Flavor symmetries

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Happy birthday Matthias

Flavor sum rules

Based on M. Gavrilova, YG, S. Schacht, 2205.12975

Flavor physics

- The goal: overconstraing the CKM matrix
- For that we need that

Number of theory parameters $<$ Number of observable

- The problem (in many cases): QCD
  - Calculate (Lattice)
  - Use approximate symmetries to reduce the number of parameters

Flavor sum rules

- Relation between amplitudes $\rightarrow$ less parameters
- The approximate symmetries are SU(3) and its subgroups: isospin ($u \leftrightarrow d$), U-spin ($d \leftrightarrow s$), and V-spin ($u \leftrightarrow s$)
- The breaking are $O(1\%)$ for isospin and $O(30\%)$ for U-spin and V-spin
- U-spin is "nicer" because the d and s has the same electric charge.
- Is U-spin useful?

Example: D sum rules

$$D \to P^+ P^- \qquad P = \pi, K$$

- In the U-spin limit all 4 amplitudes are the same
- Experimentally

$$\frac{A(KK)}{A(\pi\pi)} = 1.82 \qquad \frac{A(K\pi)}{A(\pi K)} = 1.15 \qquad \frac{A(\pi K)}{A(\pi\pi)} = 1.26$$

$$\frac{A(KK) + A(\pi\pi)}{A(K\pi) + A(\pi K)} = 1.04$$

- The last relation is valid up to 2nd order, while the first three are valid only to 1st order

Higher order sum rules

- A formal expansion in ϵ
  - To what order we can expand?
  - How to do the expansion?
- How practical higher order sumrules are?
  - Can we do precision physics with U-spin?

This talk is on the mathematical structure of U-spin sumrules to all orders

The traditional way

The U-spin set

- A U-spin set is a set of amplitudes (processes) that are related by U-spin
- An amplitude is defined by the U-spin properties of the
  - initial state
  - final state
  - Hamiltonian
- U-spin limit Hamiltonian

$$H = \sum_m f_{u,m} H_m^u$$

  - $f_{u,m}$: Weak parameters (CKM matrix elements)
  - H_m^u: operators

The standard way

- Two reasons for sum rules
  - Relations between the CKM parameters
  - The matrix element is independent on m (Wigner-Eckart theorem)

$$A_j = \sum C_{j\alpha} X_\alpha$$

    - $C_{j\alpha}$: m-dependent number (CG)
    - X_α: m-independent reduced matrix element
- Example D decays
  - One sumrule is due to $V_{us} \approx -V_{cd}$
  - Two sumrules are due to the m-independence of the $\Delta U = 1$ matrix element

The breaking expansion

- The reason is the mass different: $m_s \neq m_d$
- The small parameters is at most

$$\epsilon \sim \frac{m_s - m_d}{\Lambda_{\rm QCD}} \approx 0.3$$

- It contributes to H with H_ϵ a spurion with $u = 1$, $m = 0$
- We add it to H as

$$H^{(b+1)} = H^{(b)} \otimes H_\epsilon$$

Getting the sum rules

- Find the rotation matrix from the physical basis to the U-spin basis up to a specific order, b
- Find the null space of that matrix

Example

$$\mathcal{A}_j = f_{u,m} \sum_{\alpha} C_{j\alpha} X_\alpha$$

X_α is a short notation for reduced matrix elements

- Below is the matrix $C_{j\alpha}$ up to $b = 2$
- To find the **sum rules** one needs to find the null space of the matrix $C_{j\alpha}^T$

Example: $C_b \to L_b P^+ P^-$

Decay amplitude	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9
$A\left(\Lambda_c^+ \to \Sigma^+ K^- K^+\right)$	$\frac{1}{3}$	$-\frac{2}{3}$	0	$\frac{1}{\sqrt{10}}$	$-\frac{1}{3\sqrt{2}}$	$\frac{\sqrt{2}}{3}$	0	0	0
$A\left(\Xi_c^+ \to p\pi^-\pi^+\right)$	$\frac{1}{3}$	$-\frac{2}{3}$	0	$-\frac{1}{\sqrt{10}}$	$\frac{1}{3\sqrt{2}}$	$-\frac{\sqrt{2}}{3}$	0	0	0
$A\left(\Lambda_c^+ \to \Sigma^+ \pi^- \pi^+\right)$	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{10}}$	$-\frac{1}{3\sqrt{2}}$	$-\frac{1}{3\sqrt{2}}$	$\frac{1}{\sqrt{6}}$	0	0
$A\left(\Xi_c^+ \to pK^-K^+\right)$	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{10}}$	$\frac{1}{3\sqrt{2}}$	$\frac{1}{3\sqrt{2}}$	$-\frac{1}{\sqrt{6}}$	0	0
$A\left(\Lambda_c^+ \to \Sigma^+ \pi^- K^+\right)$	$\frac{\sqrt{2}}{3}$	$-\frac{1}{3\sqrt{2}}$	$-\frac{1}{\sqrt{6}}$	$\frac{2}{3\sqrt{5}}$	0	0	0	$\frac{1}{3\sqrt{2}}$	$\frac{1}{\sqrt{6}}$
$A\left(\Xi_c^+ \to pK^-\pi^+\right)$	$\frac{\sqrt{2}}{3}$	$-\frac{1}{3\sqrt{2}}$	$-\frac{1}{\sqrt{6}}$	$-\frac{2}{3\sqrt{5}}$	0	0	0	$-\frac{1}{3\sqrt{2}}$	$-\frac{1}{\sqrt{6}}$
$A\left(\Lambda_c^+ \to pK^-\pi^+\right)$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{10}}$	$-\frac{1}{3\sqrt{2}}$	$-\frac{1}{3\sqrt{2}}$	$-\frac{1}{\sqrt{6}}$	0	0
$A\left(\Xi_c^+ \to \Sigma^+ \pi^- K^+\right)$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{10}}$	$\frac{1}{3\sqrt{2}}$	$\frac{1}{3\sqrt{2}}$	$\frac{1}{\sqrt{6}}$	0	0
$A\left(\Lambda_c^+ \to pK^-K^+\right)$	$\frac{\sqrt{2}}{3}$	$-\frac{1}{3\sqrt{2}}$	$\frac{1}{\sqrt{6}}$	$\frac{2}{3\sqrt{5}}$	0	0	0	$\frac{1}{3\sqrt{2}}$	$-\frac{1}{\sqrt{6}}$
$A\left(\Xi_c^+ \to \Sigma^+ \pi^- \pi^+\right)$	$\frac{\sqrt{2}}{3}$	$-\frac{1}{3\sqrt{2}}$	$\frac{1}{\sqrt{6}}$	$-\frac{2}{3\sqrt{5}}$	0	0	0	$-\frac{1}{3\sqrt{2}}$	$\frac{1}{\sqrt{6}}$
$A\left(\Lambda_c^+ \to p\pi^-\pi^+\right)$	$\frac{\sqrt{2}}{3}$	$\frac{\sqrt{2}}{3}$	0	$\frac{2}{3\sqrt{5}}$	0	0	0	$-\frac{\sqrt{2}}{3}$	0
$A\left(\Xi_c^+ \to \Sigma^+ K^- K^+\right)$	$\frac{\sqrt{2}}{3}$	$\frac{\sqrt{2}}{3}$	0	$-\frac{2}{3\sqrt{5}}$	0	0	0	$\frac{\sqrt{2}}{3}$	0
$A\left(\Lambda_c^+ \to p\pi^-K^+\right)$	1	0	0	$\frac{1}{\sqrt{10}}$	$\frac{1}{\sqrt{2}}$	0	0	0	0
$A\left(\Xi_c^+ \to \Sigma^+ K^- \pi^+\right)$	1	0	0	$-\frac{1}{\sqrt{10}}$	$-\frac{1}{\sqrt{2}}$	0	0	0	0

Decay amplitude	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}	X_{18}	X_{19}	X_{20}
$A\left(\Lambda_c^+ \to \Sigma^+ K^- K^+\right)$	$-\frac{1}{2\sqrt{15}}$	$-\frac{1}{2\sqrt{15}}$	$\frac{1}{\sqrt{15}}$	0	$-\frac{1}{2\sqrt{5}}$	$\frac{1}{6}$	$-\frac{1}{3}$	0	0	0	0
$A\left(\Xi_c^+ \to p\pi^-\pi^+\right)$	$-\frac{1}{2\sqrt{15}}$	$-\frac{1}{2\sqrt{15}}$	$\frac{1}{\sqrt{15}}$	0	$-\frac{1}{2\sqrt{5}}$	$\frac{1}{6}$	$-\frac{1}{3}$	0	0	0	0
$A\left(\Lambda_c^+ \to \Sigma^+ \pi^- \pi^+\right)$	$-\frac{1}{2\sqrt{15}}$	$-\frac{1}{2\sqrt{15}}$	$-\frac{1}{2\sqrt{15}}$	$\frac{1}{2\sqrt{5}}$	$-\frac{1}{2\sqrt{5}}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{2\sqrt{3}}$	0	0	0
$A\left(\Xi_c^+ \to pK^-K^+\right)$	$-\frac{1}{2\sqrt{15}}$	$-\frac{1}{2\sqrt{15}}$	$-\frac{1}{2\sqrt{15}}$	$\frac{1}{2\sqrt{5}}$	$-\frac{1}{2\sqrt{5}}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{2\sqrt{3}}$	0	0	0
$A\left(\Lambda_c^+ \to \Sigma^+ \pi^- K^+\right)$	0	$-\frac{2}{3}\sqrt{\frac{2}{15}}$	$\frac{1}{3}\sqrt{\frac{2}{15}}$	$\frac{1}{3}\sqrt{\frac{2}{5}}$	0	0	0	0	$-\frac{1}{3}\sqrt{\frac{2}{3}}$	$\frac{1}{3\sqrt{6}}$	$\frac{1}{3\sqrt{2}}$
$A\left(\Xi_c^+ \to pK^-\pi^+\right)$	0	$-\frac{2}{3}\sqrt{\frac{2}{15}}$	$\frac{1}{3}\sqrt{\frac{2}{15}}$	$\frac{1}{3}\sqrt{\frac{2}{5}}$	0	0	0	0	$-\frac{1}{3}\sqrt{\frac{2}{3}}$	$\frac{1}{3\sqrt{6}}$	$\frac{1}{3\sqrt{2}}$
$A\left(\Lambda_c^+ \to pK^-\pi^+\right)$	$-\frac{1}{2\sqrt{15}}$	$-\frac{1}{2\sqrt{15}}$	$-\frac{1}{2\sqrt{15}}$	$-\frac{1}{2\sqrt{5}}$	$-\frac{1}{2\sqrt{5}}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{2\sqrt{3}}$	0	0	0
$A\left(\Xi_c^+ \to \Sigma^+ \pi^- K^+\right)$	$-\frac{1}{2\sqrt{15}}$	$-\frac{1}{2\sqrt{15}}$	$-\frac{1}{2\sqrt{15}}$	$-\frac{1}{2\sqrt{5}}$	$-\frac{1}{2\sqrt{5}}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{2\sqrt{3}}$	0	0	0
$A\left(\Lambda_c^+ \to pK^-K^+\right)$	0	$-\frac{2}{3}\sqrt{\frac{2}{15}}$	$\frac{1}{3}\sqrt{\frac{2}{15}}$	$-\frac{1}{3}\sqrt{\frac{2}{5}}$	0	0	0	0	$-\frac{1}{3}\sqrt{\frac{2}{3}}$	$\frac{1}{3\sqrt{6}}$	$-\frac{1}{3\sqrt{2}}$
$A\left(\Xi_c^+ \to \Sigma^+ \pi^- \pi^+\right)$	0	$-\frac{2}{3}\sqrt{\frac{2}{15}}$	$\frac{1}{3}\sqrt{\frac{2}{15}}$	$-\frac{1}{3}\sqrt{\frac{2}{5}}$	0	0	0	0	$-\frac{1}{3}\sqrt{\frac{2}{3}}$	$\frac{1}{3\sqrt{6}}$	$-\frac{1}{3\sqrt{2}}$
$A\left(\Lambda_c^+ \to p\pi^-\pi^+\right)$	0	$-\frac{2}{3}\sqrt{\frac{2}{15}}$	$-\frac{2}{3}\sqrt{\frac{2}{15}}$	0	0	0	0	0	$-\frac{1}{3}\sqrt{\frac{2}{3}}$	$-\frac{1}{3}\sqrt{\frac{2}{3}}$	0
$A\left(\Xi_c^+ \to \Sigma^+ K^- K^+\right)$	0	$-\frac{2}{3}\sqrt{\frac{2}{15}}$	$-\frac{2}{3}\sqrt{\frac{2}{15}}$	0	0	0	0	0	$-\frac{1}{3}\sqrt{\frac{2}{3}}$	$-\frac{1}{3}\sqrt{\frac{2}{3}}$	0
$A\left(\Lambda_c^+ \to p\pi^-K^+\right)$	$\frac{1}{2\sqrt{15}}$	$-\frac{1}{2}\sqrt{\frac{3}{5}}$	0	0	$\frac{1}{2\sqrt{5}}$	$\frac{1}{2}$	0	0	0	0	0
$A\left(\Xi_c^+ \to \Sigma^+ K^- \pi^+\right)$	$\frac{1}{2\sqrt{15}}$	$-\frac{1}{2}\sqrt{\frac{3}{5}}$	0	0	$\frac{1}{2\sqrt{5}}$	$\frac{1}{2}$	0	0	0	0	0

Note, CKM-free amplitudes

$b = 0$ (X_1–X_3) $b = 1$ (X_4–X_9) $b = 2$ (X_{10}–X_{20})

The U-spin expansion

The basics

- We consider only one u in H^u_m
- Any U-spin system can be constructed from doublets
- We first study systems of doublets. Then we get the rest by "combining" doublets
- The movement of irreps between initial/final state and the Hamiltonian does not change the structure of the sum rules (“crossing symmetry”)
- We consider a system with all the U-spin doublets in the final state.

U-spin pairs

We order the doublets in arbitrary but defined order. Each physics amplitude is a set of $+$ and $-$

$$A_j = (-, +, +, ..., -), \qquad A_j = \sum C_{j\alpha} X_\alpha$$

We define "U-spin conjugation" $d \leftrightarrow s$ (or $+ \leftrightarrow -$)

$$\bar{A}_j = (+, -, -, ..., +), \qquad \bar{A}_j = \sum (-1)^b C_{j\alpha} X_\alpha$$

- α is a multi-index that include b
- X_α are the reduced matrix elements. Each X_α has a specific b
- The $(-1)^b$ factor is not trivial and important

Basis change

$$A_j = \sum C_{j\alpha} X_\alpha \qquad \bar{A}_j = \sum (-1)^b C_{j\alpha} X_\alpha$$

Then we do a basis change

$$a_j \equiv A_j - \bar{A}_j \qquad s_j \equiv A_j + \bar{A}_j$$

- a_j has only terms that are odd in b
- s_j has only terms that are even in b
- We have decoupling: a-type and s-type sumrules
- At leading order, $a_i = 0$ (Grounu, 2000)
- All sumrules are of the form

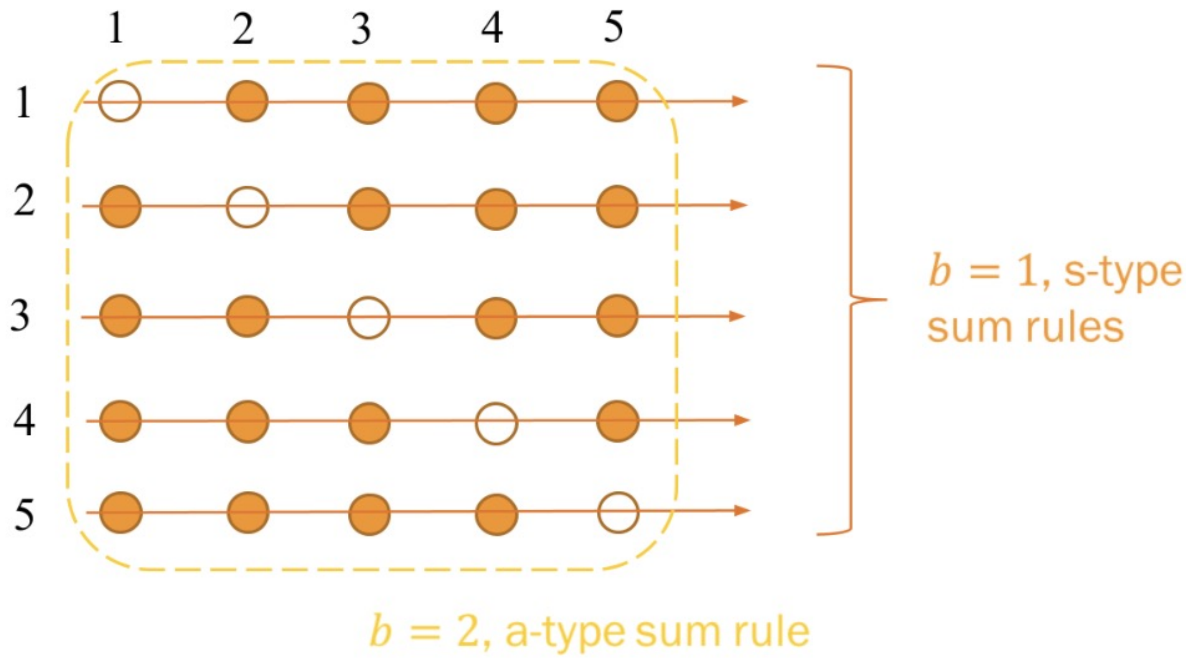
$$\sum a_j = 0 \qquad \sum s_j = 0$$

Coordinate notaion

$$A_j = (-,+,+,-,-,+) \equiv (3,4)$$

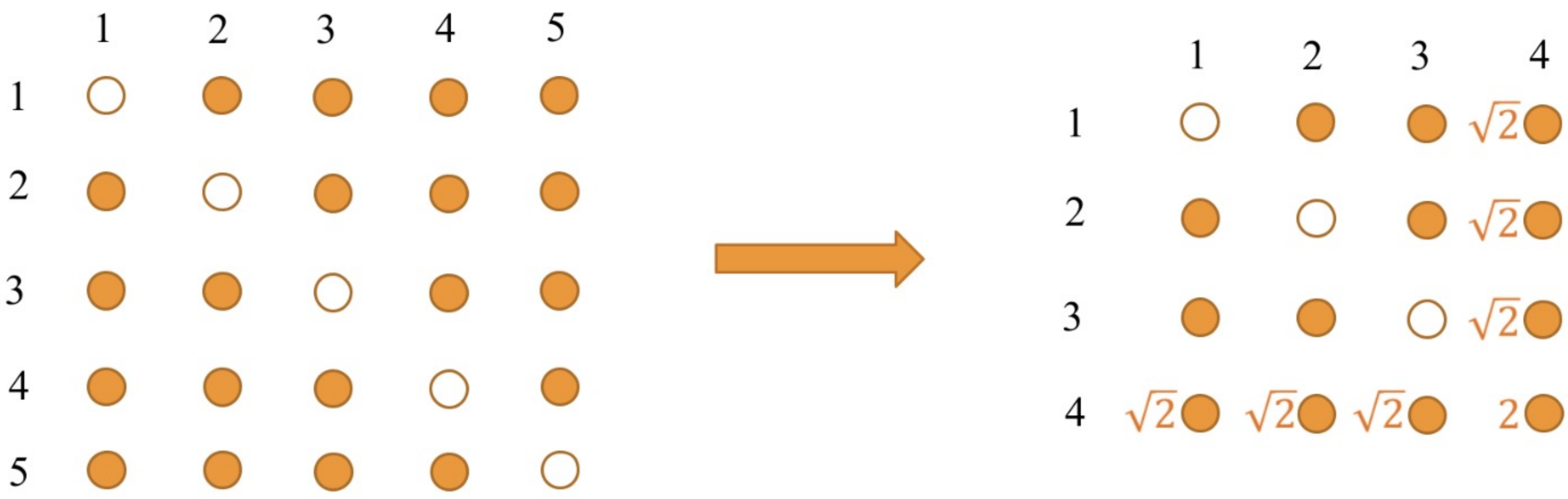
- The locations of the $-$ without the first one
- It has a length of $d = n/2 - 1$
- The U-spin set is mapped to a d-dimensional lattice
- Results:
  - We can read the sumrules very easily from the lattice
  - The highest order that we have a sumrule is d
  - We know the number of sumrules at each order
  - There is only one sumrule in the highest order

The lattice



- Points represent zero order a-type sumrules
- Lines represent 1st order s-type sumrules
- The plane represents the 2nd order a-type sumrule

Generalization: 4 doublets and a triplet



- Points represent zero order a-type sumrules
- Lines represent 1st order s-type sumrules
- The plane represents the 2nd order a-type sumrule

Generalization sumrules

- Sum rules valid up to $b = 0$

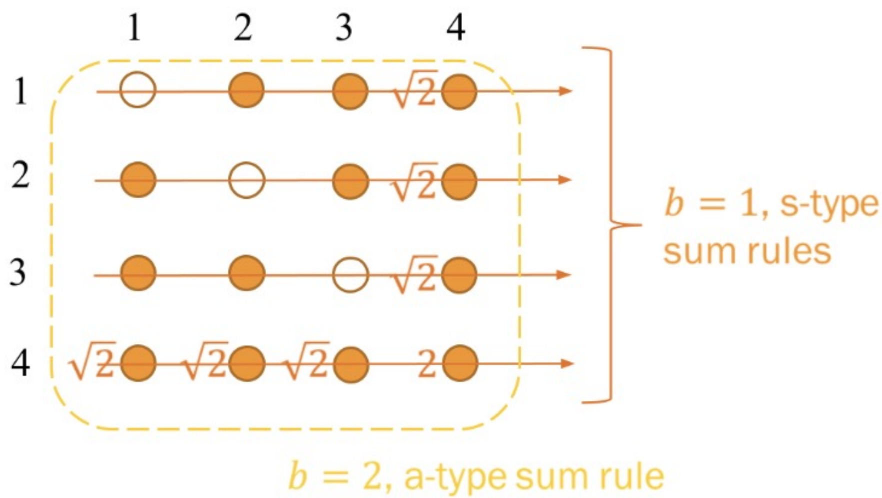
$$a_{(1,2)} = a_{(1,3)} = a_{(1,4)} = a_{(2,3)} = a_{(2,4)} = a_{(3,4)} = a_{(4,4)} = 0$$

- Sum rules valid up to $b = 1$

$$s_{(1,2)} + s_{(1,3)} + \sqrt{2}s_{(1,4)} = 0$$
$$s_{(1,2)} + s_{(2,3)} + \sqrt{2}s_{(2,4)} = 0$$
$$s_{(1,3)} + s_{(2,3)} + \sqrt{2}s_{(3,4)} = 0$$
$$s_{(1,4)} + s_{(2,4)} + s_{(3,4)} + \sqrt{2}s_{(4,4)} = 0$$

- Sum rules valid up to $b = 2$

$$a_{(1,2)} + a_{(1,3)} + a_{(2,3)} + a_{(4,4)} + \sqrt{2}a_{(1,4)} + \sqrt{2}a_{(2,4)} + \sqrt{2}a_{(3,4)} = 0$$



The traditional way

$$\mathcal{A}_j = f_{u,m} \sum_{\alpha} C_{j\alpha} X_\alpha$$

X_α is a short notation for reduced matrix elements

Example: $C_b \to L_b P^+ P^-$

- Below is the matrix $C_{j\alpha}$ up to $b=2$
- To find the **sum rules** one needs to find the null space of the matrix $C_{j\alpha}^T$

Decay amplitude	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9
$A\left(\Lambda_c^+ \to \Sigma^+ K^- K^+\right)$	$\frac{1}{3}$	$-\frac{2}{3}$	0	$\frac{1}{\sqrt{10}}$	$-\frac{1}{3\sqrt{2}}$	$\frac{\sqrt{2}}{3}$	0	0	0
$A\left(\Xi_c^+ \to p \pi^- \pi^+\right)$	$\frac{1}{3}$	$-\frac{2}{3}$	0	$-\frac{1}{\sqrt{10}}$	$\frac{1}{3\sqrt{2}}$	$-\frac{\sqrt{2}}{3}$	0	0	0
$A\left(\Lambda_c^+ \to \Sigma^+ \pi^- \pi^+\right)$	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{10}}$	$-\frac{1}{3\sqrt{2}}$	$-\frac{1}{3\sqrt{2}}$	$\frac{1}{\sqrt{6}}$	0	0
$A\left(\Xi_c^+ \to p K^- K^+\right)$	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{10}}$	$\frac{1}{3\sqrt{2}}$	$\frac{1}{3\sqrt{2}}$	$-\frac{1}{\sqrt{6}}$	0	0
$A\left(\Lambda_c^+ \to \Sigma^+ \pi^- K^+\right)$	$\frac{\sqrt{2}}{3}$	$-\frac{1}{3\sqrt{2}}$	$-\frac{1}{\sqrt{6}}$	$\frac{2}{3\sqrt{5}}$	0	0	0	$\frac{1}{3\sqrt{2}}$	$\frac{1}{\sqrt{6}}$
$A\left(\Xi_c^+ \to p K^- \pi^+\right)$	$\frac{\sqrt{2}}{3}$	$-\frac{1}{3\sqrt{2}}$	$-\frac{1}{\sqrt{6}}$	$-\frac{2}{3\sqrt{5}}$	0	0	0	$-\frac{1}{3\sqrt{2}}$	$-\frac{1}{\sqrt{6}}$
$A\left(\Lambda_c^+ \to p K^- \pi^+\right)$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{10}}$	$-\frac{1}{3\sqrt{2}}$	$-\frac{1}{3\sqrt{2}}$	$-\frac{1}{\sqrt{6}}$	0	0
$A\left(\Xi_c^+ \to \Sigma^+ \pi^- K^+\right)$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{10}}$	$\frac{1}{3\sqrt{2}}$	$\frac{1}{3\sqrt{2}}$	$\frac{1}{\sqrt{6}}$	0	0
$A\left(\Lambda_c^+ \to p K^- K^+\right)$	$\frac{\sqrt{2}}{3}$	$-\frac{1}{3\sqrt{2}}$	$\frac{1}{\sqrt{6}}$	$\frac{2}{3\sqrt{5}}$	0	0	0	$\frac{1}{3\sqrt{2}}$	$-\frac{1}{\sqrt{6}}$
$A\left(\Xi_c^+ \to \Sigma^+ \pi^- \pi^+\right)$	$\frac{\sqrt{2}}{3}$	$-\frac{1}{3\sqrt{2}}$	$\frac{1}{\sqrt{6}}$	$-\frac{2}{3\sqrt{5}}$	0	0	0	$-\frac{1}{3\sqrt{2}}$	$\frac{1}{\sqrt{6}}$
$A\left(\Lambda_c^+ \to p \pi^- \pi^+\right)$	$\frac{\sqrt{2}}{3}$	$\frac{\sqrt{2}}{3}$	0	$\frac{2}{3\sqrt{5}}$	0	0	0	$-\frac{\sqrt{2}}{3}$	0
$A\left(\Xi_c^+ \to \Sigma^+ K^- K^+\right)$	$\frac{\sqrt{2}}{3}$	$\frac{\sqrt{2}}{3}$	0	$-\frac{2}{3\sqrt{5}}$	0	0	0	$\frac{\sqrt{2}}{3}$	0
$A\left(\Lambda_c^+ \to p \pi^- K^+\right)$	1	0	0	$\frac{1}{\sqrt{10}}$	$\frac{1}{\sqrt{2}}$	0	0	0	0
$A\left(\Xi_c^+ \to \Sigma^+ K^- \pi^+\right)$	1	0	0	$-\frac{1}{\sqrt{10}}$	$-\frac{1}{\sqrt{2}}$	0	0	0	0

Decay amplitude	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}	X_{18}	X_{19}	X_{20}
$A\left(\Lambda_c^+ \to \Sigma^+ K^- K^+\right)$	$-\frac{1}{2\sqrt{15}}$	$-\frac{1}{2\sqrt{15}}$	$\frac{1}{\sqrt{15}}$	0	$-\frac{1}{2\sqrt{5}}$	$\frac{1}{6}$	$-\frac{1}{3}$	0	0	0	0
$A\left(\Xi_c^+ \to p \pi^- \pi^+\right)$	$-\frac{1}{2\sqrt{15}}$	$-\frac{1}{2\sqrt{15}}$	$\frac{1}{\sqrt{15}}$	0	$-\frac{1}{2\sqrt{5}}$	$\frac{1}{6}$	$-\frac{1}{3}$	0	0	0	0
$A\left(\Lambda_c^+ \to \Sigma^+ \pi^- \pi^+\right)$	$-\frac{1}{2\sqrt{15}}$	$-\frac{1}{2\sqrt{15}}$	$-\frac{1}{2\sqrt{15}}$	$\frac{1}{2\sqrt{5}}$	$-\frac{1}{2\sqrt{5}}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{2\sqrt{3}}$	0	0	0
$A\left(\Xi_c^+ \to p K^- K^+\right)$	$-\frac{1}{2\sqrt{15}}$	$-\frac{1}{2\sqrt{15}}$	$-\frac{1}{2\sqrt{15}}$	$\frac{1}{2\sqrt{5}}$	$-\frac{1}{2\sqrt{5}}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{2\sqrt{3}}$	0	0	0
$A\left(\Lambda_c^+ \to \Sigma^+ \pi^- K^+\right)$	0	$-\frac{2}{3}\sqrt{\frac{2}{15}}$	$\frac{1}{3}\sqrt{\frac{2}{15}}$	$\frac{1}{3}\sqrt{\frac{2}{5}}$	0	0	0	0	$-\frac{1}{3}\sqrt{\frac{2}{3}}$	$\frac{1}{3\sqrt{6}}$	$\frac{1}{3\sqrt{2}}$
$A\left(\Xi_c^+ \to p K^- \pi^+\right)$	0	$-\frac{2}{3}\sqrt{\frac{2}{15}}$	$\frac{1}{3}\sqrt{\frac{2}{15}}$	$\frac{1}{3}\sqrt{\frac{2}{5}}$	0	0	0	0	$-\frac{1}{3}\sqrt{\frac{2}{3}}$	$\frac{1}{3\sqrt{6}}$	$\frac{1}{3\sqrt{2}}$
$A\left(\Lambda_c^+ \to p K^- \pi^+\right)$	$-\frac{1}{2\sqrt{15}}$	$-\frac{1}{2\sqrt{15}}$	$-\frac{1}{2\sqrt{15}}$	$-\frac{1}{2\sqrt{5}}$	$-\frac{1}{2\sqrt{5}}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{2\sqrt{3}}$	0	0	0
$A\left(\Xi_c^+ \to \Sigma^+ \pi^- K^+\right)$	$-\frac{1}{2\sqrt{15}}$	$-\frac{1}{2\sqrt{15}}$	$-\frac{1}{2\sqrt{15}}$	$-\frac{1}{2\sqrt{5}}$	$-\frac{1}{2\sqrt{5}}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{2\sqrt{3}}$	0	0	0
$A\left(\Lambda_c^+ \to p K^- K^+\right)$	0	$-\frac{2}{3}\sqrt{\frac{2}{15}}$	$\frac{1}{3}\sqrt{\frac{2}{15}}$	$-\frac{1}{3}\sqrt{\frac{2}{5}}$	0	0	0	0	$-\frac{1}{3}\sqrt{\frac{2}{3}}$	$\frac{1}{3\sqrt{6}}$	$-\frac{1}{3\sqrt{2}}$
$A\left(\Xi_c^+ \to \Sigma^+ \pi^- \pi^+\right)$	0	$-\frac{2}{3}\sqrt{\frac{2}{15}}$	$\frac{1}{3}\sqrt{\frac{2}{15}}$	$-\frac{1}{3}\sqrt{\frac{2}{5}}$	0	0	0	0	$-\frac{1}{3}\sqrt{\frac{2}{3}}$	$\frac{1}{3\sqrt{6}}$	$-\frac{1}{3\sqrt{2}}$
$A\left(\Lambda_c^+ \to p \pi^- \pi^+\right)$	0	$-\frac{2}{3}\sqrt{\frac{2}{15}}$	$-\frac{2}{3}\sqrt{\frac{2}{15}}$	0	0	0	0	0	$-\frac{1}{3}\sqrt{\frac{2}{3}}$	$-\frac{1}{3}\sqrt{\frac{2}{3}}$	0
$A\left(\Xi_c^+ \to \Sigma^+ K^- K^+\right)$	0	$-\frac{2}{3}\sqrt{\frac{2}{15}}$	$-\frac{2}{3}\sqrt{\frac{2}{15}}$	0	0	0	0	0	$-\frac{1}{3}\sqrt{\frac{2}{3}}$	$-\frac{1}{3}\sqrt{\frac{2}{3}}$	0
$A\left(\Lambda_c^+ \to p \pi^- K^+\right)$	$\frac{1}{2\sqrt{15}}$	$-\frac{1}{2}\sqrt{\frac{3}{5}}$	0	0	$\frac{1}{2\sqrt{5}}$	$\frac{1}{2}$	0	0	0	0	0
$A\left(\Xi_c^+ \to \Sigma^+ K^- \pi^+\right)$	$\frac{1}{2\sqrt{15}}$	$-\frac{1}{2}\sqrt{\frac{3}{5}}$	0	0	$\frac{1}{2\sqrt{5}}$	$\frac{1}{2}$	0	0	0	0	0

Note, CKM-free amplitudes

$b=0$ (X_1–X_3) $b=1$ (X_4–X_9) $b=2$ (X_{10}–X_{20})

Conclusions

flavor sum rules

Flavor sumrules have very nice structure