Inclusive Semi-Leptonic Decays

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P H
Particle Physics Phenomenology after the Higgs Discovery

Contents

Introduction

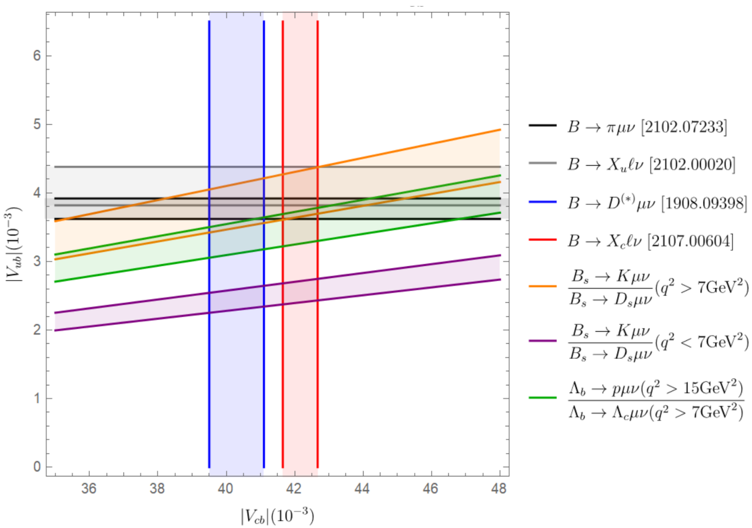
Flavour Physics has become “en vogue” due to the Flavour Anomalies:

- Lepton Universality Violation in rare B decays
- Anomalies in cc interactions in semi-tauonic decays
- Rates and angular distributions FCNC decays

... but there are also “old” anomalies

- Kaon CPV: ϵ'/ϵ
- CPV in Charm decays
- V_{xb} inclusive vs. exclusive

We should not be too disappointed after Dec. 20th,
there will always be some anomalies in flavour physics to be discussed.



Keri Vos, TRR 257 annual meeting

Inclusive V_{cb} : Heavy Quark Expansion

Heavy Quark Expansion = Operator Product Expansion
(Chay, Georgi, Bigi, Shifman, Uraltsev, Vainstain, Manohar. Wise, Neubert, M,...)

$$
\begin{aligned}
\Gamma &\propto \sum_X (2\pi)^4 \delta^4(P_B - P_X) |\langle X | \mathcal{H}_{eff} | B(v) \rangle|^2 \\
&= \int d^4x \, \langle B(v) | \mathcal{H}_{eff}(x) \mathcal{H}^\dagger_{eff}(0) | B(v) \rangle \\
&= 2 \, \mathrm{Im} \int d^4x \, \langle B(v) | T\{\mathcal{H}_{eff}(x) \mathcal{H}^\dagger_{eff}(0)\} | B(v) \rangle \\
&= 2 \, \mathrm{Im} \int d^4x \, e^{-im_b v\cdot x} \langle B(v) | T\{\widetilde{\mathcal{H}}_{eff}(x) \widetilde{\mathcal{H}}^\dagger_{eff}(0)\} | B(v) \rangle
\end{aligned}
$$

- Last step: $b(x) = b_v(x) \exp(-im_v vx)$,
  corresponding to $p_b = m_b v + k$
  Expansion in the residual momentum k

- Perform an “OPE”: m_b is much larger than any scale appearing in the matrix element

$$\int d^4x e^{-im_b vx} T\{\widetilde{\mathcal{H}}_{eff}(x)\widetilde{\mathcal{H}}_{eff}^\dagger(0)\} = \sum_{n=0}^{\infty} \left(\frac{1}{2m_Q}\right)^n C_{n+3}(\mu)\mathcal{O}_{n+3}$$

$\rightarrow$ The rate for $B \to X_c \ell \bar{\nu}_\ell$ can be written as

$$\Gamma = \Gamma_0 + \frac{1}{m_Q}\Gamma_1 + \frac{1}{m_Q^2}\Gamma_2 + \frac{1}{m_Q^3}\Gamma_3 + \cdots$$

- The Γ_i are power series in $\alpha_s(m_Q)$:
  $\rightarrow$ Perturbation theory!
- Works also for differential rates!

- Γ_0 is the decay of a free quark ("Parton Model")
- Γ_1 vanishes due to Heavy Quark Symmetries
- Γ_2 is expressed in terms of two parameters

$$2M_H\mu_\pi^2 = -\langle H(v)|\bar{Q}_v(iD)^2 Q_v|H(v)\rangle$$
$$2M_H\mu_G^2 = \langle H(v)|\bar{Q}_v\sigma_{\mu\nu}(iD^\mu)(iD^\nu)Q_v|H(v)\rangle$$

μ_π: Kinetic energy and μ_G: Chromomagnetic moment

- Γ_3 two more parameters

$$2M_H\rho_D^3 = -\langle H(v)|\bar{Q}_v(iD_\mu)(ivD)(iD^\mu)Q_v|H(v)\rangle$$
$$2M_H\rho_{LS}^3 = \langle H(v)|\bar{Q}_v\sigma_{\mu\nu}(iD^\mu)(ivD)(iD^\nu)Q_v|H(v)\rangle$$

ρ_D: Darwin Term and ρ_{LS}: Spin-Orbit Term

- Γ_4 and Γ_5 have been computed Bigi, Uraltsev, Turczyk, TM, ...

Structure of the HQE

- Structure of the expansion (@ tree):

$$
\begin{aligned}
d\Gamma = & \; d\Gamma_0 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^2 d\Gamma_2 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 d\Gamma_3 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^4 d\Gamma_4 \\
& + d\Gamma_5 \left(a_0 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^5 + a_2 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 \left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^2 \right) \\
& + \ldots + d\Gamma_7 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 \left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^4
\end{aligned}
$$

- $d\Gamma_3 \propto \ln(m_c^2/m_b^2)$
- Power counting $m_c^2 \sim \Lambda_{\rm QCD} m_b$

Determination of the HQE Parameters

- m_b, m_c, μ_π, μ_G, ρ_D etc. are determined from data
- Spectra: Hadronic invariant mass, Charegd lepton energy, Hadronic Energy
- However: There are corners in Phase Space where the OPE breaks down

Moments of the spectra can be computed in the HQE

m_b^{kin}	$\overline{m}_c(3\,\mathrm{GeV})$	μ_π^2	ρ_D^3	μ_G^2	ρ_{LS}^3	$\mathrm{BR}_{c\ell\nu}$	$10^3\lvert V_{cb}\rvert$
4.553	0.987	0.465	0.170	0.332	-0.150	10.65	42.21
0.020	0.013	0.068	0.038	0.062	0.096	0.16	0.78

WITHOUT MASS CONSTRAINTS

$$m_b^{kin}(1\mathrm{GeV}) - 0.85\,\overline{m}_c(3\mathrm{GeV}) = 3.714 \pm 0.018\,\mathrm{G}$$

Alberti, Healey, Nandi, Gambino arXiv 1411.6560, presented at MITP *Challenges in semileptonic B decays* in 2015

- Includes HQE parameters up to $1/m^3$ and full α_s/m_Q^2

QCD Corrections

For a massless final-state quark:

$$\Gamma_0 = \frac{G_F^2 |V_{cb}|^2}{192\pi^3} m_b^5 \left(1 + \sum_{k=1}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^k g_k \right) = \frac{G_F^2 |V_{cb}|^2}{192\pi^3} m_b^5 \left(1 + \frac{\alpha_s}{\pi} g_1 + \cdots \right)$$

What is the mass m_b?

- Start with the pole mass $m_b = m_b^{\mathrm{pole}}$
- This yields a large g_1
- In fact, this leads in general to a bad behavior of the perturbative series
- Perturbative series is “asymptotic”: Looks like a convergent series, but at some order k

$$g_k \sim k!$$

Renormalon Problem (of the Pole mass)

- Problem for a precision calculation!
- Switch to a "proper mass" m_b^{kin}:
  This has a perturbative relation to the pole mass

$$m_b^{\mathrm{kin}}(\mu) = m_b^{\mathrm{pole}}\left(1 + \sum_{k=1}^{\infty}\left(\frac{\alpha_s}{\pi}\right)^k m_k(\mu)\right) = m_b^{\mathrm{pole}}\left(1 + \frac{\alpha_s}{\pi} m_1(\mu) + \cdots\right)$$

- Insert this

$$\Gamma_0 = \frac{G_F^2 |V_{cb}|^2}{192\pi^3} (m_b^{\mathrm{kin}}(\mu))^5 \left(1 + \frac{\alpha_s}{\pi}(g_1 - m_1(\mu)) + \cdots\right)$$

- m_b^{kin} is much better known as the pole mass
- The perturbative series converges better: $|g_1 - m_1| \ll g_1$

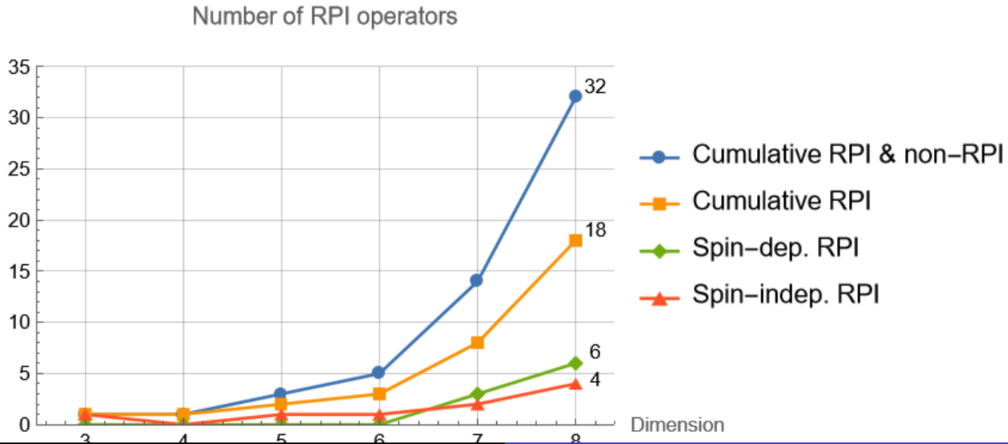
Present state of the $b \to c$ semileptonic Calculations

- Tree level terms up to and including $1/m_b^5$ known
  Bigi, Zwicky, Uraltsev, Turczyk, Vos, Milutin, ThM, ...
- $\mathcal{O}(\alpha_s)$ and full $\mathcal{O}(\alpha_s^2)$ for the partonic rate and spectra are known
  Melnikov, Czarnecki, Pak
- $\mathcal{O}(\alpha_s^3)$ to the partonic rate known (Fael, Schonwald, Steinhauser: 2011.13654)
- $\mathcal{O}(\alpha_s)$ for $1/m_b^2$ is known for rates and spectra
  Becher, Boos, Lunghi, Gambino, Pivovarov, Rosenthal, Alberti
- $\mathcal{O}(\alpha_s)$ for $1/m_b^3$ is known for rates and spectra Pivovarov, Moreno, ThM
- In the pipeline:
  - Estimation of Duality Violation

$\boxed{\text{We are moving towards a TH-uncertainty of 1\% in } V_{cb,incl}\text{!}}$

Recent Development: Reducing the Number of HQE Parameters

New Idea based on an old observation: Reparametrization Invariance Problem:
Number of HQE parameters in higher orders!



Reparametrization Invariance: (Dugan, Golden, Grinstein, Chen, Luke, Manohar...)

$$R(q) = \int d^4x\, e^{iqx}\, T[\bar{Q}(x)\Gamma q(x)\ \bar{q}(0)\Gamma^\dagger Q(0)]$$

and replace $Q(x) = \exp(-im(v\cdot x))Q_v(x)$

$$R(S) = \int d^4x\, e^{-iSx}\, T[\bar{Q}_v(x)\Gamma q(x)\ \bar{q}(0)\Gamma^\dagger Q_v(0)]$$

with $S = mv - q$. These expressions are independent of v! Perform the HQE

$$R(S) = \sum_{n=0}^{\infty} \left[C^{(n)}_{\mu_1\cdots\mu_n}(S)\right]_{\alpha\beta} \bar{Q}_{v,\alpha}(iD_{\mu_1}\cdots iD_{\mu_n})Q_{v,\beta}$$

All this is still invariant under reparametrization of v: (as long as the sum is not truncated)

$$\delta_{\rm RP}\, v_\mu = \delta v_\mu \quad \text{with} \quad v\cdot\delta v = 0$$

$$\delta_{\rm RP}\, iD_\mu = -m\delta v_\mu$$

$$\delta_{\rm RP}\, Q_v(x) = im(x\cdot\delta v)Q_v(x) \quad \text{in particular} \quad \delta_{\rm RP}\, Q_v(0) = 0\ .$$

The RP connects different orders in $1/m$, which yields the master relation between the coefficients $n = 0, 1, 2, \ldots$

$$\boxed{\delta_{\mathrm{RP}} C^{(n)}_{\mu_1\cdots\mu_n} = m\,\delta v^\alpha \left(C^{(n+1)}_{\alpha\mu_1\cdots\mu_n} + C^{(n+1)}_{\mu_1\alpha\mu_2\cdots\mu_n} + \cdots + C^{(n+1)}_{\mu_1\cdots\mu_n\alpha} \right)}$$

Use these coefficients, integrate over phase space,
get a total rate $\Gamma = \mathrm{Im}\langle B|R|B\rangle = \mathrm{Im}\langle R\rangle$
The coeffcients of the OPE will depend only on v

$$R = \sum_{n=0}^{\infty} c^{(n)}_{\mu_1\cdots\mu_n}(v) \otimes \bar{Q}_v (iD_{\mu_1} \cdots iD_{\mu_n}) Q_v$$

and satisfy the master relation between different orders in the HQE

Making use of RPI ...

- RPI is a consequence of Lorentz invariance of QCD
- RPI is an exact symmetry:
  the relations must hold to all order in α_s
- Resummation of towers of terms from different orders
- For Lorentz invariant observables:
  - The master relations are identical for all observables
  - “Rigid” relations between coefficients
  - Reduction of HQE parameters due to RPI

HQE parameters (for the total rate) to $O(1/m^4)$

$$2m_H\mu_3 = \langle H(p)|\bar{Q}_v Q_v|H(p)\rangle = \langle \bar{Q}_v Q_v\rangle \qquad \mu_3 = 1 + \frac{\mu_\pi^2 - \mu_G^2}{2m_Q^2}$$

$$2m_H\mu_G = \langle \bar{Q}_v (iD^\mu)(iD^\nu)(-i\sigma_{\mu\nu}) Q_v\rangle$$

$$2m_H\rho_D = \langle \bar{Q}_v \left[(iD^\mu), \left[\left((ivD) + \frac{(iD)^2}{2m}\right), (iD_\mu)\right]\right] Q_v\rangle$$

$$2m_H r_G^4 = \langle \bar{Q}_v [(iD_\mu), (iD_\nu)][(iD^\mu), (iD^\nu)] Q_v\rangle \qquad \langle G^2\rangle$$

$$2m_H r_E^4 = \langle \bar{Q}_v [(ivD), (iD_\mu)][(ivD), (iD^\mu)] Q_v\rangle \qquad \langle \vec{E}^2\rangle$$

$$2m_H s_B^4 = \langle \bar{Q}_v [(iD_\mu), (iD_\alpha)][(iD^\mu), (iD_\beta)](-i\sigma^{\alpha\beta}) Q_v\rangle \qquad \langle (\vec{B}\times\vec{B})\cdot\vec{\sigma}\rangle$$

$$2m_H s_E^4 = \langle \bar{Q}_v [(ivD), (iD_\alpha)][(ivD), (iD_\beta)](-i\sigma^{\alpha\beta}) Q_v\rangle \qquad \langle (\vec{E}\times\vec{E})\cdot\vec{\sigma}\rangle$$

$$2m_H s_{qB}^4 = \langle \bar{Q}_v [iD_\mu, [iD^\mu, [iD_\alpha, iD_\beta]]](-i\sigma^{\alpha\beta}) Q_v\rangle \qquad \langle \Box\vec{\sigma}\cdot\vec{B}\rangle$$

Alternative V_{cb} Determination

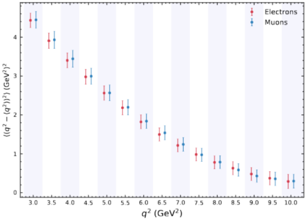
The leptonic invariant mass is RPI: and so are

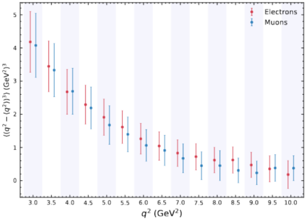
$$\frac{1}{\Gamma_0}\int d\hat{q}^2 (\hat{q}^2)^n \frac{d\Gamma}{d\hat{q}^2} \quad \text{and} \quad \frac{1}{\Gamma_0}\int_{\hat{q}^2_{\rm cut}} d\hat{q}^2 (\hat{q}^2)^n \frac{d\Gamma}{d\hat{q}^2}$$

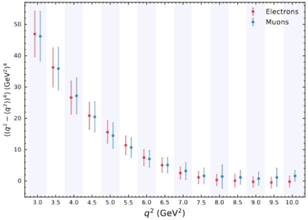
$$\begin{aligned}
Q_1 &= \frac{3}{10}\mu_3 - \frac{7}{5}\frac{\mu_G^2}{m_b^2} + \frac{\tilde{\rho}_D^3}{m_b^3}(19 + 8\log\rho) - \frac{r_E^4}{m_b^4}\left(\frac{1292}{45} + \frac{40}{3}\log\rho\right) - \frac{s_B^4}{m_b^4}(8 + 2\log\rho) \\
&+ \frac{13}{120}\frac{s_{qB}^4}{m_b^4} + \frac{s_E^4}{m_b^4}\left(\frac{63}{5} + 4\log\rho\right) + \frac{r_G^4}{m_b^4}\left(\frac{827}{45} + \frac{22}{3}\log\rho\right), \qquad (4.10) \\
Q_2 &= \frac{2}{15}\mu_3 - \frac{16}{15}\frac{\mu_G^2}{m_b^2} + \frac{\tilde{\rho}_D^3}{m_b^3}\left(\frac{358}{15} + 8\log\rho\right) - \frac{r_E^4}{m_b^4}\left(\frac{2888}{45} + \frac{64}{3}\log\rho\right) - \frac{s_B^4}{m_b^4}\left(\frac{259}{15} + 4\log\rho\right) \\
&+ \frac{s_{qB}^4}{m_b^4}\left(\frac{251}{180} + \frac{1}{3}\log\rho\right) + \frac{s_E^4}{m_b^4}\left(\frac{908}{45} + \frac{16}{3}\log\rho\right) + \frac{r_G^4}{m_b^4}\left(\frac{1373}{45} + \frac{28}{3}\log\rho\right), \qquad (4.11)
\end{aligned}$$

Data on q^2 Moments I

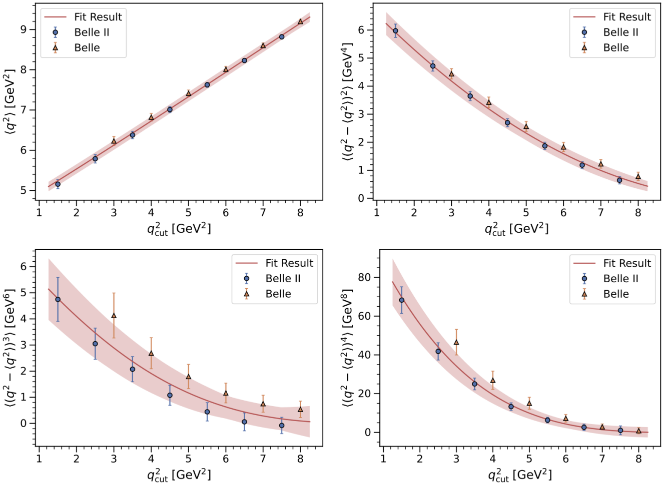
Belle Collaboration [2109.01685, 2105.08001]







Data on q^2 Moments II



2205.10274 (Bernlochner et al.)

$\Longrightarrow$ New V_{cb} Determination

$$R^*(q^2_{\text{cut}}) \qquad \langle (q^2)^n \rangle_{\text{cut}}$$

$$\downarrow$$

$$\mu_3, \mu_G^2, \tilde\rho_D^3, r_E^4, r_G^4, s_E^4, s_B^4, s_{qB}^4, m_b, m_c$$

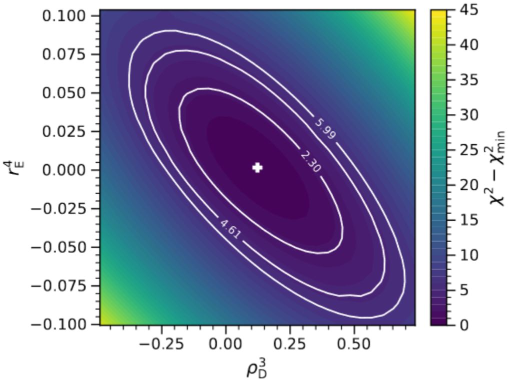
$$\downarrow$$

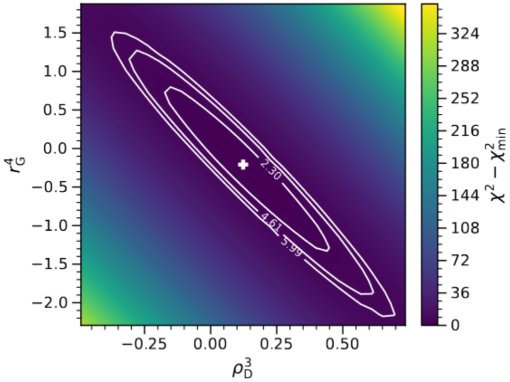
$$\mathrm{Br}(\bar B \to X_c \ell \bar\nu) \propto \frac{|V_{cb}|^2}{\tau_B} \left[\Gamma_{\mu_3}\mu_3 + \Gamma_{\mu_G}\frac{\mu_G^2}{m_b^2} + \Gamma_{\tilde\rho_D}\frac{\tilde\rho_D^3}{m_b^3} \right.$$
$$\left. + \Gamma_{r_E}\frac{r_E^4}{m_b^4} + \Gamma_{r_G}\frac{r_G^4}{m_b^4} + \Gamma_{s_B}\frac{s_B^4}{m_b^4} + \Gamma_{s_E}\frac{s_E^4}{m_b^4} + \Gamma_{s_{qB}}\frac{s_{qB}^4}{m_b^4} \right]$$

$$\downarrow$$

$$V_{cb} = (41.69 \pm 0.63) \cdot 10^{-3}$$

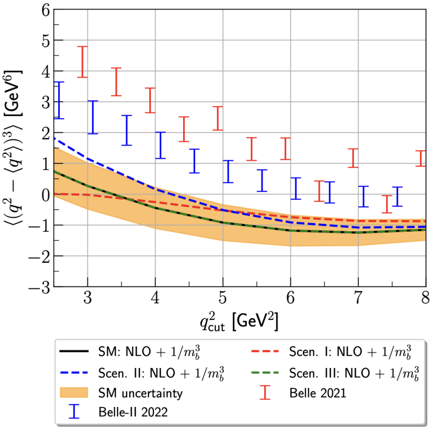
- Agrees with previous determinations
- It includes a data driven determination of the $1/m^4$ HQE Parameters
- $1/m^4$ turns our to be small $\rightarrow$ good fot the HQE

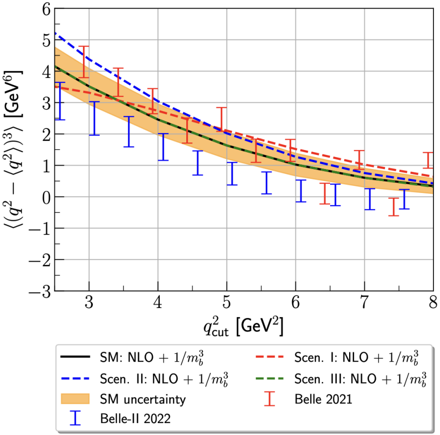




Interesting side remark: The value of ρ_D:

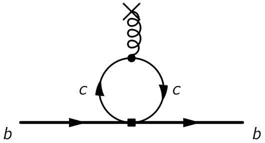
- Gambino et al.: $\rho_D = (0.185 \pm 0.031) GeV^3$ (kinetic scheme)
- Bernlochner et al. $\rho_D = (0.03 \pm 0.02) GeV^3$ (kinetic scheme)

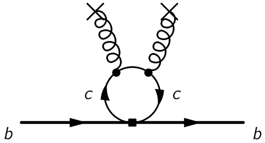




Under investigation ...

- Move on to dim-8 operators:
- Study of the $1/m_b^3 \times 1/m_c^2$ terms
- "Intrinsic" charm contributions:





$$
\begin{aligned}
\langle \bar{c}_\alpha \gamma^\nu c_\beta \rangle_A = & +\frac{2}{3}\frac{1}{(4\pi)^2}\ln\left(\frac{m_b^2}{m_c^2}\right)[D_\kappa\,,\,G^{\kappa\nu}] \\
& +\frac{i}{240\pi^2 m_c^2}\Big(13\left[D^\kappa\,,\,\left[G_{\lambda\nu}\,,\,G^{\lambda,\kappa}\right]\right]+8i\left[D^\kappa\,,\,\left[D^\lambda\,,\,[D_\lambda\,,\,G_{\kappa\nu}]\right]\right] \\
& \qquad -4i\left[D^\lambda\,,\,[D^\kappa\,,\,[D_\lambda\,,\,G_{\kappa\nu}]]\right]\Big)_{\beta\alpha}+\cdots
\end{aligned}
$$

$$
\langle \bar{c}_\alpha \gamma_\nu \gamma_5 c_\beta \rangle_A = +\frac{1}{48\pi^2 m_c^2}\left(2\left\{\left[D_\kappa\,,\,G^{\kappa\lambda}\right]\,,\,\tilde{G}_{\nu\lambda}\right\}+\left\{\left[D_\kappa\,,\,\tilde{G}_{\nu\lambda}\right]\,,\,G^{\kappa\lambda}\right\}\right)_{\beta\alpha}+\cdots
$$

Inclusive V_{ub}

- Problem: Cuts needed to suppress charmed decays
- Forces us into corners of phase space, where the usual OPE breaks down
- Expansion parameter $\Lambda_{\rm QCD}/(m_b - 2E_\ell)$
- Instead of HQE Parameters: Shape Functions $f(\omega)$

$$2M_B f(\omega) = \langle B(v)|\bar{b}_v \delta(\omega + i(n \cdot D))|B(v)\rangle$$

- Universal for all heavy-to-light decays
- Systematics: SoftCollinearEffectiveTheory calculation
  - Several subleading shape functions
  - perturbative QCD corrections

Shape Functions

- Shape function vs. local OPE: Moment Expansion

$$f(\omega) = \delta(\omega) + \frac{\mu_\pi^2}{6m_b^2}\delta''(\omega) - \frac{\rho_D^3}{18m_b^3}\delta'''(\omega) + \cdots$$

- Perturbative “jetlike” contributions: Convolution

$$S(\omega, \mu) = \int dk \; C_0(\omega - k, \mu) f(k)$$

- Charged Lepton Energy Spectrum (H: hard QCD corrections)

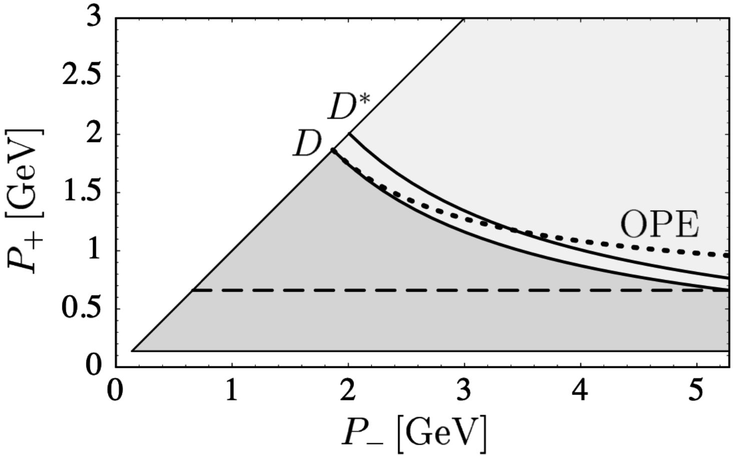
$$\frac{d\Gamma}{dy} = \frac{G_F^2 |V_{ub}^2| m_b^5}{96\pi^3} \int d\omega \, \Theta(m_b(1-y)-\omega) H(\mu) S(\omega, \mu)$$

Approaches

- Obtaining the Shape functions:
  - From Comparison with $B \to X_s \gamma$
  - From the knowledge of (a few) moments
  - From modeling
- QCD based:
  - BLNP (Bosch, Lange, Neubert, Paz)
  - GGOU (Gambino, Giordano, Ossola, Uraltsev)
  - SIMBA (Tackmann, Tackmann, Lacker, Liegti, Stewart ...)
- QCD inspired:
  - Dressed Gluon Exponentiation (Andersen, Gardi)
  - Analytic Coupling (Aglietti, Ricciardi et al.)
- Attempts to avoid the shape functions (Bauer Ligeti, Luke ...)

Bosch Lange Neubert Paz Approach 2004/2005

Study the triple differential rate in the variables $P_\ell = M_B - 2E_\ell$ and $P_\pm = E_X \mp |\vec{P}_X|$



$$\frac{d^3\Gamma}{dP_+ dP_- dP_l} = \frac{G_F^2 |V_{ub}|^2}{16\pi^3}(M_B - P_+)\Big[(P_- - P_+)(M_B - P_- + P_l - P_+)\mathcal{F}_1 + (M_B - P_-)(P_- - P_+)\mathcal{F}_2 + (P_- - P_l)(P_l - P_+)\mathcal{F}_3\Big] ,$$

At leading power in $1/m$ and α_s:

$$\mathcal{F}_1^{(0)\mathrm{OPE}}(y, P_+) = \delta(p_+) + \frac{1}{6}\mu_\pi^2 \delta''(p_+) - \frac{\rho_D^3}{18}\delta'''(p_+) + \dots ,$$
$$\mathcal{F}_2^{(0)\mathrm{OPE}}(y, P_+) = \mathcal{F}_3^{(0)}(y, P_+) = 0 ,$$

with $p_+ = P_+ - \bar{\Lambda}$ and $y = (P_- - P_+)/(M_B - P_+)$

Include QCD corrections to the leading power:

- Multiplication with a Hard Function
- Convolution with a jet function

$$\mathcal{F}_1^{(0)\mathrm{fact}}(y, P_+) = H(y, \mu_f) \int d\hat{\omega}\, p_- J(p_-(P_+ - \hat{\omega}), \mu_f) \hat{S}(\hat{\omega}, \mu_f)$$

- Deal with the radiative tail (convloution ansatz als SIMBA)

$$\hat{S}(\hat{\omega}, \mu_0) = \int_0^{\hat{\omega}} d\hat{k}\, S^{(\mathrm{part})}(\hat{\omega} - \hat{k}, \mu_0) \hat{F}(\hat{k})$$

with a partonic function $S^{(\mathrm{part})}$

- Finally: construct a Model $\hat{F}(\hat{k})$

What can be improved in the BLNP approach?

- Include higher moments into the shape function model
- Include the NLO QCD corrections consistently
- Switch to the kinetic scheme to link to $b \to c$
- Improve the modelling of the shape function: for example

Naively:

$$M_n = \int d\omega\, \omega^n f(\omega) = \frac{1}{2M_B} \langle B | \bar{h}_v (inD)^n h_v | B \rangle = c_n \Lambda_{\rm QCD}^n \quad \text{with} \quad c_n \sim \mathcal{O}(1)$$

Most of the model shape functions do not satisfy this: e.g. the model from Mattias and myself:

$f(\omega) = \frac{32}{\pi^2 \bar{\Lambda}} (1 - \frac{\omega}{\bar{\Lambda}})^2 \exp\left(-\frac{4}{\pi} (1 - \frac{\omega}{\bar{\Lambda}})^2 \right)$ has stronlgy growing c_n as $n \to \infty$

This is on its way!

Some final and Personal Remarks