Flavour in the SMEFT

Sophie Renner, University of Glasgow

Based mainly on Machado, SR, Sutherland 2210.09316

Pushing the limits of theoretical physics, Mainz, 11th May 2023

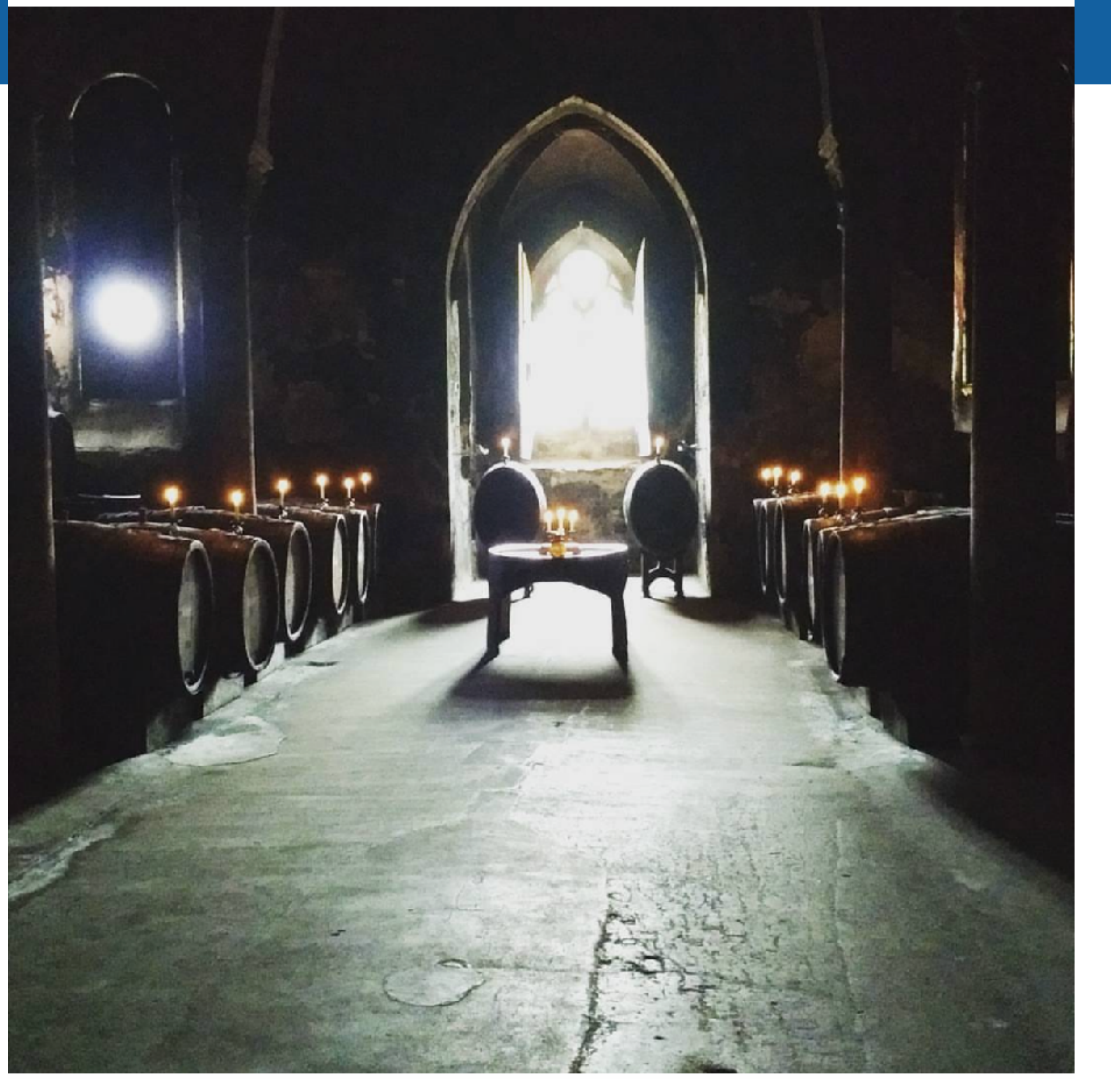
Effective Field Theories for Collider Physics, Flavor Phenomena and Electroweak Symmetry Breaking

12 Sept 2016, 09:00 → 15 Sept 2016, 18:00 Europe/Berlin

Burg Crass

Matthias Neubert (JGU Mainz)

onlyhereforpugs
Kloster Eberbach



onlyhereforpugs 346w
Wine tasting in a monastery...

1066unicorn 346w
Jealous
Reply

jessdavies140 346w
#hardatwork
Reply

SMEFT for BSM physics

Effective theory parameterising effects of heavy new physics respecting the full SM gauge group, and containing a Higgs doublet

$$\mathcal{L}_{\rm NP} = \frac{1}{\Lambda^2}\sum_i C_i^{(6)}\mathcal{O}_i^{(6)} + \frac{1}{\Lambda^4}\sum_i C_i^{(8)}\mathcal{O}_i^{(8)} + \dots$$

ADVANTAGES

Can reproduce effects of heavy new physics at low energies

Model independent

Language to interpret experimental results

Can connect scales via anomalous dimension matrix

(Alonso), Jenkins, Manohar, Trott
1308.2627, 1310.4838, 1312.2014

CHALLENGES

Too many parameters to deal with (2499 at dimension 6)

Sometimes opaque connection between operators and observables

Flavour in the SMEFT

Flavour is responsible for most of the parameters...

If there were one generation...	**76 real parameters**	(baryon number conserving)
With three generations...	**2499 real parameters**	(baryon number conserving)

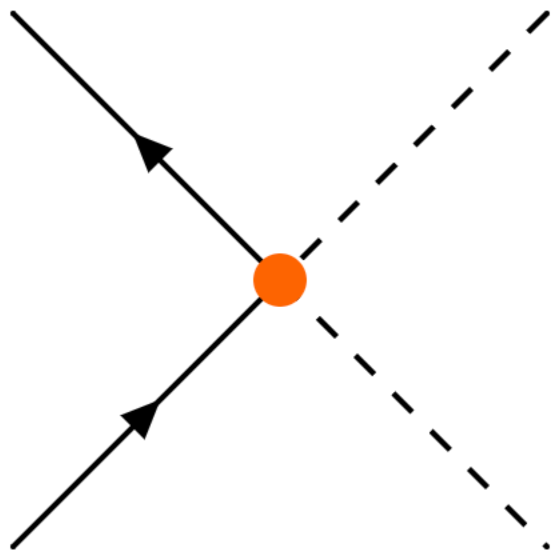
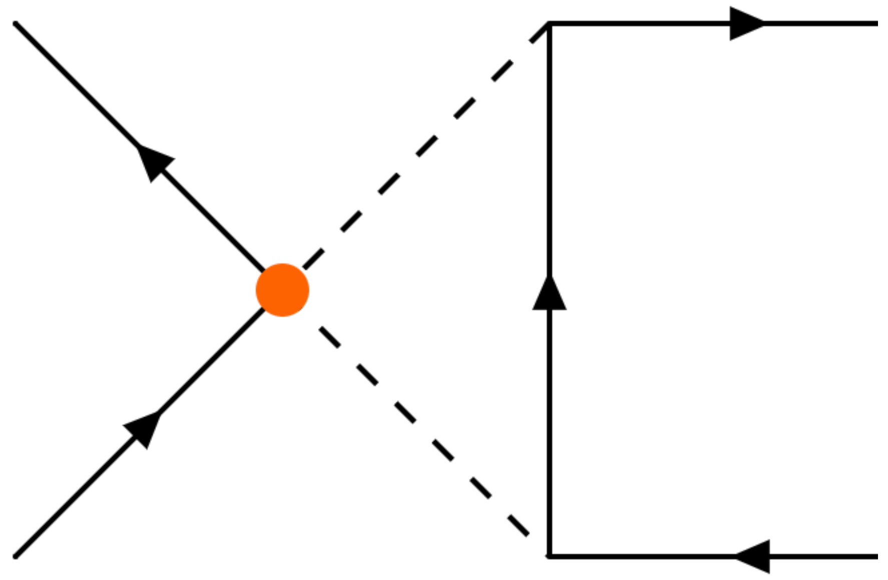
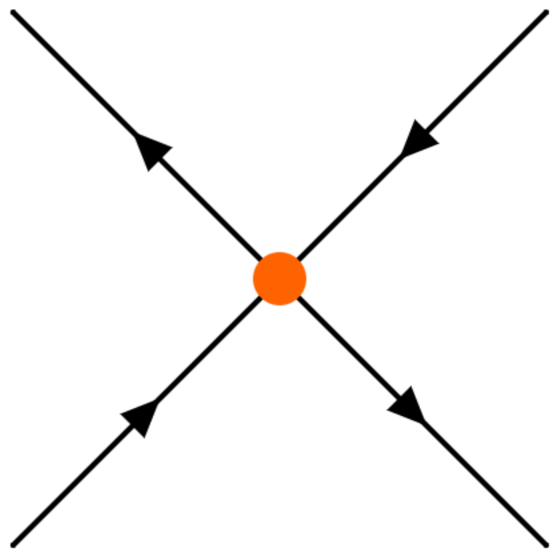
A way to narrow down the problem is to identify *categories of important operators*

e.g.

- Operators that contribute (at tree or loop level) to a class of observables
- Operators invariant under CP or flavour symmetries
  D'Ambrosio, Giudice, Isidori, Strumia, hep-ph/0207036
  Faroughy, Isidori, Wilsch, Yamamoto 2005.05366
  Greljo, Palavric, Thomsen 2203.09561
- Operators that are created at tree level by simple/motivated UV models
  e.g. Einhorn, Wudka 1307.0478
  Craig, Jiang, Li, Sutherland 2001.00017

Anomalous dimension matrix

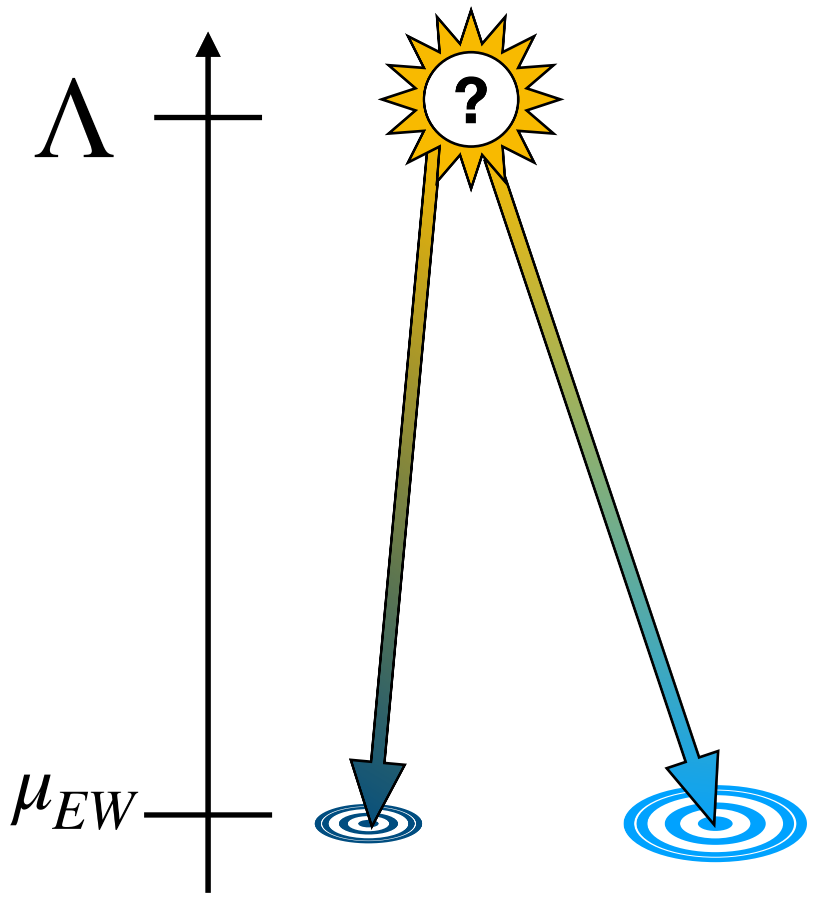
At one loop, SMEFT operators can mix into one another

e.g.  can be put into a loop:  which gives a divergent contribution to: 

Gives an off-diagonal contribution to the anomalous dimension matrix

$$\frac{\mathrm{d}C_{\mathcal{O}_i}}{\mathrm{d}\ln\mu} = \sum_j \frac{1}{16\pi^2}\gamma_{ij}C_{\mathcal{O}_j}$$

Categories generally won't be conserved over scales



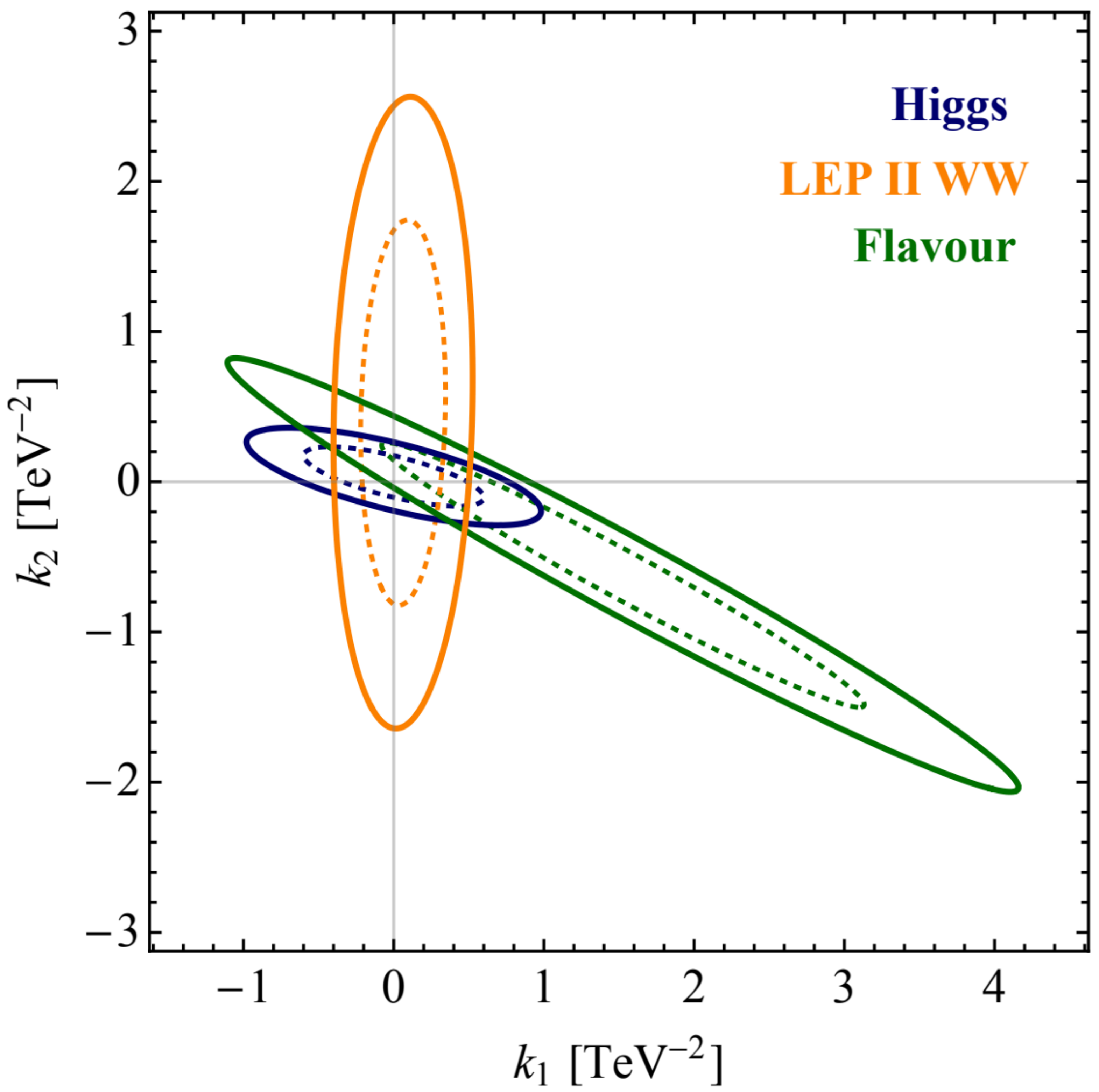
Different Wilson coefficients at different scales

Flavour fights back

No matter how "flavourless" the initial assumptions, flavour effects appear radiatively

e.g. full flavour symmetry at Λ

$$U(3)^5$$



Flavour can put meaningful constraints on the class of operators that enter Z pole measurements

$$\{C_{HWB}, C_{HD}, C_{Hl}^{(1)}, C_{Hl}^{(3)}, C_{Hq}^{(1)}, C_{Hq}^{(3)}, C_{Hu}, C_{Hd}, C_{He}, C_{ll}'\}$$

Hurth, SR, Shepherd 1903.00500
Aoude, Hurth, SR, Shepherd 2003.05432

How to make sense of it all?

Non-renormalisation theorems

Anomalous dim matrix of dimension 6 SMEFT has many zeroes ➡ Seems clear that there must be reasons for this

(Alonso), Jenkins, Manohar, Trott
1308.2627, 1310.4838, 1312.2014

Non-renormalisation theorems provide symmetry- or kinematics-based explanations for zeroes

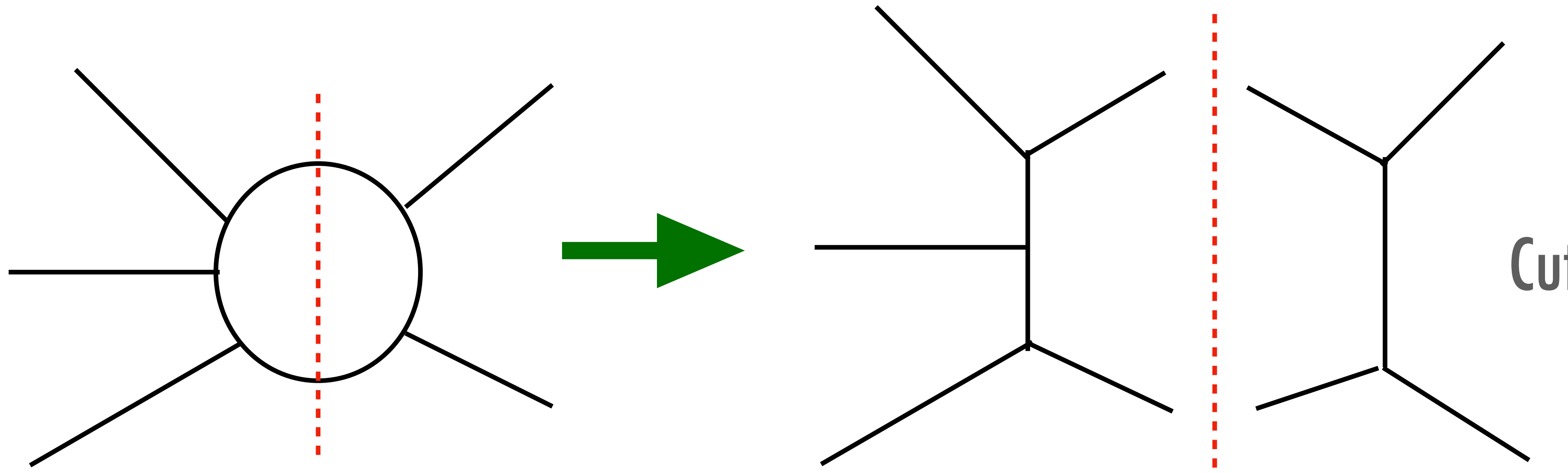
Allow us to find categories that remain distinct over scales
i.e. they do not mix into each other under renormalisation group flow

Then can study subsets independently

Anomalous dimensions via tree amplitudes

Cutkosky's rule: 2-cuts isolate the discontinuities of the amplitude

$\Longrightarrow$ <u>can deduce divergences</u>



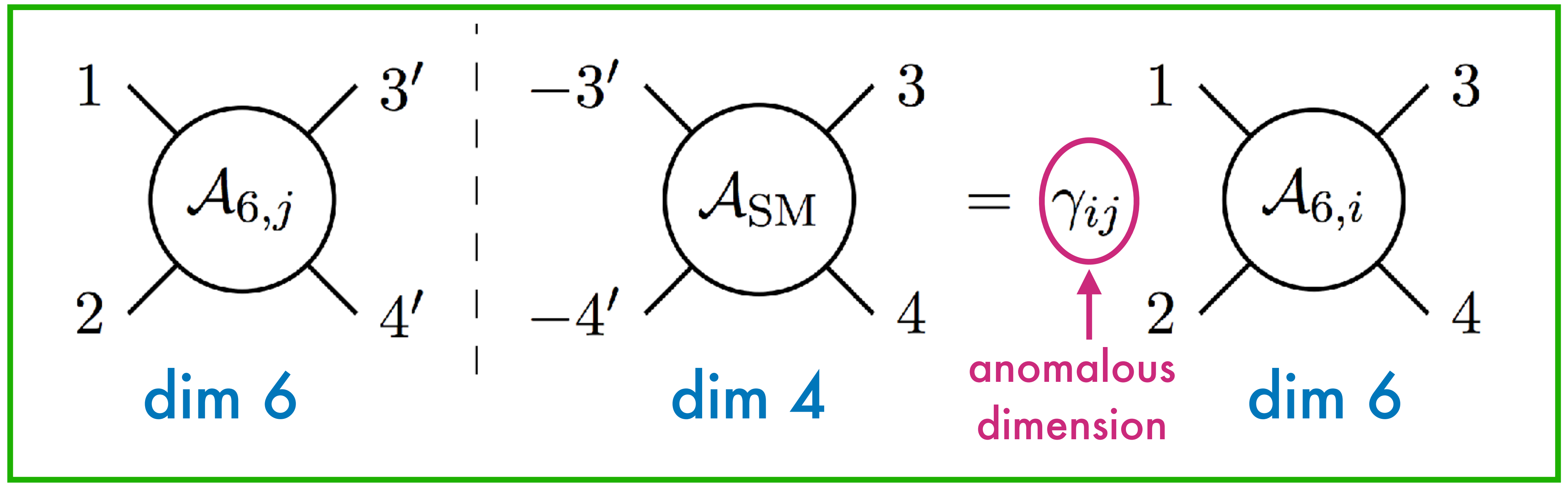
Cutkosky, J. Math. Phys 1, 429 (1960)

internal lines go onshell

These 2-cuts can be used to isolate the *UV divergent piece**

*IR divergences in self-renormalisation require some care

$\Longrightarrow$ schematically:



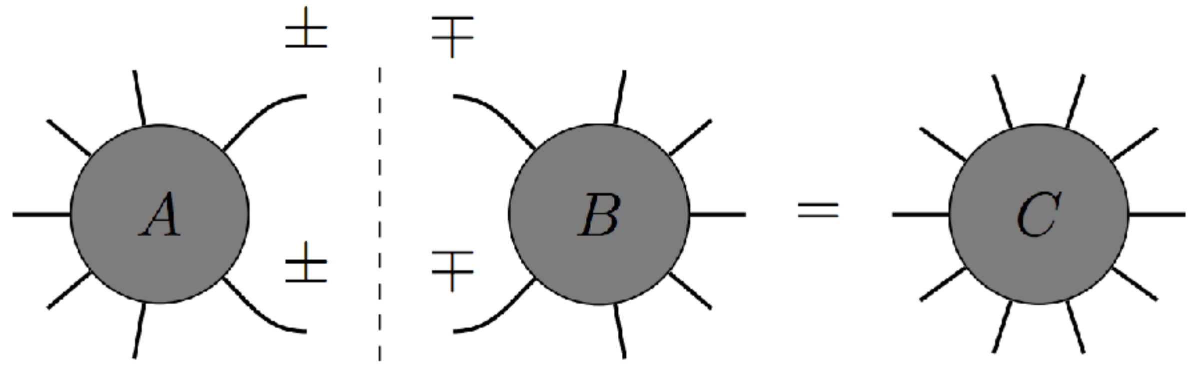
Caron-Huot, Wilhelm 1607.06448
Jiang, Ma, Shu, 2005.10261
Baratella, Fernandez, Pomarol, 2005.07129
Elias Miró, Ingoldby, Riembau, 2005.06983

All momenta are defined *ingoing* $\Longrightarrow$ lines on either side of the cut have opposite momenta and helicity

If we know the properties of the dim 6 and dim 4 amplitudes on the LHS, we can understand which amplitudes can be produced on the RHS

Helicity and non-renormalisation

Label amplitudes by number of legs n and total helicity $\sum h$

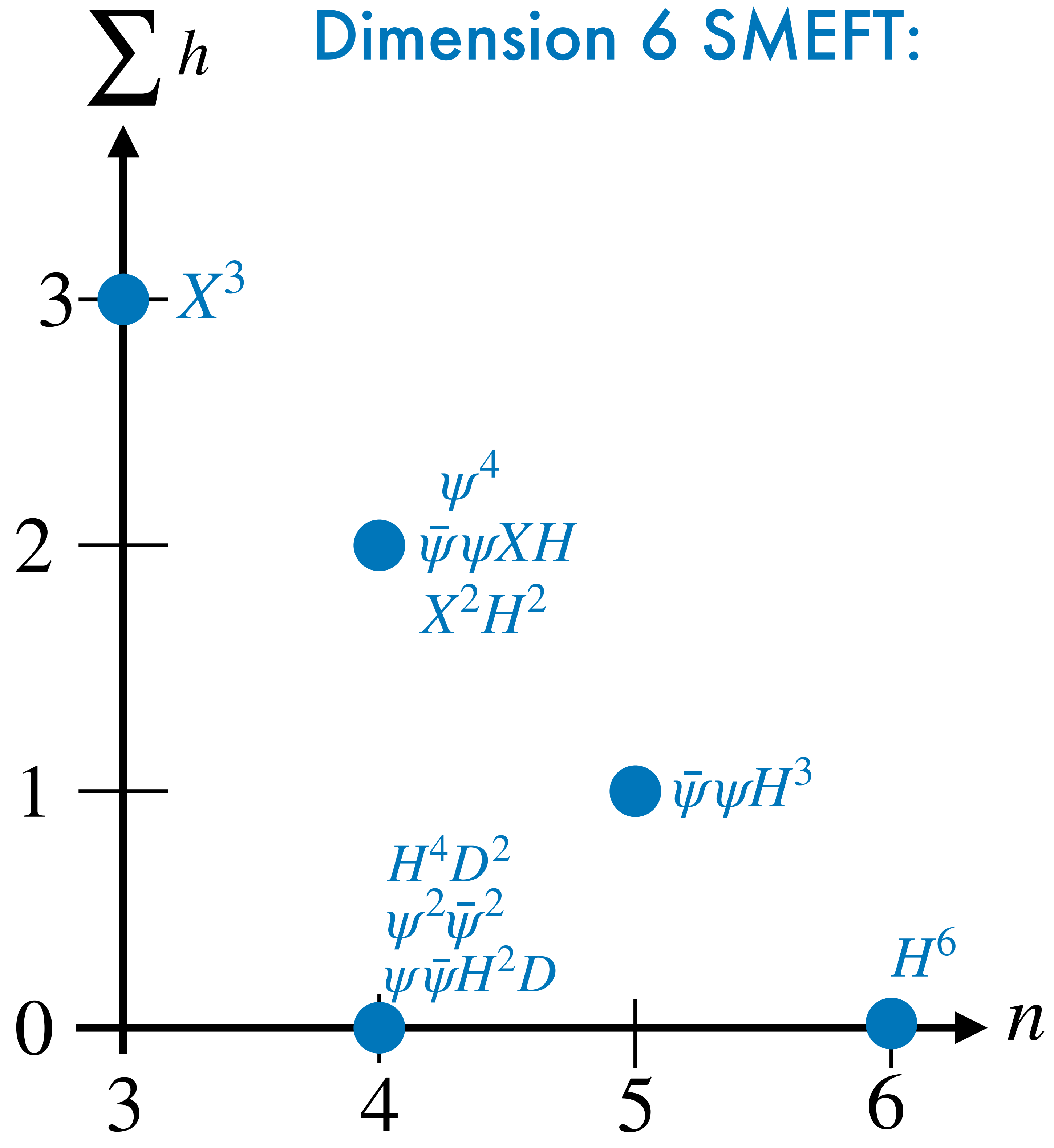


$n_C = n_A + n_B - 4$ and $\sum h_C = \sum h_A + \sum h_B$

$X = B_{\mu\nu}, G^A_{\mu\nu}, W^I_{\mu\nu}$

$\psi = Q, u, d, L, e$

Dimension 6 SMEFT:



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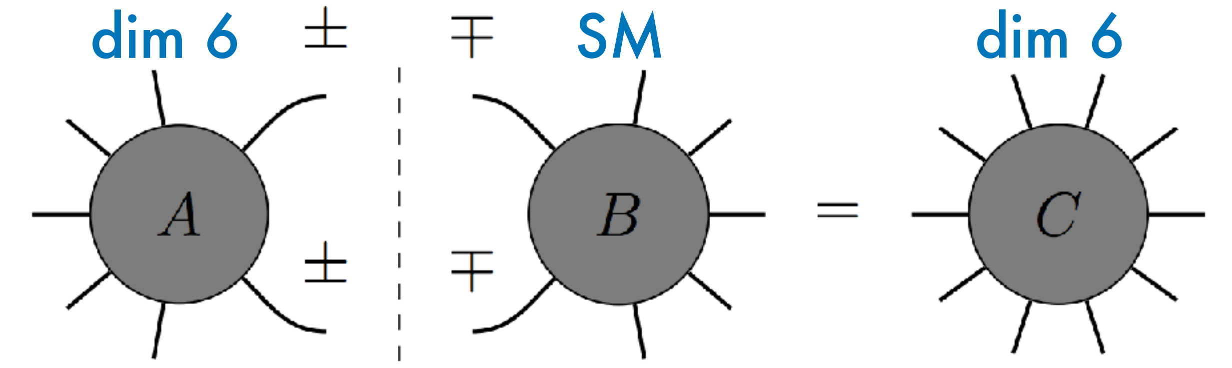
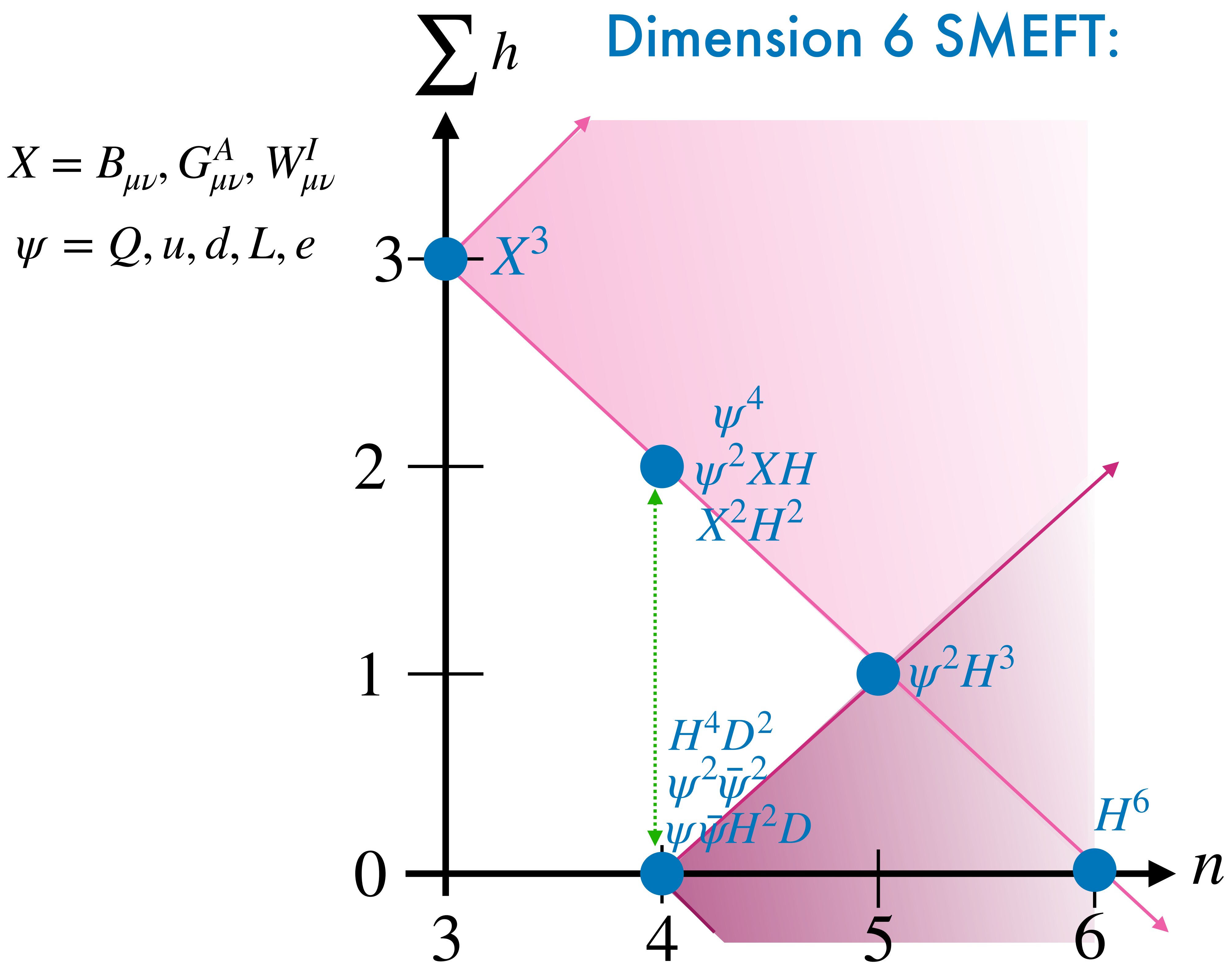


$$n_C = n_A + n_B - 4 \quad \text{and} \quad \sum h_C = \sum h_A + \sum h_B$$



All SM tree amplitudes (*) lie in the cone defined by

$$\left|\sum h\right| \leq n - 4$$

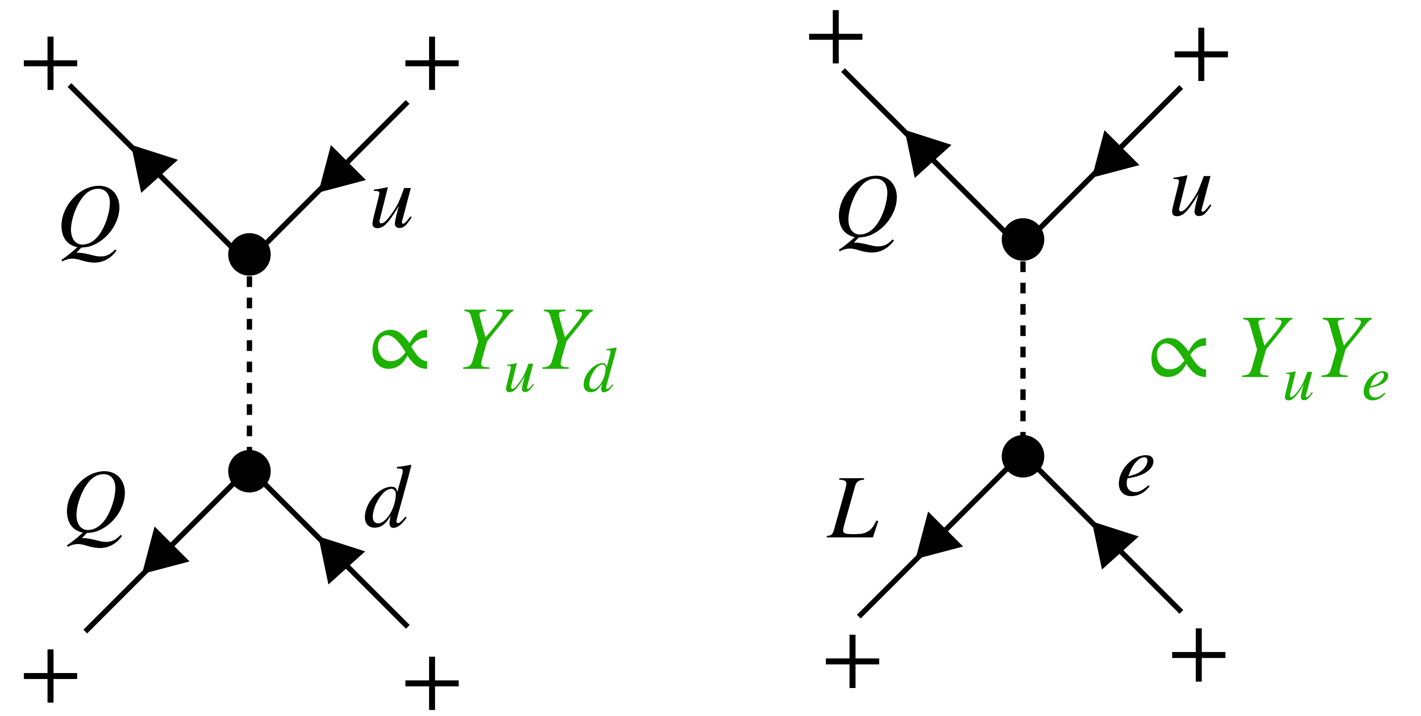
So from any operator, can only run into operators on or within the cone at one loop

Alonso, Jenkins, Manohar 1409.0868
Cheung, Shen 1505.01844

(*) exceptions:

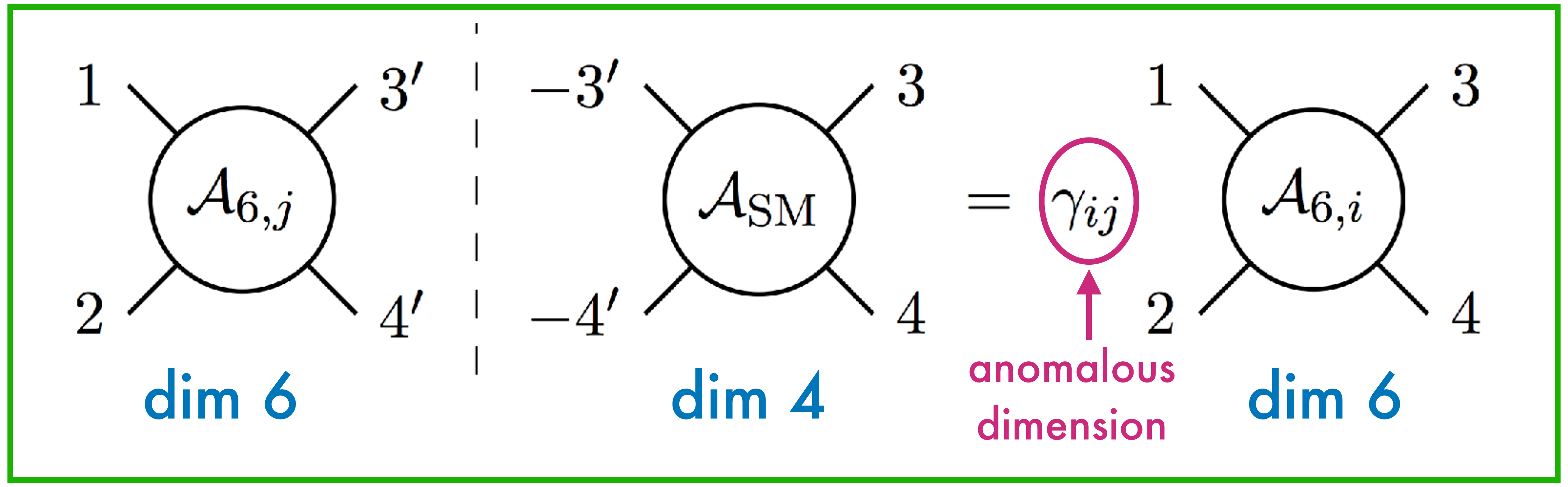
$$\left|\sum h\right| = 2 \qquad n = 4$$

Always suppressed by a small Yukawa



Going further: gauge and flavour

We have non-renormalisation theorems based on helicity, i.e. the kinematical part of the amplitudes



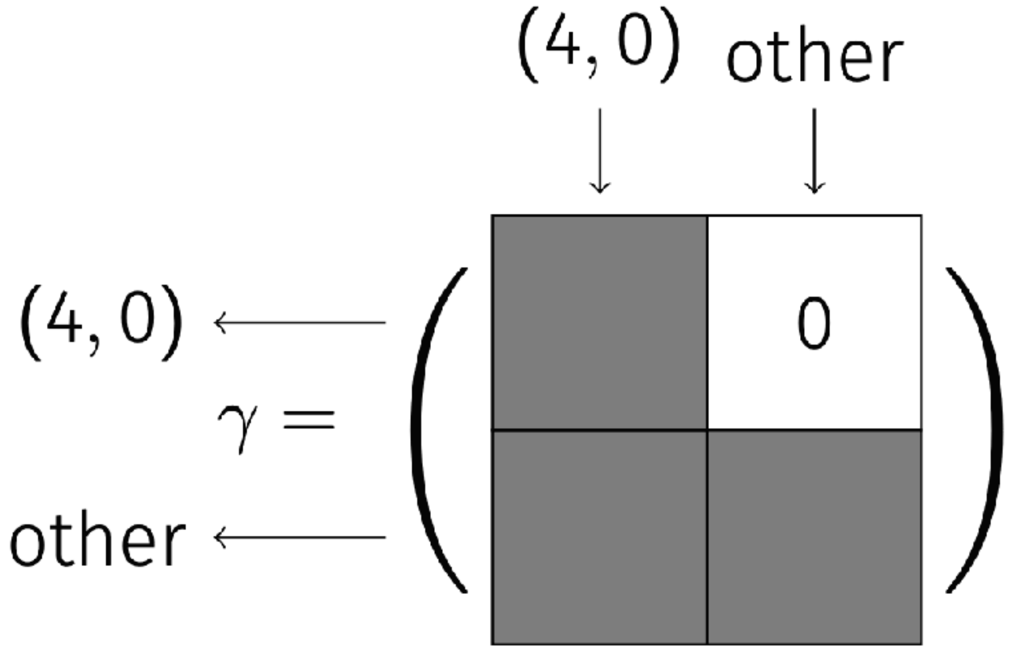
Amplitudes factorise:

(kinematics) × (gauge) × (flavour) $\Longrightarrow$

Each entry of γ_{ij} factorises:

(kinematics) × (gauge) × (flavour)

Can we find good categories for the gauge and flavour parts of the operators, that are conserved under running?



Focus on the (4,0) operators: 1460 parameters

H^4D^2	$\psi\bar{\psi}H^2D$	$\psi^2\bar{\psi}^2$
4 Higgs operators	2 Higgs, 2 fermion operators	4 fermion operators
O_{HD} $O_{H\Box}$	e.g. O_{Hu}, $O_{Hl}^{(1,3)}$ etc	(All except $O_{lequ}^{(1,3)}$ and $O_{quqd}^{(1,8)}$)

Flavour decomposition: irreps

Most operators have flavour matrices as Wilson coefficients

Can decompose these general matrices in any basis that is convenient

Natural choice: irreps of SM flavour group

$$SU(3)^5 = SU(3)_Q \times SU(3)_u \times SU(3)_d \times SU(3)_L \times SU(3)_e$$

SM fermions are in triplet irreps under their group

$SU(3)^5$ preserved by gauge interactions

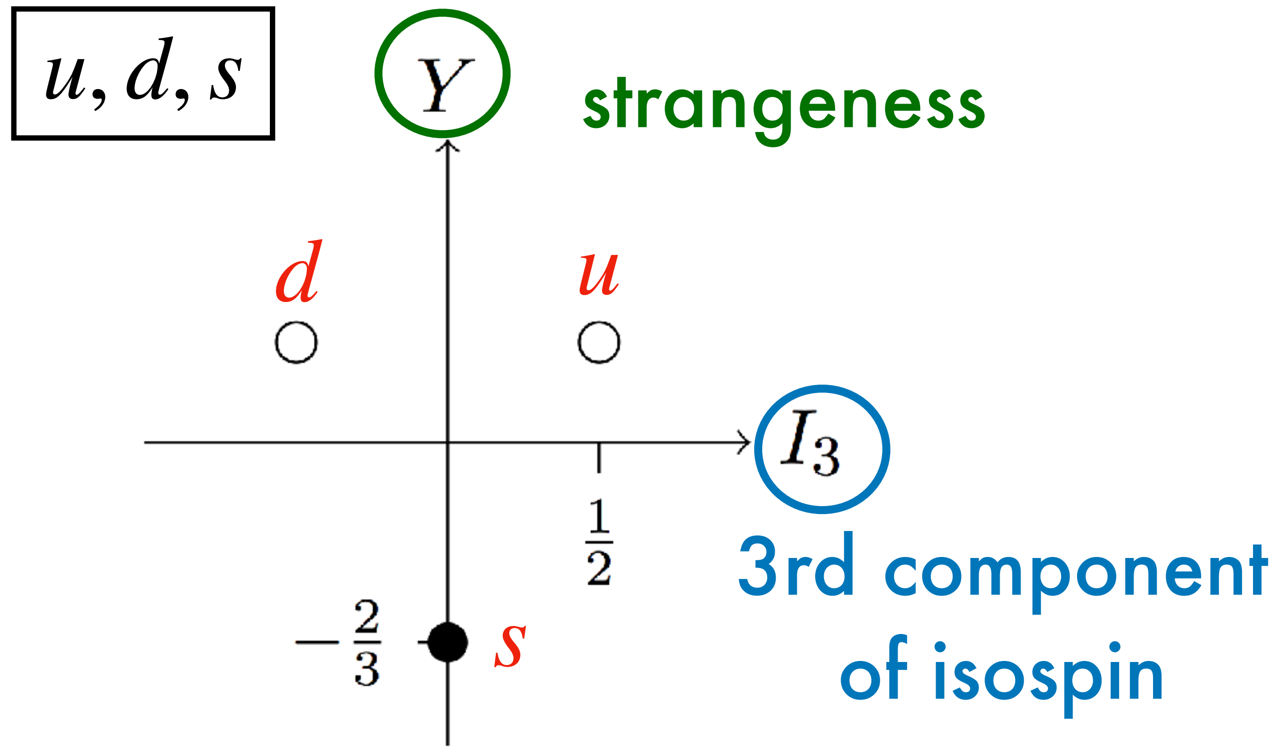
Operator type	Wilson coeff	Irrep decomposition
H^4D^2	c	$1_F \ (\forall F)$
$H^2\psi_F^2 D$	c_q^p	$3_F \otimes \bar{3}_F = 1_F \oplus 8_F$
$\psi^2\bar{\psi}^2 : (\bar{\psi}_{F_1}\psi_{F_1})(\bar{\psi}_{F_2}\psi_{F_2})$	c_{qs}^{pr}	$3_{F_1} \otimes \bar{3}_{F_1} \otimes 3_{F_2} \otimes \bar{3}_{F_2} = (1_{F_1} \otimes 1_{F_2}) \oplus (1_{F_1} \otimes 8_{F_2}) \oplus (8_{F_1} \otimes 1_{F_2}) \oplus (8_{F_1} \otimes 8_{F_2})$
$\psi_F^2\bar{\psi}_F^2$: symmetric	$c_{(qs)}^{(pr)}$	$(3_F \otimes \bar{3}_F)_{sym} \otimes (3_F \otimes \bar{3}_F)_{sym} = 1_F \oplus 8_F \oplus 27_F$
$\psi_F^2\bar{\psi}_F^2$: antisymmetric	$c_{[qs]}^{[pr]}$	$(3_F \otimes \bar{3}_F)_{antisym} \otimes (3_F \otimes \bar{3}_F)_{antisym} = 1_F \oplus 8_F$

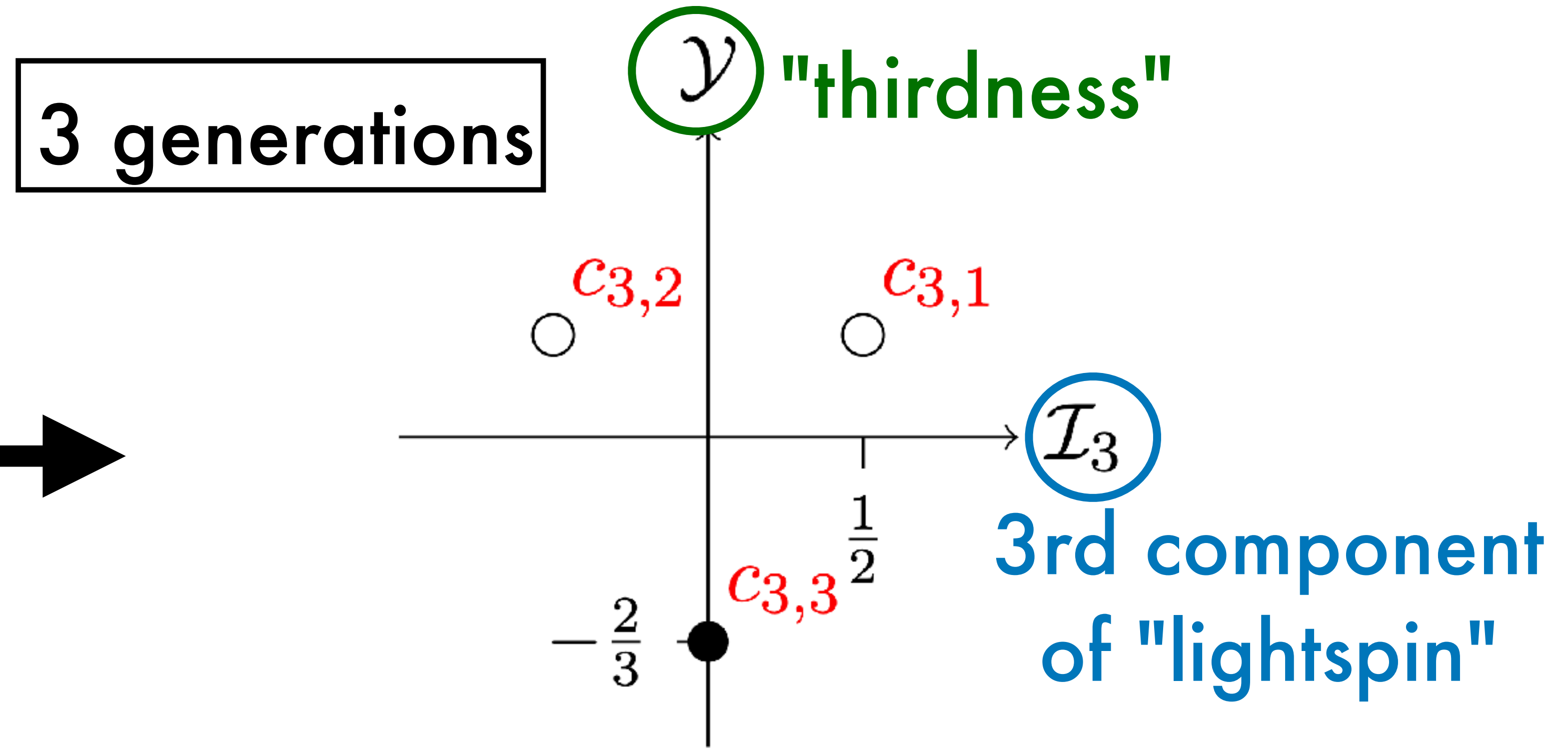
Flavour decomposition: quantum numbers

To label the components of the irreps, can use conventions developed for the $SU(3)$ of light flavours u, d, s in the 1960s

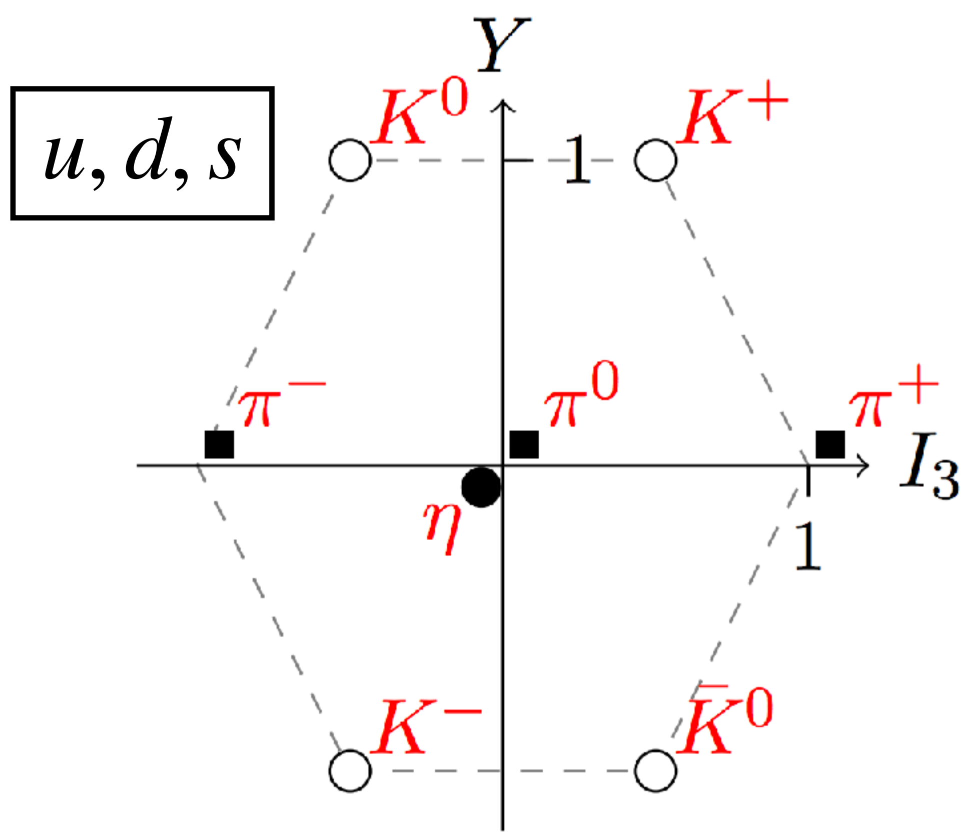
de Swart, Rev. Mod, Phys. 35 (1963) 916-939

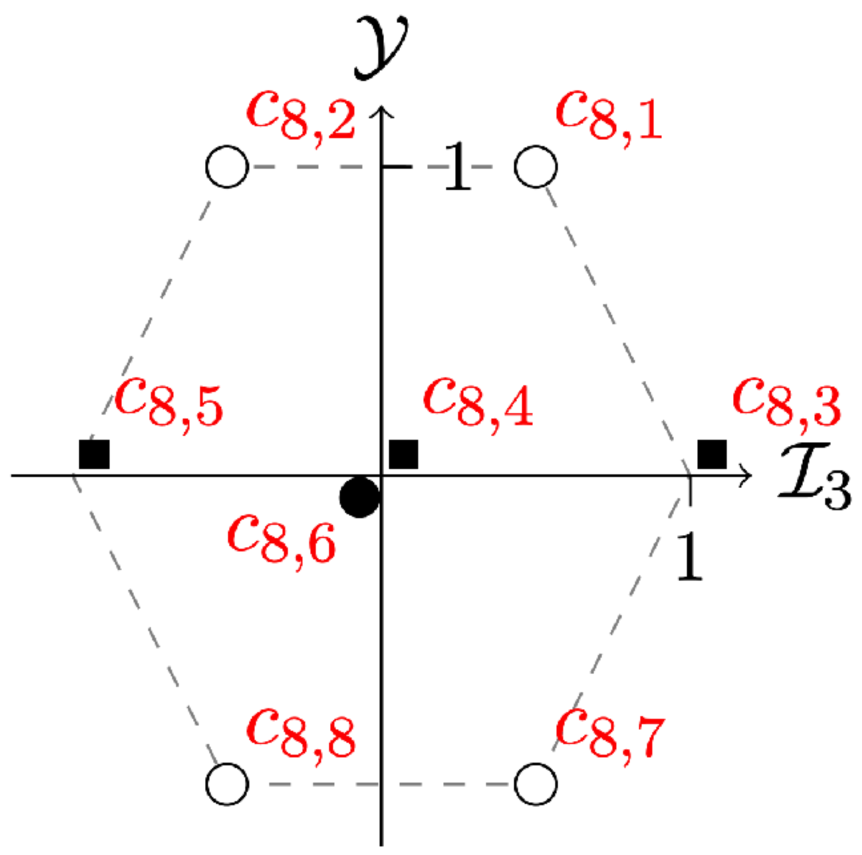
Triplet 3_F





Octet 8_F





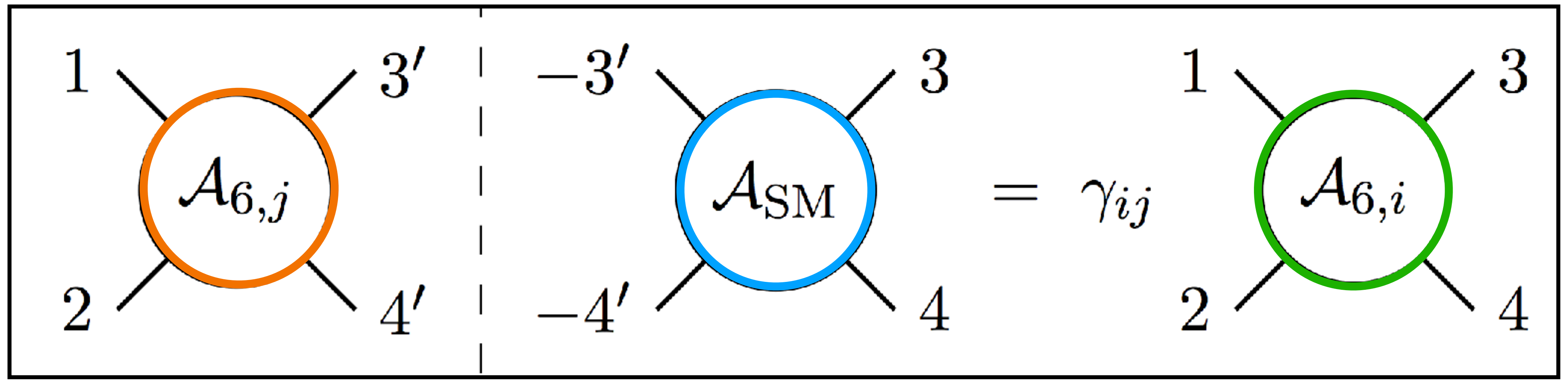
Total $\mathcal{I}$ key:

$\bullet = 0,\ \circ = \frac{1}{2},\ \blacksquare = 1$

4 quantum numbers for each species: $\{d, \mathcal{I}, \mathcal{I}_3, \mathcal{Y}\}_F$

Selection rules

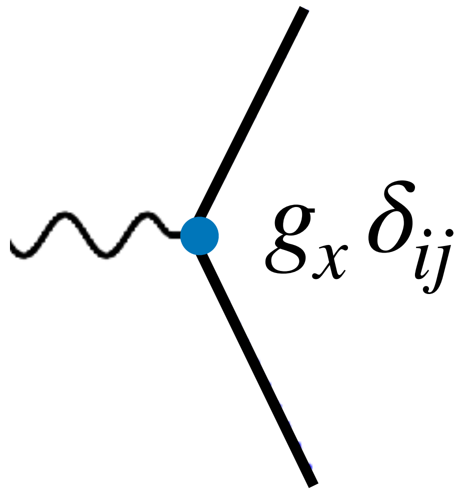
We now know the flavour quantum numbers of all the dim 6 operators...



So if we understand what flavour quantum numbers are zero in SM amplitudes...

Then we can deduce which are preserved in the running

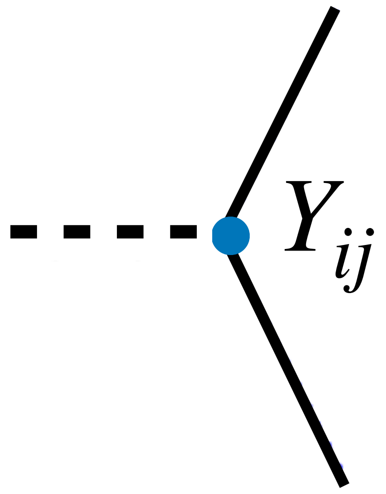
SM gauge coupling



Flavour singlet,
all quantum numbers = 0

$SU(3)^5$ preserved

SM Yukawa coupling



If just y_3 $\{I_L, I_R, I_{3L}, I_{3R}, Y_L + Y_R\} = 0$

$SU(2)_R \times SU(2)_L \times U(1)_{L+R}$ preserved

If all y_i $\{I_{3L} + I_{3R}, Y_L + Y_R\} = 0$*

$U(1)^2_{L+R}$ preserved*

(*in a basis where Yukawas are diagonalised)

Block-diagonalising γ via flavour decomposition

If we class Wilson coefficients by their flavour quantum numbers, we can trivially block-diagonalise γ

Block sizes depend on which Yukawa couplings we neglect

Only gauge couplings

All flavour quantum numbers are conserved

$\{d, \mathcal{I}, \mathcal{I}_3, \mathcal{Y}\}_F$

4 quantum numbers for each species F

Gauge couplings and top Yukawa

Conserves everything but $\{d_{irrep}\}_{\{Q,u\}}$

SM flavour symm broken

$SU(3)_Q \times SU(3)_u \to SU(2)_Q \times SU(2)_u \times U(1)_{Q+u}$

All (Gauge couplings and all Yukawas)

Only Y_{L+e} and $I_{3,L+e}$ are conserved

(equivalent to two individual lepton numbers)

neglecting more parameters ⟷ neglecting fewer parameters

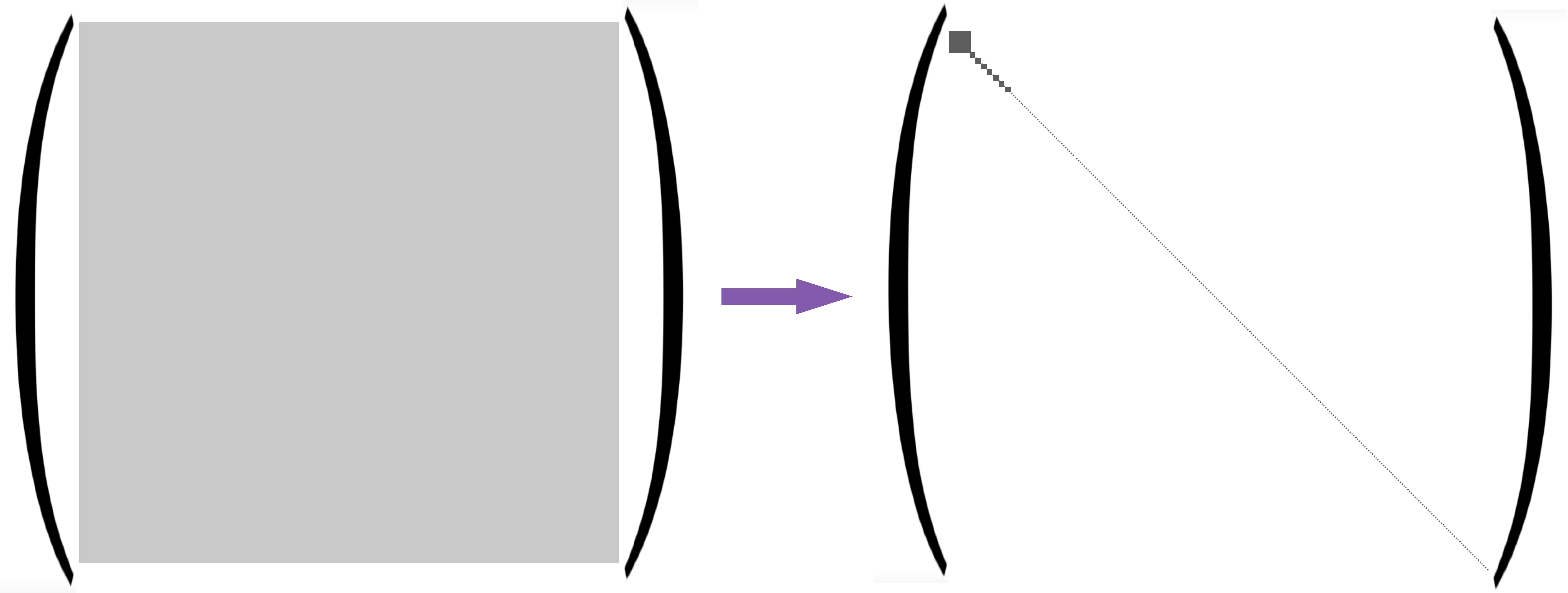
Blocks under different approximations

Gauge couplings and top Yukawa

Conserved: $\{\mathcal{I},\mathcal{I}_3,\mathcal{Y}\}_{\{Q,u\}}$, $\{d,\mathcal{I},\mathcal{I}_3,\mathcal{Y}\}_{\{d,L,e\}}$

(4,0) block before flavour decomposition

1460×1460



(4,0) block after flavour decomposition

Largest block size 61×61 (contains all the flavour singlets plus things with $\mathcal{I}_{\{Q,u\}} = 0$.)

Block size	61	17	13	12	8	2	1
Multiplicity	1	7	8	15	8	217	498

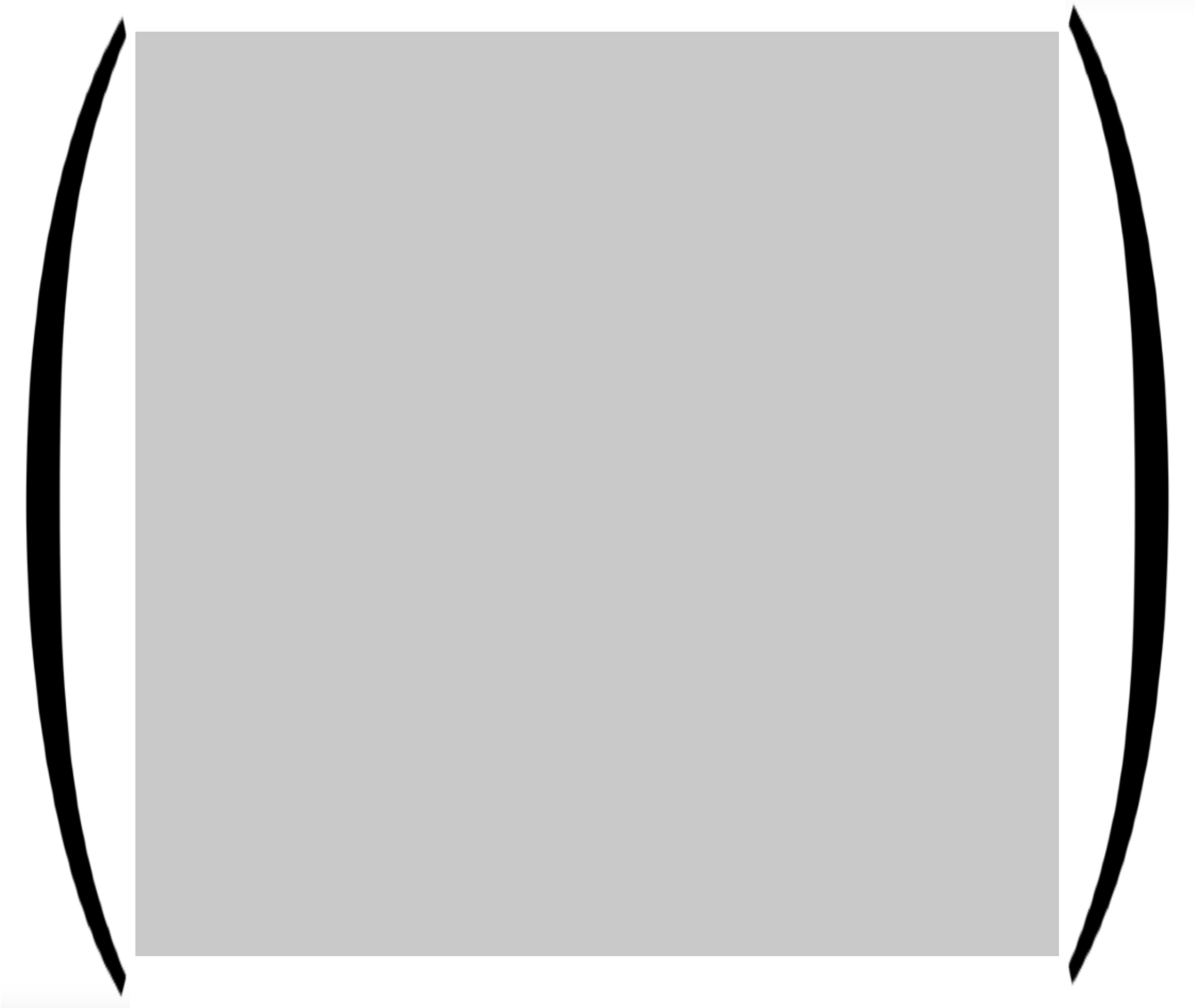
Blocks under different approximations

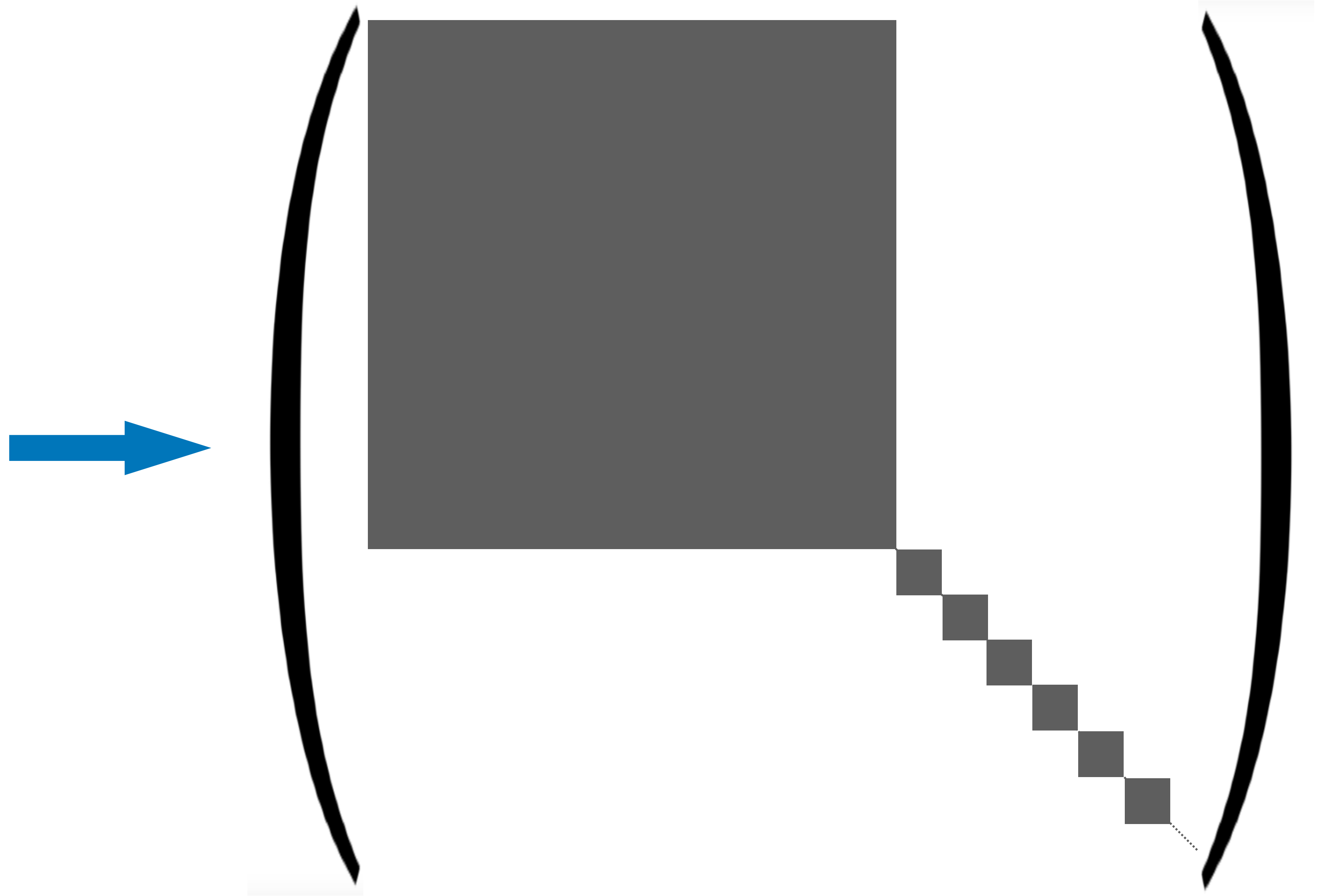
All (Gauge couplings and all Yukawas)

Conserved: $\mathcal{I}_{3,L} + \mathcal{I}_{3,e},\ \mathcal{Y}_L + \mathcal{Y}_e$

(4,0) block before flavour decomposition

1460×1460





(4,0) block after flavour decomposition

Largest block size

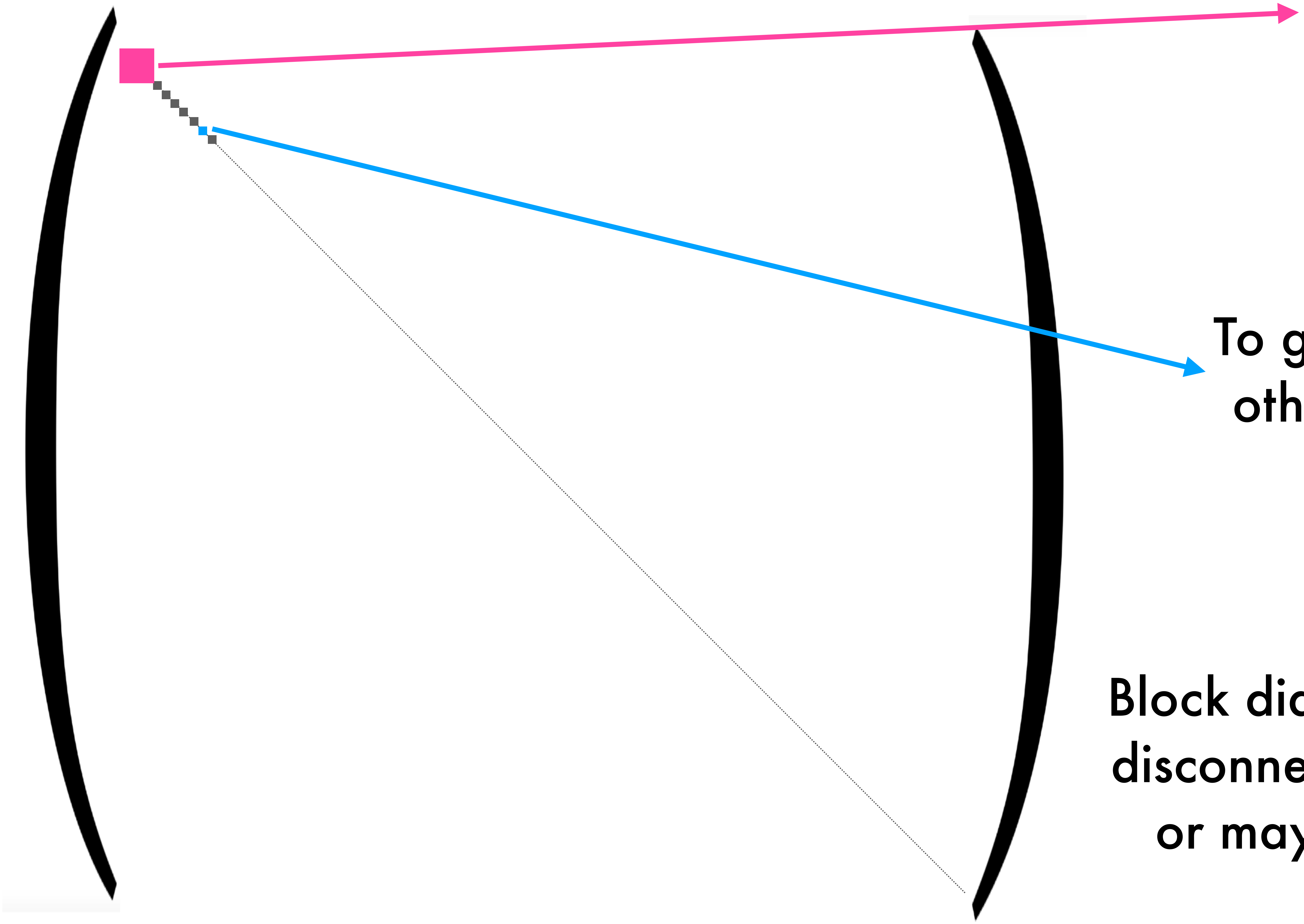
932×932

6 4×4 blocks

6 3×3 blocks

Block size	932	81	4	3
Multiplicity	1	6	6	6

Invariant categorisations: step by step approach



We could start by focussing on this subset of broadly flavour conserving coefficients

To go further, we can also look at other subsets containing flavour-violating coefficients

Block diagonalisation ensures they are theoretically disconnected. Depending on observables they may or may not be experimentally disconnected too

Pheno uses

Which coefficients can be induced by running from any given coefficient (including flavour structure), or vice versa?

e.g. the lepton flavour non-universal part of the operator

$$\mathcal{L}_{\rm NP} = \frac{C}{\Lambda^2}\left((\bar{Q}'_3\gamma^\mu Q'_3)(\bar{L}'_3\gamma_\mu L'_3) + (\bar{Q}'_3\gamma^\mu\sigma^I Q'_3)(\bar{L}'_3\gamma_\mu\sigma^I L'_3)\right)$$

which can be responsible for LFUV in B decays

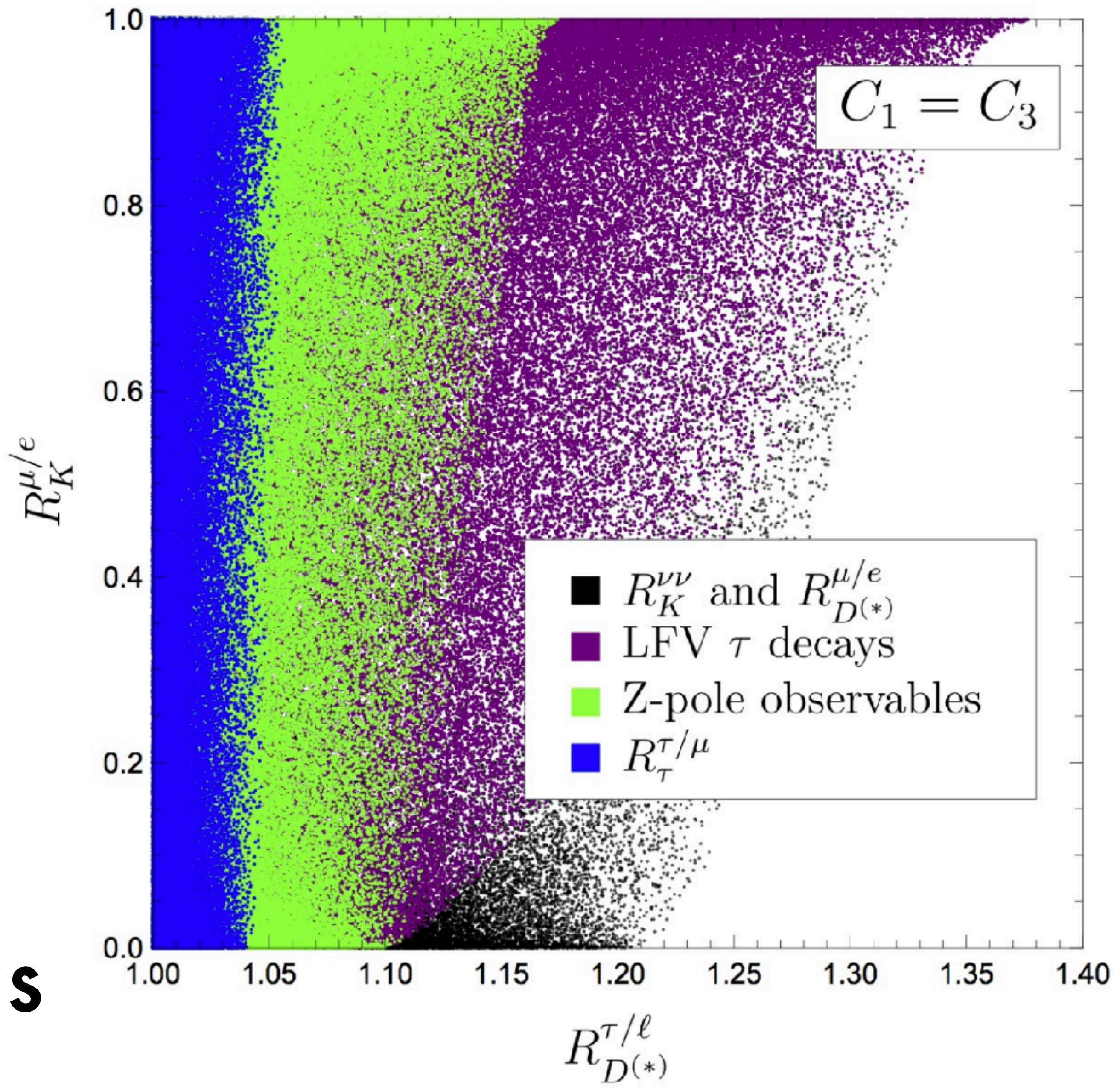
Clebsch-Gordan coefficients

$$\left.\begin{matrix} c_{1,1,8,6} \\ c_{8,6,8,6} \end{matrix}\right\} \begin{matrix} 12\times 12 \\ \text{block} \end{matrix}$$

Mixes with the $c_{8,6}$ lepton octet components of:

$$C_{LQ}^{(1)}\,(\times 2),\ C_{LQ}^{(3)}(\times 2),\ C_{Lu}\,(\times 2),\ C_{Ld},\ \underbrace{C_{LL}\,(\times 2),\ C_{Le}}_{\tau\ \text{decays}},\ \underbrace{C_{HL}^{(1)},\ C_{HL}^{(3)}}_{\text{LFUV in Z couplings}}$$

Feruglio, Paradisi, Pattori, 1606.00524 & 1705.00929

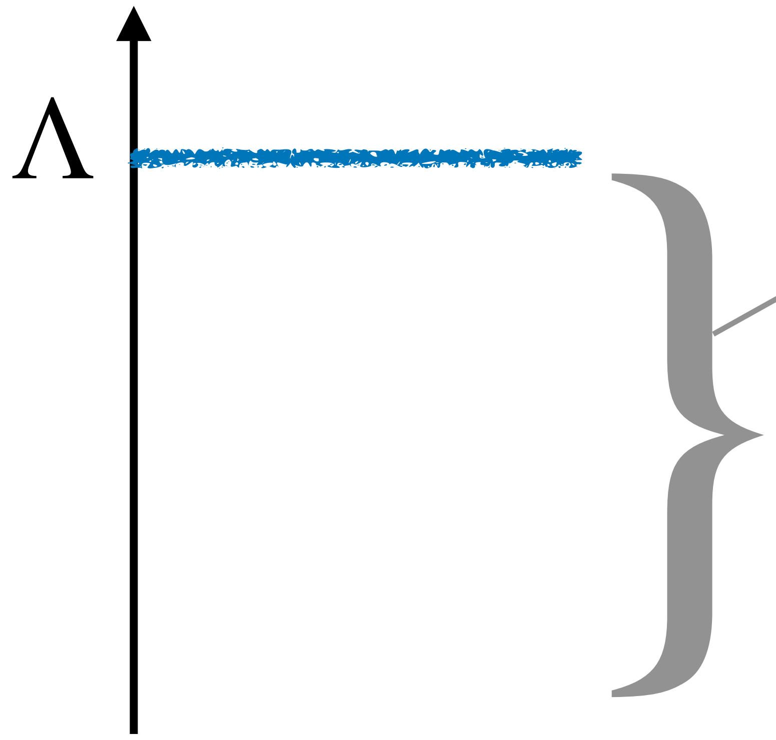


Beyond the SMEFT?

New light degrees of freedom change the game by adding new terms to the anomalous dim matrix

e.g. EFT containing a light axion-like particle

Galda, Neubert, SR 2105.01078



Here particle content is SM+ALP

Only difference between SMEFT and ALP EFT is the addition of ONE more degree of freedom below Λ

ALP interactions with SM particles begin at dim 5

More helicity amplitudes

Beyond the SMEFT?

New light degrees of freedom change the game by adding new terms to the anomalous dim matrix

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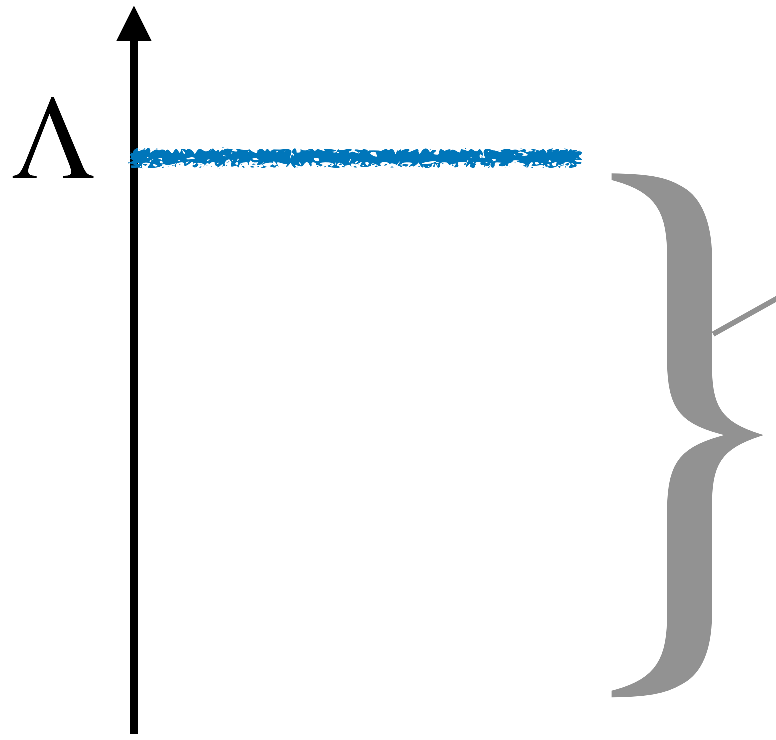

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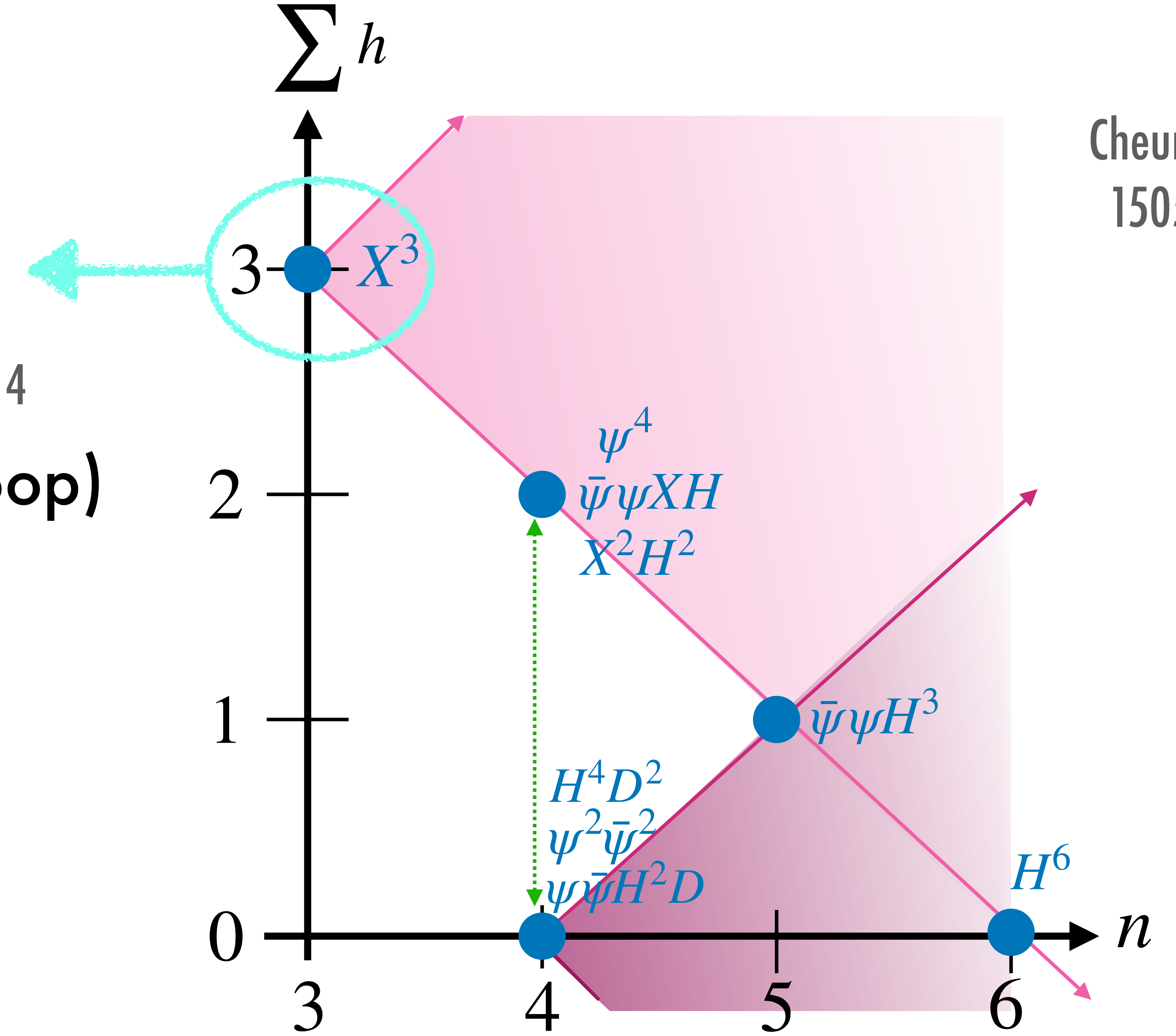


Cheung, Shen, 1505.01844

e.g. in SMEFT, X^3 operators are only self-renormalised

Alonso, Jenkins, Manohar, Trott 1312.2014

$$\dot{C}_G = (12c_{A,3} - 3b_{0,3})\, g_3^2 C_G$$
$$\dot{C}_W = (12c_{A,2} - 3b_{0,2})\, g_2^2 C_W$$

$\Longrightarrow$ if zero at Λ, zero at m_W (to 1-loop)

But in ALP EFT, same operators are renormalised by ALP-boson interactions:

e.g. $$\frac{d}{d\ln\mu} C_G(\mu) \ni \frac{8 g_s}{(4\pi f)^2} C_{GG}^2$$

$\Longrightarrow$ unavoidable in an ALP theory if $C_{GG} \neq 0$

Summary and outlook

- SMEFT phenomenology is complicated by its huge parameter space
- γ_{SMEFT} is large, non-diagonal, & flavourful
- Using a symmetry-based flavour decomposition, achieve simple block diagonalisation of (4,0) operators
- Blocks allow you to understand closed subsets of parameters and narrow in on loop-level pheno

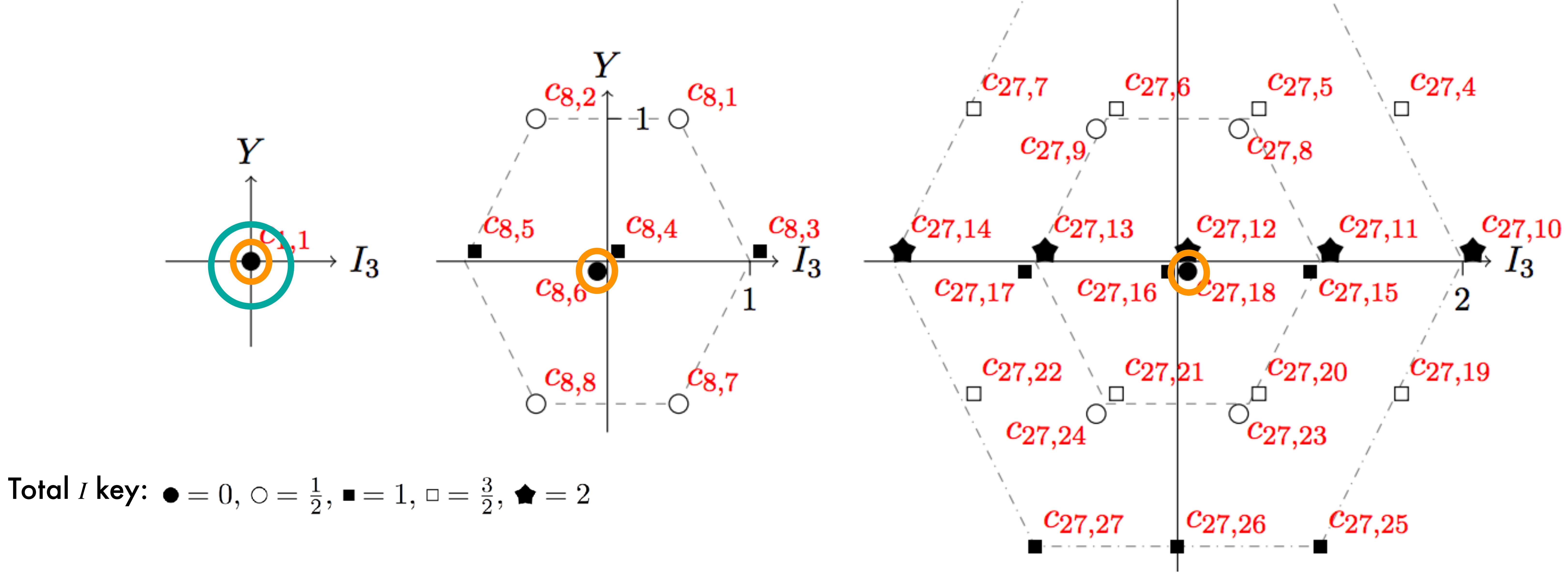
and of course...

Happy birthday Matthias!

Backups…

Flavour symmetry subsets

4 quantum numbers: $\{d_{irrep}, I, I_3, Y\}$



This is a fully general decomposition which does not restrict form of Wilson coefficients

But, since it is couched in flavour symmetry irreps,

easy to identify the subsets of coefficients that are invariant under exact flavour symmetries

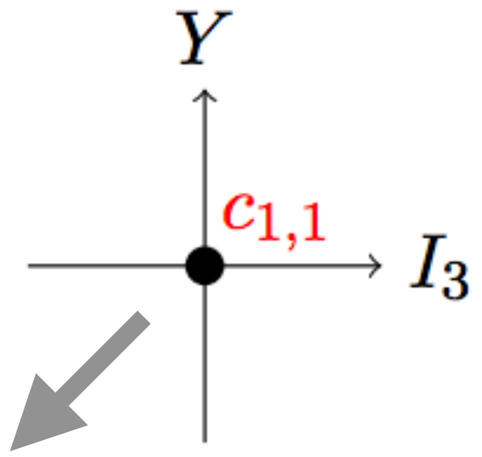
e.g.

Exact $U(3)$ symmetry: just singlets

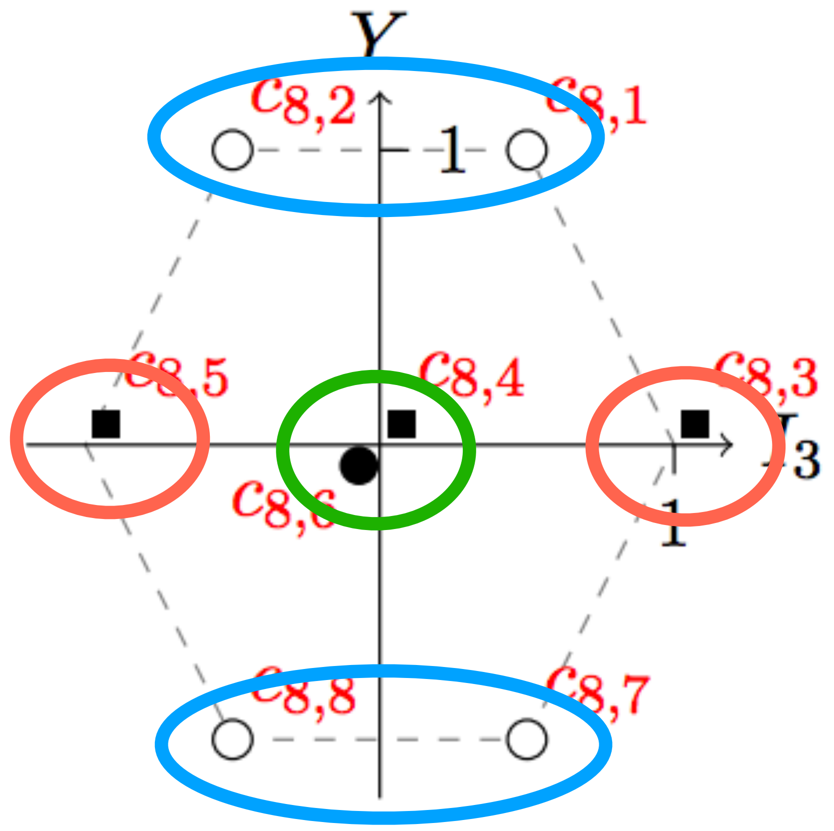
Exact $U(2)$ symmetry: just $I = 0$

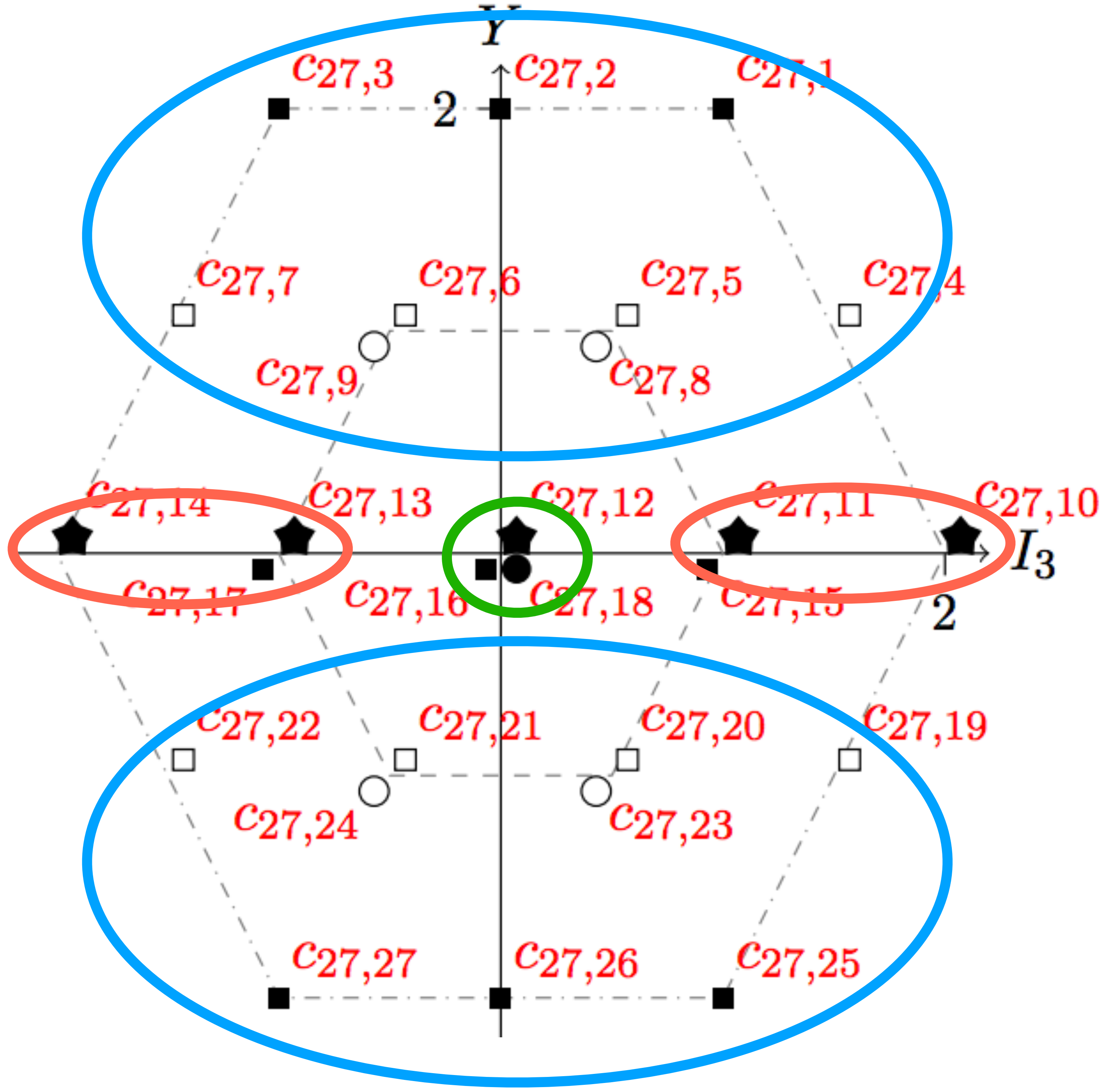
Flavour quantum numbers and pheno

For each flavour: **4 quantum numbers:** $\{d_{irrep}, I, I_3, Y\}$



Singlet:
flavour conserving &
flavour universal





Total I key: $\bullet = 0$, $\circ = \frac{1}{2}$, $\blacksquare = 1$, $\square = \frac{3}{2}$, $\bigstar = 2$

$d_{irrep} > 1,\ \{I_3, Y\} = 0$
Flavour conserving but non-universal

$I_3 > 0,\ Y = 0$
Flavour changing in first two generations only

$Y \neq 0$
Flavour changing involving 3rd generation

For 27-plet, the larger the values of I_3 and Y, the more flavour violating

Invariant categorisations: a minimal parameter set

e.g. Assume that the flavour breaking we see in the SM is dominant

i.e. NP respects (at least) $U(2)_Q \times U(2)_u \times U(3)^3$ (and CP)

So within the (4,0) block we need the 61 parameters with

$$\mathcal{I}_{\{Q,u\}} = 0,\ d_{\{d,L,e\}} = 1$$

+ other operator coefficients $\underbrace{C_W\ C_G}_{(3,3)}$ $\underbrace{C_{tB}\ C_{tW}\ C_{tG}}_{(4,2)}$ $\underbrace{C_{HB}\ C_{HW}\ C_{HG}\ C_{HWB}}_{(4,2)}$ $\underbrace{C_{tH}}_{(5,1)}$ $\underbrace{C_H}_{(6,0)}$ = 11 parameters

$\left(n, \sum h\right) =$ (3,3) (4,2) (4,2) (5,1) (6,0)

72 total

agrees with Greljo, Palavric, Thomsen 2203.09561 Table 1

If we neglect y_b and smaller, this set is complete across scales

This is a consistent choice for global fits

The non-(4,0) operators do not run into the (4,0) block

Invariant categorisations: a minimal parameter set

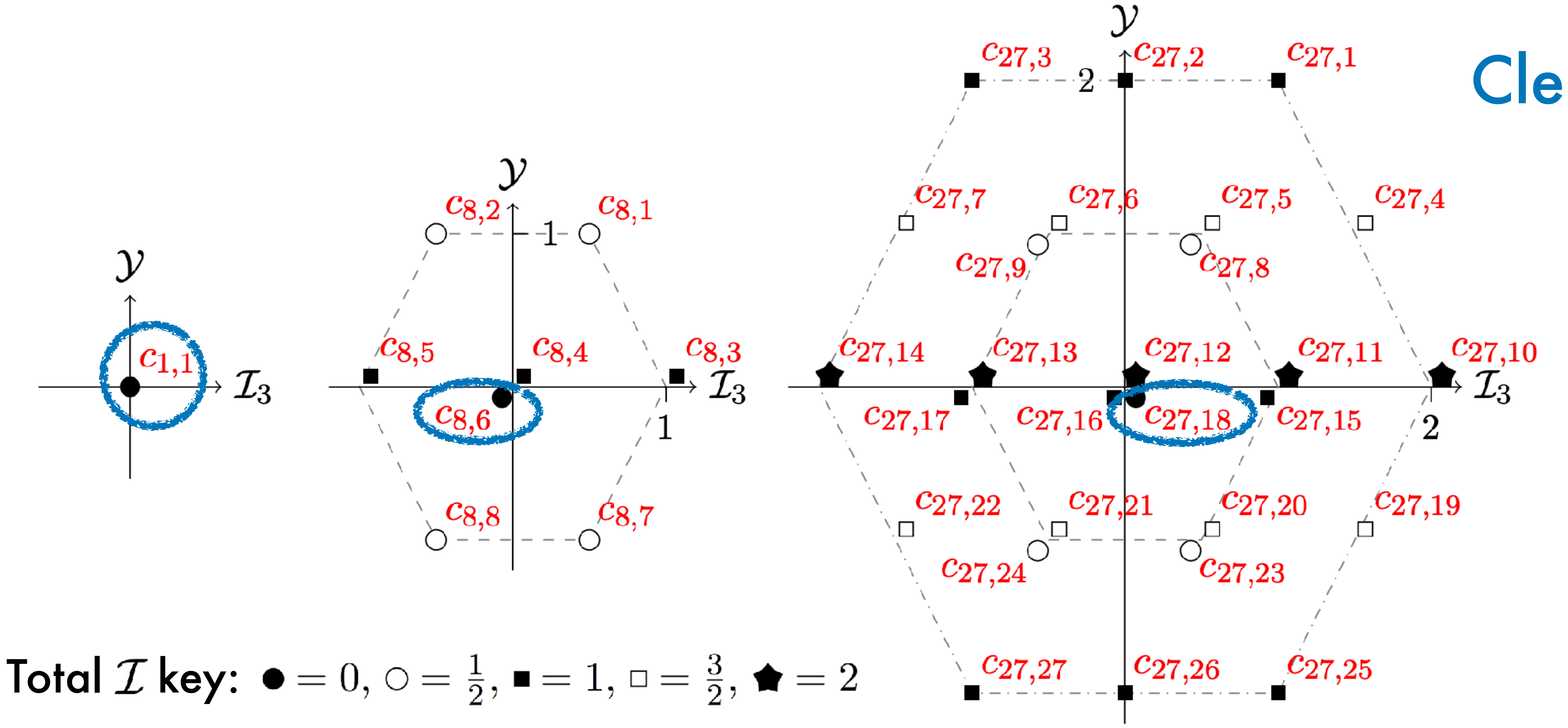
e.g. Assume that the flavour breaking we see in the SM is dominant

i.e. NP respects (at least) $U(2)_Q \times U(2)_u \times U(3)^3$

So within the (4,0) block we need the 61 parameters with

$$\mathcal{I}_{\{Q,u\}} = 0,\; d_{\{d,L,e\}} = 1$$

Not just counting!
Can see the explicit parameters



Clebsch-Gordan decompositions of the parameters:

$$c_{8,6} = \sqrt{\frac{1}{6}}\left(-c_1^1 - c_2^2 + 2c_3^3\right)$$

$$c_{27,18} = \sqrt{\frac{1}{30}}\left(c_{11}^{11} + 2c_{(12)}^{(12)} + c_{22}^{22} - 6c_{(13)}^{(13)} - 6c_{(23)}^{(23)} + 3c_{33}^{33}\right)$$

So, for example, as well as the full singlet piece of C_{qe}, we need $\frac{1}{\sqrt{3}}\left(-C_{qe}^{11ii} - C_{qe}^{22ii} + 2C_{qe}^{33ii}\right)$

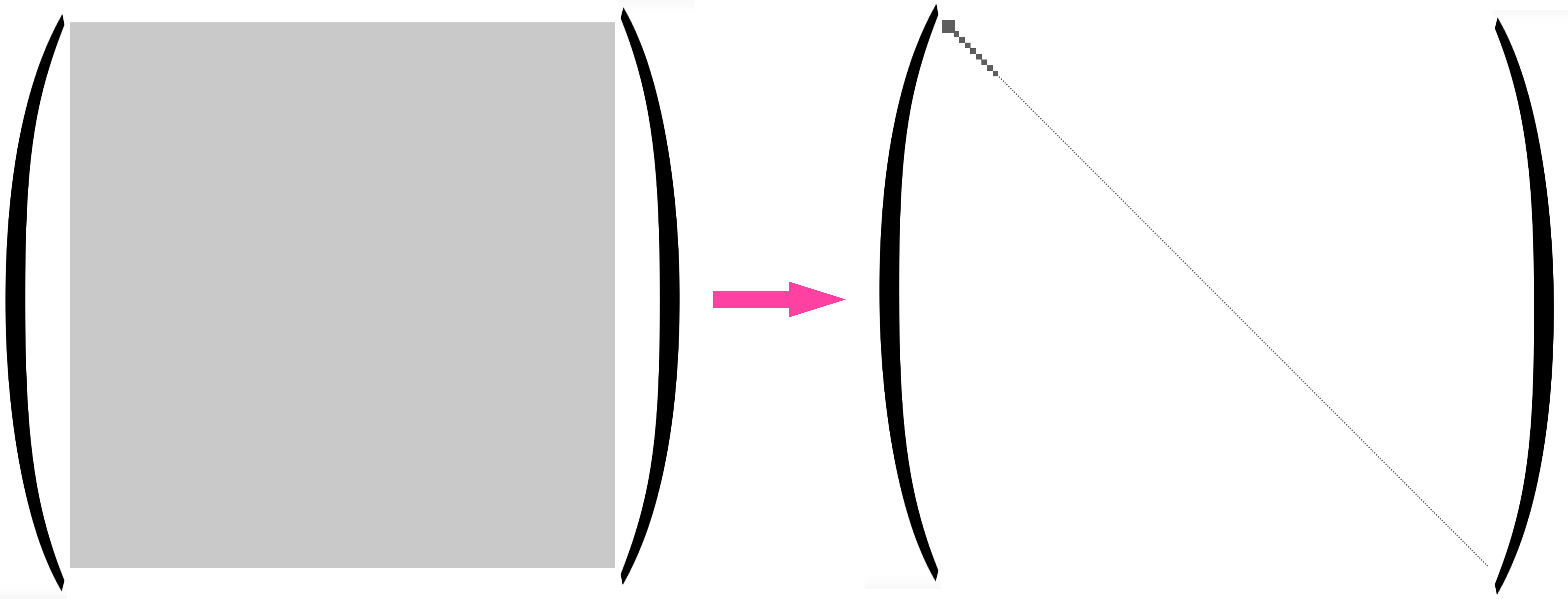
Blocks under different approximations

Only gauge couplings

All flavour quantum numbers conserved: $\{d, \mathcal{I}, \mathcal{I}_3, \mathcal{Y}\}_{\{Q,u,d,L,e\}}$

(4,0) block before flavour decomposition

1460×1460



(4,0) block after flavour decomposition

Largest block size 34×34 (contains all the flavour singlets)

Block size	34	13	9	6	2	1
Multiplicity	1	8	24	8	256	546

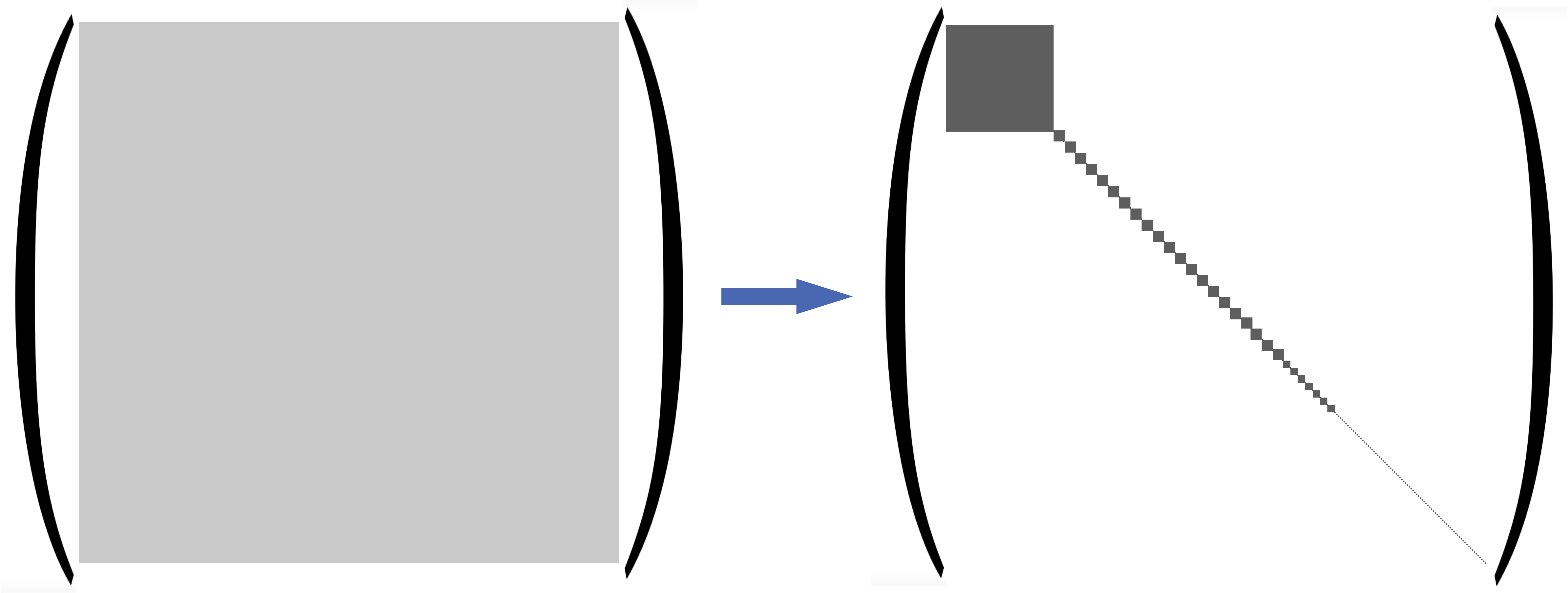
Blocks under different approximations

Gauge couplings and all 3rd generation Yukawas (full CKM)

Conserved: $\{I, I_3, Y\}_{\{u,d,L,e\}}$

(4,0) block
before flavour
decomposition

1460×1460



(4,0) block
after flavour
decomposition

Largest block size
292×292

49 2×2 blocks

321 1×1 blocks