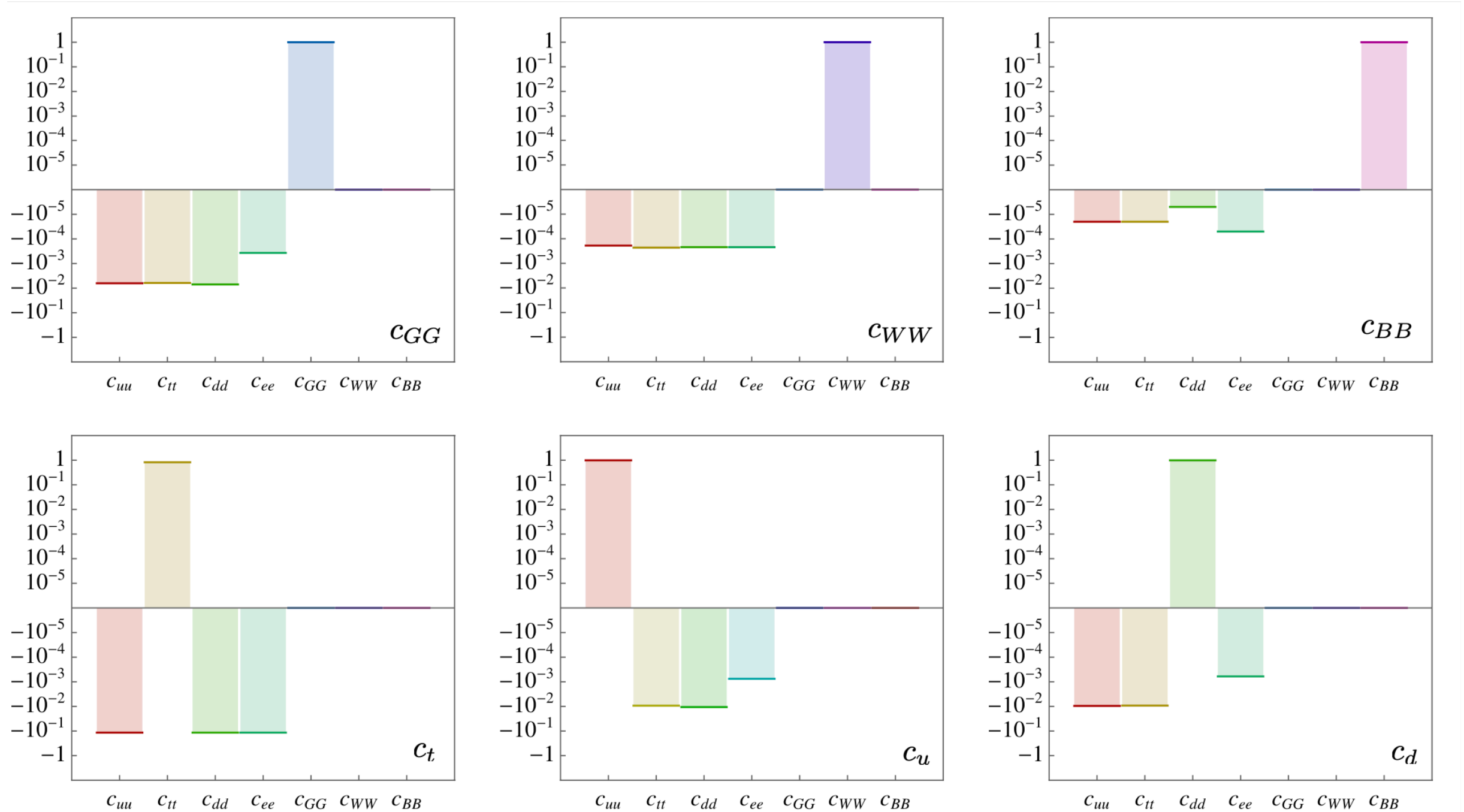
$$c_{e_i e_i}(m_t) \simeq c_{e_i e_i}(\Lambda) + 0.116 c_{tt}(\Lambda) - \left[0.37 \tilde{c}_{GG}(\Lambda) + 0.22 \tilde{c}_{WW}(\Lambda) + 0.05 \tilde{c}_{BB}(\Lambda) \right] \cdot 10^{-3}.$$

where we have defined

$$c_{f_i f_i}(\mu) \equiv [k_f(\mu)]_{ii} - [k_F(\mu)]_{ii}$$

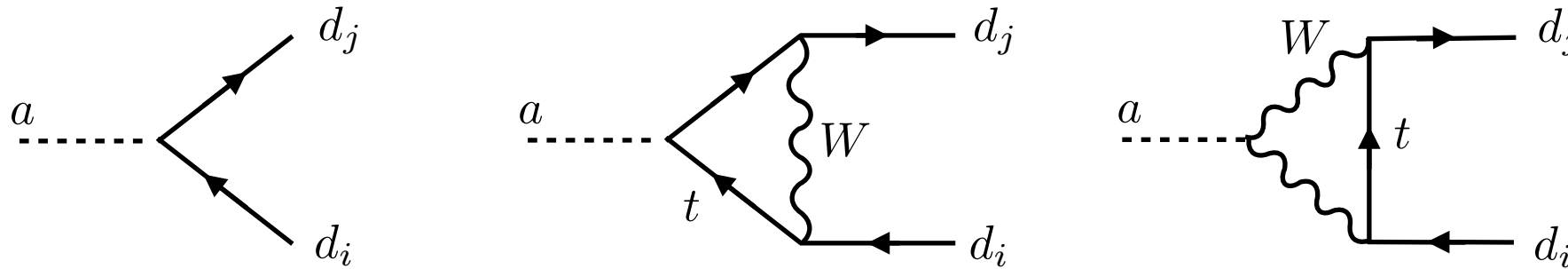
Flavor diagonal ALP-fermion couplings

ALP fermion couplings at the weak scale for $f = 1 \text{ TeV}$



Flavor off-diagonal ALP-fermion couplings

Flavor violation can come from the UV theory or from the SM



Assuming MFV (only $y_t \neq 0$) for $f = 1$ TeV

$$[k_U(\mu_w)]_{ij} = [k_u(\mu_w)]_{ij} = [k_d(\mu_w)]_{ij} = [k_E(\mu_w)]_{ij} = [k_e(\mu_w)]_{ij} = 0$$

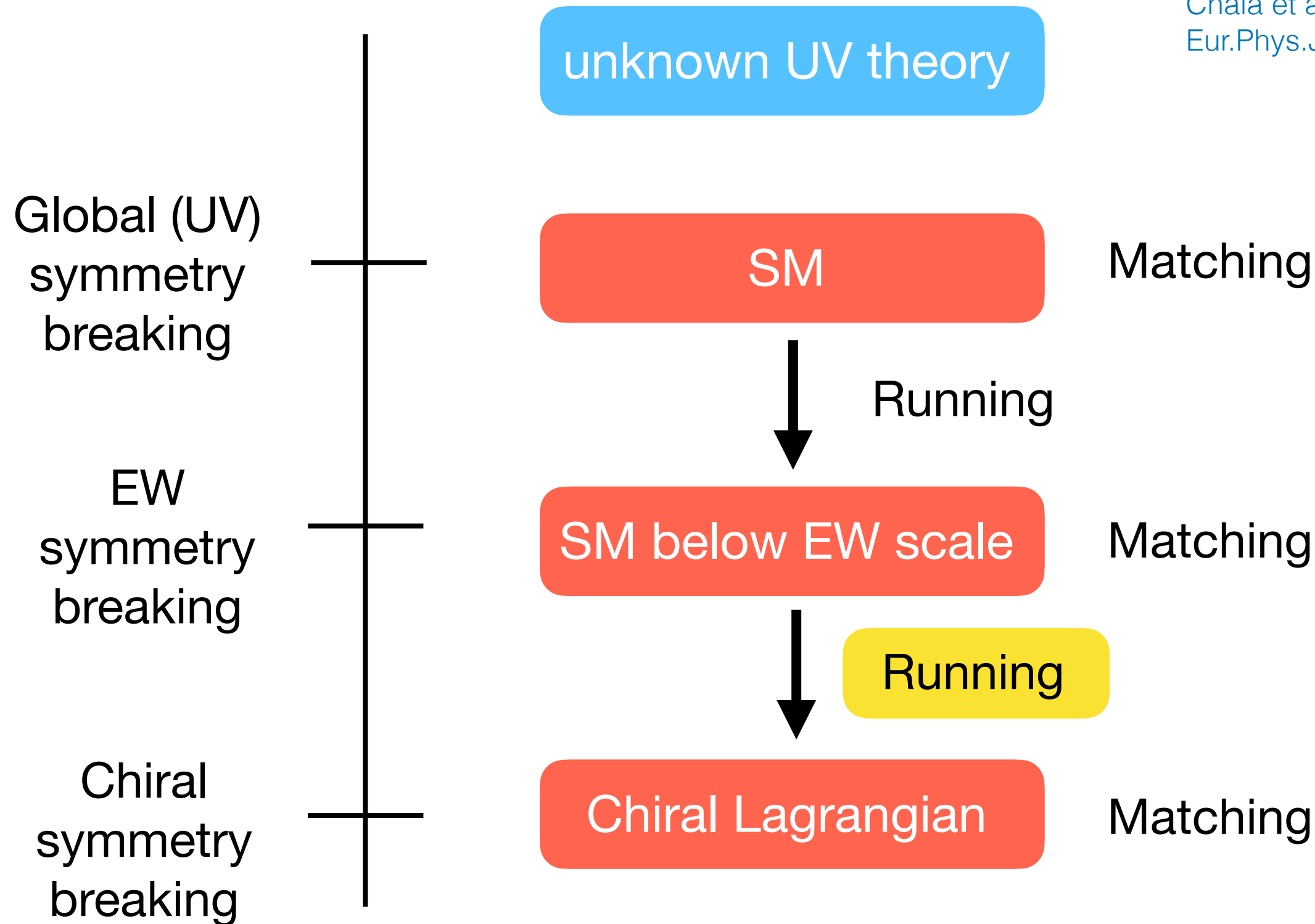
$$[k_D(m_t)]_{ij} \simeq \cancel{[k_D(\Lambda)]_{ij}} + 0.019 V_{ti}^* V_{tj} \left[c_{tt}(\Lambda) - 0.0032 \tilde{c}_{GG}(\Lambda) - 0.0057 \tilde{c}_{WW}(\Lambda) \right]$$

ALPs at different scales

MB, Neubert, Renner, Schnubel,
Thamm, *JHEP* 04 (2021) 063

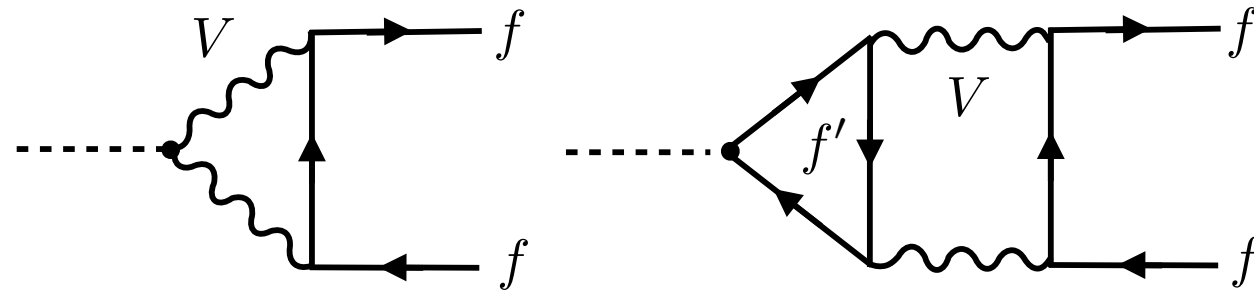
MB, Neubert, Renner, Schnubel,
Thamm, [2102.13112](#), PRL. 127

Chala et al.,
Eur.Phys.J.C 81 (2021) 2



Running below the EW scale

Running below the weak scale affects only flavor-diagonal ALP fermion couplings (running to 2 GeV)



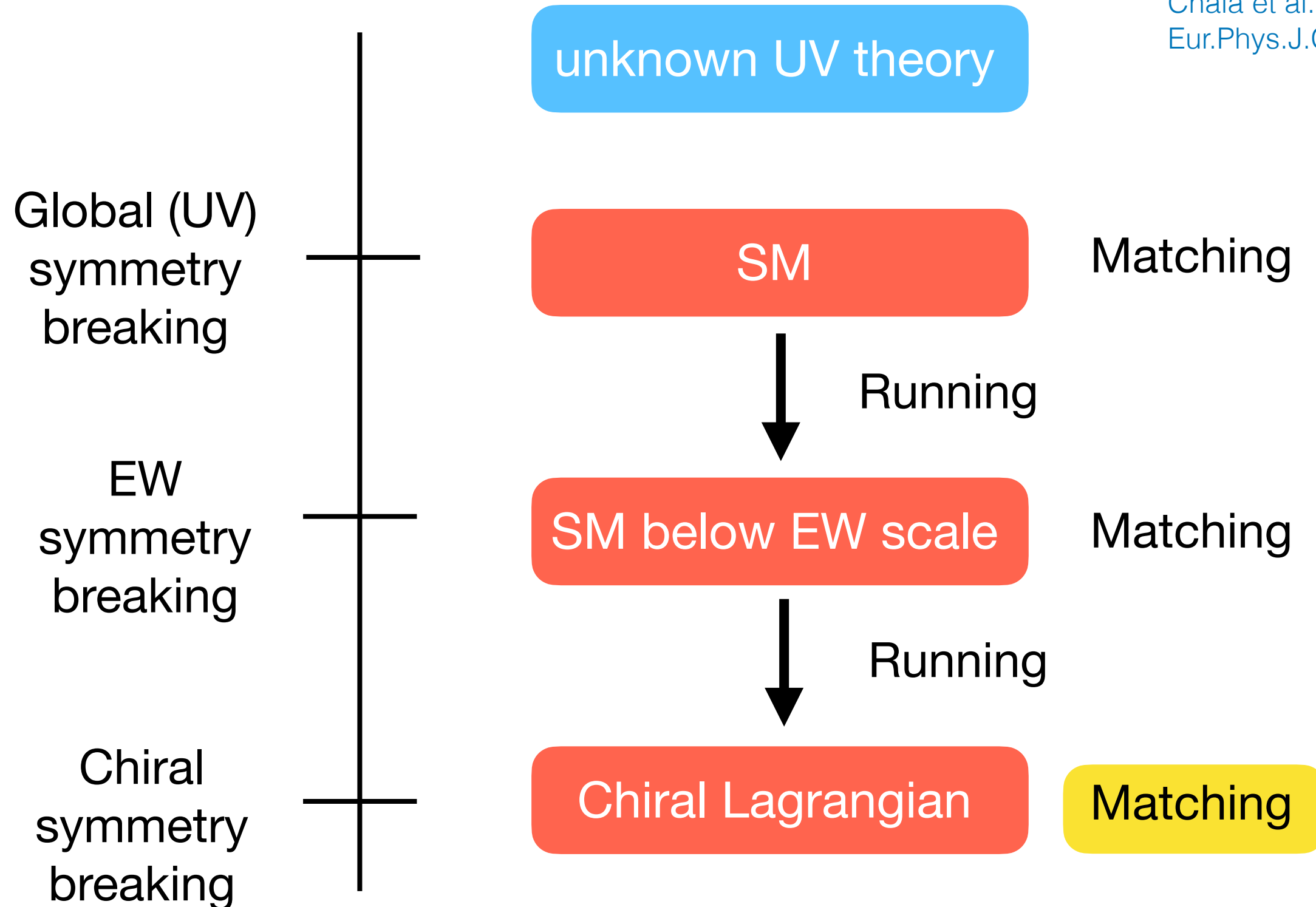
$$\begin{aligned}
 c_{qq}(\mu_0) &= c_{qq}(m_t) + \left[3.0 \tilde{c}_{GG}(\Lambda) - 1.4 c_{tt}(\Lambda) - 0.6 c_{bb}(\Lambda) \right] \cdot 10^{-2} \\
 &\quad + Q_q^2 \left[3.9 \tilde{c}_{\gamma\gamma}(\Lambda) - 4.7 c_{tt}(\Lambda) - 0.2 c_{bb}(\Lambda) \right] \cdot 10^{-5}, \\
 c_{\ell\ell}(\mu_0) &= c_{\ell\ell}(m_t) + \left[3.9 \tilde{c}_{\gamma\gamma}(\Lambda) - 4.7 c_{tt}(\Lambda) - 0.2 c_{bb}(\Lambda) \right] \cdot 10^{-5}.
 \end{aligned}$$

ALPs at different scales

MB, Neubert, Renner, Schnubel,
Thamm, *JHEP* 04 (2021) 063

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Thamm, [2102.13112](#), PRL. 127

Chala et al.,
Eur.Phys.J.C 81 (2021) 2



Matching to the chiral Lagrangian

The chiral Lagrangian + ALP then reads

$$\begin{aligned}\mathcal{L}_{\text{eff}}^{\chi} = & \frac{f_{\pi}^2}{8} \text{Tr}[\mathbf{D}^{\mu} \mathbf{\Sigma} (\mathbf{D}_{\mu} \mathbf{\Sigma})^{\dagger}] + \frac{f_{\pi}^2}{4} B_0 \text{Tr}[\hat{\mathbf{m}}_q(a) \mathbf{\Sigma}^{\dagger} + \text{h.c.}] \\ & + \frac{1}{2} \partial^{\mu} a \partial_{\mu} a - \frac{m_{a,0}^2}{2} a^2 + \hat{c}_{\gamma\gamma} \frac{\alpha}{4\pi} \frac{a}{f} F_{\mu\nu} \tilde{F}^{\mu\nu},\end{aligned}$$

where $\mathbf{\Sigma} = \exp(i\sqrt{2}\mathbf{\Pi}/f_{\pi})$

$$\mathbf{\Pi} = \lambda_b \pi^b = \begin{pmatrix} \pi_0 + \sqrt{\frac{1}{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi_0 + \sqrt{\frac{1}{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -2\sqrt{\frac{1}{3}}\eta \end{pmatrix}.$$

Matching to the chiral Lagrangian

This matching is not new. It has first been performed by Georgi, Kaplan and Randall, but they used

$$i\mathbf{D}_\mu \Sigma = i\partial_\mu \Sigma + e A_\mu [\mathbf{Q}, \Sigma]$$

missing the additional contribution from the ALP in the current:

$$i\mathbf{D}_\mu \Sigma = i\partial_\mu \Sigma + e A_\mu [\mathbf{Q}, \Sigma] + \frac{\partial_\mu a}{f} \left(\hat{\mathbf{k}}_Q \Sigma - \Sigma \hat{\mathbf{k}}_q \right)$$

This is an important omission and can be cross-checked by demanding independence of physical observables from the unphysical kappa parameters.

Matching to the chiral Lagrangian

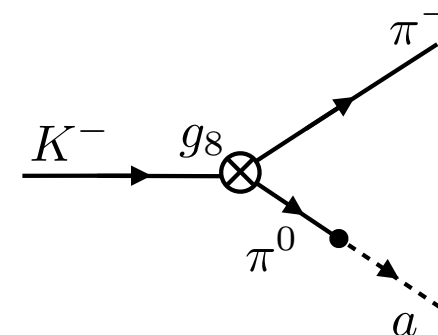
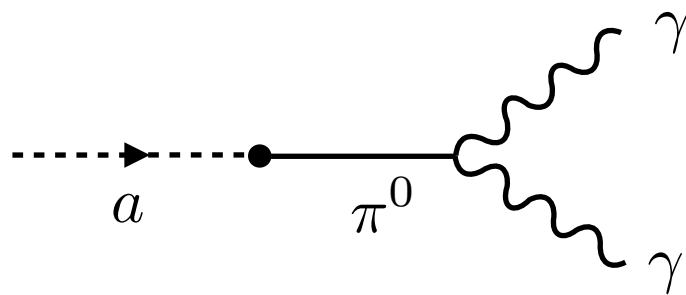
The chiral Lagrangian + ALP then reads

$$\begin{aligned}\mathcal{L}_{\text{eff}}^{\chi} = & \frac{f_{\pi}^2}{8} \text{Tr}[\mathbf{D}^{\mu} \Sigma (\mathbf{D}_{\mu} \Sigma)^{\dagger}] + \frac{f_{\pi}^2}{4} B_0 \text{Tr}[\hat{\mathbf{m}}_q(a) \Sigma^{\dagger} + \text{h.c.}] \\ & + \frac{1}{2} \partial^{\mu} a \partial_{\mu} a - \frac{m_{a,0}^2}{2} a^2 + \hat{c}_{\gamma\gamma} \frac{\alpha}{4\pi} \frac{a}{f} F_{\mu\nu} \tilde{F}^{\mu\nu},\end{aligned}$$

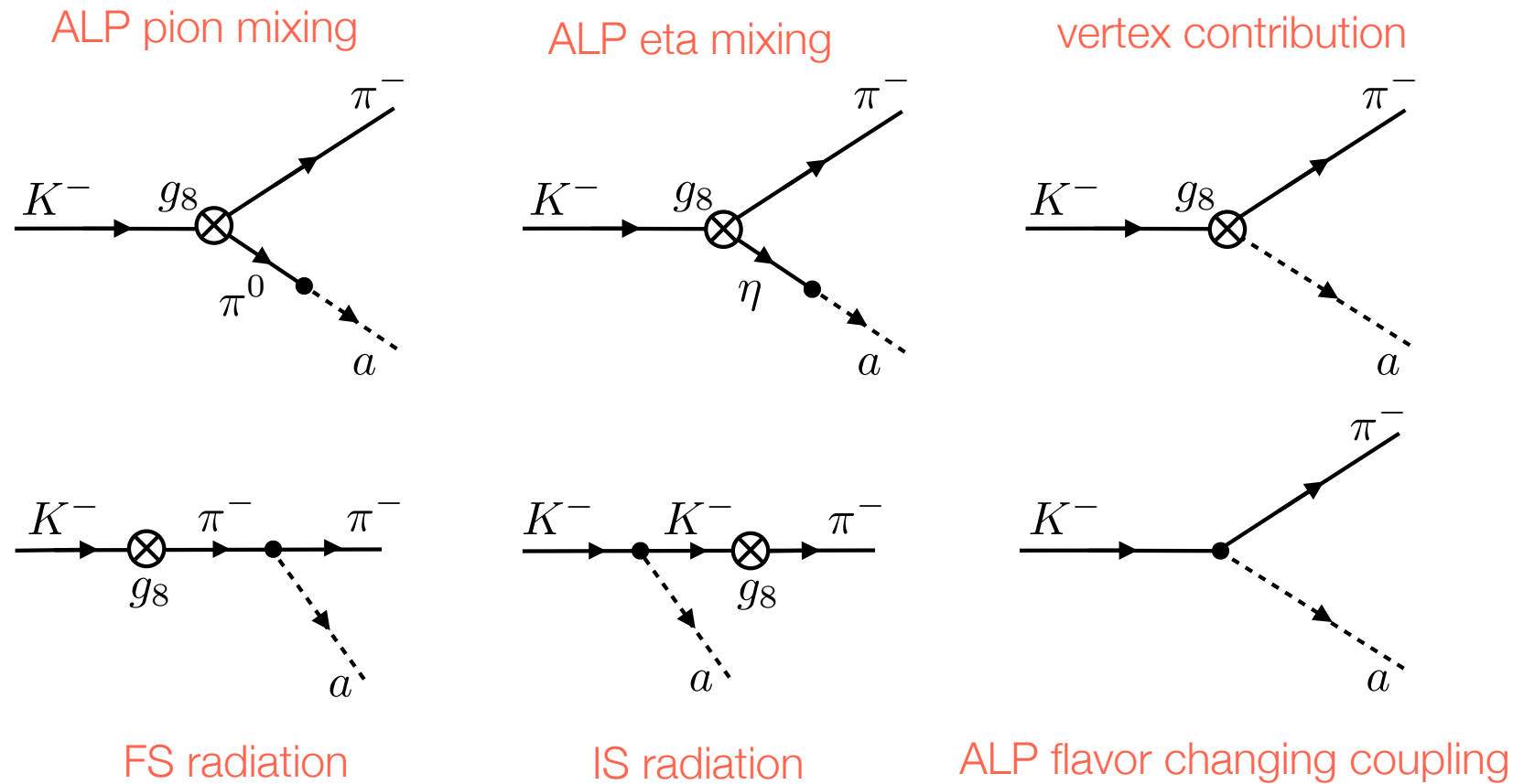
Turning on the weak interactions give rise to flavor changing couplings involving the ALP

$$\mathcal{L}_{\text{weak}} = -\frac{4G_F}{\sqrt{2}} V_{ud}^* V_{us} g_8 [L_{\mu} L^{\mu}]^{32}$$

$$L_{\mu} \equiv -\frac{if_{\pi}^2}{4} [\Sigma \hat{D}_{\mu} \Sigma^{\dagger}]$$



$K^- \rightarrow \pi^- a$ decays reconsidered



The kappa dependence reads

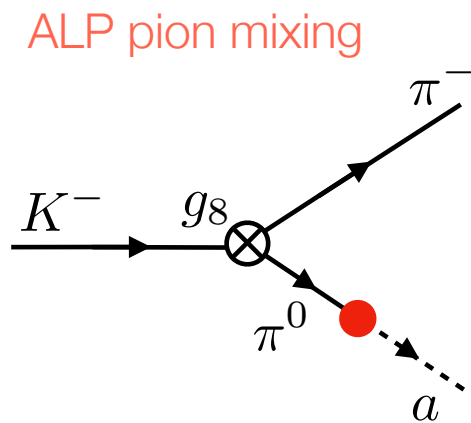
$$\mathcal{L}_{\text{weak}} = -\frac{4G_F}{\sqrt{2}} V_{ud}^* V_{us} g_8 [L_\mu L^\mu]^{32}$$

$$D_1 \ni \frac{N_8}{2f} c_{GG} (\kappa_u - \kappa_d) (m_\pi^2 - m_a^2),$$

$$D_2 \ni -\frac{N_8}{6f} c_{GG} (2m_K^2 + m_\pi^2 - 3m_a^2) (\kappa_u + \kappa_d - 2\kappa_s)$$

$$D_3 \ni \frac{N_8}{f} c_{GG} (m_a^2 (\kappa_s - \kappa_d) + m_K^2 (\kappa_d + \kappa_u) + m_\pi^2 (-\kappa_u - \kappa_s))$$

$K^- \rightarrow \pi^- a$ decays reconsidered



$$\pi^0 = \pi_{\text{phys}}^0 + \theta_{\pi a} a_{\text{phys}}$$

$$\mathcal{A}_{K^+ \rightarrow a \pi^+} \neq \theta \mathcal{A}_{K^+ \rightarrow \pi^0 \pi^+}$$

In the literature ALP (or axion) pion mixing is sometimes introduced as a measurable quantity, but this is not correct

$$\theta_{\pi a} = \frac{f_\pi}{2\sqrt{2}f} \left[\frac{m_a^2 (\hat{c}_{uu} - \hat{c}_{dd})}{m_\pi^2 - m_a^2} - \frac{m_\pi^2 \Delta_\kappa}{m_\pi^2 - m_a^2} \right]$$

$$c_{qq} = (k_q - k_Q), \quad \Delta_\kappa = 4c_{GG} \frac{m_u \kappa_u - m_d \kappa_d}{m_d + m_u}$$

$K^- \rightarrow \pi^- a$ decays reconsidered

The full amplitude is completely general

$$\begin{aligned} i\mathcal{A}_{K^- \rightarrow \pi^- a} = & \frac{N_8}{4f} \left[16c_{GG} \frac{(m_K^2 - m_\pi^2)(m_K^2 - m_a^2)}{4m_K^2 - m_\pi^2 - 3m_a^2} \right. \\ & + 6(c_{uu} + c_{dd} - 2c_{ss}) m_a^2 \frac{m_K^2 - m_a^2}{4m_K^2 - m_\pi^2 - 3m_a^2} \\ & + (2c_{uu} + c_{dd} + c_{ss}) (m_K^2 - m_\pi^2 - m_a^2) + 4c_{ss} m_a^2 \\ & \left. + (k_d + k_D - k_s - k_S) (m_K^2 + m_\pi^2 - m_a^2) \right] \\ & - \frac{m_K^2 - m_\pi^2}{2f} [k_q + k_Q]^{23}. \end{aligned}$$

The GKR paper only considered a gluon couplings and in that case the (wrong) result is smaller by a factor

$$\frac{m_u}{2(m_u + m_d)} = 0.16$$

$K^- \rightarrow \pi^- a$ decays reconsidered

Including all contributions one finds numerically

$$\begin{aligned} i\mathcal{A}(K^- \rightarrow \pi^- a) = & 10^{-11} \text{ GeV} \left[\frac{1 \text{ TeV}}{f} \right] e^{i(\delta_8 + \gamma_s)} \left[-3.3 c_{GG} - 1.6 (c_u^{ii}(\Lambda) + c_d^{ii}(\Lambda)) + 3.2 c_Q^{ii}(\Lambda) \right. \\ & \left. + 6.8 \cdot 10^{-4} c_{WW} + 4.1 \cdot 10^{-5} c_{BB} - 1.1 \cdot 10^{-3} c_L^{ii}(\Lambda) + 1.2 \cdot 10^{-4} c_e^{ii}(\Lambda) \right] \\ & + 10^{-11} \text{ GeV} \left[\frac{1 \text{ TeV}}{f} \right] e^{-i\beta_s} \left[-0.24 c_{GG} - 0.37 c_d^{ii}(\Lambda) + 76 c_u^{ii}(\Lambda) - 75 c_Q^{ii}(\Lambda) \right. \\ & \left. - 0.12 c_{WW} - 6.3 \cdot 10^{-4} c_{BB} + 1.6 \cdot 10^{-2} c_L^{ii}(\Lambda) - 1.9 \cdot 10^{-3} c_e^{ii}(\Lambda) \right], \end{aligned}$$

For a given benchmark the prediction can be read off, e.g.

- only c_{GG} : The “chiral contribution” (g_8) dominates
- only c_{WW} : The “RGE” contribution dominates

$K^- \rightarrow \pi^- a$ decays reconsidered

We can now use these results to put limits on ALPs. Lets consider the case of a pure SU(2) coupling in the UV

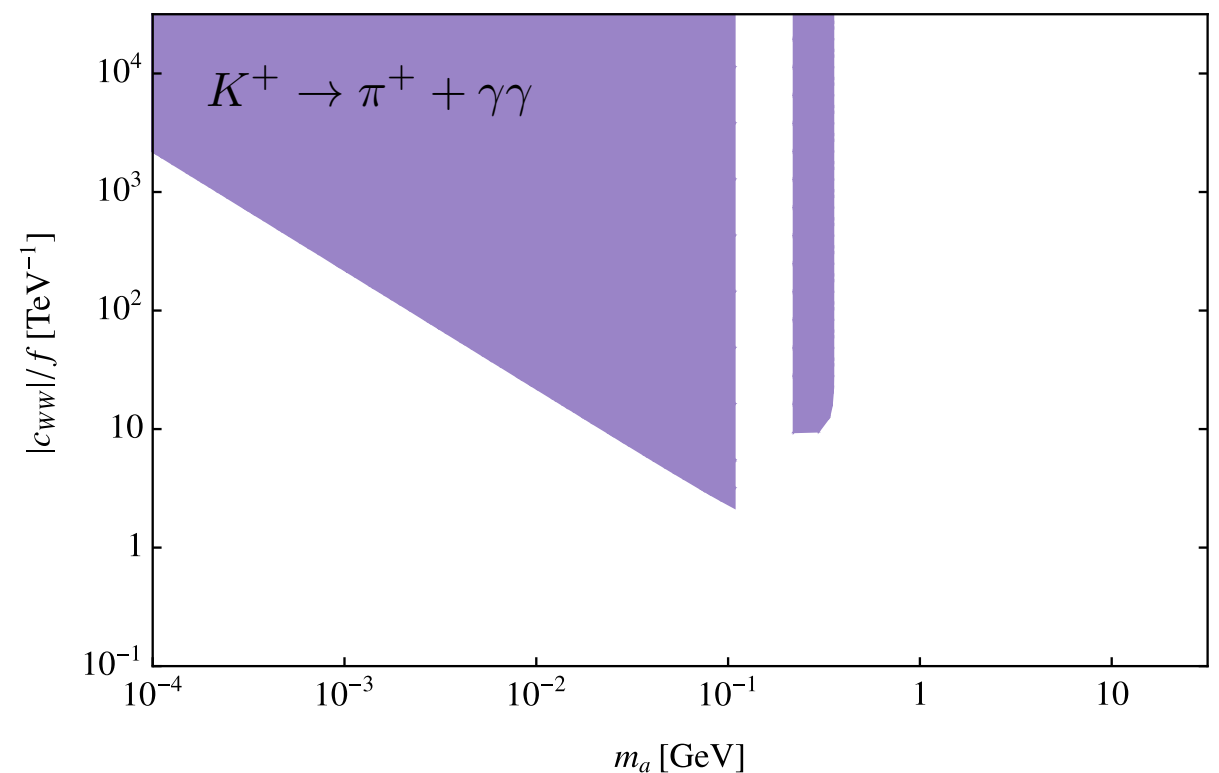
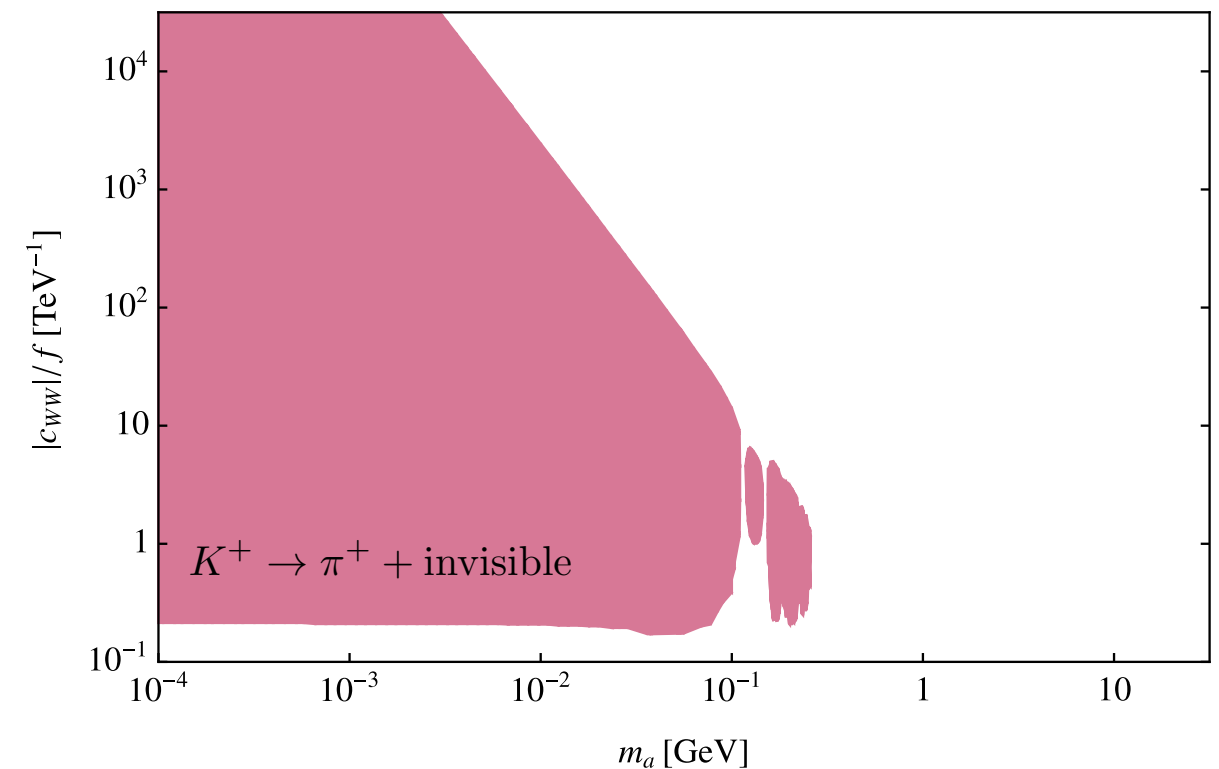
Current limits from

$\text{Br}(K^+ \rightarrow \pi^+ X)$	$0 < m_{\nu\nu} < 261$ (*)	(search)
$\text{Br}(K^+ \rightarrow \pi^+ X)$	$110 < m_X < 155$	(search)
$\text{Br}(K_L \rightarrow \pi^0 \bar{\nu}\nu)$	$0 < m_{\nu\nu} < 261$	$< 3.0 \times 10^{-9}$

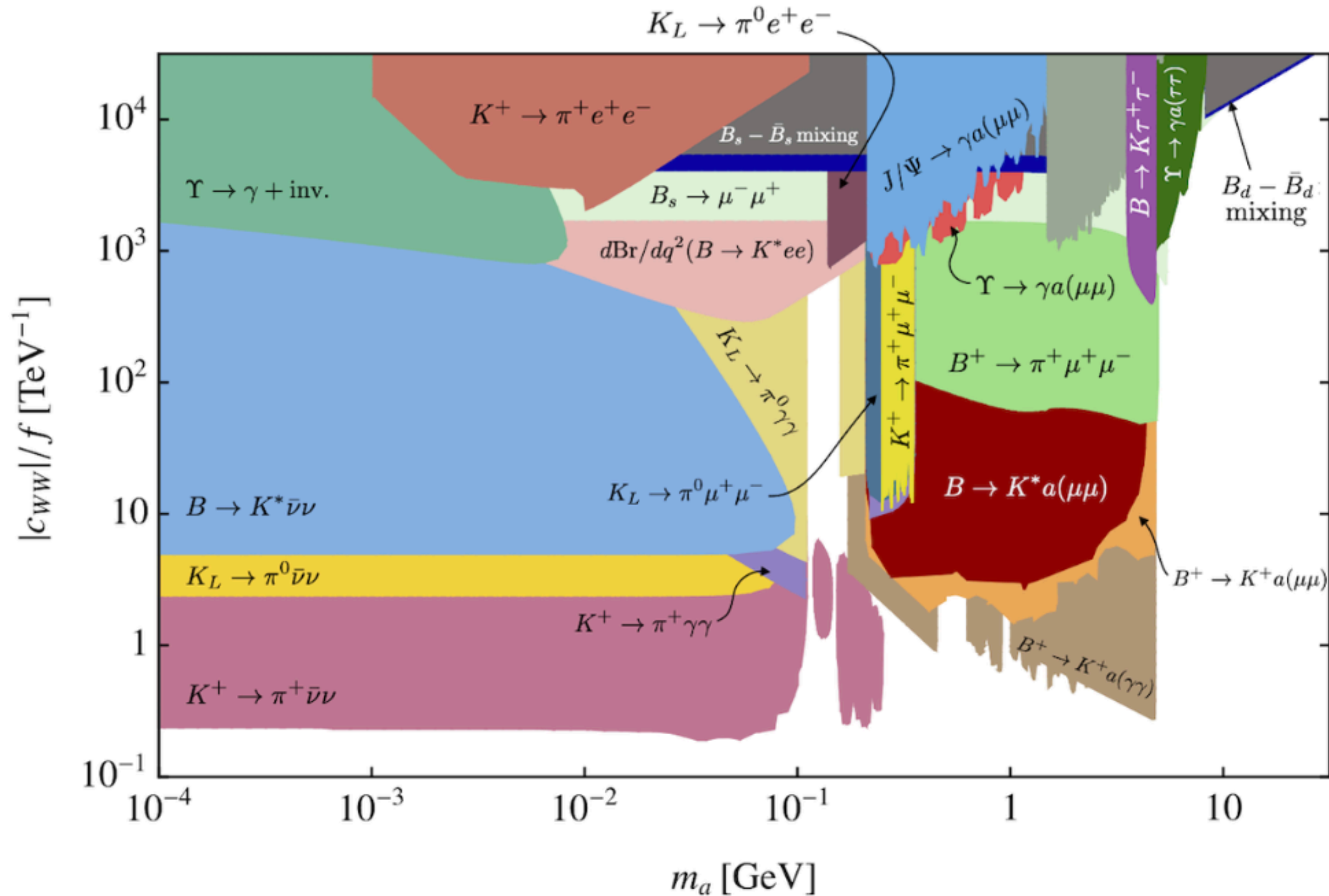
NA62, 2103.15389
NA62, JHEP **02** (2021)
KOTO, PRL **122** (2019)

$\text{Br}(K^+ \rightarrow \pi^+ \gamma\gamma)$	$m_{\gamma\gamma} < 108$	$< 8.3 \times 10^{-9}$
$\text{Br}(K^+ \rightarrow \pi^+ \gamma\gamma)$	$220 < m_{\gamma\gamma} < 354$	$(9.65 \pm 0.63) \times 10^{-7}$

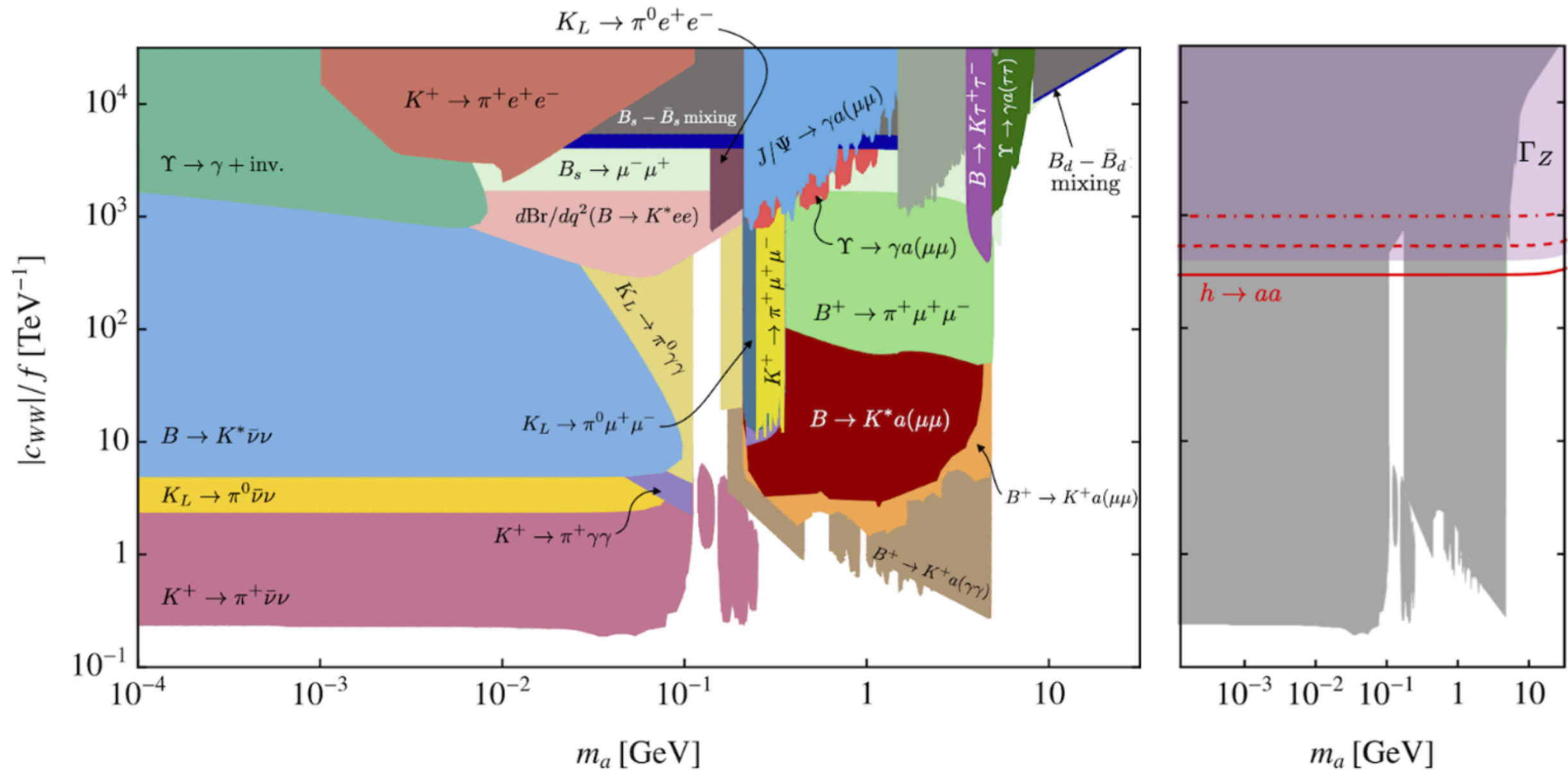
E949, Phys Lett **B623** (2005)
NA62 Phys. Lett **B536** (2014)



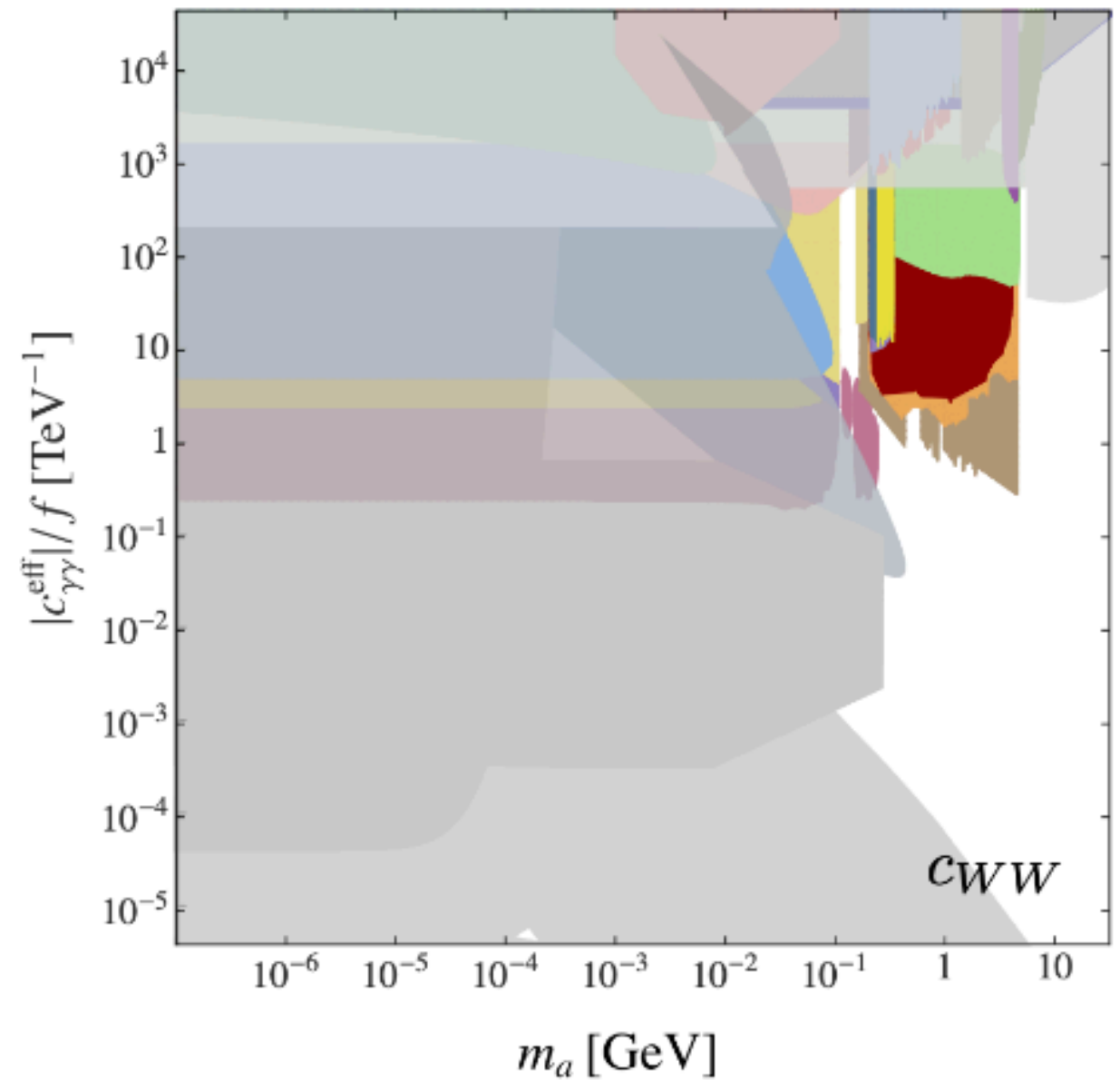
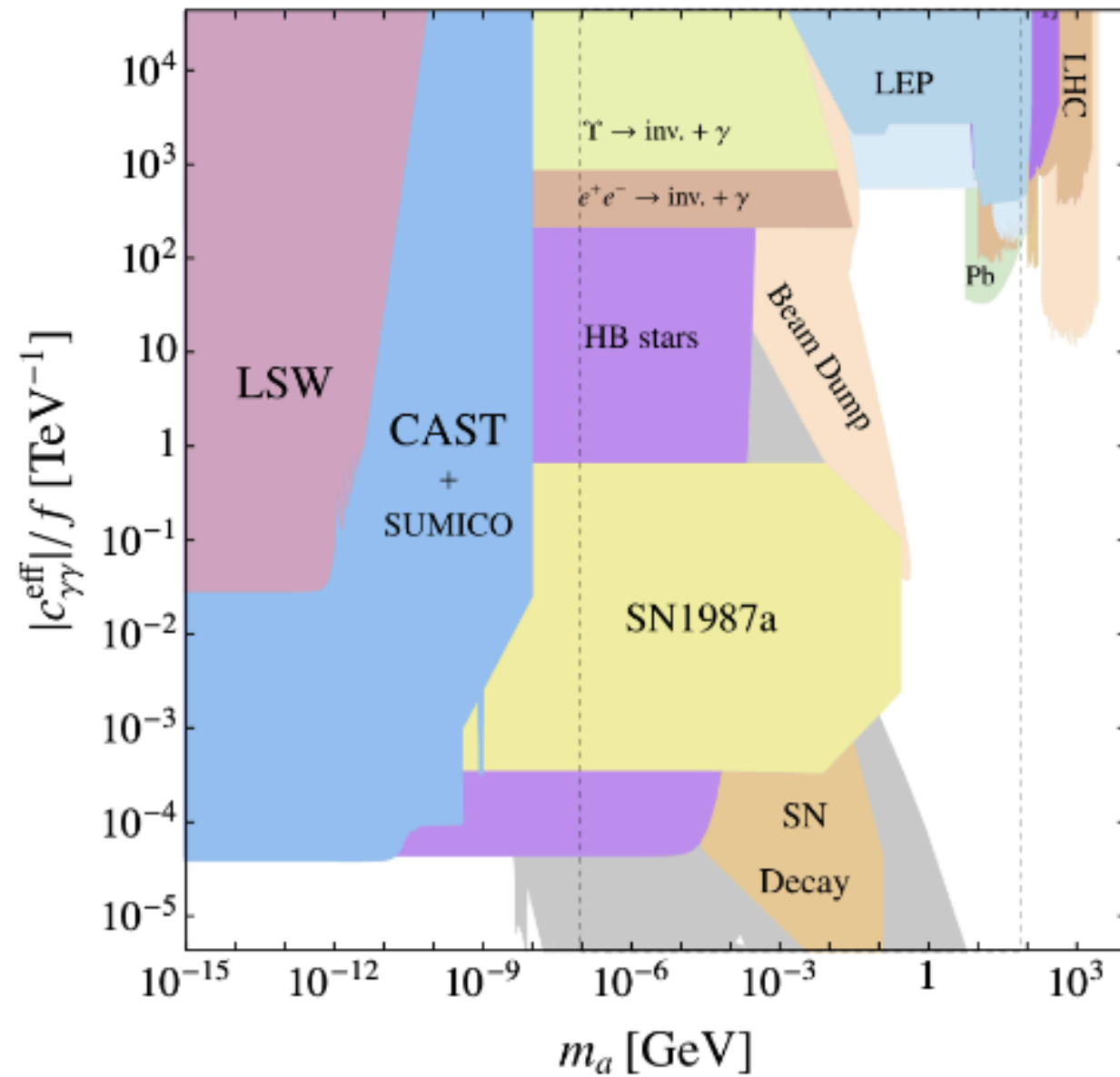
Flavor bounds on ALPs



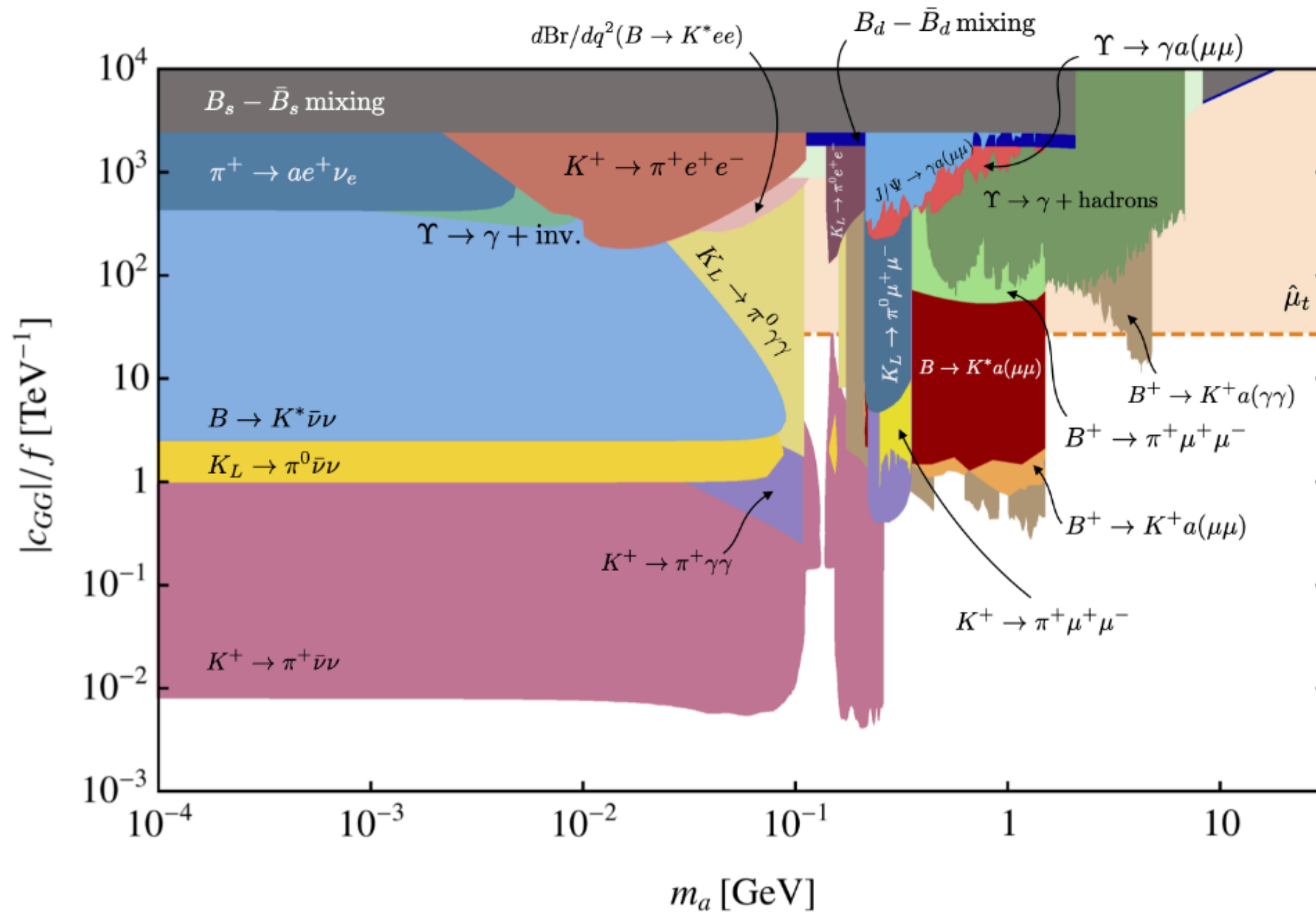
Flavor bounds on ALPs



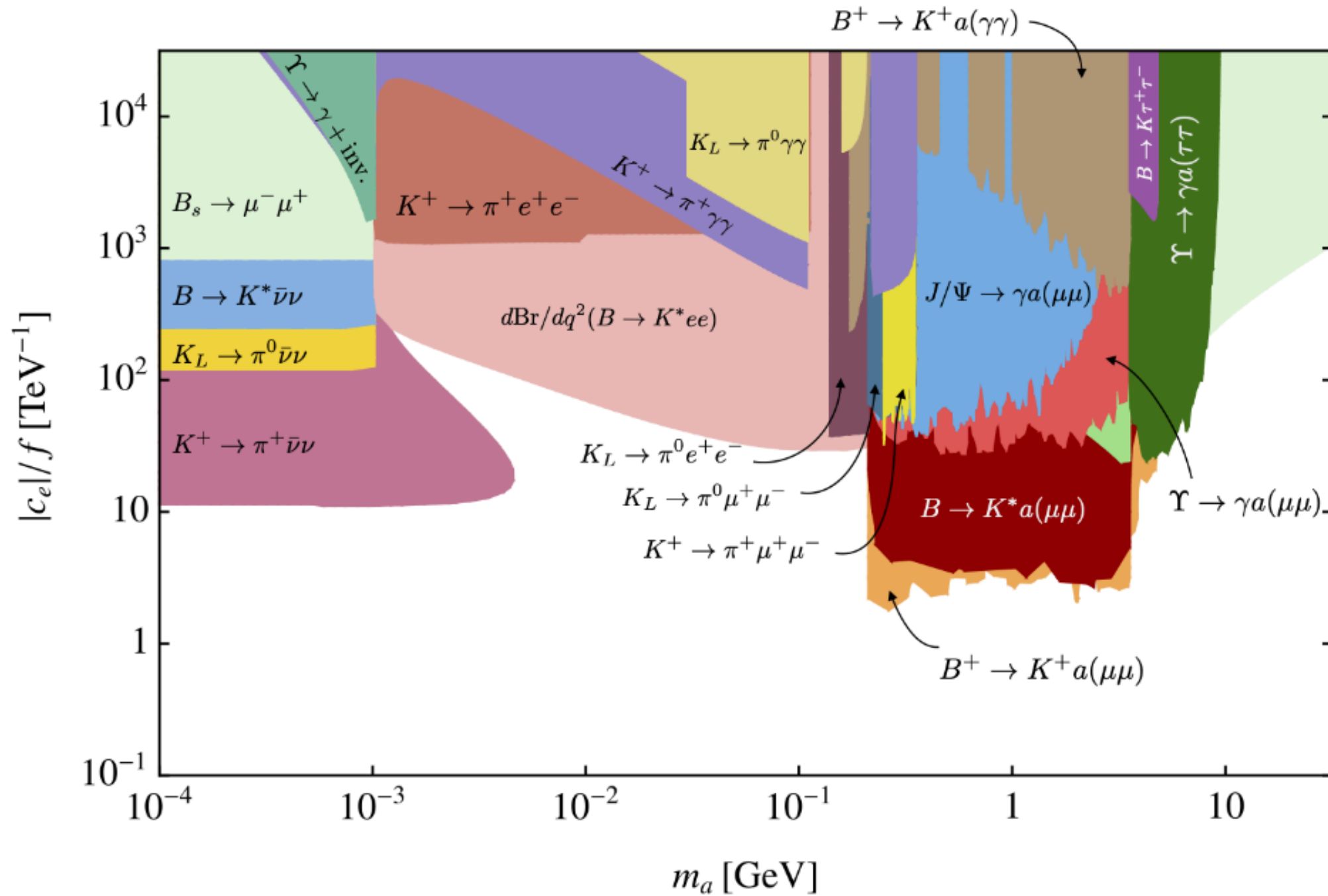
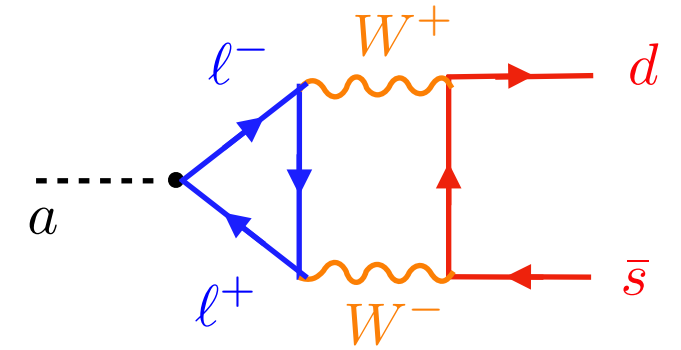
Flavor bounds vs other bounds



Flavor bounds vs other bounds



Flavor bounds vs other bounds



Conclusions

An axion could be the only light remnant of a heavy new physics sector out of reach of the LHC

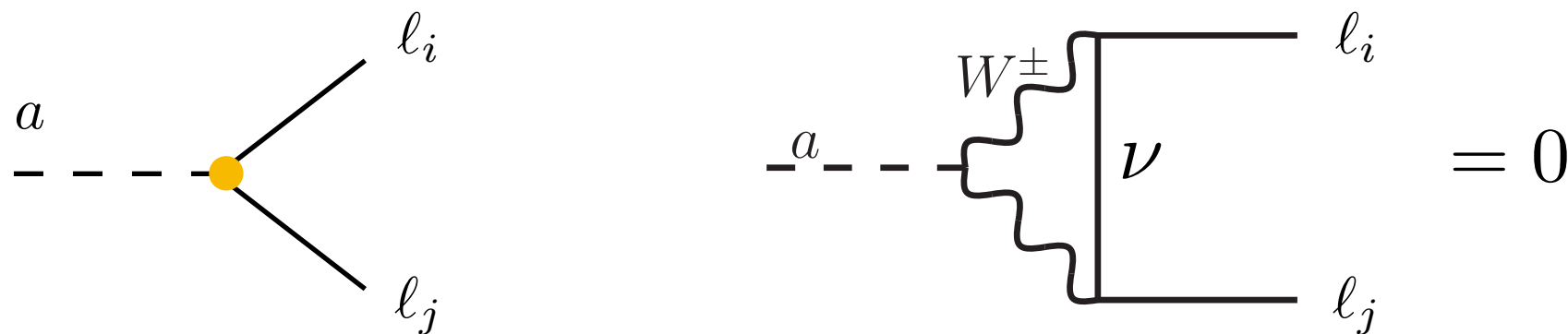
Flavor bounds uniquely constrain axionlike particles with masses between 100 MeV and 10 GeV

In the coming years searches for light new physics will probe a large range of parameter space where we've never looked

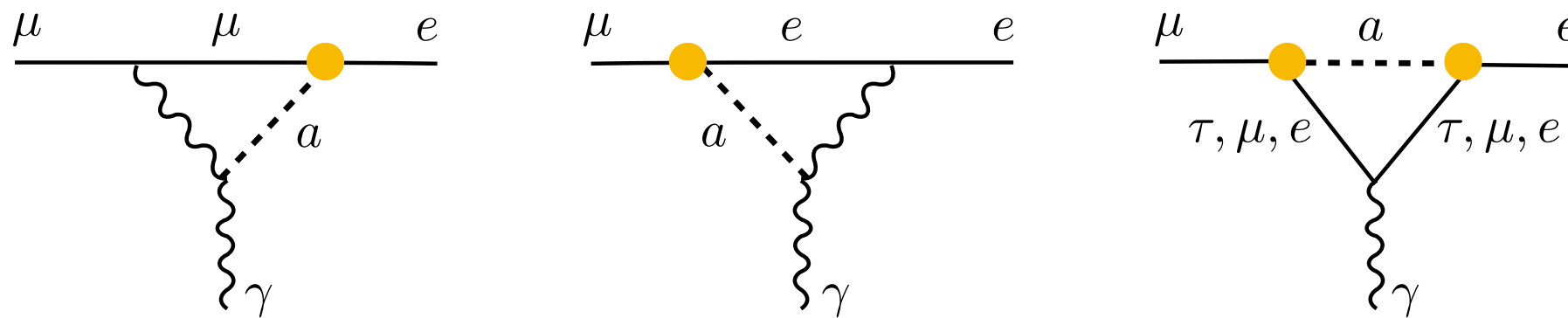
Backup

Flavour-violating couplings to leptons

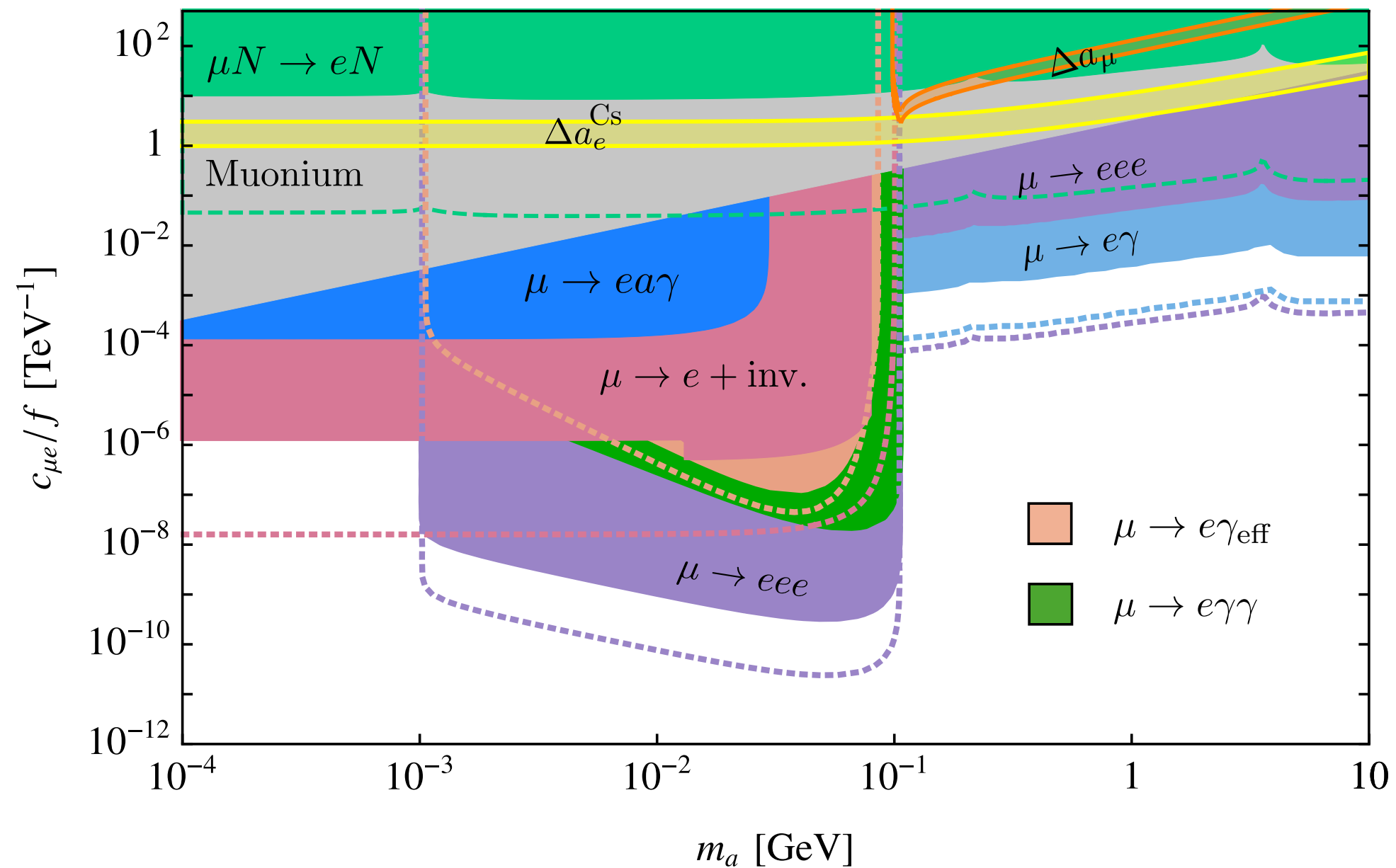
Without tree-level flavour violating couplings to leptons there are no loop-induced LFV ALP couplings, because the SM conserves lepton flavour



If they are present they induce dipole moments



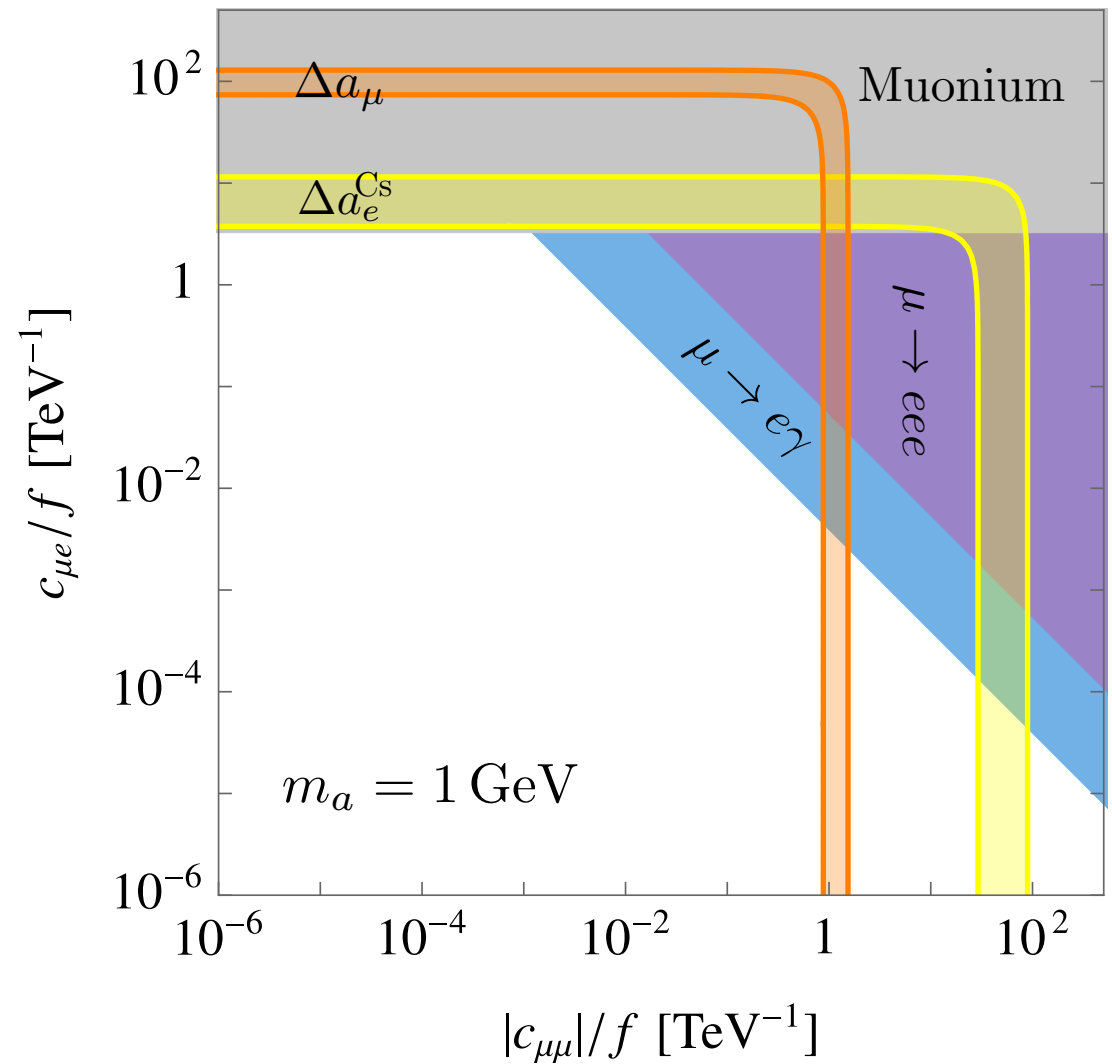
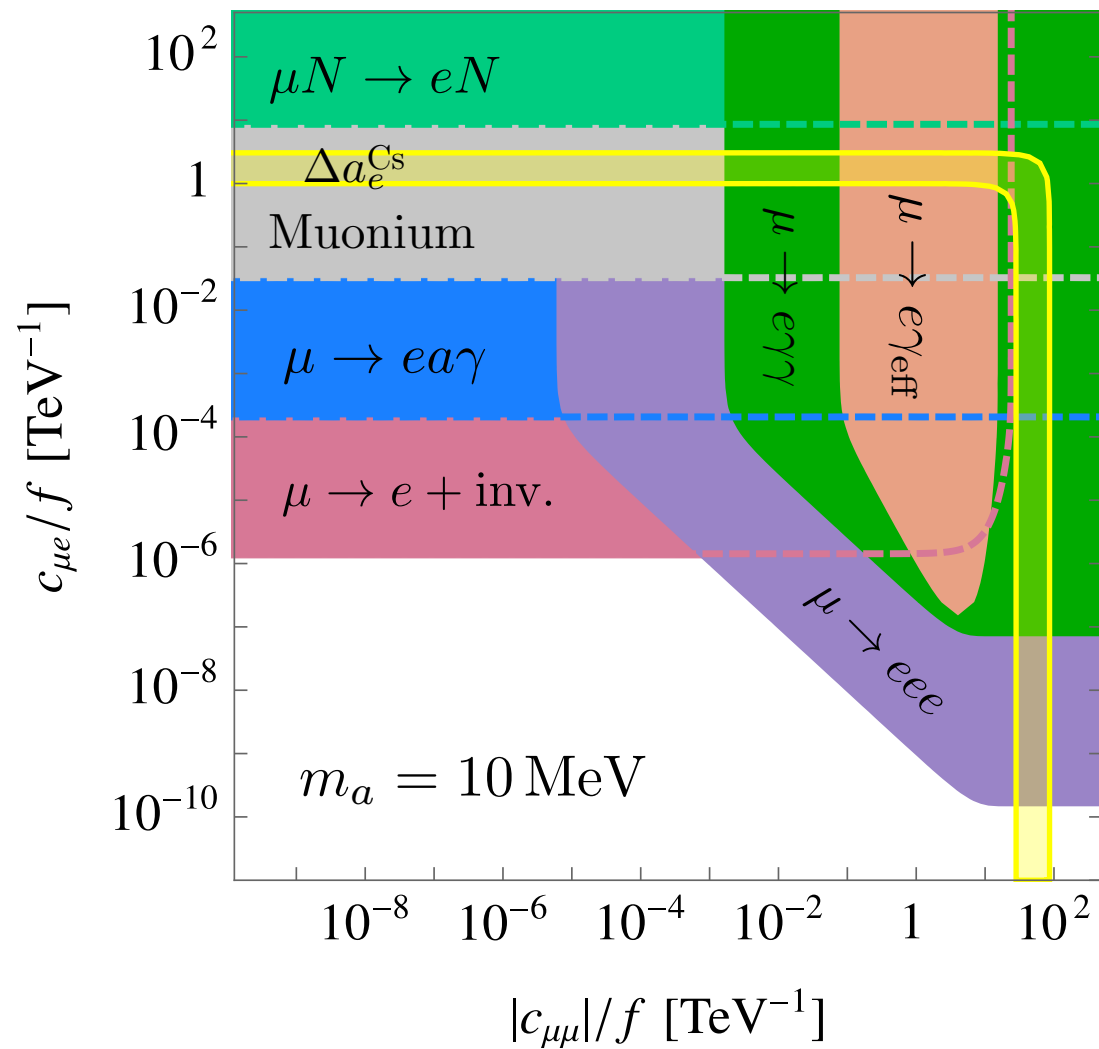
Bounds from mu-e couplings



$$c_{\mu e} = \sqrt{|(k_e)_{\mu\mu}|^2 + |(k_E)_{\mu\mu}|^2}$$

MB, Neubert, Renner,
Schnubel, Thamm, 21....

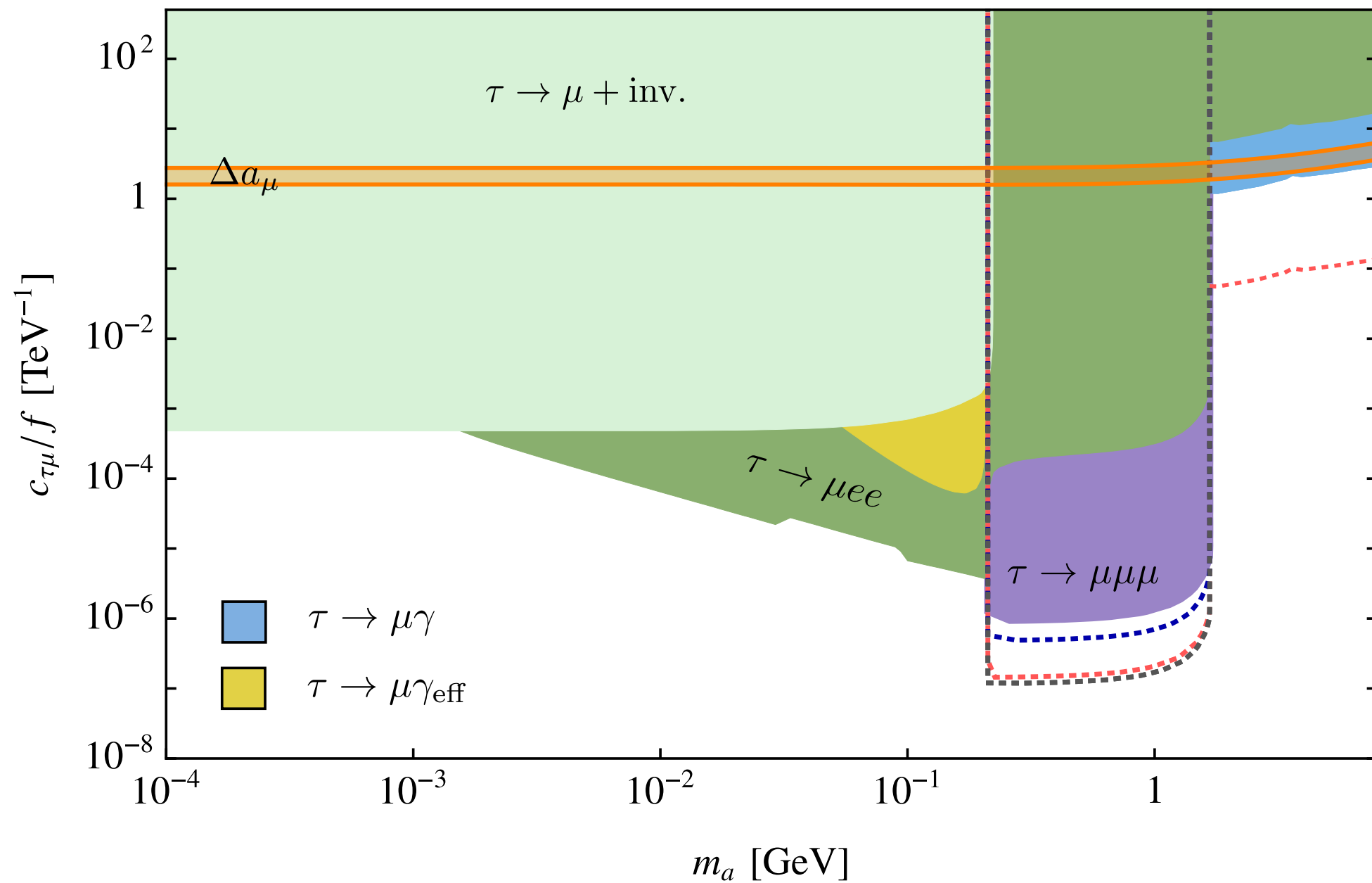
Bounds from mu-e couplings



$$c_{\mu e} = \sqrt{|(k_e)_{\mu\mu}|^2 + |(k_E)_{\mu\mu}|^2}$$

MB, Neubert, Renner,
Schnubel, Thamm, 21....

Bounds from tau-mu couplings



Bounds from tau-mu couplings

