Strong-CP with and without Gravity

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Gravity:

Newton $\rightarrow$ Einstein $\rightarrow$ QFT

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}$$

in quantum theory

graviton

$$h_{\mu\nu} = \frac{\langle \hat{h}_{\mu\nu} \rangle}{M_P}$$

Planck mass $\sim 10^{19}$ GeV

$\hat{h}_{\mu\nu} \rightarrow$ particle with

Spin $= 2$, $m = 0$

S-matrix is the only existing formulation of quantum gravity.

Organic (but not limited to) string theory.

This puts severe restrictions on vacuum landscape and, in particular, excludes: (see, G.D., 2012 02133 [hep-th] 2209 14219 [hep-ph])

⊛ de Sitter, (meta)stable

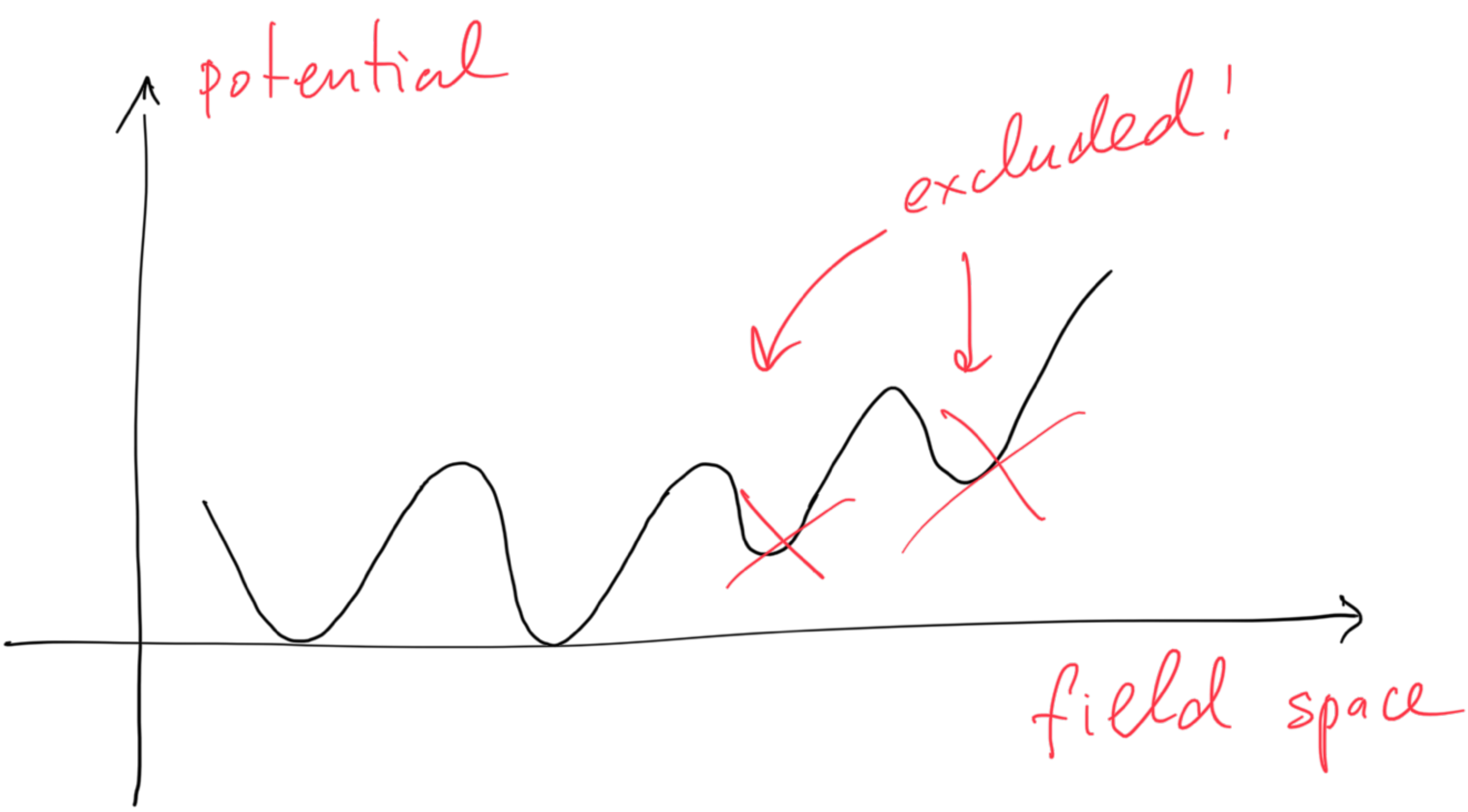
⊛ Landscapes that support eternal inflation (Vilenking '83, Linde '86)

G.D., Gomez '13, '14 + Zell '17

⊛ Big crunch cosmologies;

⊛ All cosmologies with non-S-matrix-vacua.

This fixes our framework:
EFT of S-matrix theory
defined on asymptotic
S-matrix vacuum of
Minkowski



First immediate implication:
S-matrix gravity nullifies an outstanding cosmological puzzle:

Cosmological term is Einstein's equation

$$G_{\mu\nu} = T_{\mu\nu} + g_{\mu\nu}\Lambda$$

$\Lambda \to = 0$

Dark energy = New physics

Prediction: Equation of state

$$w > -1$$

(In fact, arguments indicate

$$w + 1 \simeq \frac{1}{260}$$
)

Implication for strong-CP puzzle

$$\mathcal{L} = \mathcal{L}_{QCD} + \bar{\Theta} F\tilde{F}$$

$$\bar{\Theta} = \Theta + \text{arg.det.} M_q$$

$$F\tilde{F} \equiv \varepsilon^{\mu\nu\alpha\beta} \partial_\mu C_{\nu\alpha\beta}$$

chern-Simons 3-form

$$C_{\nu\alpha\beta} \equiv tr\left(A_{[\nu}\partial_\alpha A_{\beta]} + \frac{2}{3} A_{[\nu} A_\alpha A_{\beta]}\right)$$

$$A_\mu \equiv A_\mu^a T^a \leftarrow \text{gluon matrix}$$

$$U = e^{-i\omega^a T^a}$$

Gauge redundancy:

$$A_\mu \to U A_\mu U^+ + U^+ \partial_\mu U$$

$$C_{\mu\nu\alpha} \to C_{\mu\nu\alpha} + \partial_{[\mu}\Omega_{\nu\alpha]}$$

$$\Omega_{\nu\alpha} = tr\, A_{[\mu}\partial_{\nu]}\omega$$

$\bar{\Theta}$ is physical and contributes to EDMN.

The current bound

$$d_n < 2.9 \times 10^{-26} \, \mathrm{cm}$$

(Baker et. al. hep-ex/0602020)

translates an bound

$$|\bar{\Theta}| \lesssim 10^{-9}$$

Thus, we live in a vacuum with very small $\bar{\Theta}$.

This is the strong-CP puzzle; formulated as naturalness problem.

$\bar{\Theta}$ is physical due to topological susceptibility of vacuum (TSV)

$$\langle F\tilde{F}, F\tilde{F}\rangle_{p\to 0} \equiv$$

$$\equiv \lim_{p\to 0} \int d^4x\, e^{ipx} \langle T[F\tilde{F}(x), F\tilde{F}(0)]\rangle = \text{const} \neq 0$$

Then, Källén-Lehmann spectral representation:

$$\langle C, C\rangle = \frac{1}{p^2} + \sum_{m\neq 0} \frac{\rho(m^2)}{p^2 - m^2}$$

massless 3-form!

The exact vacuum
is captured by effective theory:

$$\mathcal{L} = K(E)$$

Algebraic function of $E \equiv dC$

Equation of motion

$$\partial_\mu \left(\frac{\partial K}{\partial E} \right) = 0$$

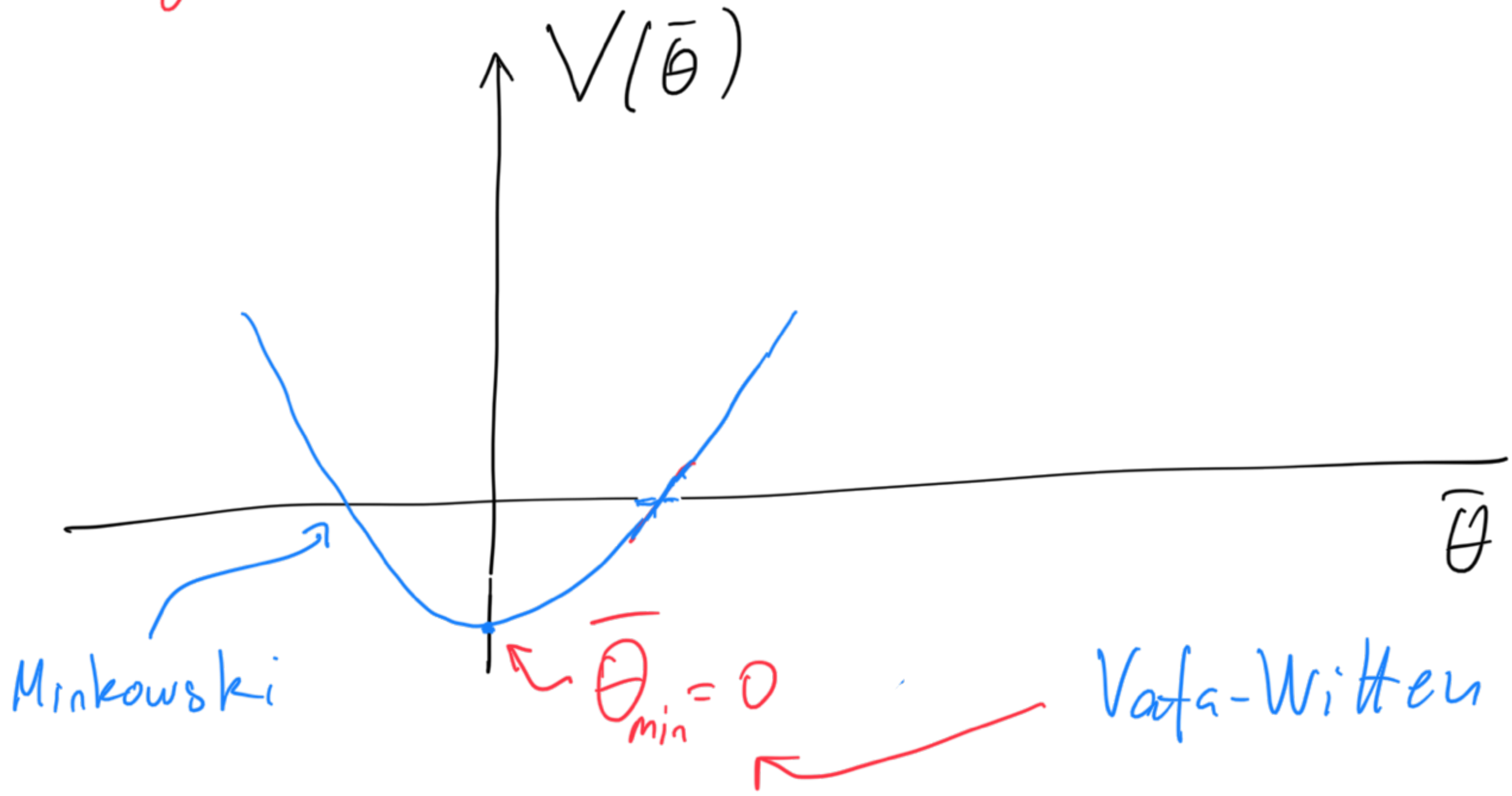
Vacuum solution is arbitrary constant

$$E = \bar{\Theta}$$

constant "electric" field of Chern-Simons
3-form C

These are the celebrated
Θ-vacua of QCD

The θ-vacua are <u>not</u> degenerate



If one $\bar{\theta}$ is Minkowski, the others <u>are not</u>.

This is excluded by S-matrix gravity:

θ-vacua must be eliminated by consistency.

G.D, Gomez, Zell '18
G.D, '22

Gravity = Axion.

Must be exact!

Axion (Weinberg; Wilczek '78)

eliminates $\bar{\theta}$-vacua by making $\bar{\theta}$ dynamical:

$$\mathcal{L}_a = (\partial_\mu a)^2 - \left(\frac{a}{f_a} - \bar{\theta}\right) F\tilde{F}$$

Equivalently, it Higgses the 3-form

G.D., hep-th/0507215

$$\langle CC \rangle = \cancel{\frac{1}{p^2}} + \sum_{m\neq 0} \frac{\rho(m^2)}{p^2 - m^2}$$

But for this, the axion shift symmetry $a \to a + \text{const.}$

must be protected exactly modulo QCD anomaly.

In Peccei-Quinn:

$$\Phi = |\Phi| e^{i\frac{a}{f_a}}$$

Global $U(1)_{PQ}$ - symmetry.

$$\Phi \to e^{i\alpha}\Phi \longrightarrow \frac{a}{f_a} \to \frac{a}{f_a} + \alpha$$

Not protected against explicit breaking!

E.g. by operators:

$$\left(\Phi^{+}\right)^{m} \Phi^{n} \qquad m \neq n.$$

S-matrix demands that they must vanish to <u>all-orders</u>.

In Peccei-Quinn formulation of axion as of Goldstone of global $U(1)_{PQ}$

$$\Phi = |\Phi| e^{i\frac{a}{f_a}},$$

$\bar{\Theta}$ is <u>uncalculable</u>:

It is sensitive to arbitrary continuous deformations of the theory by $U(1)_{PQ}$-violating operators

$$(\Phi^+)^m \Phi^n \qquad m \neq n$$

This is incompatible with S-matrix (and thus, with gravity).

This favors the alternative pure-gauge formulation of QCD axion: G.D., hep-th/0507215

All we need is to introduce a single degree of freedom $B_{\mu\nu}$, with a proper gauge charge under QCD:

$$B_{\mu\nu} \to B_{\mu\nu} + \frac{1}{f_a}\Omega_{\mu\nu}$$

$$C_{\alpha\mu\nu} \to C_{\alpha\mu\nu} + \partial_{[\alpha}\Omega_{\mu\nu]}$$

$$\Omega^{(x)}_{\mu\nu} = tr\, A_{[\mu}\partial_{\nu]}\omega(x)$$

QCD gauge redundancy

In this theory the axion is an intrinsic part of QCD.

It is protected by gauge symmetry undre arbitrary local deformations of the theory.

Theory:

$$\mathcal{L} = \mathcal{L}_{QCD} + \bar{\theta} F \tilde{F} +$$
$$+ \frac{1}{f_a^2} \left(C - f_a \, dB \right)^2$$

$\bar{\theta}$ is unphysical to all orders in operator expansion

Axion $B_{\mu\nu}$ becomes a longitudinal (Stückelberg) polarization of the 3-form $C_{\mu\nu\alpha}$ and they compose a massive 3-form

$$C^{(massive)}_{\mu\nu\alpha} \equiv C_{\mu\nu\alpha} - f_a \partial_{[\mu} B_{\nu\alpha]}$$

3-form is "Higgsed"

and

the pole at $p^2 = 0$ is removed:

$$\langle C\, C \rangle = \frac{1}{p^2 + m_a^2} + \dots$$

Correspondigly, $\bar{\Theta}$ is unphysical against arbitrary deformations.

EFT:

$$\mathcal{L} = K(E) + \bar{\theta} E + \frac{1}{f_a^2}\left(C - f_a\, dB\right)^2$$

$$\partial_\mu \frac{\partial K}{\partial E} + \frac{1}{f_a^2} {}^*(C - dB)_\mu = 0$$

$$\partial^\mu\, {}^*(C - dB)_\mu = 0$$

vacuum: $E = 0 \longleftrightarrow \bar{\theta}$ unphysical

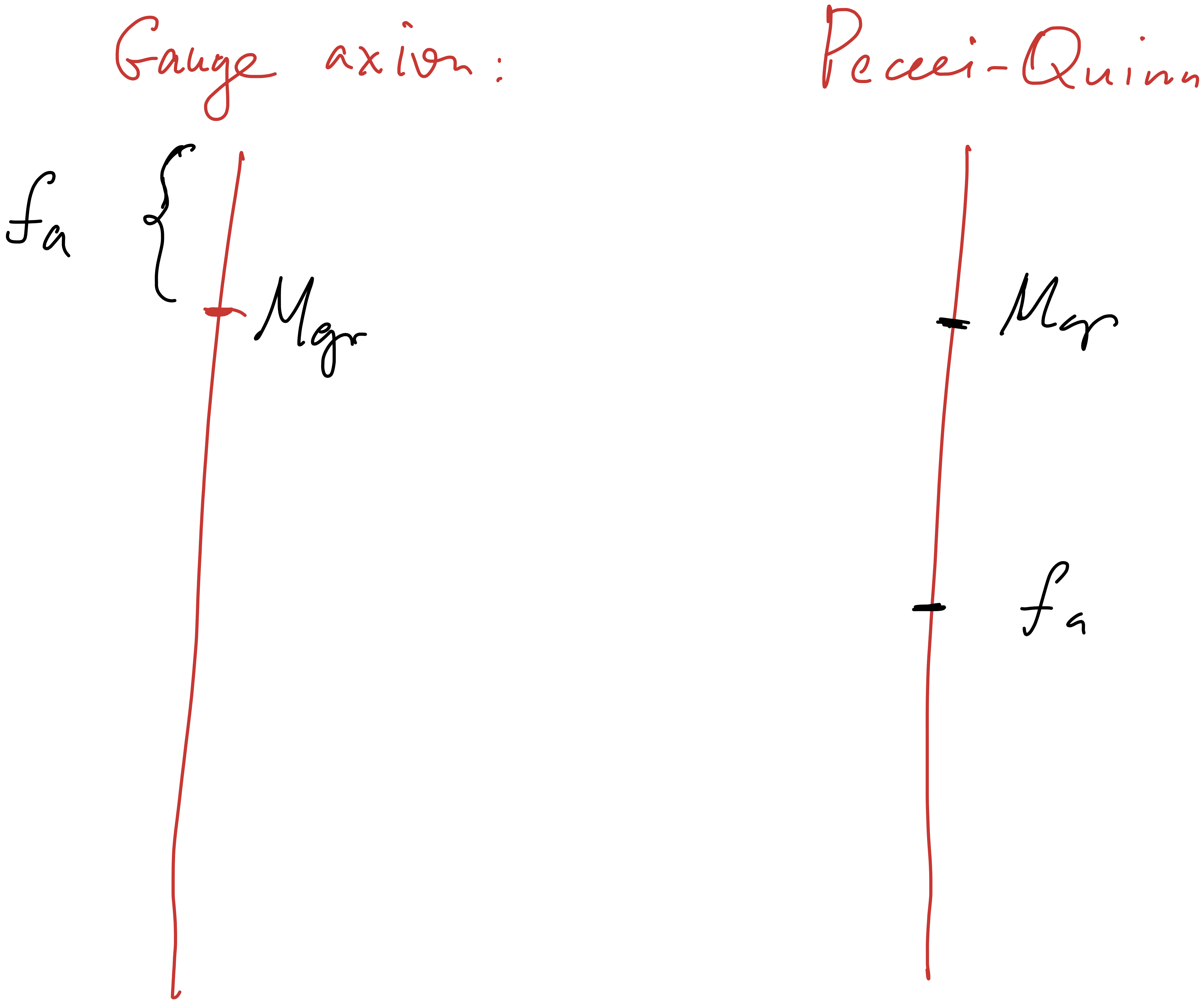
This is insensitive to deformation by arbitrary local operators.

General proof, see, G.D., hep-th/0507215

for some explicit examples, see: Sakhelashvili, 2110.03386 [hep-th].

Thus, S-matrix motivates gauge formulation of QCD axion. This suggests that

$f_a \gtrsim M_{gr} \equiv$ scale of gravity



The advantage in calculability:

Gauge axion predicts: $\underline{\underline{\bar{\theta} = 0}}$.

The weak contribution to EDMN is too small for near-future detection

$d_n \sim 10^{-31-32}$ cm   Shabalin '79
Ellis, Gaillard '79

Thus, a near-future detection of EDMN will be a signal for new CP-violating physics beyond <u>Standar Model</u>.

$$\mathcal{L} = K(E) + m_a^2 (C + \tilde{C} - dB)^2$$

$$+ \tilde{K}(\tilde{E})$$

$\hookrightarrow \tilde{E} \equiv d\tilde{C}$

A candidate in gravity, gravitational Chern-Simons

$$\tilde{C} \equiv \Gamma \wedge d\Gamma + \frac{2}{3} \Gamma \wedge \Gamma \wedge \Gamma$$

$$\tilde{E} \equiv d\tilde{C} = R\tilde{R} = \epsilon^{\alpha\beta\mu\nu} R^{i}_{j\,\alpha\beta} R^{j}_{i\,\mu\nu}$$

If in pure gravity $\langle R\tilde{R}, R\tilde{R} \rangle_{q \to 0} \neq 0$,

gravity must provide additional axion.

OR

anomalous chiral symmetry of neutrino

$$\nu \to e^{i\alpha\gamma_5}\,\nu$$

can neutralize TVS of gravity, resulting into $m_\nu \neq 0$.

G. D., Folkerts, Franca 1312.7273 [hep-th]
G. D., Funcke, 1602.03191 [hep-ph]

Each 3-form provided by gravity must be accompanied either by axion or a chiral fermion.

QCD axion is safe by consistency of S-matrix gravity.