Could $\mu \leftrightarrow e$ observations distinguish models?

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[LFV = Lepton Flavour-Changing, NC contact interaction = FCNC for leptons. And NP is heavy.]

1. NP is somewhere...can look everywhere/under the lampost/chase ambulances... or, $[m_{\nu}] \Rightarrow \text{LFV}$ is an NP signature that *exists* !

2. intro $\begin{cases} \text{exptal constraints} + \text{why } \mu \leftrightarrow e? \\ \text{are } \mu \rightarrow e\gamma, \mu \rightarrow e\overline{e}e, \mu A \rightarrow eA \text{ sufficient for discovery}? \end{cases}$

- 3. if we see $\mu \rightarrow e\gamma$, $\mu \rightarrow e\overline{e}e$, $\mu A \rightarrow eA$ —can we distinguish models?*E.g.*:
 - type II seesaw
 - a singlet leptoquark (for R_D^*)
- 4. summary

what we know about LFV : bounds/upcoming reach $(\Delta LF = 1, \Delta QF = 0)$ $(\Delta LF = \Delta QF = 1)$, $(\Delta LF = 2)$

some processes	current constraints on BR	future sensitivities
$\mu \! \rightarrow \! e \gamma$	$< 4.2 \times 10^{-13}$	$6 imes 10^{-14}$ (MEG) $ ightarrow$
$\mu \rightarrow e \bar{e} e$	$< 1.0 imes 10^{-12}$ (SINDRUM)	10^{-16} (202x, Mu3e)
$\mu A \to eA$	$< 7 imes 10^{-13}$ Au, (sindrumii)	$10^{-(16 ightarrow?)}$ (Mu2e,COMET)
		$10^{-(18 ightarrow ?)}$ (prism/prime/enigma
$\tau \rightarrow \{e,\mu\}\gamma$	$< 3.3 \ 4.4 \times 10^{-8}$	$f_{ew} \times 10^{-9}$ (Bolle II)
$\tau \to c\bar{c}c \mu \bar{\mu} \mu c \bar{\mu} \mu$	$< 0.5, 4.4 \times 10$ $< 1.5 2.7 \times 10^{-8}$	$f_{OW} \times 10^{-9}$ (Delle II IICh2)
$\gamma \rightarrow eee, \mu\mu\mu, e\mu\mu$	$< 1.0 - 2.7 \times 10$	$IEW \times IO$ (Belle-II, LHCD?)
$\tau \rightarrow \left\{ \begin{array}{c} e \\ \mu \end{array} \right\} \left\{ \pi, \rho, \phi, \ldots \right\}$	$\lesssim {\rm few} \times 10^{-8}$	few $ imes 10^{-9}$ (Belle-II)
$h \to \tau^{\pm} \ell^{\mp}$	$< 1.5, 2.2 imes 10^{-3}$ (Atlas/cms)	$< 2.4 imes 10^{-4}$ (ILC)
$h \to \mu^{\pm} e^{\mp}$	$< 6.1 imes 10^{-5}$ (ATLAS/CMS)	$2.1 imes 10^{-5}$ (ILC)
$Z \to e^{\pm} \mu^{\mp}$	$< 7.5 imes 10^{-7}$ (atlas)	
$Z \to l^{\pm} \tau^{\mp}$	$< imes 10^{-7}$ (ATLAS)	
$K^+ \to \pi^+ \bar{\mu} e$	$< 1.3 imes 10^{-11}$ (E865)	10^{-12} (NA62)
muonium		



nucl. phys. \approx WIMP scattering \supset SpinIndep $\sim A^2$ -enhanced (+SD, neglect) coherent, all ops interfere \Leftrightarrow one op per target for $\{e_L, e_R\}$



 $O_{Aheavy\perp}$ = indep. combo of 4fermion ops probed by heavy targets (Au)

Are $\mu \rightarrow e\gamma$, $\mu \rightarrow e\overline{e}e$, $\mu A \rightarrow eA$ sufficient for discovery? probe few ops— if $\Delta F_Q=0$, $\mu \rightarrow e$ occurs, will it contribute to $\mu \rightarrow e\gamma$, $\mu \rightarrow e\overline{e}e$ or $\mu A \rightarrow eA$?

Probably yes: SM loops ensure almost every $\Delta QF = 0$, $\mu \rightarrow e$ interaction with ≤ 4 legs, contributes $\gtrsim \mathcal{O}(10^{-3})$ to amplitudes $\mu \rightarrow e\gamma$, $\mu \rightarrow e\overline{e}e$ and/or $\mu A \rightarrow eA$ (not $\overline{e}\mu G\widetilde{G}$, $\overline{e}\mu F\widetilde{F}$, $\overline{e}\gamma\mu F\partial F...$)



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if see $\mu \rightarrow e\gamma$, $\mu \rightarrow e\overline{e}e$, or $\mu A \rightarrow eA$...?can distinguish models?

...studied "top-down" for decades...

EFT recipe to study this: (not scan model space—no measure)

- data is a "12-d" ellipse in coefficient-space (in an ideal theorist's world)
- \bullet with RGEs, can take ellipse to $\Lambda_{\rm NP}$
- are there parts of ellipse that model *cannot* fill? If yes, model can be distinguished/ruled out by $\mu \leftrightarrow e$ data.

Apply recipe:

1) type II seewaw 2) singlet LQ for R_D^*

Many questions... . LFV @ loop \leftrightarrow tree? LFV for singlets? NParticles interact with q? LFV not for m_{ν} ...?

Type II seesaw — add SU(2) triplet scalar \vec{T}



$$\mathcal{L} \supset \left([Y]_{\alpha\beta} \, \overline{\ell_{\alpha}^c} \varepsilon \vec{\tau} \cdot \vec{T} \ell_{\beta} + M_T \lambda_H \ H \varepsilon \vec{\tau} \cdot \vec{T^*} H + \text{h.c.} \right) + \dots$$
get $[m_{\nu}]$ by matching:



Type II seesaw — add SU(2) triplet scalar \vec{T}

$$\mu$$
 e

$$\mathcal{L} \supset \left([Y]_{\alpha\beta} \, \overline{\ell_{\alpha}^c} \varepsilon \vec{\tau} \cdot \vec{T} \ell_{\beta} + M_T \lambda_H \ H \varepsilon \vec{\tau} \cdot \vec{T^*} H + \text{h.c.} \right) + \dots \qquad \mu$$
get $[m_{\nu}]$ by matching:

then expect:



Type II seesaw — add SU(2) triplet scalar \vec{T}

$$\mu$$
 e

 $\mathcal{L} \supset \left([Y]_{\alpha\beta} \, \overline{\ell_{\alpha}^c} \varepsilon \vec{\tau} \cdot \vec{T} \ell_{\beta} + M_T \lambda_H \ H \varepsilon \vec{\tau} \cdot \vec{T^*} H + \text{h.c.} \right) + \dots \qquad \mu$ get $[m_{\nu}]$ by matching:

$$\begin{array}{c}
\nu \\
\nu \\
\overline{T} \\
\mu \\
H
\end{array} \qquad [m_{\nu}]_{\alpha\beta} \sim \frac{[Y]_{\alpha\beta} \lambda_H M_T v^2}{M_T^2} \sim 0.03 \text{ eV} \times [Y]_{\alpha\beta} \frac{\lambda_H}{10^{-12}} \frac{\text{TeV}}{M_T}
\end{array}$$

then expect:



Type II seesaw: predictions

(recall 12 (complex) operator coefficients $\begin{cases} C_{DR}, C_{VLL}^{e\mu ee}, C_{VLR}^{e\mu ee}, C_{SRR}^{e\mu ee}, C_{AlightL}, C_{AheavyR} \\ C_{DL}, C_{VRL}^{e\mu ee}, C_{VRR}^{e\mu ee}, C_{SLL}^{e\mu ee}, C_{AlightL}, C_{AheavyR} \end{cases}$

Type II seesaw: predictions

 $\begin{array}{l} (\text{recall 12 (complex) operator coefficients} \left\{ \begin{array}{c} C_{DR}, \ C_{VLL}^{e\mu ee}, \ C_{VLR}^{e\mu ee}, \ C_{SRR}^{e\mu ee}, \ C_{AlightL}, \ C_{AheavyR} \\ C_{DL}, \ C_{VRL}^{e\mu ee}, \ C_{VRR}^{e\mu ee}, \ C_{SLL}^{e\mu ee}, \ C_{AlightL}, \ C_{AheavyR} \\ \end{array} \right. \\ \text{e seven coefficients for LFV-involving-singlet-leptons are negligeable} \\ (\text{predicted by all } m_{\nu} \text{ models where NP interacts with doublets}) \\ \text{Test by polarising } \mu. \end{array}$

• any of C_{DR} , $C_{VLL}^{e\mu ee}$ or $C_{Al,L}$ ($\propto C_{VLR}^{e\mu ee}$) can vanish, sometimes both 4f. ($\Rightarrow \mu \rightarrow e\gamma$ or $\mu A \rightarrow eA$ can vanish, but not $\mu \rightarrow e\overline{e}e$ ($\propto |dipole|^2 + |4l|^2$)

...umm. How to represent?

Type II seesaw: predictions

 $\begin{array}{l} (\text{recall 12 (complex) operator coefficients} \left\{ \begin{array}{c} C_{DR}, \ C_{VLL}^{e\mu ee}, \ C_{VLR}^{e\mu ee}, \ C_{SRR}^{e\mu ee}, \ C_{AlightL}, \ C_{AheavyR} \\ C_{DL}, \ C_{VRL}^{e\mu ee}, \ C_{VRR}^{e\mu ee}, \ C_{SLL}^{e\mu ee}, \ C_{AlightL}, \ C_{AheavyR} \\ \end{array} \right. \\ \bullet \text{ seven coefficients for LFV-involving-singlet-leptons are negligeable} \\ (\text{predicted by all } m_{\nu} \text{ models where NP interacts with doublets}) \\ \end{array}$

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veery

prelim

model lives in red pie slice expt excludes blue region outside pie plate vert. axis $\sim C_{DL}(m_{\mu}) \leftrightarrow \mu \rightarrow e\gamma$; horiz. axis $C_{VLL}^{e\mu ee}(m_{\mu})$; C_{Al} vanishes.

A leptoquark (for R_{D^*})

SU(2) singlet scalar LQ, mass m_{LQ} , interactions to all flavours of l and q:

 $(-\lambda_L^{lr}\overline{\ell}_l\varepsilon q_r^c + \lambda_R^{lr}\overline{e}_l u_r^c)S + h.c.$

 \star generates scalar (+ vector) $\mu A \rightarrow eA$ operators at tree

 $(\mu A \!
ightarrow \! eA$ specially sensitive to scalar ops)

* generates LFV operators for singlet leptons as well as doublets

 \Rightarrow it can fill the whole ellipse? Consistent with any $\mu \leftrightarrow e$ observation? Not quite: not generate $(\bar{e}P_{R,L}\mu)(\bar{e}P_{R,L}e)$ (dim8 in SMEFT), detectable to $\mu \rightarrow e\bar{e}e$.

Summary

upcoming searches for $\mu \to e\gamma$, $\mu \to e\overline{e}e$ and $\mu A \to eA$ will have excellent sensitivity $(\Lambda_{\rm NP} \gtrsim 10^4 v)$ to \sim a dozen operator coefficients.

Loop effects described by (leading order) RGEs ensure that almost every $\mu \to e$ operator (chiral basis) with ≤ 4 legs (in broken EW), contributes to amplitudes for $\mu \to e\gamma$, $\mu \to e\overline{e}e$ and/or $\mu A \to eA$, suppressed by $\gtrsim O(10^{-3})$. Can even have interesting sensitivity to products of some ($\mu \to \tau$) × ($\tau \to e$) interactions!

Prospects for distinguishing between models using $\mu \rightarrow e$ observations can be explored in EFT (using operator basis motivated by observables...). More later on what we learn :)

Backup

theoretical expectations for LFV rates

1. put m_{ν} in loops with EW bosons (SM for Dirac m_{ν} , in EFT for Majorana):

$$BR_{LFV} \gtrsim \left|\frac{m_{\nu}^2}{16\pi^2 v^2} \log \frac{\Lambda_{\rm NP}}{v}\right|^2 \gtrsim 10^{-55}$$



ugh... but m_{ν} , LFV different dependance on NP scale Λ_{NP} (and cplgs λ ?):

$$m_{\nu} \sim \frac{\lambda^2 v^2}{\Lambda_{\rm NP}} \quad , \qquad \sqrt{BR_{LFV}} \sim \frac{\lambda^2 v^2}{\Lambda_{\rm NP}^2}$$

2. input other relation for λ and Λ_{NP} : $\Delta m_H^2 \Big|_{NP} \sim \frac{\lambda^2 \Lambda_{NP}^2}{16\pi^2} < v^2$

$$\Rightarrow \quad BR_{LFV} \gtrsim \left|\frac{1}{16\pi^2} \left[\frac{m_{\nu}}{\pi v}\right]^{4/3}\right|^2 \sim 10^{-38}$$

$$\lambda \bigcirc \lambda$$

still too small... but used same coupling λ for LNV and LFV...

3. many models separate LFV from LNV (SUSY seesaw, type II and inverse seesaws, NP for flavour anomalies...) and predict $BR_{LFV} \lesssim \text{expt.}$ Study such models here.

What are $\mathcal{O}_{Alight}, \mathcal{O}_{Aheavy\perp}$?

$$\mathcal{O}_{Alight,X} \sim 0.7(\overline{e}P_X\mu) \Big[(\overline{u}u) + (\overline{d}d) + ... \Big] + 0.13(\overline{e}\gamma^{\alpha}P_X\mu) \Big[(\overline{u}\gamma_{\alpha}u) + (\overline{d}\gamma_{\alpha}d) \Big]$$
$$\mathcal{O}_{Aheavy\perp,X} \simeq (\overline{e}\gamma^{\alpha}P_X\mu) \Big[0.56(\overline{u}\gamma_{\alpha}u) + 0.8(\overline{d}\gamma_{\alpha}d) \Big] + ...$$

obtained by matching nucleons to quarks, then writing $\mathcal{O}_{Aheavy,X} = \mathcal{O}_{Alight,X} + \epsilon \mathcal{O}_{Aheavy\perp,X}$ where ϵ calculable misalignement $\simeq 5\%$.

problem: scalar density of u quarks in $N \in \{n, p\} \simeq$ scalar density of d quarks \Rightarrow with current theory uncertainties in $\mu A \rightarrow eA$, measuring C_S^n and C_S^p only allows to determine $C_S^u + C_S^d$ (but not $C_S^u - C_S^d$).

 $(L \leftrightarrow R \text{ not identical in SMEFT, but not worry})$

take observable-motivated basis to Λ_{NP} ?

1. $\mu \to e\gamma$ measures $C_{D,R}(m_{\mu})$ solving RGEs gives $\vec{C}(m_{\mu}) = \vec{C}(m_W) G(m_{\mu}, m_W)$, \Rightarrow define $\vec{v}_{\mu \to e\gamma}(m_{\mu}, \Lambda)$ such that:

$$\begin{split} C_{DR}(m_{\mu}) &= \vec{C}(\Lambda) \cdot \vec{v}_{\mu \to e\gamma}(m_{\mu}, \Lambda) \\ C_{D,X}(m_{\mu}) &= C_{D,X}(m_{W}) \left(1 - 16 \frac{\alpha_{e}}{4\pi} \ln \frac{m_{W}}{m_{\mu}} \right) \\ &\quad - \frac{\alpha_{e}}{4\pi e} \ln \frac{m_{W}}{m_{\mu}} \left(-8 \frac{m_{\tau}}{m_{\mu}} C_{T,XX}^{\tau\tau} + C_{S,XX}^{\mu\mu} + C_{2loop} \right) \\ &\quad + 16 \frac{\alpha_{e}^{2}}{2e(4\pi)^{2}} \ln^{2} \frac{m_{W}}{m_{\mu}} \left(\frac{m_{\tau}}{m_{\mu}} C_{S,XX}^{\tau\tau} \right) \\ &\quad - 8\lambda^{a_{T}} \frac{\alpha_{e}}{4\pi e} \ln \frac{m_{W}}{2 \text{ GeV}} \left(-\frac{m_{s}}{m_{\mu}} C_{T,XX}^{ss} + 2 \frac{m_{c}}{m_{\mu}} C_{T,XX}^{cc} - \frac{m_{b}}{m_{\mu}} C_{T,XX}^{bb} \right) f_{TD} \\ &\quad + 16 \frac{\alpha_{e}^{2}}{3e(4\pi)^{2}} \ln^{2} \frac{m_{W}}{2 \text{ GeV}} \left(\sum_{u,c} 4 \frac{m_{q}}{m_{\mu}} C_{S,XX}^{qq} + \sum_{d,s,b} \frac{m_{q}}{m_{\mu}} C_{S,XX}^{qq} \right) \end{split}$$

all coeffs on right side $C(m_W)$ (basis vectors rotate and change length with scale) $\lambda = \alpha_s(m_W)/\alpha_s(2\text{GeV}) \simeq 0.44$, $f_{TS} \simeq 1.45$, $a_S = 12/23$, $a_T = -4/23$.

Counting constraints in space of ~ 100 operators

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Count constraints: (write
$$\delta \mathcal{L} = C_{Lorentz,XY}^{flavour} / v^n \mathcal{O}_{Lorentz,XY}^{flav}$$
, $X, Y \in \{L, R\}$)

 $\mu \rightarrow e\gamma$: $BR(\mu \rightarrow e\gamma) = 384\pi^2(|C_{D,L}|^2 + |C_{D,R}|^2) \Rightarrow 2 \text{ constraints}$

 $\mu \rightarrow e\bar{e}e$: (e relativistic \approx chiral, neglect interference between e_L, e_R)

$$BR = \frac{|C_{S,LL}|^2}{8} + 2|C_{V,RR} + 4eC_{D,L}|^2 + (64\ln\frac{m_{\mu}}{m_e} - 136)|eC_{D,L}|^2 + |C_{V,RL} + 4eC_{D,L}|^2 + \{L \leftrightarrow R\} \implies 6 \text{ more constraints}$$

 $\mu A \rightarrow eA : (S_A^N, V_A^N = \text{integral over nucleus A of } N \text{ distribution} \times \text{lepton wavefns, different for diff. } A)$ $BR_{SI} \sim Z^2 |V_A^p \tilde{C}_{V,L}^p + S_A^p \tilde{C}_{S,R}^p + V_A^n \tilde{C}_{V,L}^n + S_A^b \tilde{C}_{S,R}^n + D_A C_{D,R}|^2 + |L \leftrightarrow R|^2$ $BR_{SD} \sim |\tilde{C}_A^N + 2\tilde{C}_T^N|^2$

SI bds on Au, Ti, (+ SD on ?Ti, Au?) $\Rightarrow 4 + 2$ more constraints future: improved theory, 3SI+2SD targets $\Rightarrow 6+4$ constraints

is 12-20 constraints on ~ 100 operators a problem?

 $\mu A
ightarrow eA$: most sensitive process, expt + th





target (Z=13,A=27, J=5/2)

- μ^- captured by Al nucleus, tumbles down to 1s. ($r\sim Zlpha/m_\mu \stackrel{_>}{_\sim} r_{Al}$)
- in SM: muon "capture" $\mu + p \rightarrow \nu + n$, or decay-in-orbit
- LFV: μ interacts with \vec{E} , nucleons (via $\tilde{C}^{N}_{\Gamma,X}(\bar{e}\Gamma P_X N)(\bar{N}\Gamma N)$), converts to e

$$\mu \rightarrow \mathcal{O} \xrightarrow{p} \mu \xrightarrow{p} \mu \xrightarrow{n} \Gamma = \{I, \gamma_5, \gamma^{\alpha}, \gamma^{\alpha}\gamma_5, \sigma\}$$

$$\mu \rightarrow \mathcal{O} \xrightarrow{e_L} \mu \xrightarrow{p} \mu \xrightarrow{p} \mu \xrightarrow{n} \Gamma = \{I, \gamma_5, \gamma^{\alpha}, \gamma^{\alpha}\gamma_5, \sigma\}$$

$$\Gamma = \{S, P, V, A, T\}$$

$$\approx \text{WIMP scattering on nuclei}$$
1) "Spin Independent" rate $\propto A^2$ (amplitude $\propto \sum_N \propto A$)

$$BR_{SI} \sim Z^2 |\sum ... \tilde{C}_{SI}|^2 , \quad \tilde{C}_{SI} \in \{\tilde{C}_V^p, \tilde{C}_S^p, \tilde{C}_N^n, \tilde{C}_S^n, C_D\}$$

2) "Spin Dependent" rate $\sim \Gamma_{SI}/A^2$ (sum over $N \propto$ spin of only unpaired nucleon) $BR_{SD} \sim ... |\tilde{C}_A^N + 2\tilde{C}_T^N|^2$ CiriglianoDavidsonKuno

HoferichterEtal

Can't we do without RGEs, etc?

in discovery mode for LFV+electroweak loops are small...include later?

counterex: $\mu A \rightarrow eA$ in model giving tensor $2\sqrt{2}G_F C_T^{uu}(\overline{e}\sigma P_R\mu)(\overline{u}\sigma u)$ at weak scale

1: forget loops quark tensor matches to nucleon spin $\bar{N}\gamma\gamma_5N$: $(N \in \{n, p\})$

 $\Rightarrow BR(\mu A \to eA) \approx BR_{SD} \approx \frac{1}{2} |C_T^{uu}|^2$ (CiriglianoDKuno) Hoferichter etal

2: include QED loops $m_W \rightarrow 2$ GeV:



Then, scalar ops have enhanced nuclear matrix elements, and are SpinIndep:

$$BR(\mu A
ightarrow eA) pprox BR_{SI} \sim Z^2 |2C_T^{uu}|^2 \sim 10^3 BR_{SD}$$

loops can change Lorentz structure/external legs \Rightarrow different operator whose coefficient better constrained. Important for $\mu \rightarrow e$. (?not $\tau \rightarrow l$?)

operator list:Kuno-Okada, +CiriglianoKitanoOTuzon Operator basis $m_{ au} o m_W$: ~ 90 operators operators operators operators operators operators operator basis manchengLiMatis

Add QCD×QED-invar operators, representing all 3,4 point interactions of μ with e and flavourdiagonal combination of γ , g, u, d, s, c, b. $Y \in L$, R.

 $(\overline{e}\gamma^{\alpha}P_{Y}\mu)(\overline{e}\gamma_{\alpha}P_{Y}e) \qquad (\overline{e}\gamma^{\alpha}P_{Y}\mu)(\overline{e}\gamma_{\alpha}P_{X}e)$ $(\overline{e}P_Y\mu)(\overline{e}P_Ye)$ dim 6 $(\overline{e}\gamma^{\alpha}P_{Y}\mu)(\overline{\mu}\gamma_{\alpha}P_{X}\mu) \qquad (\overline{e}\gamma^{\alpha}P_{Y}\mu)(\overline{\mu}\gamma_{\alpha}P_{X}\mu)$ $(\overline{e}P_Y\mu)(\overline{\mu}P_Y\mu)$ $(\overline{e}\gamma^{\alpha}P_{Y}\mu)(\overline{f}\gamma_{\alpha}P_{Y}f) \qquad (\overline{e}\gamma^{\alpha}P_{Y}\mu)(\overline{f}\gamma_{\alpha}P_{X}f)$ $(\overline{e}P_Y\mu)(\overline{f}P_Xf) \qquad f \in \{u, d, s, c, b, \tau\}$ $(\overline{e}P_Y\mu)(\overline{f}P_Yf)$ $(\overline{e}\sigma P_Y \mu)(\overline{f}\sigma P_Y f)$ $\frac{1}{m_t} (\overline{e} P_Y \mu) G_{\alpha\beta} G^{\alpha\beta} \qquad \frac{1}{m_t} (\overline{e} P_Y \mu) G_{\alpha\beta} \widetilde{G}^{\alpha\beta}$ dim 7 $\frac{1}{m_t} (\overline{e} P_Y \mu) F_{\alpha\beta} F^{\alpha\beta} \qquad \frac{1}{m_t} (\overline{e} P_Y \mu) F_{\alpha\beta} \widetilde{F}^{\alpha\beta} \qquad \dots zzz...but \sim 90 \text{ coeffs!}$ $(P_X, P_Y = (1 \pm \gamma_5)/2), \text{ all operators with coeff } -2\sqrt{2}G_FC.$