The Muon $(g-2)$ — What's going on?

Hartmut Wittig

Institute for Nuclear Physics, Helmholtz Institute Mainz, and PRISMA+ Cluster of Excellence,
Johannes Gutenberg-Universität Mainz

Pushing the Limits of Theoretical Physics
10th Anniversary of MITP
8 May 2023

PRISMA+ HIM Helmholtz-Institut Mainz JOHANNES GUTENBERG UNIVERSITÄT MAINZ JG|U

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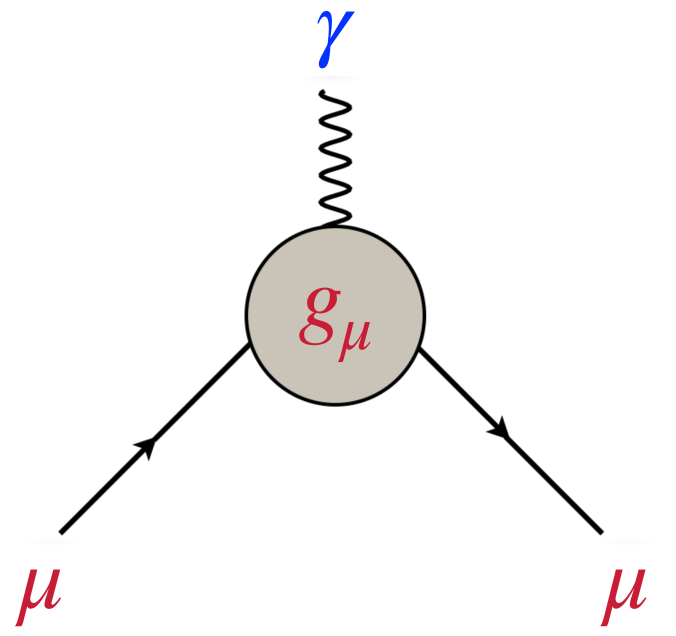

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Sensitive probe of Physics beyond the Standard Model

$$a_\mu = a_\mu^{\rm QED} + a_\mu^{\rm weak} + a_\mu^{\rm strong} + a_\mu^{\rm BSM}\,?$$

$$a_\ell^{\rm BSM} \propto m_\ell^2/M_{\rm BSM}^2\,, \qquad \ell = e,\,\mu,\,\tau$$

$\rightarrow$ confront precision measurements with SM prediction

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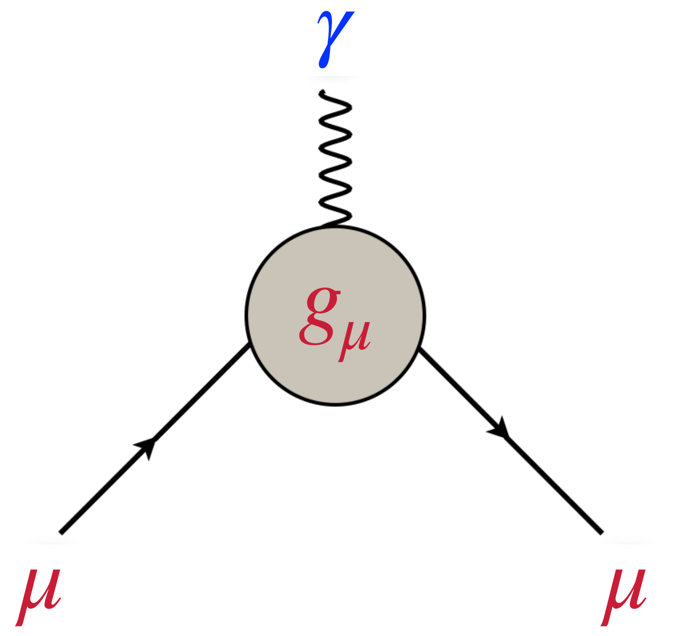

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Pillar of research programme of PRISMA⁺ Cluster of Excellence

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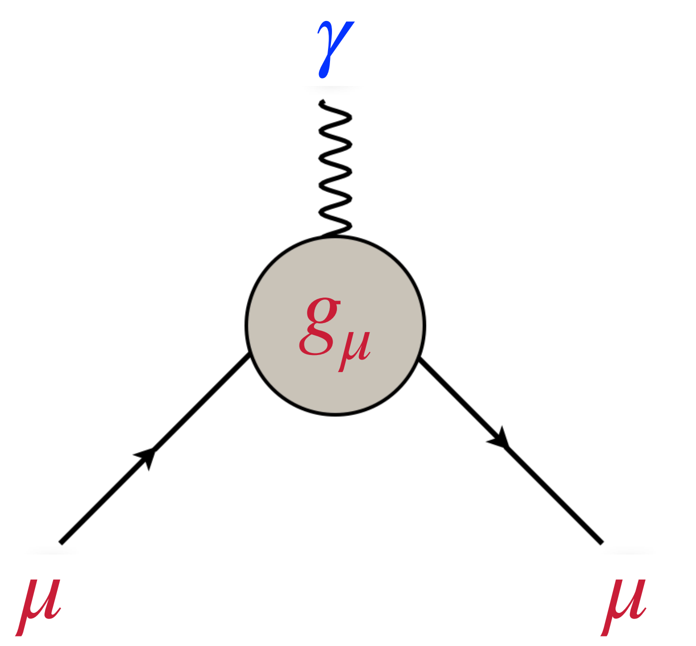

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Direct measurement: E989@Fermilab	**Martin Fertl**
Cross section measurements: $e^+e^- \rightarrow$ hadrons	**Achim Denig**
Lattice QCD calculations: HVP and HLbL	**Harvey Meyer, H.W.**
Data-driven formalism for HLbL	**Marc Vanderhaeghen**
Model building; BSM interpretation	**Matthias Neubert**

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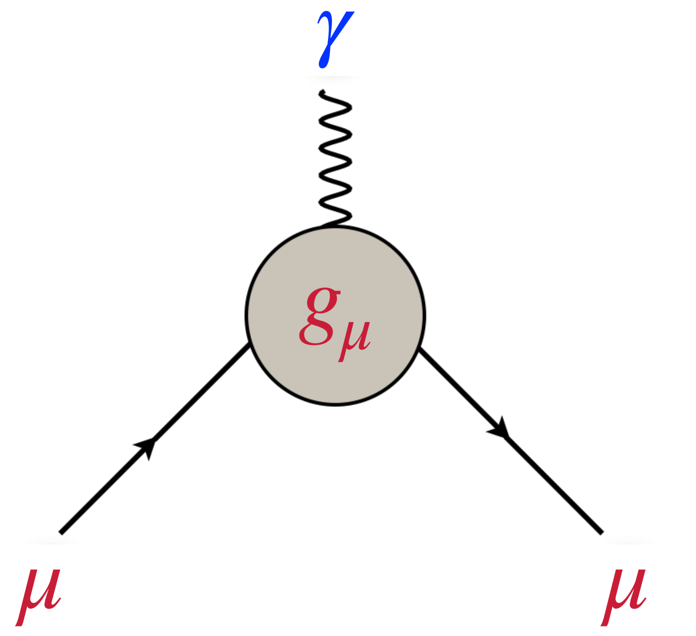

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TOPICAL WORKSHOPS

Hadronic contributions to the muon anomalous magnetic moment: strategies for improvements of the accuracy of the theoretical prediction

Tom Blum **Univ. of Connecticut**, Simon Eidelman **INP Novosibisrk**, Fred Jegerlehner **HU Berlin**, Dominik Stöckinger **TU Dresden**, Achim Denig, Marc Vanderhaeghen **JGU Mainz**

April 1–5, 2014, JGU Campus Mainz

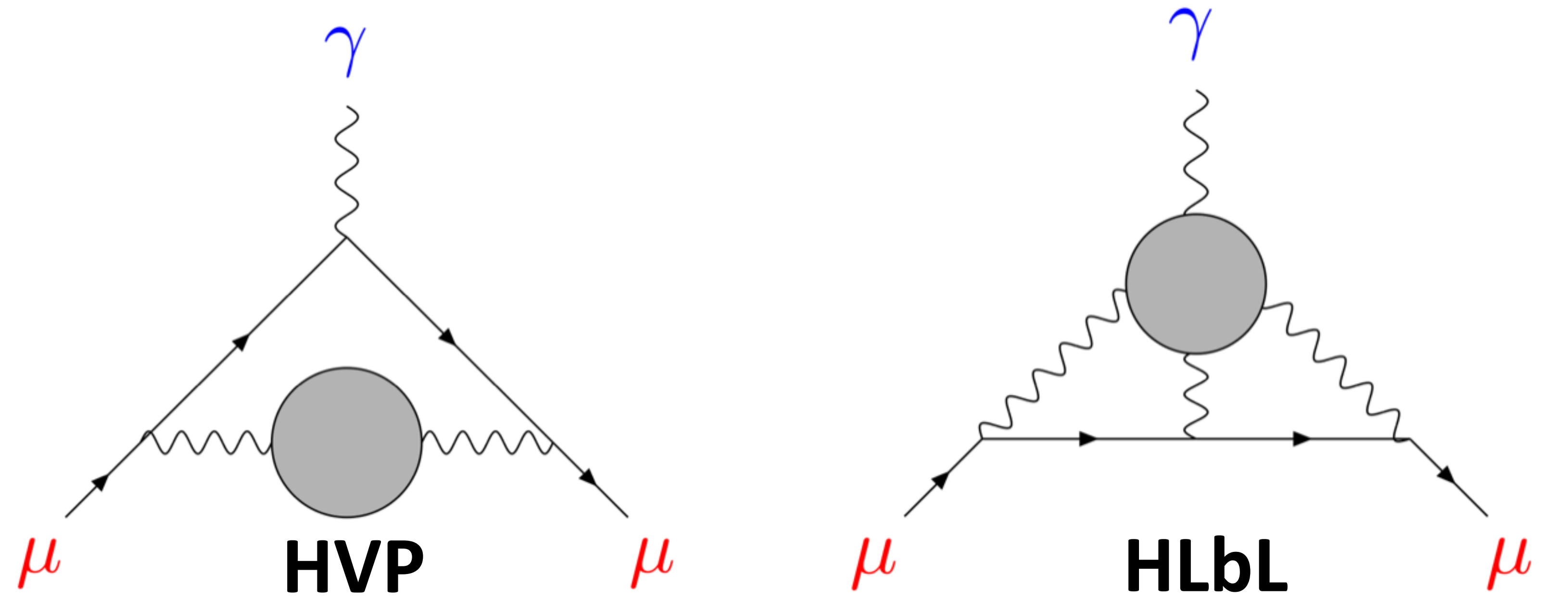
Mainz Institute for Theoretical Physics

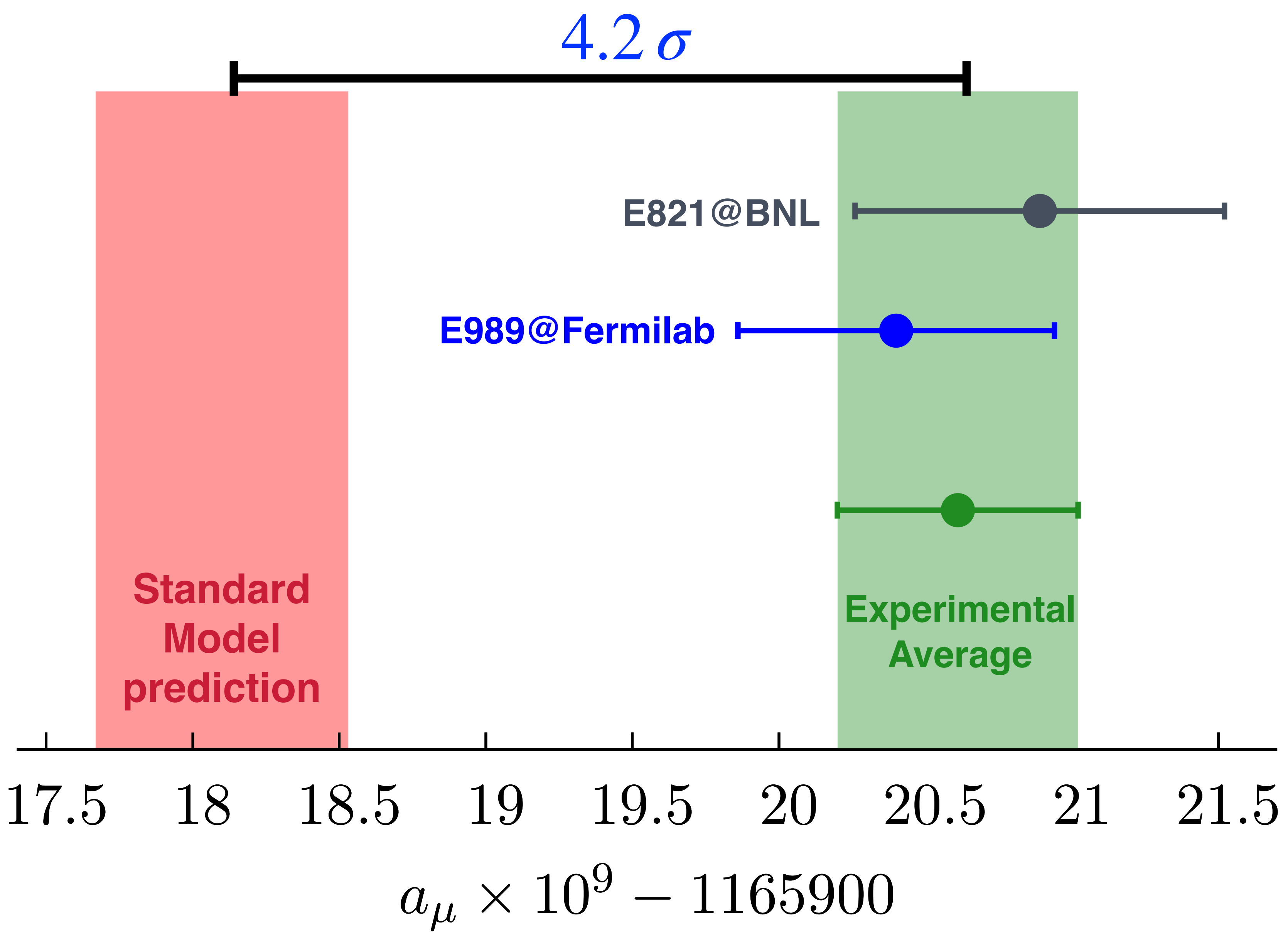
Standard Model prediction

White Paper of "$g-2$ Theory Initiative" (2020)

- Overall precision of $0.37\,\mathrm{ppm}$ *[Aoyama et al., Phys. Rep. 887 (2020) 1]*
- Error dominated by hadronic vacuum polarisation (HVP) and light-by-light scattering (HLbL)
- HVP evaluated using "data-driven" approach based on dispersion integrals and hadronic cross sections

$$\Rightarrow \quad a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}} = (25.1 \pm 5.9) \cdot 10^{-10} \quad [4.2\sigma]$$





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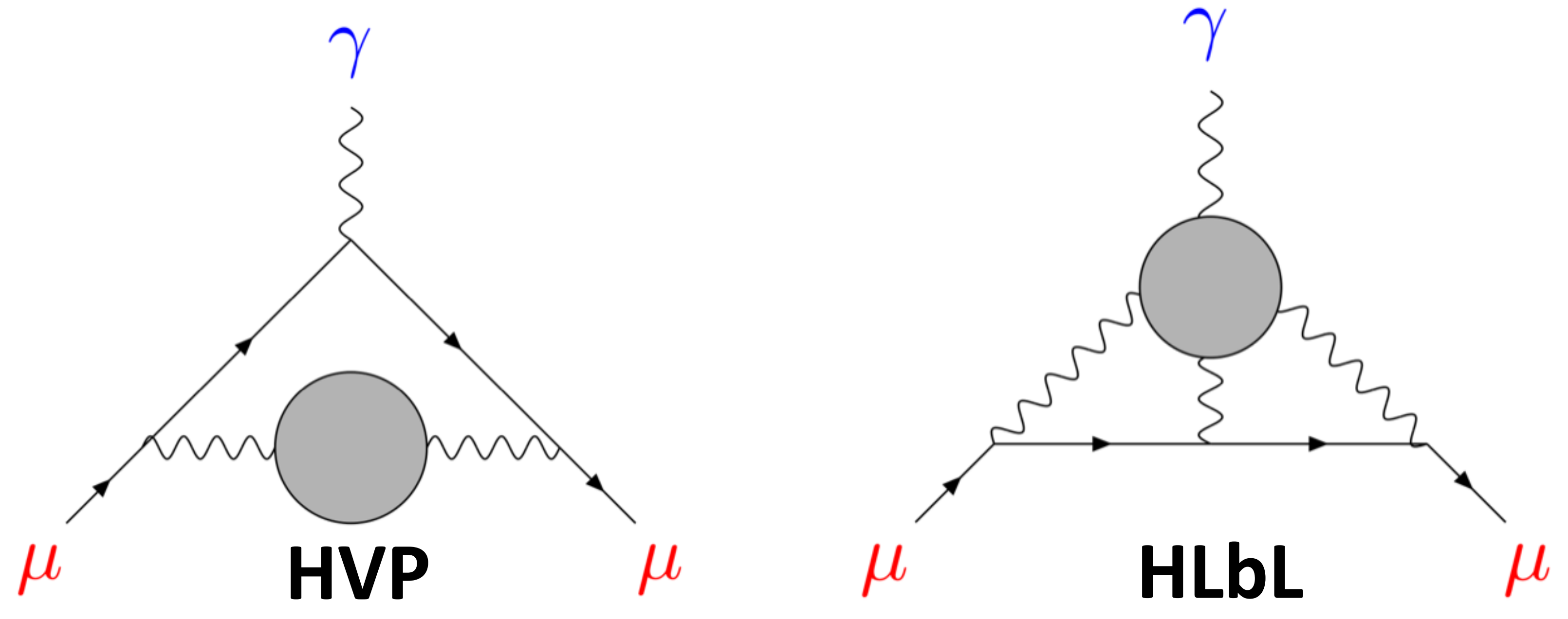

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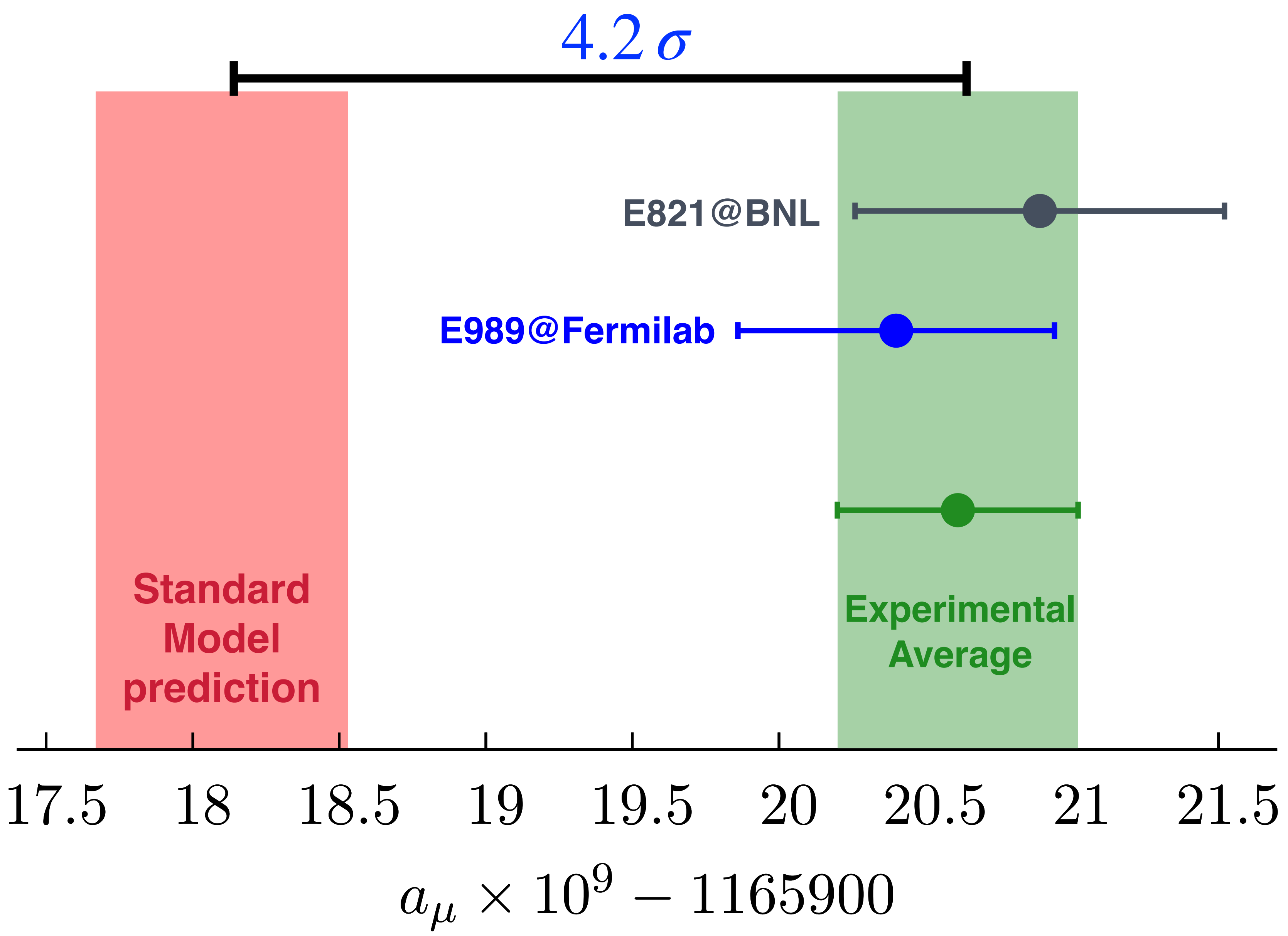

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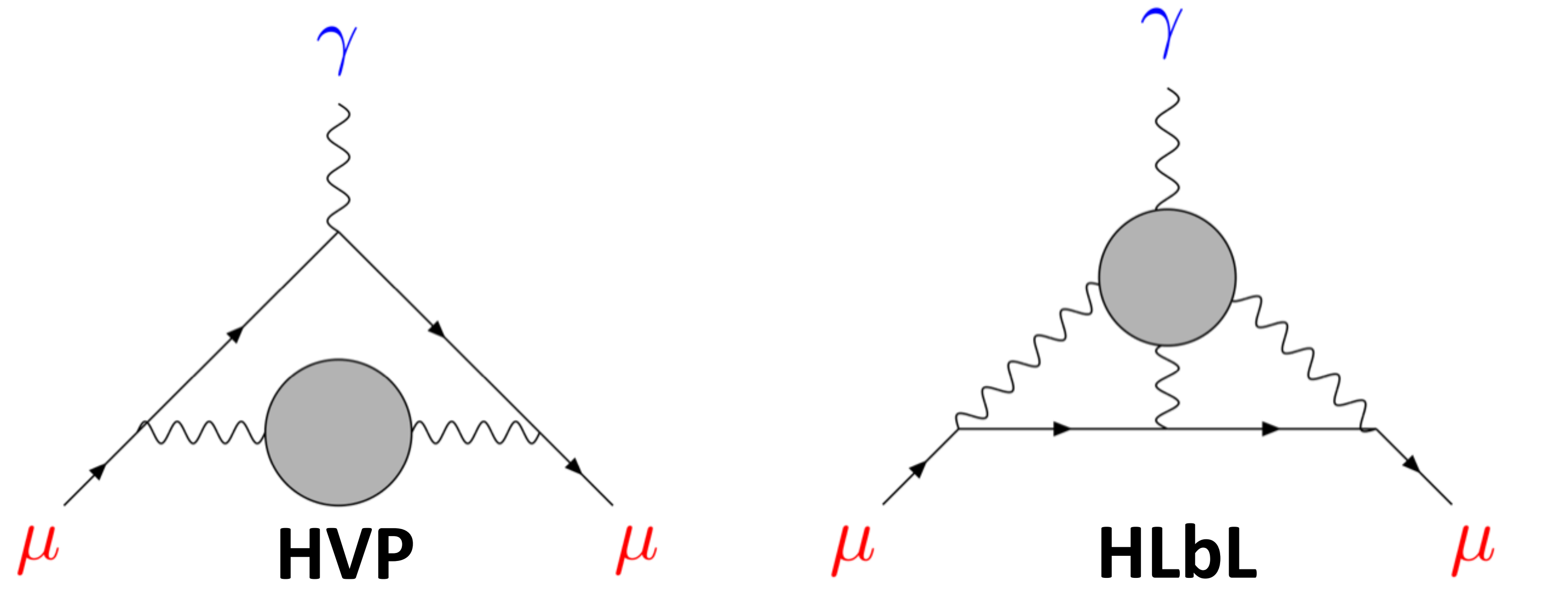

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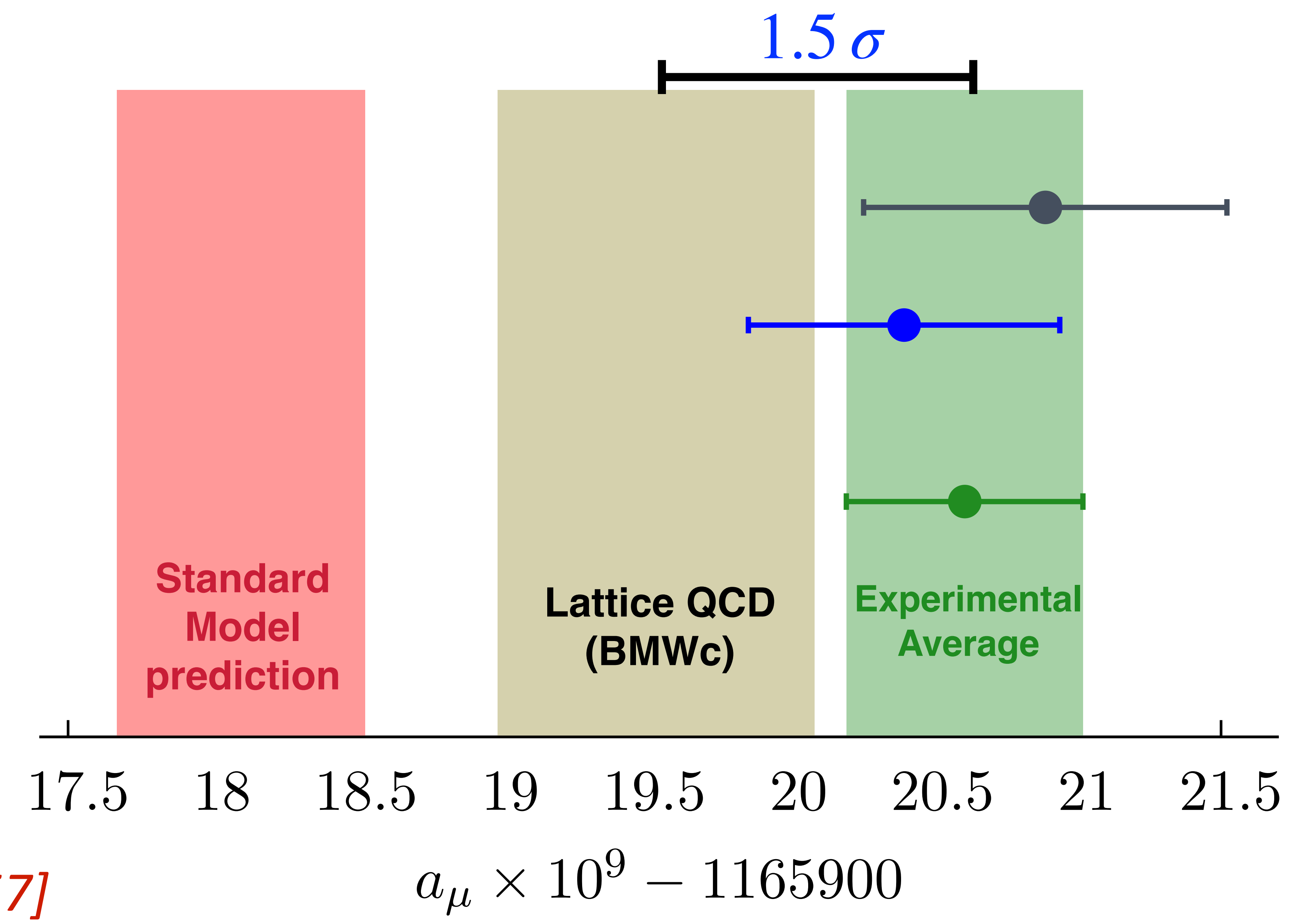

[Borsányi et al. (BMW Collab.), Nature 593 (2021) 7857]

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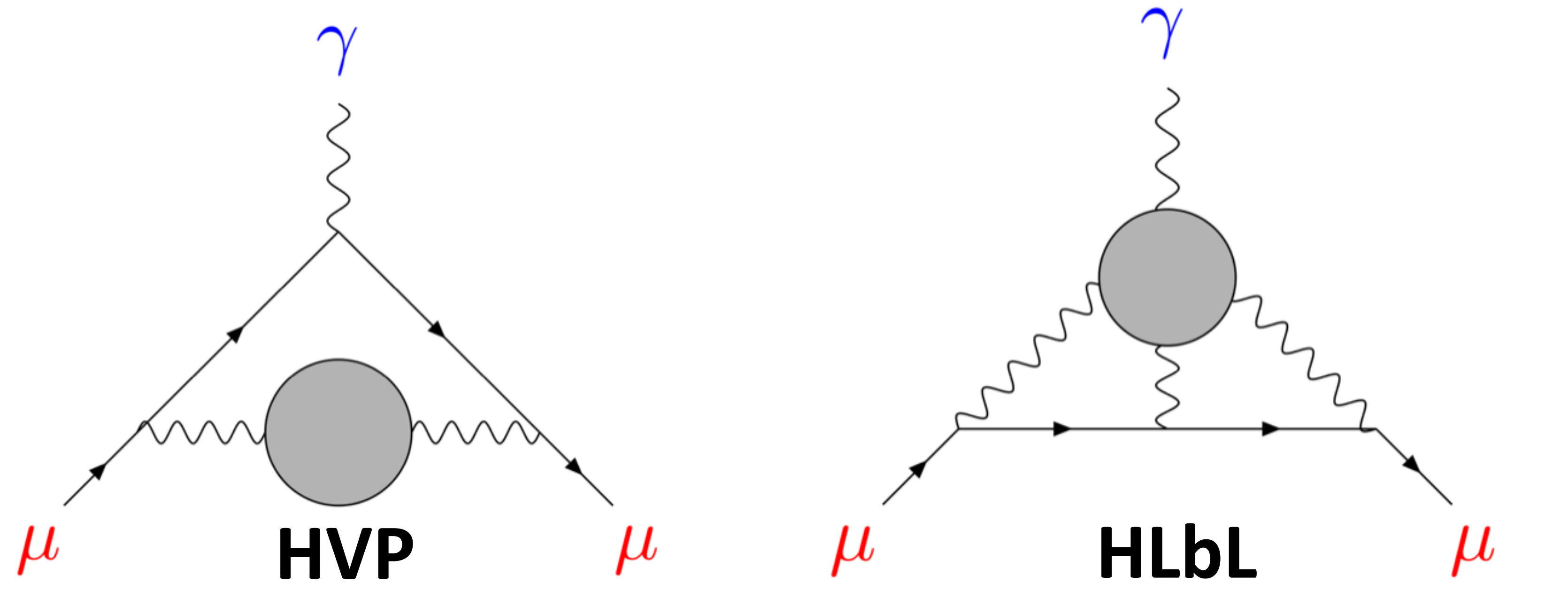

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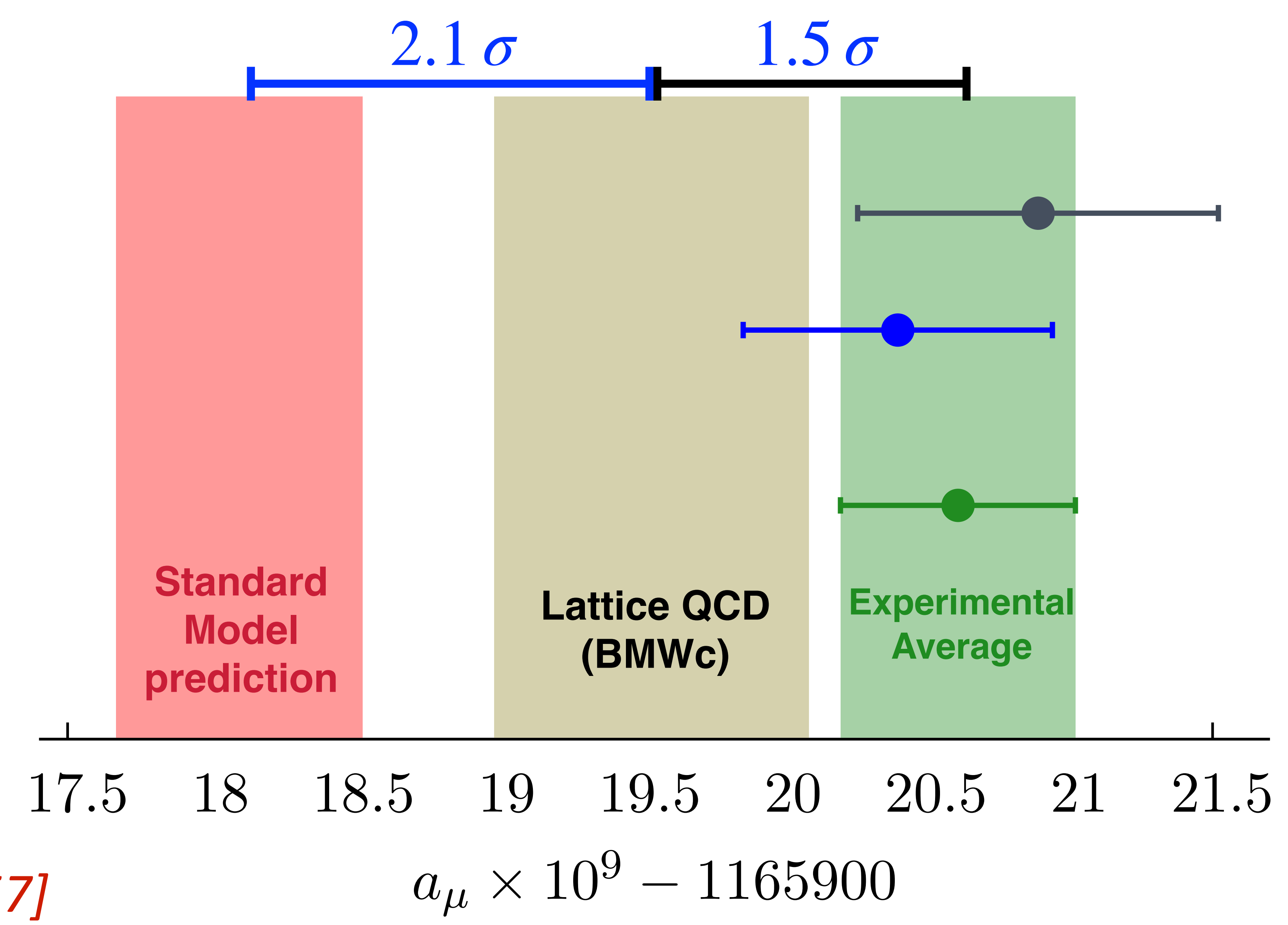

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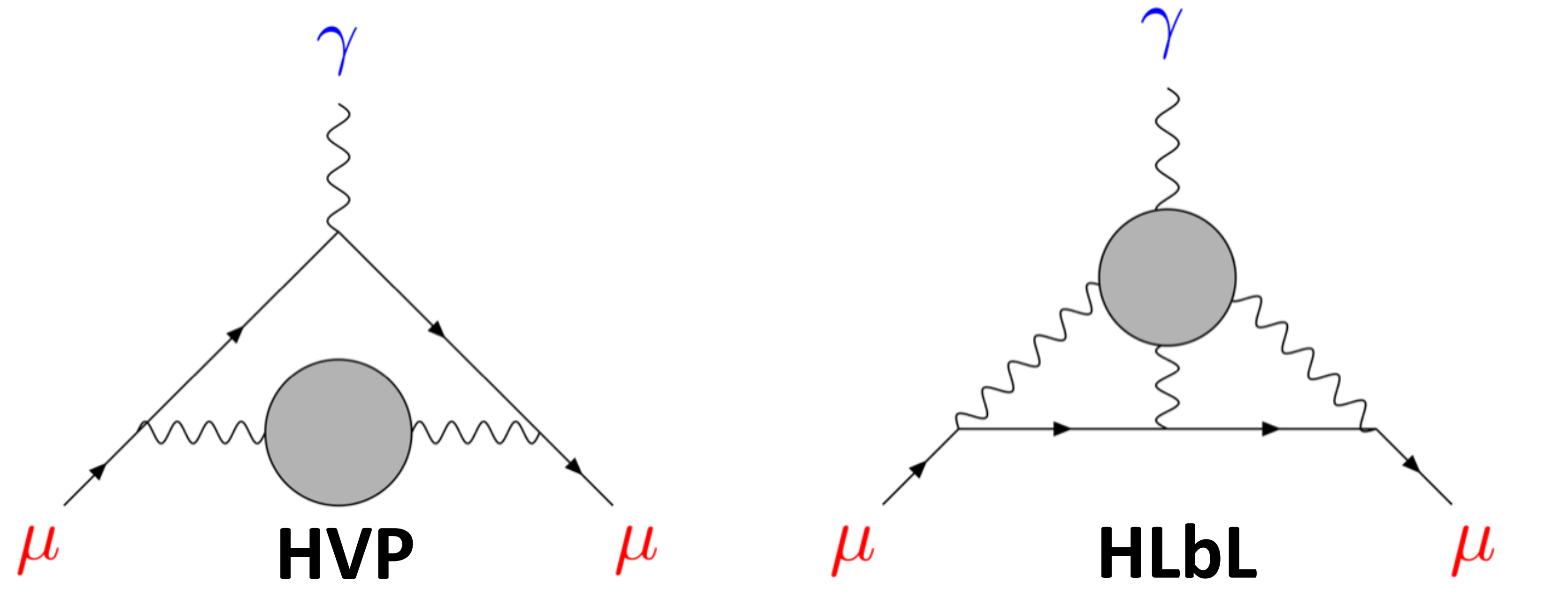

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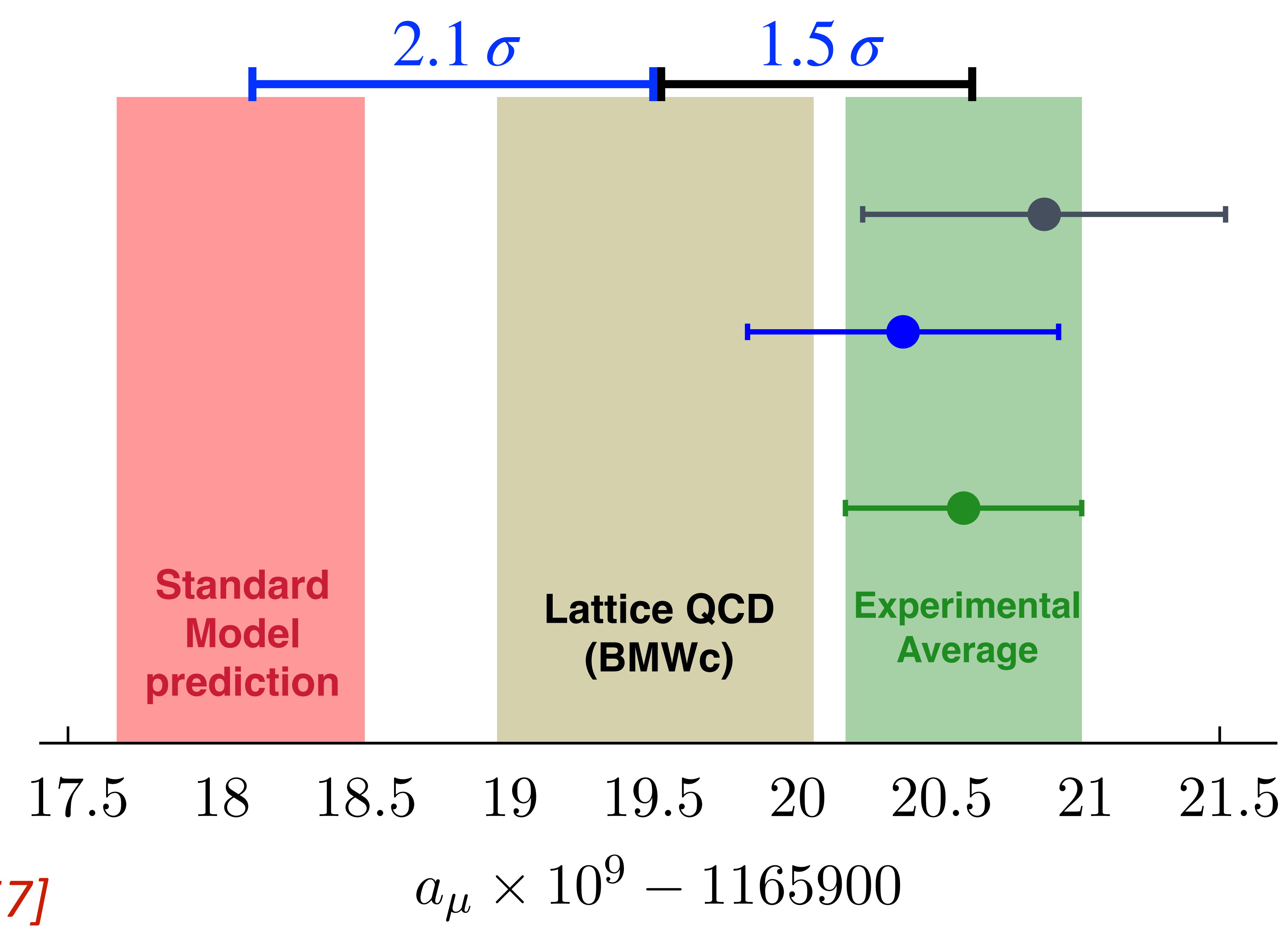
Requires independent confirmation



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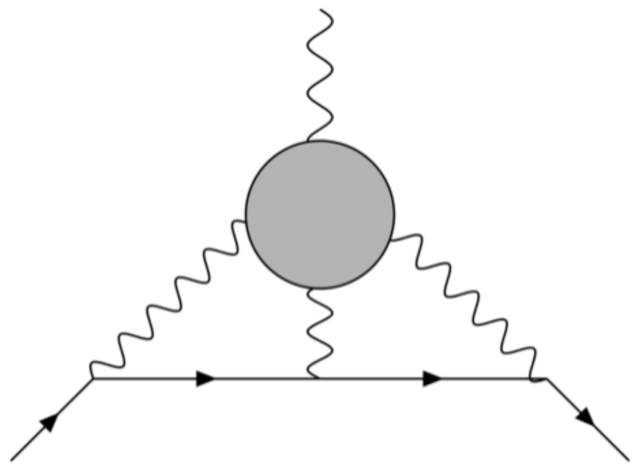
Hadronic light-by-light scattering



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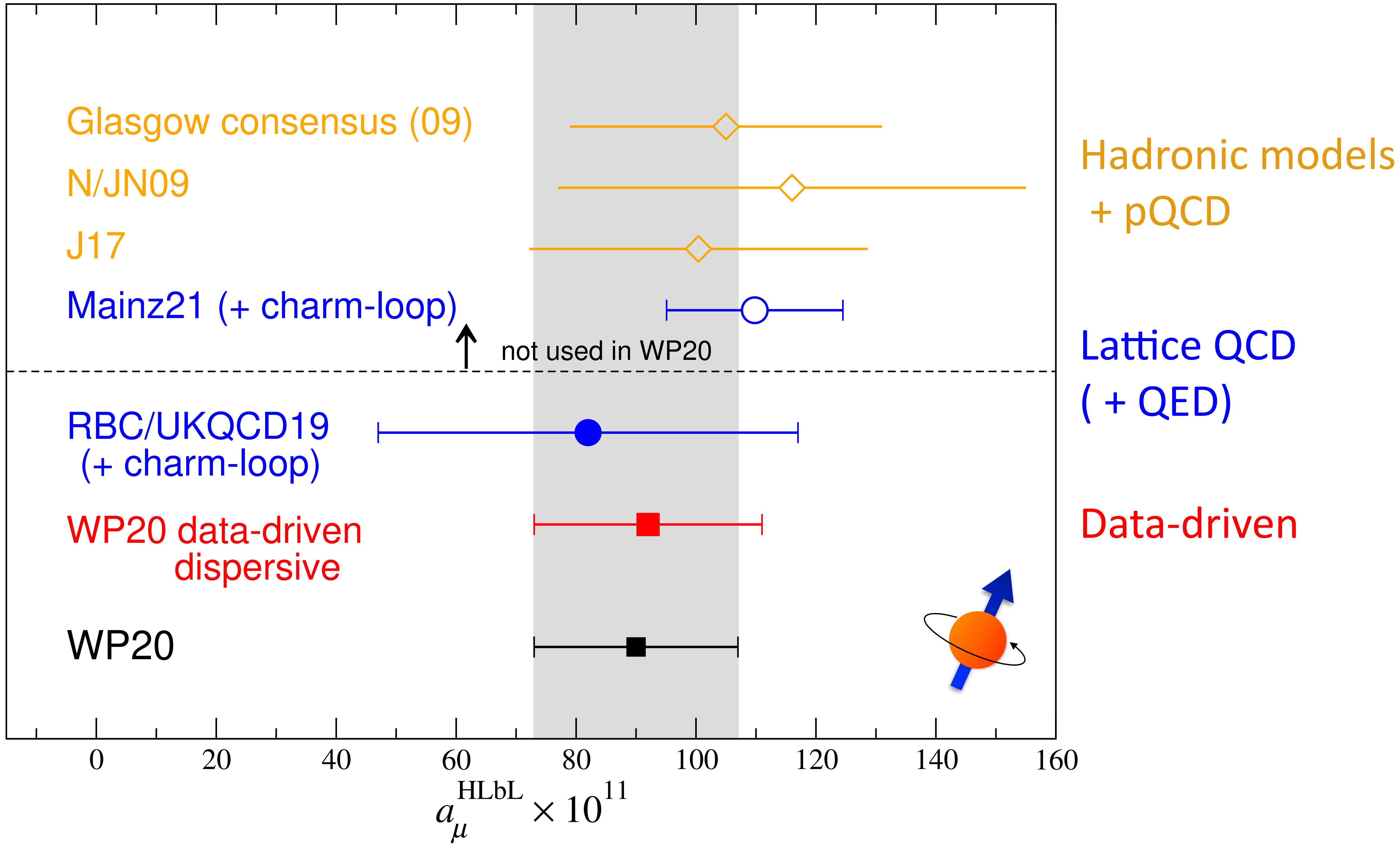

Hadronic models, data-driven method and Lattice QCD produce consistent results

White paper recommended value:

$$a_\mu^{\rm hlbl} = (92 \pm 18) \cdot 10^{-11}$$

Recent lattice calculations:

$$a_\mu^{\rm hlbl} = \begin{cases} (109.6 \pm 14.7) \cdot 10^{-11} & \text{Mainz} \\ (124.7 \pm 15.2) \cdot 10^{-11} & \text{RBC/UKQCD} \end{cases}$$

[Chao et al., EPJC 81 (2021) 651; EPJC 82 (2022) 664; Blum et al., arXiv:2304.04423]

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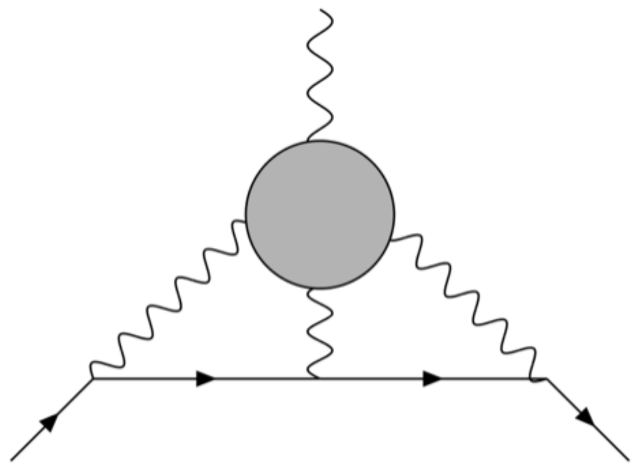

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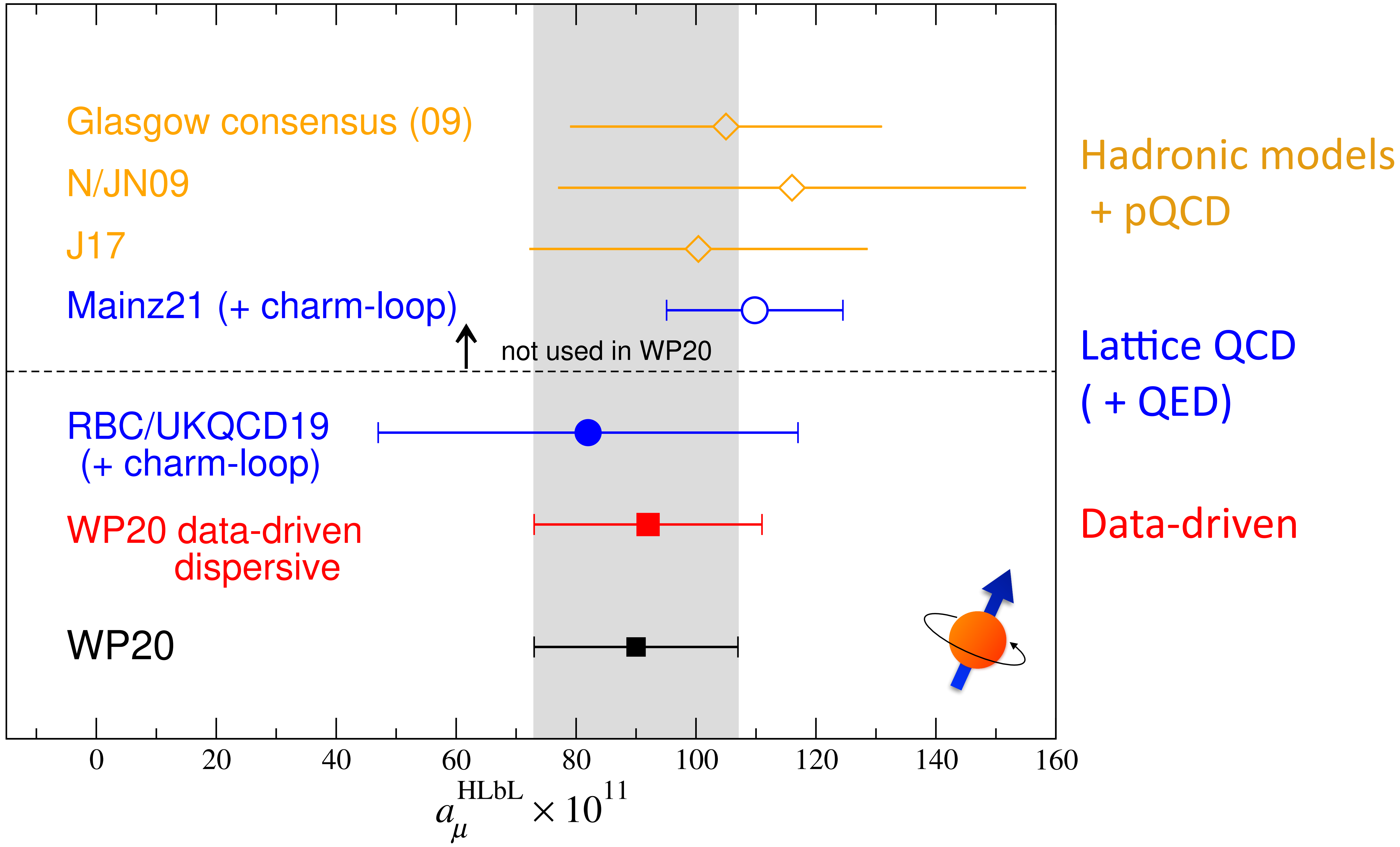

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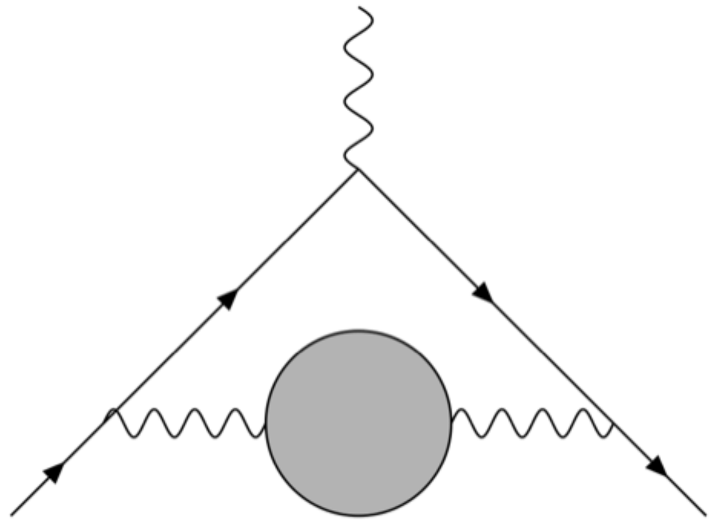
$a_\mu^{\rm hlbl}$: **Uncontroversial** — contributes 0.15 ppm to the total SM uncertainty of 0.37 ppm

→ Focus on refinements and further reduction of uncertainty

Hadronic vacuum polarisation: Data-driven approach



Express hadronic vacuum polarisation as a dispersion integral:

$$a_\mu^{\rm hvp} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{m_{\pi^0}^2}^{\infty} ds\, \frac{R_{\rm had}(s)\hat{K}(s)}{s^2}, \qquad R_{\rm had}(s) = \frac{3s}{4\pi\,(\alpha(s))^2}\,\sigma(e^+e^- \to \text{hadrons})$$

"*R*-ratio"

- Use experimental data for $R_{\rm had}(s)$ in the low-energy regime ("data-driven approach")

  → SM prediction affected by experimental uncertainties

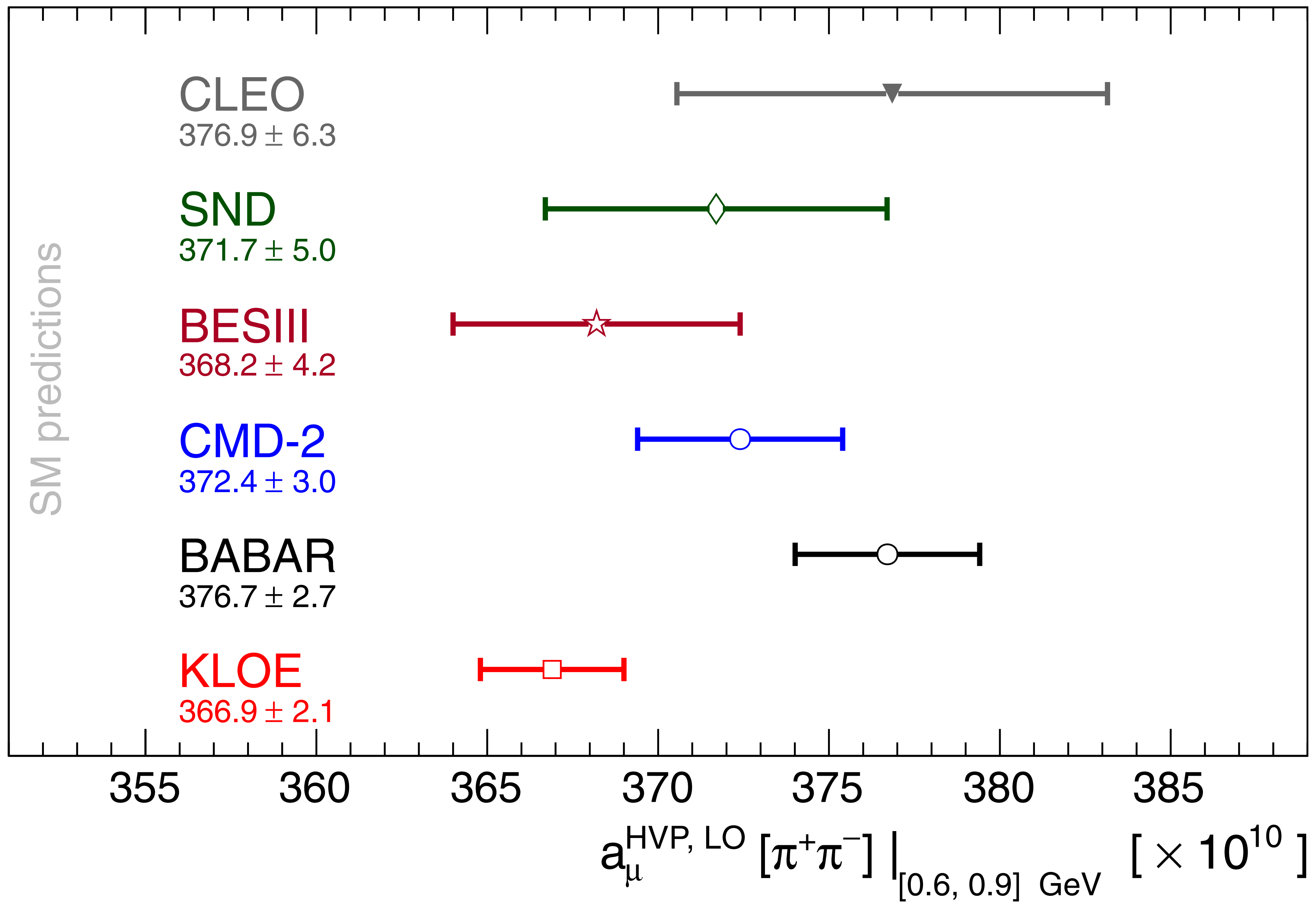
- White Paper recommended value (2020):

$$a_\mu^{\rm hvp,\,LO} = 693.1(2.8)_{\rm exp}(2.8)_{\rm syst}(0.7)_{\rm DV+QCD} \times 10^{-10}$$

$$= 693.1(4.0) \times 10^{-10} \qquad [0.6\%]$$

(accounts for tensions in the data and differences between analyses)

$e^+e^- \to \pi^+\pi^-$



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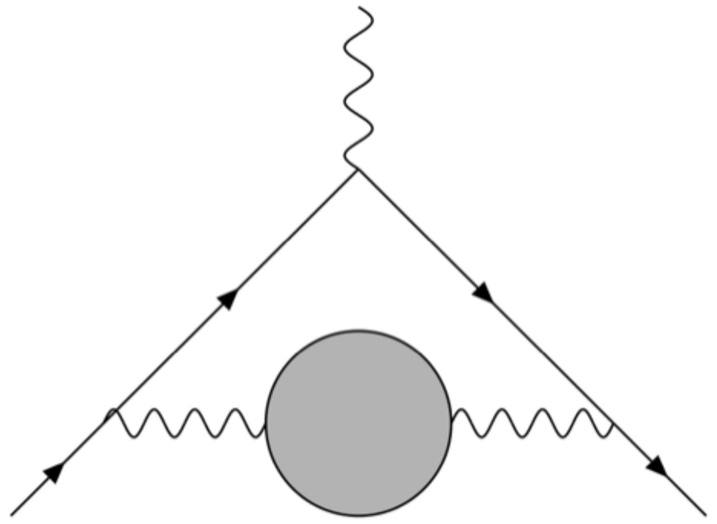

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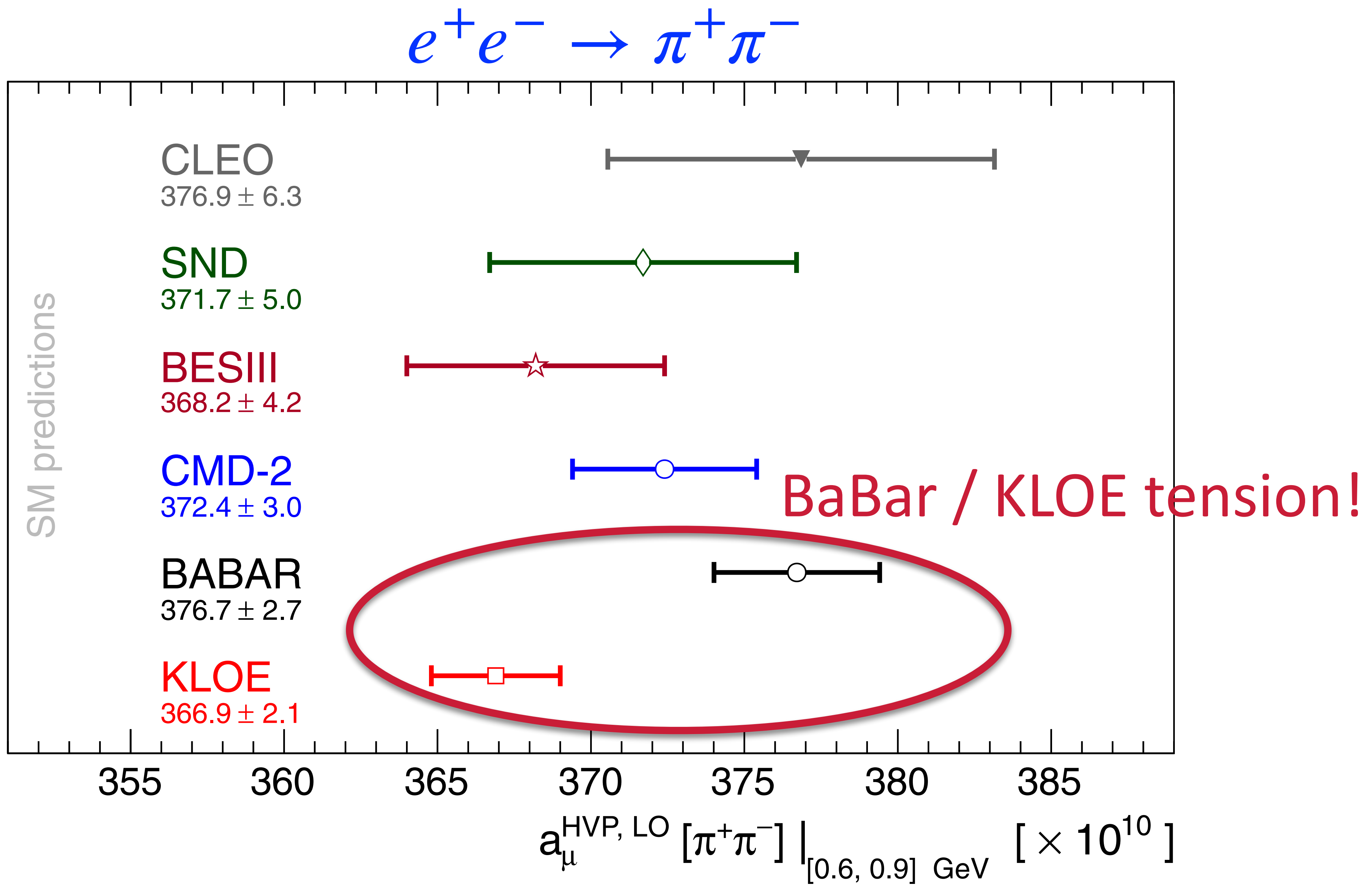

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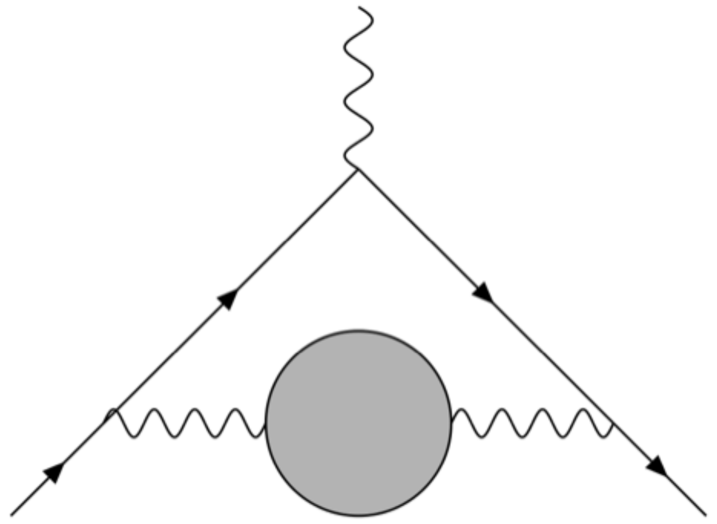

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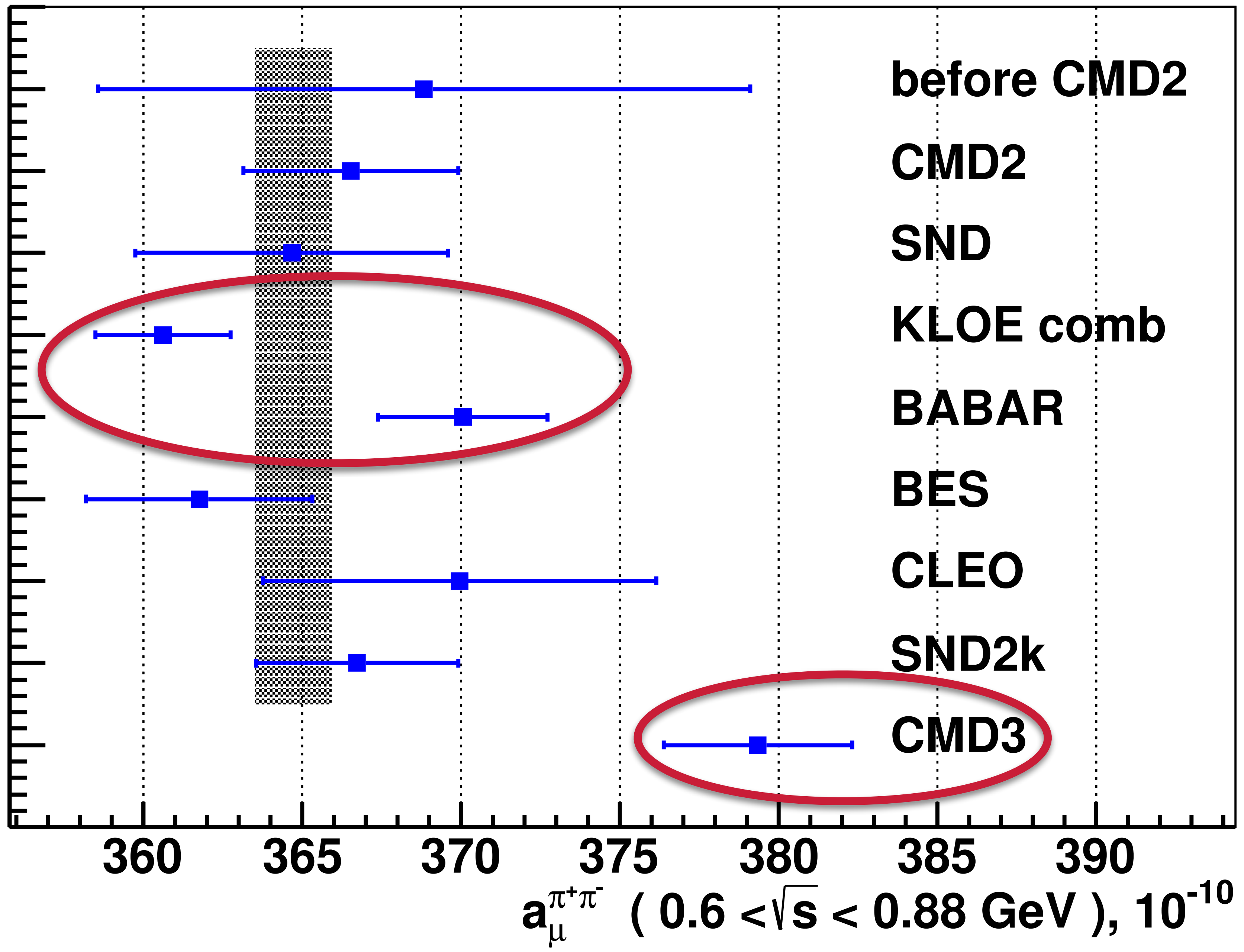
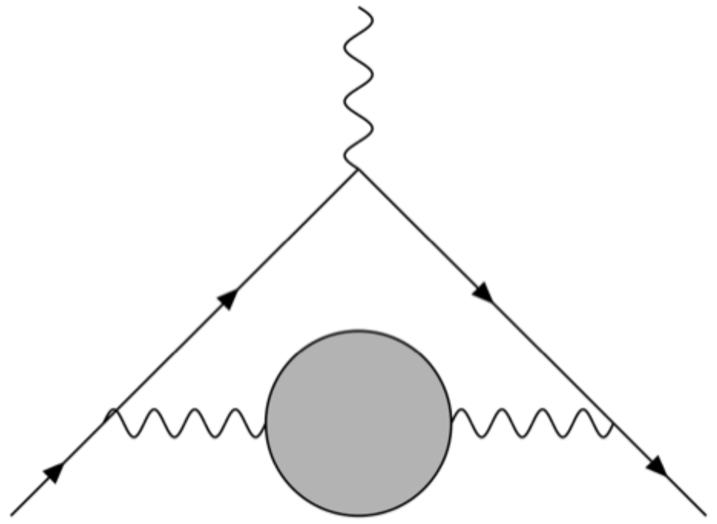
- Recent results in the $\pi^+\pi^-$ channel by CMD-3:
  - → further tension among e^+e^- data



[Ignatov et al. (CMD-3 Collab.), arXiv:2302.08834]

Hadronic vacuum polarisation: Lattice QCD



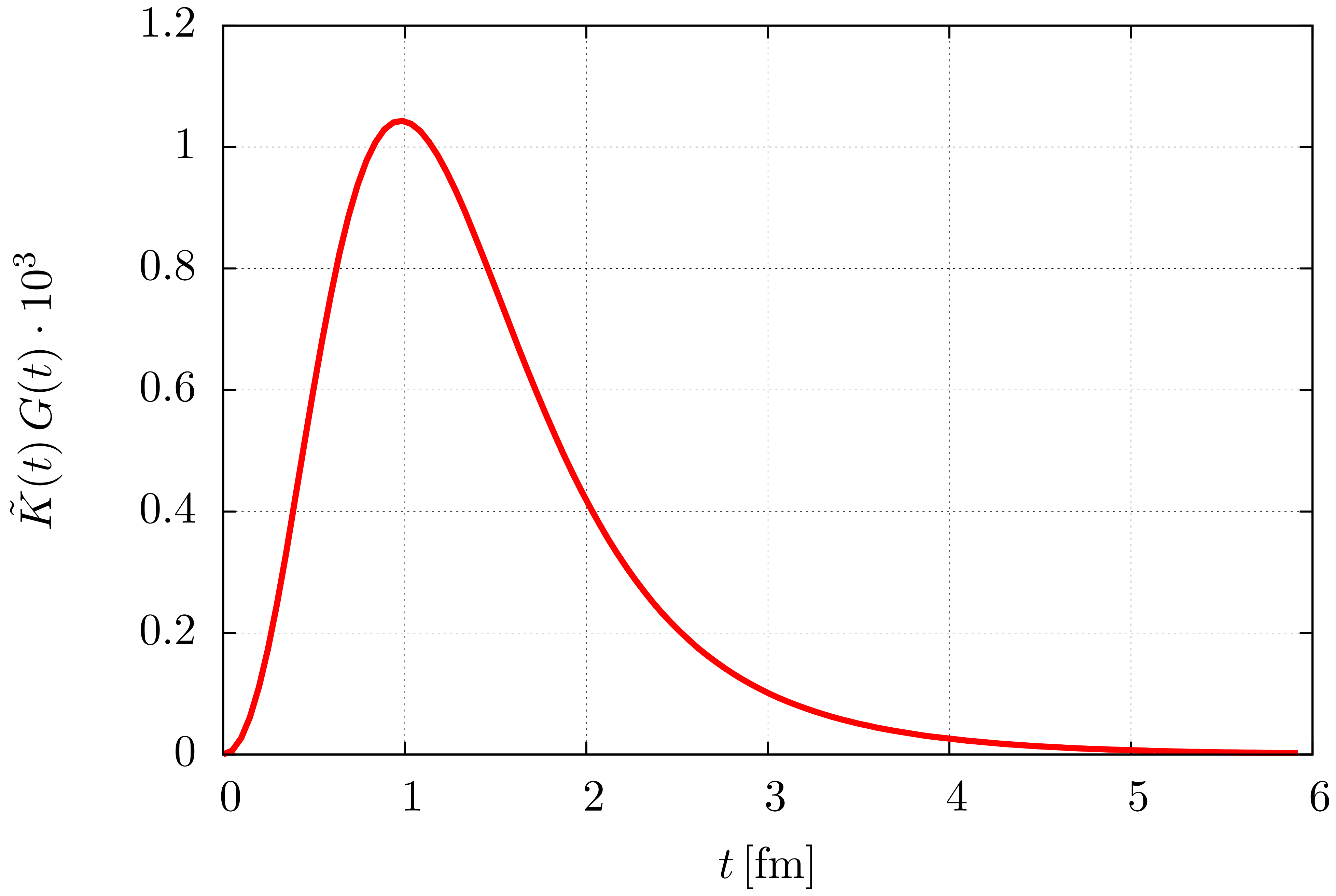
Lattice QCD does NOT determine the R-ratio from first principles

Time-momentum representation (TMR): *[Bernecker & Meyer EPJA 47 (2011) 148]*

$$a_\mu^{\rm hvp} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \tilde{K}(t)\, G(t), \quad G(t) = -a^3 \sum_{\vec{x}} \left\langle J_k^{\rm e.m.}(\vec{x},t) J_k^{\rm e.m.}(0) \right\rangle$$

($\tilde{K}(t)$: known kernel function)

- No reliance on experimental data, except for simple input quantities → scale setting, calibration
- Not sensitive to exclusive hadronic channels



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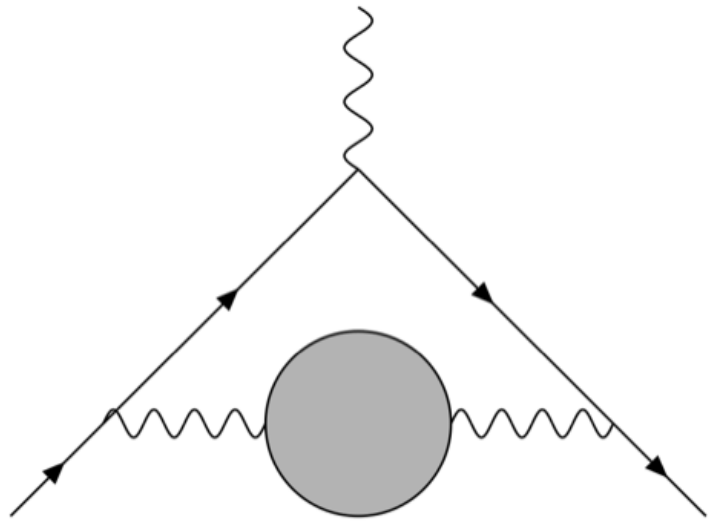

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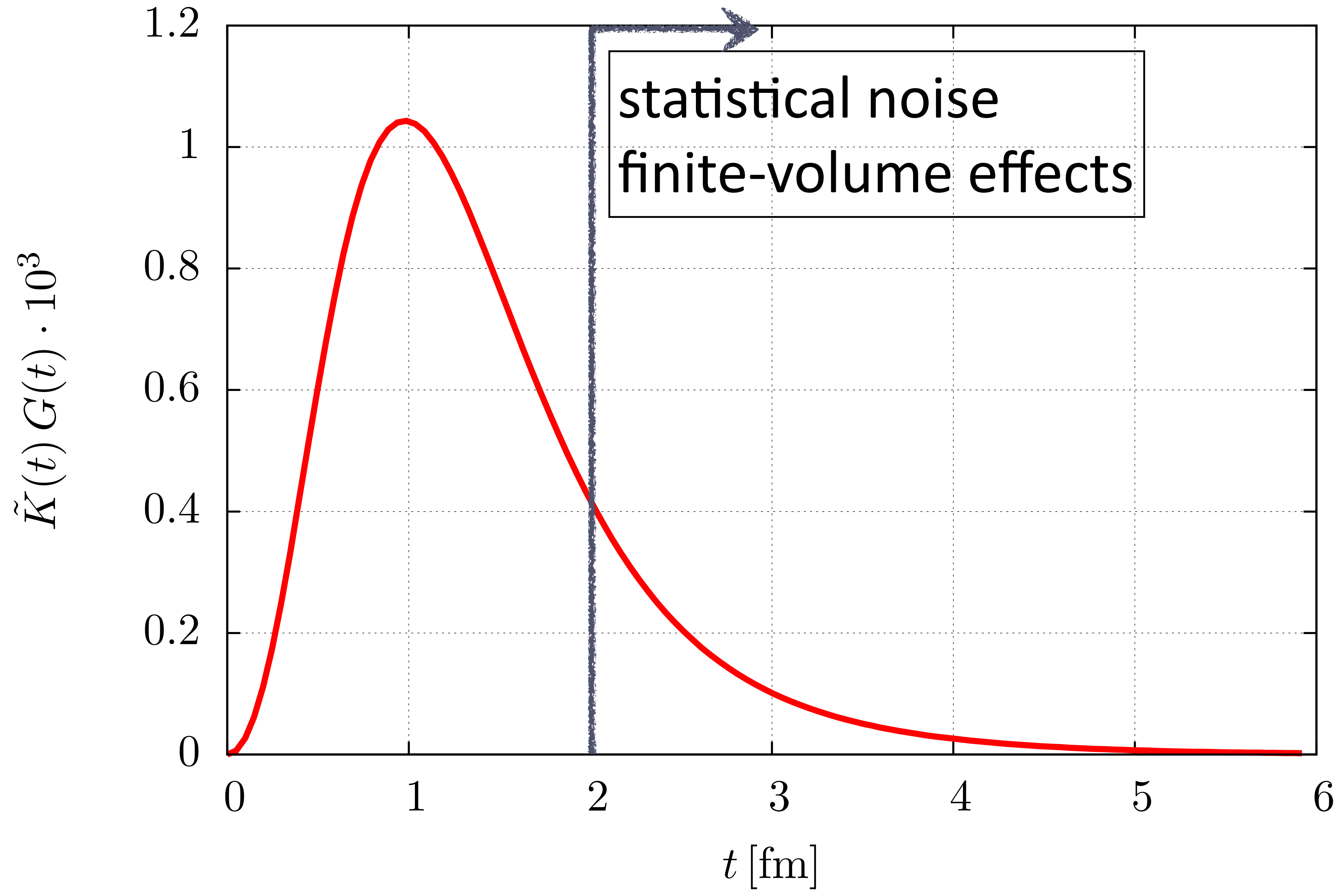

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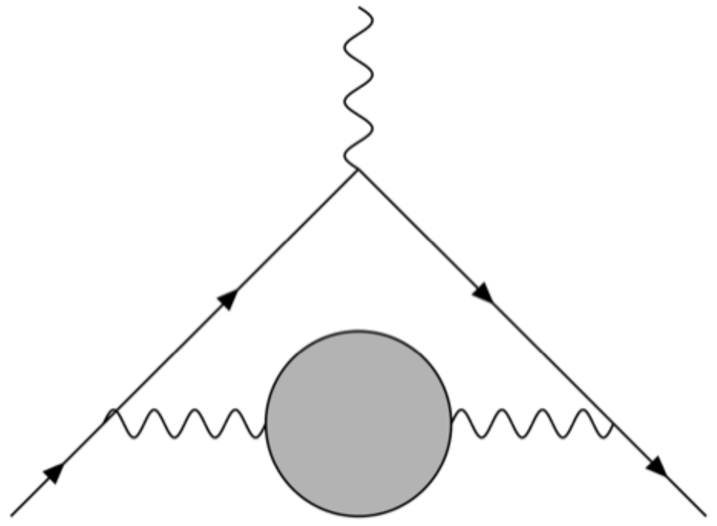

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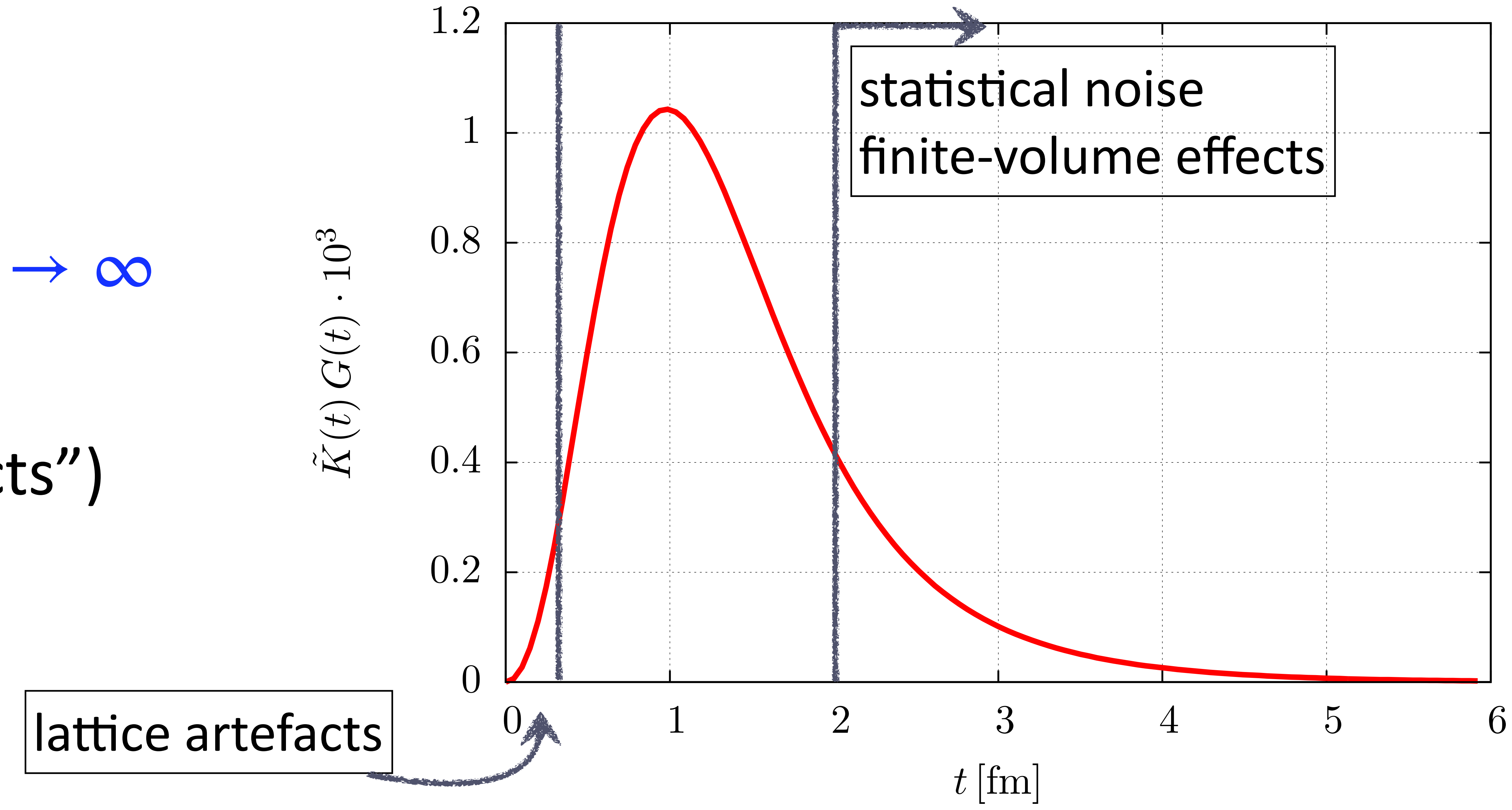

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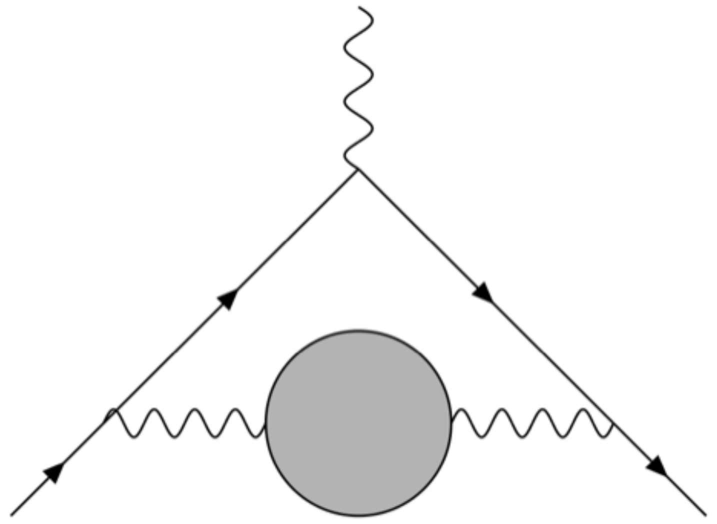

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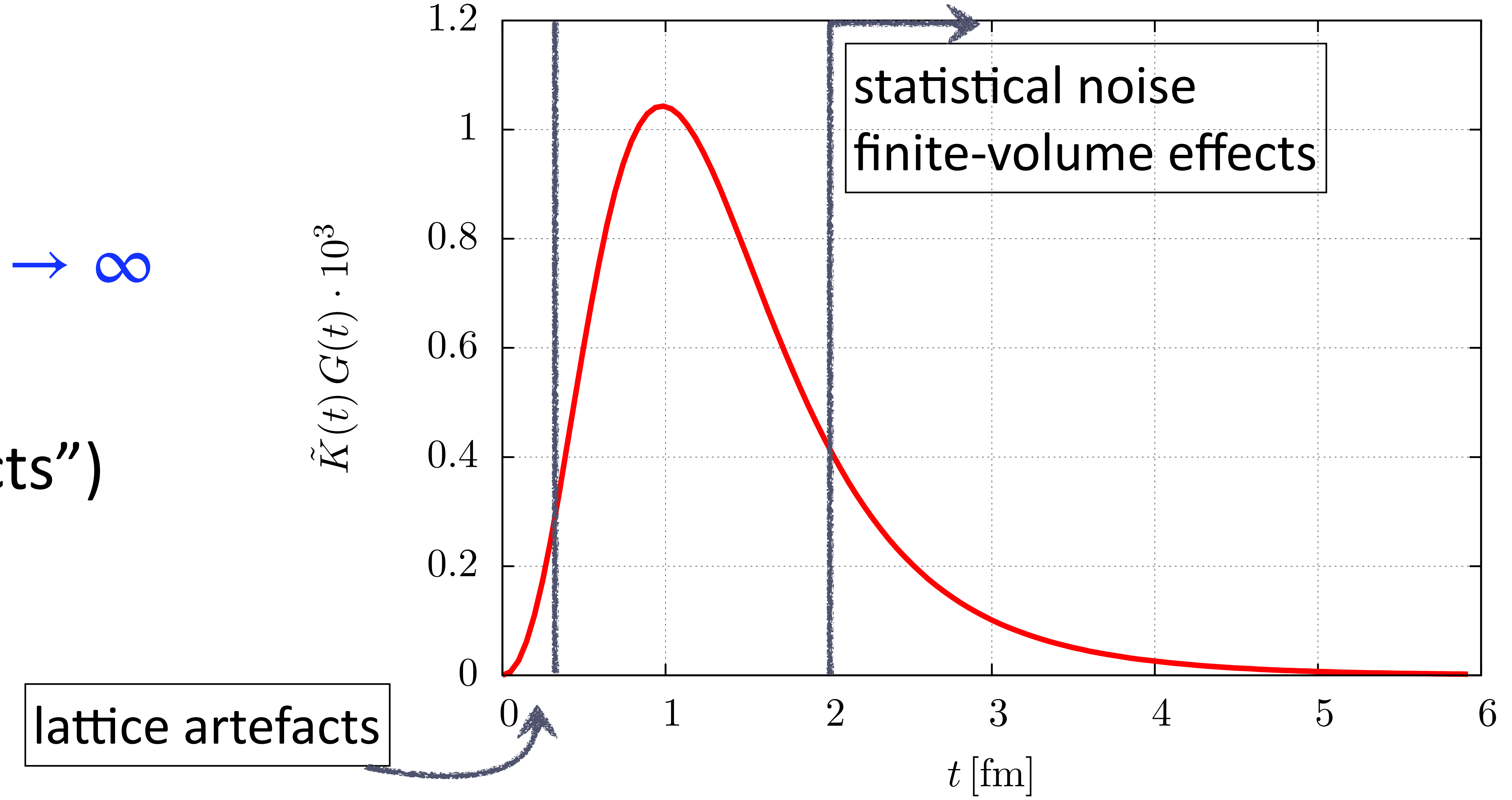

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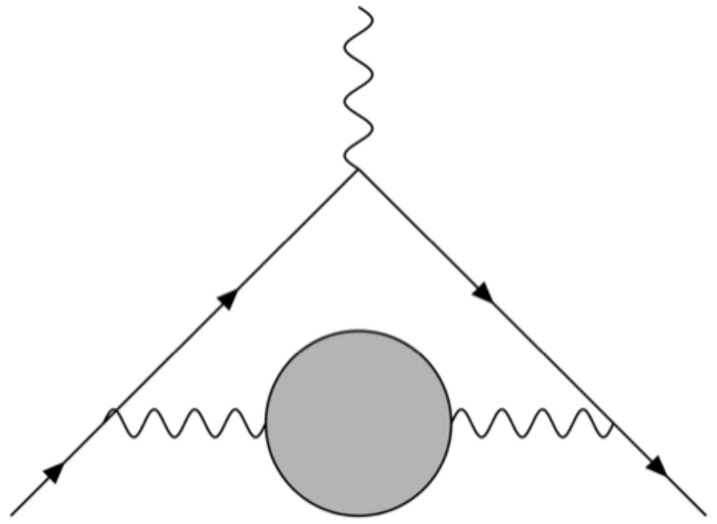

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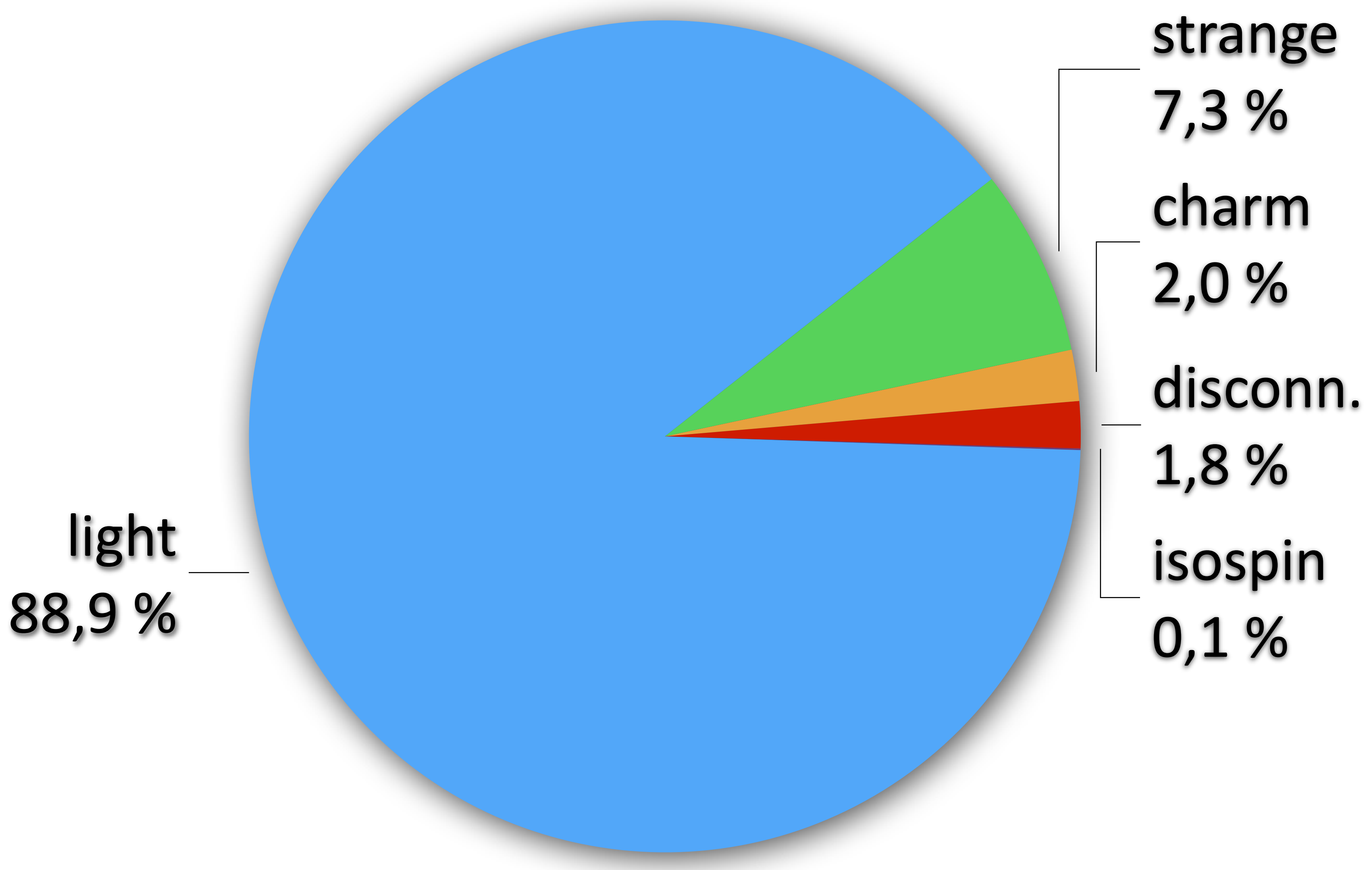
Light-quark connected contribution dominates



Discretisations of the quark action

computational cost

Rooted staggered quarks:

- remnant fermion doublers — “tastes”
- correct analytically for taste-induced lattice artefacts
- used by: BMW, Fermilab-HPQCD-MILC, ABGP,…

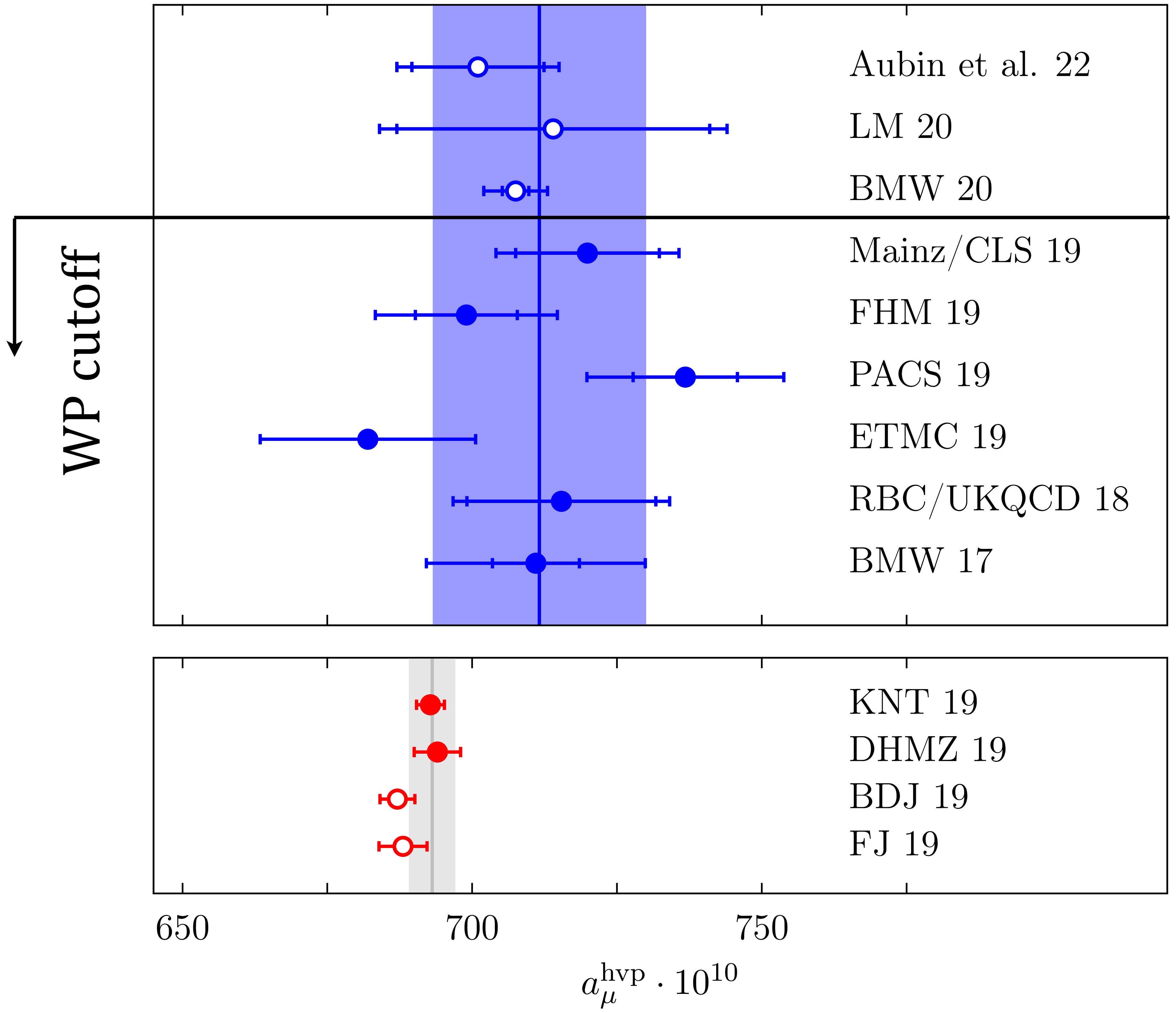
Wilson quarks:

- no doublers; chiral symmetry broken explicitly
- “exceptional configurations”: negative eigenvalues of Wilson-Dirac operator
- used by: Mainz/CLS, ETM, PACS

Domain wall /overlap quarks:

- no doublers; chiral symmetry breaking exponentially small
- live in five dimensions (dwf)
- evaluate sign function of “conventional” action (ovlp)
- used by: RBC/UKQCD, χQCD,…

HVP in Lattice QCD

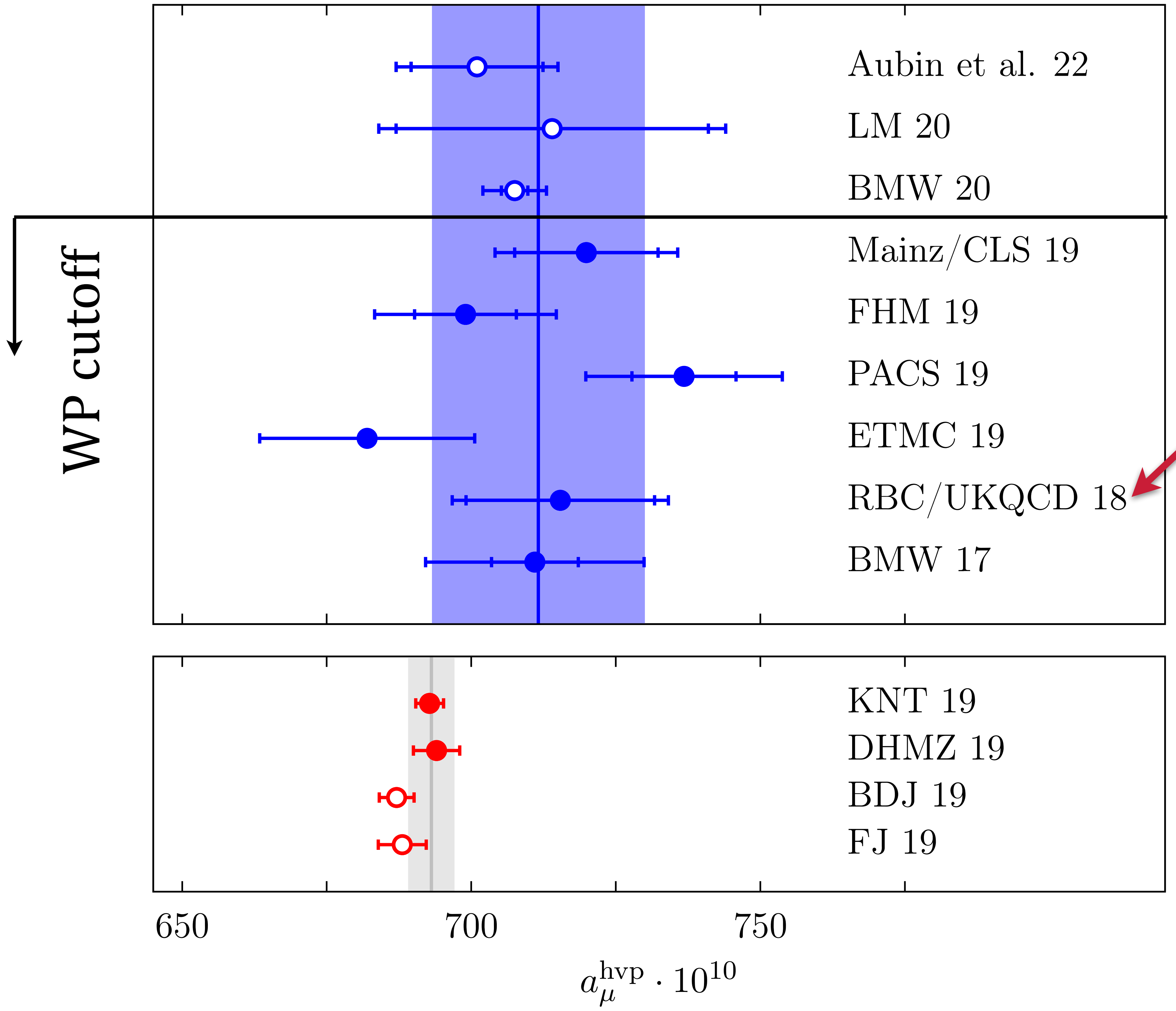


White Paper:

R-ratio: $a_\mu^{\rm hvp,\,LO} = (693.1 \pm 4.0) \cdot 10^{-10}$ [0.6%]

LQCD: $a_\mu^{\rm hvp,\,LO} = (711.6 \pm 18.4) \cdot 10^{-10}$ [2.6%]

HVP in Lattice QCD



RBC/UKQCD *[Blum et al., Phys. Rev. Lett. 121 (2018) 022003]*

- Domain wall fermions
- Two ensembles: $a = 0.114,\ 0.084\ \mathrm{fm}$ at m_π^{phys}
- Leading isospin-breaking corrections included
- Naive continuum extrapol'n in a^2 including estimated a^4-term

$$a_\mu^{\mathrm{hvp,\,LO}} = (715.4 \pm 16.3 \pm 9.2) \cdot 10^{-10} \quad [2.6\%]$$

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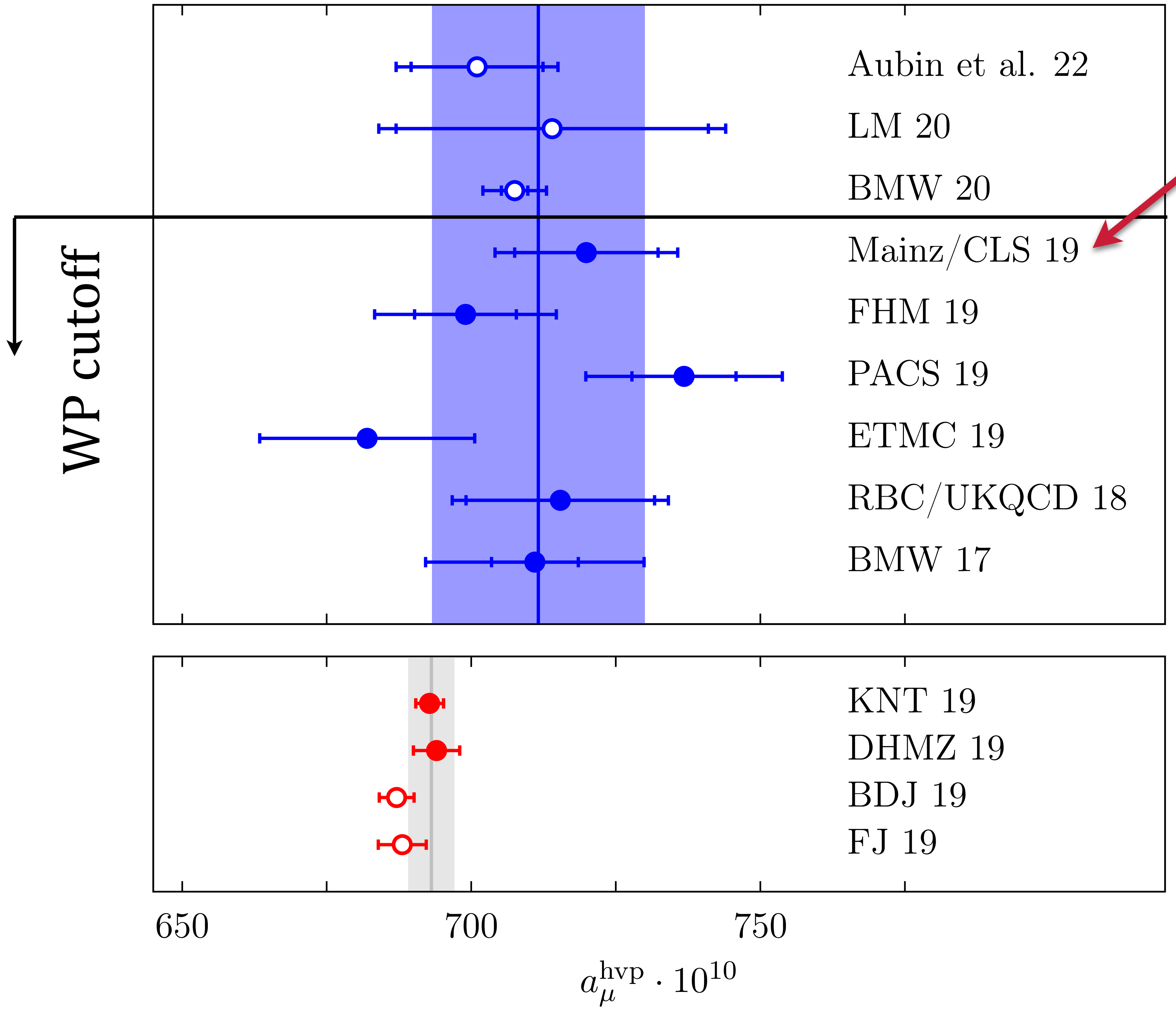
HVP in Lattice QCD



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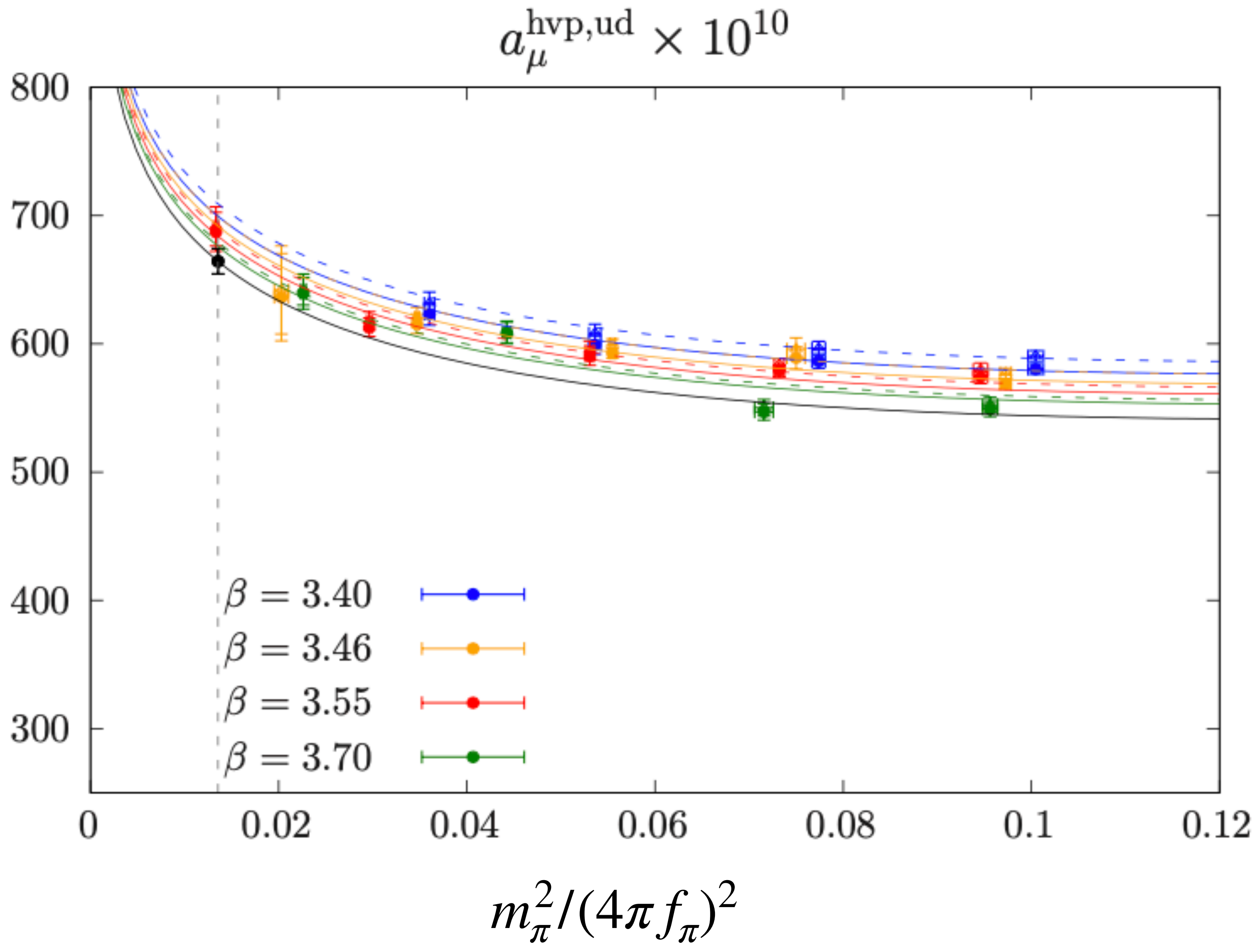
Mainz/CLS

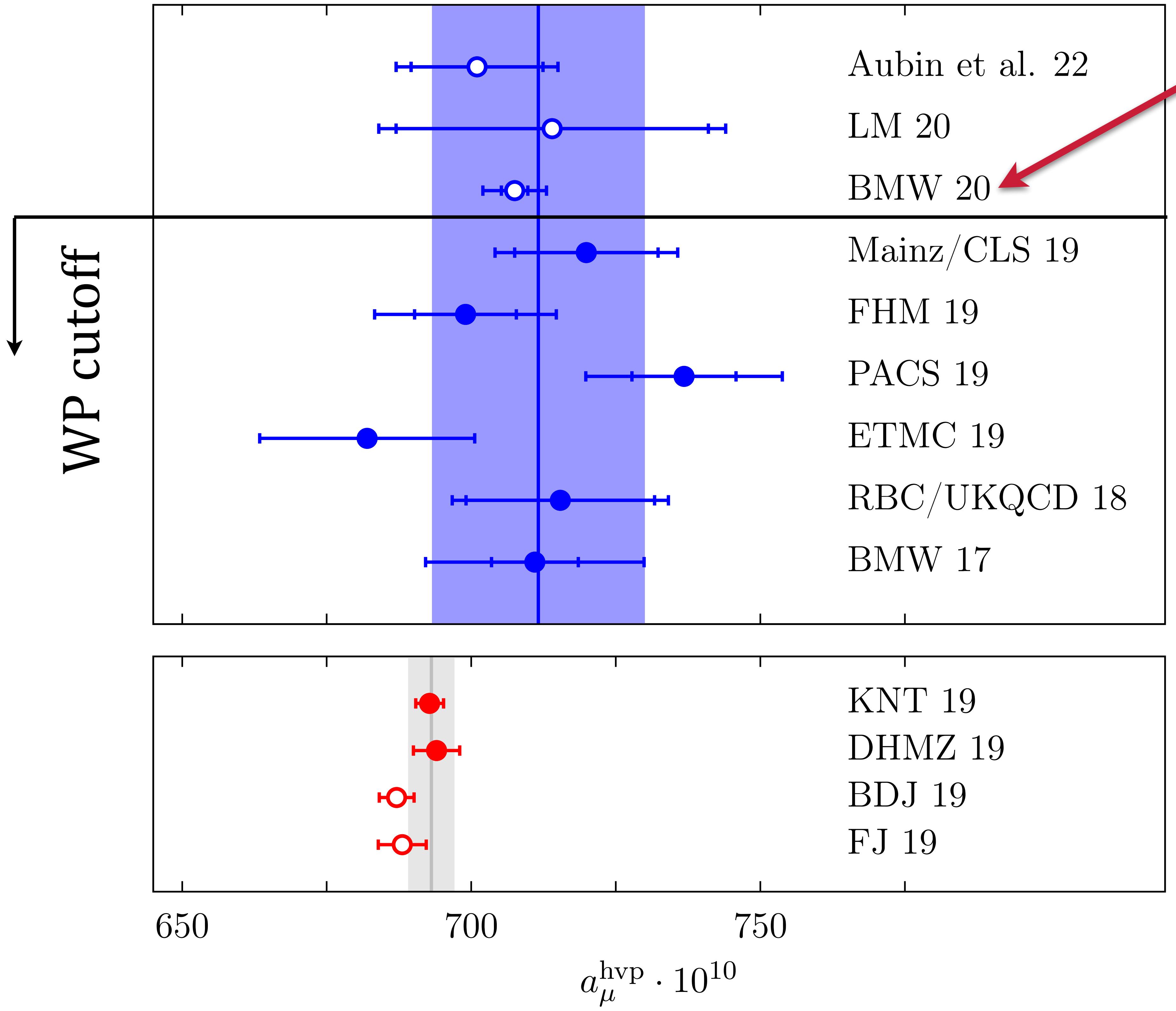
[Gérardin et al., Phys. Rev. D 100 (2019) 014510]

- $\mathrm{O}(a)$ improved Wilson fermions
- Four lattice spacings: $a = 0.085 - 0.050\,\mathrm{fm}$
- Pion masses $m_\pi = 130 - 420\,\mathrm{MeV}$
- Isospin-breaking correction by ETMC added to error
- Simultaneous chiral and continuum extrapolation



$a_\mu^{\rm hvp,\,LO} = (720.0 \pm 12.6 \pm 9.9) \cdot 10^{-10}$ [2.2%]

HVP in Lattice QCD



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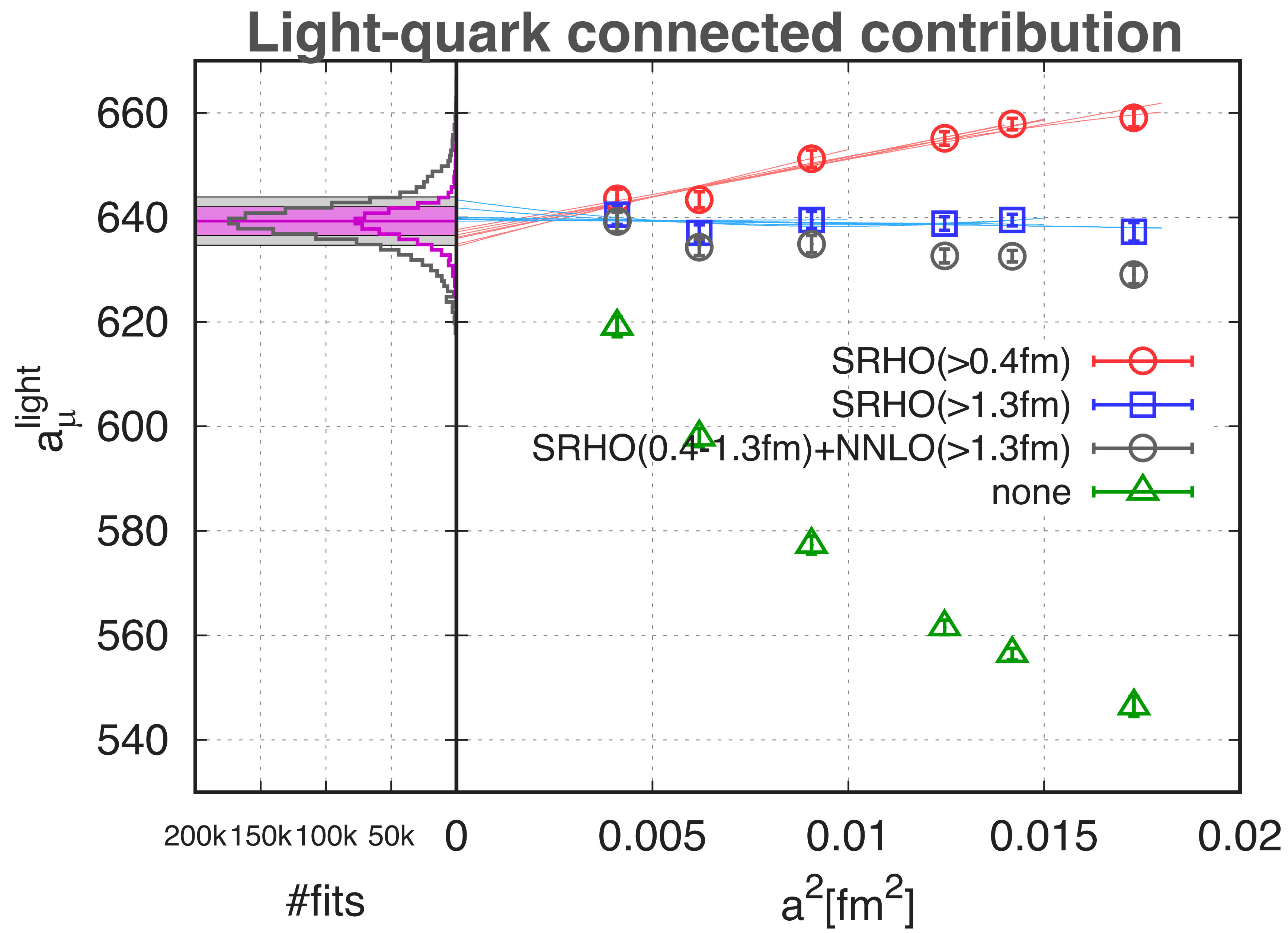
BMWc [Borsányi et al., Nature 593 (2021) 7857]

- Rooted staggered fermions
- Six lattice spacings: $a = 0.132 - 0.064\,\text{fm}$
- Physical pion mass throughout
- Correct for taste-breaking before continuum extrapol'n
- Final result selected from distribution of different fits



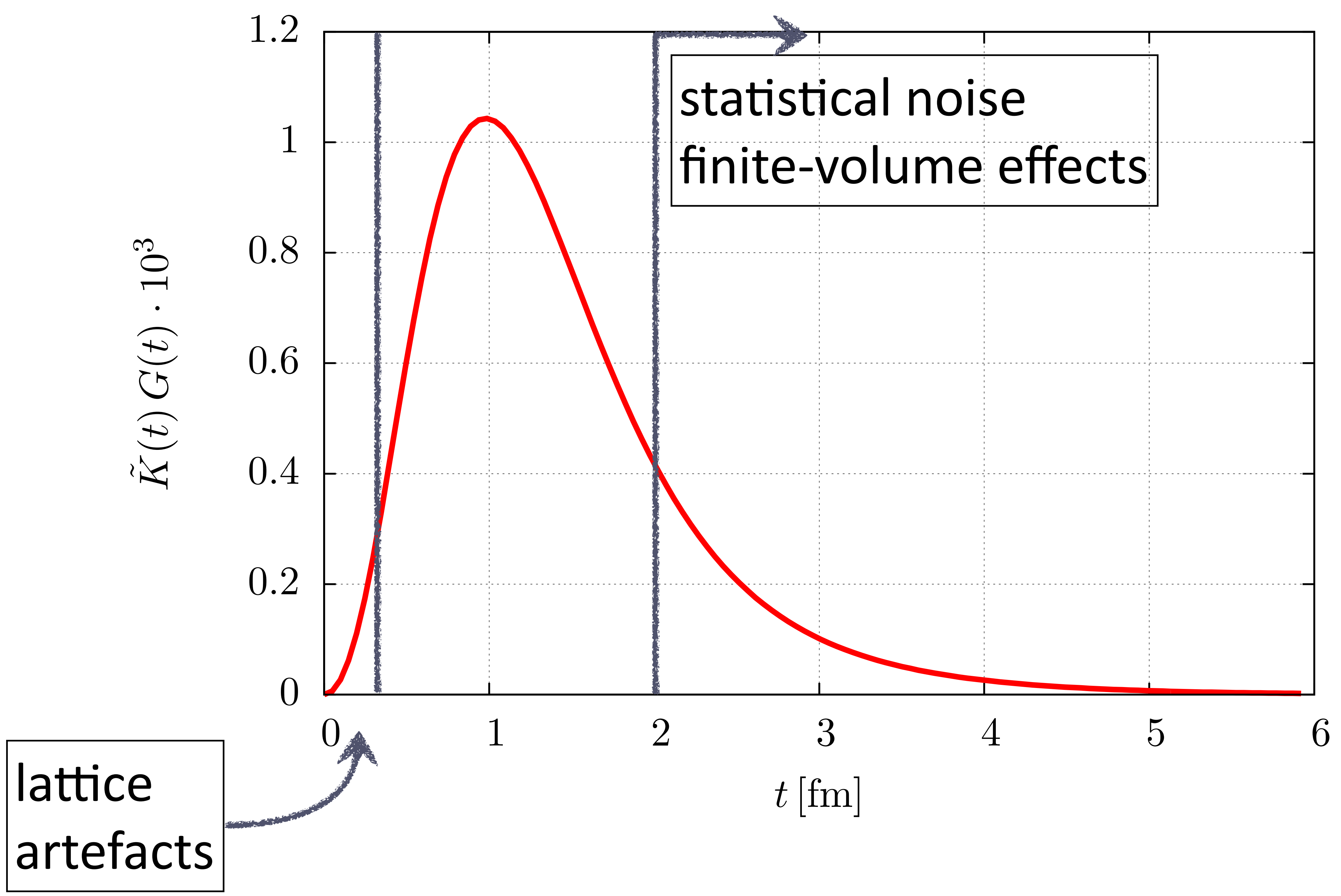
$a_\mu^{\rm hvp,\,LO} = (707.5 \pm 2.3 \pm 5.0) \cdot 10^{-10}$ [0.8%]

Window observables

[Blum et al., Phys. Rev. Lett. 121 (2018) 022003]

Idea: restrict integration to "unproblematic" regions

→ reduce statistical fluctuations and systematic effects



$$a_\mu^{\rm hvp,\,win} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \tilde{K}(t)\, G(t)\, W(t;\, t_0,\, t_1)$$

Intermediate-distance window:

$$W^{\rm ID}(t;\, t_0,\, t_1) = \Theta(t,\, t_0, \Delta) - \Theta(t,\, t_1, \Delta)$$

$$\Theta(t,\, t', \Delta) = \tfrac{1}{2}\left[1 + \tanh(t - t')/\Delta\right]$$

$$t_0 = 0.4\,\text{fm},\;\; t_1 = 1.0\,\text{fm},\;\; \Delta = 0.15\,\text{fm}$$

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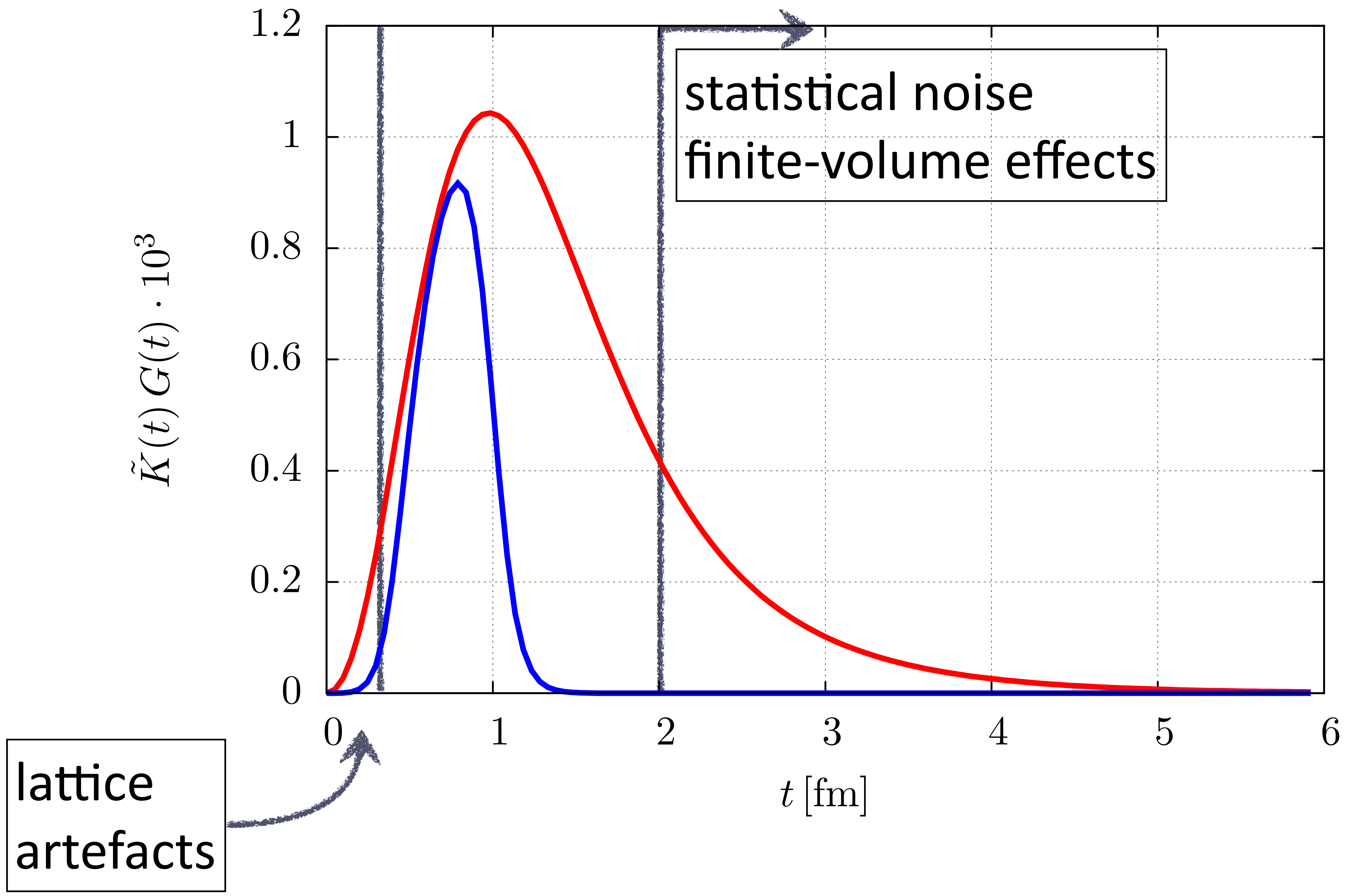

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- Finite-volume correction reduced to 0.25%
- Uncertainty dominated by statistics

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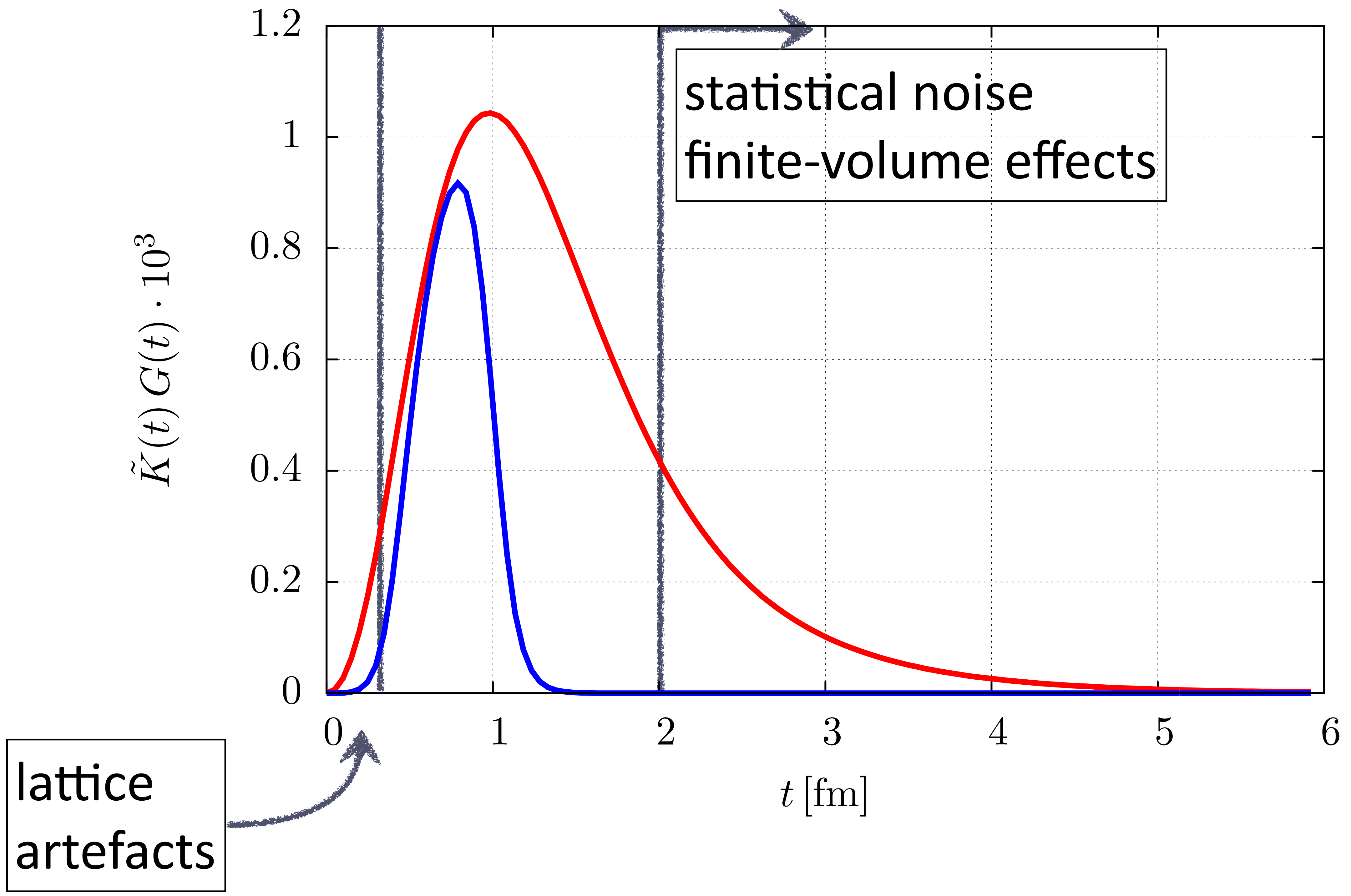

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→ Benchmark quantity for sub-contribution of HVP

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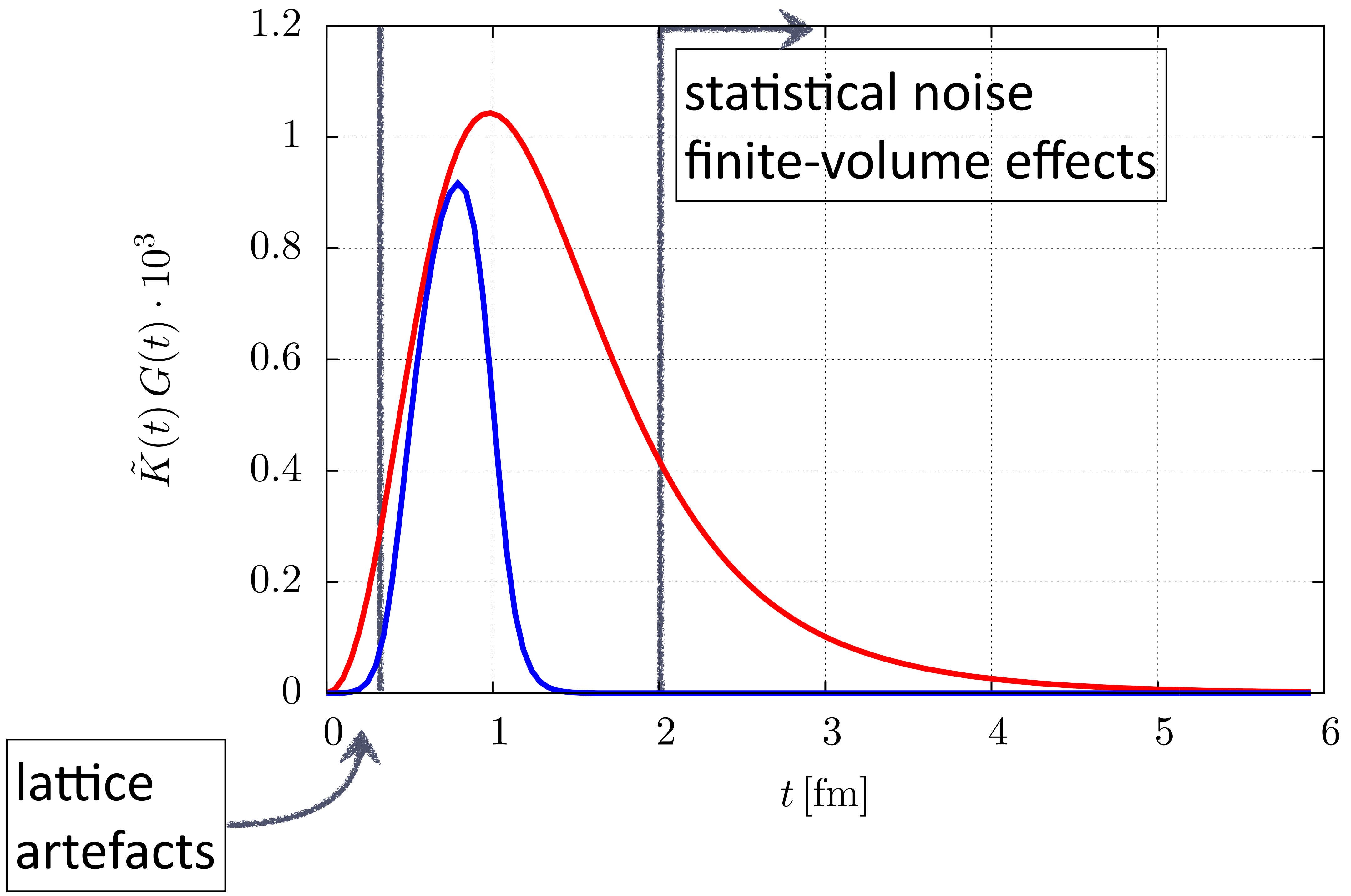

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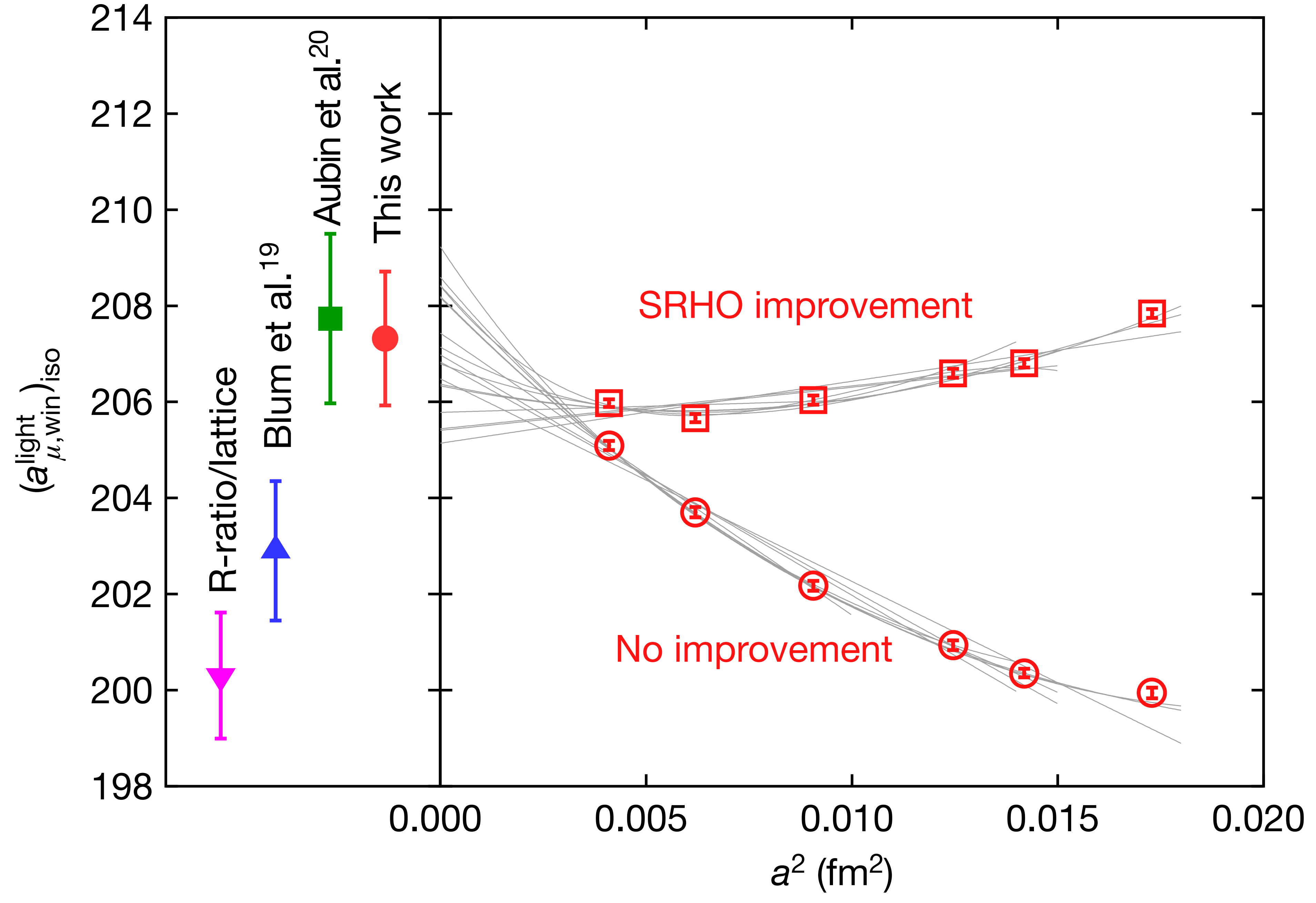
→ Benchmark quantity for sub-contribution of HVP

Data-driven approach: $a_\mu^{\mathrm{win}} = (229.4 \pm 1.4)\cdot 10^{-10}$ [Colangelo et al., Phys Lett B833 (2022) 137313]

(Excluding the 2023 CMD-3 result for $e^+e^- \to \pi^+\pi^-$)

Intermediate window observable in Lattice QCD

BMWc: Rooted staggered quarks



$$a_\mu^{\text{win,ud}} = (207.3 \pm 0.4 \pm 1.3) \cdot 10^{-10}$$

[Borsányi et al., Nature 593 (2021) 7857]

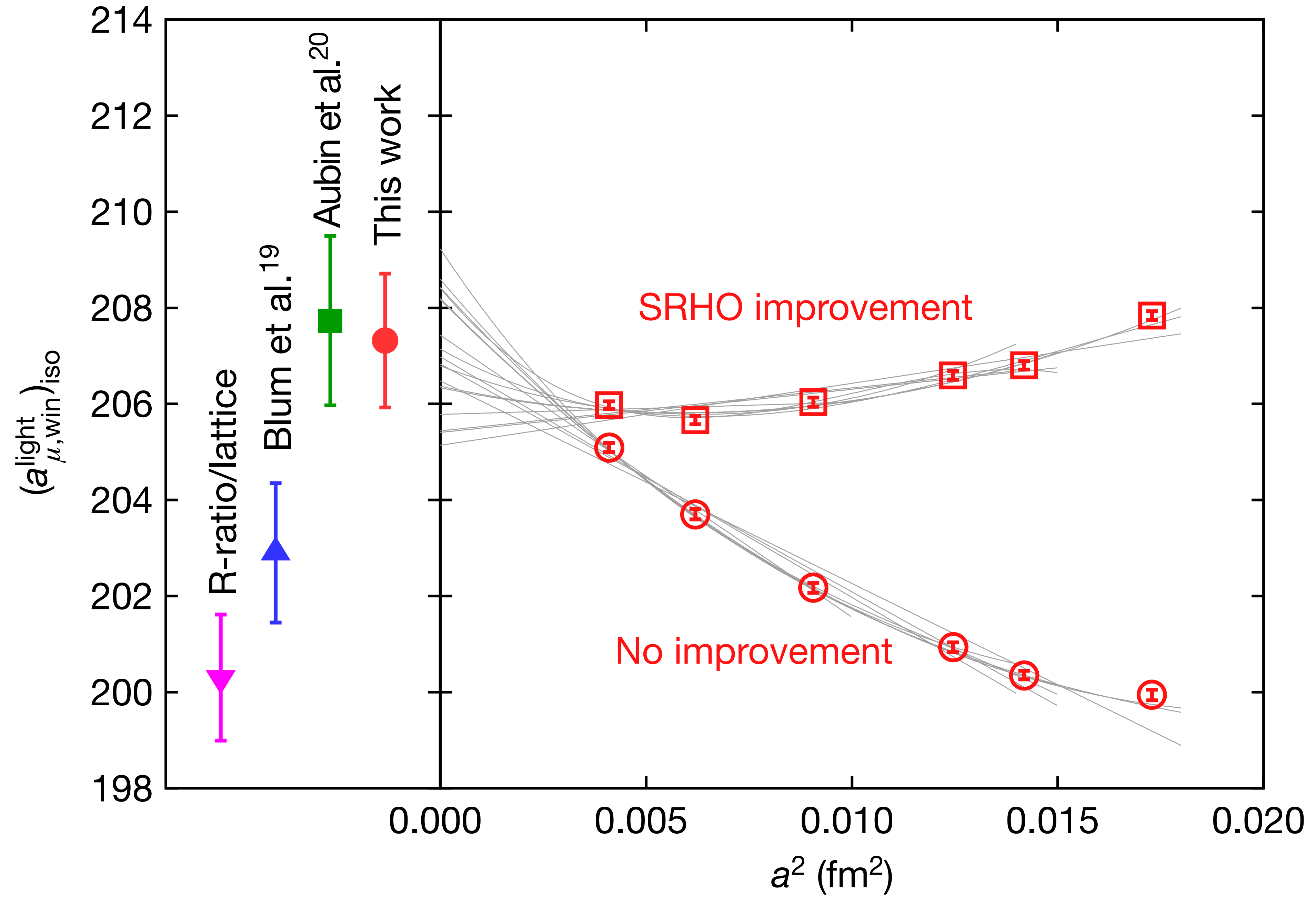
Mainz/CLS: $\mathrm{O}(a)$ improved Wilson quarks

- Extension to six lattice spacings:
  $a = 0.099 - 0.035\,\text{fm}$
- Pion masses $m_\pi = 130 - 420\,\text{MeV}$
- Two discretisations of the vector current: local and conserved
- Simultaneous chiral and continuum extrapolation
- Isospin-breaking correction included

[Cè et al., Phys Rev D106 (2022) 114502]

Intermediate window observable in Lattice QCD

BMWc: Rooted staggered quarks



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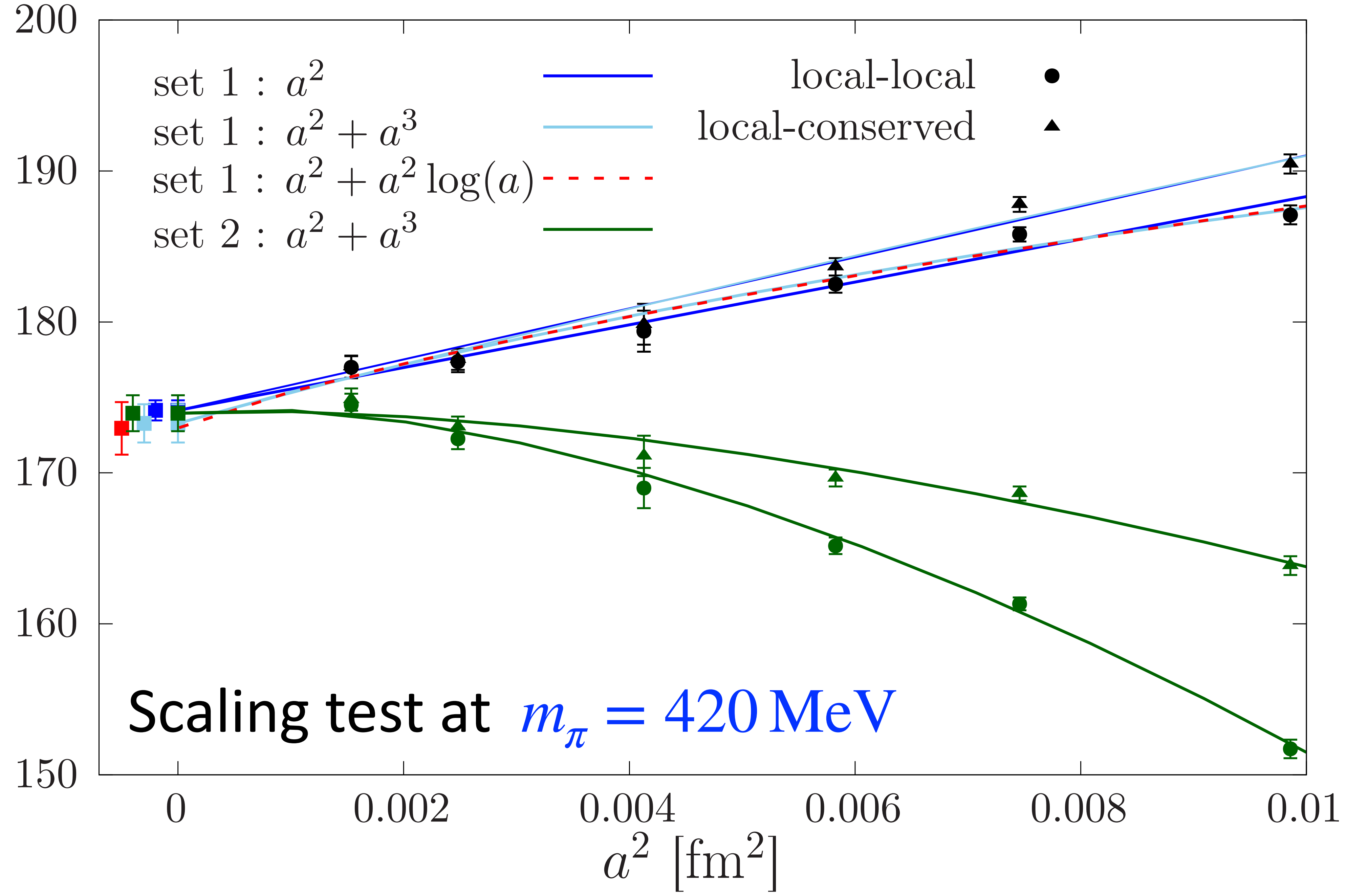

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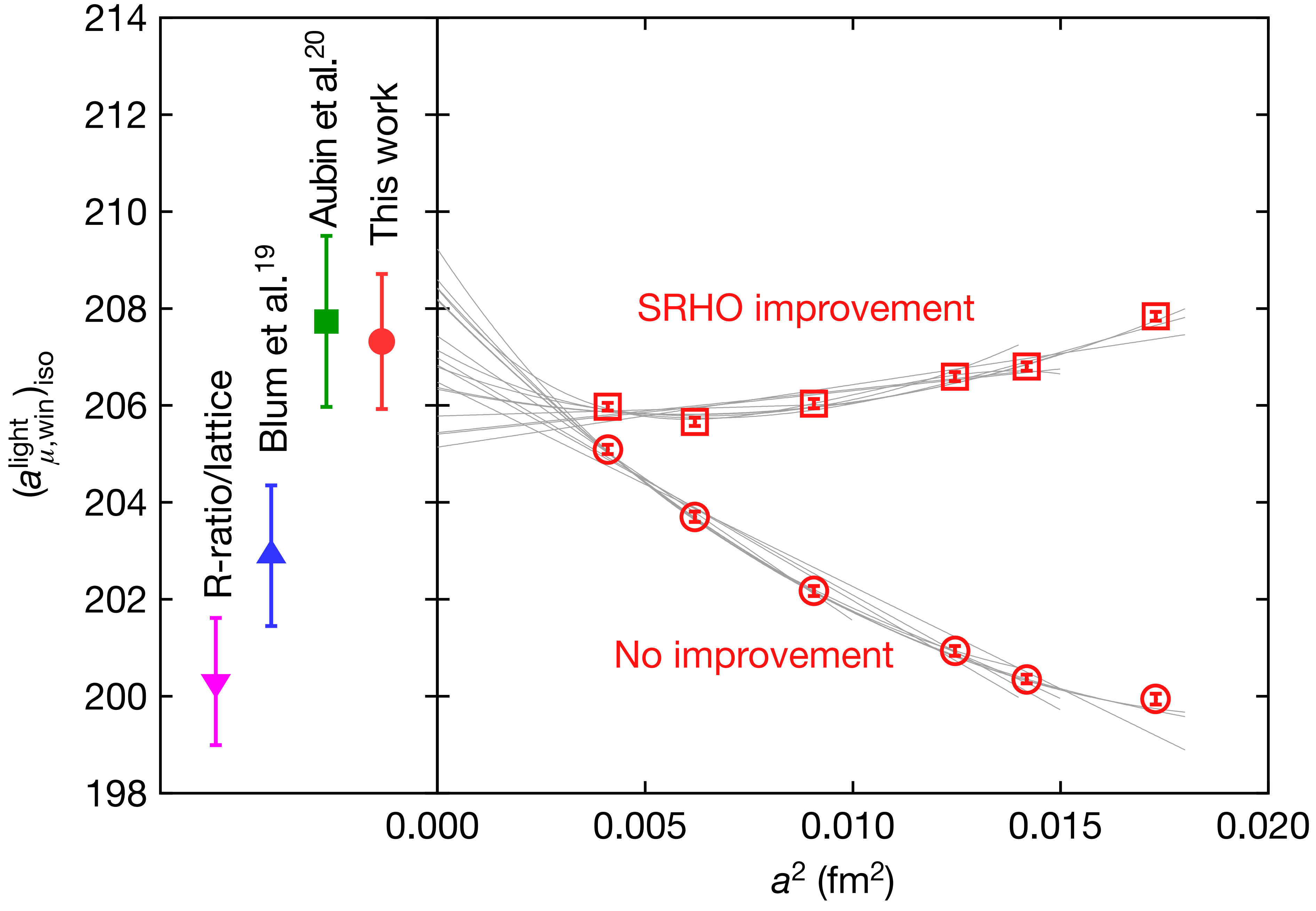

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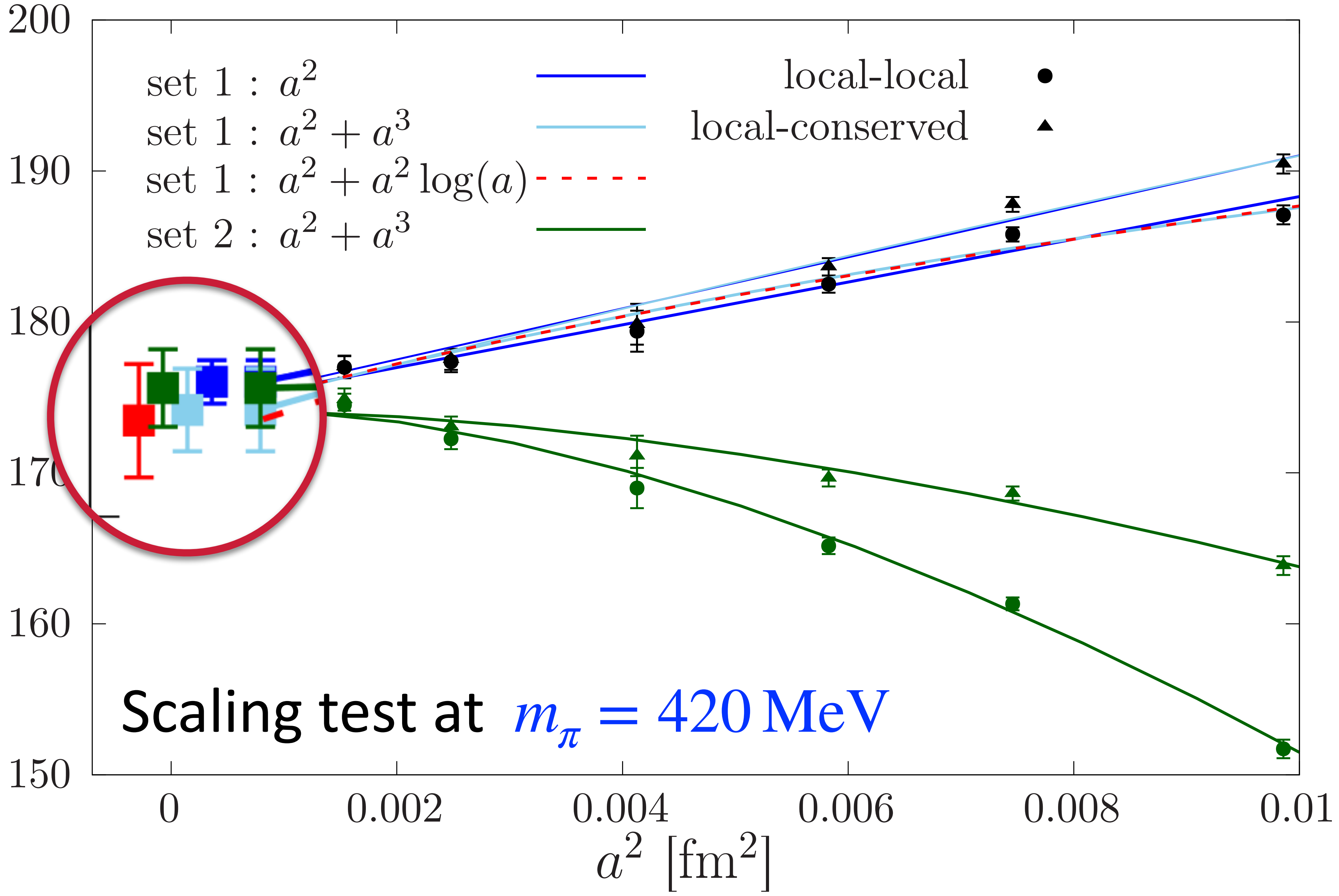

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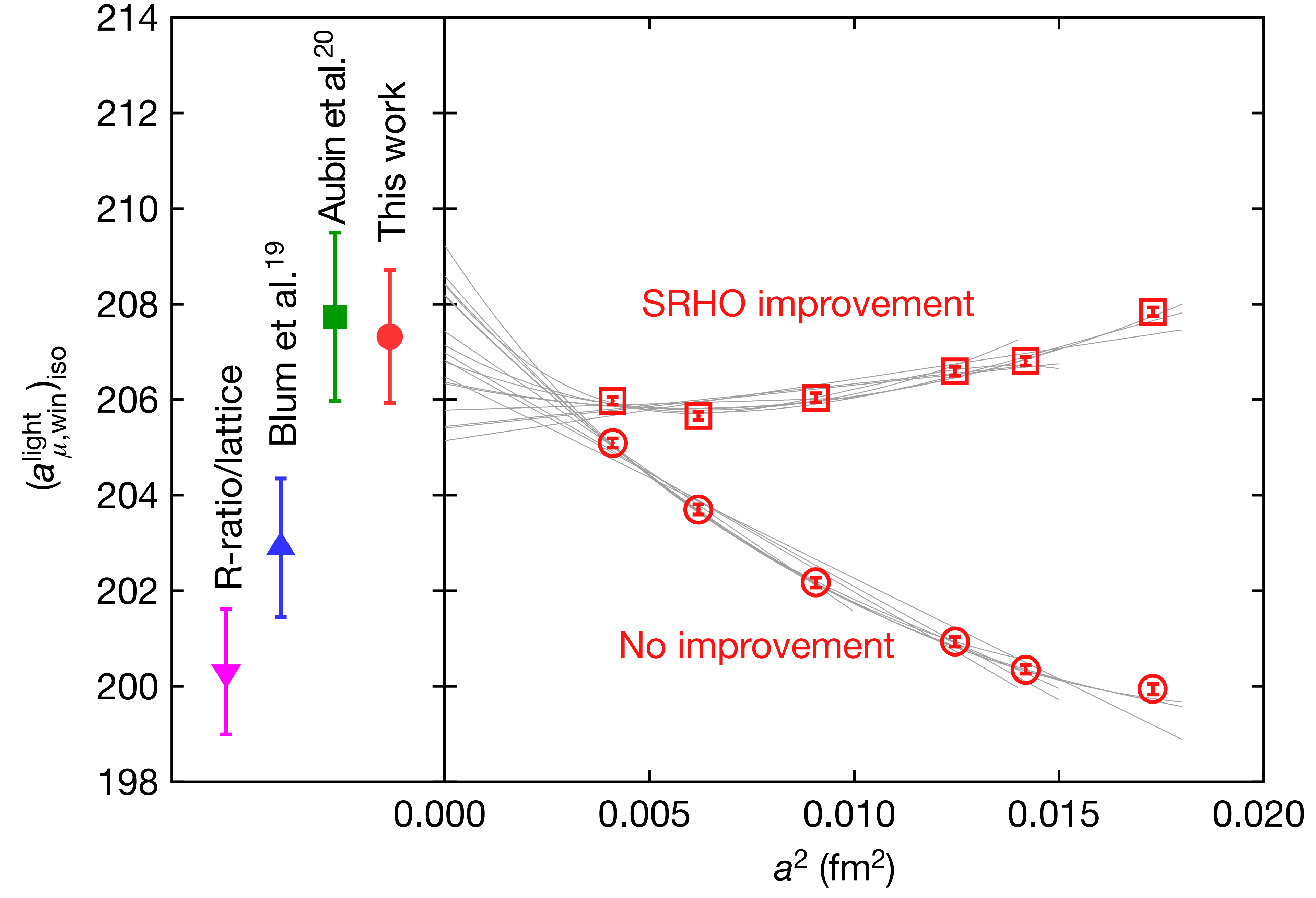

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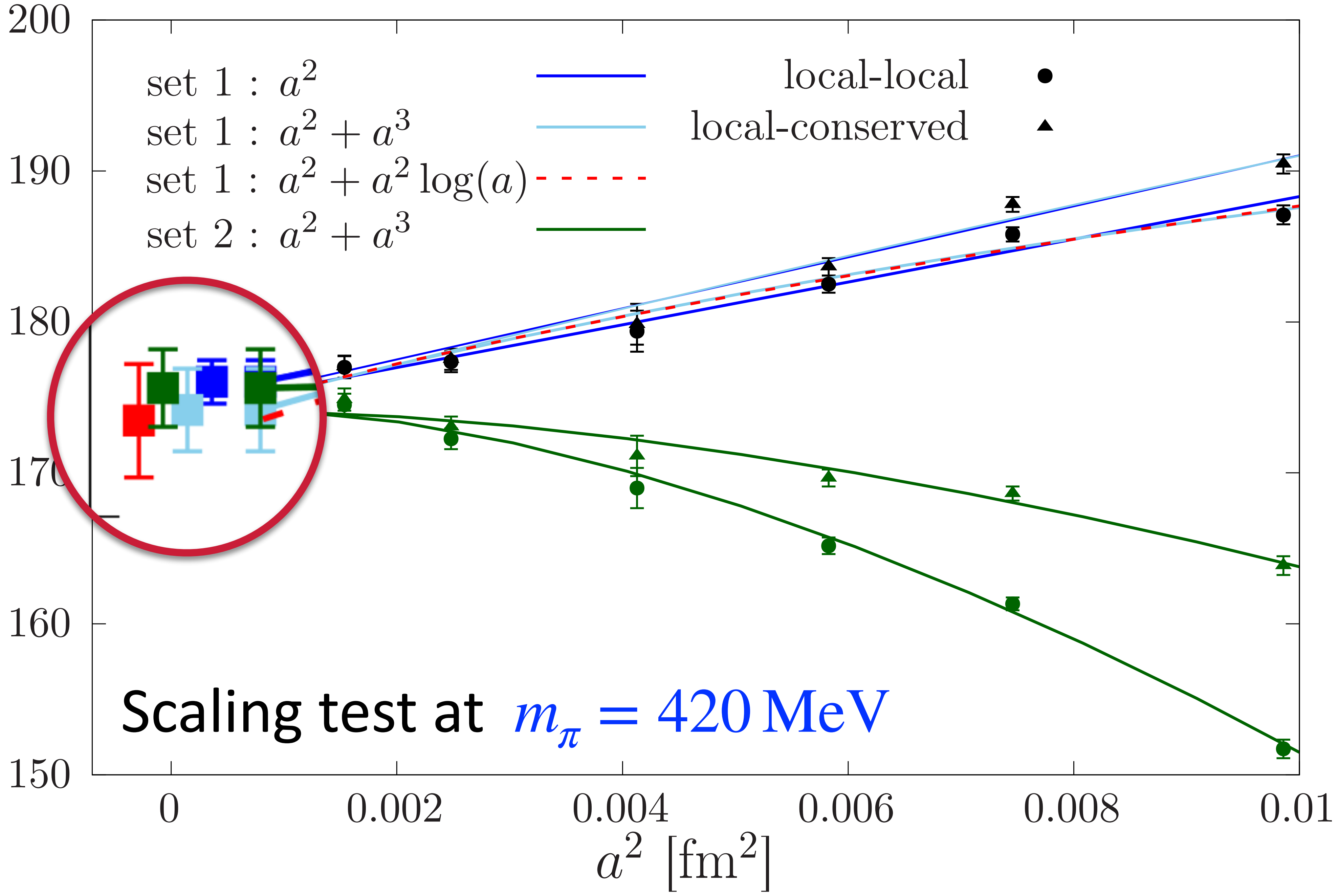
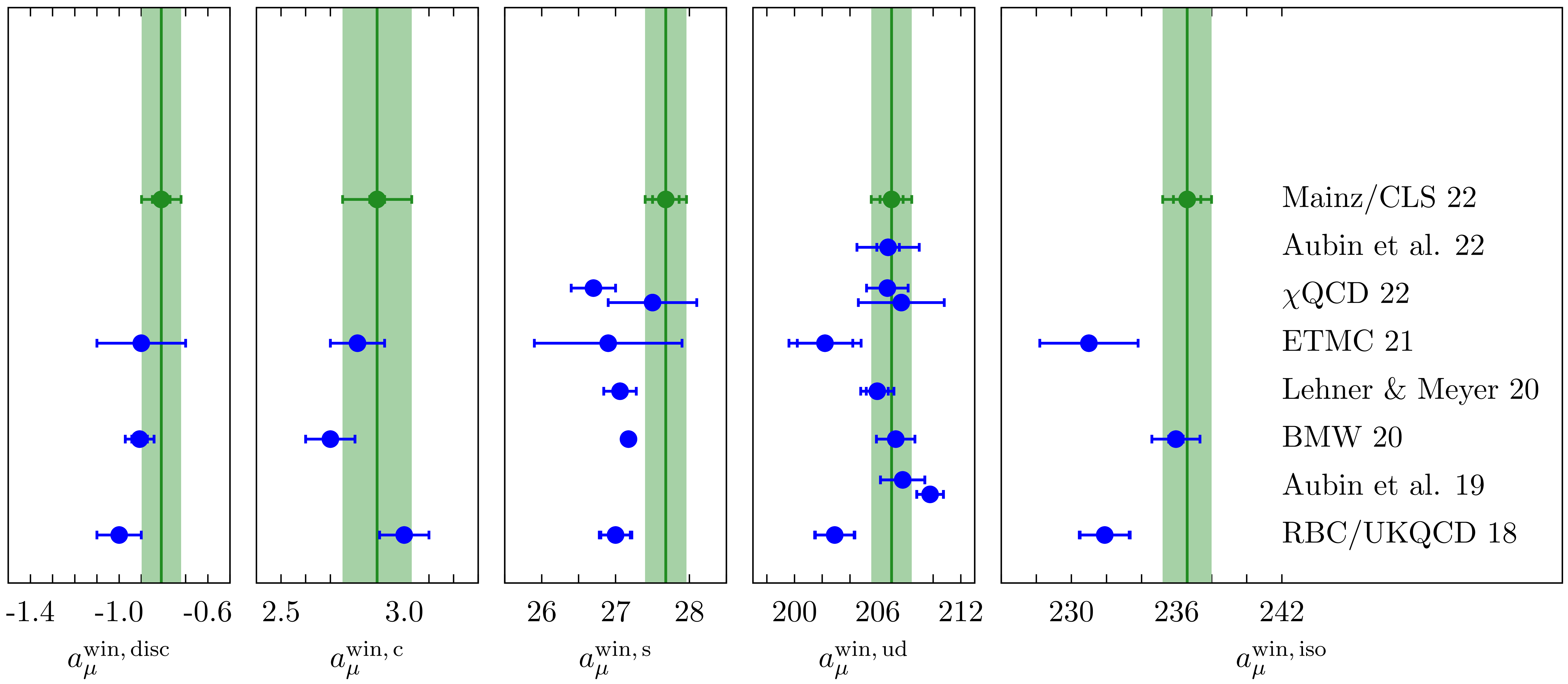


$$a_\mu^{\mathrm{win,ud}} = (207.0 \pm 0.8 \pm 1.2) \cdot 10^{-10}$$

[Cè et al., Phys Rev D106 (2022) 114502]

Intermediate window observable in Lattice QCD

Results for individual quark flavours / quark-disconnected contribution in isospin limit



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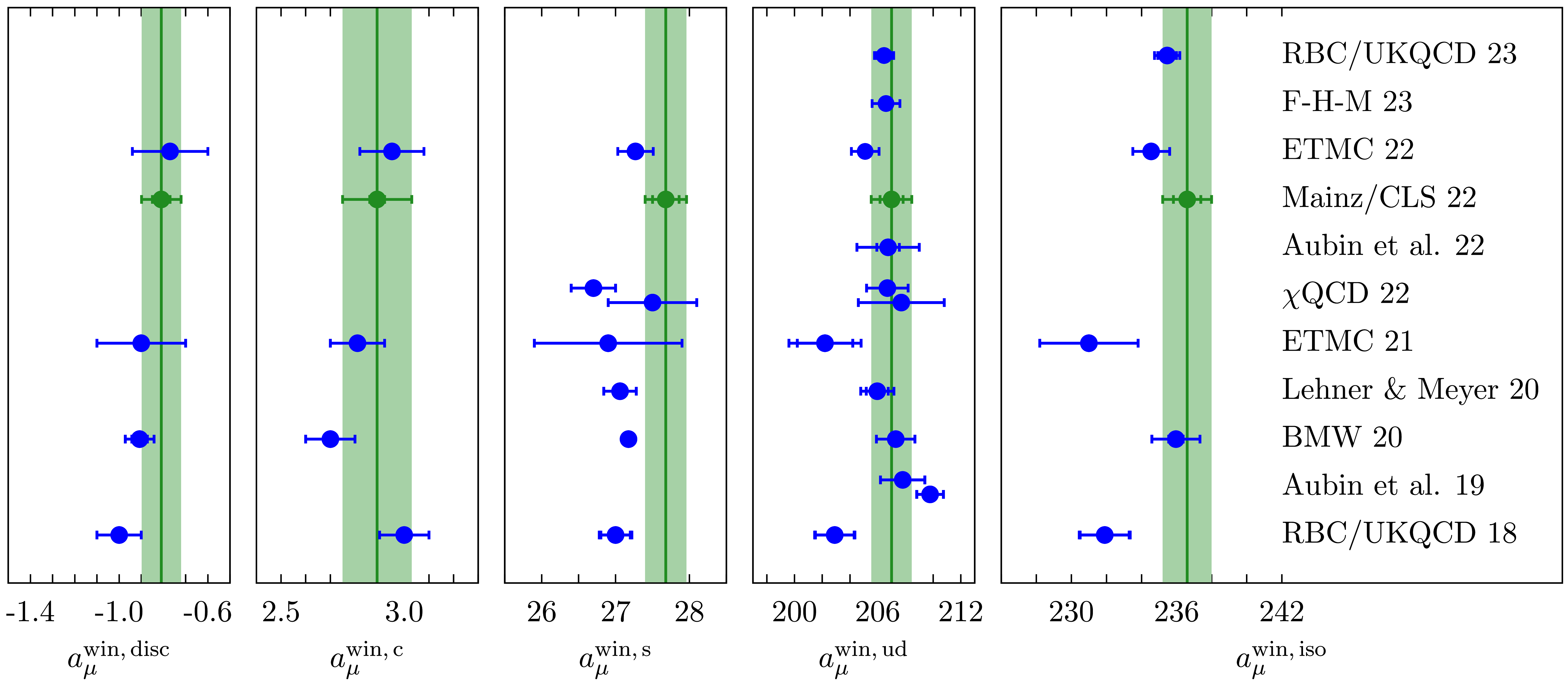

[Blum et al., arXiv:2301.08696]

[Bazavov et al., arXiv:2301.08274]

[Alexandrou et al., PRD 107 (2023) 074506]

[Cè et al., Phys Rev D106 (2022) 114502]

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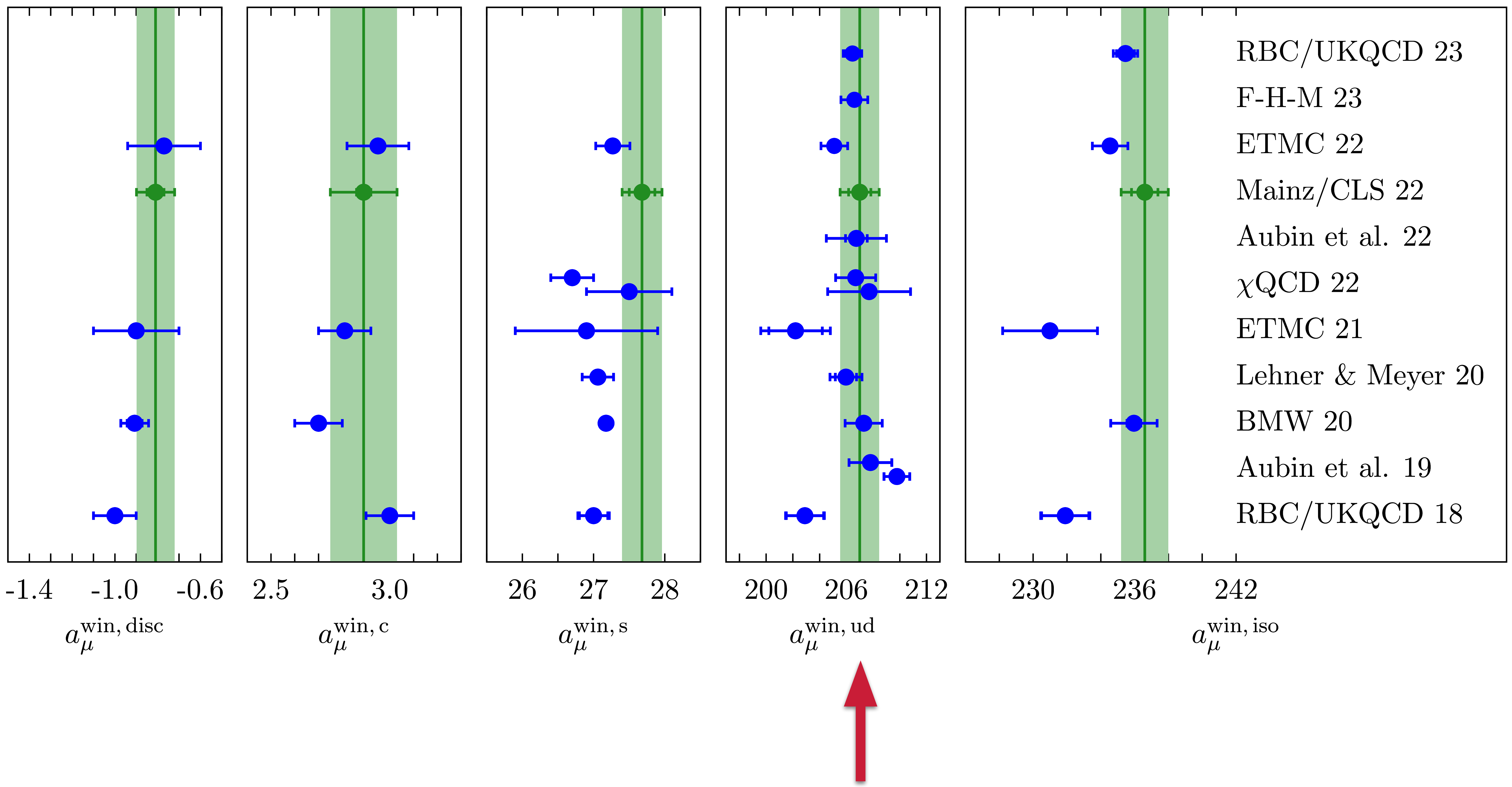


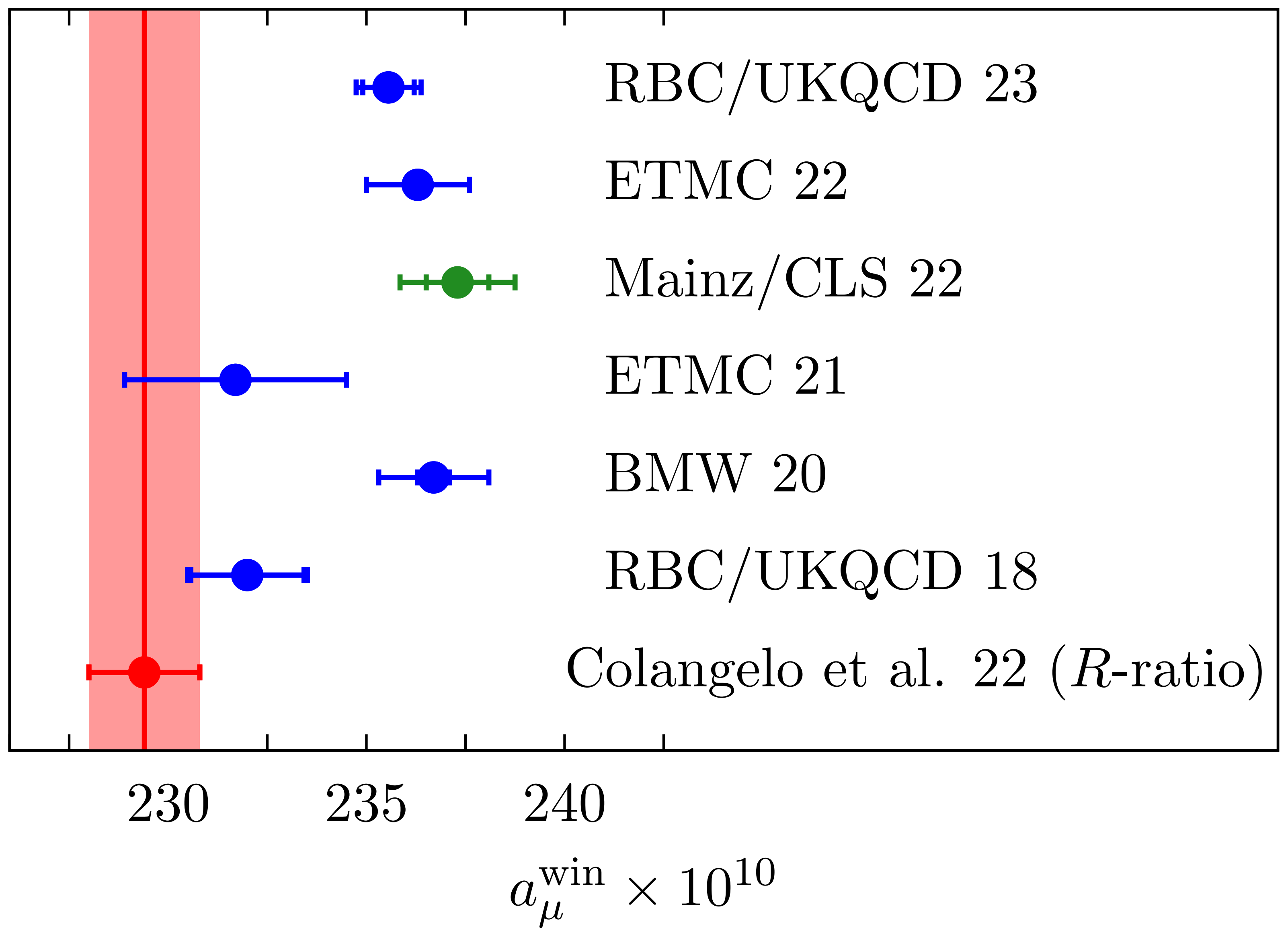
[Blum et al., arXiv:2301.08696]
[Bazavov et al., arXiv:2301.08274]
[Alexandrou et al., PRD 107 (2023) 074506]
[Cè et al., Phys Rev D106 (2022) 114502]

Result for the dominant, light-quark connected contribution confirmed for wide range of different discretisations with sub-percent precision

Intermediate window observable: Comparison with R-ratio

R-ratio estimate: $a_\mu^{\rm win} = (229.4 \pm 1.4) \cdot 10^{-10}$

Mainz/CLS 22: $a_\mu^{\rm win} = (237.30 \pm 1.46) \cdot 10^{-10}$



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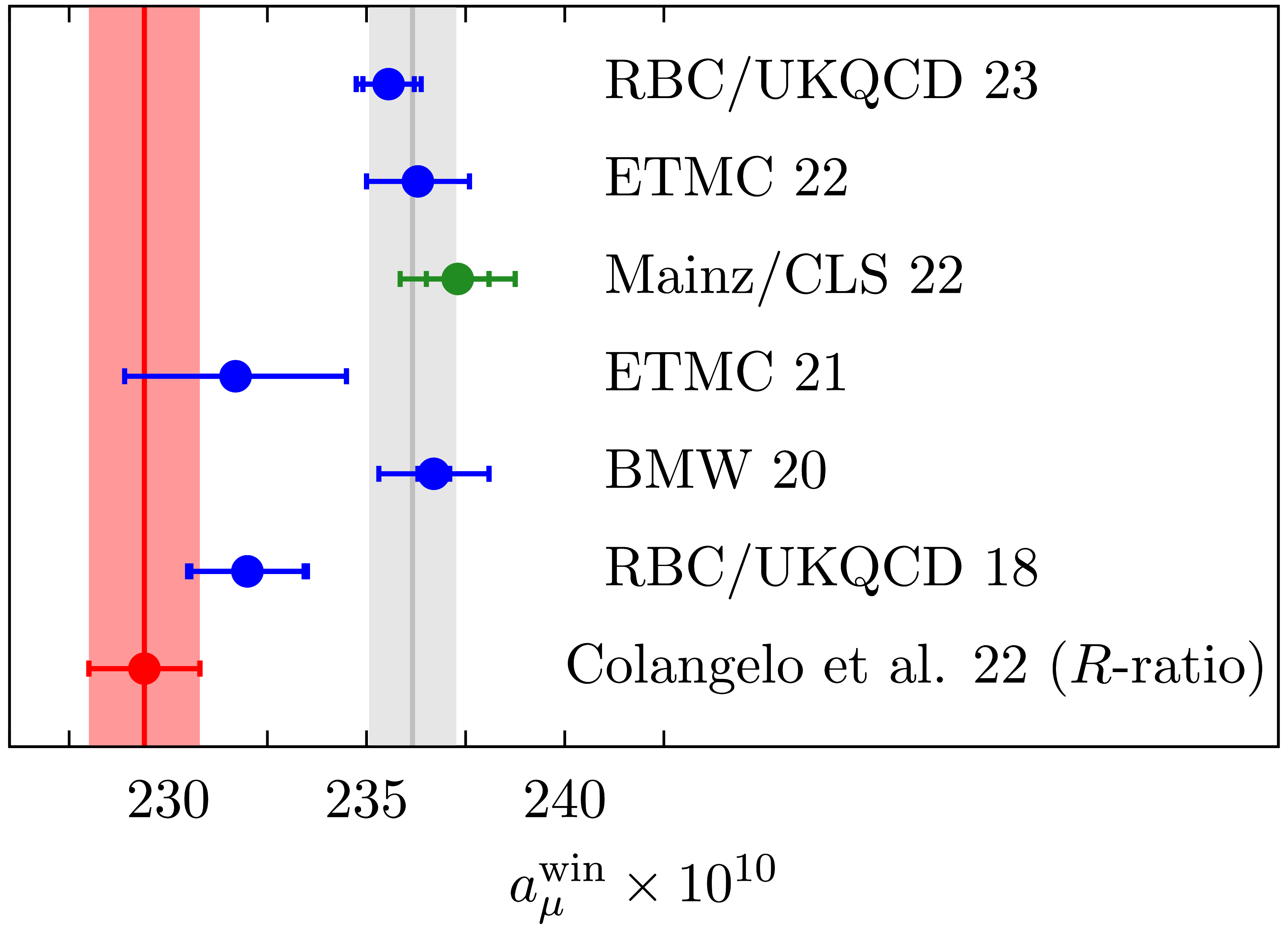
Lattice average: $a_\mu^{\rm win} = (236.16 \pm 1.09) \cdot 10^{-10}$

(RBC/UKQCD 23, ETMC 22, Mainz/CLS 22, BMW 20)



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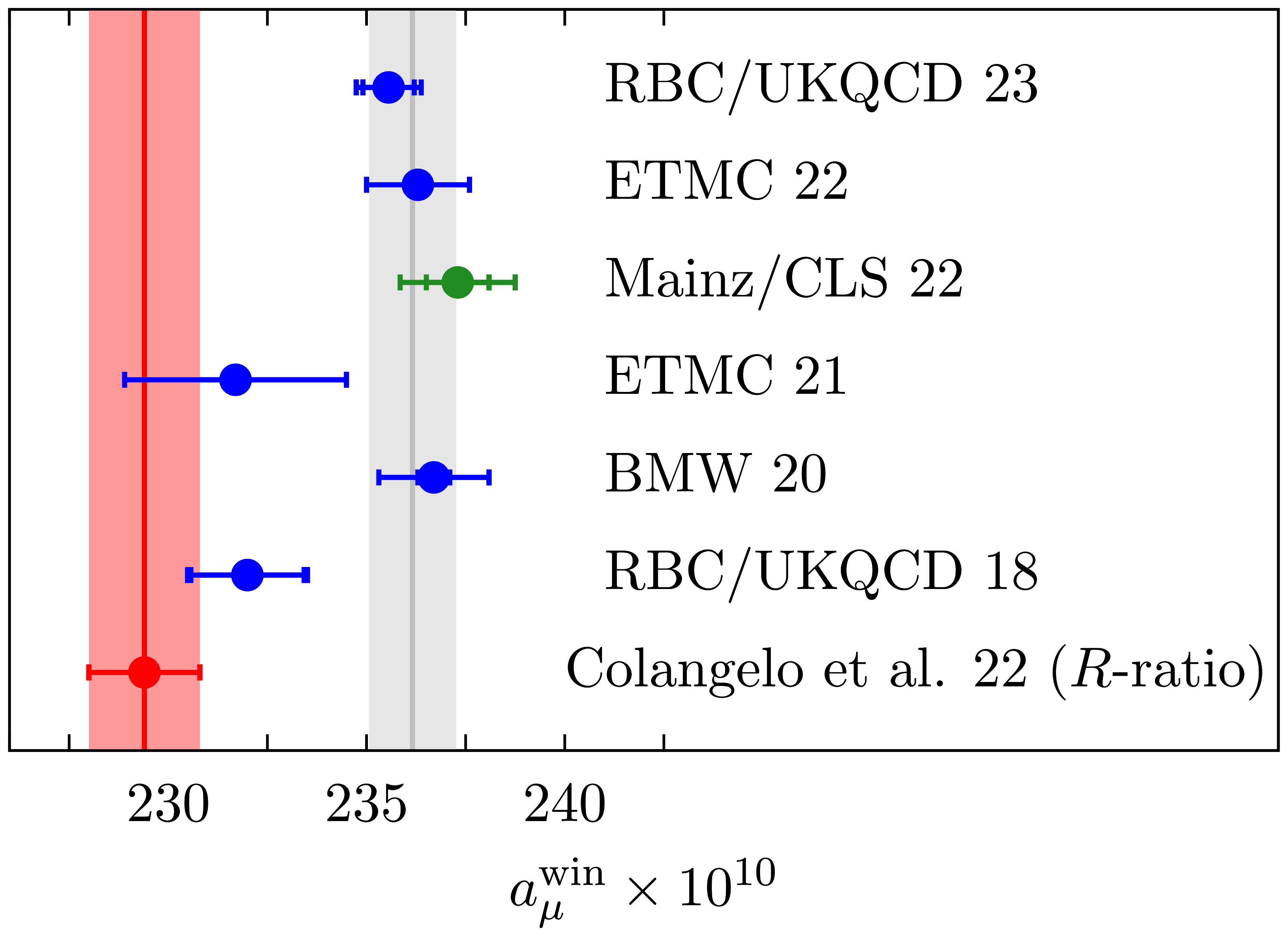

$$\Rightarrow \quad a_\mu^{\text{win}}\Big|_{\text{Lat-av.}} - a_\mu^{\text{win}}\Big|_{R\text{-ratio}} = (6.8 \pm 1.8) \cdot 10^{-10} \quad [3.8\,\sigma]$$

- Confirmed tension between lattice QCD and e^+e^- data (prior to 2023) for sub-contribution to HVP

Intermediate window observable: Comparison with R-ratio

R-ratio estimate: $a_\mu^{\rm win} = (229.4 \pm 1.4) \cdot 10^{-10}$

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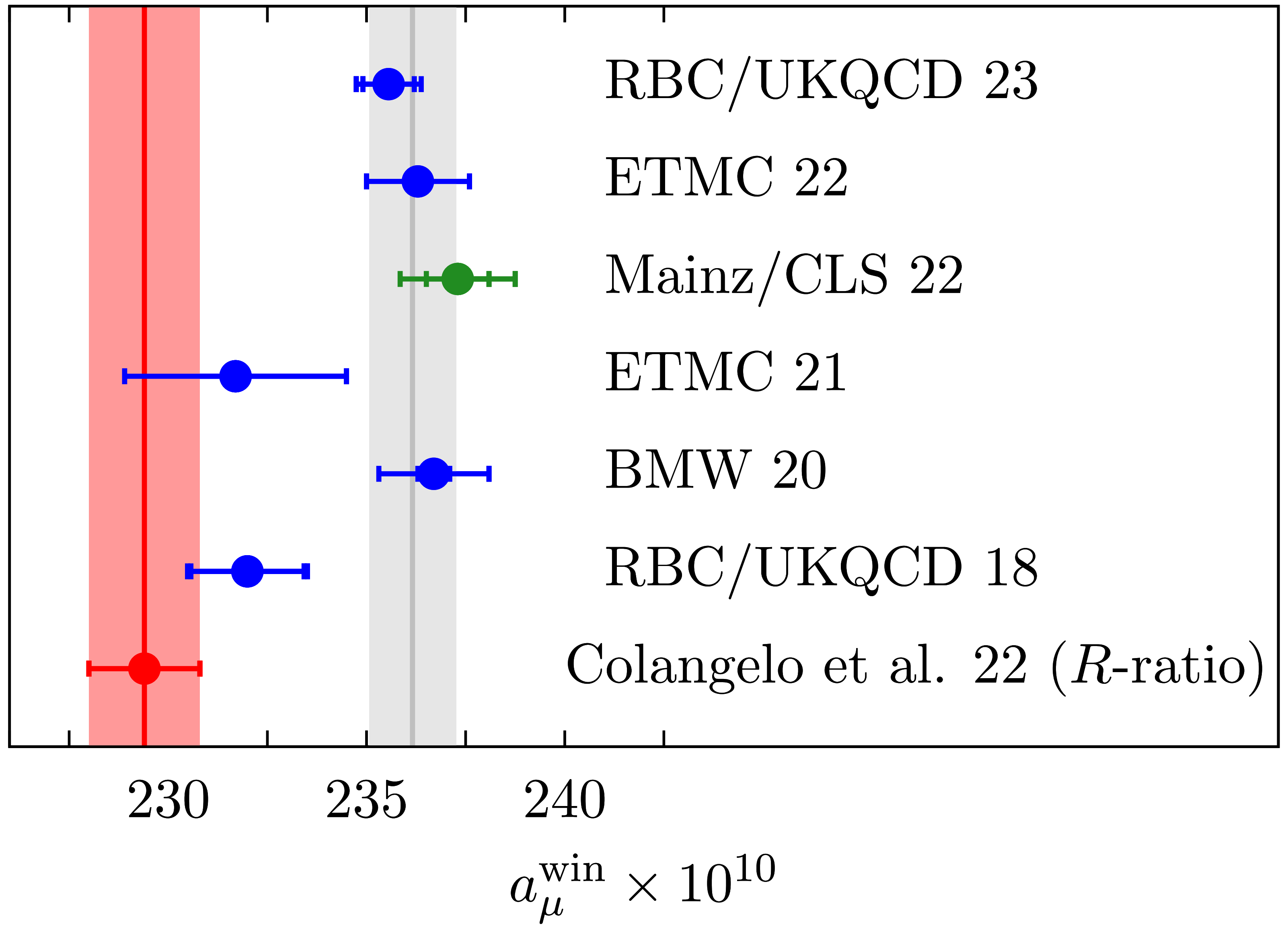

$$\Rightarrow \quad a_\mu^{\rm win}\Big|_{\rm Lat-av.} - a_\mu^{\rm win}\Big|_{R-\rm ratio} = (6.8 \pm 1.8) \cdot 10^{-10} \quad [3.8\,\sigma]$$

- Confirmed tension between lattice QCD and e^+e^- data (prior to 2023) for sub-contribution to HVP
- Subtract R-ratio prediction for $a_\mu^{\rm win}$ from White Paper estimate and replace by lattice average:

$$a_\mu^{\rm exp} - a_\mu^{\rm SM}\Big|_{\rm Lat-av.}^{\rm win} = (18.3 \pm 5.9) \cdot 10^{-10} \quad [3.1\,\sigma]$$

Intermediate window observable: Comparison with R-ratio

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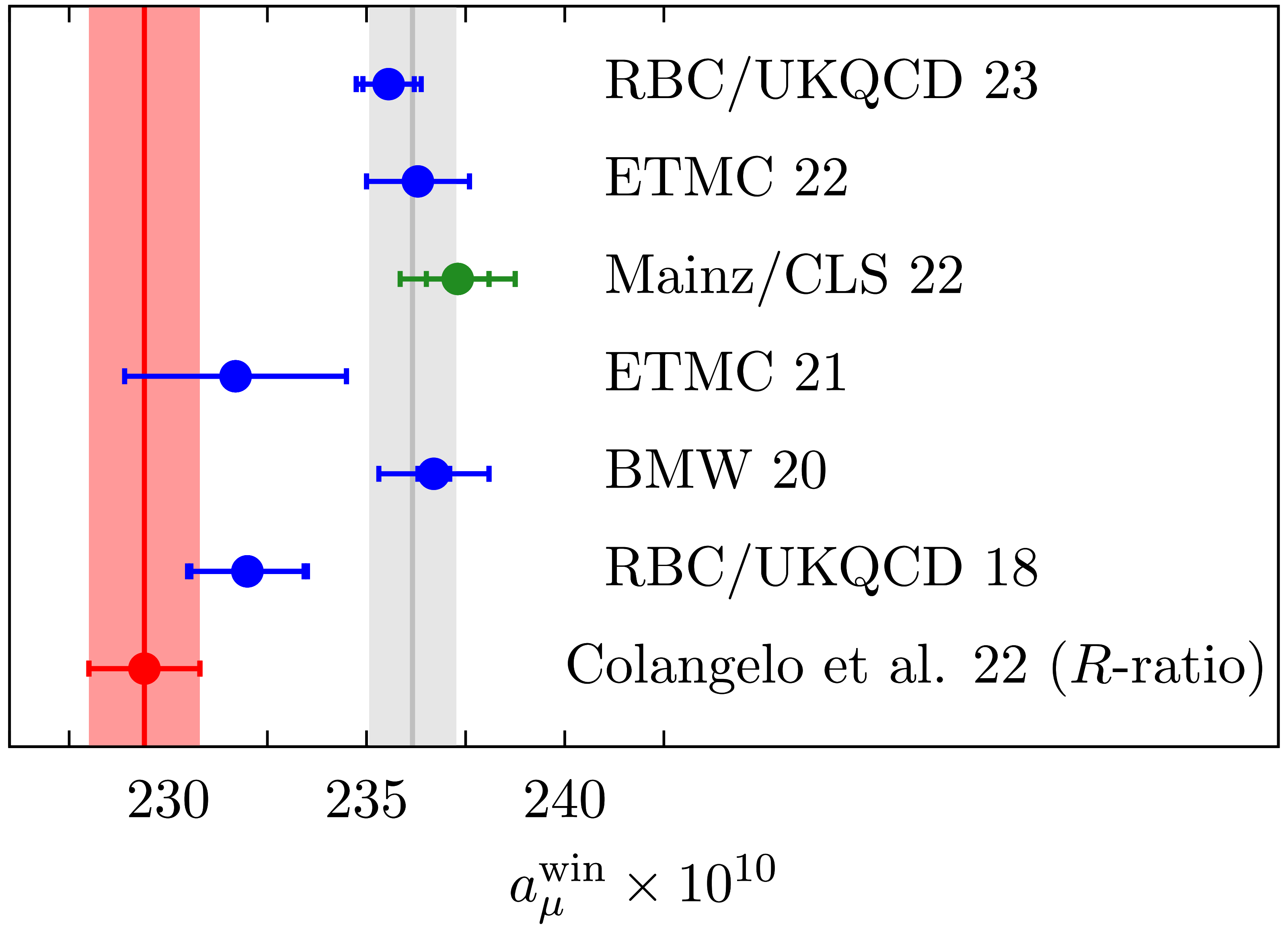


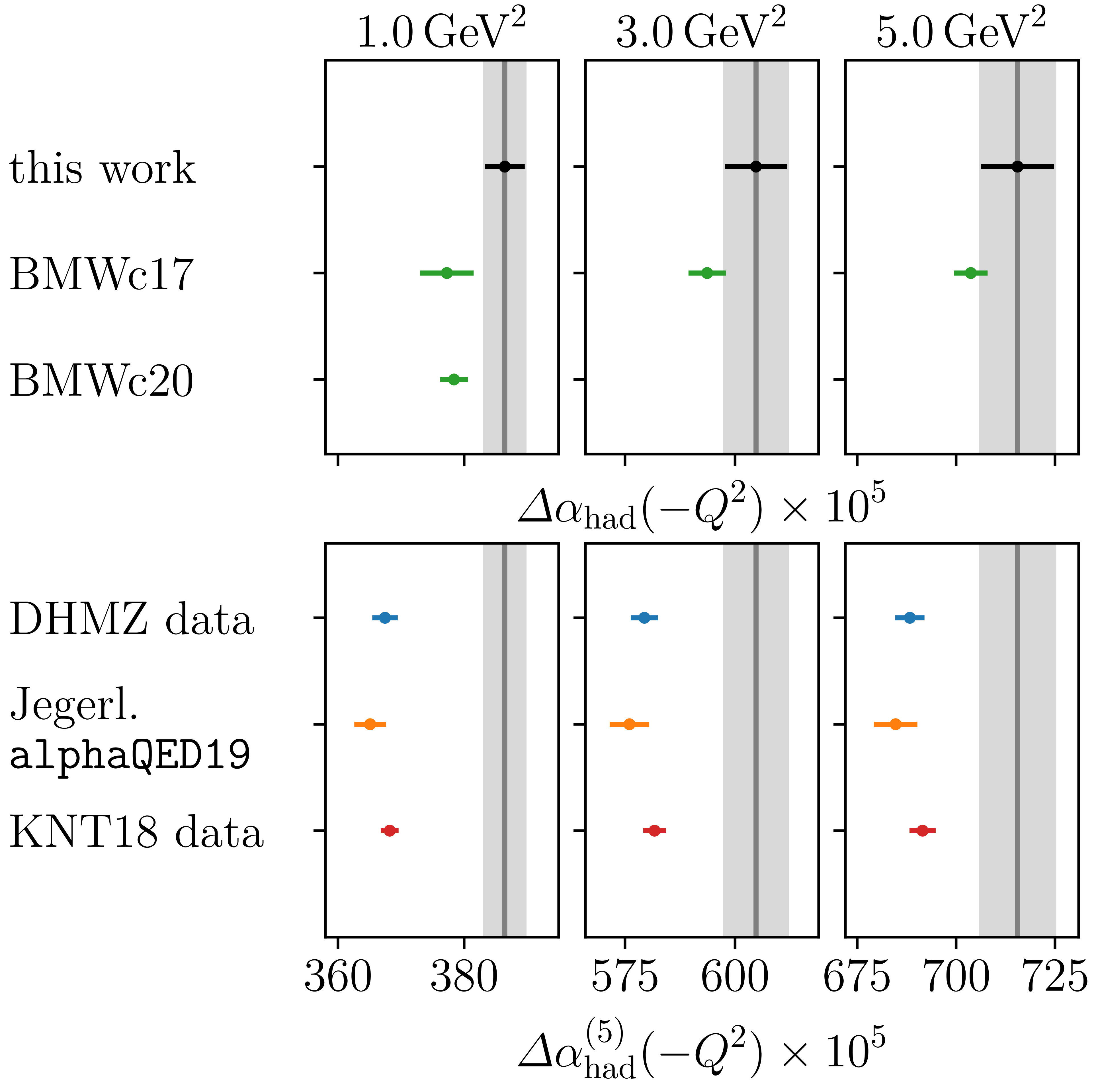
$$\Rightarrow \quad a_\mu^{\rm win}\Big|_{\rm Lat-av.} - a_\mu^{\rm win}\Big|_{R-{\rm ratio}} = (6.8 \pm 1.8) \cdot 10^{-10} \quad [3.8\,\sigma]$$

- Confirmed tension between lattice QCD and e^+e^- data (prior to 2023) for sub-contribution to HVP
- Two-pion contribution less dominant for intermediate window observable

$$\sqrt{s} = 600 - 900\,{\rm MeV}: \qquad \frac{\text{“}R(s)^{\rm lat}\text{”}}{R(s)^{e^+e^-}} = 1 + \epsilon \quad \Rightarrow \quad \frac{(a_\mu^{\rm hvp})^{\rm lat}}{(a_\mu^{\rm hvp})^{e^+e^-}} \approx \frac{(a_\mu^{\rm win})^{\rm lat}}{(a_\mu^{\rm win})^{e^+e^-}} = 1 + 0.6\epsilon$$

[Mainz/CLS, Cè et al., et al., PRD 106 (2022) 114502]

Relation to the hadronic running of electromagnetic coupling



Dispersion integral: $$\Delta\alpha^{(5)}_{\rm had}(q^2) = -\frac{\alpha\, q^2}{3\pi}\,\fint_{m^2_{\pi^0}}^{\infty} ds\,\frac{R(s)}{s(s-q^2)}$$

Lattice QCD:

$$\Delta\alpha_{\rm had}(-Q^2) = \frac{\alpha}{\pi}\frac{1}{Q^2}\int_0^\infty dt\, G(t)\left[Q^2t^2 - 4\sin^2\left(\tfrac{1}{2}Q^2t^2\right)\right]$$

- Direct lattice calculation of $\Delta\alpha(-Q^2)$ on the same gauge ensembles used in Mainz/CLS 22

  [Cè et al., JHEP 08 (2022) 220, arXiv:2203.08676]

- Tension of $\sim 3\sigma$ observed with data-driven evaluation of $\Delta\alpha_{\rm had}(-Q^2)$ for $Q^2 \gtrsim 3\,{\rm GeV}^2$

  $\rightarrow$ consistent with tension for window observable

Comparison with perturbative Adler function

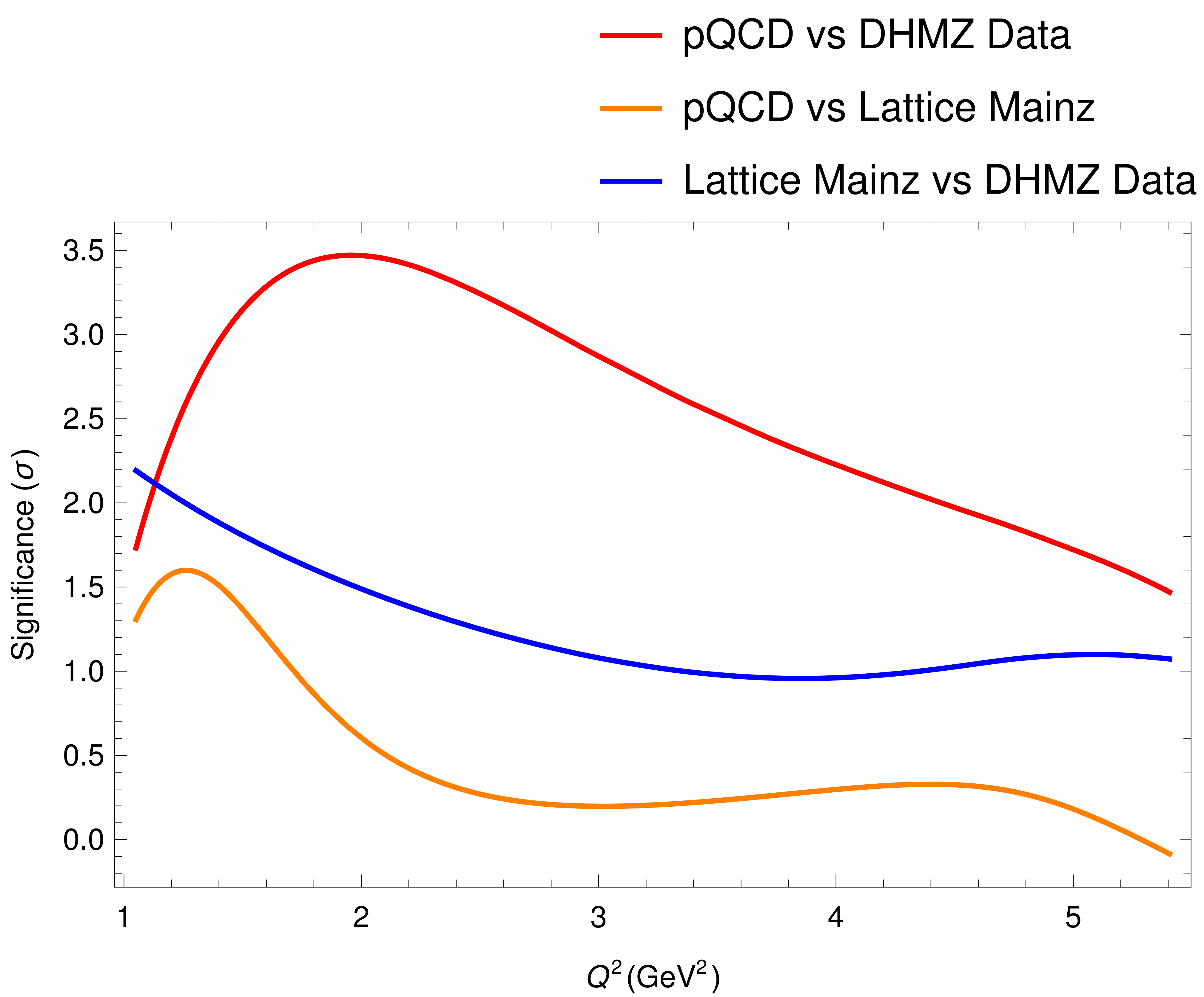
Adler function:

$$D(-s) = \frac{3\pi}{\alpha}\, s\, \frac{d}{ds}\, \Delta\alpha_{\text{had}}(s)$$

- Known in massive QCD perturbation theory at four loops
- Data-driven evaluation of $D(Q^2)$ via R-ratio:

$$D(Q^2) = Q^2 \int_{m_{\pi^0}^2}^{\infty} ds\, \frac{R(s)}{(s+Q^2)^2}$$

- Determine $D(Q^2)$ from lattice calculation of $\Delta\alpha_{\text{had}}(Q^2)$



Good agreement between perturbative and lattice QCD for $Q^2 \gtrsim 2\,\text{GeV}^2$

Slight tension of $1\text{–}2\sigma$ between data-driven evaluation and QCD

[Davier, Díaz-Calderón, Malaescu, Pich, Rodríguez-Sánchez, Zhang, JHEP 04 (2023) 067, arXiv:2302.01359]

Evaluation of $\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$ and comparison with EW precision data

Adler function approach, aka. "Euclidean split technique"

$$\Delta\alpha_{\rm had}^{(5)}(M_Z^2) = \Delta\alpha_{\rm had}^{(5)}(-Q_0^2) \quad \leftarrow \text{ lattice QCD}$$

$$+[\Delta\alpha_{\rm had}^{(5)}(-M_Z^2) - \Delta\alpha_{\rm had}^{(5)}(-Q_0^2)] \quad \leftarrow \text{ perturbative Adler function}$$

$$+[\Delta\alpha_{\rm had}^{(5)}(M_Z^2) - \Delta\alpha_{\rm had}^{(5)}(-M_Z^2)] \quad \leftarrow \text{ pQCD}$$

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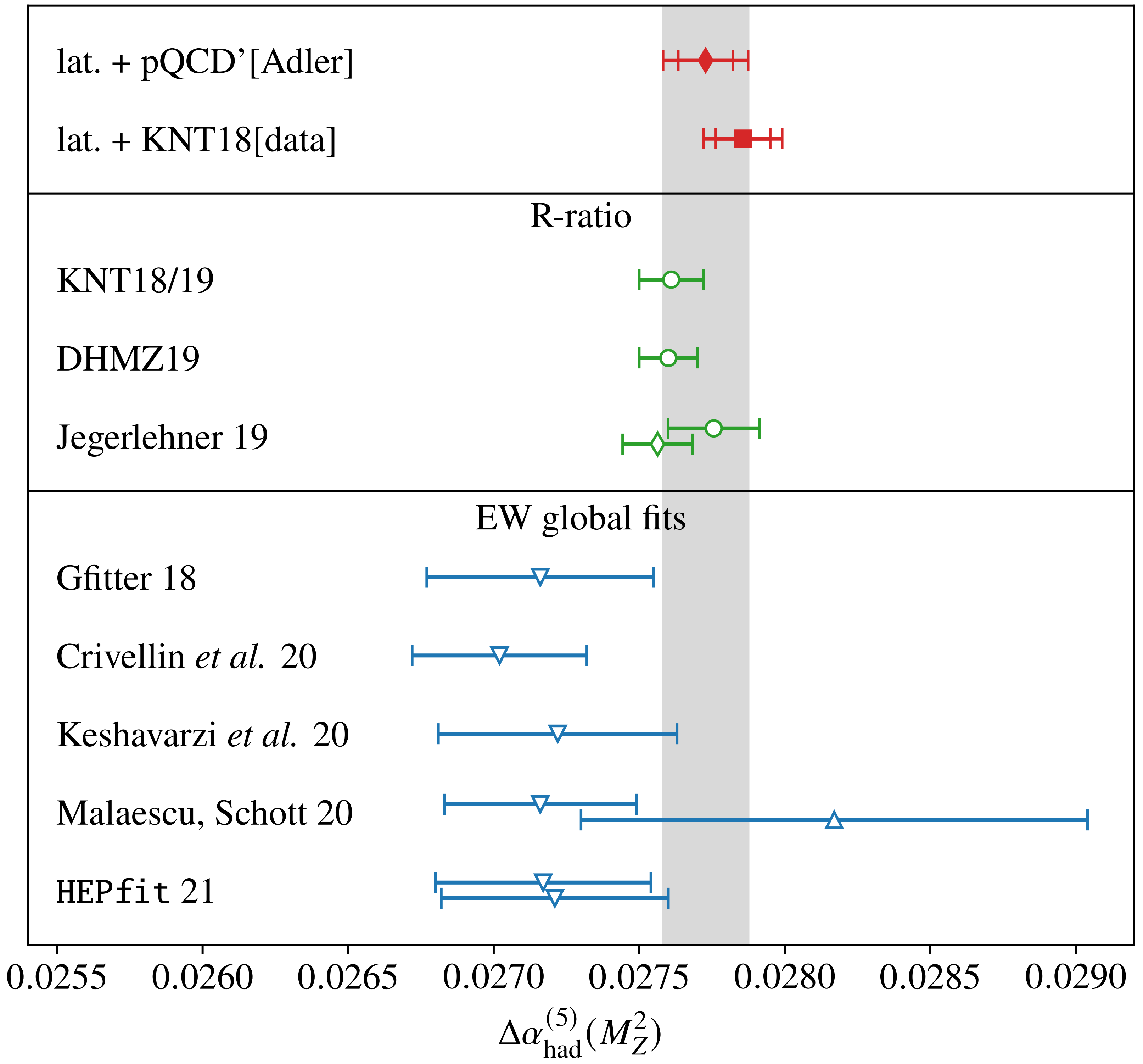
$$+[\Delta\alpha_{\rm had}^{(5)}(-M_Z^2) - \Delta\alpha_{\rm had}^{(5)}(-Q_0^2)]$$

$$+[\Delta\alpha_{\rm had}^{(5)}(M_Z^2) - \Delta\alpha_{\rm had}^{(5)}(-M_Z^2)]$$

$$\Rightarrow \quad \Delta\alpha_{\rm had}^{(5)}(M_Z^2) = 0.027\,73(9)_{\rm lat}(2)_{\rm btm}(12)_{\rm pQCD}$$

[Cè et al., JHEP 08 (2022) 220, arXiv:2203.08676]



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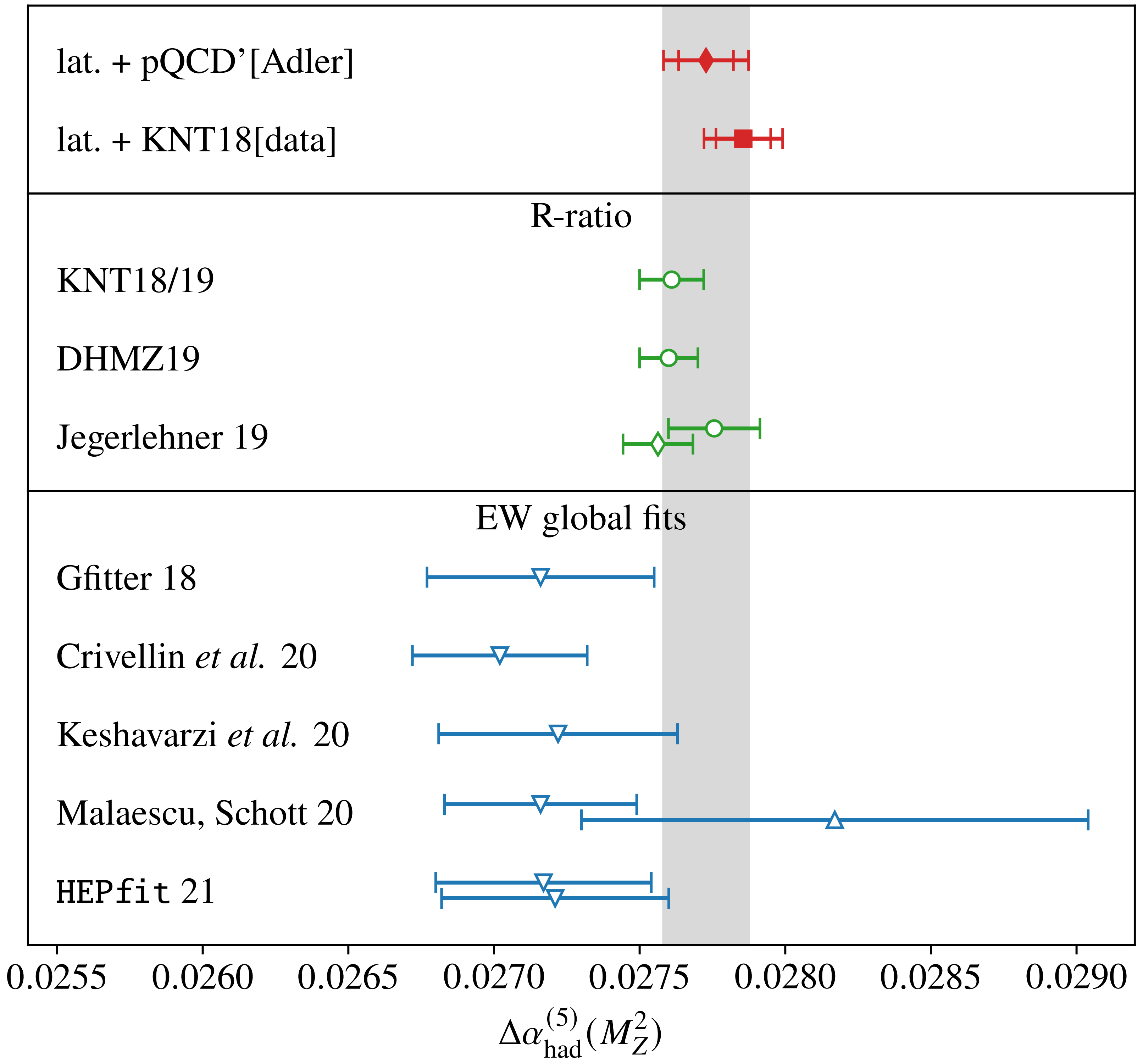
$$+[\Delta\alpha_{\rm had}^{(5)}(-M_Z^2) - \Delta\alpha_{\rm had}^{(5)}(-Q_0^2)]$$
$$+[\Delta\alpha_{\rm had}^{(5)}(M_Z^2) - \Delta\alpha_{\rm had}^{(5)}(-M_Z^2)]$$

$$\Rightarrow \quad \Delta\alpha_{\rm had}^{(5)}(M_Z^2) = 0.027\,73(9)_{\rm lat}(2)_{\rm btm}(12)_{\rm pQCD}$$

[Cè et al., JHEP 08 (2022) 220, arXiv:2203.08676]

- Agreement between lattice QCD and R-ratio estimate
- Contradiction with tension observed at low energies?



Evaluation of $\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$ and comparison with EW precision data

Adler function approach, aka. "Euclidean split technique"

$$\Delta\alpha_{\rm had}^{(5)}(M_Z^2) = \Delta\alpha_{\rm had}^{(5)}(-Q_0^2)$$
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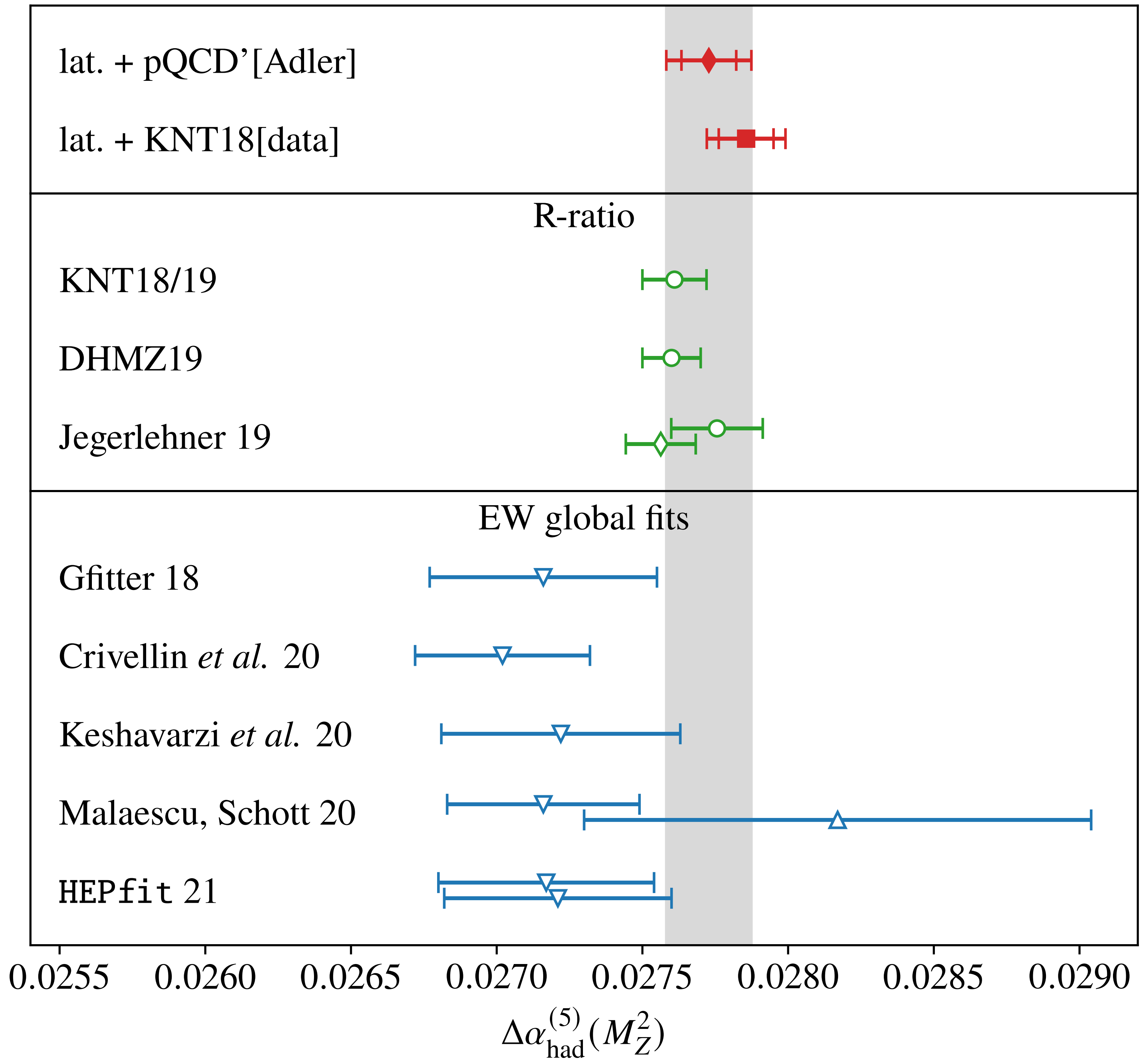
$$\Rightarrow \quad \Delta\alpha_{\rm had}^{(5)}(M_Z^2) = 0.027\,73(9)_{\rm lat}(2)_{\rm btm}(12)_{\rm pQCD}$$

[Cè et al., JHEP 08 (2022) 220, arXiv:2203.08676]



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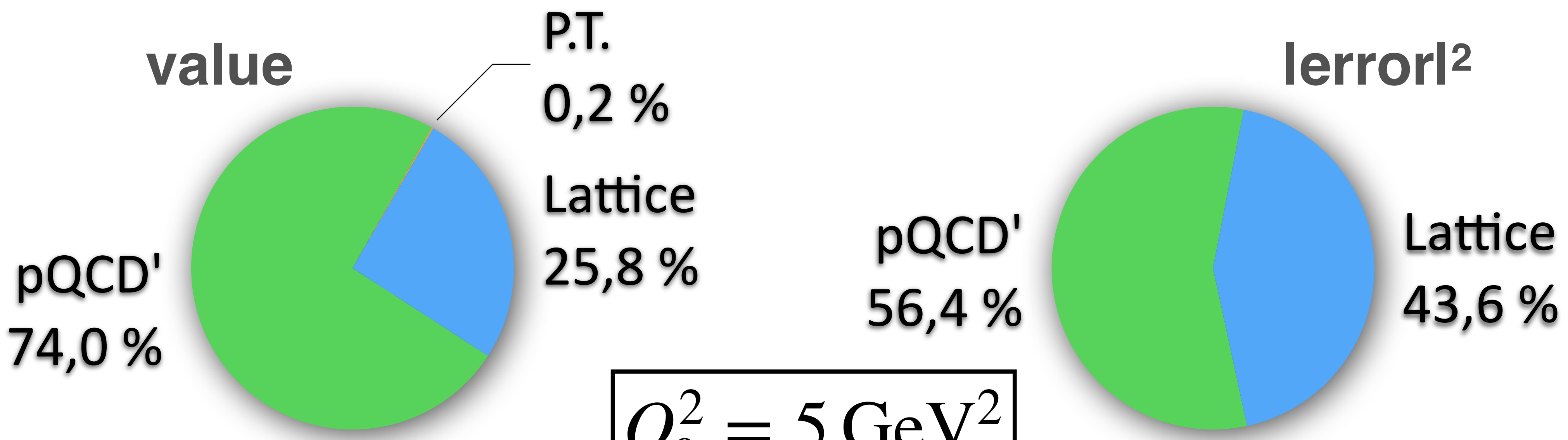
  Not in the correlated difference!



$Q_0^2 = 5\,\mathrm{GeV}^2$

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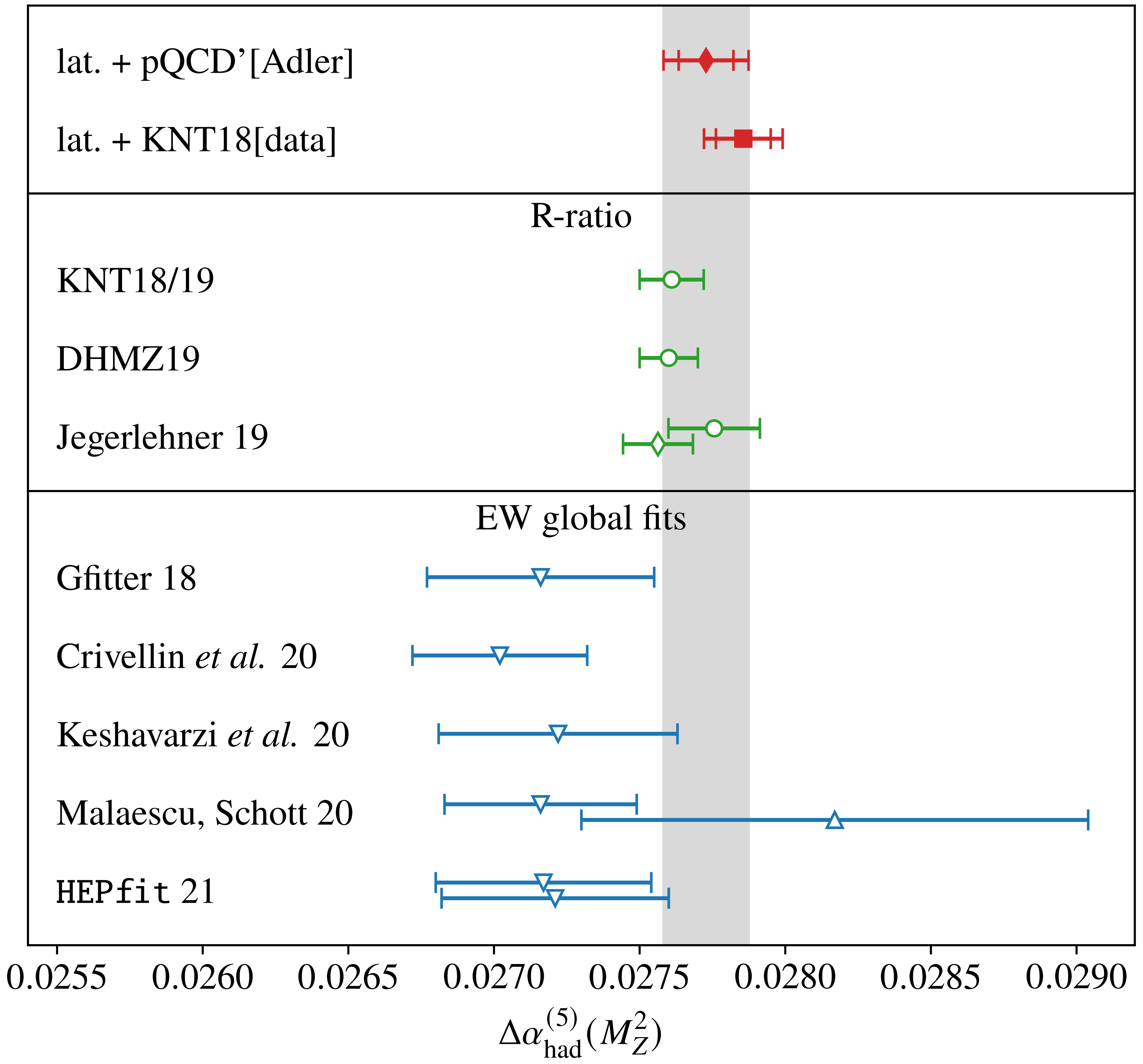

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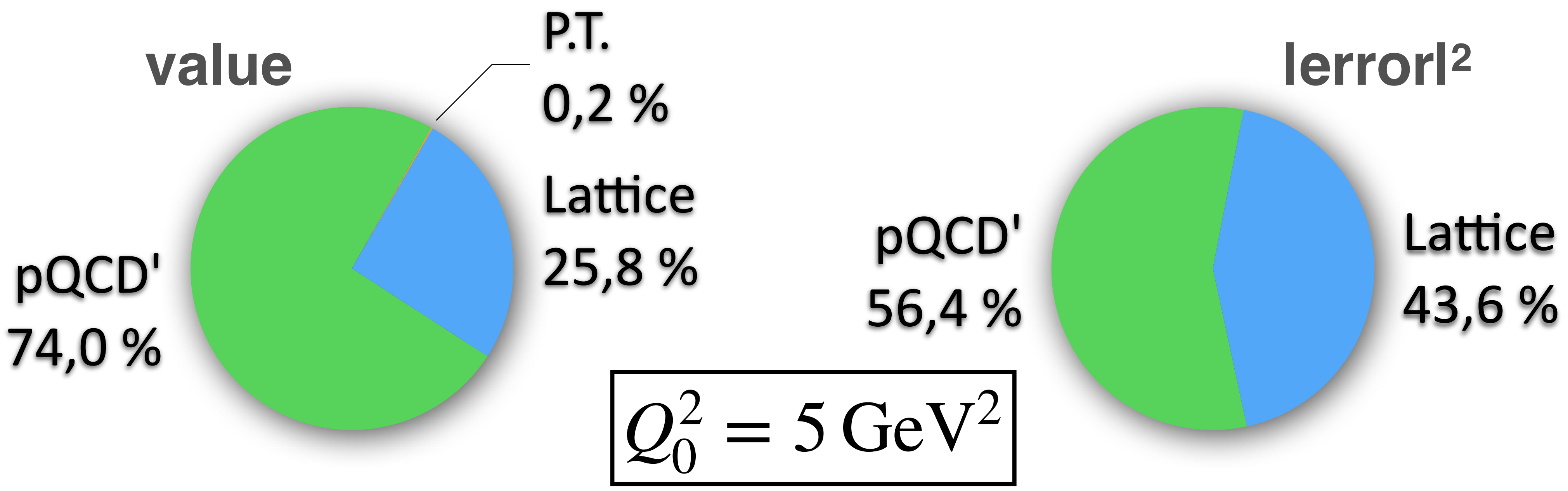
  Not in the correlated difference!

  No inconsistency with global electroweak fit



Summary — Conclusions — Outlook

Observed tensions:

- HVP: tension of $2.1\,\sigma$ between e^+e^- data* and single lattice calculation
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Larger value of HVP is not excluded by EW precision data

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*pre-2023

Summary — Conclusions — Outlook

Deviation of order $10 \cdot 10^{-10}$ between SM and experiment is a large one!

Precision must be increased further

Update of Fermilab E989 expected $\gtrsim$ June 2023

Muon g–2 Theory Initiative

Sixth Plenary Workshop

Bern, Switzerland, September 4–8, 2023

UNIVERSITÄT BERN
AEC ALBERT EINSTEIN CENTER FOR FUNDAMENTAL PHYSICS

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Spares

Evaluation of the dispersion integral

Many different groups and analyses (DHMZ, KNT, FJ, CHHKS, BHLS,…)

Disagreement for some exclusive channels

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	−0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	−0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	−0.12
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
K_SK_L	12.82(0.06)(0.18)(0.15)	13.04(19)	−0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	−0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
[1.8, 3.7] GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	−1.00
J/ψ, $\psi(2S)$	7.76(12)	7.84(19)	−0.08
[3.7, ∞) GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\text{HVP, LO}}$	$694.0(1.0)(3.5)(1.6)(0.1)_\psi(0.7)_{\text{DV+QCD}}$	692.8(2.4)	1.2

Merging procedure: average of individual results + theoretical constraints + conservative error estimate (reflecting tensions in the data, differences in procedures)

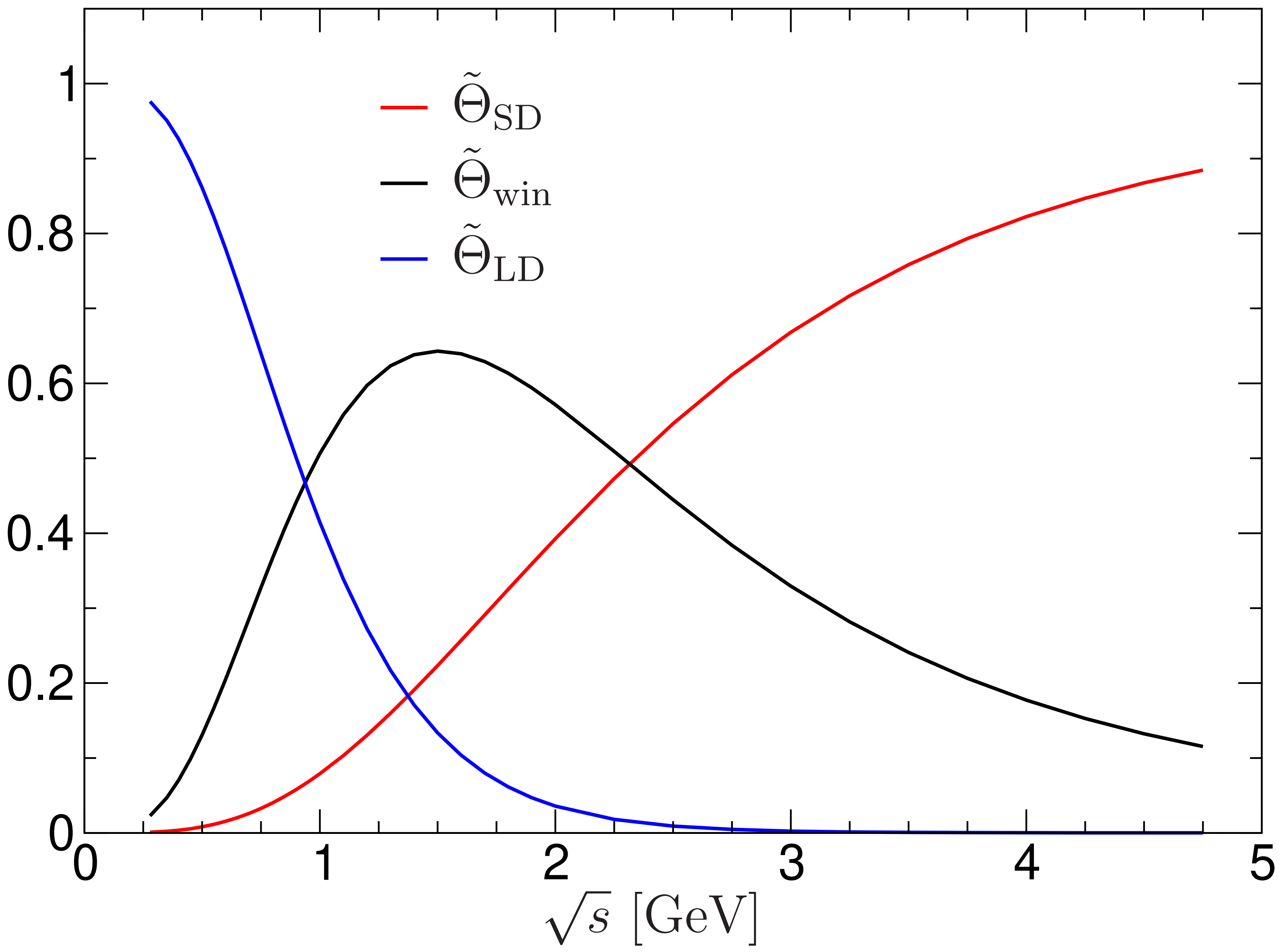
$$a_\mu^{\text{hvp, LO}} = 693.1(2.8)_{\text{exp}}(2.8)_{\text{syst}}(0.7)_{\text{DV+QCD}} \times 10^{-10} = 693.1(4.0) \times 10^{-10} \qquad [0.6\%]$$

Window observables: Comparison with R-ratio

Starting point: $$G(t) = \frac{1}{12\pi^2}\int_{m_{\pi^0}^2}^{\infty} d(\sqrt{s})\,R(s)\,s\,\mathrm{e}^{-\sqrt{s}t}$$ [RBC/UKQCD 2018]

Insert $G(t)$ into expression for time-momentum representation:

$$a_\mu^{\mathrm{hvp,\,ID}} = \left(\frac{\alpha}{\pi}\right)^2 \int_{m_{\pi^0}^2}^{\infty} d(\sqrt{s})\,R(s)\,\frac{1}{12\pi^2}\,s\int_0^\infty dt\,\tilde{K}(t)\,W^{\mathrm{ID}}(t;\,t_0,\,t_1)\,\mathrm{e}^{\sqrt{s}t}$$



Intermediate window from R-ratio following procedure for WP estimate:

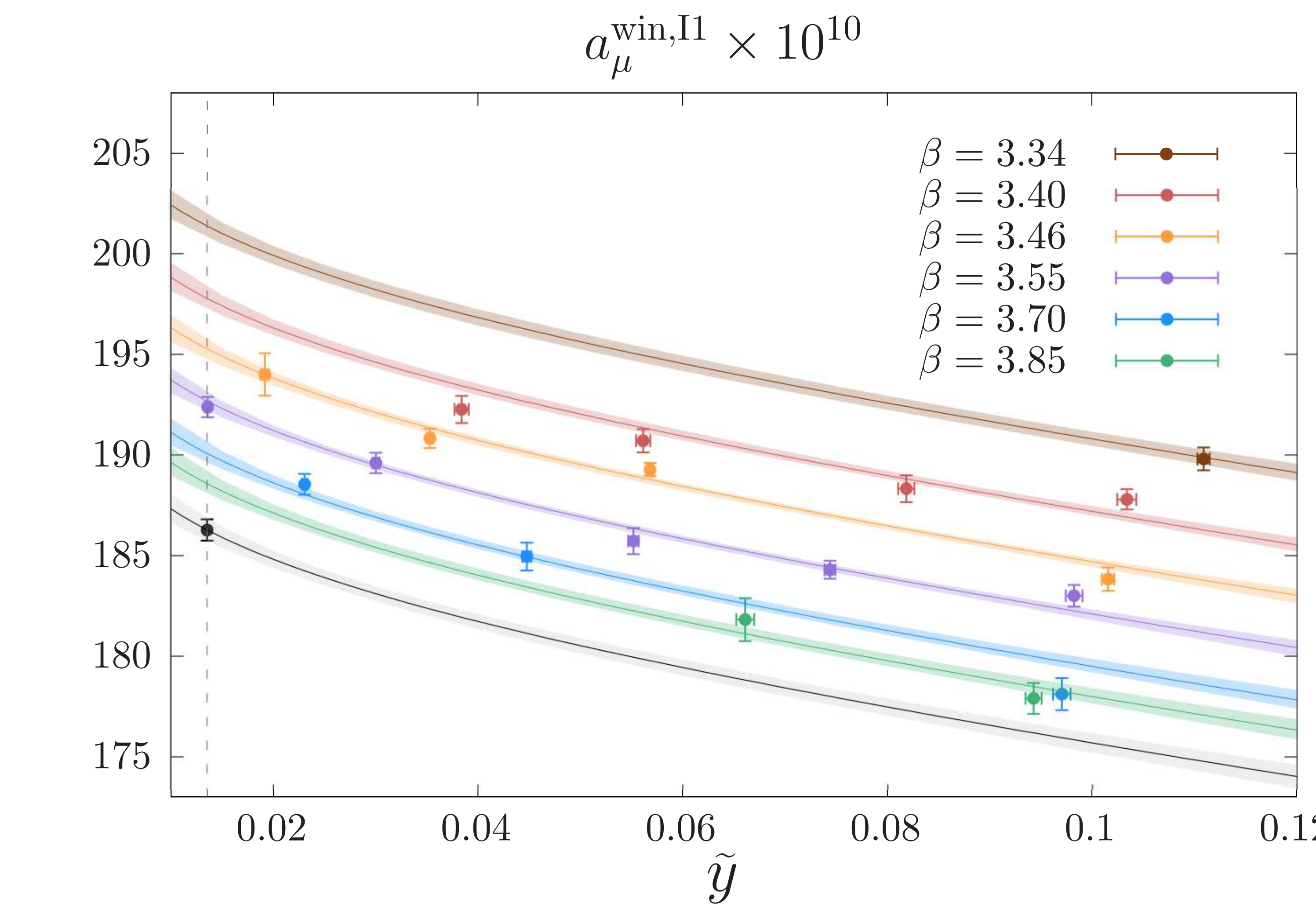
$$a_\mu^{\mathrm{hvp,\,ID}} \equiv a_\mu^{\mathrm{win}} = (229.4 \pm 1.4)\cdot 10^{-10}$$

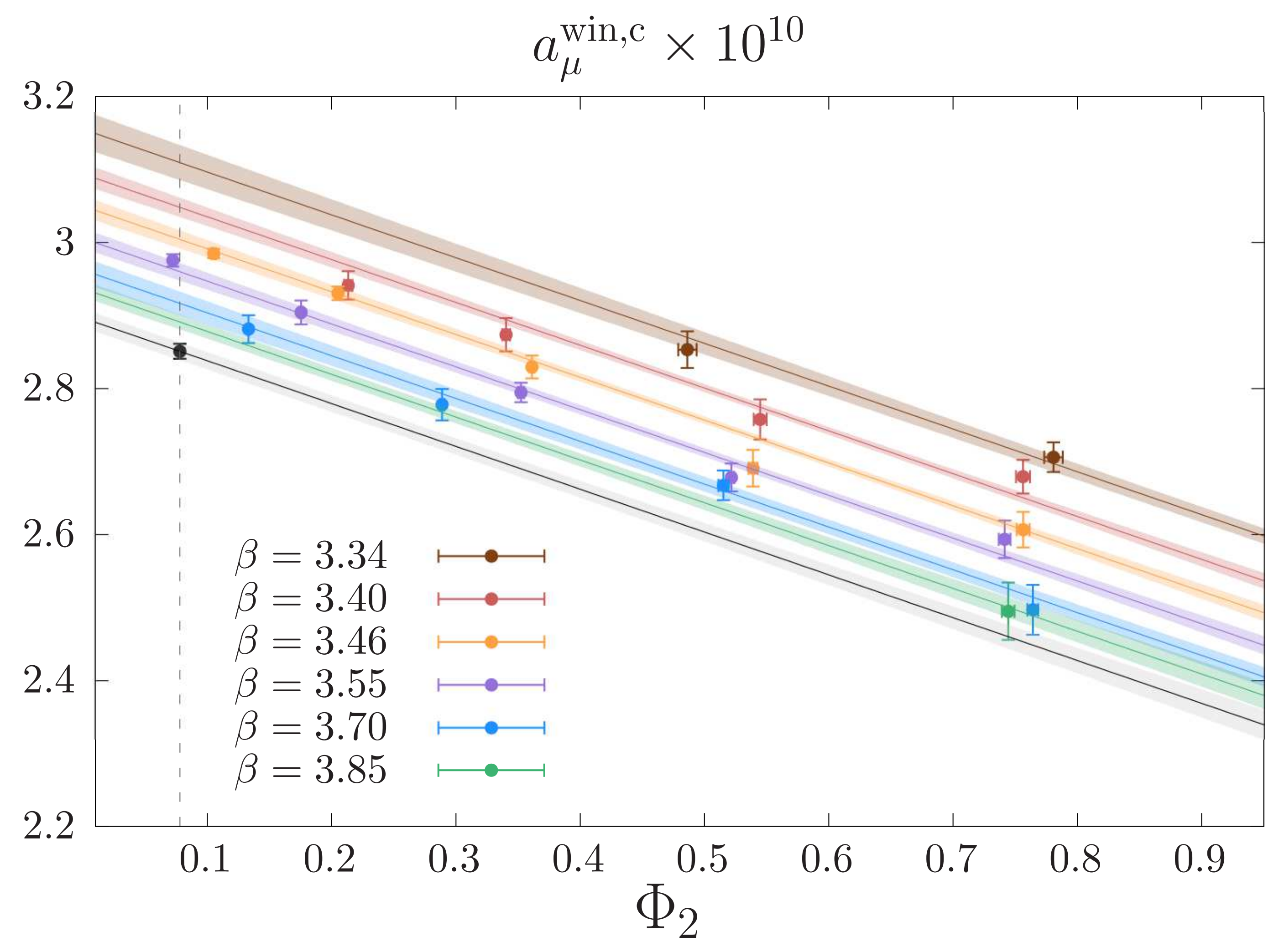
Finer decomposition allows for more detailed studies of energy dependence

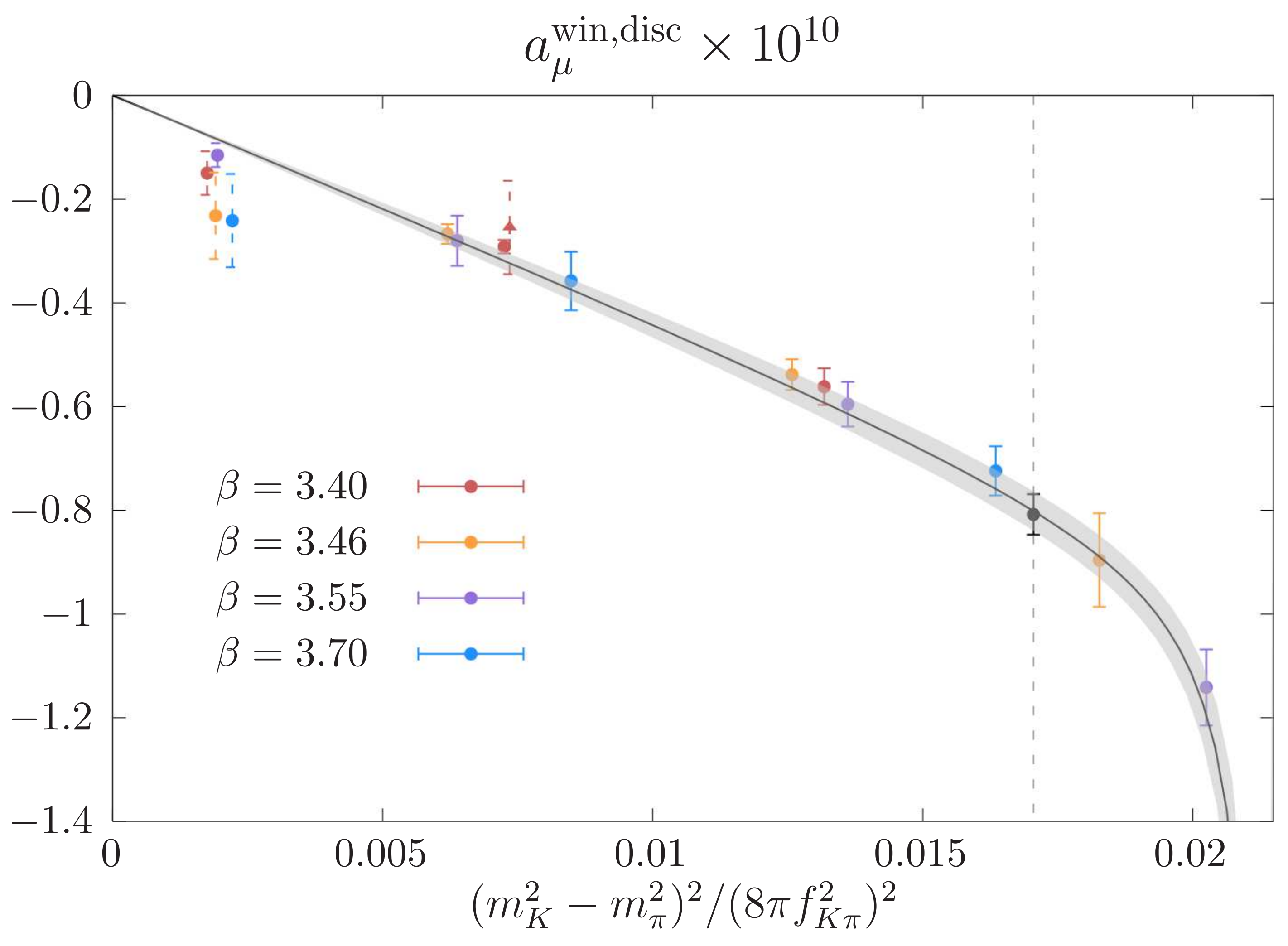
[Colangelo et al., Phys Lett B833 (2022) 137313]

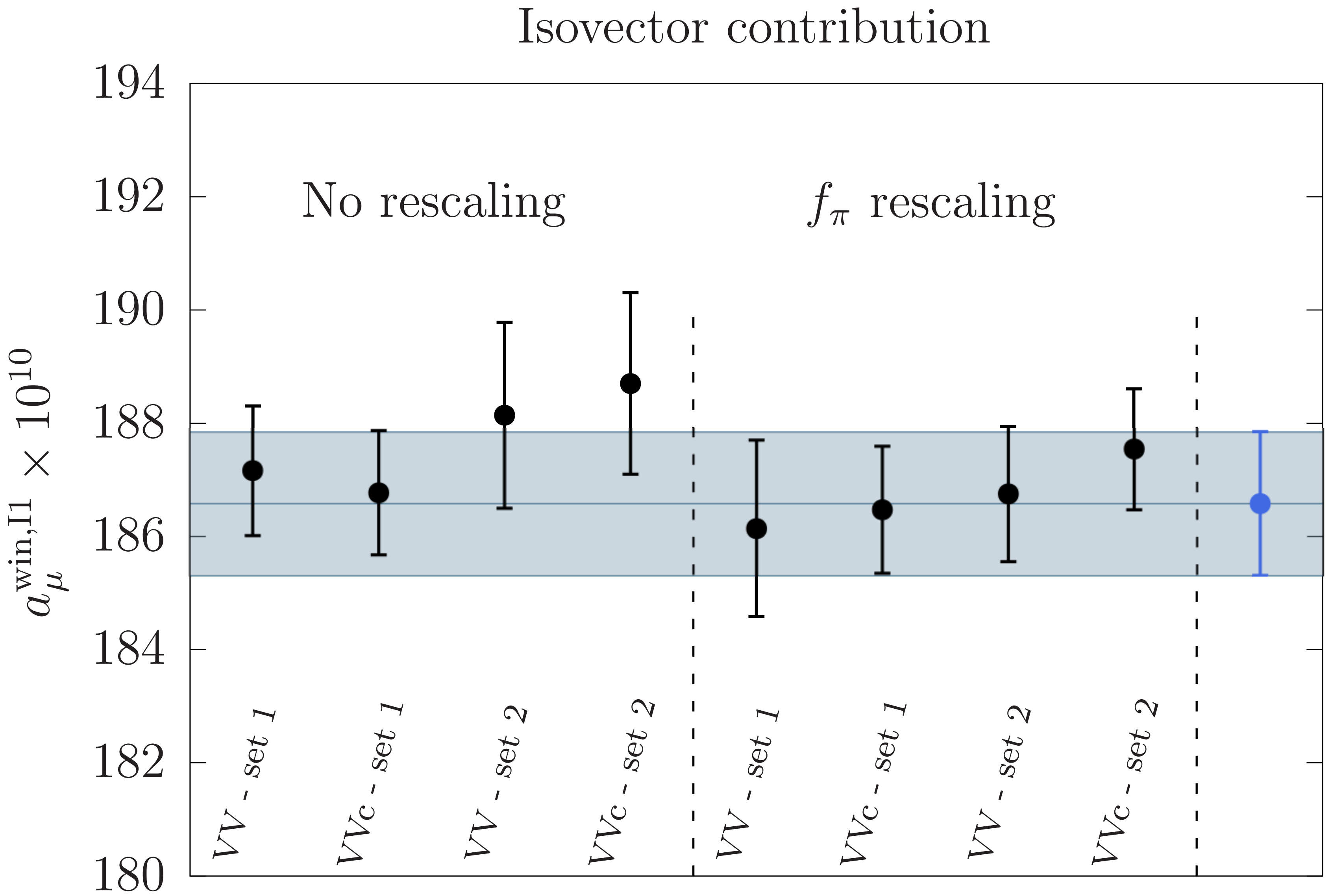
Mainz/CLS: Results at the physical point

[Cè et al., Phys Rev D106 (2022) 114502]









$$a_\mu^{\rm win,I1} = (186.30 \pm 0.75_{\rm stat} \pm 1.08_{\rm syst}) \times 10^{-10}\,,$$

$$a_\mu^{\rm win,I0} = a_\mu^{\rm win,I0,\not{c}} + a_\mu^{\rm win,c} = (50.30 \pm 0.23_{\rm stat} \pm 0.32_{\rm syst}) \times 10^{-10}\,,$$

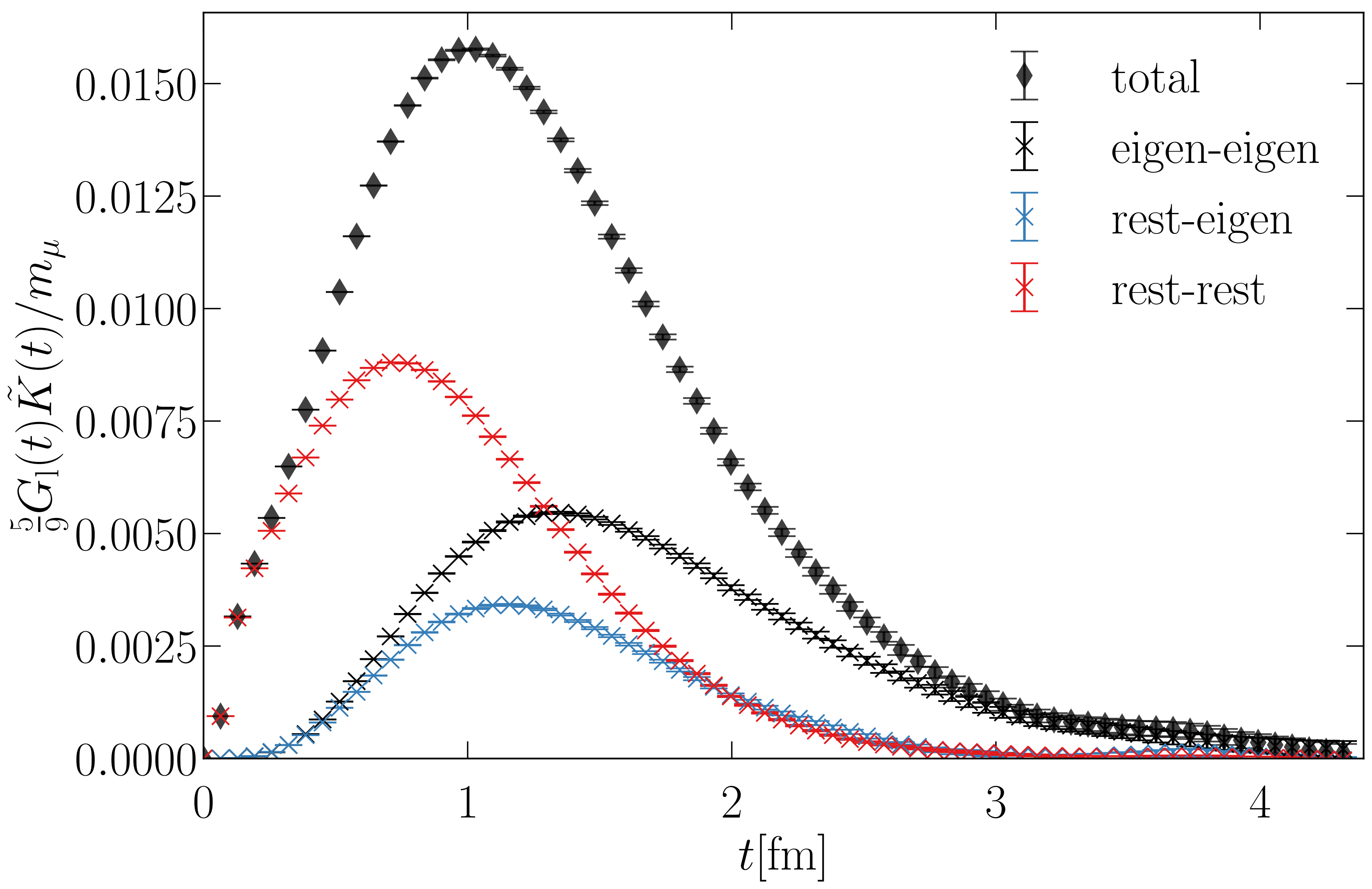
$$a_\mu^{\rm win,iso} = a_\mu^{\rm win,I1} + a_\mu^{\rm win,I0} = (236.60 \pm 0.79_{\rm stat} \pm 1.13_{\rm syst} \pm 0.05_{\rm Q}) \times 10^{-10}$$

Include shift of $+(0.70 \pm 0.47) \cdot 10^{-10}$ due to isospin-breaking:

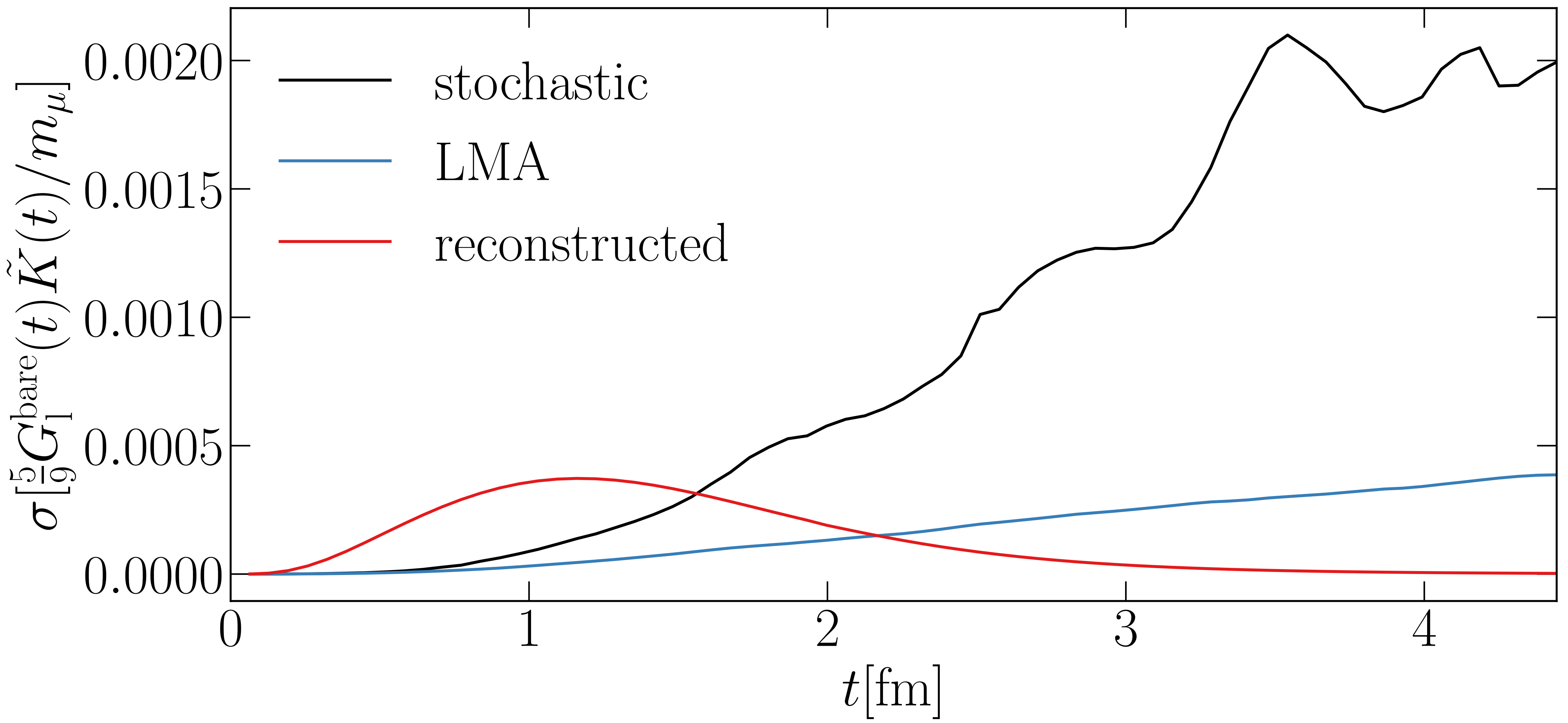
$$a_\mu^{\rm win} = (237.30 \pm 0.79_{\rm stat} \pm 1.13_{\rm syst} \pm 0.05_{\rm Q} \pm 0.47_{\rm IB}) \times 10^{-10}$$

Mainz/CLS: Noise reduction and the HVP contribution

Deflation techniques: Low-mode averaging



Low-mode averaging vs. spectral reconstruction



$m_\pi \approx 130\,\mathrm{MeV}$ at $a = 0.066\,\mathrm{fm}; \quad 96^3 \cdot 192$

Euclidean split technique and the Adler function

Adler function: $$D(-s) = \frac{3\pi}{\alpha}\, s\, \frac{d}{ds}\, \Delta\alpha_{\rm had}(s)$$

$D(Q^2)$ known in massive QCD perturbation theory at three loops

$$\left[\Delta\alpha_{\rm had}^{(5)}(-M_Z^2) - \Delta\alpha_{\rm had}^{(5)}(-Q_0^2)\right]_{\rm pQCD/Adler} = \frac{\alpha}{3\pi}\int_{Q_0^2}^{M_Z^2} \frac{dQ^2}{Q^2}\, D(Q^2)$$

Relation of $D(Q^2)$ and R-ratio: $$D(Q^2) = Q^2 \int_{m_{\pi^0}^2}^{\infty} ds\, \frac{R(s)}{(s+Q^2)^2}$$

Direct DR: $$\left[\Delta\alpha_{\rm had}^{(5)}(-M_Z^2) - \Delta\alpha_{\rm had}^{(5)}(-Q_0^2)\right]_{\rm DR} = \frac{\alpha(M_Z^2 - Q_0^2)}{3\pi}\int_{m_{\pi^0}^2}^{\infty} ds\, \frac{R(s)}{(s+Q_0^2)(s+M_Z^2)}$$

Perturbation theory: $$\left[\Delta\alpha_{\rm had}^{(5)}(M_Z^2) - \Delta\alpha_{\rm had}^{(5)}(-M_Z^2)\right] = 0.000\,045(2)$$ *[Jegerlehner, CERN Yellow Report, 2020]*

Euclidean split technique: relative contributions to $\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z^2)$