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Super-leading logarithms from effective field theory

Thomas Becher
University of Bern

Festsymposium “The EFT Kaleidoscope”
on the occasion of the 60th birthday of Matthias Neubert
MITP Mainz, May 11, 2023

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A long-standing and super-pleasant collaboration with a leading physicist on the resummation of large logarithms from effective field theory

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University of Bern

Festsymposium “The EFT Kaleidoscope”
on the occasion of the 60th birthday of Matthias Neubert
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A 20 year journey to exclusivity

Super-leading logarithms from effective field theory

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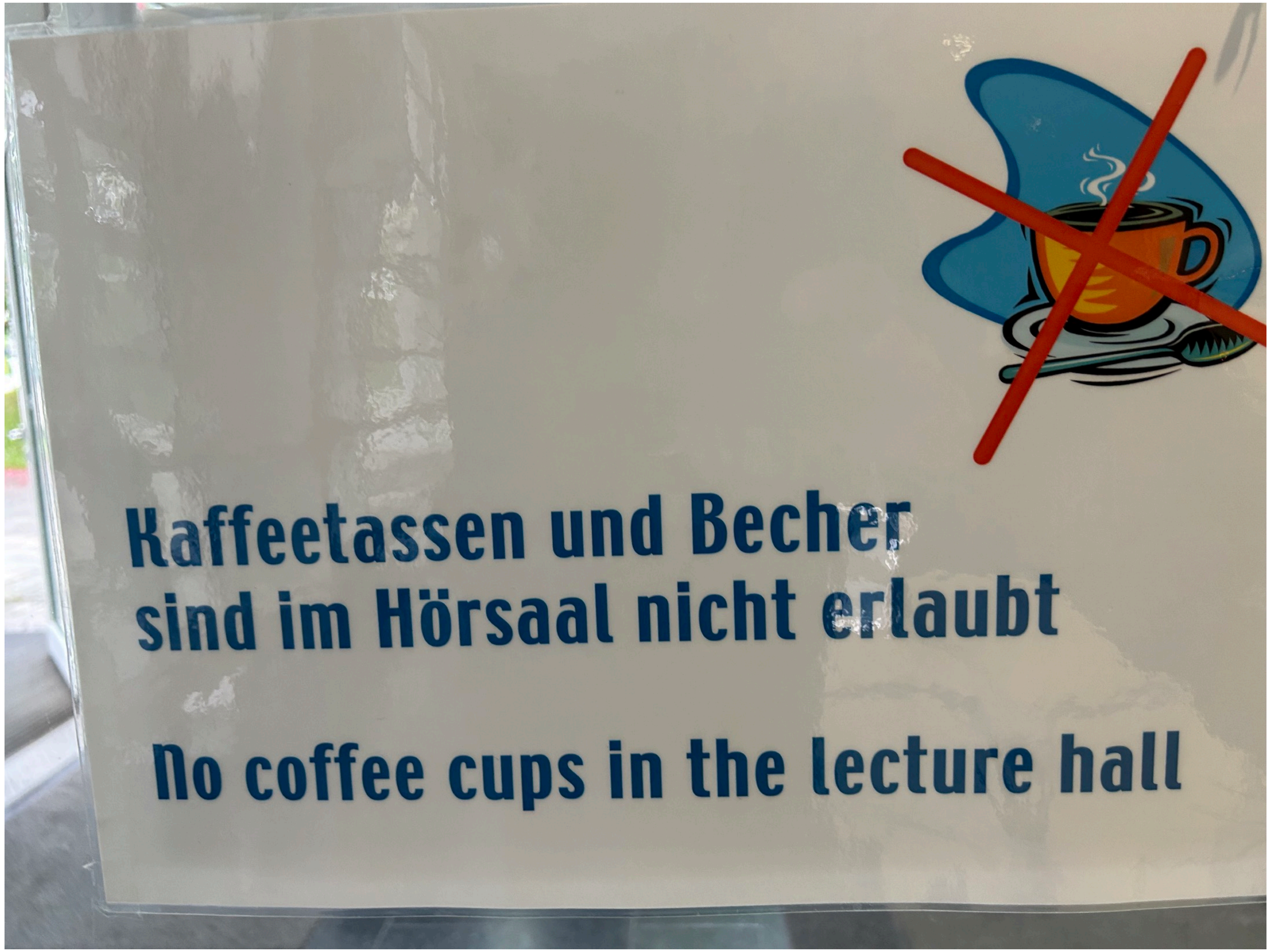
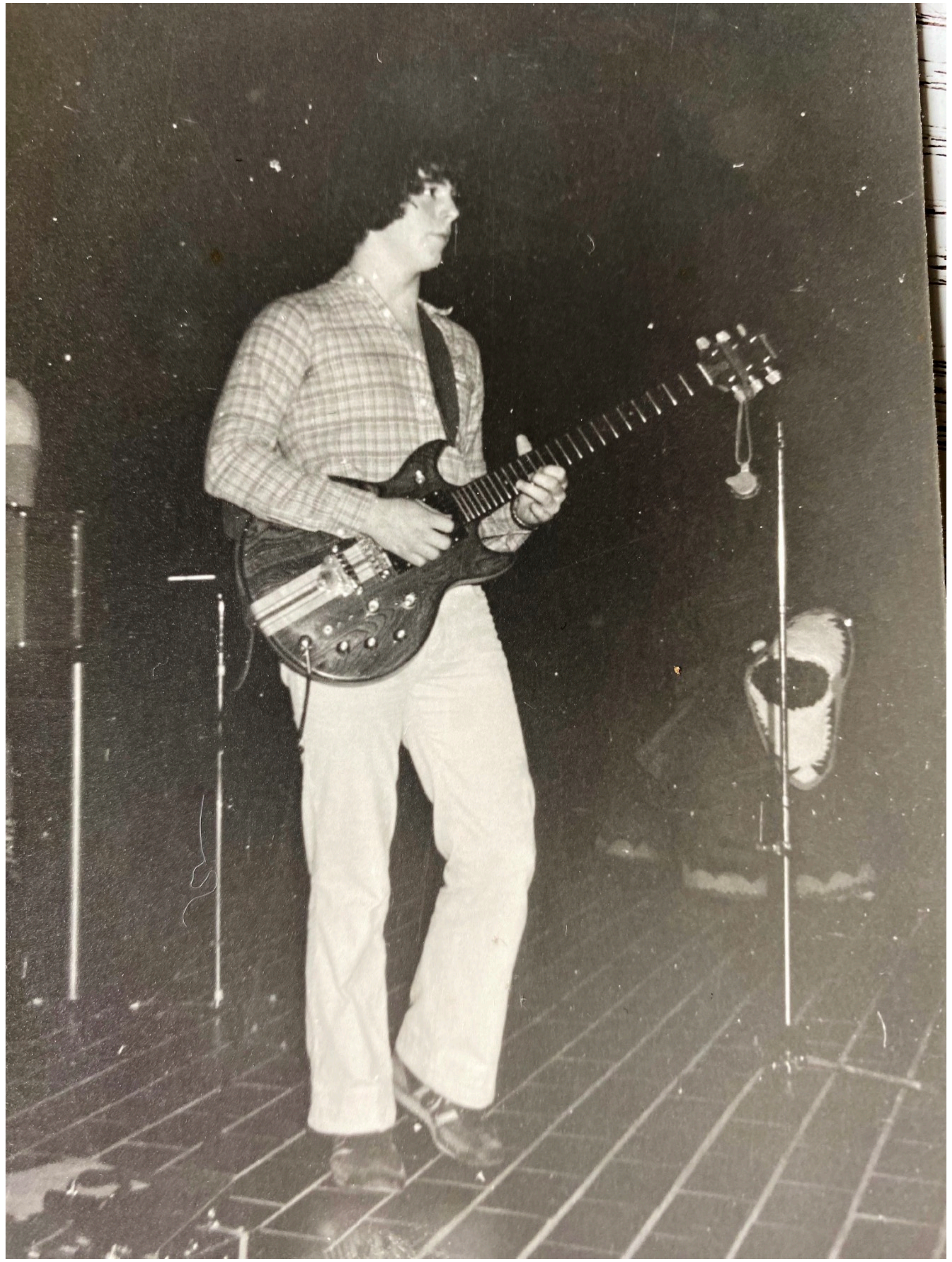
Festsymposium "The EFT Kaleidoscope"
on the occasion of the 60th birthday of Matthias Neubert
MITP Mainz, May 11, 2023

23 years,
36 common papers,
5700 email messages …



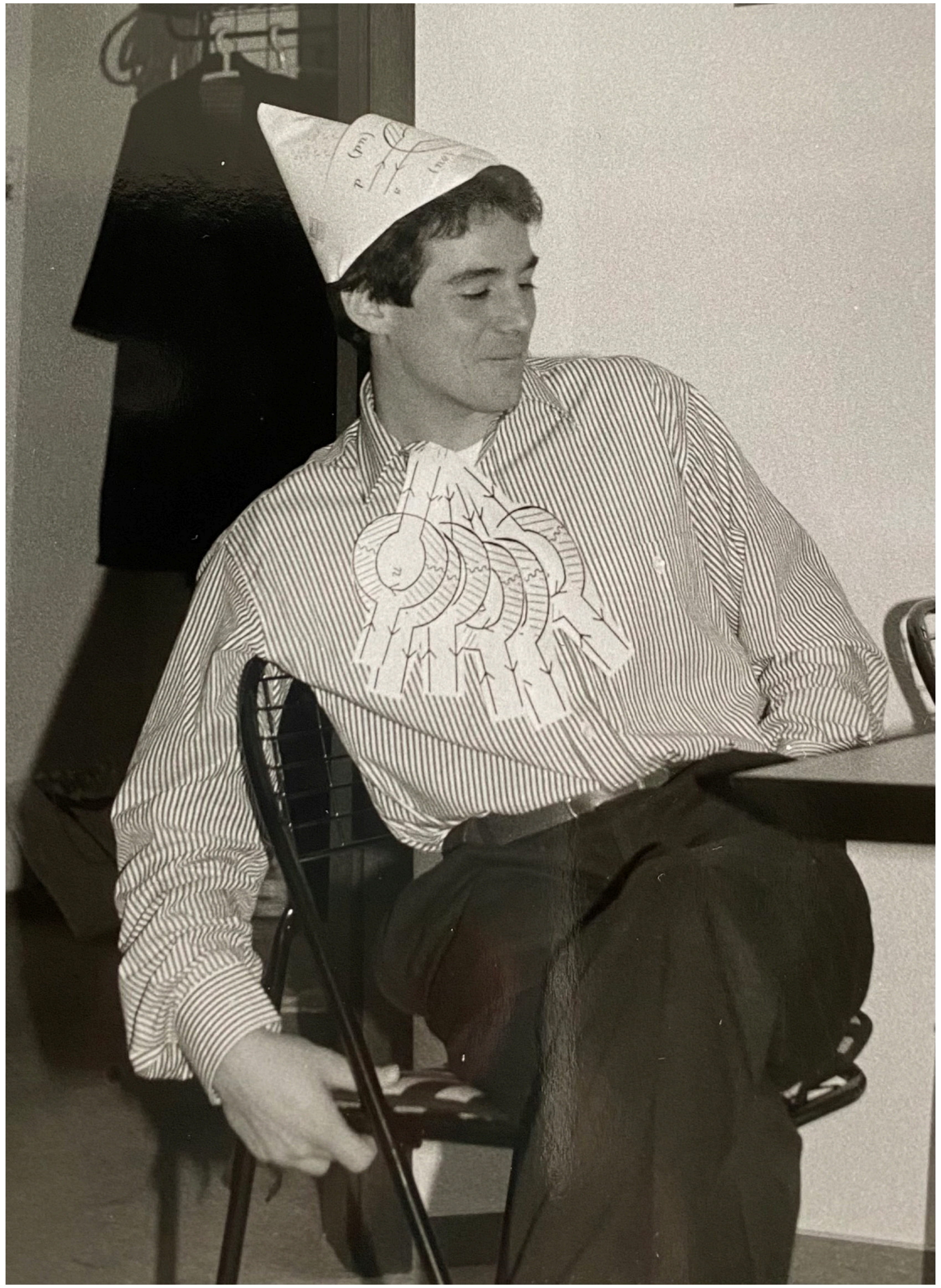
from Hartmut Wittig

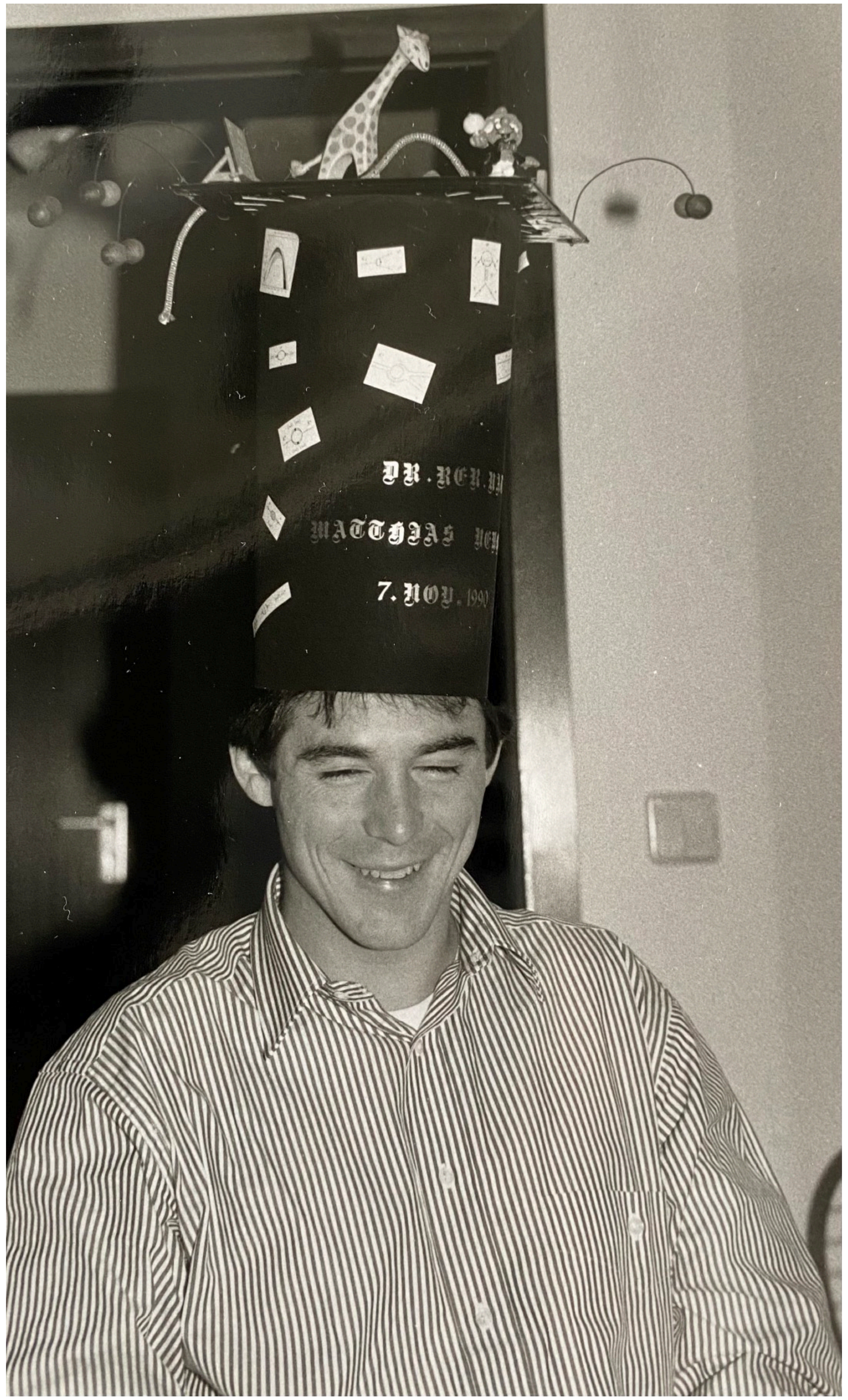
Source: Tina Neubert



High-school band ``solution”, Siegen, < 1980

High-school band ``solution”, Siegen, < 1980





PhD in Heidelberg, 1990

postdoc at SLAC, 1992

SLAC–PUB–6263
June 1993
T/E

Heavy Quark Symmetry

Matthias Neubert[1]
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94309

Abstract

We review the current status of heavy quark symmetry and its applications to weak decays of hadrons containing a single heavy quark. After an introduction to the underlying physical ideas, we discuss in detail the formalism of the heavy quark effective theory, including a comprehensive treatment of symmetry breaking corrections. We then illustrate some nonperturbative approaches, which aim at a dynamical, QCD-based calculation of the universal form factors of the effective theory. The main focus is on results obtained using QCD sum rules. Finally, we perform an essentially model-independent analysis of semileptonic B meson decays in the context of the heavy quark effective theory.

(to appear in Physics Reports)

[1]Supported by the Department of Energy under contract DE-AC03-76SF00515.

CERN (1993-1998)

Jungfraujoch, April 2011

Jungfraujoch, April 2011

Jungfraujoch, April 2011

Jungfraujoch, April 2011

Jungfraujoch, April 2011

Cornell (1999-2006)

On Sun, 13 Feb 2000, Thomas Becher wrote:

Lieber Herr Neubert,

Ich bin eben von meinem Besuch am Forschungszentrum Juelich zurueckgekehrt und kann Ihnen noch meine Ankunftszeit in Ithaca mitteilen. […] Ich fliege am Montag, dem 6. Maerz in Zuerich ab und […] werde Groessenordnung 21:00 in Ithaca eintreffen und kann dann meine Wohnung beziehen.

Ich habe einen Stadtplan von Cornell und weiss, wo das Newman Laboratory liegt. Ich waere froh, wenn Sie mir das Stockwerk und die Nummer ihres Bueros angeben koennten, sodass ich Sie am Dienstag morgen finde.

Mit freundlichen Gruessen,

Thomas Becher

Cornell (1999-2006)

VOLUME 83, NUMBER 10 PHYSICAL REVIEW LETTERS 6 SEPTEMBER 1999

QCD Factorization for $B \to \pi\pi$ Decays: Strong Phases and CP Violation in the Heavy Quark Limit

M. Beneke,[1] G. Buchalla,[1] M. Neubert,[2] and C. T. Sachrajda[3]
[1] *Theory Division, CERN, CH-1211 Geneva 23, Switzerland*
[2] *Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309*
[3] *Department of Physics and Astronomy, University of Southampton, Southampton SO17 1BJ, United Kingdom*
(Received 17 May 1999)

We show that, in the heavy quark limit, the hadronic matrix elements that enter B meson decays into two light mesons can be computed from first principles, including "nonfactorizable" strong interaction corrections, and expressed in terms of form factors and meson light-cone distribution amplitudes. The conventional factorization result follows in the limit when both power corrections in $1/m_b$ and radiative corrections in α_s are neglected. We compute the order-α_s corrections to the decays $B_d \to \pi^+\pi^-$, $B_d \to \pi^0\pi^0$, and $B^+ \to \pi^+\pi^0$ in the heavy quark limit and briefly discuss the phenomenological implications for the branching ratios, strong phases and CP violation.

PACS numbers: 13.25.Hw, 11.30.Er, 12.38.Bx, 12.39.Hg

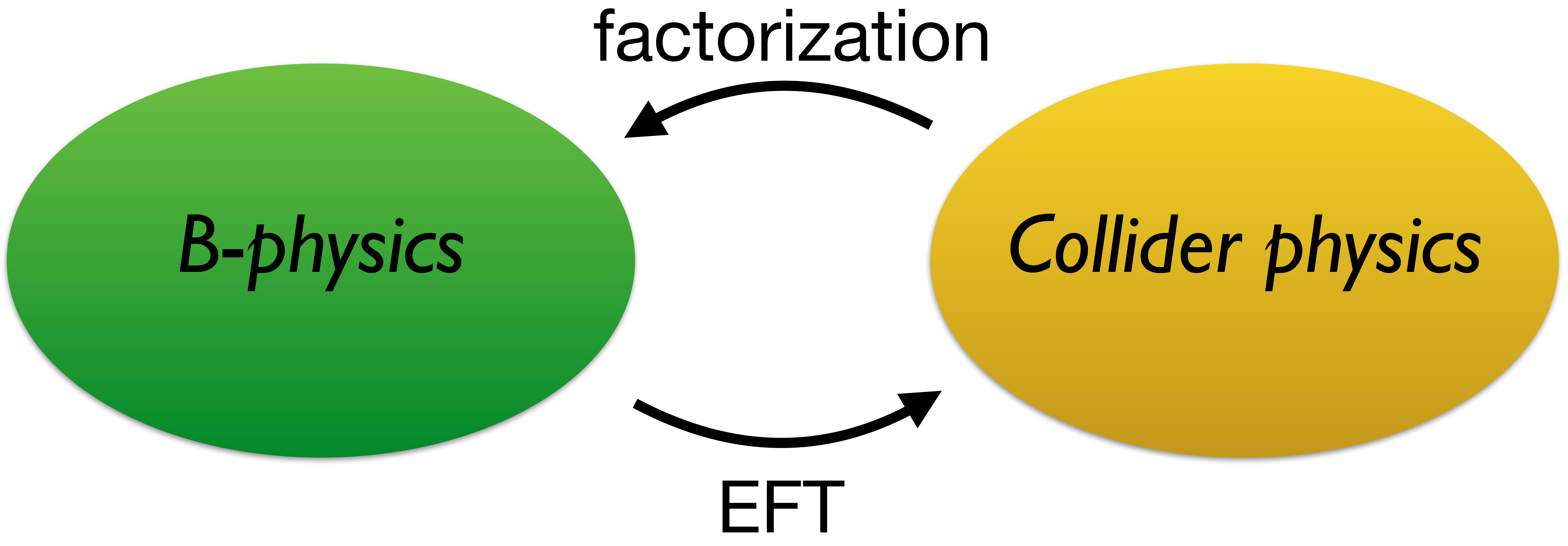
$$\langle \pi(p')\pi(q)|Q_i|\bar{B}(p)\rangle = f^{B\to\pi}(q^2)\int_0^1 dx\, T_i^{\mathrm{I}}(x)\Phi_\pi(x) + \int_0^1 d\xi\, dx\, dy\, T_i^{\mathrm{II}}(\xi,x,y)\Phi_B(\xi)\Phi_\pi(x)\Phi_\pi(y)$$

PHYSICAL REVIEW D, VOLUME 63, 114020

An effective field theory for collinear and soft gluons: Heavy to light decays

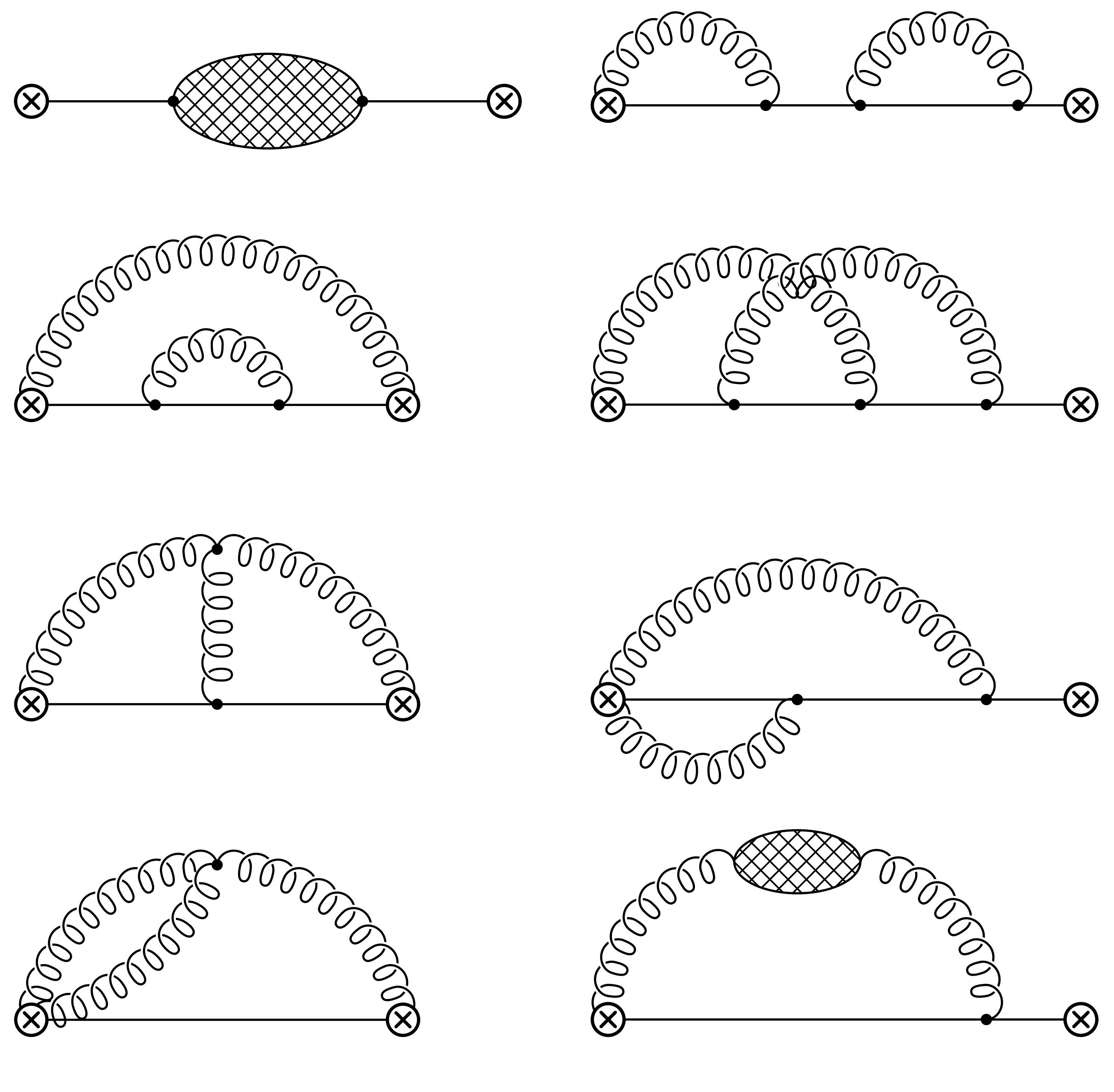
Christian W. Bauer,[1] Sean Fleming,[2] Dan Pirjol,[1] and Iain W. Stewart[1]
[1] *Physics Department, University of California at San Diego, La Jolla, California 92093*
[2] *Physics Department, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213*
(Received 30 November 2000; published 7 May 2001)

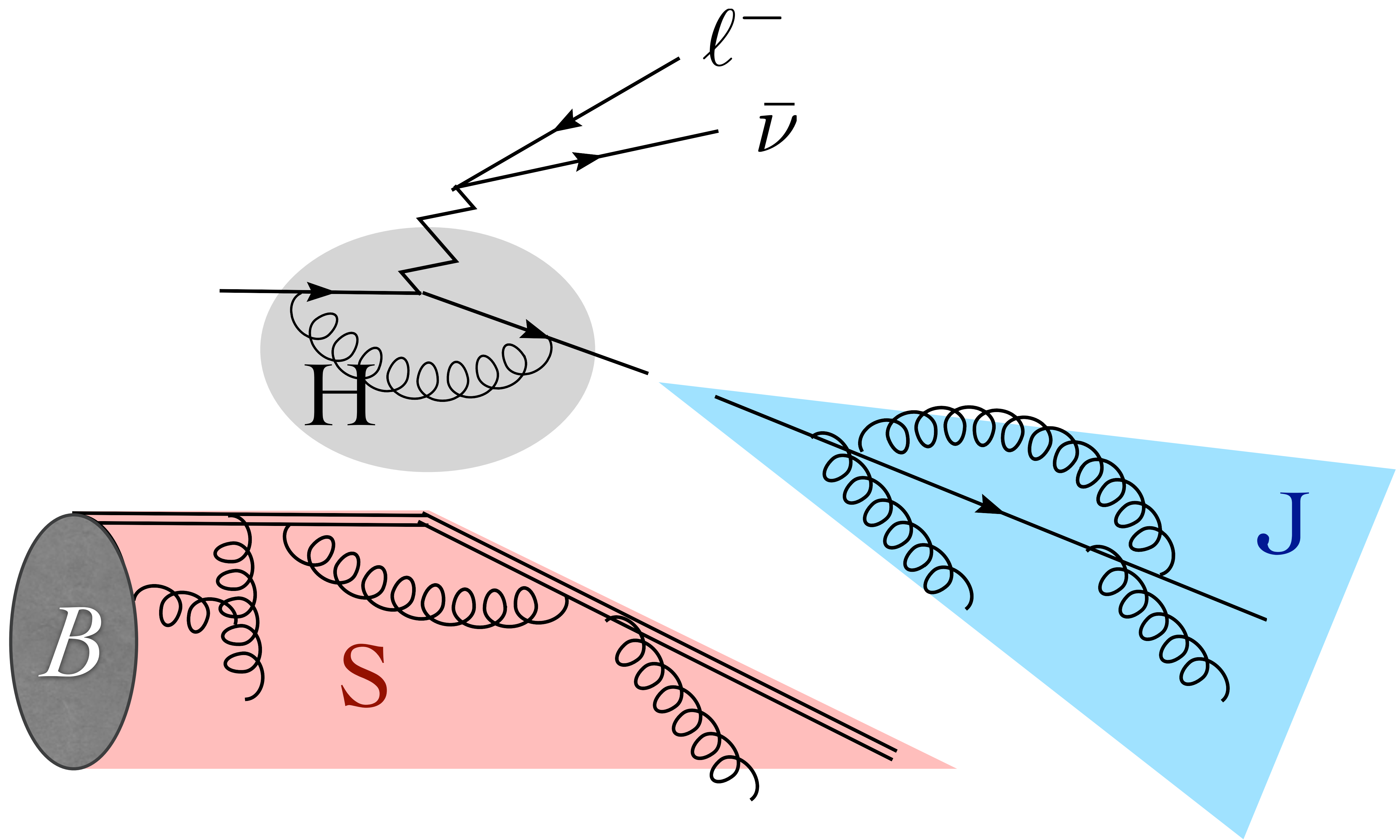
Cross fertilization



Effective field theory methods for collider physics

- Momentum regions as fields
- Factorization using operator methods
- Resummation by RG evolution in EFT
- On-shell matching: partonic processes are EFT Wilson coefficients





$$d\Gamma = H \cdot J \otimes S$$

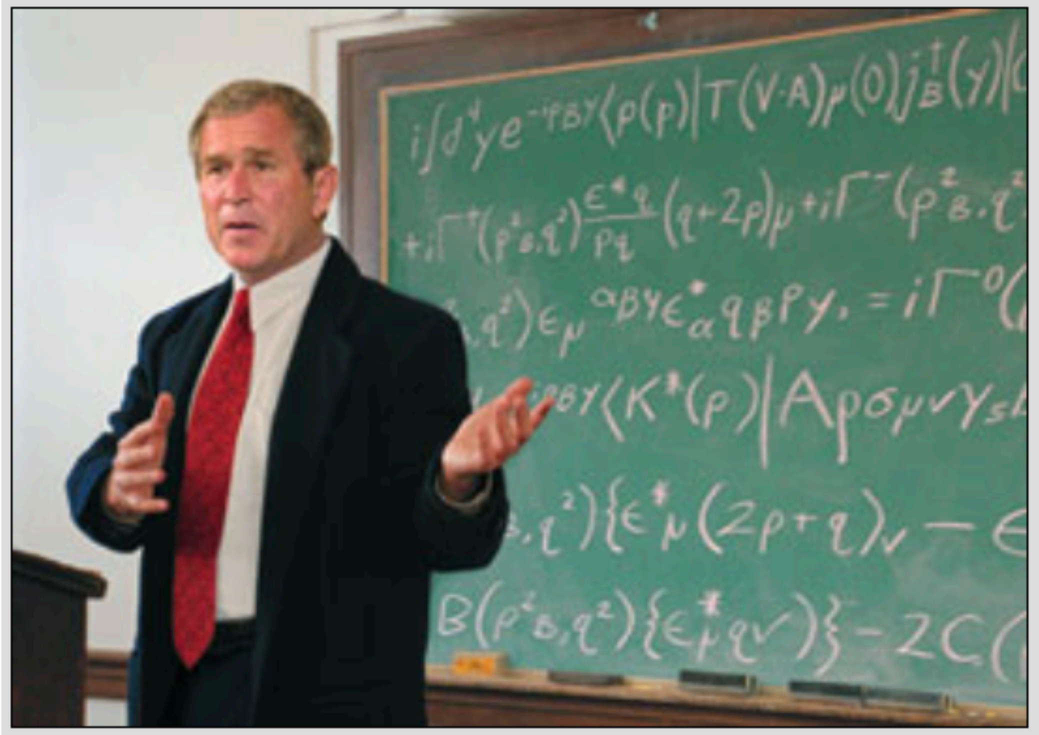
- Operator definitions of hard, jet and soft functions
- First two-loop computations in SCET (*J* and *S*)

TB, Neubert, Phys.Lett.B 633 (2006) 739, Phys.Lett.B 637 (2006) 251

the ONION 1 August 2001

Bush Finds Error In Fermilab Calculations

BATAVIA, IL--President Bush met with members of the Fermi National Accelerator Laboratory research team Monday to discuss a mathematical error he recently discovered in the famed laboratory's "Improved Determination Of Tau Lepton Paths From Inclusive Semileptonic *B*-Meson Decays" report.



Above: Bush shows Fermilab scientists where they went wrong in their calculations.

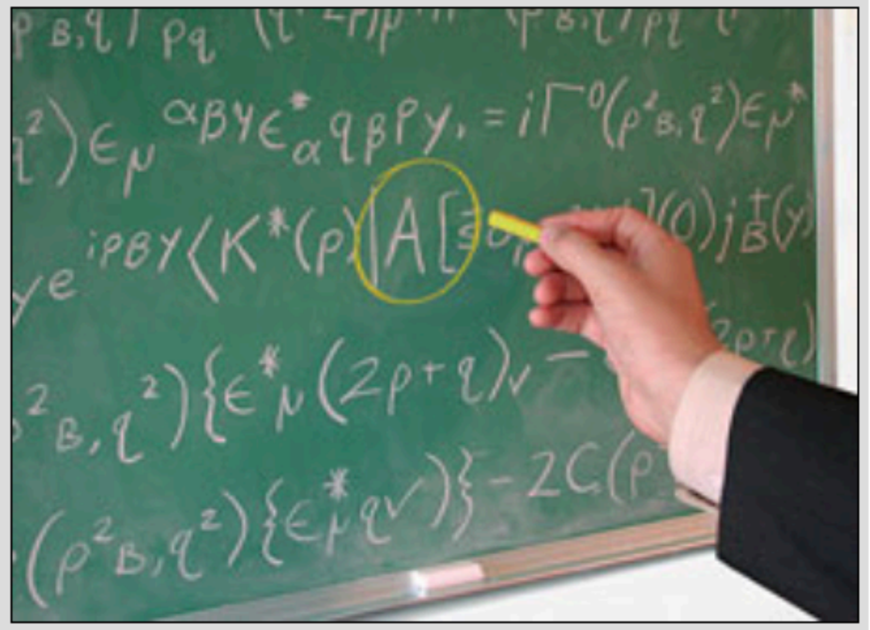
"I'm somewhat out of my depth here," said Bush, a longtime Fermilab follower who describes himself as "something of an armchair physicist." "But it seems to me that, when reducing the perturbative uncertainty in the determination of V_{ub} from semileptonic Beta decays, one must calculate the rate of Beta events with a standard dilepton invariant mass at a subleading order in the hybrid expansion. The Fermilab folks' error, as I see it, was omitting that easily overlooked mathematical transformation and, therefore, acquiring incorrectly re-summed logarithmic corrections for the b-quark mass. Obviously, such a miscalculation will result in a precision of less than 25 percent in predicting the resulting path of the tau lepton once the value for any given decaying tau neutrino is determined."

The Bush correction makes it possible for scientists to further study the tau lepton, a subatomic particle formed by the collision of a tau neutrino and an atomic nucleus.

Bush resisted criticizing the Fermilab scientists responsible for the error, saying it was "actually quite small" and that "anyone could have made the mistake."

"High-energy physics is a complex and demanding field, and even top scientists drop a decimal point or two every now and then," Bush said. "Also, I might hasten to add that what I pointed out was more a correction of method than of mathematics. Experimental results on the Tevatron accelerator would have exposed the error in time, anyway."

Fermilab director Michael Witherell said the president was being too modest "by an order of magnitude."



Above: Bush circles the crucial misstep.

"In addition to gently reminding us that even the best minds in the country are occasionally fallible, President Bush has saved his nation a few million dollars," Witherell said. "We would have made four or five runs on the particle accelerator with faulty data before figuring out what was wrong. But, thanks to Mr. Bush, we're back on track."

"It's true, I dabbled in the higher maths during my Yale days," said Bush, who spent three semesters as an assistant to Drs. Kasha and Slaughter at Yale's renowned Sloane High-Energy Physics Lab. "But I didn't have the true gift for what Gauss called 'the musical language in which is spoken the very universe.' If I have any gift at all, it's my instinct for process and order."

Continued Bush: "As much as I enjoyed studying physics at Yale, by my junior year it became apparent that I could far better serve humanity through a career in statecraft."

While he says he is "flattered and honored" by the tau-neutrino research team's request that he review all subsequent Fermilab publications on lepton-path determination, Bush graciously declined the "signal honor."

"This sort of thing is best left to the likes of [Thomas] Becher and [Matthias] Neubert, not a dilettante such as myself," Bush said. "I just happened to have some time on the plane coming back from the European G8 summit, decided to catch up on some reading, and spotted one rather small logarithmic branching-ratio misstep in an otherwise flawless piece of scientific scholarship. Anyone could have done the same."

the ONION 1 August 2001

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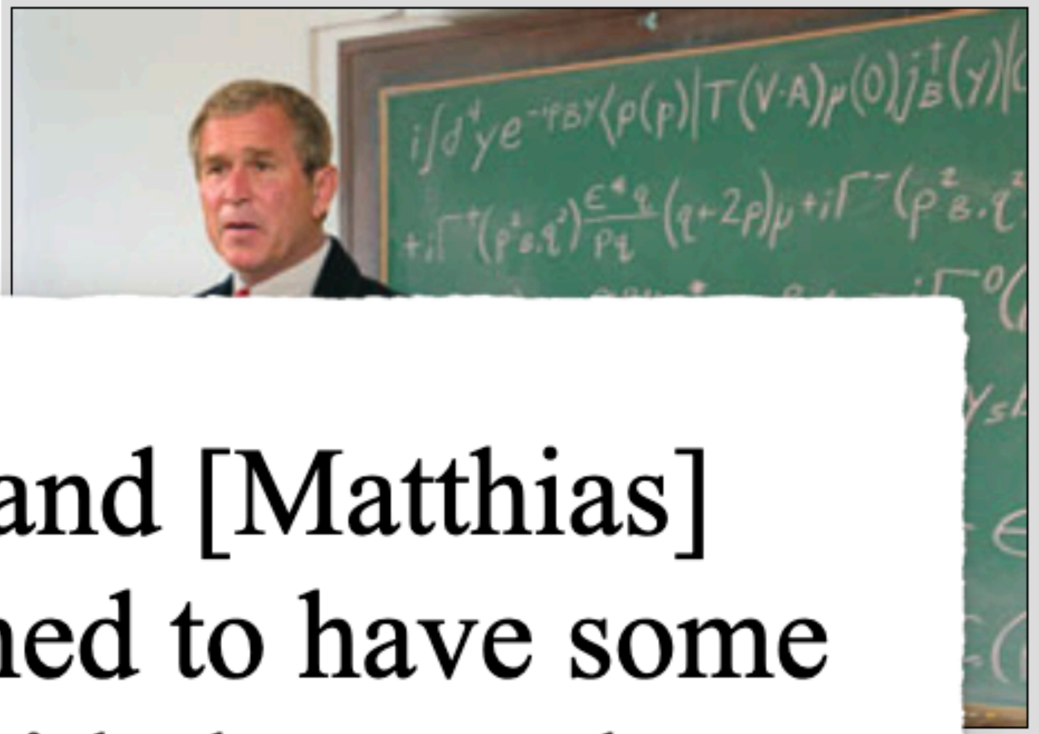

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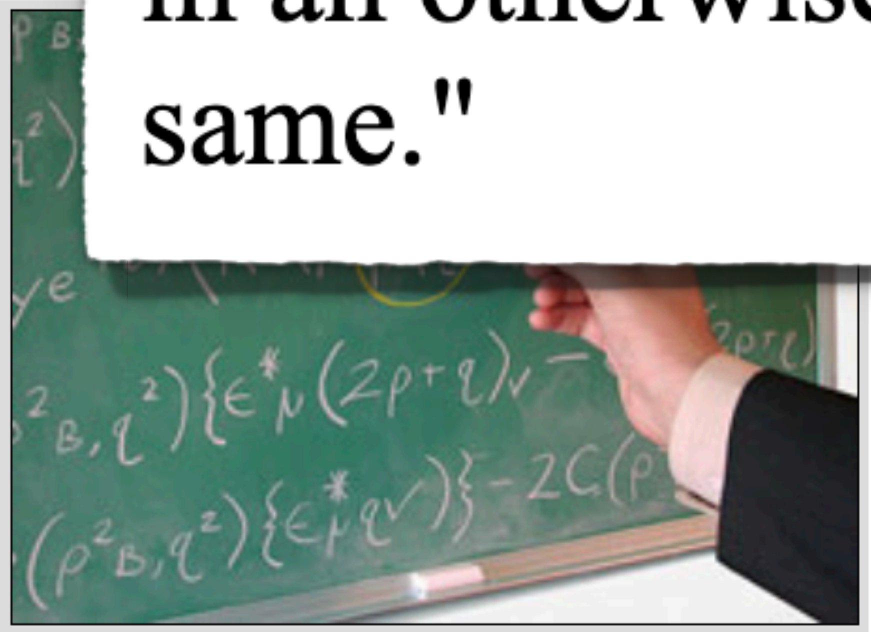

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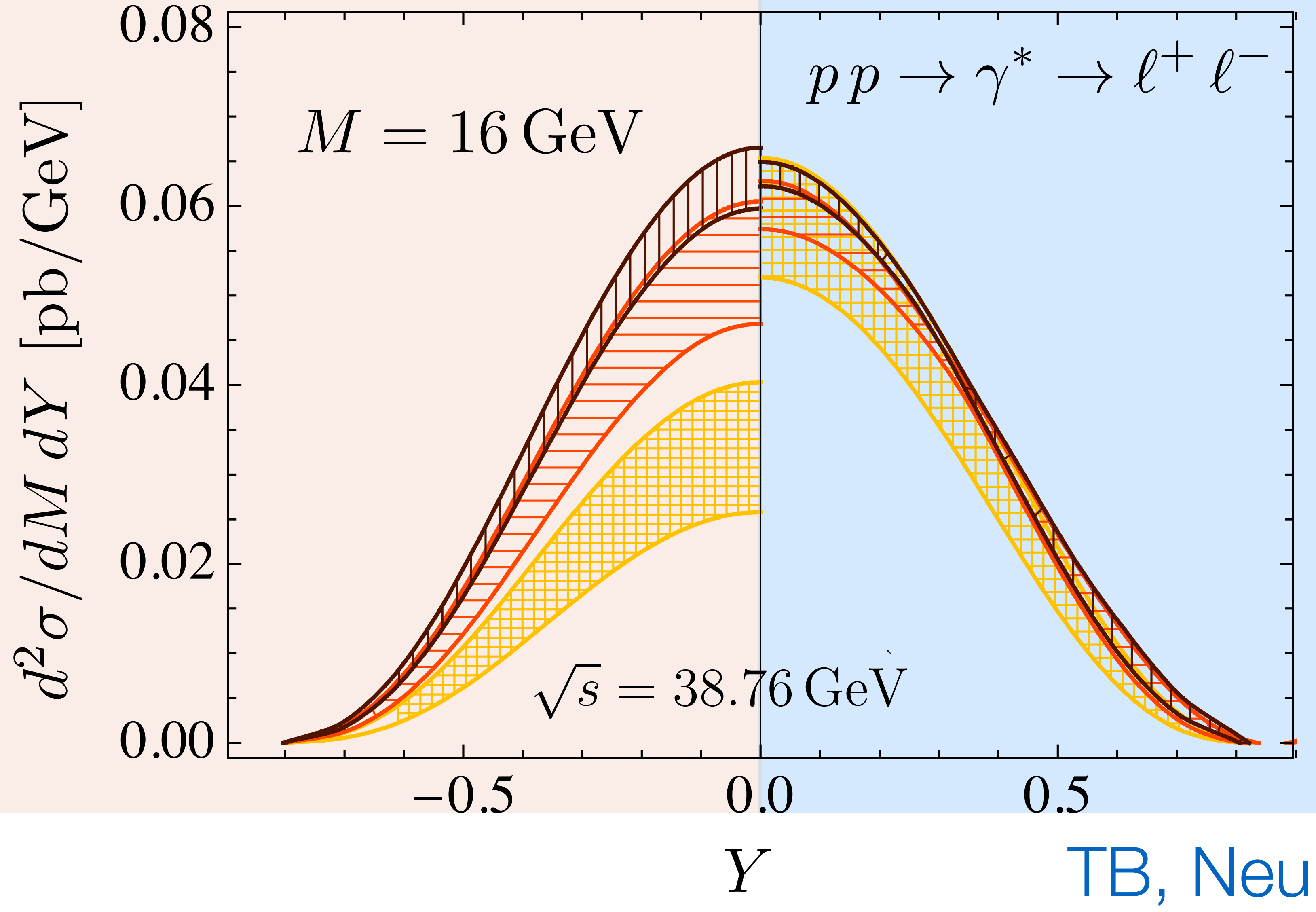
Threshold resummation

TB, Neubert, Phys.Rev.Lett. 97 (2006) 082001

fixed order
up to NNLO

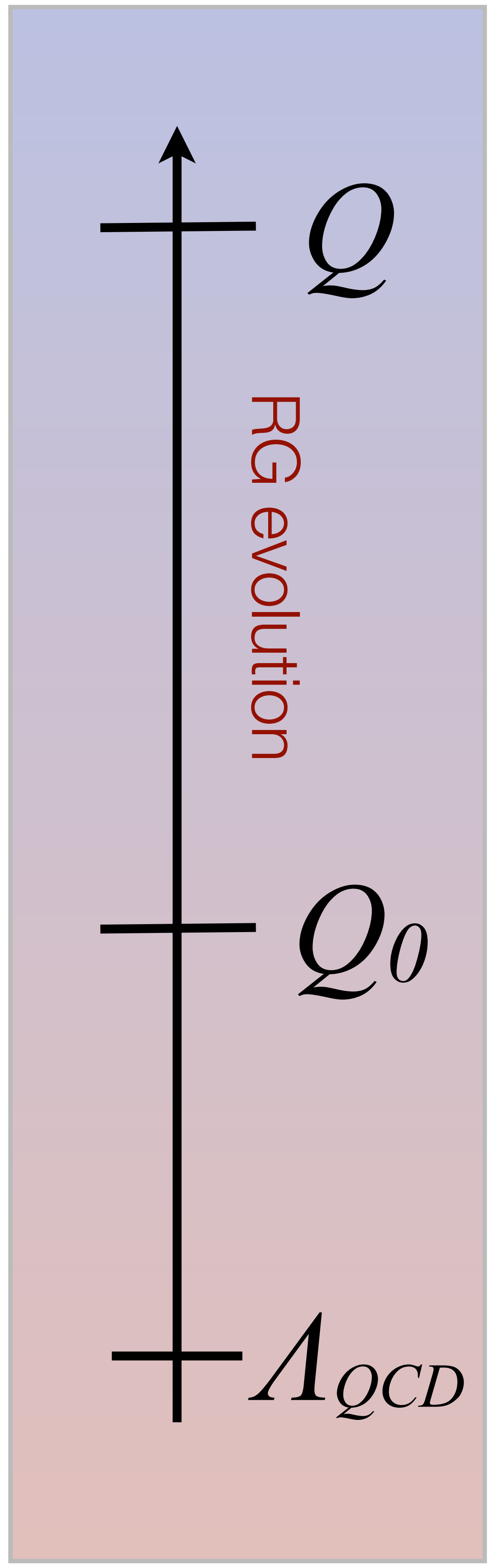


resummed+matched
up to N^{3}LL + NNLO

TB, Neubert, Xu JHEP 07 (2008) 030

Threshold resummation in momentum space from EFT

- RG evolution between physical scales avoids Landau-pole singularities
- No need for numerical Mellin inversion



Aspen February 2006

3rd SCET workshop, Tucson March 2006

Infrared Singularities of *n*-Leg Amplitudes

TB, Neubert, Phys.Rev.Lett. 102 (2009) 162001, JHEP 06 (2009) 081

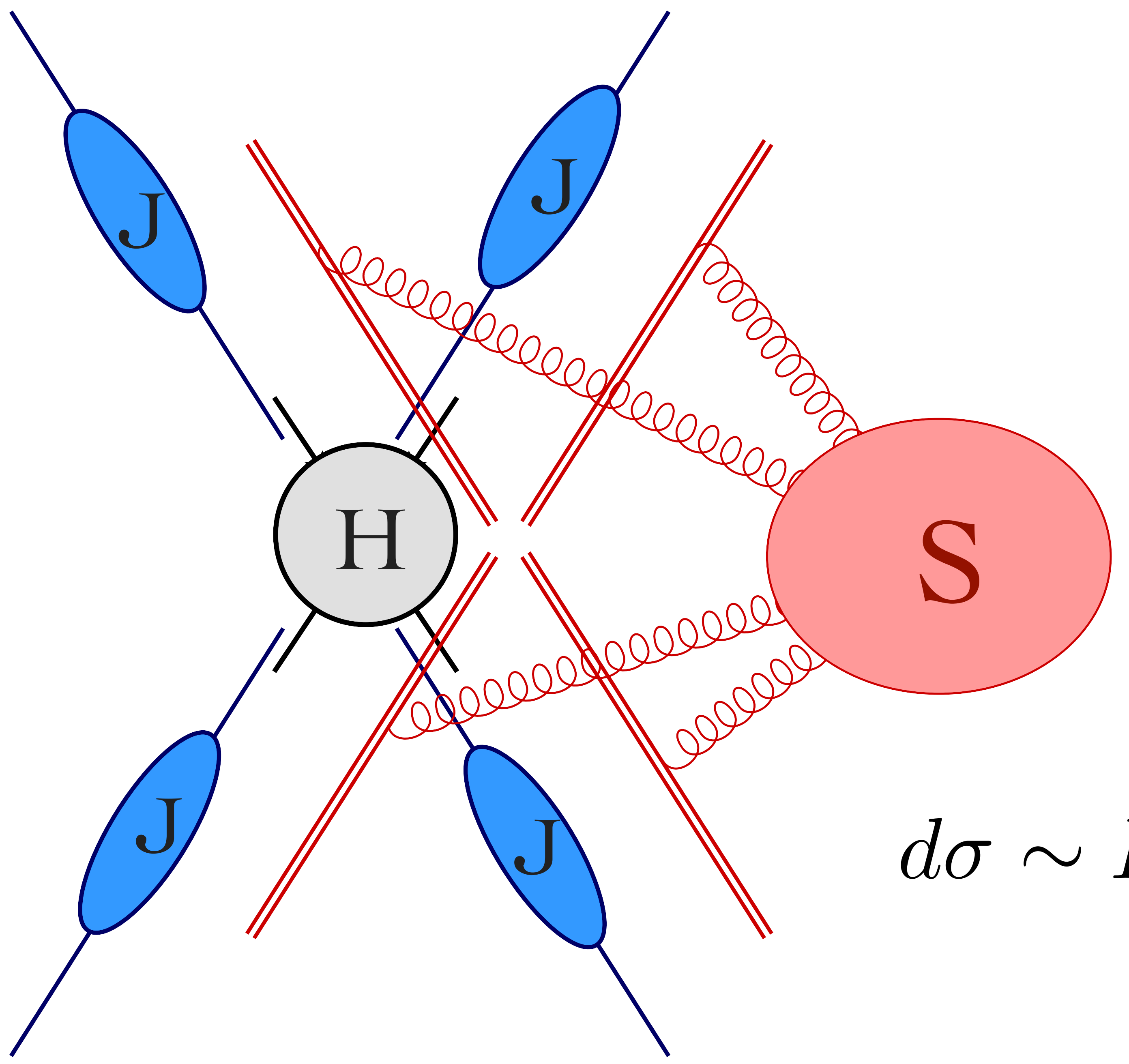
On-shell amplitudes are EFT Wilson coefficients

- Correspondence of IR to UV divergences. Renormalization

$$|\mathcal{M}_n(\{\underline{p}\},\mu)\rangle = \lim_{\epsilon\to 0} \boldsymbol{Z}^{-1}(\epsilon,\{\underline{p}\},\mu)\,|\mathcal{M}_n(\epsilon,\{\underline{p}\})\rangle$$

- Z-factor from anomalous dimension

$$\boldsymbol{Z}(\epsilon,\{\underline{p}\},\mu) = \mathbf{P}\exp\left[\int_\mu^\infty \frac{d\mu'}{\mu'}\,\boldsymbol{\Gamma}(\{\underline{p}\},\mu')\right]$$



TB, Neubert, Phys.Rev.Lett. 102 (2009) 162001,
JHEP 06 (2009) 081

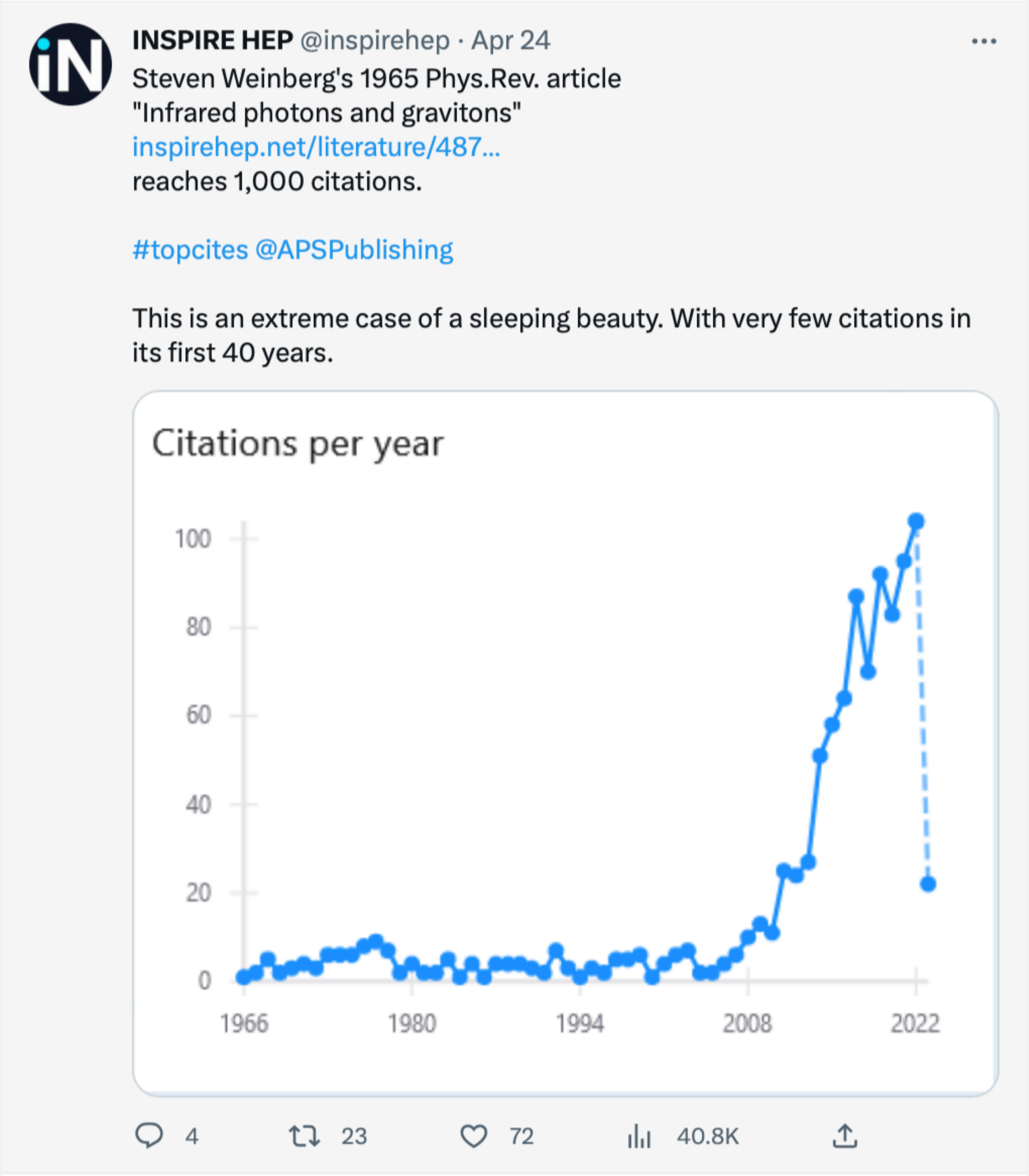
Gardi, Magnea *JHEP* 03 (2009) 079
+Dixon JHEP 02 (2010) 081

…

$$d\sigma \sim H(\{s_{ij}\},\mu)\prod_i J_i(M_i^2,\mu)\otimes S(\{\Lambda_{ij}^2\},\mu)$$

Strong constraints on the anomalous dimension $\boldsymbol{\Gamma}$

- Soft-collinear factorization
- Collinear limits, forward limit, …
- Non-abelian exponentiation for S

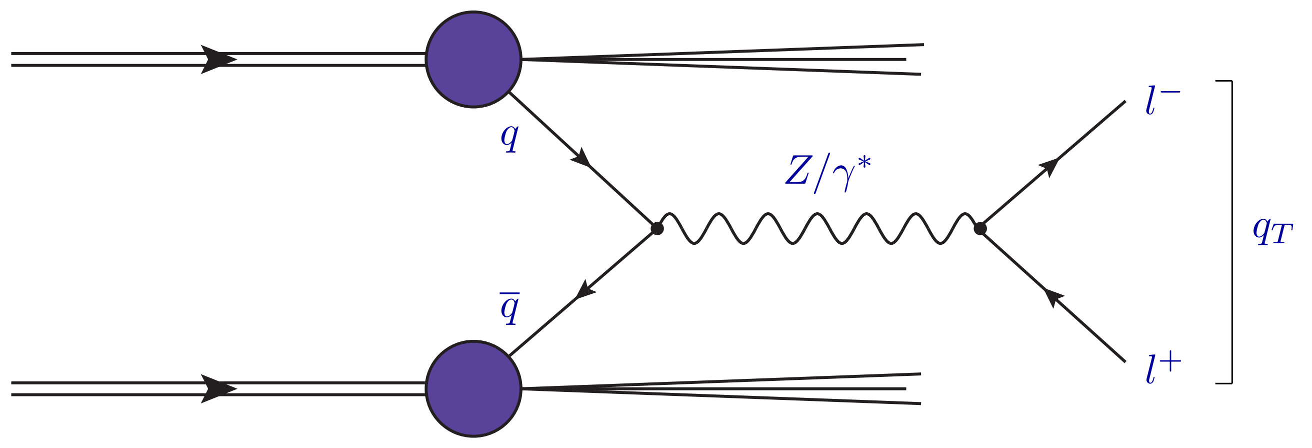


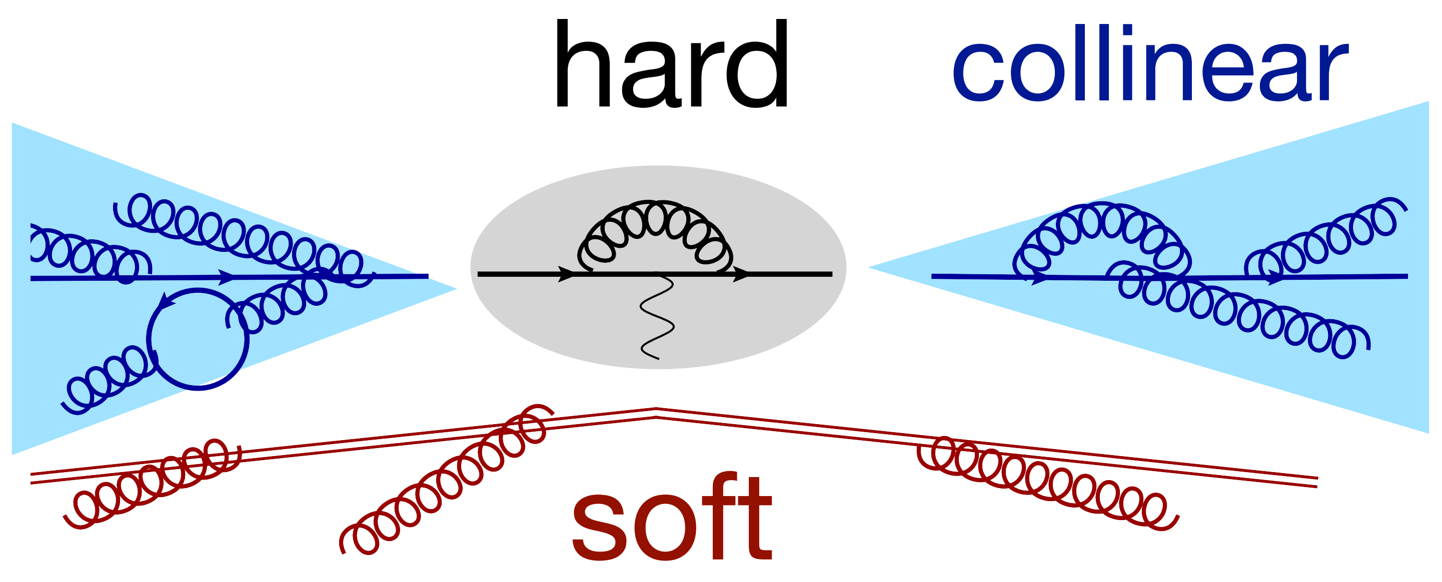
a very active reseach area!

see talks by
Einan, Leonardo,
Lorenzo and Ze Long

Rigi October 2008

q_T resummation for Z production





- Factorization theorem

$$\frac{d\sigma}{dq_T dy} = \sum_{ab=q,\bar{q}} \int d^2x_\perp \, e^{-iq_\perp \cdot x_\perp} H_{ab}(Q^2,\mu) \mathcal{B}_a(\xi_1, x_T^2, \mu) \, \mathcal{B}_b(\xi_2, x_T^2, \mu) \mathcal{S}(x_T^2, \mu)$$

- Ingredients fulfil RG evolution equations
- S corresponds to emissions off initial hard $q\bar{q}$-pair
- Additional large logarithms in BBS

Collinear Anomaly

TB, Neubert, Eur.Phys.J.C 71 (2011)

$\mathcal{J}$ and $\mathcal{S}$ need additional regulator α

pure power

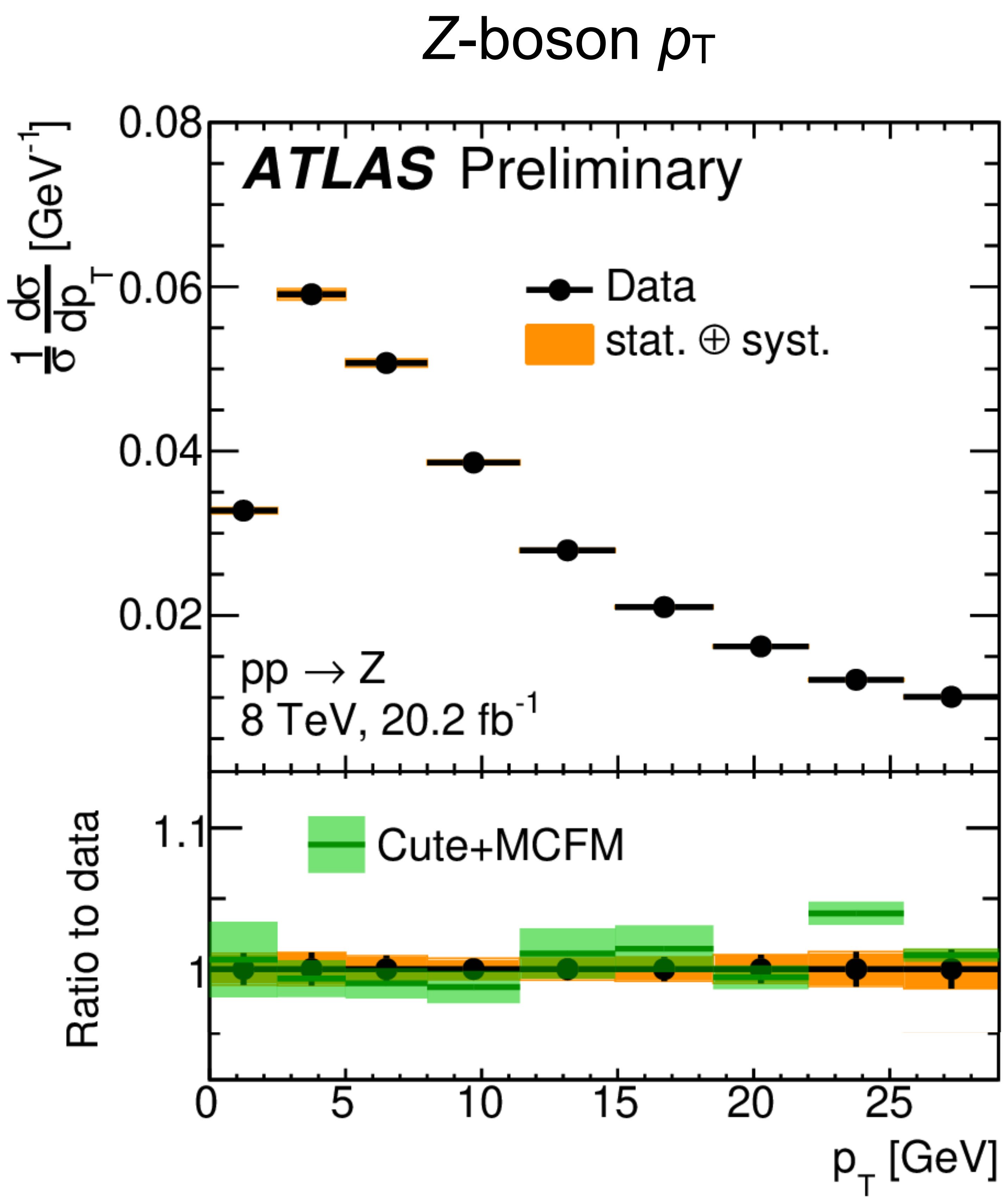
anomaly exponent

$$\mathcal{J}(\mu)\,\mathcal{J}(\mu)\,\mathcal{S}(\mu) = \left(\frac{Q^2}{\mu^2}\right)^{-F(\mu)} W(\mu)$$

but product well-defined in limit $\alpha \to 0$

remainder Q-indep

Anomaly: additional regularization breaks rescaling symmetry of SCET, which would guarantee Q-independence. Symmetry is not recovered in the limit $\alpha \to 0$.



CuTe TB and Neubert, Wilhelm
JHEP 02 (2012) 124

Event based resummation TB, Hager 2019

CuTe-MCFM TB and Neumann ’20
$\mathrm{N^4LL_p}$ + NNLO Neumann, Campbell ’22

see also: Chen et. al., 2203.01565
Radish+NNLOJet

Comparison plot from ALTAS LHC Seminar by S. Camarda, April 18, 2023

Resummations up to $\mathrm{N^4LL}$ (3-loop matching, 4-loop anomalous dimensions!), but only for very inclusive observables (`global event shapes”)

Channel islands, Santa Barbara, 2011

Traditional resummation methods (such as SCET) restricted to global observables which do not involve angular cuts on hadronic radiation.

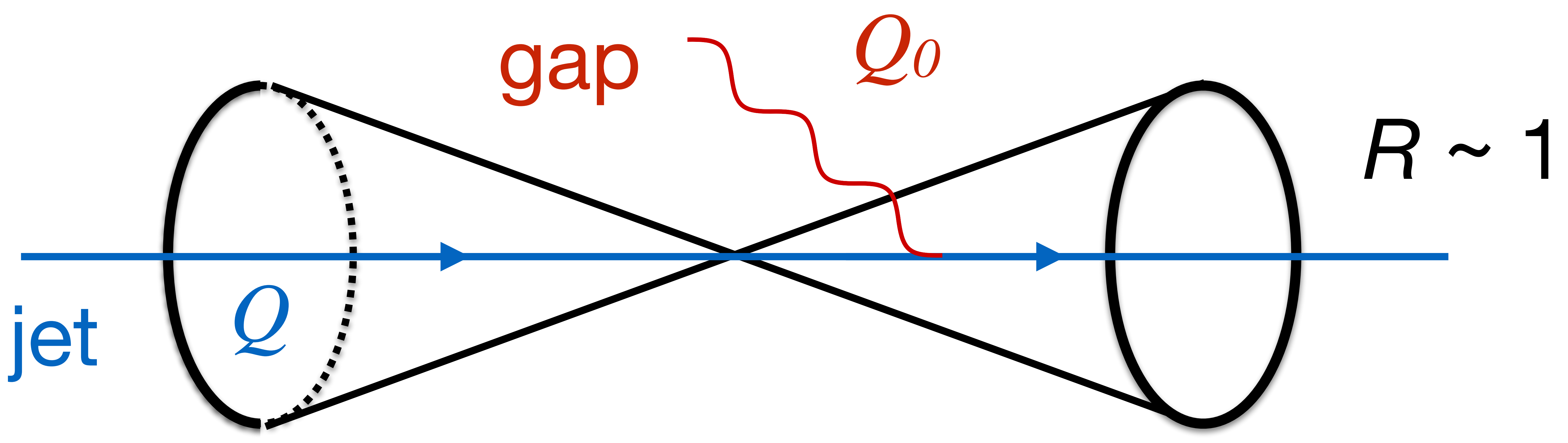
Non-global observables such as

- jet cross sections or isolation-cone cross sections (relevant for γ production)

involve very intricate structure of soft radiation

- secondary emissions: non-global logarithms (NGLs) Dasgupta, Salam ’02
- hadronic collisions: complex phases & breakdown of color coherence: super-leading logarithms SLL Forshaw, Kyrieleis, Seymour ’06

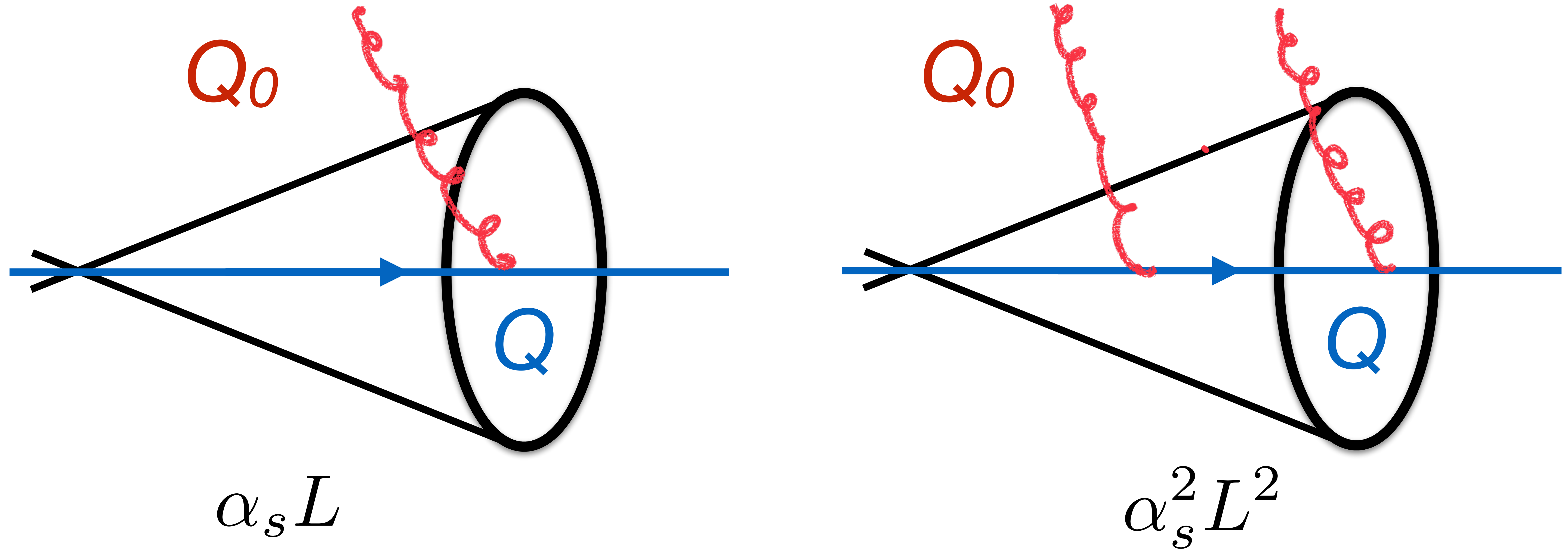
Simplest example of non-global observable: gap between jets aka interjet energy flow aka rapidity slice



→ large logarithms $\alpha_s^n L^m$ with $L = \ln(Q/Q_0)$

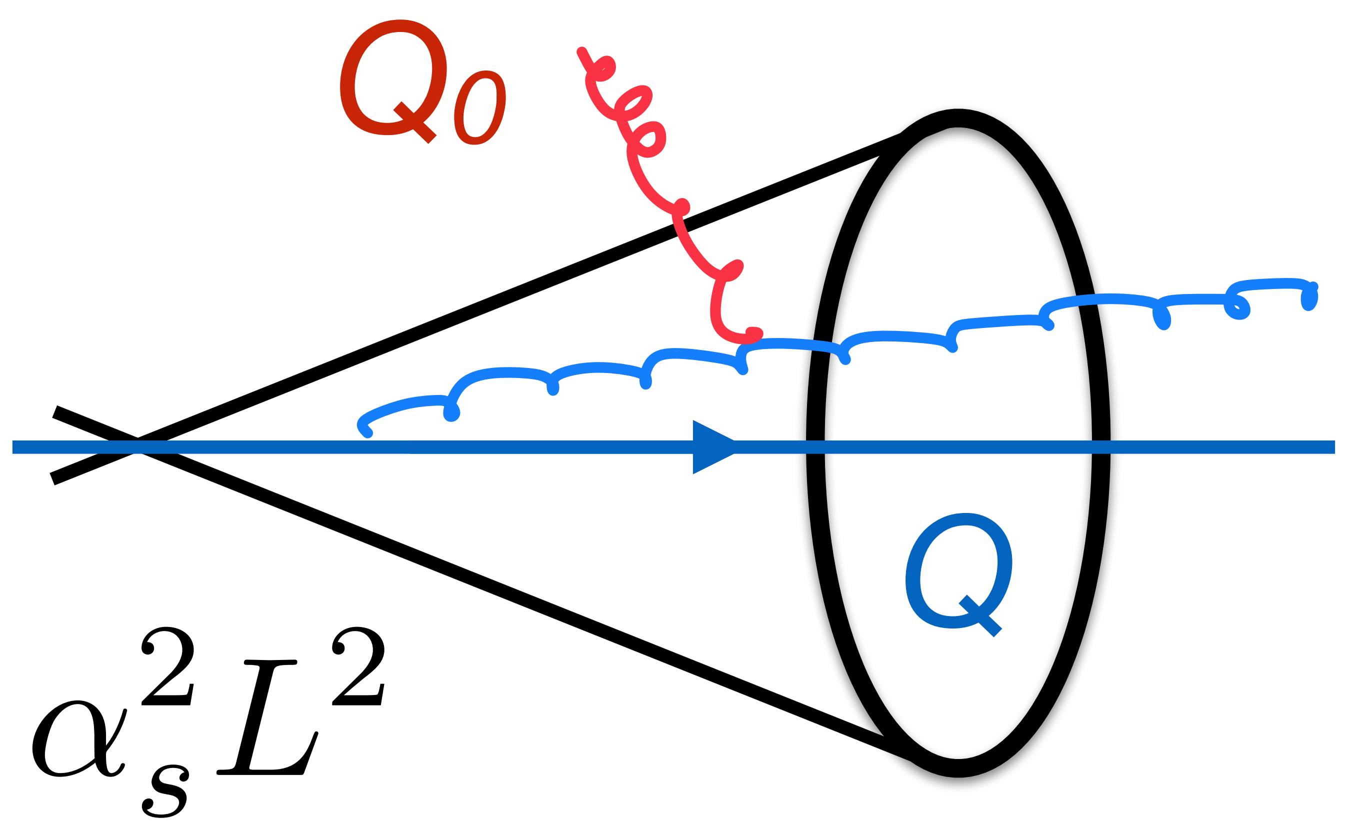
Will discuss case of large cone radius R ~ 1.

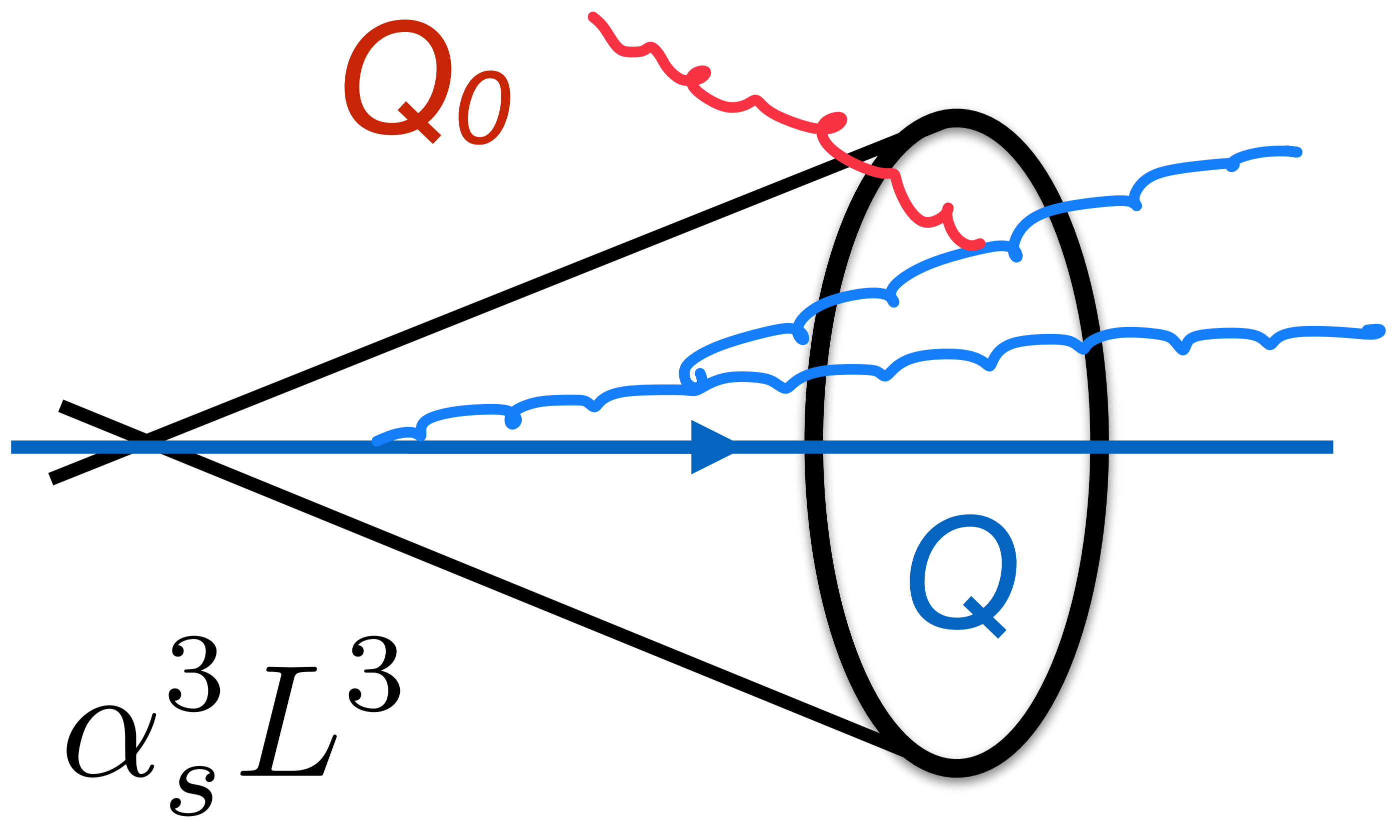
Global Logarithms



- In (massive) QED, logarithmic terms would exponentiate: full result is exponential of one loop!
- For global observables in QCD, non-abelian higher-order corrections ("non-abelian exponentiation")

Non-global logarithms (NGLs)





- Soft gluons from secondary emissions inside the jets
- Not captured by standard resummation methods. Even leading NGLs $(\alpha_s L)^n$ do not simply exponentiate!
- At large N_c leading NGLs can be obtained with parton shower Dasgupta, Salam '02 or by solving a non-linear integral equation Banfi, Marchesini, Smye '02, the BMS equation

Factorization for gap between jets in e^+e^-

TB, Neubert, Rothen, Shao Phys.Rev.Lett. 116 (2016) 19, 192001, see also Caron-Huot '15

Hard function
m hard partons along
fixed directions $\{n_1, \ldots, n_m\}$
$\boldsymbol{\mathcal{H}}_m \propto |\mathcal{M}_m\rangle\langle\mathcal{M}_m|$

Soft function
squared amplitude
with *m* Wilson lines

$$\sigma(Q, Q_0) = \sum_{m=2}^{\infty} \langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, Q, \mu) \otimes \boldsymbol{\mathcal{S}}_m(\{\underline{n}\}, Q_0, \mu) \rangle$$

color trace

integration over directions

Soft emissions are obtained from the matrix elements of the Wilson-line operators

$$\boldsymbol{S}_1(n_1)\,\boldsymbol{S}_2(n_2)\;\ldots\;\boldsymbol{S}_m(n_m)|\mathcal{M}_m(\{\underline{p}\})\rangle$$

Wilson lines: $\boldsymbol{S}_i(n_i) = \exp\left[ig_s\int_0^\infty ds\, n_i\cdot A^a(sn_i)\,\boldsymbol{T}_i^a\right]$

(For outgoing particle! Incoming has integration from -∞ to 0)

To get the amplitudes with additional soft partons, one takes the matrix element of the multi-Wilson-line operators:

$$\langle X_s|\,\boldsymbol{S}_1(n_1)\;\ldots\;\boldsymbol{S}_m(n_m)\,|0\rangle$$

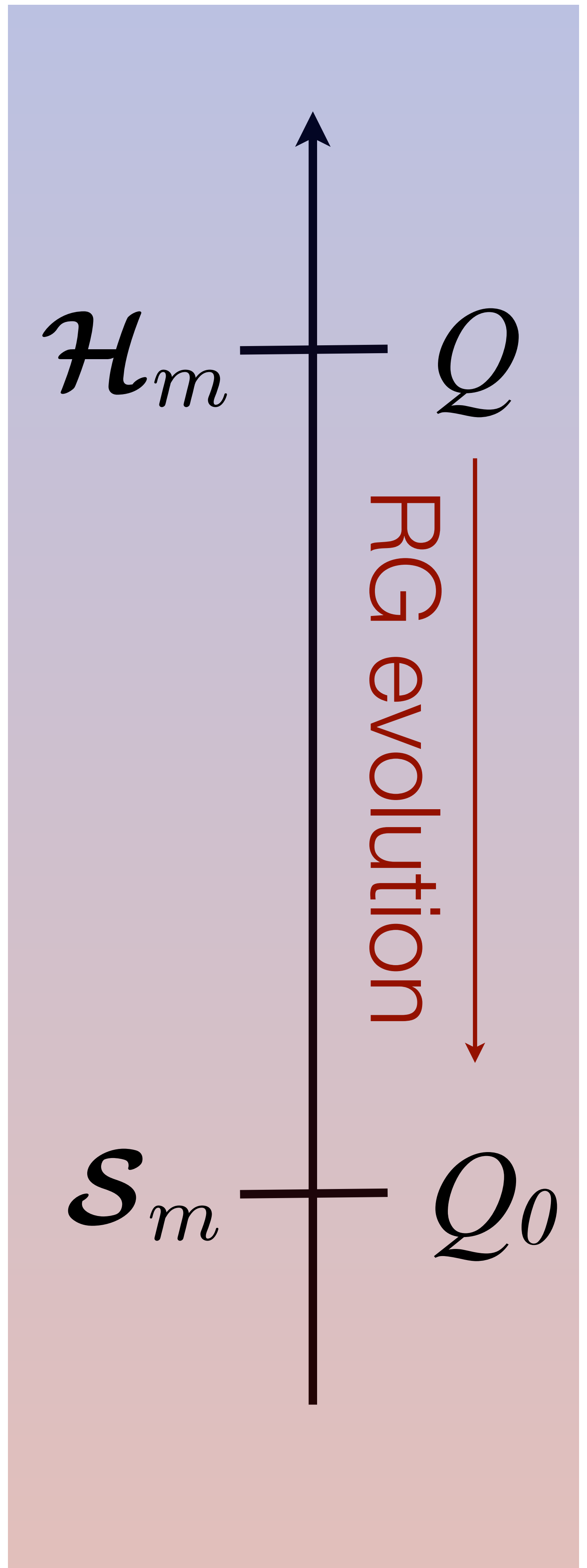
Resummation by RG evolution

Wilson coefficients fulfill RG equations

$$\frac{d}{d\ln\mu}\boldsymbol{\mathcal{H}}_m(Q,\mu) = -\sum_{l=2}^{m}\boldsymbol{\mathcal{H}}_l(Q,\mu)\,\boldsymbol{\Gamma}^H_{lm}(Q,\mu)$$

1. Compute $\mathcal{H}_\mathrm{m}$ at a characteristic high scale $\mu_h \sim Q$
2. Evolve $\mathcal{H}_\mathrm{m}$ to the scale of low energy physics $\mu_s \sim Q_0$
3. Evaluate $\mathcal{S}_\mathrm{m}$ at low scale $\mu_s \sim Q_0$

Avoids large logarithms $\alpha_s^n \ln^n(Q/Q_0)$ of scale ratios which spoil convergence of perturbation theory.



RG = Parton Shower

- Ingredients for LL

$$\boldsymbol{\mathcal{H}}_2(\mu = Q) = \sigma_0$$
$$\boldsymbol{\mathcal{H}}_m(\mu = Q) = 0 \text{ for } m > 2$$
$$\boldsymbol{\mathcal{S}}_m(\mu = Q_0) = 1$$

$$\boldsymbol{\Gamma}^{(1)} = \begin{pmatrix} \boldsymbol{V}_2 & \boldsymbol{R}_2 & 0 & 0 & \dots \\ 0 & \boldsymbol{V}_3 & \boldsymbol{R}_3 & 0 & \dots \\ 0 & 0 & \boldsymbol{V}_4 & \boldsymbol{R}_4 & \dots \\ 0 & 0 & 0 & \boldsymbol{V}_5 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

- RG

$$\frac{d}{dt}\boldsymbol{\mathcal{H}}_m(t) = \boldsymbol{\mathcal{H}}_m(t)\boldsymbol{V}_m + \boldsymbol{\mathcal{H}}_{m-1}(t)\boldsymbol{R}_{m-1}\,.$$

shower evolution time

$$t \equiv t(\mu_h, \mu_s) = \int_{\alpha_s(\mu_s)}^{\alpha_s(\mu_h)} \frac{d\alpha}{\beta(\alpha)}\frac{\alpha}{4\pi}$$

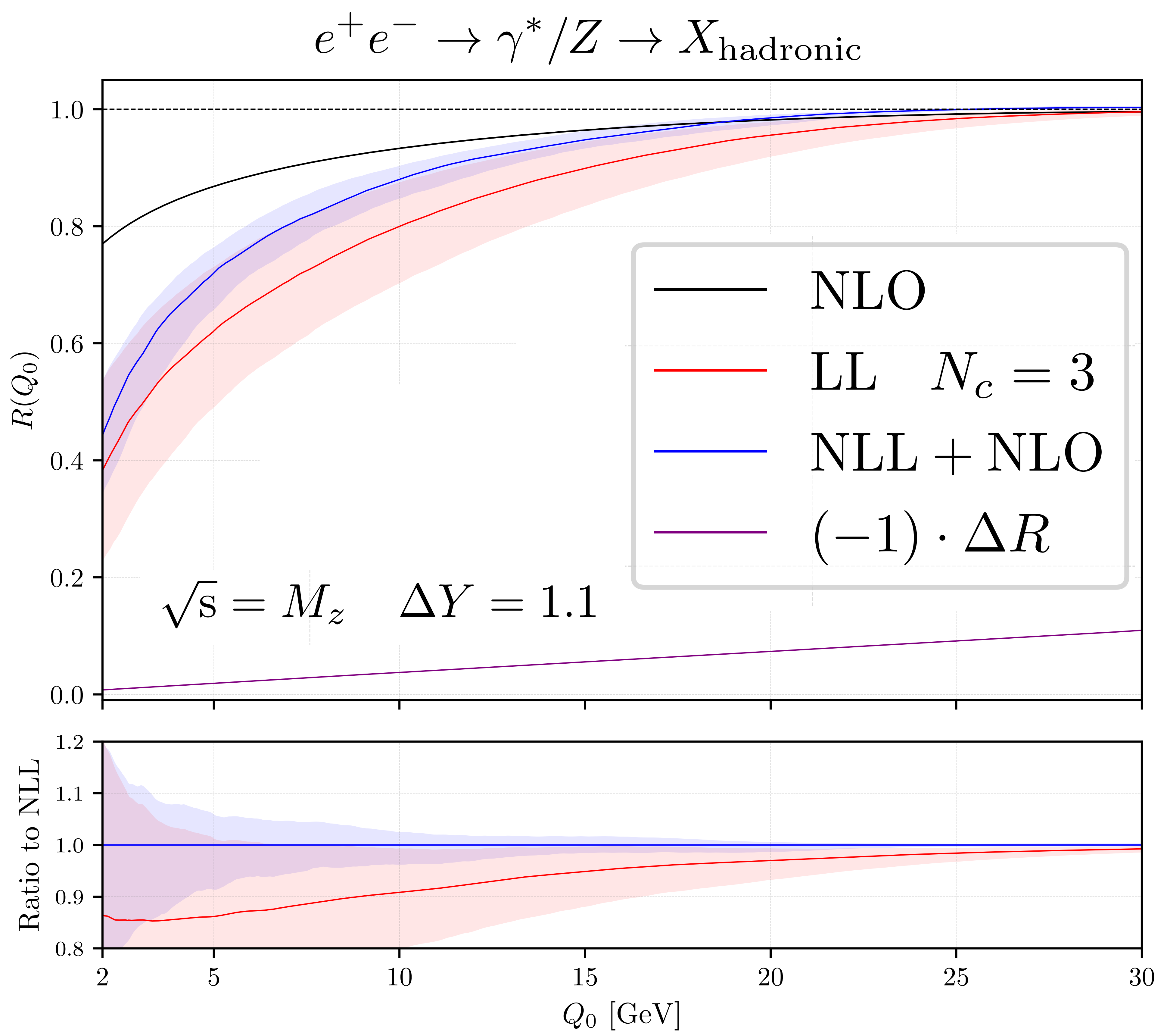
- equivalent to parton shower equation

$$\boldsymbol{\mathcal{H}}_m(t) = \boldsymbol{\mathcal{H}}_m(t_1)e^{(t-t_1)V_n} + \int_{t_1}^{t} dt'\boldsymbol{\mathcal{H}}_{m-1}(t')\boldsymbol{R}_{m-1}e^{(t-t')\boldsymbol{V}_n}$$

Progress on NGLs

- PanScales, a general-purpose shower, which correctly resums leading large-N_c NGLs (and global logs!) Dasgupta, Dreyer, Hamilton, Monni, Salam and Soyez ‘20, + … ,’21 Alaric Herren, Höche, Krauss, Reichelt, Schoenherr ‘22
- Finite-N_c results for leading NGLs in e^+e^- Hatta, Ueda ’13 + Hagiwara ‘15 based on Weigert ’03; De Angelis, Forshaw and Plätzer ’20
- First NLL numerical results in the large-N_c limit
  - Extension of BMS framework to NLL (2104.06416) and numerical implementation in MC code `Gnole` (2111.02413) Banfi, Dreyer, Monni
  - Two-loop anomalous dimension in factorization framework TB, Rauh, Xu, 2112.02108; implementation into shower code TB, Schalch, Xu, in preparation

Next-to-leading non-global logarithms



Ingredients:

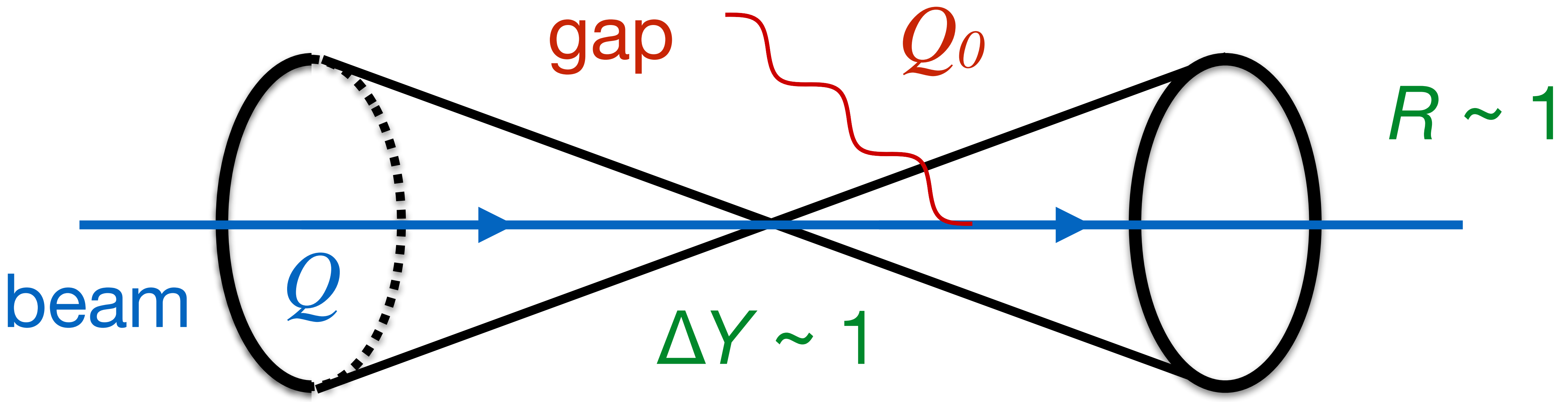
- $N_c = 3$ leading logs obtained from Hatta, Ueda '13.
- Two-loop anomalous dimension $\boldsymbol{\Gamma}^{(2)}$ TB, Rauh, Xu, '21
- Implementation of $\boldsymbol{\Gamma}^{(2)}$ in parton shower framework TB, Schalch, Xu, in preparation

Corrections scale as $\mathcal{O}(\alpha_s^2)$ or $\mathcal{O}(\alpha_s/N_c^2)$ terms.
First NGL resummation at this accuracy level!

Super-Leading Logs (SLLs)

Forshaw, Kyrieleis, Seymour ’06 ’08

Analyze gap between jets at hadron collider, cone around beam direction



Large logarithms $\alpha_s^n L^m$ with $L = \ln(Q/Q_0)$

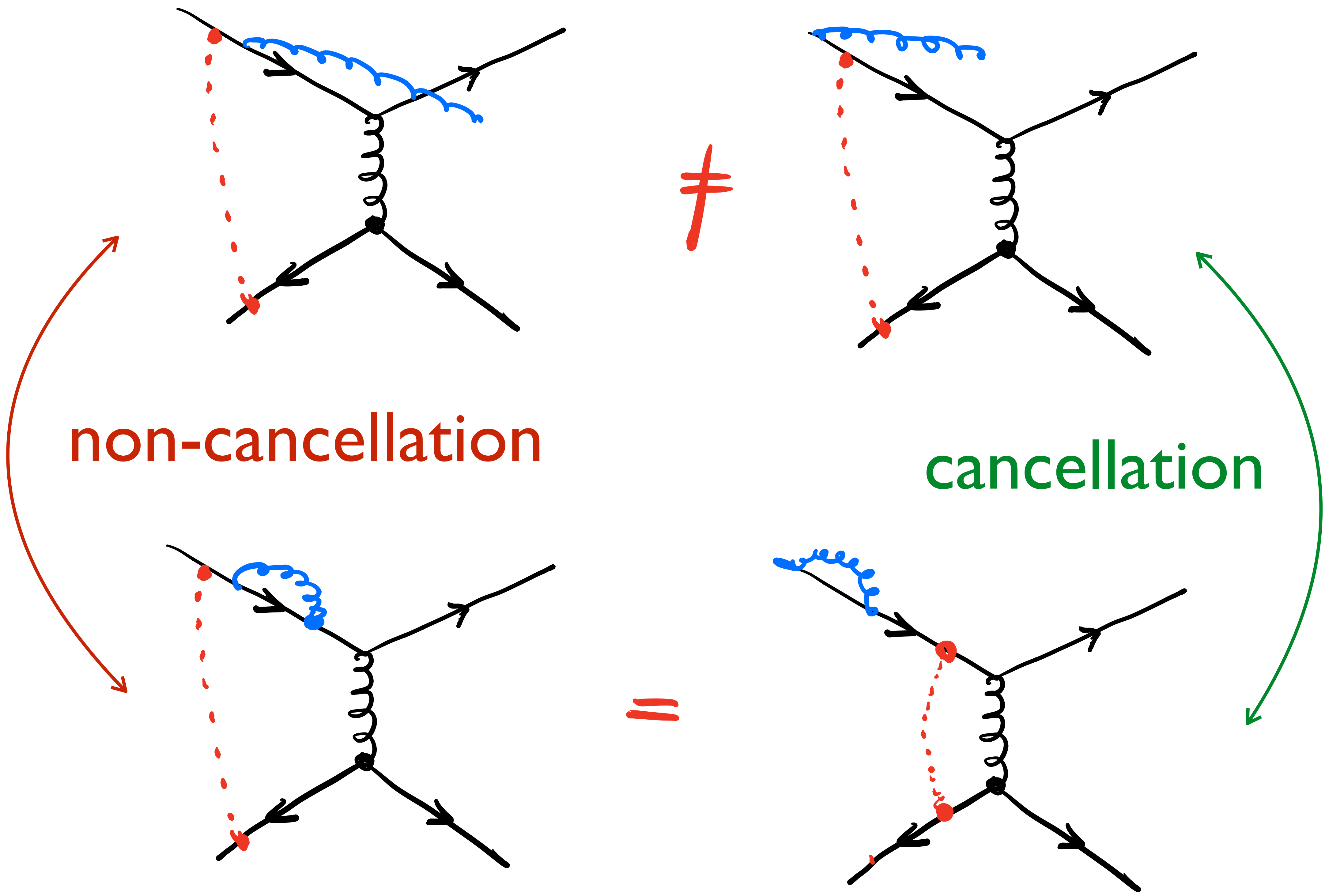
- e^+e^- : $m \leq n$, leading logs $m = n$
- pp : $\alpha_s L,\ \alpha_s^2 L^2,\ \alpha_s^3 L^3,\ \alpha_s^4 L^5 \ldots,\ \alpha_s^{3+n} L^{3+2n}$

missing in large-N_c parton showers!
(Deductor? Soper and Nagy … ’19)

Non-cancellation of collinear logs

Forshaw, Kyrieleis, Seymour ’06 ’08; Catani, de Florian, Rodrigo ’11, …

Double logarithms due to soft+collinear configurations.



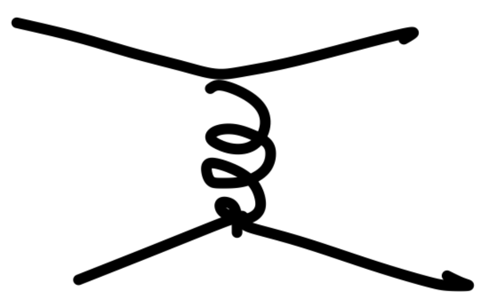
Blue: collinear emission. Red: Glauber/Coulomb phase

Note: Glauber phases cancel in e^+e^- and in large-N_c limit

Earlier results on SLLs

Since effect first arises at $O(\alpha_s^4)$, only few results

- Discovery of effect, computation of first SLL in gaps between jets for $qq \to qq$ Forshaw, Kyrieleis, Seymour '06
- Colour space calculation of leading SLL Forshaw, Kyrieleis, Seymour '08
    - Note that SLLs vanish in the large-N_c limit.
- Diagrammatic calculation, first *two* orders, different channels *qq, qg, gg* Keates and Seymour '09



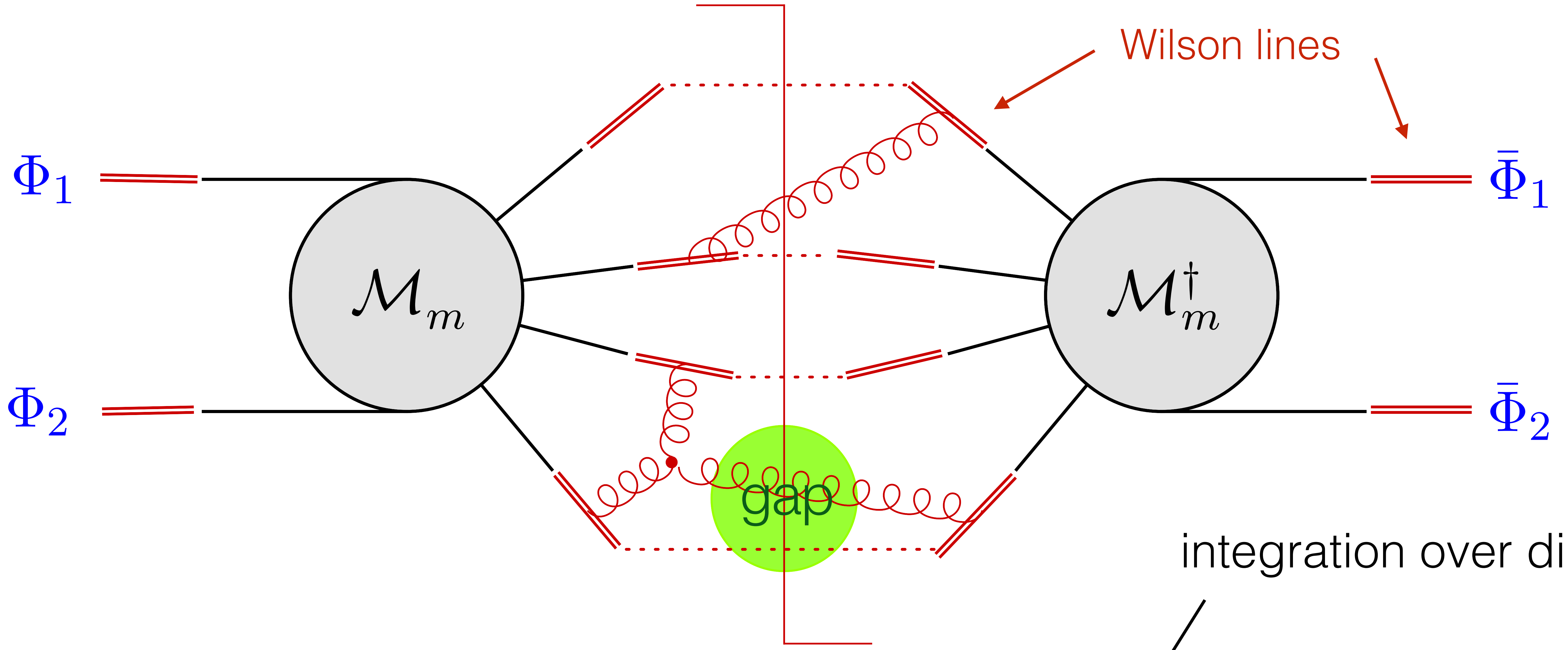
$$S_O^{(4)} = \left(\frac{\alpha_s}{4\pi}\right)^4 L_Q^5 \, \Delta Y \, \pi^2 \, \frac{8}{15} \left(3N_c^2 - 4\right) \sigma_0 \,,$$

$$S_O^{(5)} = \left(\frac{\alpha_s}{4\pi}\right)^5 L_Q^7 \, \Delta Y \, \pi^2 \, \frac{4}{315} \, N_c \left(-27N_c^2 + 44\right) \sigma_0$$

Glauber $(i\pi)^2$

Factorization for hadronic collisions

TB, Neubert, Shao Phys.Rev.Lett. 127 (2021) 21, 212002 + Stillger, in preparation



integration over directions

$$\sigma_{2\to M}(Q_0) = \int dx_1 \int dx_2 \sum_{m=2+M}^{\infty} \langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, x_1, x_2, s, \mu) \otimes \boldsymbol{\mathcal{W}}_m(\{\underline{n}\}, Q_0, x_1, x_2, \mu) \rangle$$

Hard functions
m hard partons along
fixed directions $\{n_1, \ldots, n_m\}$

$$\boldsymbol{\mathcal{H}}_m \propto |\mathcal{M}_m\rangle\langle\mathcal{M}_m|$$

Soft + collinear function
squared amplitude
for *m* Wilson lines
+collinear fields

Remarks

Glauber

s+*c* interactions

Stewart, Rothstein '16

- Effective theory

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SCET}} = \mathcal{L}_{c_1} + \mathcal{L}_{c_2} + \mathcal{L}_s + \mathcal{L}_G$$

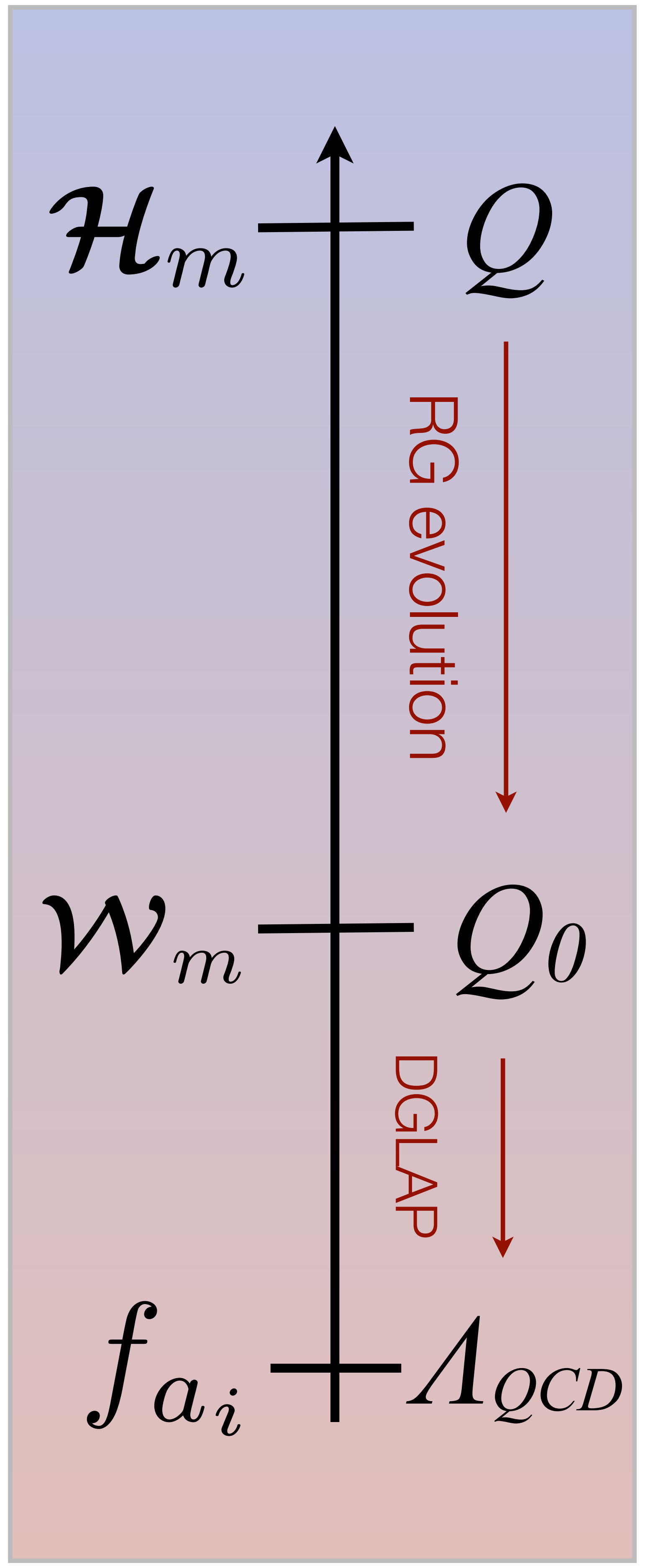
  - Additional regulator
  - Low energy matrix elements $\boldsymbol{\mathcal{W}}_m$ will suffer from rapidity logarithms

- RG evolution

Mellin convolution

$$\frac{d}{d\ln\mu}\boldsymbol{\mathcal{H}}_m(\{\underline{n}\},s,\mu) = -\sum_{l=2+M}^{m}\boldsymbol{\mathcal{H}}_l(\{\underline{n}\},s,\mu)\star\boldsymbol{\Gamma}^H_{lm}(\{\underline{n}\},s,\mu)$$

  - mixes multiplicities + colors!



One-loop anomalous dimension

$$\mathbf{\Gamma}^H\left(\{\underline{n}\},\xi_1,\xi_2,s,\mu\right)=\frac{\alpha_s}{4\pi}\mathbf{\Gamma}^{(1)}=\frac{\alpha_s}{4\pi}\begin{pmatrix}\boldsymbol{V}_k & \boldsymbol{R}_k & 0 & 0 & \dots \\ 0 & \boldsymbol{V}_{k+1} & \boldsymbol{R}_{k+1} & 0 & \dots \\ 0 & 0 & \boldsymbol{V}_{k+2} & \boldsymbol{R}_{k+2} & \dots \\ 0 & 0 & 0 & \boldsymbol{V}_{k+3} & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots\end{pmatrix}$$

k: number of partons at Born-level

- Split into soft(+collinear) and purely collinear

$$\mathbf{\Gamma}^{(1)}(\xi_1,\xi_2)=\mathbf{\Gamma}_1^C(\xi_1)\delta(1-\xi_2)+\delta(1-\xi_1)\mathbf{\Gamma}_2^C(\xi_2)+\delta(1-\xi_1)\,\delta(1-\xi_2)\,\mathbf{\Gamma}^S$$

- Split soft part

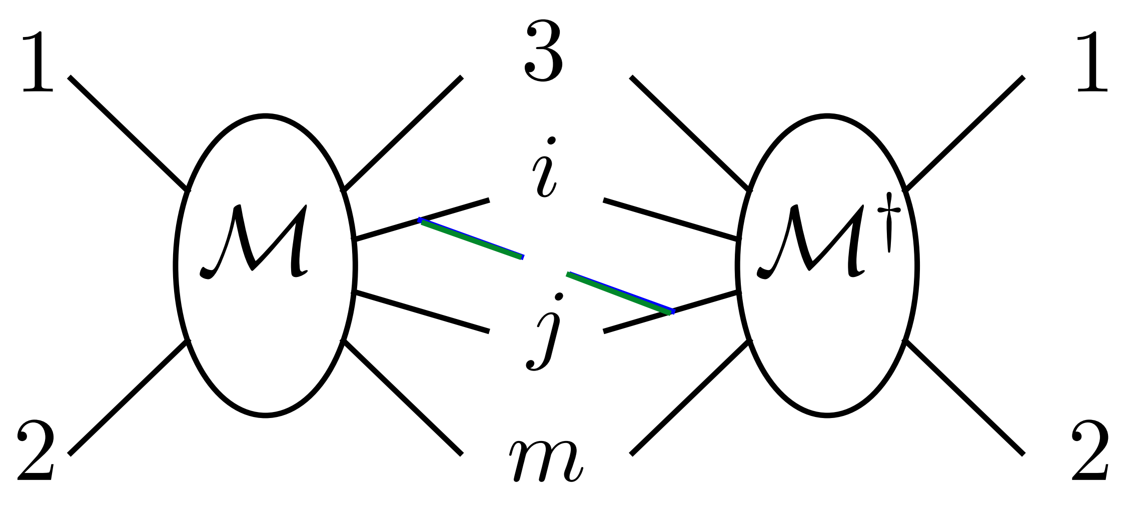
$$\mathbf{\Gamma}^S=\overline{\mathbf{\Gamma}}+\mathbf{\Gamma}^G+\mathbf{\Gamma}^c\ln\frac{\mu^2}{\hat{s}}$$

wide-angle soft   Glauber   cusp: soft+collinear

see also Forshaw, Holguin, and Plätzer '19

Soft wide-angle emissions $\overline{\Gamma}$

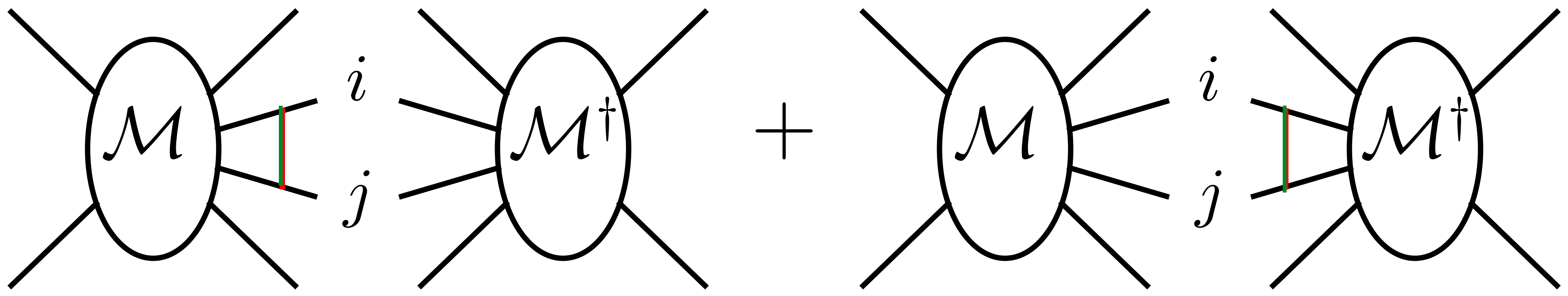
$$\mathcal{H}_m \overline{\boldsymbol{R}}_m = \sum_{(ij)}$$



$$\overline{\boldsymbol{R}}_m = -4 \sum_{(ij)} \boldsymbol{T}_{i,L} \circ \boldsymbol{T}_{j,R}\, \overline{W}_{ij}^{m+1}\, \Theta_{\text{hard}}(n_{m+1})$$

extra hard parton!

$$\mathcal{H}_m \overline{\boldsymbol{V}}_m = \sum_{(ij)}$$



$$\overline{\boldsymbol{V}}_m = 2 \sum_{(ij)} \left(\boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,L} + \boldsymbol{T}_{i,R} \cdot \boldsymbol{T}_{j,R} \right) \int \frac{d\Omega(n_k)}{4\pi}\, \overline{W}_{ij}^{k}$$

soft dipole

$$W_{ij}^{q} = \frac{n_i \cdot n_j}{n_i \cdot n_q\, n_j \cdot n_q}$$

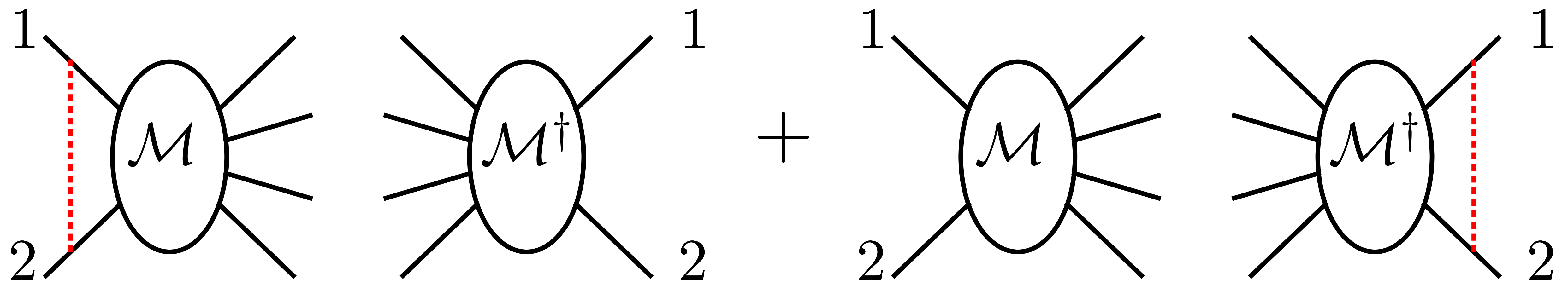
soft dipole with collinear subtraction

$$\overline{W}_{ij}^{q} = W_{ij}^{q} - \frac{1}{n_i \cdot n_q}\delta(n_i - n_q) - \frac{1}{n_j \cdot n_q}\delta(n_j - n_q)$$

see Forshaw, Holguin, and Plätzer ‘19

Glauber term $\boldsymbol{\Gamma}^G$

$$\boldsymbol{\mathcal{H}}_m\, \boldsymbol{V}^G =$$



$$\boldsymbol{V}^G = -8i\pi\left(\boldsymbol{T}_{1,L}\cdot\boldsymbol{T}_{2,L} - \boldsymbol{T}_{1,R}\cdot\boldsymbol{T}_{2,R}\right)$$

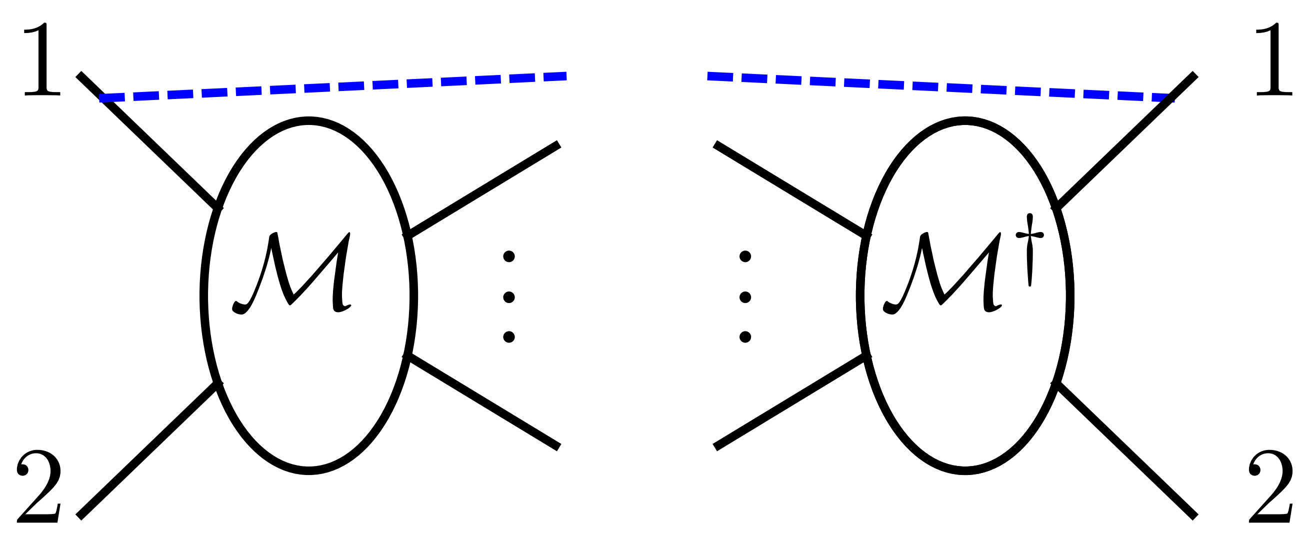
Used color conservation $\sum_i \boldsymbol{T}_i = 0$ to simplify Glauber terms in 1 + 2 → 3 + … + *m*

Π_{ij} = 1 if both inc./out.

$$\sum_{(ij)}\left(\boldsymbol{T}_{i,L}\cdot\boldsymbol{T}_{j,L} - \boldsymbol{T}_{i,R}\cdot\boldsymbol{T}_{j,R}\right)\Pi_{ij} = 4\left(\boldsymbol{T}_{1,L}\cdot\boldsymbol{T}_{2,L} - \boldsymbol{T}_{1,R}\cdot\boldsymbol{T}_{2,R}\right)$$

(Soft+)Collinear Cusp Term $\mathbf{\Gamma}^c$

$$\boldsymbol{\mathcal{H}}_m\, \boldsymbol{R}_1^c \; =$$



$$\boldsymbol{R}_i^c = -4\boldsymbol{T}_{i,L} \circ \boldsymbol{T}_{i,R}\, \delta(n_{m+1} - n_i)$$
$$\boldsymbol{V}_i^c = 4C_i\, \mathbf{1}$$

- Only present for initial-state partons i=1,2. Final state terms cancel!
- Multiplied by $\ln \frac{\mu^2}{\hat{s}}$ → double logarithms!

Computation of SLLs

Cannot use large N_c: compute order by order

$$\left\langle \boldsymbol{\mathcal{H}}_4\, \boldsymbol{U}(\{\underline{n}\},\mu_s,\mu_h) \,\hat{\otimes}\, \mathbf{1} \right\rangle = \left\langle \boldsymbol{\mathcal{H}}_4\, \mathbf{P}\exp\left[\int_{\mu_s}^{\mu_h} \frac{d\mu}{\mu} \boldsymbol{\Gamma}(\{\underline{n}\},\mu)\right] \hat{\otimes}\, \mathbf{1} \right\rangle$$

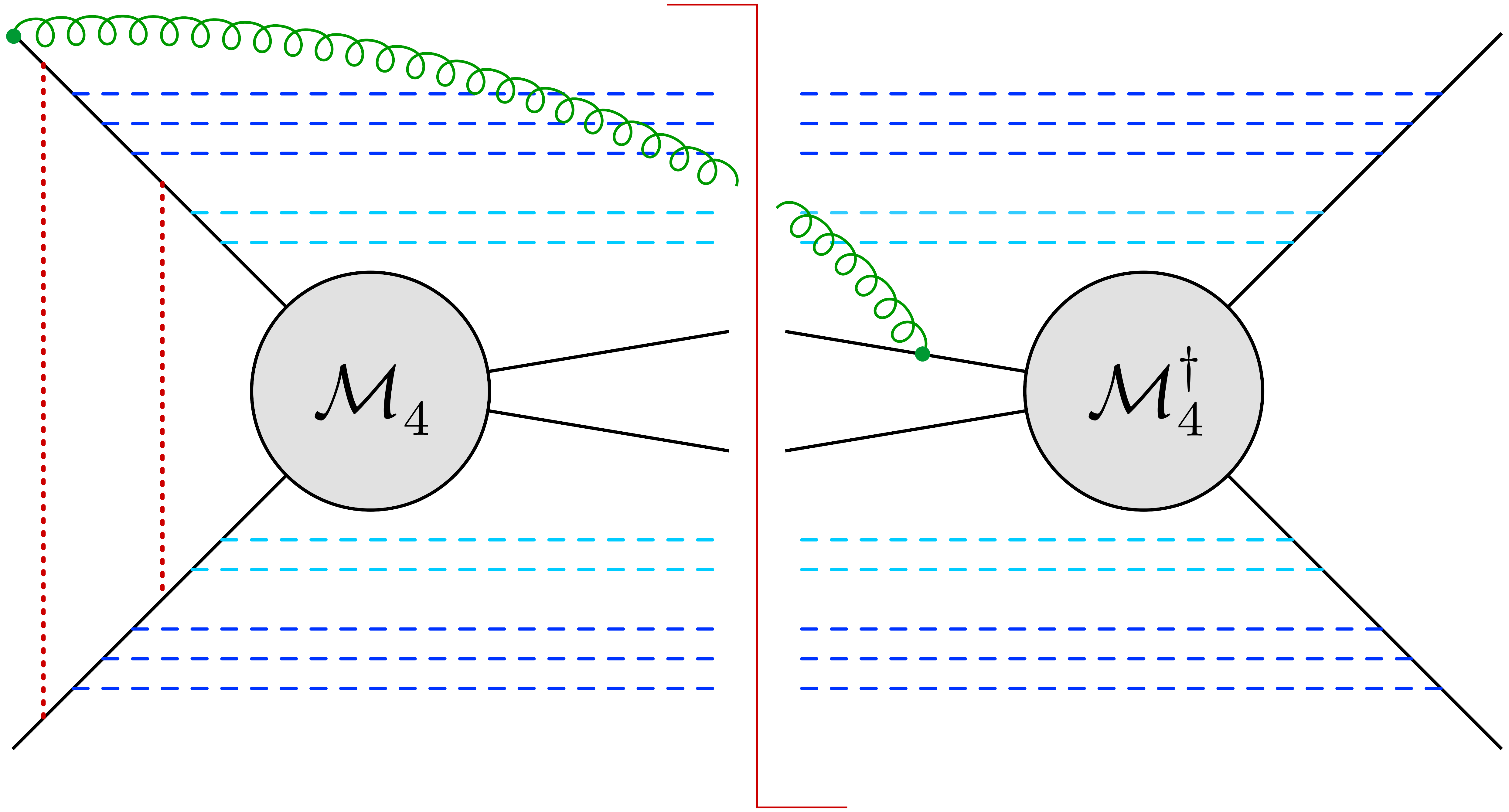
$$= \langle \boldsymbol{\mathcal{H}}_4 \rangle + \int_{\mu_s}^{\mu_h} \frac{d\mu}{\mu} \langle \boldsymbol{\mathcal{H}}_4\, \boldsymbol{\Gamma}(Q,\mu) \hat{\otimes} \mathbf{1} \rangle + \int_{\mu_s}^{\mu_h} \frac{d\mu}{\mu} \int_{\mu}^{\mu_h} \frac{d\mu'}{\mu'} \langle \boldsymbol{\mathcal{H}}_4 \boldsymbol{\Gamma}(Q,\mu)\, \boldsymbol{\Gamma}(Q,\mu') \hat{\otimes}\, \mathbf{1} \rangle + \dots$$

$\hat{\sigma}_{\rm LO}$ $\qquad$ $\alpha_s L$ $\qquad$ $\alpha_s^2 L^2$

Need products of anomalous dimensions. Each μ integral produces single log ($\overline{\boldsymbol{\Gamma}}$, $\boldsymbol{\Gamma}^G$) or double logs ($\boldsymbol{\Gamma}^c$), i.e. SLLs!

Will set μ_h=Q and μ_s=Q_0 and ignore running of α_s.

$$C_{r,n} = \langle \boldsymbol{\mathcal{H}}_4 \, (\boldsymbol{\Gamma}^c)^r \, \boldsymbol{\Gamma}^G (\boldsymbol{\Gamma}^c)^{n-r} \, \boldsymbol{\Gamma}^G \, \overline{\boldsymbol{\Gamma}} \otimes \mathbf{1} \rangle \qquad 0 \le r \le n$$

$C_{4,10} =$ 

Properties

$$[\boldsymbol{\Gamma}^c, \overline{\boldsymbol{\Gamma}}] = 0$$

$$\langle \boldsymbol{\mathcal{H}}_m \, \boldsymbol{\Gamma}^c \otimes \mathbf{1} \rangle = 0$$

$$\langle \boldsymbol{\mathcal{H}}_m \, \boldsymbol{V}^G \otimes \mathbf{1} \rangle = 0$$

+ many more diagrams: Glauber(s) on the right side, different attachents for wide-angle soft, virtuals …

Evaluation of C_{rn}

- Basic strategy: commute $\mathbf{\Gamma}^{c}$'s and $\mathbf{\Gamma}^{G}$ to the right where they vanish.
- After a lot of color algebra, one finds

$$C_{rn} = -16\,(4\pi)^2\,(4N_c)^n \sum_{i=1}^{7} v_i^r \left\langle \boldsymbol{\mathcal{H}}_{2\to M}\, \boldsymbol{Q}_i \right\rangle$$

power-like *n* and *r* dependence

- Eigenvalues

$$v_1 = 0\,, \quad v_2 = \frac{1}{2}\,, \quad v_3 = 1\,, \quad v_4 = \frac{3N_c - 2}{2N_c}\,, \quad v_5 = \frac{3N_c + 2}{2N_c}\,, \quad v_6 = \frac{2\,(N_c - 1)}{N_c}\,, \quad v_7 = \frac{2\,(N_c + 1)}{N_c}$$

- Eigenoperators are $\boldsymbol{Q}_i$ are combinations color 10 basic structures.

Eigenoperators, color structures O_i and S_i

$$\boldsymbol{Q}_1 = J_{12}\left[\frac{4N_c}{N_c^2-1}\,C_1C_2\,\boldsymbol{S}_6\right],$$

$$\boldsymbol{Q}_2 = \sum_{j=3}^{M+2} J_j\left[-\frac{N_c}{N_c^2-1}\,\boldsymbol{O}_4^{(j)}\right] + J_{12}\left[\frac{2N_c}{N_c^2-1}\,(C_1+C_2)\,\boldsymbol{S}_5 - \frac{4N_c}{N_c^2-1}\,C_1C_2\,\boldsymbol{S}_6\right],$$

$$\boldsymbol{Q}_3 = \sum_{j=3}^{M+2} J_j\left[-\frac{N_c^2}{2(N_c^2-4)}\,\boldsymbol{O}_2^{(j)}\right] + J_{12}\left[\frac{N_c^2}{N_c^2-4}\,\boldsymbol{S}_3 - \frac{N_c^2}{3}\,\boldsymbol{S}_5\right]$$

$$\begin{aligned}\boldsymbol{Q}_4 = \sum_{j=3}^{M+2} J_j&\left[\frac{1}{2}\boldsymbol{O}_1^{(j)} + \frac{N_c}{4(N_c-2)}\,\boldsymbol{O}_2^{(j)} - \frac{1}{2}\boldsymbol{O}_3^{(j)} + \frac{1}{2(N_c-1)}\,\boldsymbol{O}_4^{(j)}\right]\\ &+ J_{12}\left[\frac{1}{2}\boldsymbol{S}_1 + \frac{N_c}{4(N_c-2)}\,\boldsymbol{S}_2 - \frac{N_c}{2(N_c-2)}\,\boldsymbol{S}_3 - \frac{1}{2}\boldsymbol{S}_4\right.\\ &\quad\left.+\left((C_1+C_2)\,\frac{N_c-2}{N_c-1} + \frac{N_c(N_c-4)}{6}\right)\boldsymbol{S}_5 + \frac{2C_1C_2}{N_c-1}\,\boldsymbol{S}_6\right],\end{aligned}$$

$$\begin{aligned}\boldsymbol{Q}_5 = \sum_{j=3}^{M+2} J_j&\left[\frac{1}{2}\boldsymbol{O}_1^{(j)} + \frac{N_c}{4(N_c+2)}\,\boldsymbol{O}_2^{(j)} + \frac{1}{2}\boldsymbol{O}_3^{(j)} + \frac{1}{2(N_c+1)}\,\boldsymbol{O}_4^{(j)}\right]\\ &+ J_{12}\left[\frac{1}{2}\boldsymbol{S}_1 + \frac{N_c}{4(N_c+2)}\,\boldsymbol{S}_2 - \frac{N_c}{2(N_c+2)}\,\boldsymbol{S}_3 + \frac{1}{2}\boldsymbol{S}_4\right.\\ &\quad\left.+\left(-(C_1+C_2)\,\frac{N_c+2}{N_c+1} + \frac{N_c(N_c+4)}{6}\right)\boldsymbol{S}_5 + \frac{2C_1C_2}{N_c+1}\,\boldsymbol{S}_6\right],\end{aligned}$$

$$\boldsymbol{Q}_6 = -J_{12}\left[\frac{1}{2}\boldsymbol{S}_1 + \frac{N_c}{4(N_c-2)}\,\boldsymbol{S}_2 - \frac{1}{2}\boldsymbol{S}_4 + \frac{2C_1C_2}{N_c-1}\,\boldsymbol{S}_6\right],$$

$$\boldsymbol{Q}_7 = -J_{12}\left[\frac{1}{2}\boldsymbol{S}_1 + \frac{N_c}{4(N_c+2)}\,\boldsymbol{S}_2 + \frac{1}{2}\boldsymbol{S}_4 + \frac{2C_1C_2}{N_c+1}\,\boldsymbol{S}_6\right].$$

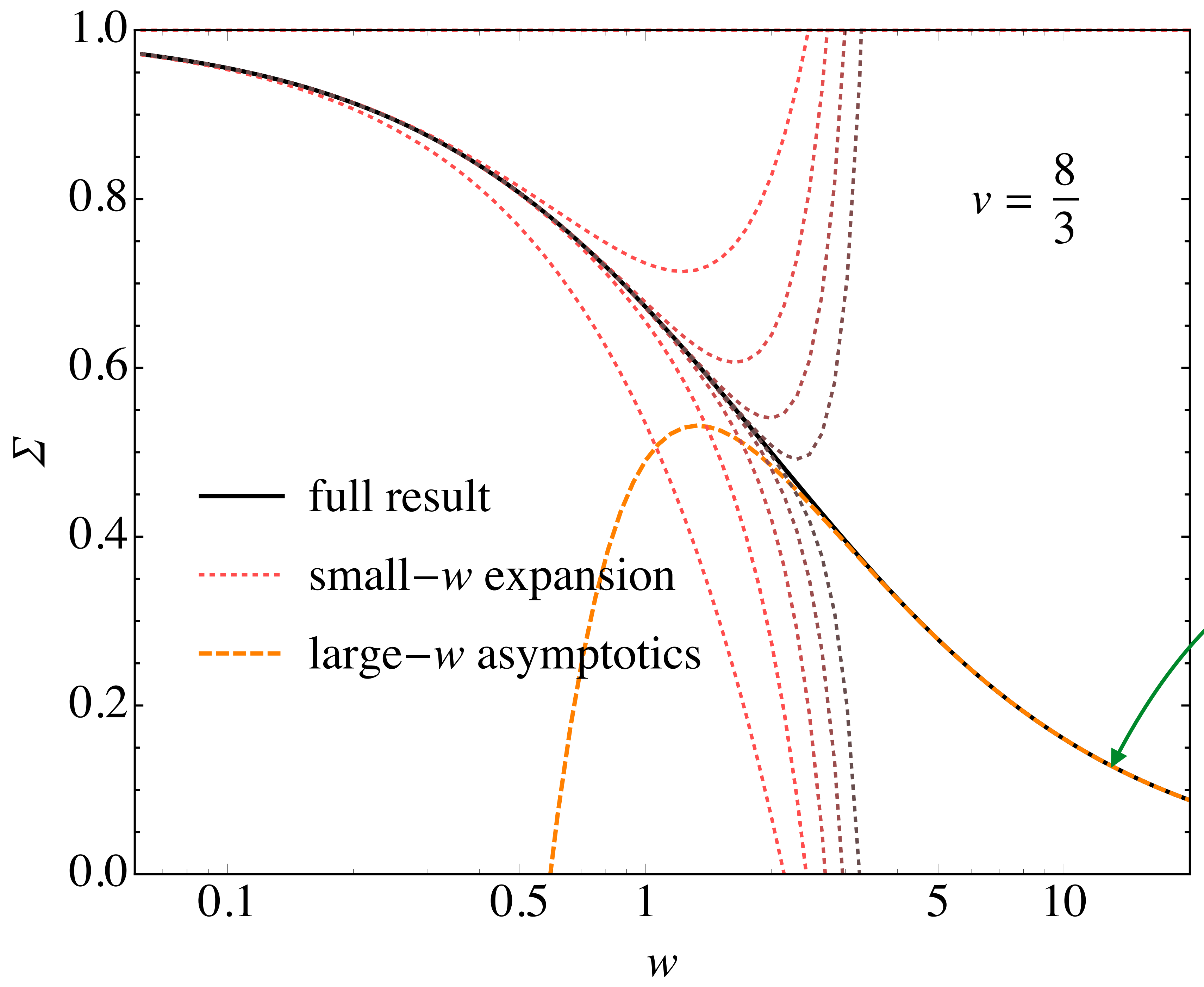
$$\begin{aligned}\boldsymbol{S}_1 &= f_{abe}f_{cde}\,\{\boldsymbol{T}_1^b,\boldsymbol{T}_1^c\}\,\{\boldsymbol{T}_2^a,\boldsymbol{T}_2^d\},\\ \boldsymbol{S}_2 &= d_{ade}d_{bce}\,\{\boldsymbol{T}_1^b,\boldsymbol{T}_1^c\}\,\{\boldsymbol{T}_2^a,\boldsymbol{T}_2^d\},\\ \boldsymbol{S}_3 &= d_{ade}d_{bce}\left[\boldsymbol{T}_2^a\left(\boldsymbol{T}_1^b\boldsymbol{T}_1^c\boldsymbol{T}_1^d\right)_+ + (1\leftrightarrow 2)\right],\\ \boldsymbol{S}_4 &= \{\boldsymbol{T}_1^a,\boldsymbol{T}_1^b\}\,\{\boldsymbol{T}_2^a,\boldsymbol{T}_2^b\},\\ \boldsymbol{S}_5 &= \boldsymbol{T}_1\cdot\boldsymbol{T}_2,\\ \boldsymbol{S}_6 &= \boldsymbol{1}.\end{aligned}$$

$$\begin{aligned}\boldsymbol{O}_1^{(j)} &= f_{abe}f_{cde}\,\boldsymbol{T}_2^a\,\{\boldsymbol{T}_1^b,\boldsymbol{T}_1^c\}\,\boldsymbol{T}_j^d - (1\leftrightarrow 2),\\ \boldsymbol{O}_2^{(j)} &= d_{ade}d_{bce}\,\boldsymbol{T}_2^a\,\{\boldsymbol{T}_1^b,\boldsymbol{T}_1^c\}\,\boldsymbol{T}_j^d - (1\leftrightarrow 2),\\ \boldsymbol{O}_3^{(j)} &= \boldsymbol{T}_2^a\,\{\boldsymbol{T}_1^a,\boldsymbol{T}_1^b\}\,\boldsymbol{T}_j^b - (1\leftrightarrow 2),\\ \boldsymbol{O}_4^{(j)} &= 2C_1\,\boldsymbol{T}_2\cdot\boldsymbol{T}_j - 2C_2\,\boldsymbol{T}_1\cdot\boldsymbol{T}_j.\end{aligned}$$

$$J_j = \int\frac{d\Omega(n_k)}{4\pi}\left(W_{1j}^k - W_{2j}^k\right)\Theta_{\text{gap}}(n_k)$$

TB, Neubert, Stillger, Shao, in preparation

All-order summation



$$w = \frac{N_c \alpha_s(\bar{\mu})}{\pi} \ln^2\left(\frac{\mu_h}{\mu_s}\right)$$

$$\Sigma(v, w) \sim \frac{1}{w}$$

- Combine C_{rn} with μ-Integrals, we can sum SLLs to all orders using

$$\Sigma(v,w) = \sum_{n=0}^{\infty} \sum_{r=0}^{n} \frac{(-4)^n \, 3! \, n!}{(2n+3)!} \frac{(2r)!}{4^r \, (r!)^2} v^r w^n = {}^{1+1}F_{2+0}\left(\begin{matrix} 1 : 1, \frac{1}{2} \, ; \\ 2, \frac{5}{2} : \quad ; \end{matrix} - w, -vw \right)$$

Resummed result

TB, Neubert, Shao Phys.Rev.Lett. 127 (2021) 21, 212002

Combine C_{rn} with μ integrals and carry out the sums.

Simplest case is $qq\to qq$ scattering with photon exchange

$J_i = \pm\Delta Y$

$$\Delta\hat{\sigma}^{(S)} = -\hat{\sigma}_B \frac{4C_F}{3\pi}\, \alpha_s^3\, L^3\, \Delta Y\; {}_2F_2\left(1,1;2,\tfrac{5}{2};-w\right)$$

with $w = \dfrac{N_c \alpha_s}{\pi} L^2$.

$\sim \dfrac{\ln w}{w}$ for large w

Note: Standard Sudakov has form e^{-cw}.

Outlook

- Resummation of leading superleading logs is now available
- Next steps and open questions
  - phenomenological applications
  - analysis of low-energy matrix elements, Glauber contributions
  - single logarithmic resummation? NGL $\times$ SLL?
  - systematics of the expansion? “exponentiation”?
  - factorization breaking

erc

Congratulations to Matthias for winning a 2nd ERC Advanced Grant **EFT4jets**
"An Effective Field Theory for Non-Global Observables at Hadron Colliders"

Albufeira 2007

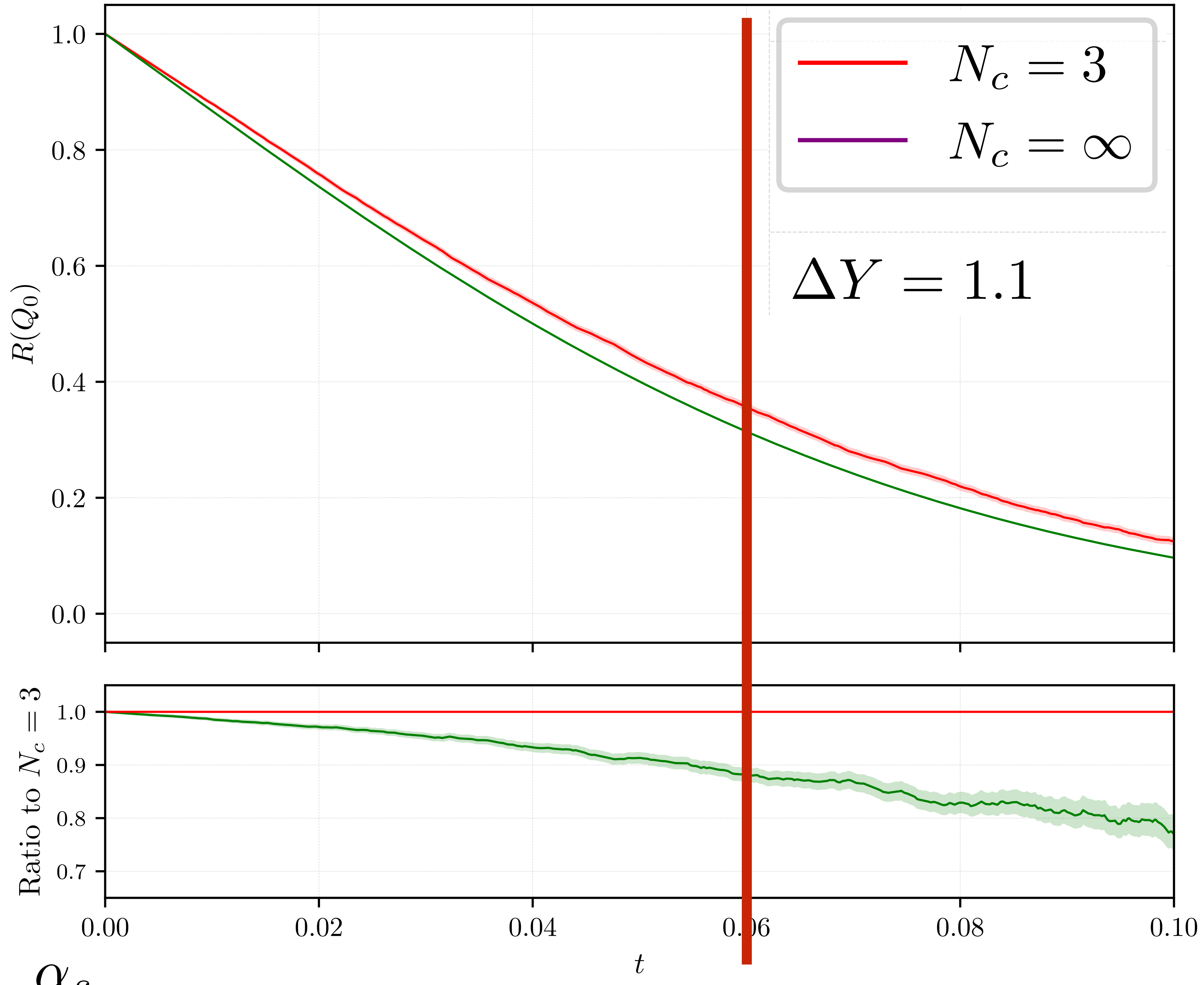
Congratulations Matthias!

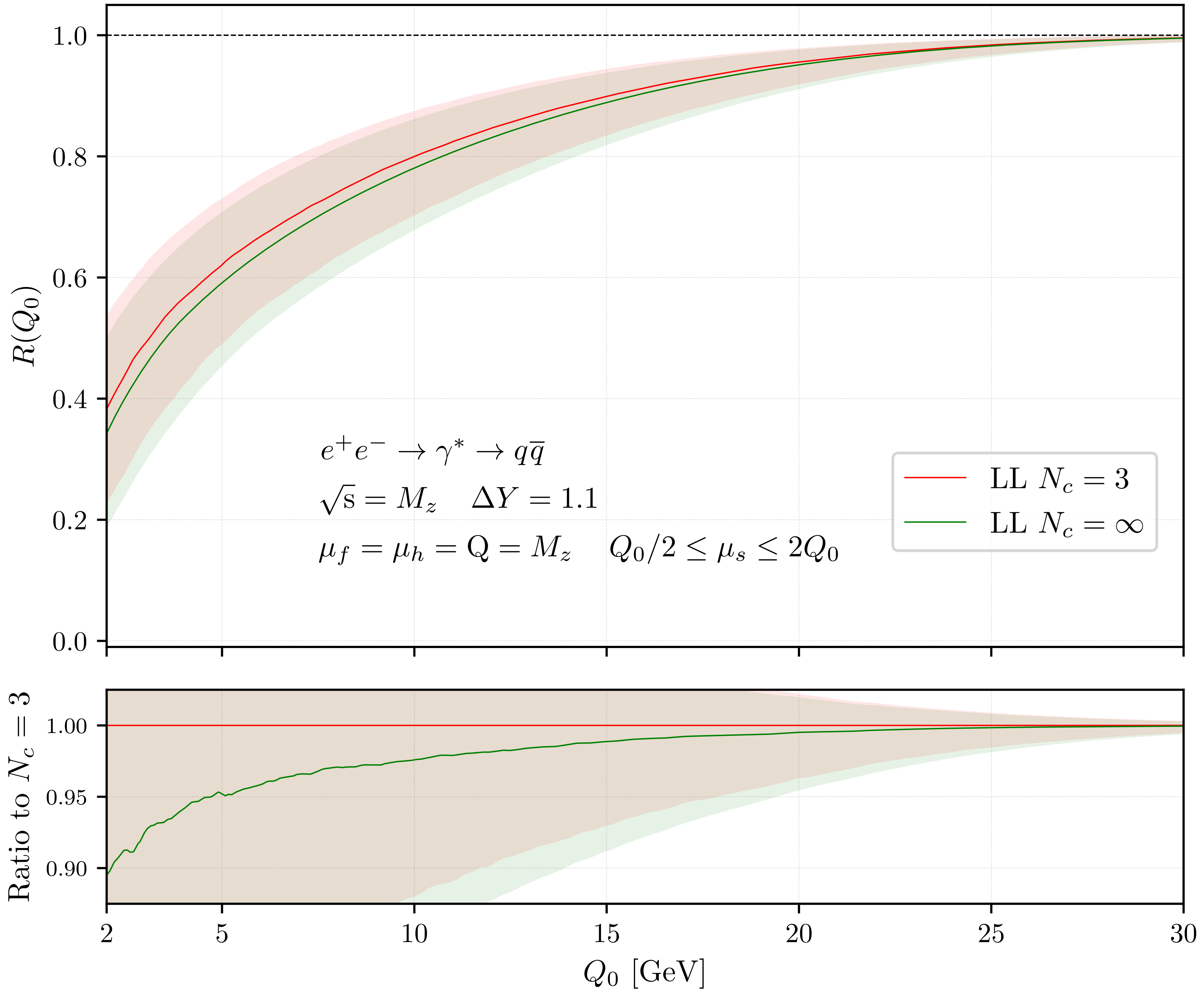
I’m looking forward to the next 20 years of collaboration!

Extra slides

Large N_c versus $N_c = 3$

Hatta, Ueda '13

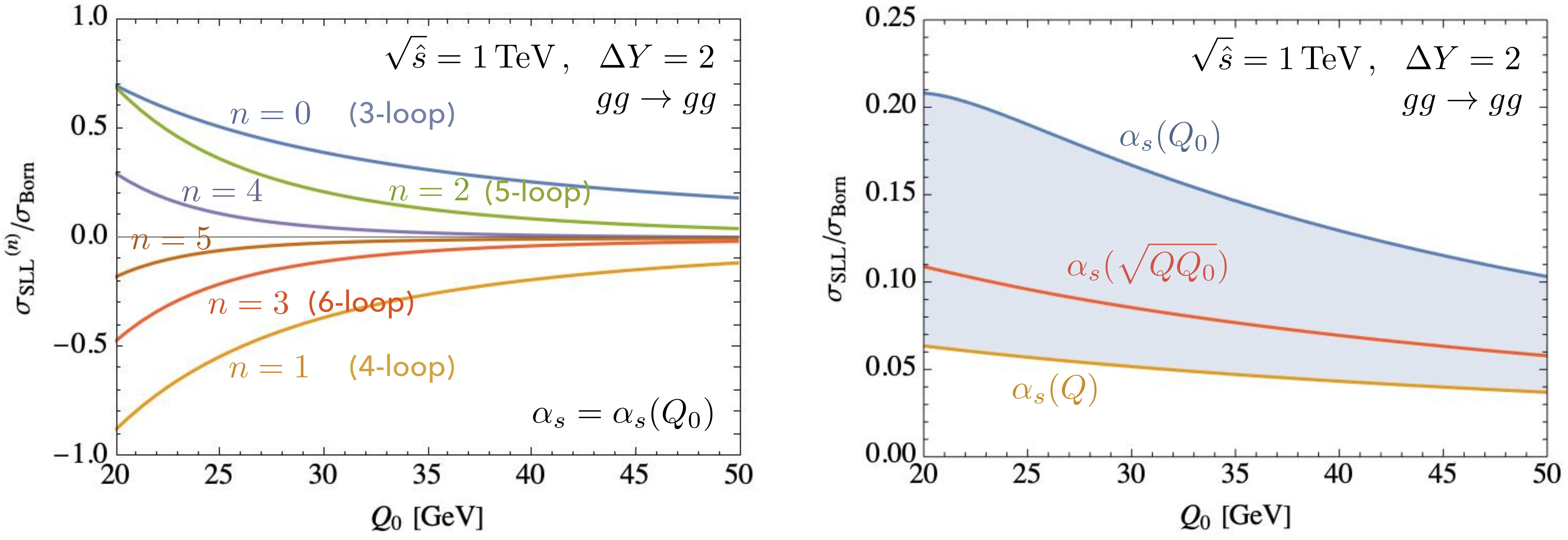




$$t = \frac{\alpha_s}{4\pi} \ln(Q/Q_0)$$

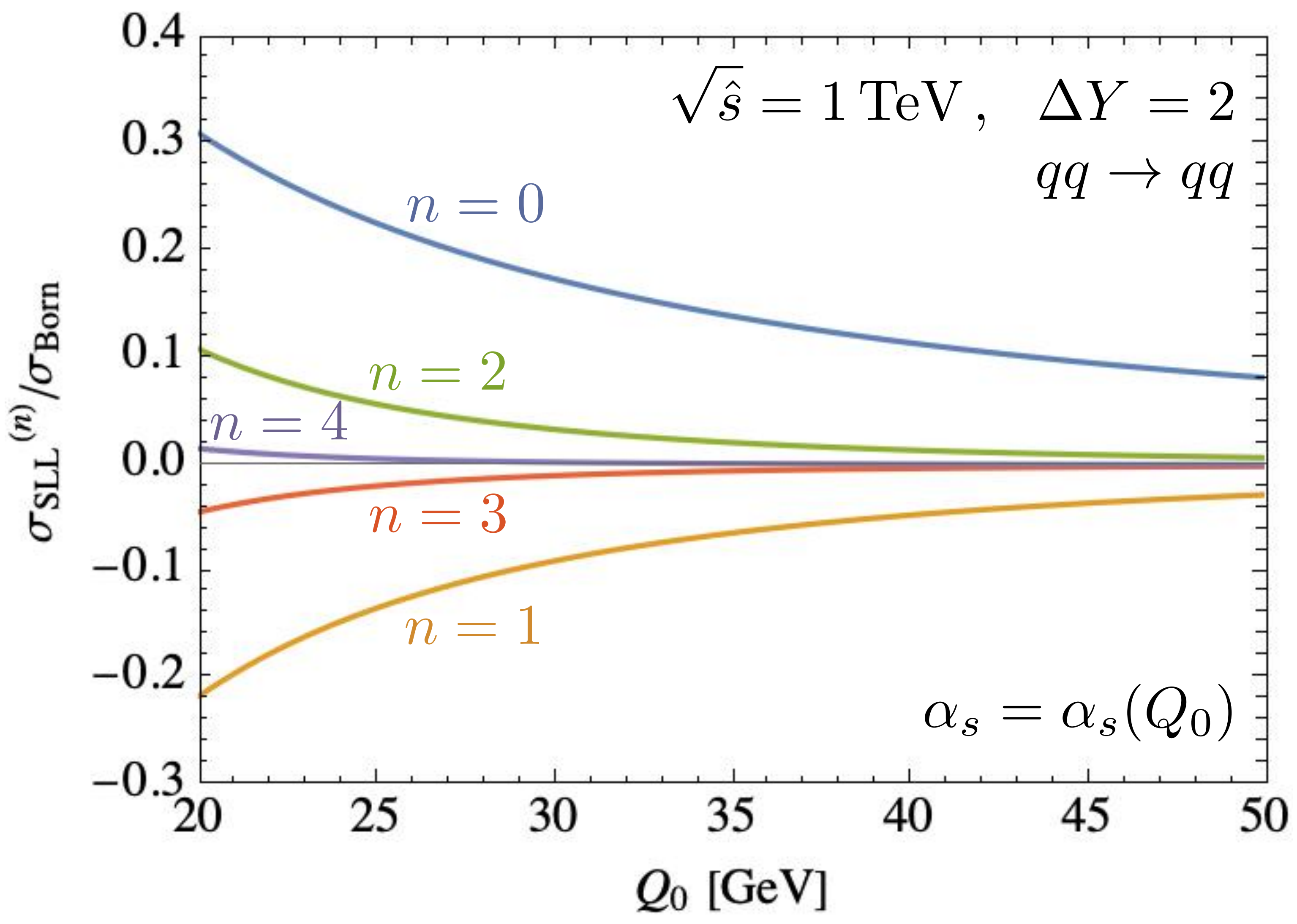
$Q_0 \sim 2$

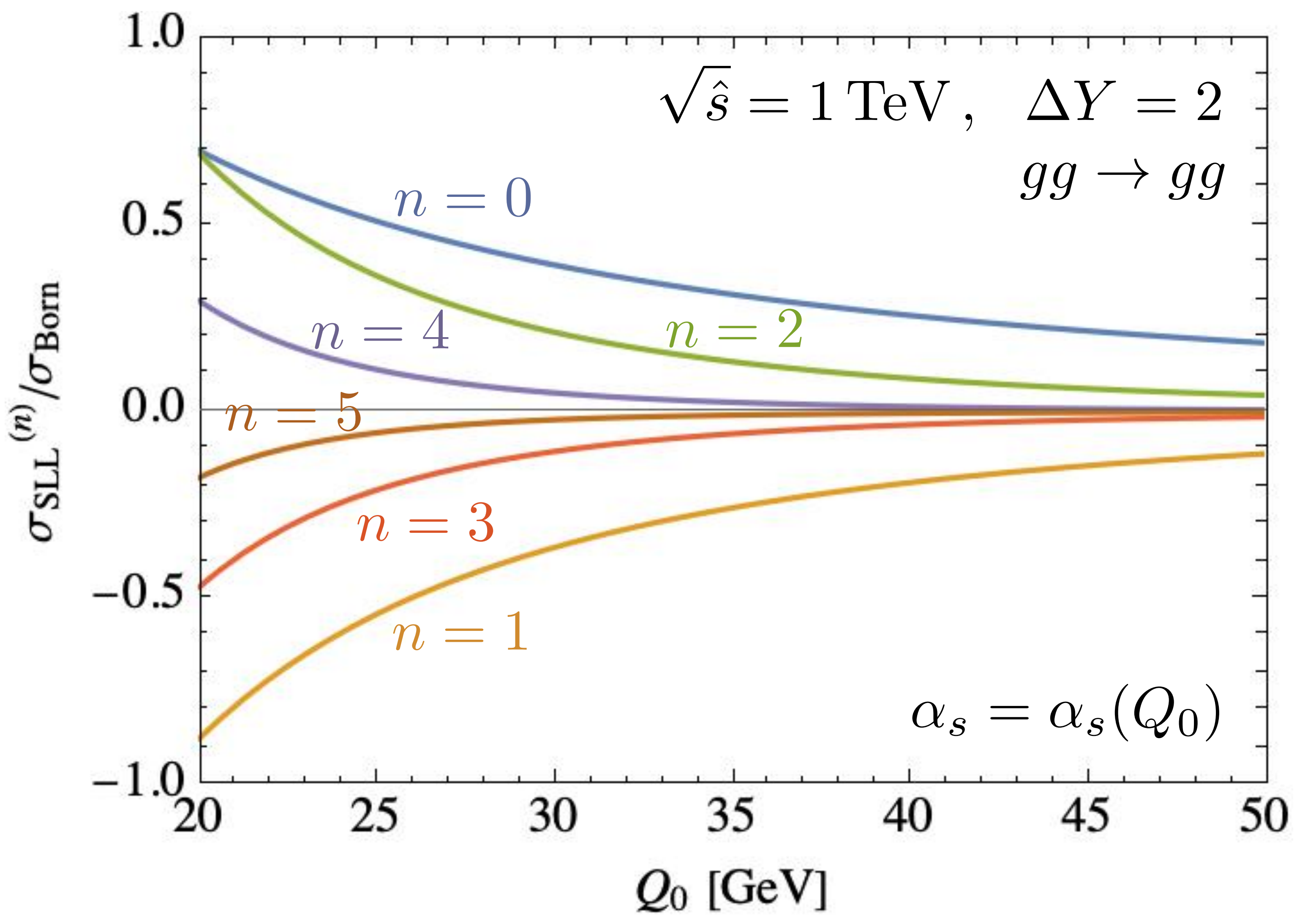
Forward gluon-gluon scattering



- Slow convergence (w~2): necessary to include eight terms (10 loops!) to converge to resummed result
- *Very sensitive* to choice of μ in α_s: should include running!

Terms of order $\alpha_s^{n+3} L^{2n+3}$

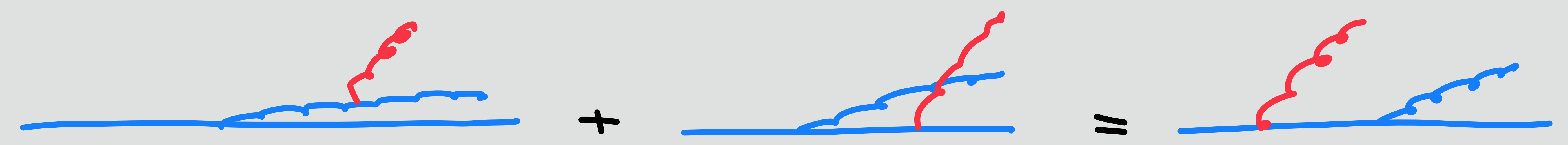




- Hard function for gluon exchange in t-channel.
- n=0 term is not SLL, but missing in large N_c limit.

Basic properties of Γ

- Color coherence: $\boldsymbol{\mathcal{H}}_m \, \boldsymbol{\Gamma}^c \, \overline{\boldsymbol{\Gamma}} = \boldsymbol{\mathcal{H}}_m \, \overline{\boldsymbol{\Gamma}} \, \boldsymbol{\Gamma}^c$



- Collinear safety $\langle \boldsymbol{\mathcal{H}}_m \, \boldsymbol{\Gamma}^c \otimes \mathbf{1} \rangle = 0$

$\langle \boldsymbol{\mathcal{H}}_m \, (\boldsymbol{R}_i^c + \boldsymbol{V}_i^c) \otimes 1 \rangle \propto \langle \boldsymbol{T}_i^a \, \boldsymbol{\mathcal{H}}_m \, \boldsymbol{T}_i^a - C_i \, \boldsymbol{\mathcal{H}}_m \rangle = 0$ cyclicity of trace

- $\langle \boldsymbol{\mathcal{H}}_m \, \boldsymbol{V}^G \otimes \mathbf{1} \rangle = 0$ cyclicity of trace

Leading SLLs

1. Want maximum number of $\mathbf{\Gamma}^c$'s at given order.
2. Need $\mathbf{\Gamma}^G$ to prevent $\mathbf{\Gamma}^c$ from commuting to the right and vanishing. Two insertions of $\mathbf{\Gamma}^G$ since cross section is real.
3. Need one emission $\overline{\mathbf{\Gamma}}$ at the end to prevent $\mathbf{\Gamma}^G$ from vanishing

Taken together, this implies that the leading SLLs at (n+3)-rd order arise from matrix elements

$$C_{rn} = \left\langle \boldsymbol{\mathcal{H}}_4 \left(\mathbf{\Gamma}^c\right)^r \boldsymbol{V}^G \left(\mathbf{\Gamma}^c\right)^{n-r} \boldsymbol{V}^G \,\overline{\mathbf{\Gamma}} \otimes \mathbf{1} \right\rangle \qquad 0 \leq r \leq n$$