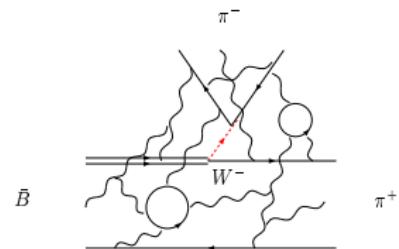
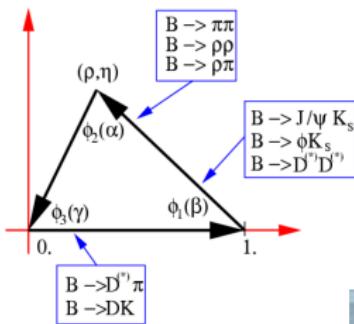


Charmless exclusive B decays

M. Beneke (TU München)

MITP Symposium “Pushing the Limits of Theoretical Physics”

Mainz, May 08 - 12, 2023



30 years of common scientific interests

- Renormalons (1994/95)
- Heavy-quark physics (1994–)
- Exclusive B decays and QCD factorization (1998–2003, 2009)

MB, G. Buchalla, M. Neubert and C.T. Sachrajda,

QCD factorization for $B \rightarrow \pi\pi$ decays: Strong phases and CP violation in the heavy quark limit,
Phys.Rev.Lett.83:1914-1917,1999 [hep-ph/9905312]

QCD factorization for exclusive, nonleptonic B meson decays: General arguments and the case of heavy light final states,
Nucl.Phys.B591:313-418,2000 [hep-ph/0006124]

QCD factorization in $B \rightarrow \pi K$, $\pi\pi$ decays and extraction of Wolfenstein parameters, Nucl.Phys.B606:245-321,2001
[hep-ph/0104110]

Comment on $B \rightarrow M_1 M_2$: Factorization, charming penguins, strong phases, and polarization, Phys.Rev.D72:098501,2005
[hep-ph/0411171]

Penguins with Charm and Quark-Hadron Duality, Eur.Phys.J.C61:439-449,2009 (arXiv:0902.4446 [hep-ph])

MB, M. Neubert,

Flavor singlet B decay amplitudes in QCD factorization, Nucl.Phys.B651:225-248,2003 [hep-ph/0210085]

QCD factorization for $B \rightarrow PP$ and $B \rightarrow PV$ decays, Nucl.Phys.B675:333-415 (2003) [hep-ph/0308039]

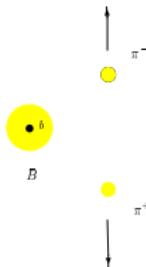
- Soft-collinear EFT (2002–)
- Next-to-leading power factorization (2017–)

Exclusive hadronic charmless (quasi)-two-body B decays

$B \rightarrow \pi^+ \pi^-, K\pi, \phi K, \dots$

Simple kinematics, complicated dynamics,
interesting physics

- CP violation
- FCNC dominated (some)
- Test of SM flavour sector

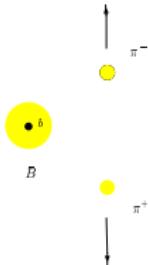


Exclusive hadronic charmless (quasi)-two-body B decays

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Simple kinematics, complicated dynamics,
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Status mid/end of 1990s

- CP violation only observed in kaon system
- **CLEO experiment:**
1995: 1st observation of FCNC B decay to $K^* \gamma$
Observation of charmless decays
 $K^+ \pi^-$ (1997), $\pi^+ \pi^-$ (2000)
No CP asymmetries

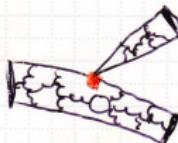
Theory

- **HQET**
- Spin-flavour symmetry of soft physics
- (Diagrammatic factorization proofs for DY, DIS)

The most popular assumption is naive factorization
[Bauer, Stech, Wirbel ...]

What does this mean?

$$\langle \pi^+ \pi^- | (\bar{u} b)_{v-A} (\bar{d} u)_{v-A} | \bar{B}_d \rangle \approx$$
$$= i f_\pi f_{(m_B^2)}^{B \rightarrow \pi} \cdot M_B^2$$



No strong phases, no CPV

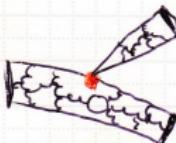
What about "non-factorizable" terms?



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No strong phases, no CPV

What about "non-factorizable"



Charming penguins in B decays

M. Ciuchini^{a,1}, E. Franco^b, G. Martinelli^b, L. Silvestrini^c

^a Theory Division, CERN, 1211 Geneva 23, Switzerland

^b Dip. di Fisica, Univ. "La Sapienza" and INFN, Sezione di Roma, P.le A. Moro, I-00185 Rome, Italy

^c Dip. di Fisica, Univ. di Roma "Tor Vergata" and INFN, Sezione di Roma II,
Via della Ricerca Scientifica 1, I-00133 Rome, Italy

Received 18 April 1997; accepted 10 June 1997

Abstract

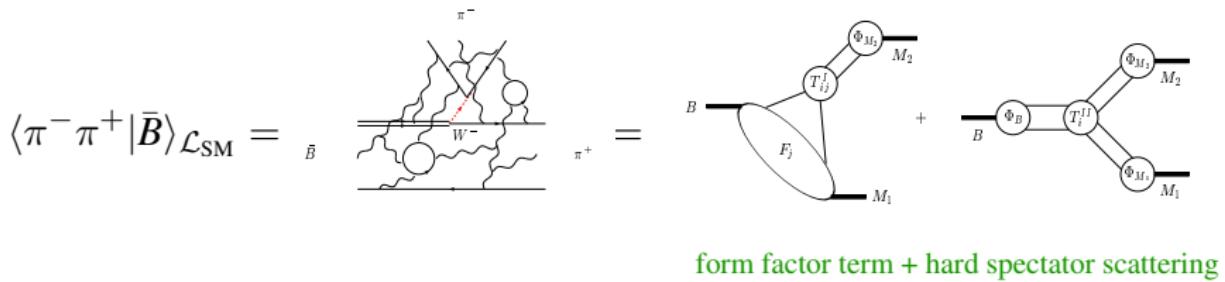
Full expressions of the $B_d^0 \rightarrow \pi^+ \pi^-$ and $B_d^0 \rightarrow \pi^0 \pi^0$ amplitudes, given in terms of matrix elements of operators of the effective weak Hamiltonian, are used to study the dependence of

leading logarithmic corrections [18–23].⁵ The state of the art in the calculation of the matrix elements of the operators is such that, given the complexity of the expressions in Eqs. (8)–(11), this turns out to be impossible. For example, factorized amplitudes are RP and scale independent, being expressed in terms of physical quantities. Thus

“QCD factorization” [MB, Buchalla, Neubert, Sachrajda, 1999-2001]

First attempt to apply collinear factorization methods from high-energy scattering to B decays, where soft physics does not cancel

- “soft-collinear” factorization
- B -meson light-cone distribution amplitude

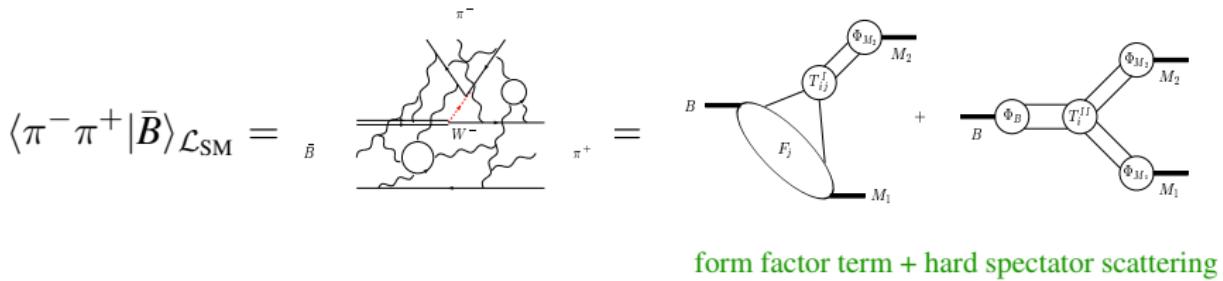


form factor term + hard spectator scattering

“QCD factorization” [MB, Buchalla, Neubert, Sachrajda, 1999-2001]

First attempt to apply collinear factorization methods from high-energy scattering to B decays, where soft physics does not cancel

- “soft-collinear” factorization
- B -meson light-cone distribution amplitude



$$\begin{aligned} \langle M_1 M_2 | C_i O_i | \bar{B} \rangle_{\mathcal{L}_{\text{eff}}} &= \sum_{\text{terms}} \color{red} C(\mu_h) \times \left\{ F_{B \rightarrow M_1} \times \underbrace{T^I(\mu_h, \mu_s)}_{1+\alpha_s+\dots} \star f_{M_2} \Phi_{M_2}(\mu_s) \right. \\ &\quad \left. + f_B \Phi_B(\mu_s) \star \left[\underbrace{T^{II}(\mu_h, \mu_I)}_{1+\dots} \star \underbrace{J^{II}(\mu_I, \mu_s)}_{\alpha_s+\dots} \right] \star f_{M_1} \Phi_{M_1}(\mu_s) \star f_{M_2} \Phi_{M_2}(\mu_s) \right\} + \mathcal{O}(1/m_b) \end{aligned}$$

Problem with multiple scales: $M_W, m_b, \sqrt{m_b \Lambda_{\text{QCD}}}, \Lambda_{\text{QCD}}$

Qualitative predictions

- Previous ad hoc factorization is the infinite heavy-quark mass limit
- Direct CP asymmetries are generically small, since strong rescattering phases are $\delta \sim \mathcal{O}(\alpha_s(m_b), \Lambda/m_b)$.

$$A_{\text{CP}}(M_1 M_2) = \underbrace{a_1 \alpha_s}_{\text{calculable!}} + \dots + \mathcal{O}(\Lambda_{\text{QCD}}/m_b)$$

- Hierarchy of branching fractions between PP, VP, PV, unless the leading perturbative term is suppressed (e.g. colour-suppressed).
- $\gamma \approx (70 \pm 5)^\circ$ from time-dependent CP asymmetries

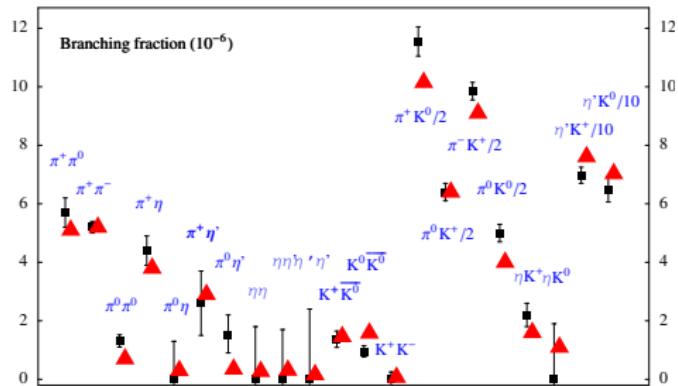
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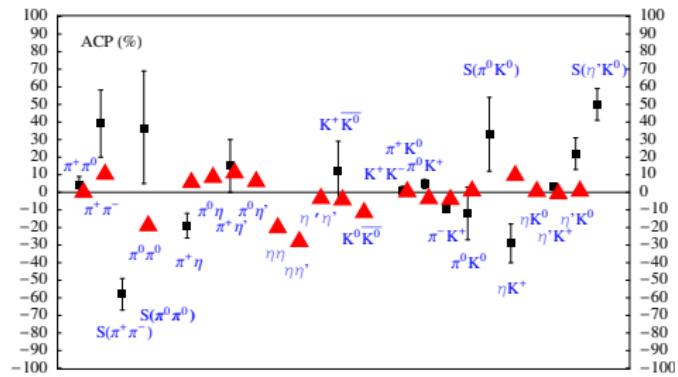
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E-mail Matthias, 18 May 1999:
“Bj doesn't believe a word of what we are saying.”



Red triangles: Theory (S4) from
MB, M. Neubert, Nucl. Phys. B675
(2003) 333



The closer view

- 2001: First measured direct CP asymmetries ($\pi^+ K^-$, then $\pi^+ \pi^-$ in 2002) small, but opposite in sign to lowest-order perturbative prediction.

$$A_{\text{CP}} = [c \times \alpha_s]_{\text{NLO}} + \mathcal{O}(\alpha_s^2, \Lambda/m_b)$$

- 2003: Colour-suppressed mode $\pi^0 \pi^0$ larger than prediction

$$C \propto a_2(\pi\pi) = 0.220 - [0.179 + 0.077i]_{\text{NLO}} + \left[\frac{r_{\text{sp}}}{0.485} \right] \left\{ [0.123]_{\text{LoSp}} + [0.072]_{\text{tw3}} \right\}$$

$$r_{\text{sp}} = \frac{9f_{M1}\hat{f}_B}{m_b f_+^{B\pi}(0)\lambda_B}$$

Still await for confirmation (soon?) from BELLE II

The closer view

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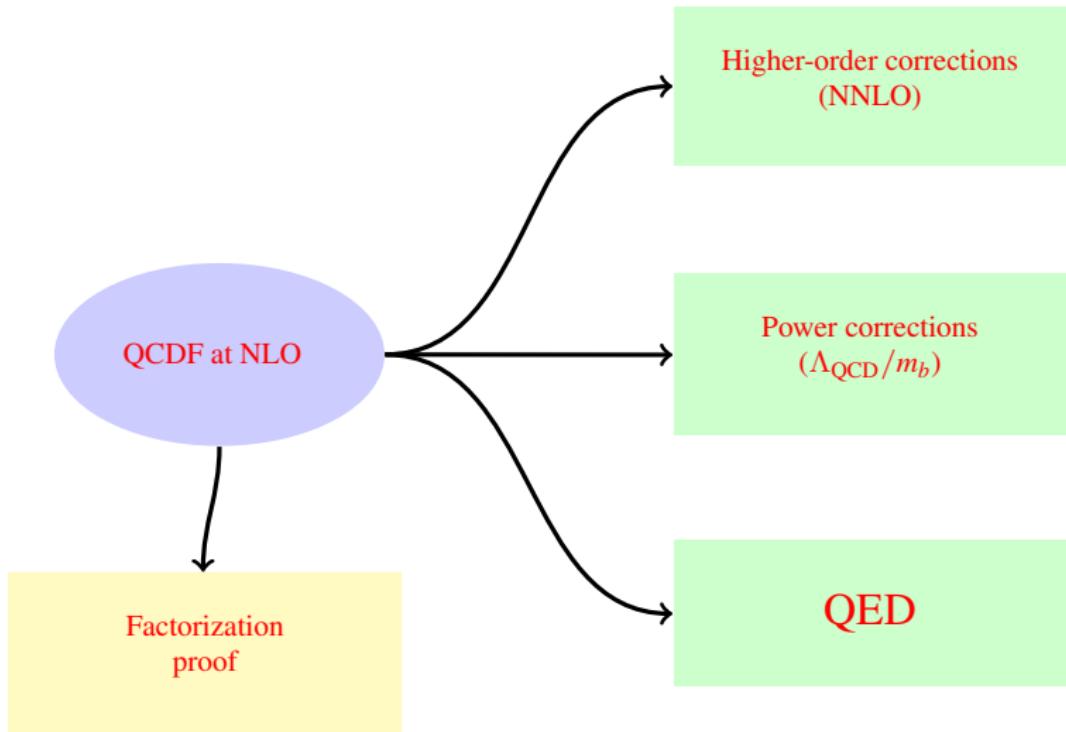
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Still await for confirmation (soon?) from BELLE II

Perturbative explanation or power correction?

Further developments



NNLO QCD factorization calculations

MB, Jäger; Kivel; Pilipp (tree); MB, Jäger; Jain, Rothstein Stewart (penguin) [2005-2007, 1-loop spectator scattering];
Bell, MB, Huber, Li [2007-2009, 2-loop tree; 2015-2020 2-loop penguin]

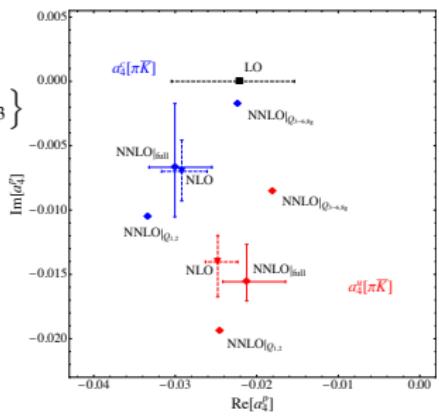
Colour-suppressed tree

$$\begin{aligned} C \equiv a_2(\pi\pi) &= 0.220 - [0.179 + 0.077i]_{\text{NLO}} - [0.031 + 0.050i]_{\text{NNLO}} \\ &\quad + \left[\frac{r_{\text{sp}}}{0.485} \right] \left\{ [0.123]_{\text{LO}_{\text{Sp}}} + [0.053 + 0.054i]_{\text{NLO}_{\text{Sp}}} + [0.072]_{\text{tw3}} \right\} \\ &= 0.26 - 0.07i \quad \rightarrow \quad 0.51 - 0.02i \quad (\text{if } 2 \times r_{\text{sp}}) \end{aligned}$$

Charming penguin

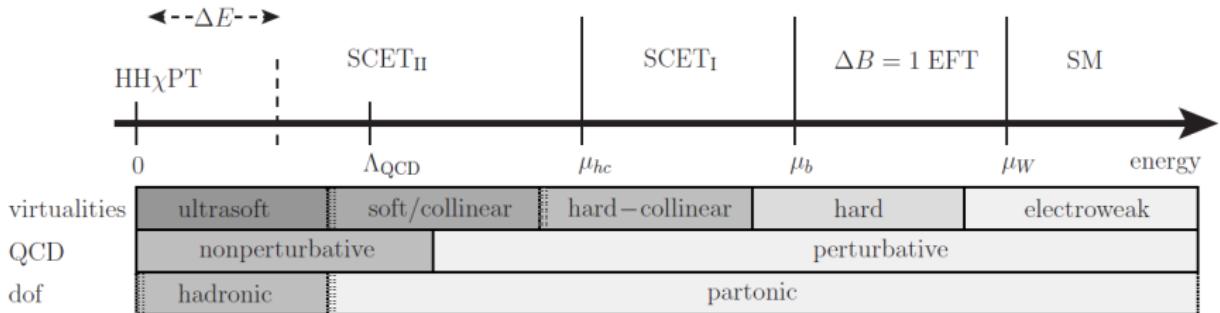
$$\begin{aligned} a_4^c(\pi\bar{K})/10^{-2} &= -2.87 - [0.09 + 0.09i]_{V_1} + [0.05 - 0.62i]_{P_1} \\ &\quad - [0.77 + 0.50i]_{P_2, Q_{1,2}} + [0.33 + 0.38i]_{P_2, Q_{3-6,8g}} \\ &\quad + \left[\frac{r_{\text{sp}}}{0.434} \right] \left\{ [0.13]_{\text{LO}} + [0.14 + 0.12i]_{\text{HV}} + [0.01 + 0.03i]_{\text{HP}} + [0.07]_{\text{tw3}} \right\} \\ &= (-3.00^{+0.45}_{-0.32}) + (-0.67^{+0.50}_{-0.39})i \end{aligned}$$

- NLO direct CP asymmetries completed only in 2020
- No anomalously large corrections.



Including QED: Scales and EFTs

Multiple scales: $m_W, m_b, \sqrt{m_b \Lambda_{\text{QCD}}}, \Lambda_{\text{QCD}}, \Delta E$

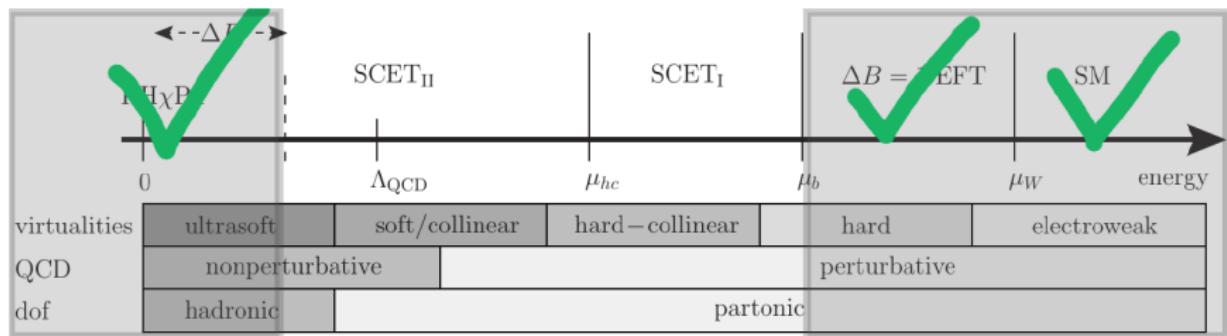


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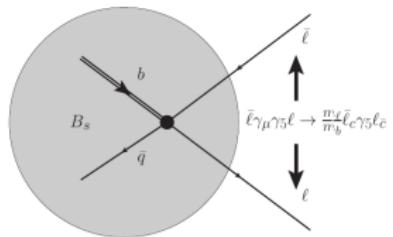
Short-distance QED at $\mu \gtrsim m_b$ can be included in the usual weak effective Lagrangian (extended Fermi theory) + renormalization group.

Far IR (ultrasoft scale) described by theory of point-like hadrons.



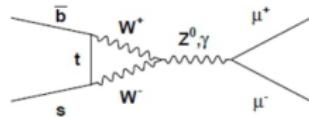
Goal: Theory for QED corrections between the scales m_b and Λ_{QCD} (structure-dependent effects).

Example: $B_s \rightarrow \mu^+ \mu^-$ [MB, Bobeth, Szafron, 2017, 2019]

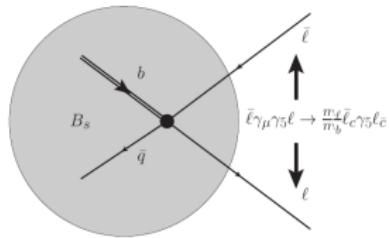


$$\langle 0 | \bar{q} \gamma^\mu \gamma_5 b | \bar{B}_q(p) \rangle$$

Local annihilation and helicity flip.

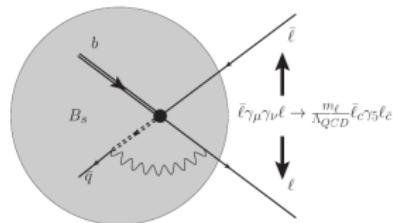


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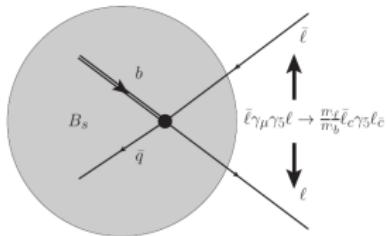
Local annihilation and helicity flip.



$$\langle 0 | \int d^4x T\{j_{\text{QED}}(x), \mathcal{L}_{\Delta B=1}(0)\} | \bar{B}_q \rangle$$

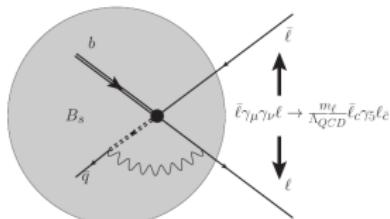
Helicity-flip and annihilation delocalized
by a hard-collinear distance

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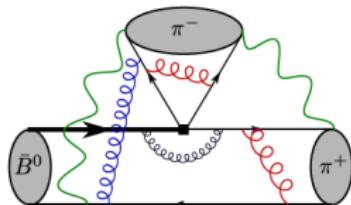
$$\langle 0 | \int d^4x T\{j_{\text{QED}}(x), \mathcal{L}_{\Delta B=1}(0)\} | \bar{B}_q \rangle$$

Helicity-flip and annihilation delocalized by a hard-collinear distance

- The virtual photon probes the B meson structure. Annihilation/helicity-suppression is “smeared out” over light-like distance $1/\sqrt{m_B \Lambda}$. Still short-distance [\rightarrow B-meson LCDA].
- Structure-dependent effects is a m_B/Λ power-enhanced and (double) logarithmically enhanced, purely virtual correction. Not the standard soft logarithms.

Including virtual QED effects into the factorization theorem

[MB, Böer, Toelstede, Vos, 2020-2022; MB, Böer, Finauri, Vos, 2021]



SCET_I operators

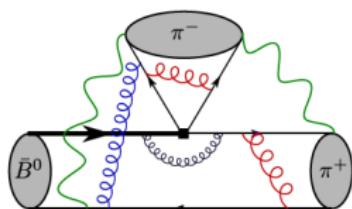
$$\mathcal{O}^I(t) = [\bar{\chi}_{\bar{C}}(tn_-) \not{p}_- \gamma_5 \chi_{\bar{C}}] [\bar{\chi}_c \mathbf{S}_{n+}^{\dagger(\mathcal{Q}_{M_2})} h_v]$$

$$\mathcal{O}^{II}(t, s) = [\bar{\chi}_{\bar{C}}(tn_-) \not{p}_- \gamma_5 \chi_{\bar{C}}] [\bar{\chi}_c \mathcal{A}_{C,\perp}(sn_+) \mathbf{S}_{n+}^{\dagger(\mathcal{Q}_{M_2})} h_v]$$

$$S_{n\pm}^{(q)} = \exp \left\{ -i Q_q e \int_0^\infty ds n_\pm A_s(sn_\pm) \right\}$$

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QCD + QED factorization formula

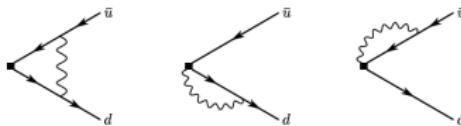
$$\begin{aligned} \langle M_1 M_2 | Q_i | \bar{B} \rangle_{\text{non-rad.}} &= \mathcal{F}_{Q_2}^{BM_1}(0) \int_0^1 du \underbrace{T_{i,Q_2}^{\text{I,QCD+QED}}(u)}_{\mathcal{O}(\alpha_{\text{em}}) \text{ corrected SD}} f_{M_2} \Phi_{M_2}(u) \\ &+ \int_{-\infty}^\infty d\omega \int_0^1 du dv T_{i,\otimes}^{\text{II,QCD+QED}}(z, u) f_B \Phi_{B,\otimes}(\omega) f_{M_1} \Phi_{M_1}(v) f_{M_2} \Phi_{M_2}(u) \end{aligned}$$

Formula retains its form, but the hadronic matrix elements are generalized. They become process-dependent through the directions and charges of the *other* particles.

LCDA of the charged pion in QCD

QCD definition

$$\langle \pi^-(p) | \bar{d}(tn_+) \frac{\not{t}_+}{2} \gamma_5 [tn_+, 0] u(0) | 0 \rangle = -\frac{in+p}{2} \int_0^1 du e^{iu(n+p)t} f_M \phi_M(u; \mu)$$



ERBL kernel [Efremov, Radyushkin, Brodsky, Lepage, 1979]

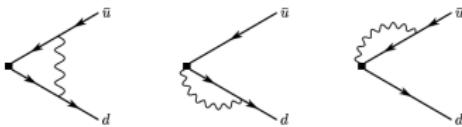
$$\Gamma(u, v; \mu) = - \left(\frac{\alpha_s C_F}{\pi} \right) \left[\left(1 + \frac{1}{v-u} \right) \frac{u}{v} \theta(v-u) + \left(1 + \frac{1}{u-v} \right) \frac{1-u}{1-v} \theta(u-v) \right]_+$$

- LCDA symmetric in $u \leftrightarrow 1 - u$
- One-loop kernel diagonalized by Gegenbauer polynomials, asymptotic behaviour $\Phi_\pi(u, \mu) \xrightarrow{\mu \rightarrow \infty} 6u(1-u)$.

LCDA of the charged pion in QCD×QED [2108.05589, MB, Böer, Toelstede, Vos]

QCD×QED

$$\langle \pi^-(p) | R_c^{(Q_M)} (\bar{d} W^{(d)}) (t n_+) \frac{\not{n}_+}{2} \gamma_5 [t n_+, 0] (W^{\dagger(u)} u)(0) | 0 \rangle = -\frac{i n_+ p}{2} \int_0^1 du e^{i u (n_+ p)_t} f_M \Phi_M(u; \mu)$$



QCD×QED kernel

$$\begin{aligned} \Gamma(u, v; \mu) = & -\frac{\alpha_{\text{em}} Q_M}{\pi} \delta(u - v) \left(Q_M \left(\ln \frac{\mu}{2E} + \frac{3}{4} \right) - Q_d \ln u + Q_u \ln \bar{u} \right) \\ & - \left(\frac{\alpha_s C_F}{\pi} + \frac{\alpha_{\text{em}}}{\pi} Q_u Q_d \right) \left[\left(1 + \frac{1}{v-u} \right) \frac{u}{v} \theta(v-u) + \left(1 + \frac{1}{u-v} \right) \frac{1-u}{1-v} \theta(u-v) \right]_+ \end{aligned}$$

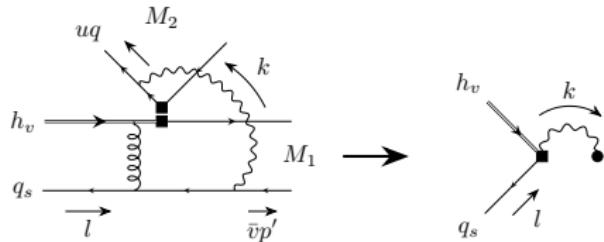
- Cusp anomalous dimension, logarithms $\ln u$, $\ln(1 - u)$ and energy dependence are a remnant of the soft physics and breaking of boost invariance.
- Gegenbauer polynomials are no longer eigenfunctions, asymptotic behaviour $\Phi_\pi(u, \mu) \xrightarrow{\mu \rightarrow \infty} 6u(1 - u)$ no longer holds. QED evolution is asymmetric and endpoint behaviour changes from linear.

B-LCDA alias soft function for $B \rightarrow M_1 M_2$ in QCD \times QED

[2204.09091, MB, Börer, Toelstede, Vos]

$$\begin{aligned} & \frac{1}{R_c^{(Q_{M_1})} R_{\bar{c}}^{(Q_{M_2})}} \langle 0 | \bar{q}_s^{(q)}(m_-) [m_-, 0]^{(q)} \not{p}_- \gamma_5 h_v(0) S_{n+}^{\dagger(Q_{M_2})}(0) S_{n-}^{\dagger(Q_{M_1})}(0) | \bar{B}_v \rangle \\ &= i F_{\text{stat}}(\mu) \int_{-\infty}^{\infty} d\omega e^{-i\omega t} \Phi_{B,\otimes}(\omega, \mu) \end{aligned}$$

- Four different soft functions depending on final state charges $00, -0, 0-, +-$.



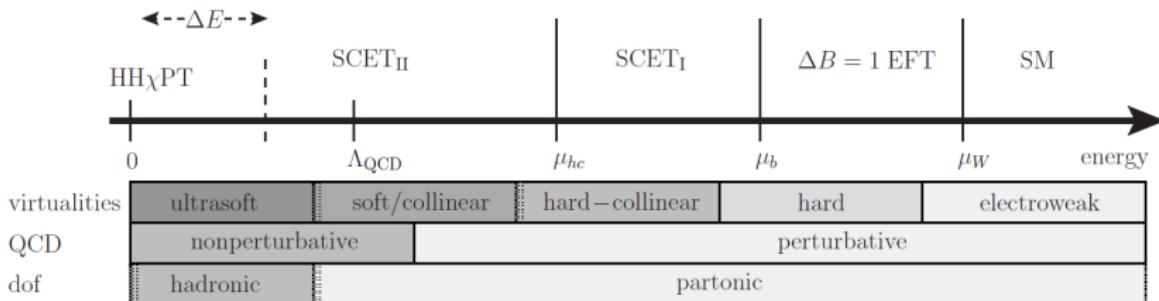
- Not really LCDAs, but soft functions: contain electromagnetic final-state rescattering. Complex-valued.
- For $Q_{M_2} \neq 0$, support in $\omega \in]-\infty, \infty[$ (different from QCD where $\omega > 0$), because two light-like directions are involved.

Photon radiation and the non-radiative amplitude as “matching coefficient”

The LCDAs are *non-perturbative* SCET_{II} matrix elements, but *infrared-divergent* w.r.t. to QED corrections.

Real photon radiation with $E_{\max} \ll \Lambda_{\text{QCD}}$ can be described in an EFT where the degrees of freedom are point-like mesons coupling with multipole couplings to ultrasoft photons (**“boosted heavy-meson/lepton theory”**).

The minimally IR-subtracted virtual non-radiative amplitude SCET_{II} amplitude is the non-perturbative matching coefficient.



Isospin-protected ratios / sum rules for the πK final states

Consider ratios / sums where some QCD uncertainties drop out.

[MB, Neubert, 2003]

$$R_L = \frac{2\text{Br}(\pi^0 K^0) + 2\text{Br}(\pi^0 K^-)}{\text{Br}(\pi^- K^0) + \text{Br}(\pi^+ K^-)} = R_L^{\text{QCD}} + \cos \gamma \text{Re } \delta_E + \delta_U$$

$$R_L^{\text{QCD}} - 1 \approx (1 \pm 2)\% \quad \delta_E \approx 0.1\% \quad \delta_U = 5.8\%$$

QED correction larger than QCD and QCD uncertainty, but short-distance QED negligible.

[Gronau, Rosner, 2006]

$$\begin{aligned} \Delta(\pi K) &\equiv A_{\text{CP}}(\pi^+ K^-) + \frac{\Gamma(\pi^- \bar{K}^0)}{\Gamma(\pi^+ K^-)} A_{\text{CP}}(\pi^- \bar{K}^0) - \frac{2\Gamma(\pi^0 K^-)}{\Gamma(\pi^+ K^-)} A_{\text{CP}}(\pi^0 K^-) \\ &\quad - \frac{2\Gamma(\pi^0 \bar{K}^0)}{\Gamma(\pi^+ K^-)} A_{\text{CP}}(\pi^0 \bar{K}^0) \equiv \Delta(\pi K)^{\text{QCD}} + \delta\Delta(\pi K) \end{aligned}$$

$$\Delta(\pi K)^{\text{QCD}} = (0.5 \pm 1.1)\% \quad \delta_\Delta(\pi K) \approx -0.4\%$$

QED correction of similar size but small.

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- Heavy-quark physics (1994–)
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- Soft-collinear EFT (2002–)
- Next-to-leading power factorization (2017–)

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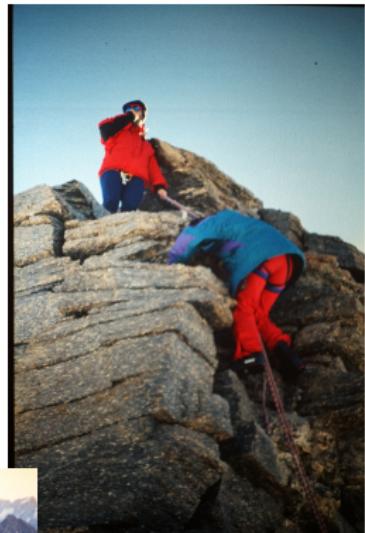
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and good friend ...



Mt Blanc du Tacul, August 1997





Gran Paradiso,
September 1997

Back-up

SCET_{II} (re-) factorization and soft rearrangement

- Leading long-range soft photon interactions between energetic charged particles can be accounted for by light-like Wilson lines:

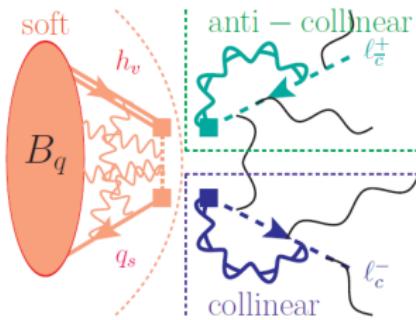
$$Y_{\pm}^{(q)} = \exp \left\{ -i Q_q e \int_0^{\infty} ds n_{\pm} A_s (sn_{\pm}) \right\}$$

- s, c, \bar{c} do not interact in SCET_{II}. Sectors are factorized.
- Anomalous dimensions should be separately well defined, but turn out to be IR divergent.

$$\tilde{\mathcal{J}}_{\mathcal{A}\chi}^{B1}(v, t) = \bar{q}_s(vn_-) Y(vn_-, 0) \frac{\not{v}_-}{2} P_L h_v(0) [Y_+^\dagger Y_-](0) [\ell_c(0)(2\mathcal{A}_{c\perp}(tn_+)P_R)\ell_{\bar{c}}(0)] = \hat{\mathcal{J}}_s \otimes \hat{\mathcal{J}}_c \otimes \hat{\mathcal{J}}_{\bar{c}}$$

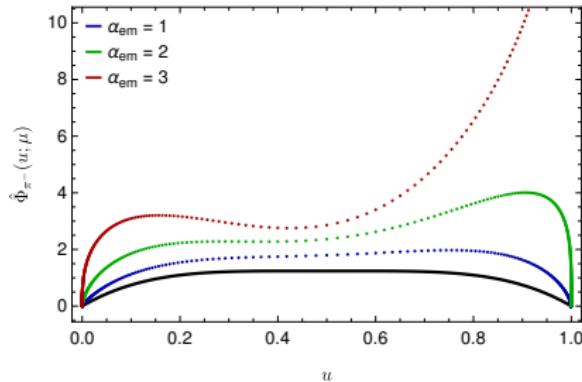
- Soft rearrangement** of the Wilson line tadpole: $\langle 0 | [Y_+^\dagger Y_-](0) | 0 \rangle \equiv R_+ R_-$

$$\hat{\mathcal{J}}_s \otimes \hat{\mathcal{J}}_c \otimes \hat{\mathcal{J}}_{\bar{c}} = \frac{\hat{\mathcal{J}}_s}{R_+ R_-} \otimes R_+ \hat{\mathcal{J}}_c \otimes R_- \hat{\mathcal{J}}_{\bar{c}}$$



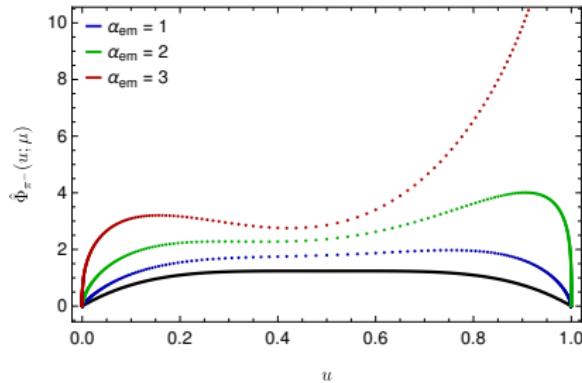
Endpoint behaviour and numerical QED effect

Large α_{em} for numerical illustration



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Example for the real world α_{em} :

$$\left\langle \bar{u}^{-1} \right\rangle_{M^-}(\mu) = \int_0^1 \frac{du}{1-u} \Phi_{M^-}(u; \mu) = 3Z_\ell(\mu) \sum_{n=0}^{\infty} a_n^{M^-}(\mu)$$

$$\left\langle \bar{u}^{-1} \right\rangle_{\pi^-}(5.3 \text{ GeV}) = 0.9997^{\text{QED}}_{\text{point charge}} (3.285^{+0.05}_{-0.05}|_{\text{LL}} - 0.020|_{\text{NLL}} + 0.017|_{\text{partonic}}^{\text{QED}})$$

$$\left\langle \bar{u}^{-1} \right\rangle_{\pi^-}(80.4 \text{ GeV}) = 0.985^{\text{QED}}_{\text{point charge}} (3.197^{+0.03}_{-0.03}|_{\text{LL}} - 0.022|_{\text{NLL}} + 0.042|_{\text{partonic}}^{\text{QED}})$$

(Initial value: 3.42 at $\mu = 1 \text{ GeV}$.

QED effects of similar size as NLL evolution for the inverse moments.

Numerical estimate of QED effects for πK final states

Non-radiative amplitude

- Electroweak scale to m_B : QED corrections to Wilson coefficients included
- m_B to μ_c : $\mathcal{O}(\alpha_{\text{em}})$ corrections to short-distance kernels included.
QED effects in form factors and LCDA not included.

Ultrasoft photon radiation

$$U(M_1 M_2) = \left(\frac{2\Delta E}{m_B} \right)^{-\frac{\alpha_{\text{em}}}{\pi}} \left(Q_B^2 + Q_{M_1}^2 \left[1 + \ln \frac{m_{M_1}^2}{m_B^2} \right] + Q_{M_2}^2 \left[1 + \ln \frac{m_{M_2}^2}{m_B^2} \right] \right) \quad (M_1, M_2 \text{ light mesons})$$

$$U(\pi^+ K^-) = 0.914$$

$$U(\pi^0 K^-) = U(K^- \pi^0) = 0.976$$

$$U(\pi^- \bar{K}^0) = 0.954 \quad [\text{for } \Delta E = 60 \text{ MeV}]$$

$$U(\bar{K}^0 \pi^0) = 1$$