Conformal Extensions of the Standard Model

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Reminder: Scales and Hierarchy Problems

1) Why are (tree level) scales vastly different?

2) Stability of vastly different scales under quantum corrections?

SM + embedding at Λ (new physics, not a regulator)

$$\delta M_H^2 = \frac{\Lambda^2}{32\pi^2 V^2}\left(6M_W^2 + 3M_Z^2 + 3M_H^2 - 12M_t^2\right) \quad \sim\Lambda^2 \gg M_H^2$$

SM + Dirac neutrino masses: no problem – just like SM

SM + Majorana neutrino masses → more scales M_i

→ **generates a HP problem for large M even if y_ν is tiny**

$$\delta m_H^2 \simeq \frac{y_\nu^2}{16\pi^2} M^2 \qquad y_\nu^2 = M m_\nu / v^2$$

→ $M \lesssim 10^7 - 10^8$ GeV ↔ **see-saw, leptogenesis, ...**

The Problem: 2 or more EXPLICIT Scales

- Renormalizable QFT with two scalars φ , Φ
  with masses m, M and a hierarchy $m << M$
- These scalars must interact since $\varphi^+\varphi$ and $\Phi^+\Phi$ are singlets
  ➔ $\lambda_{mix}(\varphi^+\varphi)(\Phi^+\Phi)$ (= portal) in addition to φ^4 and Φ^4
- Quantum corrections $\sim M^2$ drives m to the (heavy) scale M
  ➔ vastly different explicit scalar scales are generically unstable

- **Since SM Higgs exists ➔ problem: embedding with a 2nd scalar**
  - gauge extensions: LR, PS, GUTs → must be broken…
  - even for SUSY GUTS → doublet-triplet splitting...
  - also for fashionable Higgs-portal scenarios...

- **Ways out:**
- No Higgs ...
- Symmetry: SUSY, ... ➔ conformal symmetry = no explicit scales!
- Question: Is one physical scale ($\mu^2 \neq 0$) of the SM an issue?

Explaining Masses without Mass

The Lagrangian should not contain any dimensionful parameter ➔ **scale invariance**

Scale invariance is hardly broken by scale anomaly: Callan, '70; Symanzik,'70
The scale anomaly cannot directly generate a mass gap
To generate a mass gap, scale invariance has to be spontaneously broken

What about the Standard Model:

- It is a one-scale theory (←→ adding Majorana masses?)
- For $\mu^2 = 0$ increased symmetry → makes classical scale invariance exact
- Loops: log. running coupling constants break scale invariance ➔ β-functions

$$\partial^\mu J_\mu = T^\mu{}_\mu = \sum_i \beta_i \cdot \hat{O}_i + \mathcal{C}$$

$\hat{O}_i$ = dim. 4 operators $\mathcal{C}$ = Weyl anomaly ←→ curved backgrounds

- log running and quadratic divergences are different breakings of scale invariance
- Quadratic divergences ←→ second scale (cutoff Λ, heavy particle...)
  Bardeen '95 quadratic divergences as artefact of the regularization
  ➔ Λ^2 not surprising if regulator induces explicit scale!

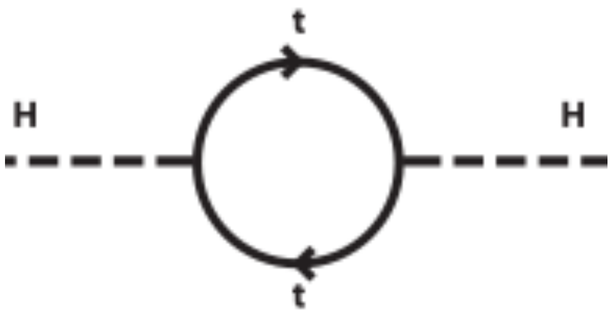
Consequence for Theories without any explicit Scale

Conformal anomaly = breaking of CS by loops

anomaly $\simeq$ trace of energy momentum tensor

$\leftrightarrow$ β-functions $\leftrightarrow$ log running $\leftrightarrow$ UV fixed points

not related to Λ^2 divergences



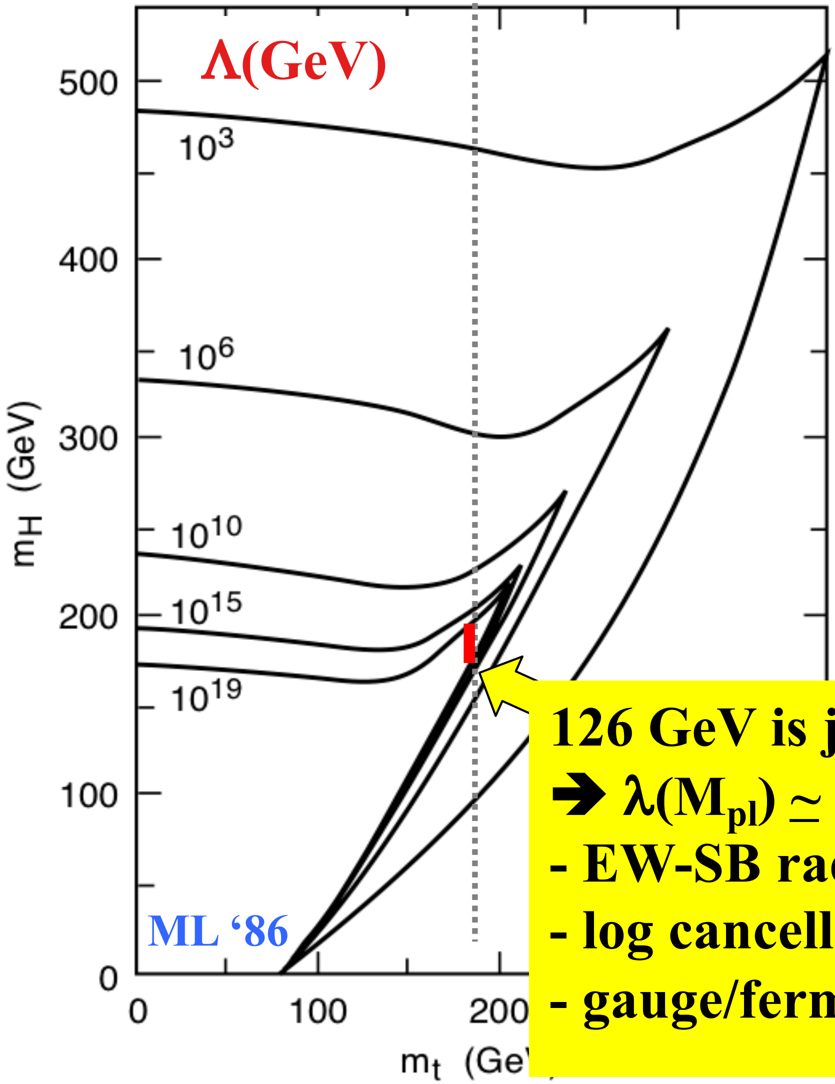
remnant protective feature of CS
naïve power counting may be wrong
➔ no Λ^2 divergence

➔ dimensional transmutation of conformal theories by log running of couplings like in chiral QCD

A remarkable Coincidence of the SM

➔ SM is a renormalizable QFT like QED w/o hierarchy problem

➔ Cutoff "Λ" has no meaning ➔ triviality, vacuum stability



126 GeV is just here!

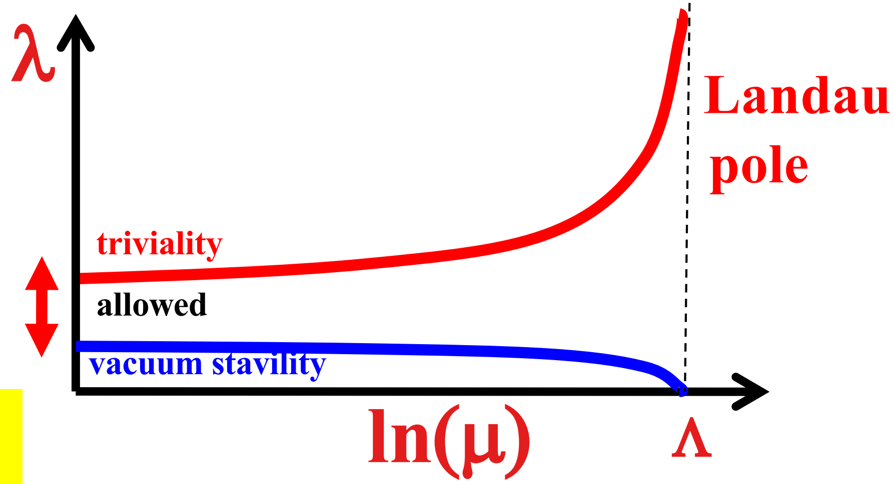
➔ $\lambda(M_{pl}) \simeq 0$

- EW-SB radiative
- log cancellations
- gauge/fermion/scalar

126 GeV < m_H < 174 GeV

SM does not exist w/o embedding

- U(1) coupling , Higgs self-coupling

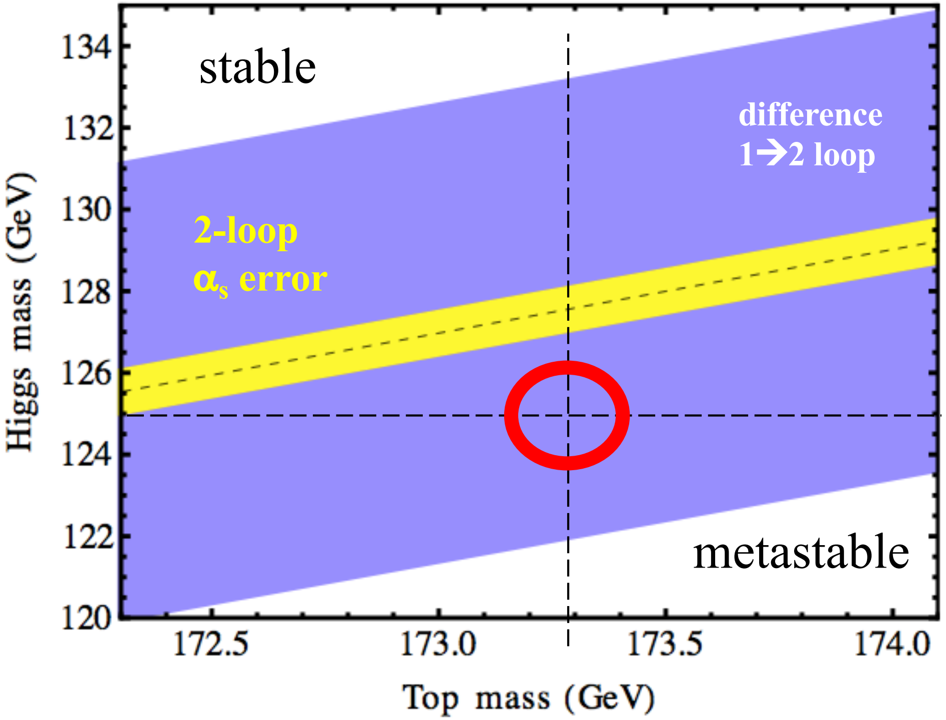


➔ RGE arguments seem to work

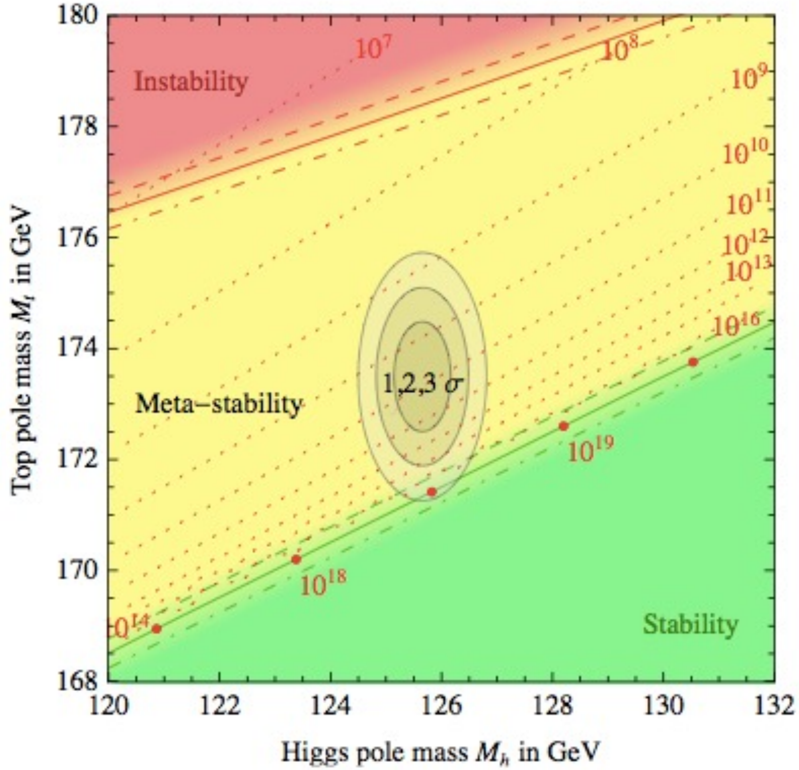
➔ but we need some embedding

Is the Higgs Potential at M_{Planck} flat?

Holthausen, ML, Lim
12 Dec 2011



Elias-Miro, Espinosa, Giudice, Isidori, Riotto, Strumia
13 Dec 2011



Experimental values indicate metastability, but,

➔ **we need to include DM, neutrino masses, …? are all errors (EX+TH) fully included?**

➔ **be cautious about claiming that metastability is established**

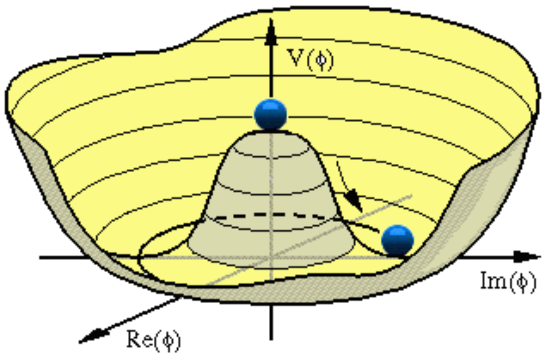
➔ **Important observation:**

- **remarkable relation between weak scale, m_t, couplings and M_{Planck} ←→ precision**
- **interplay between gauge, Higgs and top loops: log divergences – not quadartic div.**

Is there a Message?

- $\lambda(M_{Planck}) \simeq 0$? ➔remarkable log cancellations ⇐⇒ CA~ β-fcts.
  M_{planck}, M_{weak}, gauge, Higgs & Yukawa couplings are unrelated

- remember: μ is the only single scale of the SM ➔ special role
  ➔ if in addition $\mu^2 = 0$ ➔ $V(M_{Planck}) \simeq 0$
  ➔ flat Mexican hat (<1%) at the Planck scale!



➔

➔ conformal (or shift) symmetry as solution to the HP?

➔ combined conformal & EW symmetry breaking

- conceptual issues
- minimal realizations ⇐⇒ SM seems to know about high scales → bottom-up
⇐⇒ many new d.o.f. (fields, big reps.) ~ UV-instabilities

Bottom-up realizations

Why the minimalistic SM does not work

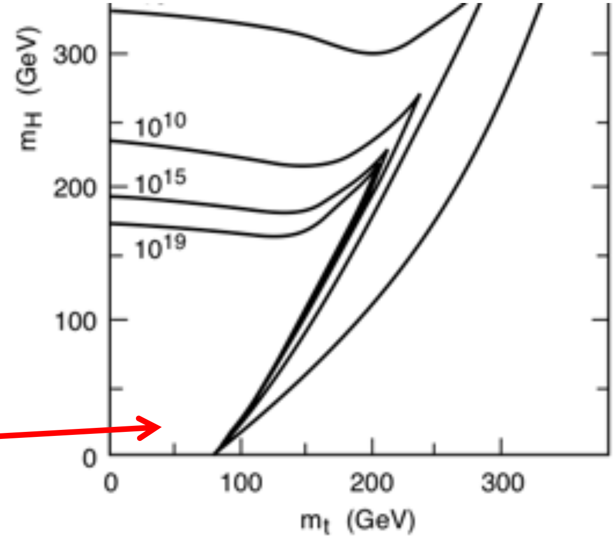
Minimalistic version: → "SM-"

SM + with $\mu = 0$ ←→ CS

Coleman Weinberg: effective potential

→ CS breaking (dimensional transmutation)

→ induces for $m_t < 79$ GeV
a Higgs mass $m_H = 8.9$ GeV

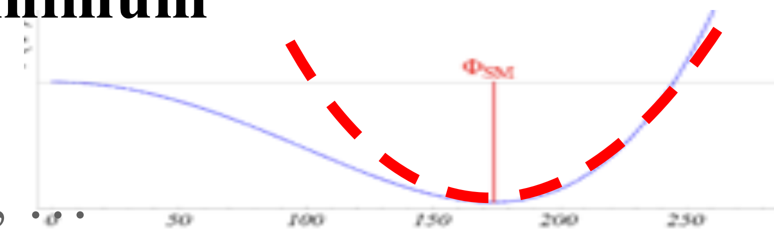


- **Success: no-scale SM → broken SM but: Higgs and top do not fit**

- **DSB for weak coupling ←→ CS= phase boundary**
  → scale set by log-running couplings ←→ gap eqn: hierarchical!

- **Reason for $m_H \ll v$: V_{eff} flat around minimum**
  ←→ $m_H \sim$ loop factor $\sim 1/16\pi^2$

AND: We need neutrino masses, dark matter, …



Realizing the Idea via Higgs Portals

- **SM scalar Φ plus some new scalar φ (or more scalars)**
- **CS → no scalar mass terms**
- **the scalar portal $\lambda_{mix}(\varphi^+\varphi)(\Phi^+\Phi)$ must exist**

→ a condensate of $<\varphi^+\varphi>$ produces $\lambda_{mix}<\varphi^+\varphi>(\Phi^+\Phi) = \mu^2(\Phi^+\Phi)$
→ effective mass term for Φ

- **no CA… → breaking only ln(Λ)**

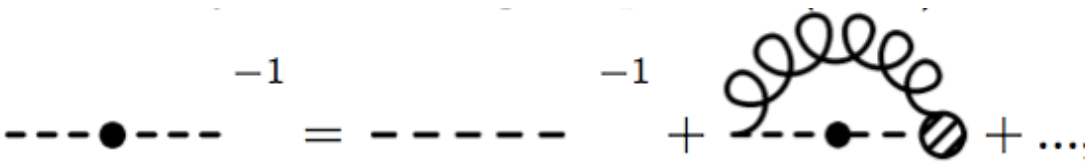
→ implies a TeV-ish condensate for φ to obtain $<\Phi> = 246$ GeV

- **Many model building possibilities / phenomenological aspects:**
  - φ could be an effective field of some hidden sector DSB
  - further particles could exist in hidden sector; e.g. confining…
  - extra hidden U(1) potentially problematic ←→ U(1) mixing
  - avoid Yukawas which couple visible and hidden sector

→phenomenology safe due to Higgs portal →suppressed TeV-ish BSM physics!

Rather minimalistic: SM + QCD Scalar S

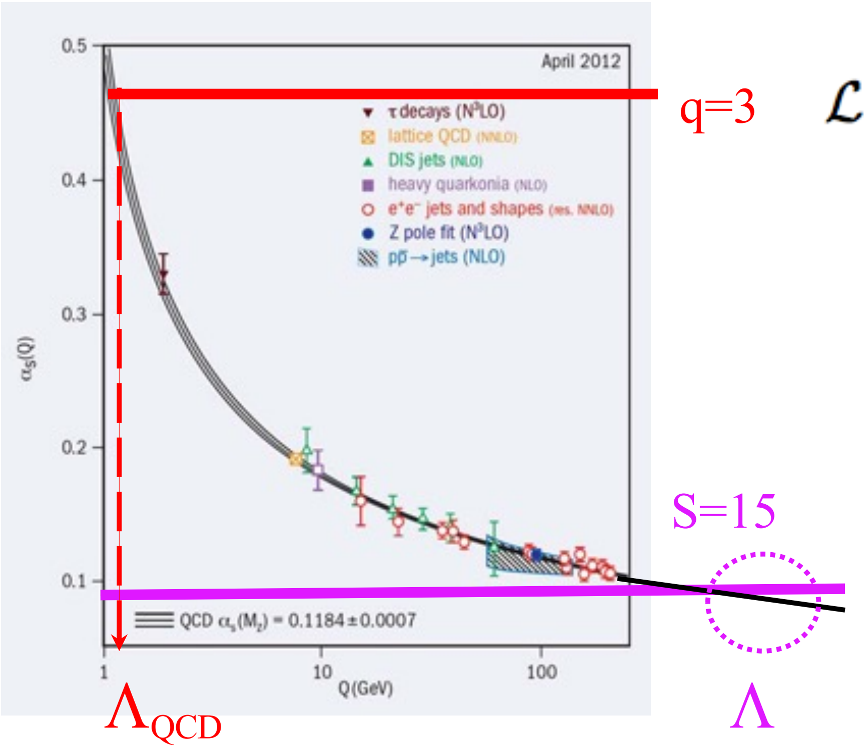
J. Kubo, K.S. Lim, ML New scalar representation S → QCD gap equation:



→ $\boxed{C_2(S)\alpha(\Lambda) \gtrsim X}$

$C_2(\Lambda)$ increases with larger representations

←→ condensation for smaller values of running α



$$\mathcal{L} = \mathcal{L}_{\mathrm{SM},m^2\to 0} + (D_{\mu,ij}S_j)^\dagger (D^\mu_{ik}S_k)$$
$$+ \lambda_{HS} H^\dagger H S^\dagger S - \lambda_{\mathbf{1}_i}\left[\bar{\boldsymbol{S}} \times \boldsymbol{S} \times \bar{\boldsymbol{S}} \times \boldsymbol{S}\right]_{\mathbf{1}_i}$$

$$\lambda_{HS}\langle S^\dagger S\rangle H^\dagger H \to \lambda_{HS}\Lambda^2 H^\dagger H$$

$$m_h^2 = 2\lambda_{HS}\Lambda^2 \qquad \frac{\lambda_h}{\lambda_{HS}} = \frac{\Lambda^2}{v^2}$$

SM ⊗ hidden $SU(3)_H$ Gauge Sector

Holthausen, Kubo, Lim, ML

- hidden $SU(3)_H$:

$$\mathcal{L}_{\rm H} = -\frac{1}{2}{\rm Tr}\ F^2 + {\rm Tr}\ \bar{\psi}(i\gamma^\mu D_\mu - yS)\psi$$

gauge fields ; $\psi = 3_H$ with $SU(3)_F$; **S = real singlet scalar**

- SM coupled by S via a Higgs portal:

$$V_{\rm SM+S} = \lambda_H (H^\dagger H)^2 + \frac{1}{4}\lambda_S S^4 - \frac{1}{2}\lambda_{HS} S^2 (H^\dagger H)$$

- no scalar mass terms
- use similarity to QCD, use NJL approximation, …
- χ–ral symmetry breaking in hidden sector: $SU(3)_L x SU(3)_R$ → $SU(3)_V$ ➔ **generation of TeV scale**

➔ **transferred into the SM sector through the singlet S**

➔ **dark pions are PGBs: naturally stable ➔ DM**

Realizing the Idea: Many more Models

SM + extra singlet or doublet: Φ, φ
Nicolai, Meissner Farzinnia, He, Ren, Foot, Kobakhidze, Volkas, Hill, …

Minimal B-L extension if SM: $SU(3)c \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$
Iso, Okada, Orikasa

Minimal LR-model: $SU(3)c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ Holthausen, ML, Schmidt

SM ⊗ $SU(N)_H$ with new N-plet in a hidden sector
Ko, Carone, Ramos, Holthausen, Kubo, Lim, ML, Hambye, Strumia , …

SM + QCD colored scalar which condenses at TeV scale Kubo, Lim, ML

SM ⊗ $[SU(2)_X \otimes U(1)_X]$
Altmannshofer, Bardeen, Bauer, Carena, Lykken

… more ...

Since the SM-only version does not work ➔ observable effects:

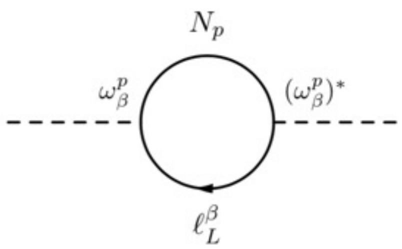
- Higgs coupling to other scalars (singlet, hidden sector, …)
- dark matter candidates ←→ hidden sectors & Higgs portals
- consequences for neutrino masses

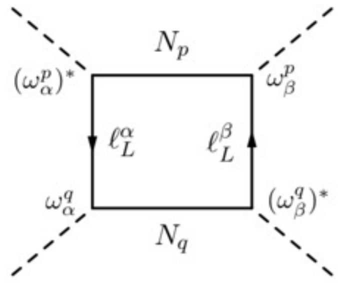
The Neutrino Option

Connection between EWSB and neutrinos ⇔ ν-hierarchy problem

Neutrino option: Brivio, Trott

➔ symmetry breaking V_{eff} from neutrino loops





Conformal Realization of the Neutrino Option: Brdar, Emonds, Helmboldt, ML

➔ conformal symmetry + V_{eff} from neutrino loops (not from Higgs portal)

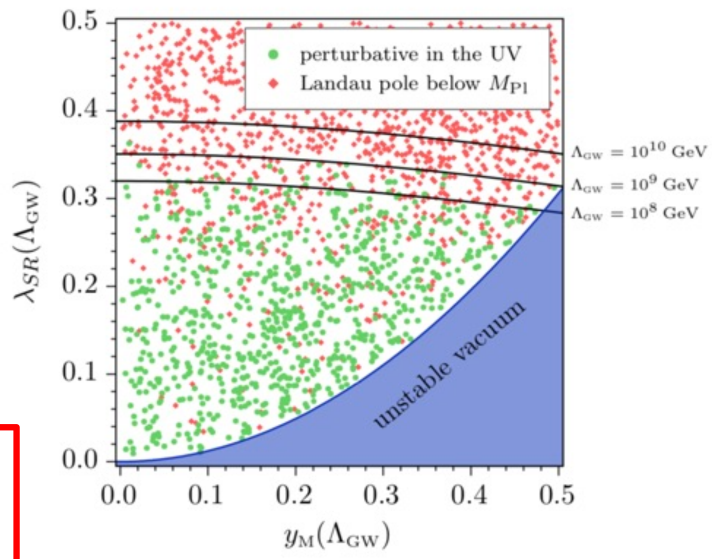
Fields: SM + 3x NR + 2x scalar SM singlets: S, R

$$\mathcal{L} \supseteq \frac{1}{2}\partial_\mu S \partial^\mu S + \frac{1}{2}\partial_\mu R \partial^\mu R + \mathrm{i}\bar{N}_R \partial\!\!\!/ N_R \\ - V(H,S,R) - \left(\frac{1}{2} y_\mathrm{M} S \bar{N}_R N_R^c + y_\nu \bar{L}\tilde{H} N_R + \mathrm{h.c.}\right)$$

➔ consistent UV-complete realization of the idea

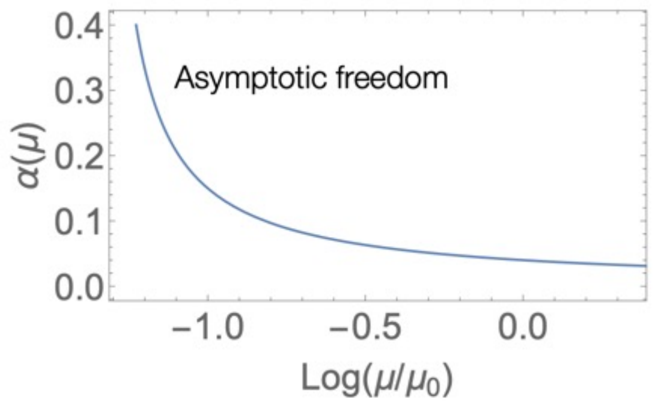
➔ very nice feature:

$$\boxed{\lambda_{HS} \ll \frac{3}{16\pi^2} y_\nu^2(\Lambda_\mathrm{GW}) \cdot y_\mathrm{M}^2(\Lambda_\mathrm{GW}) \simeq O(10^{-12})}$$

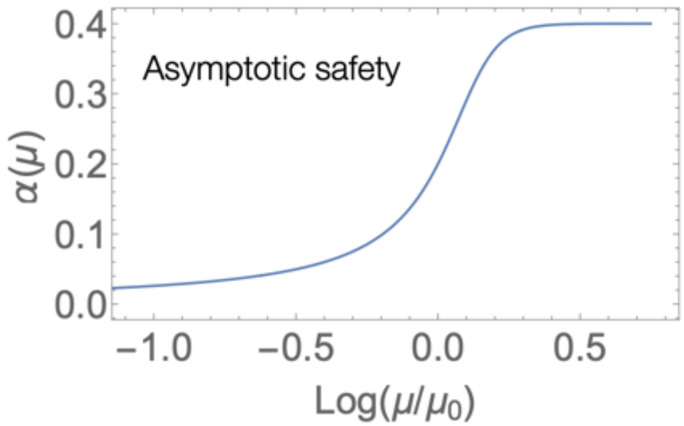


UV-Completion

Successful theories should have a meaningful UV-completion
➔ vanishing β-functions for all couplings (UV fixedpoints) ←→ restored scale symmetry



Interacting UV-fixedpoint ➔
← trivial fixedpoint



Interacting UV-fixedpoints:
- requires carefully selected particle content ➔ explanation?
- scalar self-couplings and Yukawa couplings tend to have Landau poles...

Better trivial fixedpoints:
- no fundamental scalars
- no Yukawa couplings
  ➔ all sclars composite
  ➔ automatically safe models


Conformal Little Higgs

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to appear

Conformal Symmetry & Neutrino Masses

ML, S. Schmidt and J.Smirnov

- No explicit scale → no explicit (Dirac or Majorana) mass term
  → only Yukawa couplings ⊗ generic scales

- Enlarge the Standard Model field spectrum
  like in 0706.1829 - R. Foot, A. Kobakhidze, K.L. McDonald, R. Volkas

- Consider direct product groups: SM ⊗ HS

- Two scales: $\boxed{\text{CS breaking scale at O(TeV) + induced EW scale}}$

Important consequence for fermion mass terms:
→ spectrum of Yukawa couplings ⊗TeV or ⊗EW scale
→ interesting consequences ←→ Majorana mass terms are no longer expected at the generic L-breaking scale → anywhere

Examples

$$\mathcal{M} = \begin{pmatrix} 0 & y_D\langle H\rangle \\ y_D^T\langle H\rangle & y_M\langle\phi\rangle \end{pmatrix}$$

Yukawa seesaw:

SM + ν_R + singlet

$$\langle\phi\rangle \approx \text{TeV}$$

$$\langle H\rangle \approx 1/4\,\text{TeV}$$

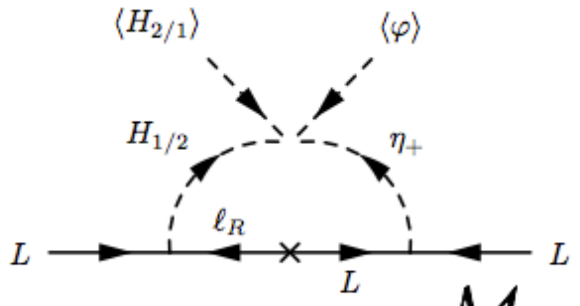
➔ generically expect a TeV seesaw

BUT: y_M can be tiny

➔ wide range of sterile masses ➔ including pseudo-Dirac case

➔ suppressed $0\nu\beta\beta$

Radiative masses



$$\mathcal{M} = m_L \quad \text{or}$$

$$\mathcal{M} = \begin{pmatrix} \mu_1 & y_D\langle H\rangle \\ y_D^T\langle H\rangle & \mu_2 \end{pmatrix}$$

➔pseudo-Dirac case

The punch line:

all usual neutrino mass terms can be generated

➔ suitable scalars required

➔ no explicit masses: all via Yukawa couplings

➔ different numerical expectations ←→ could easily explain keV masses

Conformal Symmetry & Dark Matter

➔ **see talk by Aqeel Ahmed**

Different natural and viable options:

1) eV, **keV = DM**, TeV, … sterile neutrino mass easily possible ←→ not so easy in standard see-saw's

2) New particles which are fundamental or composite DM candidates:
   - hidden sector pseudo-Goldstone-bosons
   - stable color neutral bound states from new QCD representations

➔ some look like WIMPs

➔ others are extremely weakly coupled (via Higgs portal)

➔ or even coupled to QCD (threshold suppressed…)

Including the Planck Scale

The Planck Scale from CS Breaking

Conformal Gravity (CG):

- more symmetry ➔ CG claimed to be power counting renormalizable
- CG may have a ghost... ➔ see later

Idea: Generate M_{Planck} from **conformal gravity ⊗ SU(N)**

➔ gauge assisted condensate via SU(N) field

➔ M_{Planck} becomes an effective scale

Kubo, ML, Schmitz, Yamada similar ideas: **Donoghue, Menezes, ...**

$$S_C = \int d^4x \sqrt{-g} \left[-\hat{\beta}\, S^\dagger S\, R + \hat{\gamma}\, R^2 - \frac{1}{2} \mathrm{Tr}\, F^2 + \right.$$
$$\left. + g^{\mu\nu} (D_\mu S)^\dagger D_\nu S - \hat{\lambda} (S^\dagger S)^2 + a\, R_{\mu\nu} R^{\mu\nu} + b\, R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} \right]$$

R = Ricci curvature scalar, $R_{\mu\nu}$ = Ricci tensor, $R_{\mu\nu\alpha\beta}$ = Riemann tensor

F = field-strength tensor of the $SU(N_c)$ gauge theory, **S = complex scalar in fund. rep. ➔ N_c**

➔ most general diffeomorphism invariance, gauge invariance, and global scale invariance

Condensation in SU(N_c) gauge sector

➔ **dimensional transmutation:** $\langle S^{+}S\rangle$ ➔ effective Planck mass

$$\boxed{M_{planck} = 2\,\beta\,f_0 = \frac{N_c\beta}{16\pi^2}\,(2\,\lambda\,f_0)\left(1 + 2\,\ln\frac{2\,\lambda\,f_0}{\Lambda^2}\right) \quad \text{with} \quad f_0 = \langle S^{+}S\rangle}$$

➔ Effectively normal gravity with a dynamically generated M_{Planck}

The Ghost Problem in quadratic Gravity

Unlike GR, **quadratic gravity is renormalizable** thanks to four derivatives of the metric

$$\mathcal{L}_{\text{EH}} = \sqrt{-g}\underbrace{M_{\text{pl}}^2}_{\text{dimensionful}} R \qquad \mathcal{L}_{\text{QG}} = \sqrt{-g}\Big(-\beta\phi^2 R + \gamma R^2 - \kappa C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} \Big)$$

dimensionless

Problem: Double pole ➔ **classical Ostrogradsky instability**

$$\Delta_{hh} \sim \frac{1}{p^2} - \frac{1}{(p^2 - m_{\text{gh}}^2)} \quad \Rightarrow \quad \mathcal{H} \sim c_+\pi_+^2 - c_-\pi_-^2 + \cdots$$

unbounded Hamiltonian

Leads after quantization to negative norm states ➔ **unitarity violation**

$$\begin{aligned} \left[\hat{a}_h(\mathbf{p}),\, \hat{a}_h^\dagger(\mathbf{q})\right] &= \delta^3(\mathbf{p}-\mathbf{q}) \\ \left[\hat{a}_H(\mathbf{p}),\, \hat{a}_H^\dagger(\mathbf{q})\right] &= -\delta^3(\mathbf{p}-\mathbf{q}) \end{aligned} \quad \Rightarrow \quad \sum_n \left| \langle n|S|\alpha\rangle \right|^2 \neq 1$$

breakdown of probability interpretation

Potential Solutions

- Remove ghosts from asymptotic spectrum Lee-Wick-style
  - Quantize ghosts as “fakeons” that don’t appear by definition [Anselmi 1801.00915]
  - Demonstrate ghosts are unstable with nice decay products [Donoghue, Menezes 1908.02416]

- Use alternative quantization procedures
  - Define generalized QM norm [Salvio 1907.00983]
  - Employ (non-Hermitian) *PT*-symmetric QFT [Bender, Mannheim 0706.0207]

- **Unitarity OK if interaction energies are below the ghost mass**
  ➔ conformal theories OK if ghost becomes massive after SSB
  $M_{ghost} \simeq M_{Planck}$ ➔ no unitarity violation except in the early (pre-inflation) universe
  Kubo, Kuntz 2202.08298, 2208.12832

Summary

- **Explaining masses without masses ➔ conformal symmetry**
  → inspiring SM features...
  → close, but does not work

- **SM embedings into QFTs with conformal symmetry**
  → combined conformal & electro-weak symmetry breaking
  → implications for BSM phenomenology
  → implications for Higgs couplings, neutrino physics, dark matter, …
  ➔ testable consequences: @LHC, dark matter, neutrinos

- **Planck scale generation by gauge induced breaking of conformal GR**
  → very nice phenomenology: inflation…
  → consistent quantum gravity: renormalizability!, ghosts?
  ←→ normal GR from a theory with more symmetry

Backup

Dilaton-Scalaron Inflation

Effective Jordan-frame Lagrangian:

$$\frac{\mathcal{L}^J_{\text{eff}}}{\sqrt{-g_J}} = -\frac{1}{2} B(\chi)\, M^2_{\text{Pl}}\, R_J + G(\chi)\, R^2_J + \frac{1}{2} g^{\mu\nu}_J \partial_\mu \chi\, \partial_\nu \chi - U(\chi)$$ ➔ auxiliary field Ψ ➔

$$\frac{\mathcal{L}^J_{\text{eff}}}{\sqrt{-g_J}} = -\left[\frac{1}{2} B(\chi)\, M^2_{\text{Pl}} - 2\, G(\chi)\, \psi\right] R_J + \frac{1}{2} g^{\mu\nu}_J \partial_\mu \chi\, \partial_\nu \chi - U(\chi) - G(\chi)\, \psi^2$$

Weyl rescaling: $g_{\mu\nu} = \Omega^2\, g^J_{\mu\nu}$ $\qquad \Omega^2 = e^{\Phi(\phi)}\,, \qquad \Phi(\phi) = \dfrac{\sqrt{2}\,\phi}{\sqrt{3}\, M_{\text{Pl}}}$

Einstein-frame scalar potential:

$$V(\chi, \phi) = e^{-2\,\Phi(\phi)} \left[U(\chi) + \frac{M^4_{\text{Pl}}}{16\, G(\chi)} \left(B(\chi) - e^{\Phi(\phi)} \right)^2 \right]$$

➔ Slow role inflation

➔ fits data very well!