



MATCHing Effective Theories Efficiently

Javier Fuentes-Martín
University of Granada



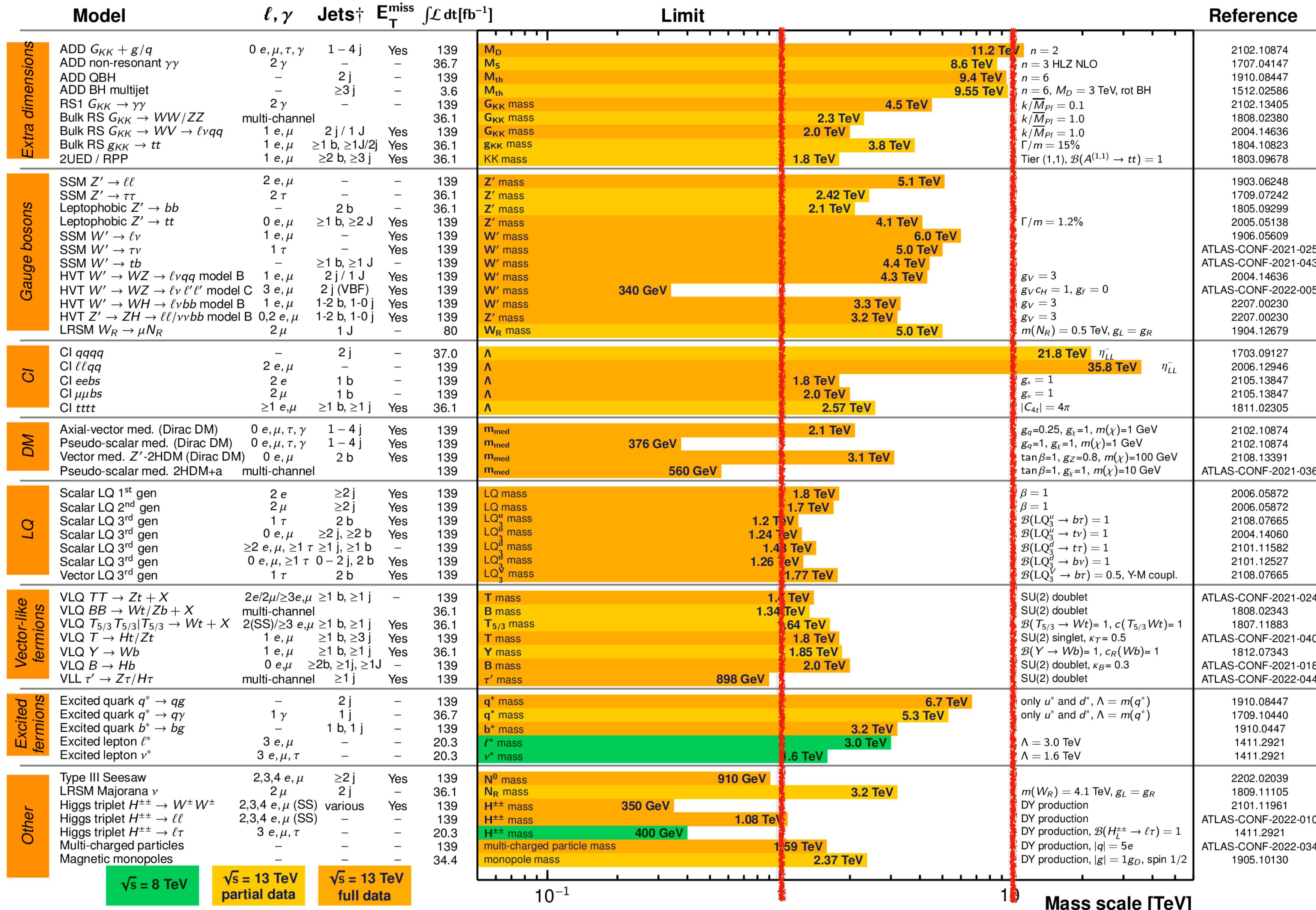
ATLAS Heavy Particle Searches* - 95% CL Upper Exclusion Limits

Status: July 2022

ATLAS Preliminary

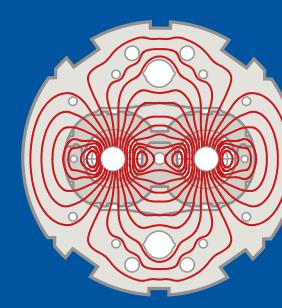
$\int \mathcal{L} dt = (3.6 - 139) \text{ fb}^{-1}$

$\sqrt{s} = 8, 13 \text{ TeV}$

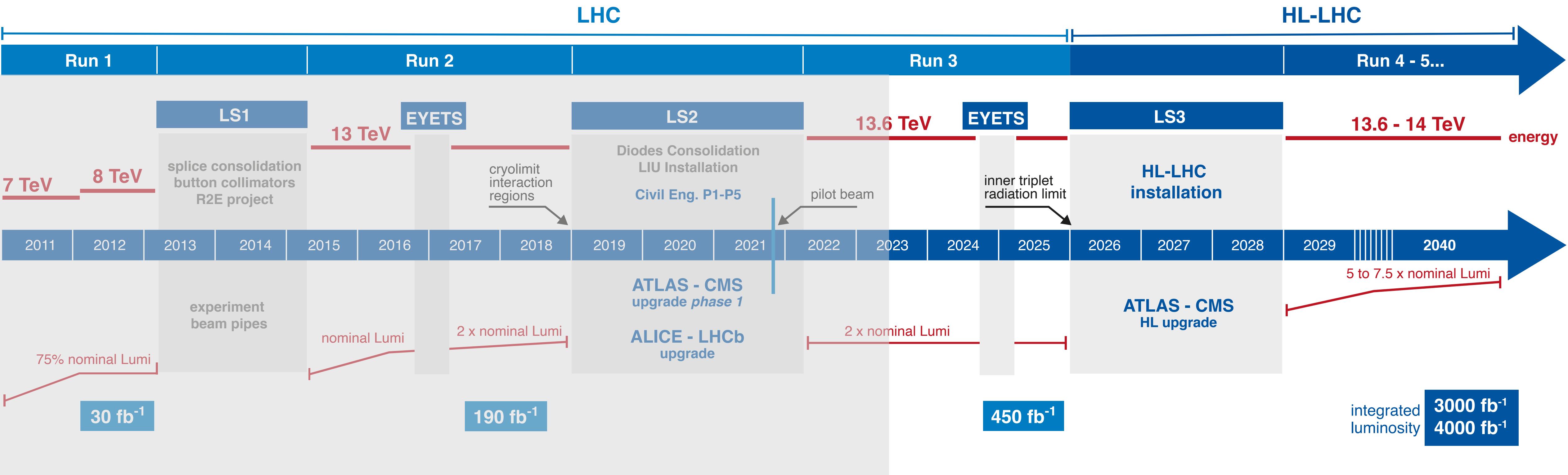


*Only a selection of the available mass limits on new states or phenomena is shown.

†Small-radius (large-radius) jets are denoted by the letter j (J).



LHC / HL-LHC Plan



- No significant increase in the energy reach, but 20x more luminosity! Multiple ongoing low-energy experiments
- Rather than searching for resonances, we can look for the low-energy footprints of new physics

The Effective Field Theory approach

EFTs are essential to interpret experimental observations

$$\mathcal{L}_{\text{EFT}}(\eta_L) = \mathcal{L}_{d=4}(\eta_L) + \sum_{\ell=0}^{\infty} \sum_{n=5}^{\infty} \sum_k \frac{C_{n,k}^{(\ell)}}{(16\pi^2)^\ell \Lambda^{n-4}} O_{n,k}(\eta_L)$$

UV physics

■ Bottom → Up

EFTs offer a **model comprehensive** (“model independent”) approach to study deviations from the SM, organized in a double expansion in E/Λ and loop orders.

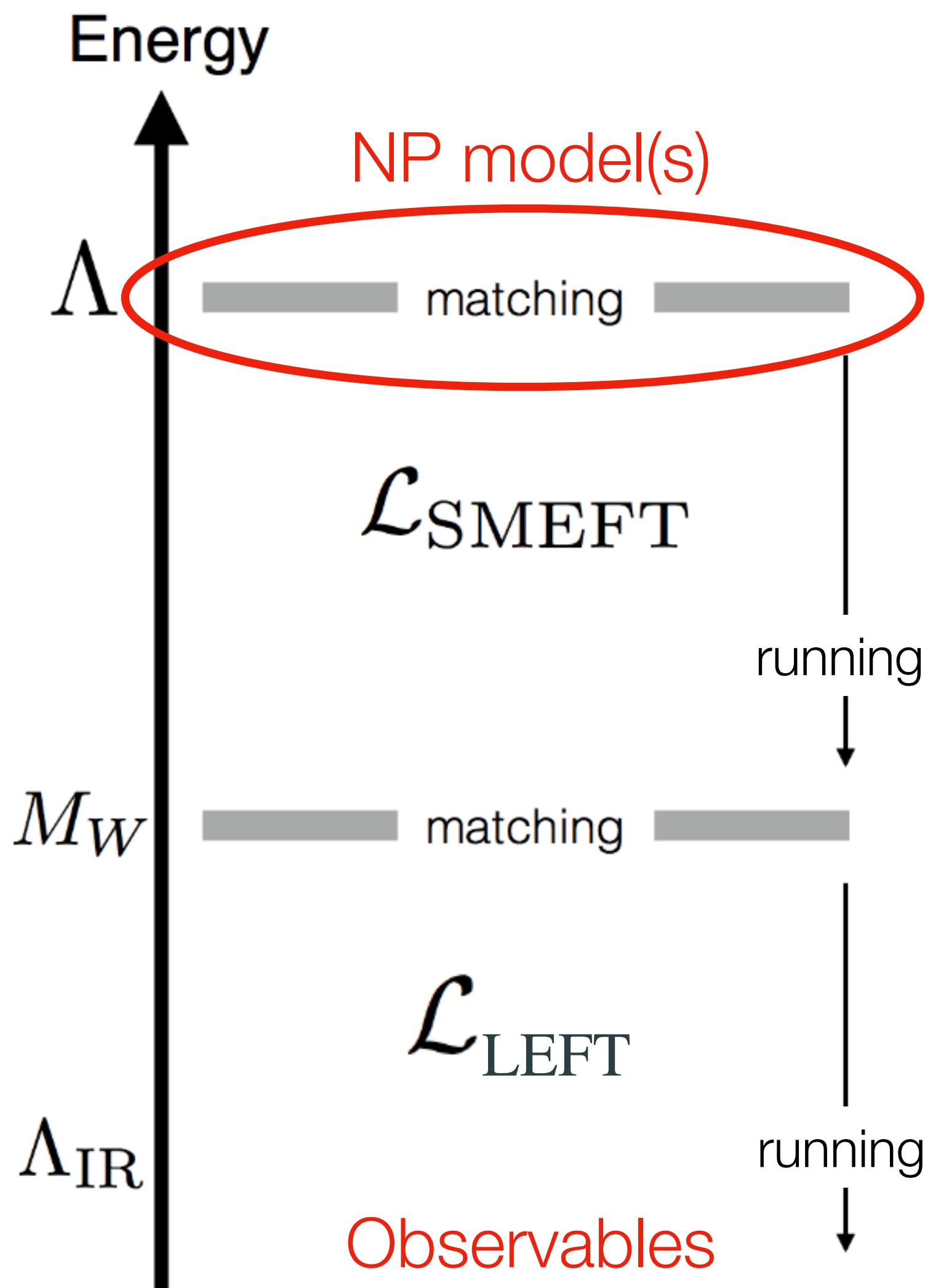
■ Top → Down

(B)SM computations of experimental observables are **multi-scale problems**:
Precision requires using EFTs (RG resummation of large logs)

Multiple BSM models share the same EFT, so many computations are **reusable** (“compute once for all”)

The vast landscape of BSM models and the repetitive nature of EFT computations call for **automated solutions**

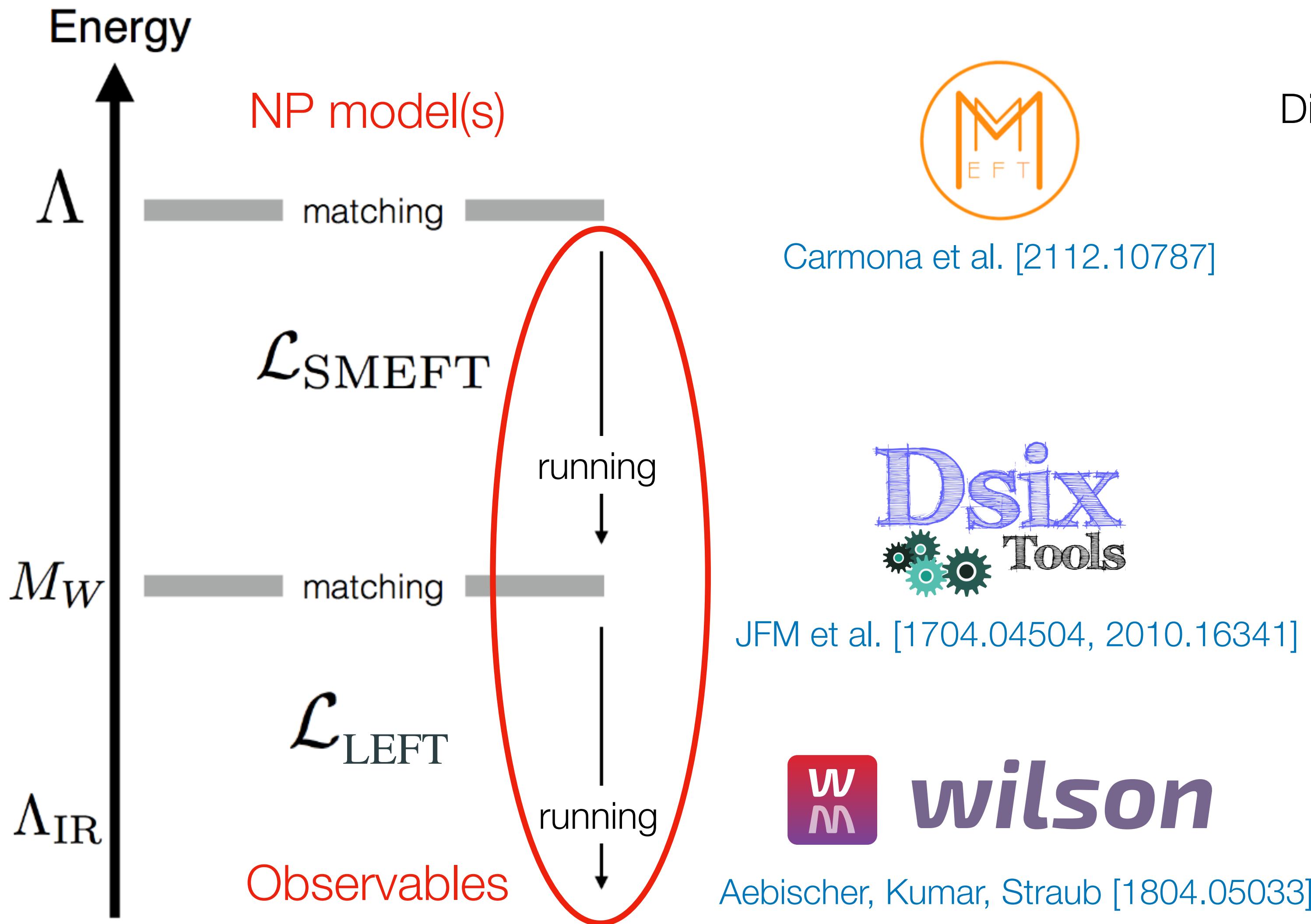
The EFT approach: recent progress towards automation



Carmona et al. [2112.10787]

Diagrammatic matching of many NP models
(requires prior knowledge of the EFT)
[d -dimensional and off-shelf]

The EFT approach: recent progress towards automation



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(requires prior knowledge of the EFT)
[d -dimensional and off-shelf]



“Hard-coded” one-loop results based on:

- Jenkins, Manohar, Trott [1308.2627, 1310.4838]
- Alonso, Jenkins, Manohar, Trott [1312.2014]
- Jenkins, Manohar, Stoffer [1709.04486]
- Dekens, Stoffer [1908.05295]
- Jenkins, Manohar, Stoffer [1711.05270]

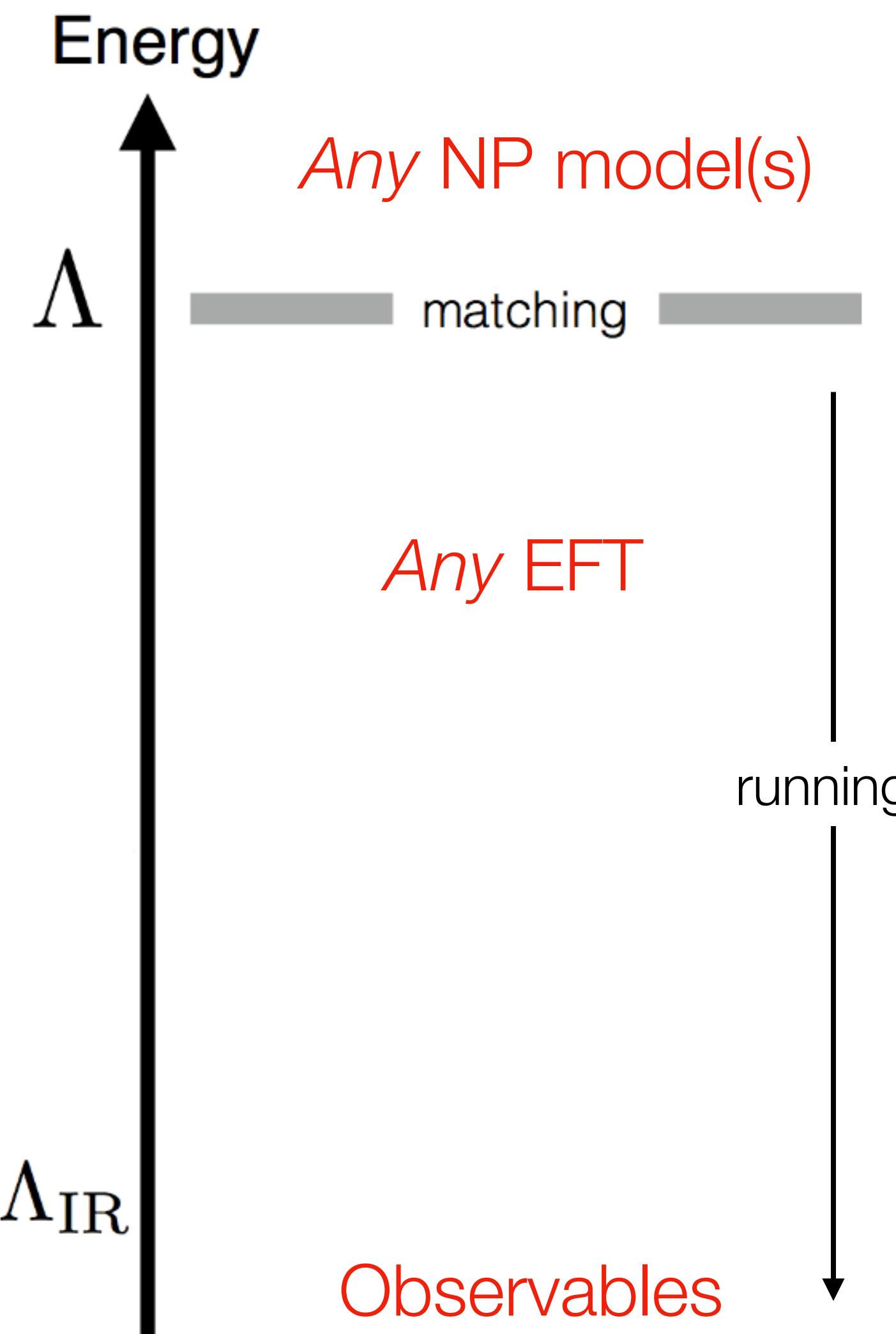


Aebischer, Kumar, Straub [1804.05033]

The EFT approach: recent progress towards automation



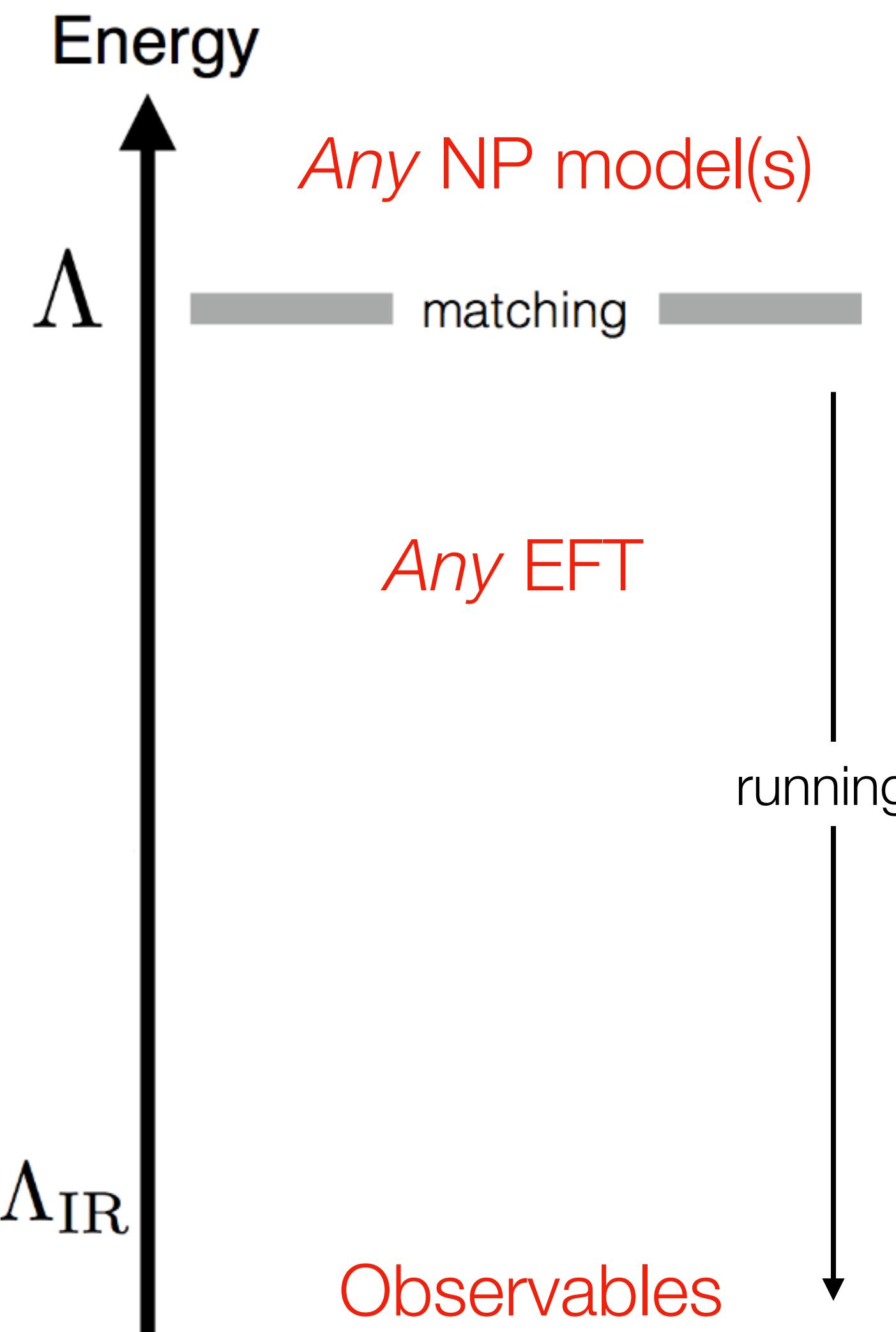
Path integral methods and Matchete



*Path-integral methods for matching and running are *ideal* for this task:*

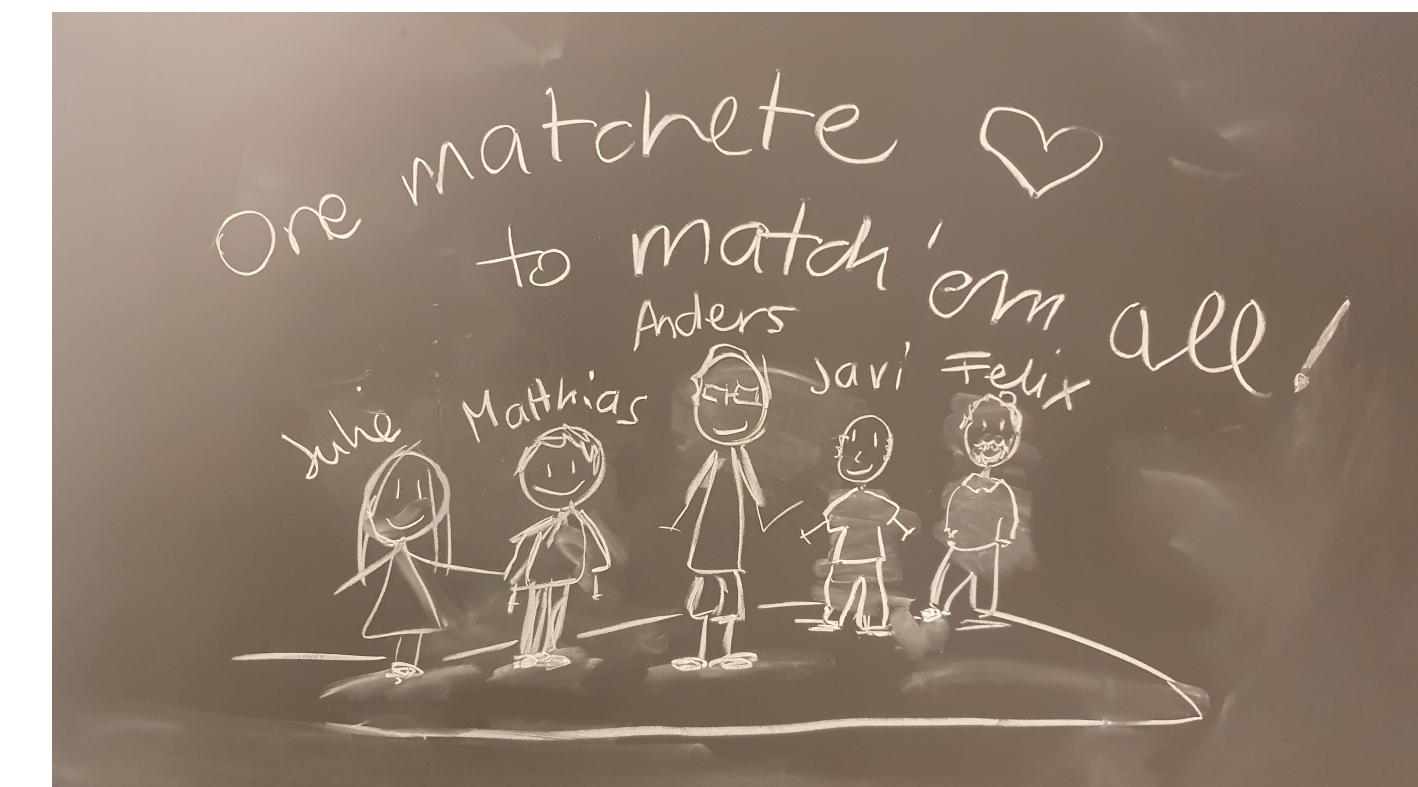
- The EFT Lagrangian comes out directly as part of the procedure
No prior knowledge of the EFT is required!
- Manifestly gauge invariant by construction
[Gaillard '86, Chan '86, Cheyette '88]

Path integral methods and Matchete



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- The EFT Lagrangian comes out directly as part of the procedure
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drawing by Claudia Cornella



Matchete mini-workshop

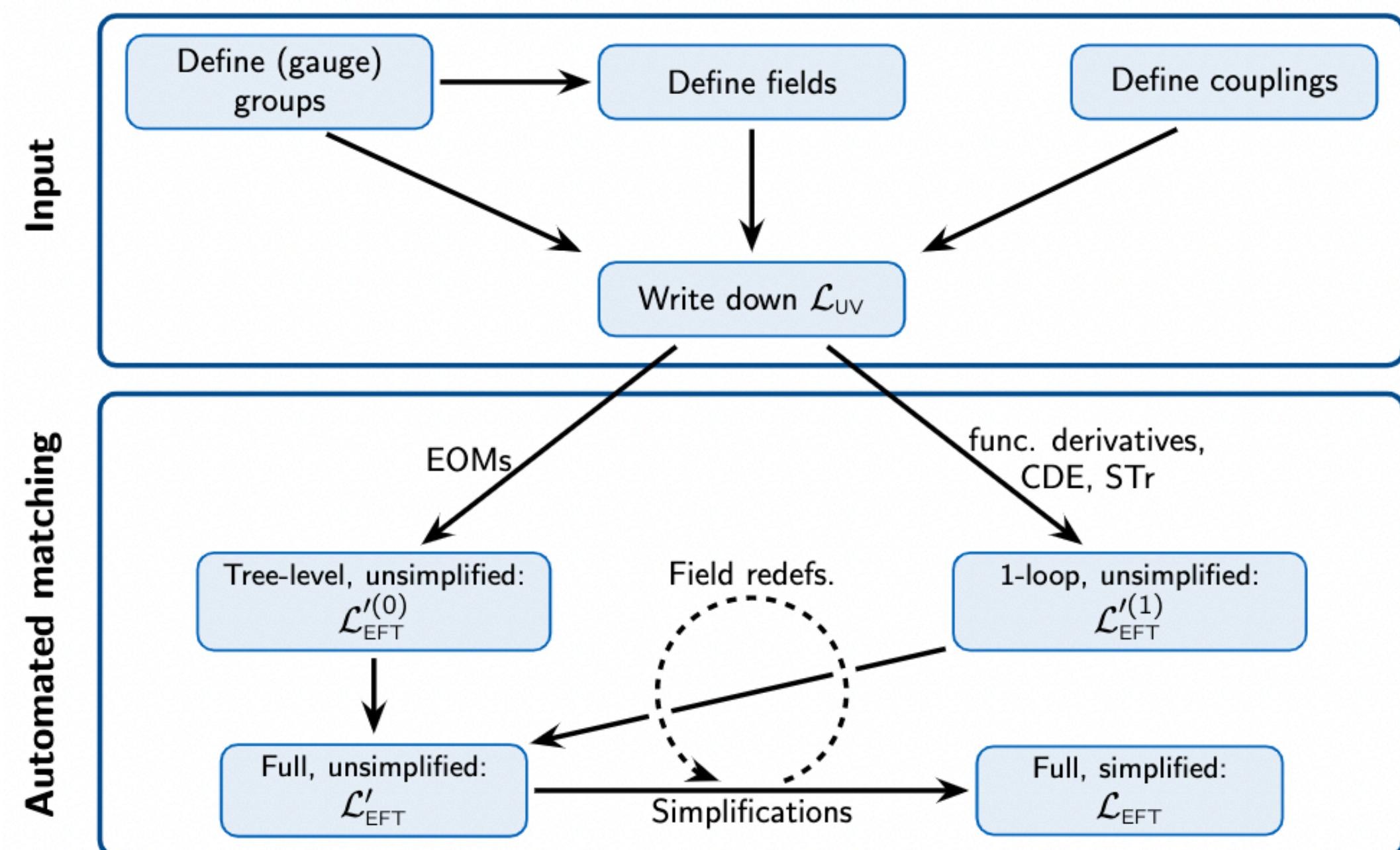
MITP, 8-12 November 2021

with M. König, J. Pagés, A. E. Thomsen, F. Wilsch

The Matchete package



is a **Mathematica package** aimed at fully automating EFT matching and RG evolution of arbitrary weakly-coupled UV theories using functional methods



[JFM, König, Pagès, Thomsen, Wilsch, [2212.04510](#)]

Proof-of-concept version (Matchete v0.1)
now publicly available:

- One-loop matching of any model with heavy scalars and/or fermions
- Simple and intuitive input/output
- Handles *all* group theory (any group and reps)
- Partial simplifications of the resulting EFT Lagrangian (IBP, field redefinitions, ...)
- SSB and heavy vectors not yet supported [w.i.p with Olgoso, Santiago, Thomsen]
- Computation of the RGE not yet available

Two BSM matching examples

SM extension with a scalar SM-singlet

$$\mathcal{L}_{\text{UV}} = \mathcal{L}_{\text{SM}} + \frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2}M^2 S^2 - \frac{\mu_S}{3!} S^3 - \frac{\lambda_S}{4!} S^4 - \frac{\lambda_{SH}}{2} (H^\dagger H) S^2 - \kappa (H^\dagger H) S \quad \text{with } M, \kappa, \mu_S \gg v_{\text{EW}}$$

Less than half a minute to compute the one-loop matching
(which was correctly determined only after several literature iterations)

[Henning, Lu, Murayama [1412.1837](#);
Ellis, Quevillon, You, Zhang [1706.07765](#);
Jiang, Craig, Li, Sutherland [1811.08878](#);
Haisch, Ruhdorfer, Salvioni, Venturini, Weiler [2003.05936](#)]

SM extension with a vector-like lepton ($E \sim (\mathbf{1}, \mathbf{1})_{-1}$)

$$\mathcal{L}_{\text{UV}} = \mathcal{L}_{\text{SM}} + i(\bar{E} \gamma_\mu D^\mu E) - m_E \bar{E} E - (y_E \ell_L H E_R + \text{h.c.}) \quad \text{with } M_E \gg v_{\text{EW}}$$

Less than a minute to compute the one-loop matching and simplify the result
(result validated against **matchmakereft**)

Let's see how it works!

An example of Matchete in action

Matchete in action

Next is a live demonstration, see the attached Mathematica notebook

Reducing the EFT Lagrangian to its basis

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 + \frac{C_1}{\Lambda^2}\phi^6 + \frac{C_2}{\Lambda^2}\phi^3\partial^2\phi + \frac{C_3}{\Lambda^2}\phi^2(\partial_\mu\phi)^2$$

Exact simplifications (linear): IBP, Dirac and group identities, commutation relations...

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 + \frac{C_1}{\Lambda^2}\phi^6 + \frac{3C_2 - C_3}{3\Lambda^2}\phi^3\partial^2\phi$$

On-shell equivalence (non-linear): Field redefinitions (sometimes equivalent to using of EOMs)

$$\phi \rightarrow \phi + \frac{3C_2 - C_3}{3\Lambda^2}\phi^3 \quad \left[\partial^2\phi = -m^2\phi - \frac{\lambda}{3!}\phi^3 + \mathcal{O}(\Lambda^{-2}) \right]$$

$$\mathcal{L} \rightarrow \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2 - \left(\frac{\lambda}{4!} + \frac{m^2(3C_2 - C_3)}{3\Lambda^2} \right)\phi^4 + \frac{18C_1 - \lambda(3C_2 - C_3)}{18\Lambda^2}\phi^6$$

Removal of evanescent operators: Formally solved

[JFM, König, Pagès, Thomsen, Wilsch, [2211.09144](#)]

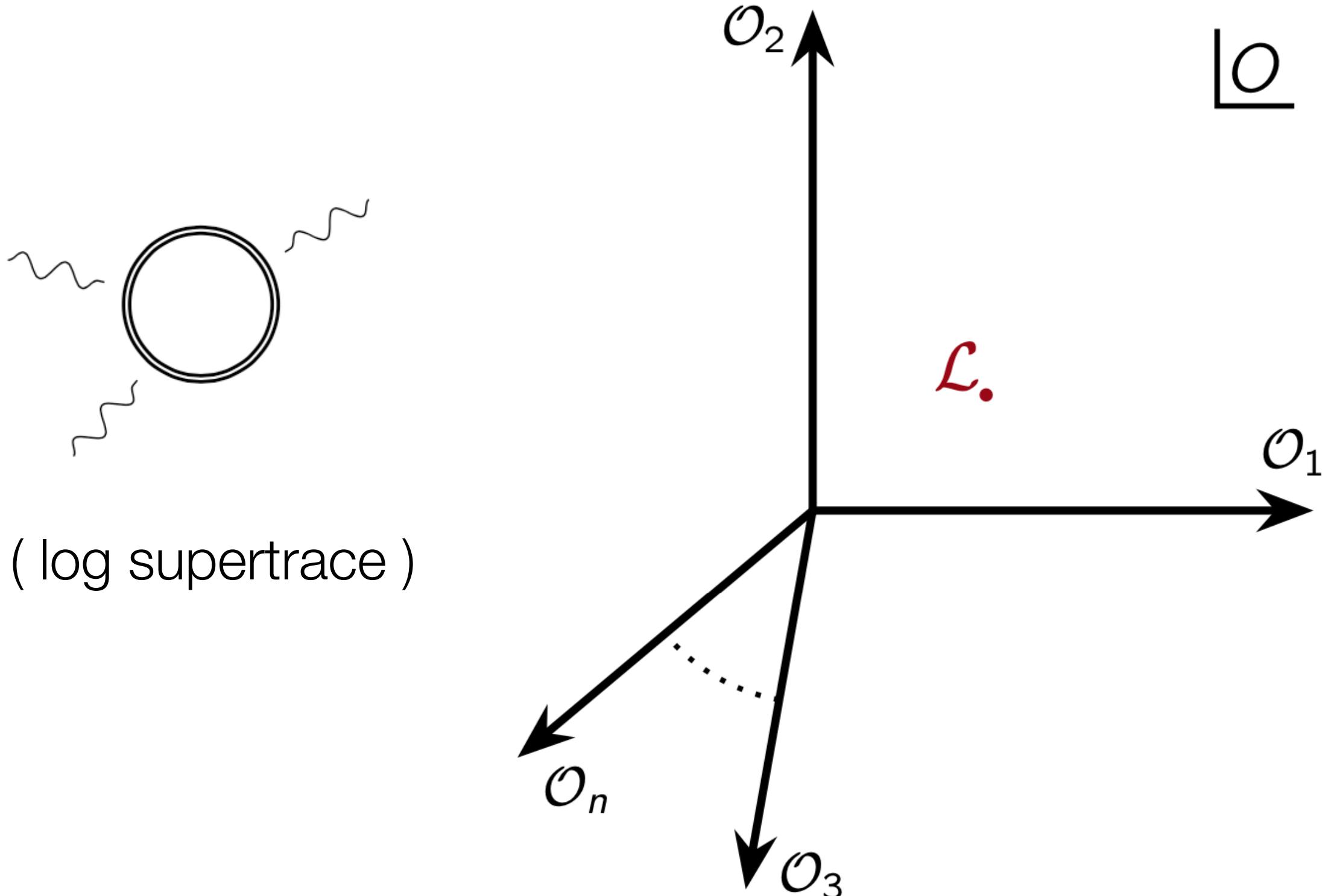
Linear simplifications

Example: Integrating out a heavy fermion in the fundamental representation of SU(3)

```
In[12]:= LEFT // NiceForm
Out[12]//NiceForm=
```

$$\begin{aligned} & \frac{7}{540} \hbar g^2 \frac{1}{M\Psi^2} (D_\rho G^{\mu\nu A})^2 + \\ & \frac{1}{40} \hbar g^2 \frac{1}{M\Psi^2} G^{\mu\nu A} D^2 G^{\mu\nu A} + \frac{7}{540} \hbar g^2 \frac{1}{M\Psi^2} D_\rho G^{\mu\nu A} D_\gamma G^{\mu\rho A} - \\ & \frac{1}{180} \hbar g^2 \frac{1}{M\Psi^2} D_\gamma G^{\mu\nu A} D_\rho G^{\mu\rho A} + \frac{1}{40} \hbar g^2 \frac{1}{M\Psi^2} G^{\mu\nu A} D_\nu D_\rho G^{\mu\rho A} + \\ & \frac{1}{40} \hbar g^2 \frac{1}{M\Psi^2} G^{\mu\nu A} D_\rho D_\nu G^{\mu\rho A} - \frac{1}{24} \hbar g^3 \frac{1}{M\Psi^2} G^{\mu\nu A} G^{\mu\rho B} G^{\nu\rho C} f^{ABC} \end{aligned}$$

$$\mathcal{L}_{\text{EFT}} = \sum_i C_i \mathcal{O}_i \in O$$



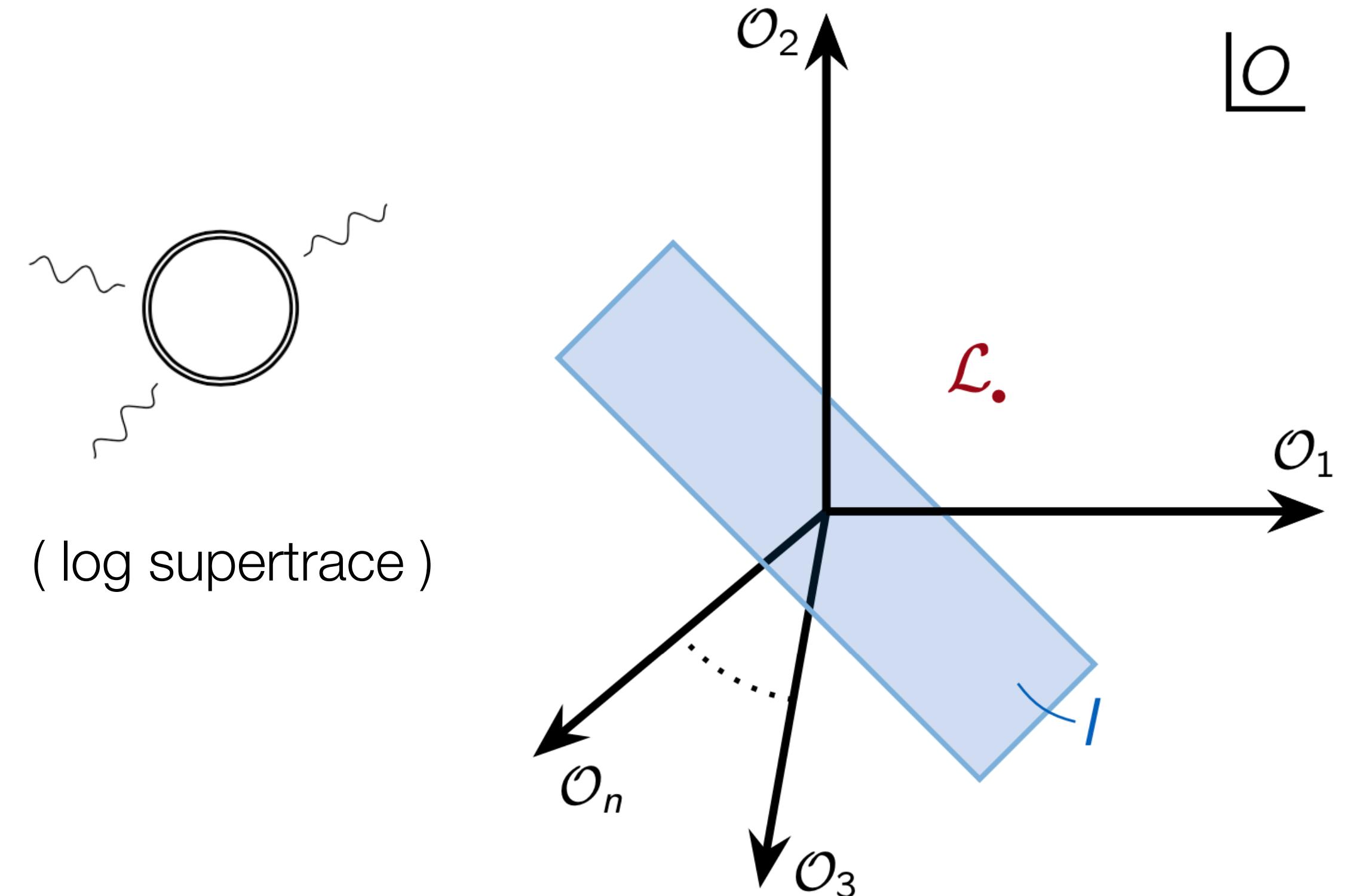
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$$\mathcal{L}_{\text{EFT}} = \sum_i C_i \mathcal{O}_i \in O$$



$I \subseteq O$ is the space of all operators identities, such as IBP relations, yielding e.g.

$$\mathcal{O}_1 + 2 \mathcal{O}_3 = 0$$

interpreted as

$$\mathcal{O}_1 + 2 \mathcal{O}_3 \in I$$

Linear simplifications

Example: Integrating out a heavy fermion in the fundamental representation of SU(3)

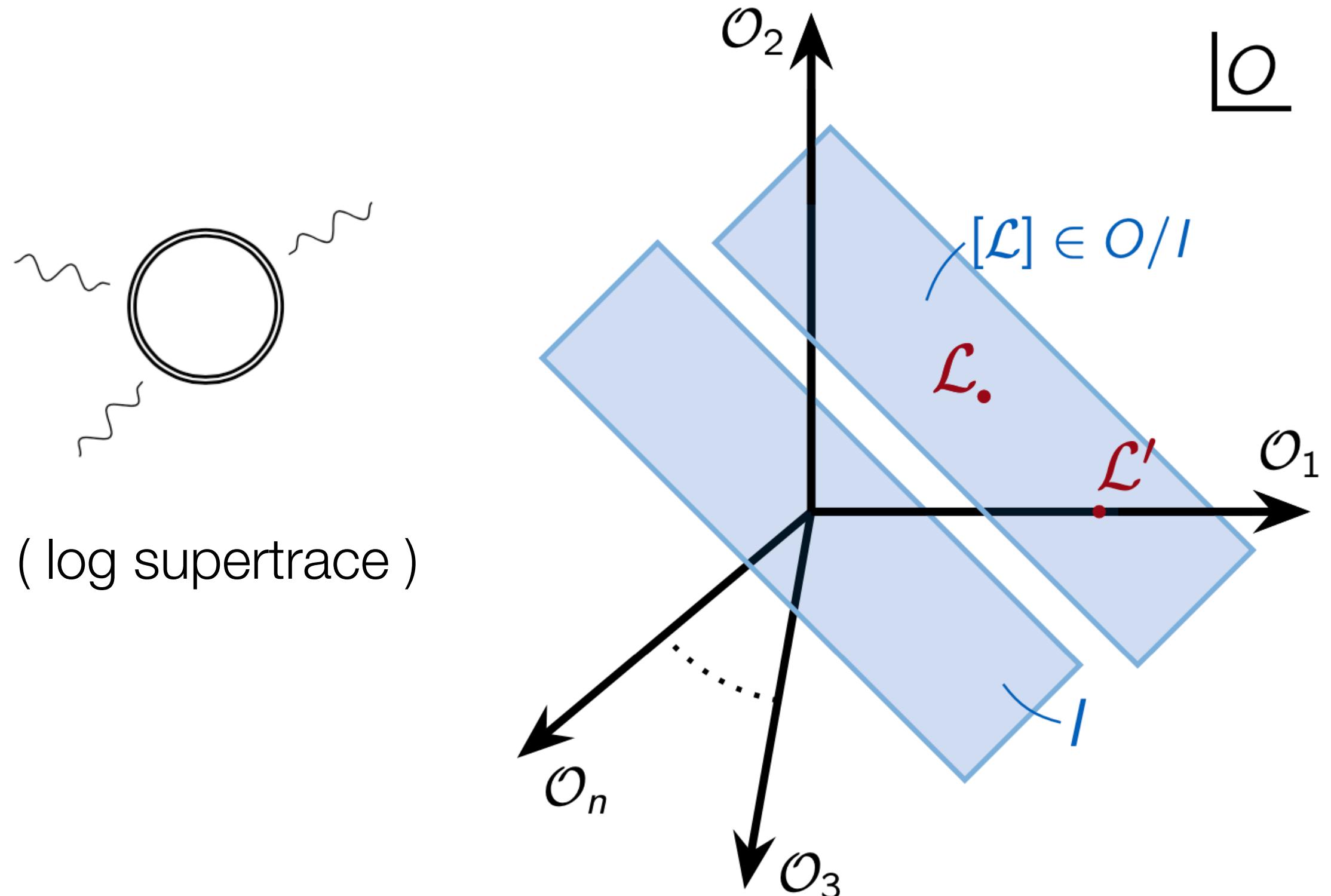
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$$\mathcal{L}_{\text{EFT}} = \sum_i C_i \mathcal{O}_i \in O$$

By gaussian elimination, we can choose a representative element for $[\mathcal{L}_{\text{EFT}}] \in O/I$ to get an EFT basis

```
In[13]:= LEFT // GreensSimplify // NiceForm
Out[13]/NiceForm=
```

$$-\frac{1}{15} \hbar g^2 \frac{1}{M\Psi^2} D_\nu G^{\mu\nu A} D_\rho G^{\mu\rho A} - \frac{1}{180} \hbar g^3 \frac{1}{M\Psi^2} G^{\mu\nu A} G^{\mu\rho B} G^{\nu\rho C} f^{ABC}$$


$I \subseteq O$ is the space of all operators identities, such as IBP relations, yielding e.g.

$$\mathcal{O}_1 + 2 \mathcal{O}_3 = 0$$

interpreted as

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Evanescent operators

Evanescent operators appear from special type of linear simplification (valid only for $d = 4$)

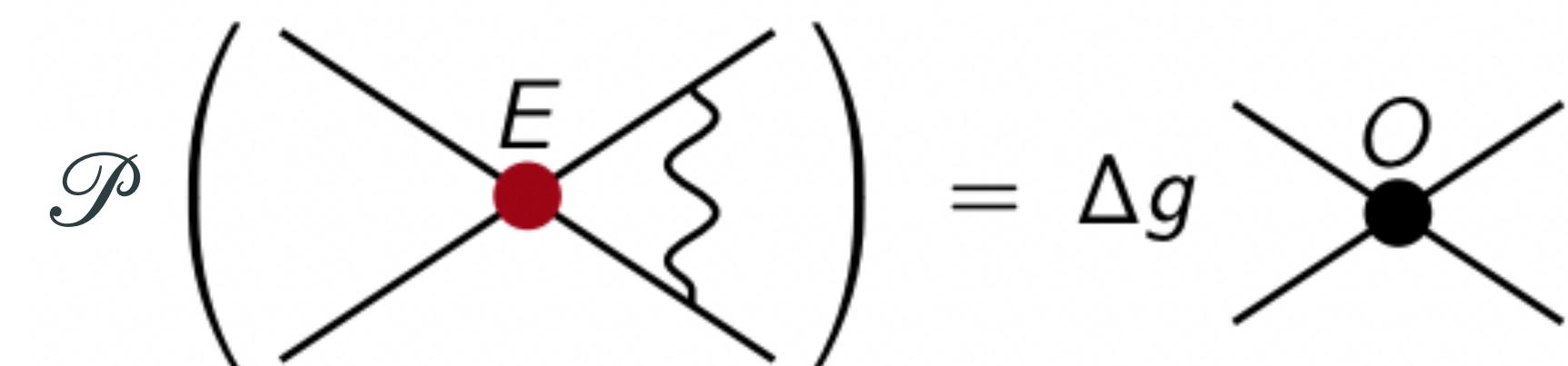
$$O_d = \underbrace{\mathcal{P} O_d}_{\text{Physical part}} + \underbrace{\mathcal{E} O_d}_{\text{Evanescent part}} \quad \mathcal{P} \equiv \text{Projection to the physical } (d=4) \text{ basis}$$

$\curvearrowleft \text{Id} - \mathcal{P}$

E.g. Fierz identities

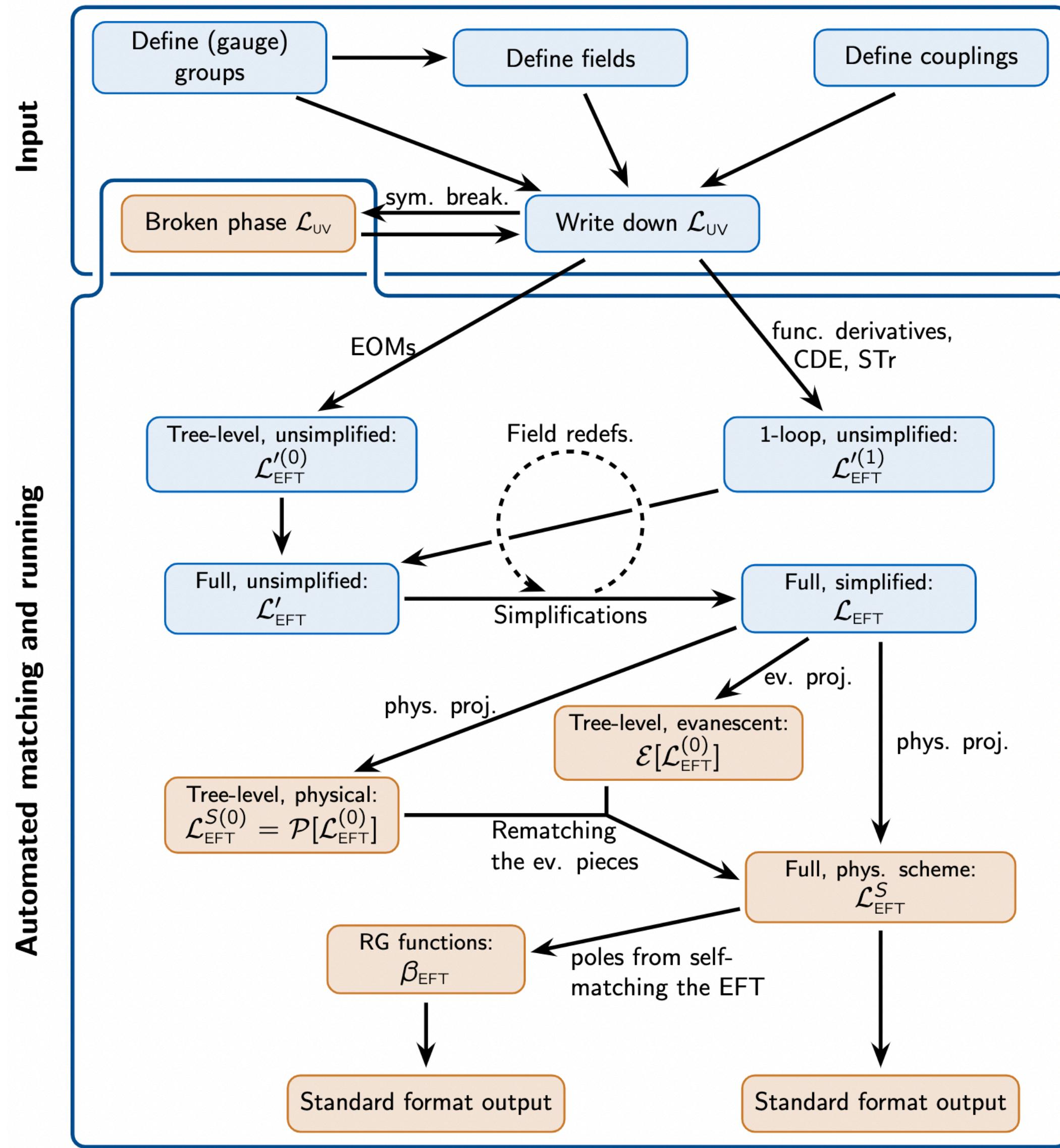
$$(\bar{\ell}_p e_r)(\bar{e}_s \ell_t) = -\frac{1}{2} (\bar{\ell}_p \gamma_\mu \ell_t)(\bar{e}_s \gamma^\mu e_r) + E_{\ell e}^{prst} \xrightarrow{\text{rank}(d-4)} (\bar{\ell}_p e_r)(\bar{e}_s \ell_t) + \frac{1}{2} (\bar{\ell}_p \gamma_\mu \ell_t)(\bar{e}_s \gamma^\mu e_r) - E_{\ell e}^{prst} \in I$$

Representative elements are chosen so evanescent operators are retained. Afterwards, these are removed by shifting the coefficients of physical operators



e.g. $E_{\ell e}^{prst} \rightarrow -\frac{g_L y_e^{ts}}{128\pi^2} Q_{eW}^{pr} + [\text{other contributions}]$

Future plans



Proof of concept already available at:
<https://gitlab.com/matchete/matchete>

Expected future functionalities include:

- Handling of evanescent contributions
- Complete basis reduction and identification
- One-loop RG computations
- Heavy vectors and symmetry breaking
- Interface with other EFT tools
(UFO / WCxf outputs)
- Other γ_5 and regularization/renormalization schemes
- Matching and running beyond one loop

Going beyond one loop

[JFM, Thomsen, Palavic, w.i.p]

The quantum effective action is the generating functional of 1PI functions

$$e^{i\Gamma_{UV}[\hat{\eta}]} = \int \mathcal{D}\eta \exp\left(iS_{UV}[\eta + \hat{\eta}] + i\int_x J\eta\right)$$

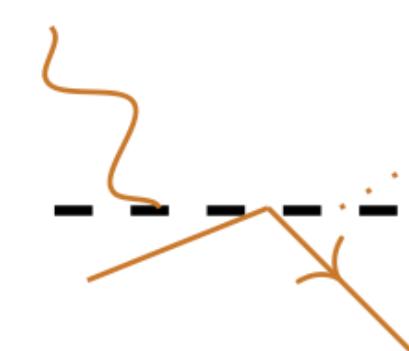
η : Quantum fields (loop lines)

$\hat{\eta}$: Classical fields (tree lines)

The loop expansion is produced with the saddlepoint approximation :

$$\begin{aligned} \Gamma_{UV}[\hat{\eta}] &= S_{UV}[\hat{\eta}] - i \ln \int \mathcal{D}\eta \exp \left[i \left(\frac{\eta^2}{2} Q[\hat{\eta}] + \frac{\eta^3}{3!} B[\hat{\eta}] + \frac{\eta^4}{4!} D[\hat{\eta}] + \dots \right) \right] \\ &= S_{UV}[\hat{\eta}] + \frac{i}{2} \log \text{(circle)} + \frac{i}{2} \text{(loop with dot)} + \frac{1}{12} \text{(triangle)} - \frac{1}{8} \text{(cross)} + \mathcal{O}(\hbar^3) \end{aligned}$$

All propagators are dressed with arbitrary background field insertions

$$Q_{IJ}[\hat{\eta}] = \frac{\delta^2 S}{\delta \eta^I \delta \eta^J} [\hat{\eta}] = \left(\text{---} \text{---} \text{---} \right)^{-1}$$


Going beyond one loop

The EFT action is given by

$$S_{\text{EFT}}[\phi] = \Gamma_{\text{UV}}[\hat{\Phi}, \phi] \Big|_{\text{hard}}$$

$$\frac{\delta \Gamma_{\text{UV}}}{\delta \Phi} \Big|_{\text{hard}} [\hat{\Phi}, \phi] = 0$$

Φ : Heavy
 ϕ : Light

“hard” denotes the part where all loop momenta are $p \sim \Lambda$ (incl. tree-level contributions) ^(*)

- Already used at one loop order [JFM, Portolés, Ruiz-Femenía, [1607.02142](#); Z. Zhang [1610.00710](#)]
- Explicit proof to two-loop order [JFM, Thomsen, Palavic, w.i.p]
- The hard region is by far the easiest to compute (only vacuum diagrams at zero external momenta)
- Enables functional matching at any loop order

^(*) Method of regions: Beneke, Smirnov, hep-ph/9711391; Jantzen, 1111.2589

Conclusions

- (Automated) EFT matching is crucial to BSM phenomenology
- Functional matching is ideal for automation (also useful for pen-and-paper computations!)
- Complete one-loop automation: Lagrangian in, fully simplified EFT Lagrangian out not yet available
 - Ongoing progress with 
- The ultimate goal is a code (or chain of codes) that fully automates
 - Matching
 - RG evolution
 - Connection to observables / fit to data

} **Multi-step matching**

Interface with other EFT pheno codes

streamlining future BSM analyses

MITP SCIENTIFIC PROGRAMS

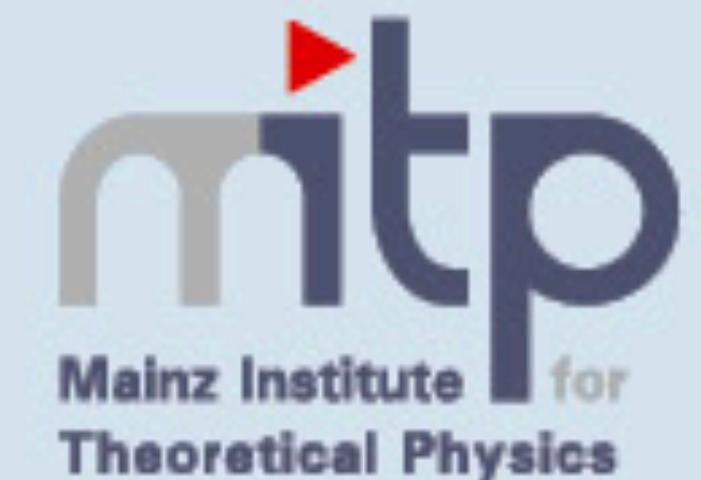
EFT 2023
Foundations &
Tools



EFT Foundations and Tools 2023
August 28 - September 8, 2023



<https://indico.mitp.uni-mainz.de/event/330>



Organized by Jason Aebischer, Adrián Carmona, Claudia Cornella, JFM, and Anders Eller Thomsen

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Thank you