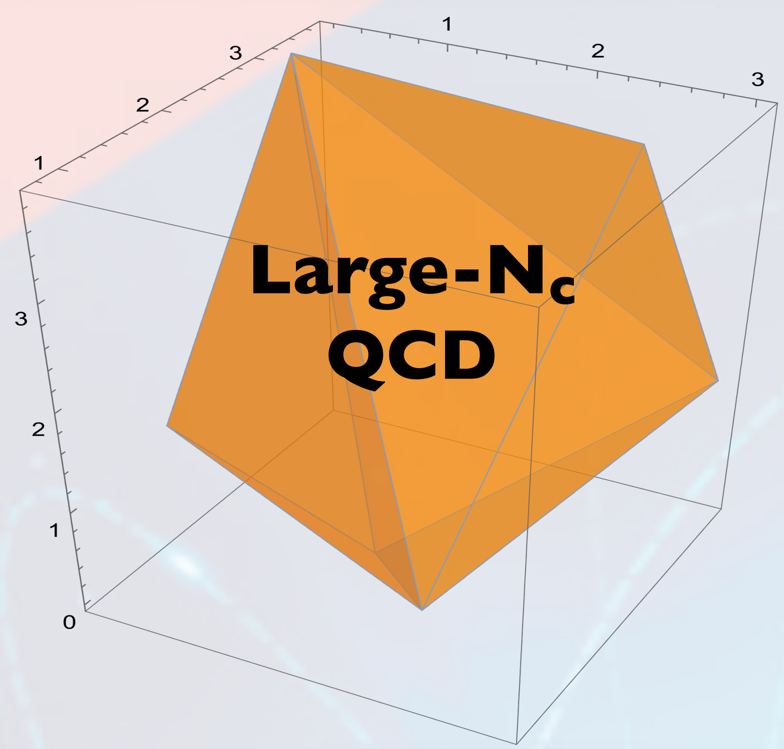
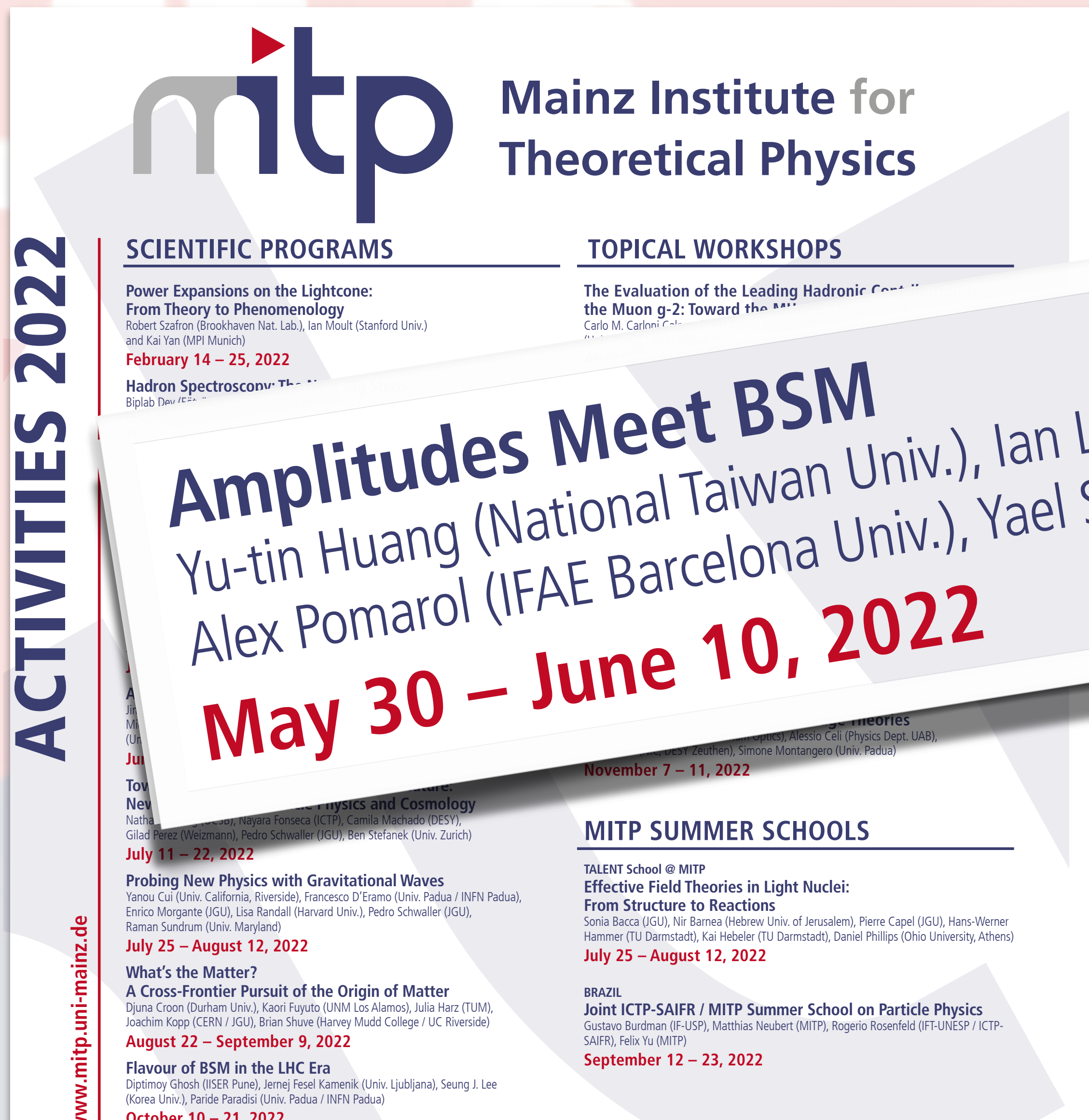
Cornering Large-N_c QCD with Positivity Bounds



Alex Pomarol, IFAE & UAB (Barcelona)

based on 2211.12488 [hep-th] with C. Fernandez, F. Riva and F. Sciotti

Cornering Large-N_c QCD with Positivity Bounds



...ol, IFAE & UAB (Barcelona)

2211.12488 [hep-th] with C. Fernandez, F. Riva and F. Sciotti

TH CERN football team ~94

Motivation

- Understand better **Strongly-coupled theories** as plays an important role in nature, e.g. QCD
- They could also play an **important role BSM:**
  - Higgs composite
  - Dark Matter
- To understand their physics, **simplifying techniques** are essential

Best examples:
- Taking $N_c \to \infty$ of $SU(N_c)$ (**large-N_c** limit)
- **Holography**: $CFT_4 \leftrightarrow AdS_5$

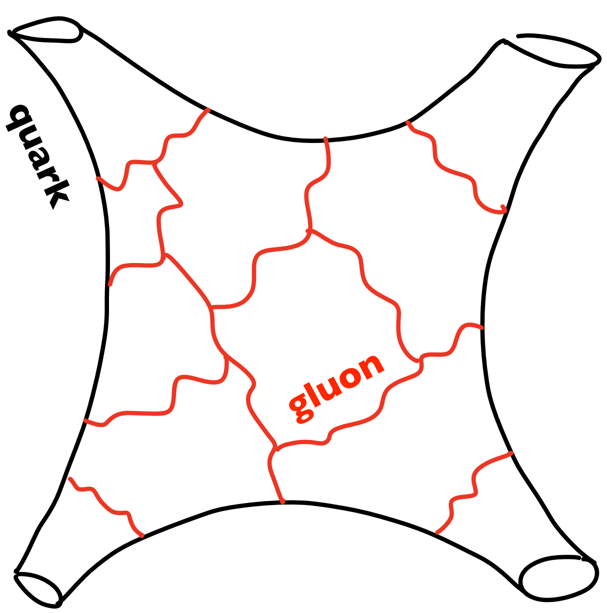
☞ **Positivity bounds** can help to better understand them

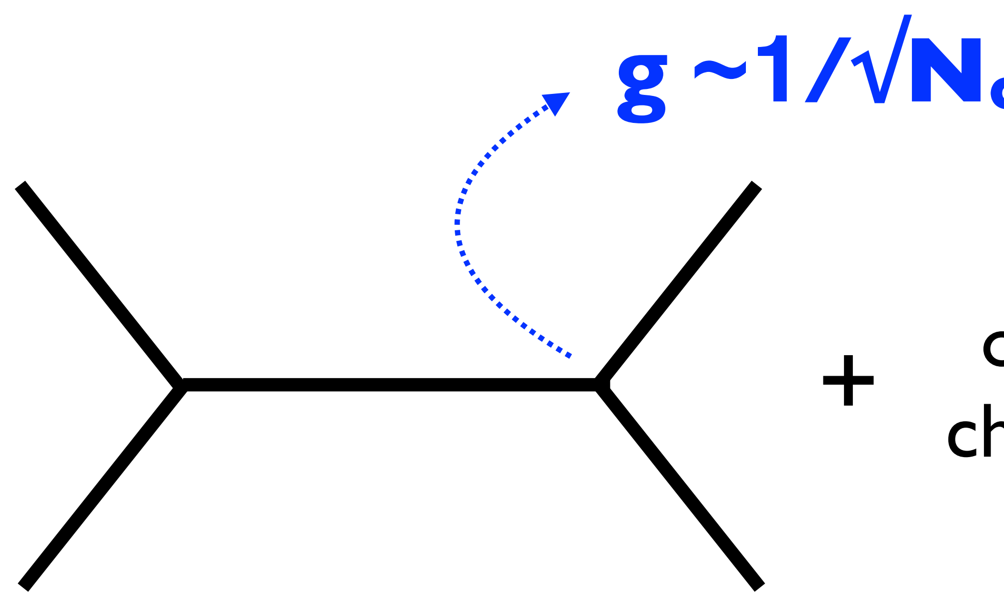
Large N_c limit

G. 't Hooft, Nucl. Phys. B 72, 461 (1974)

E. Witten, Nucl. Phys. B 160, 57 (1979)

☞ **Amplitudes are mediated by weakly-coupled color-less states (mesons)**





+ other channels

quarks, gluons
SU(N_c)

$N_c \to \infty$

mesons ($q\bar{q}$ states), glueballs,
baryons (skyrmions)

Large N_c limit

G. 't Hooft, Nucl. Phys. B 72, 461 (1974)

E. Witten, Nucl. Phys. B 160, 57 (1979)

Powerful simplification
but still difficult to get predictions

Theory of <u>infinite</u> mesons of different spin J,
with unknown couplings and masses

f_0 (J=0), ρ (J=1), f_2 (J=2), ρ_3 (J=3), …

(as in real QCD)

$$\frac{\Gamma}{m} \sim \frac{1}{3}$$

Positivity bounds

N. Arkani-Hamed, T.-C. Huang, and Y.-T. Huang, arXiv: 2012. 15849

C. de Rham, S. Melville, A. J. Tolley, and S.-Y. Zhou, arXiv: 1702.06134

B. Bellazzini, J. Elias Miro´, R. Rattazzi, M. Riembau, and F. Riva, arXiv: 2011.00037

A.Sinha and A. Zahed, arXiv: 2012.04877

A.J. Tolley, Z.-Y. Wang, and S.-Y. Zhou, arXiv: 2011.02400

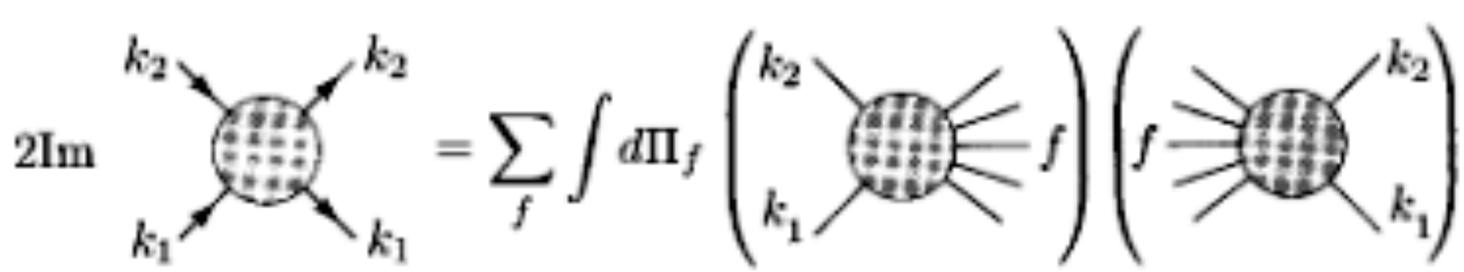
S. Caron-Huot and V. Van Duong, arXiv: 2011.02957

S. Caron-Huot, D. Mazac, L. Rastelli, and D. Simmons-Duffin, arXiv: 2102.08951

and much more…

- **Generalizations of the optical theorem**

forward limit:



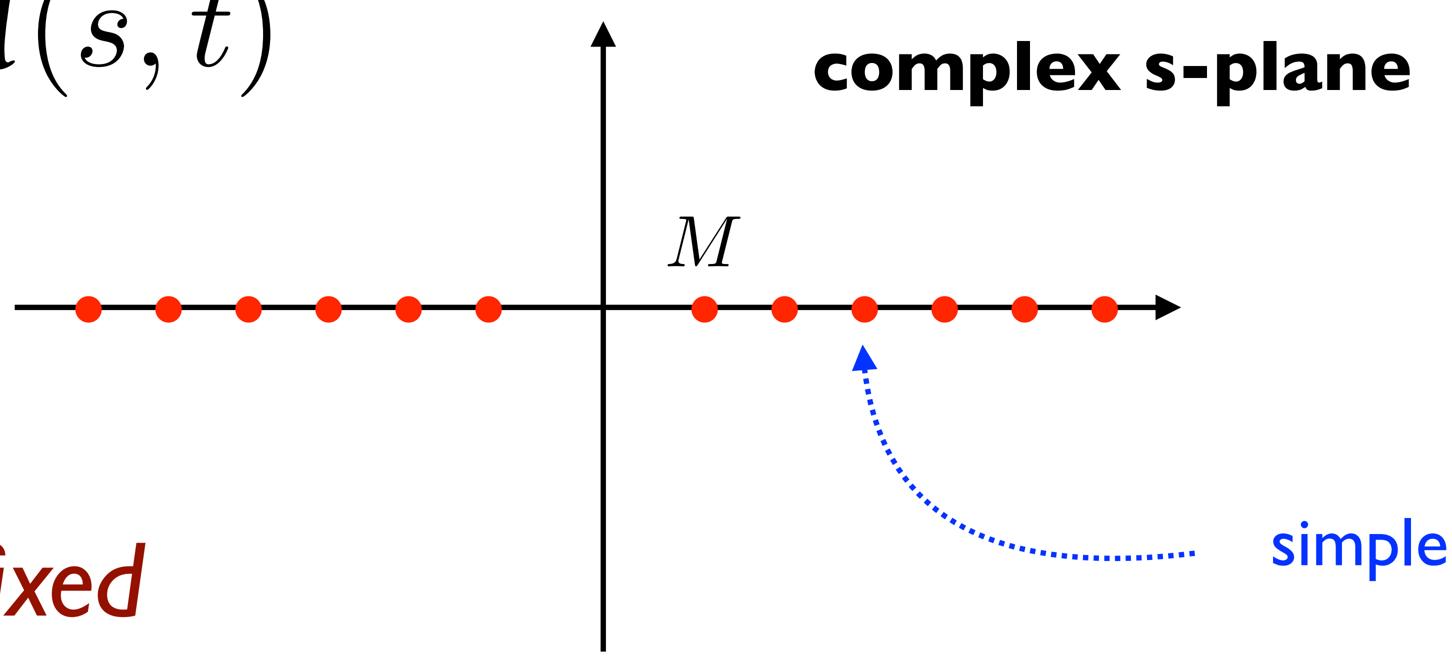
≥ 0

Peskin & Schroeder

Positivity bounds in Large N_c QCD

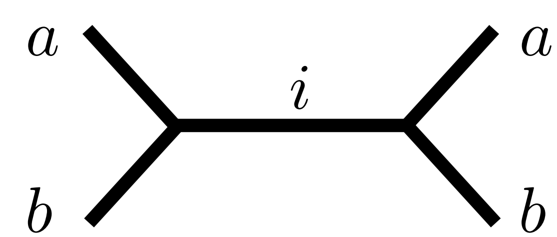
Analytical structure of amplitudes:

$\mathcal{M}(s,t)$



u fixed

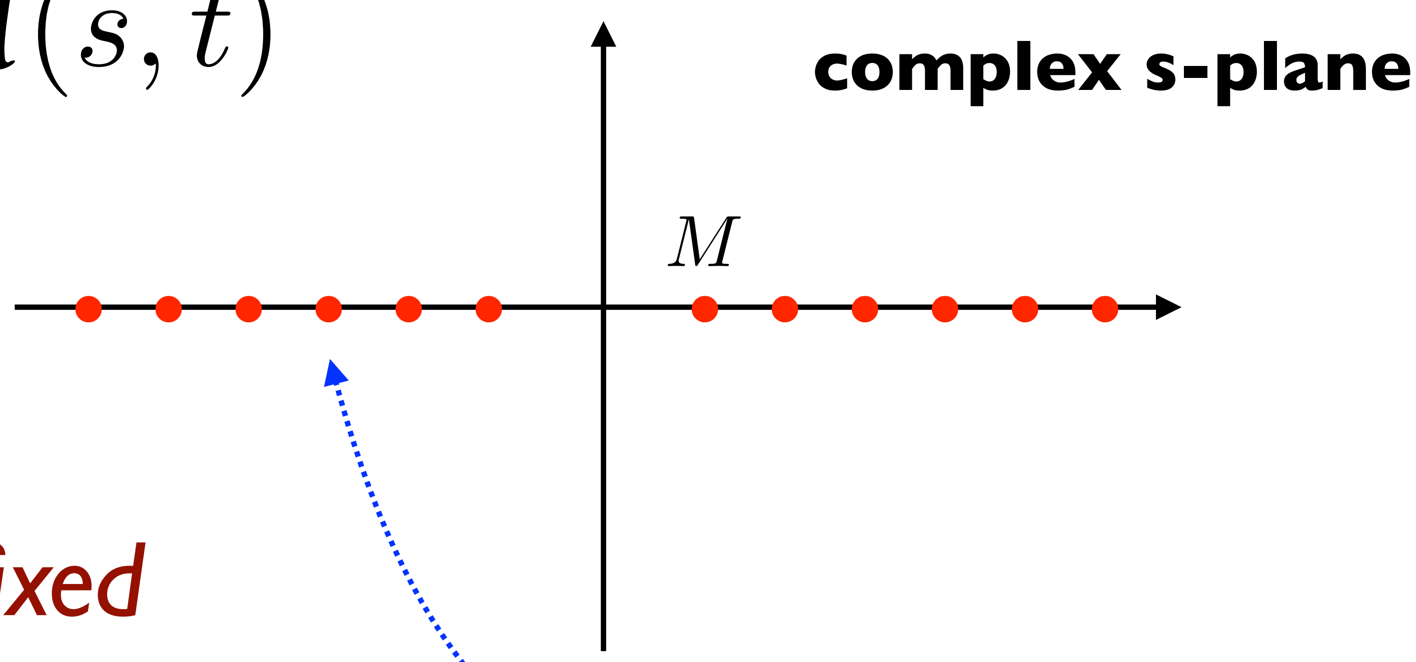
simple poles associated to the mesons:



Positivity bounds in Large N_c QCD

Analytical structure of amplitudes:

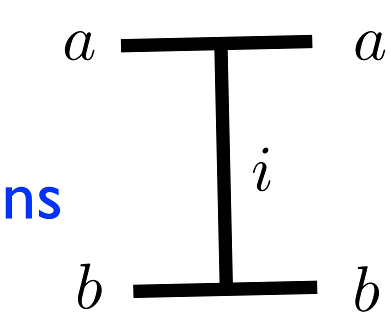
$$\mathcal{M}(s,t)$$



u fixed

$s = -t-u-4m^2$

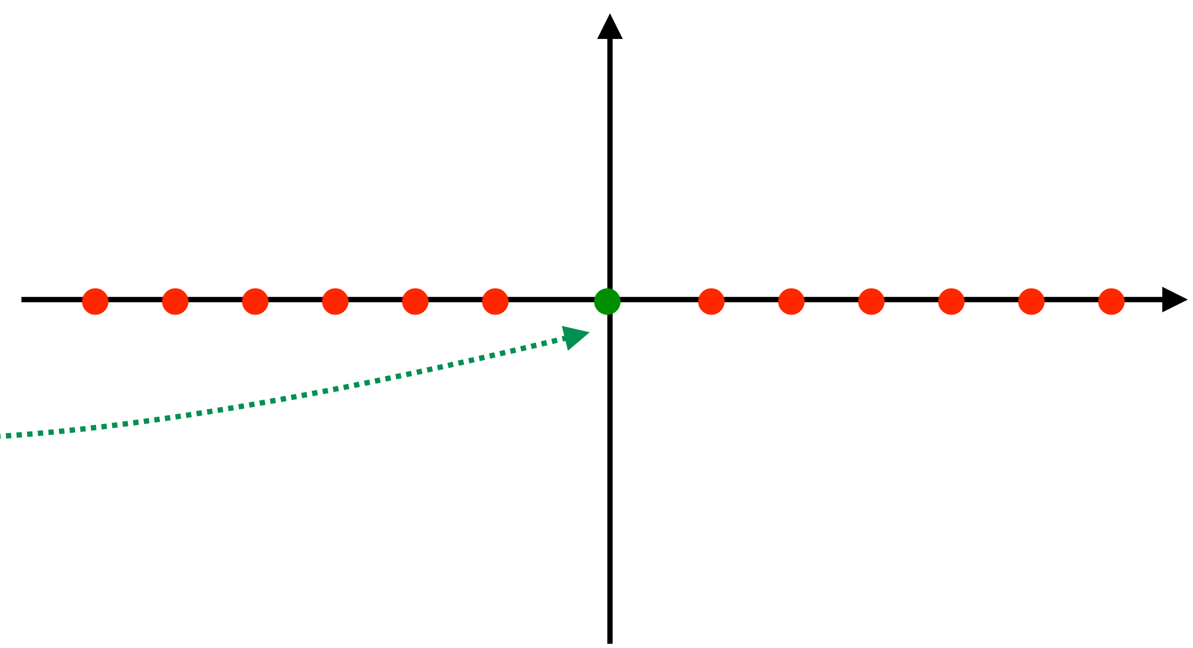
simple poles associated to the mesons



Positivity bounds in Large N_c QCD

This simple structure allows to get dispersion relations:

$$\frac{\mathcal{M}(s,t)}{s^{k+1}}$$



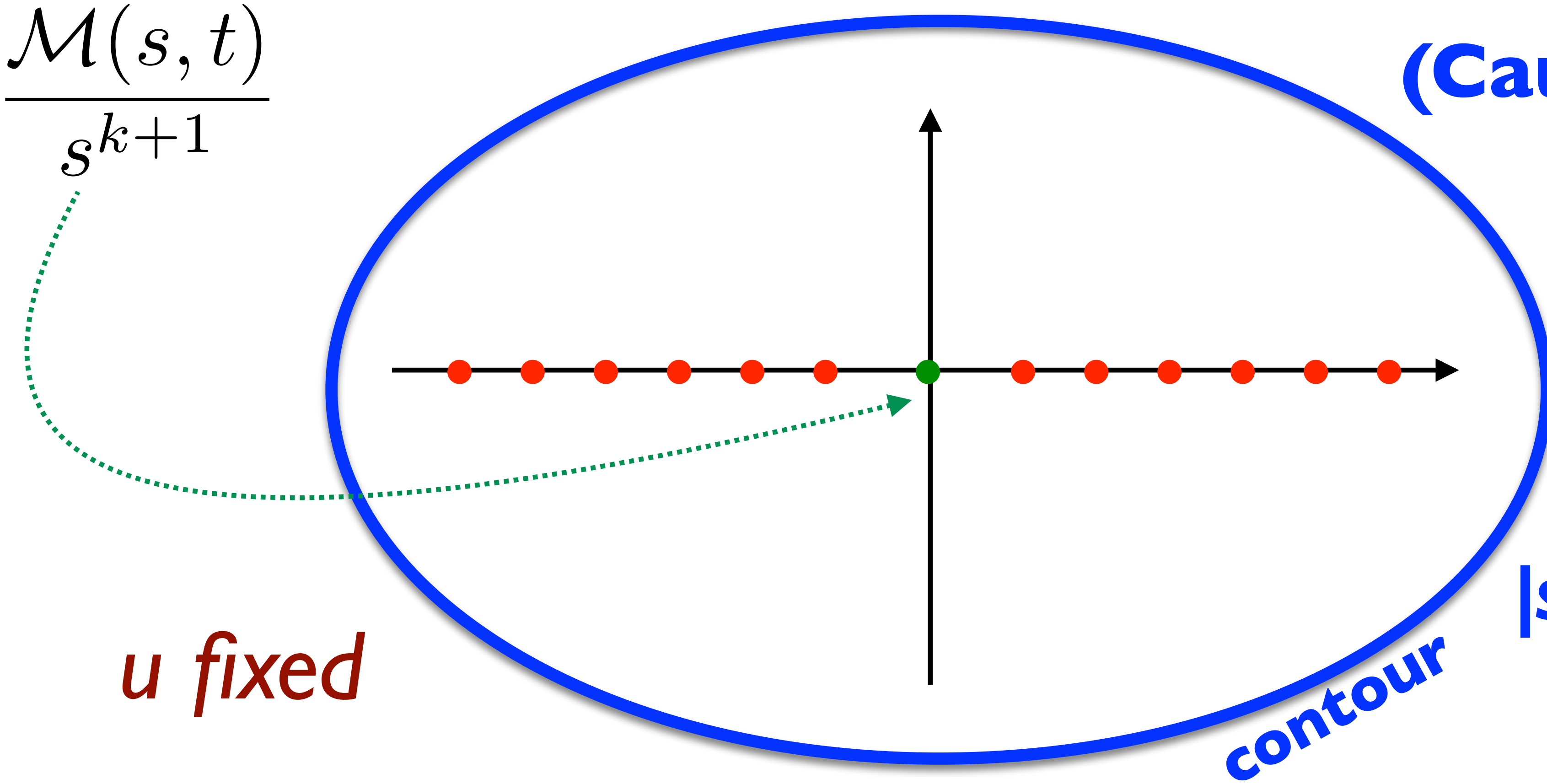
u fixed

Positivity bounds in Large N_c QCD

This simple structure allows to get dispersion relations:

$$\frac{\mathcal{M}(s,t)}{s^{k+1}}$$

(Cauchy at work)



$$\frac{\mathcal{M}(s,t)}{s^k} \to 0$$

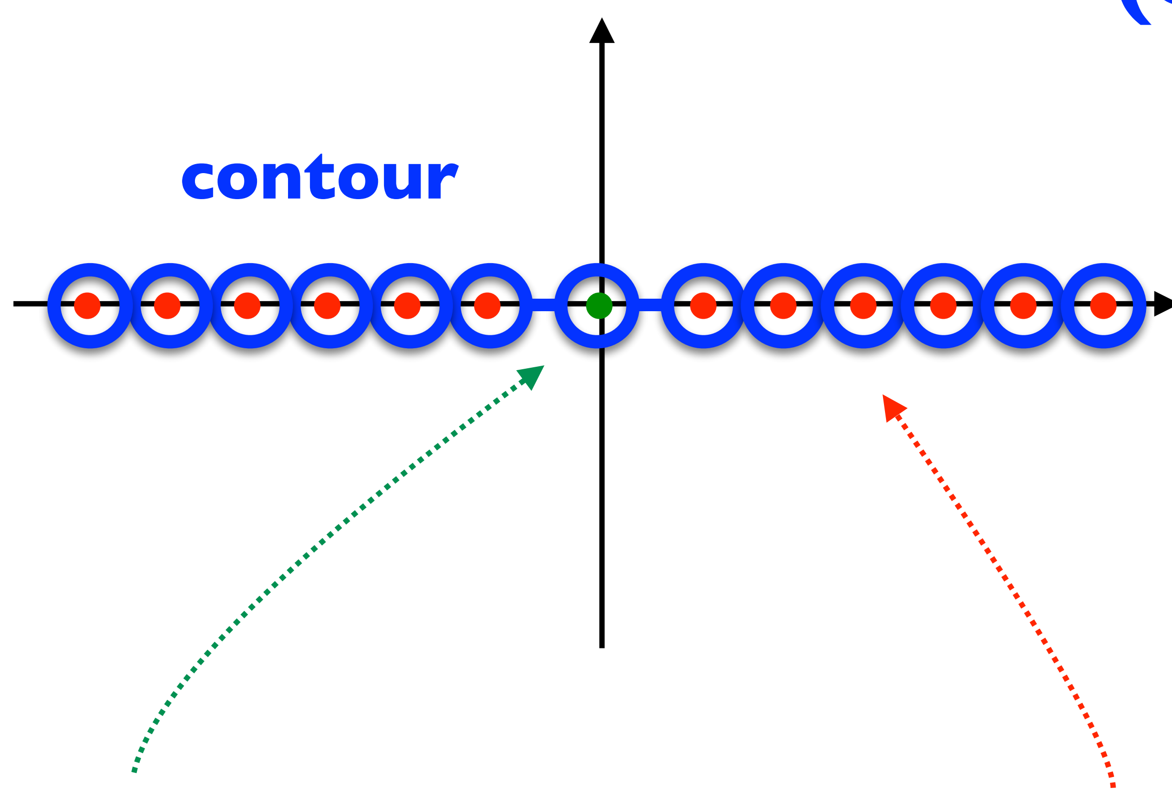
$|s| \to \infty$

Positivity bounds in Large N_c QCD

This simple structure allows to get dispersion relations:

$$\frac{\mathcal{M}(s,t)}{s^{k+1}}$$

(Cauchy at work)



u fixed

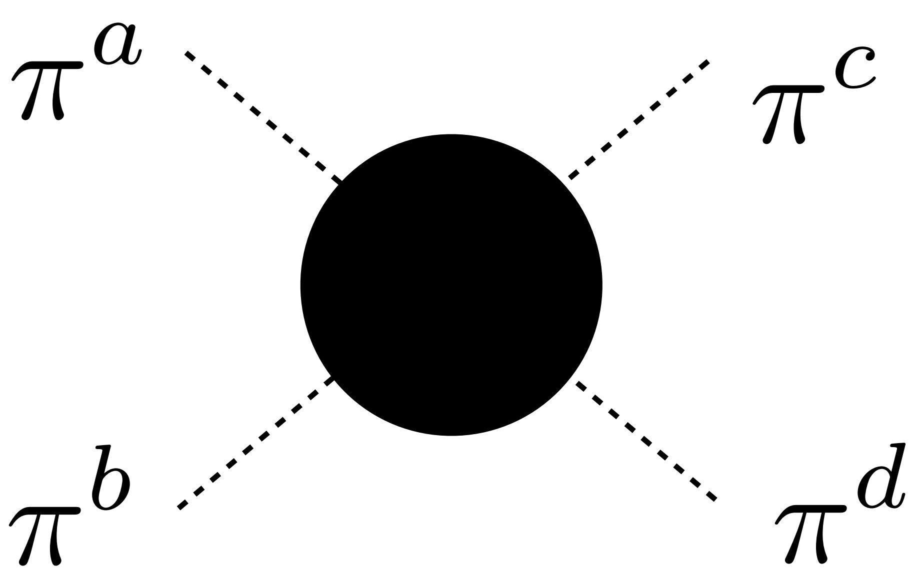
residue at the origin + sum of residues at the mass poles = 0

(low-energy EFT parameters related *to masses and couplings of mesons)*

pion-pion scattering

J. Albert and L. Rastelli, arXiv: 2203.11950

Lets take two flavors: **SU(2)** = Isospin global symmetry



$\pi^a \in \mathbf{3}$ *massless*

Extra condition from large-N_c:

Mesons = $q \;\bar{q}$

Isospin = I = $1/2 \otimes 1/2$ = $0, 1$

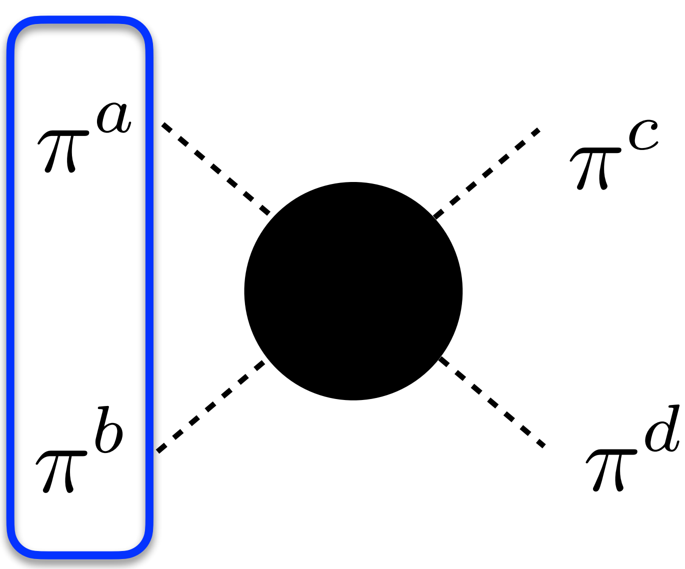
☞ **no I =2 states**

Extra condition from large-N$_c$:

Mesons = $q \ \bar{q}$

Isospin = I = 1/2⊗1/2 = 0, 1

☞ **no I =2 states**



I =2

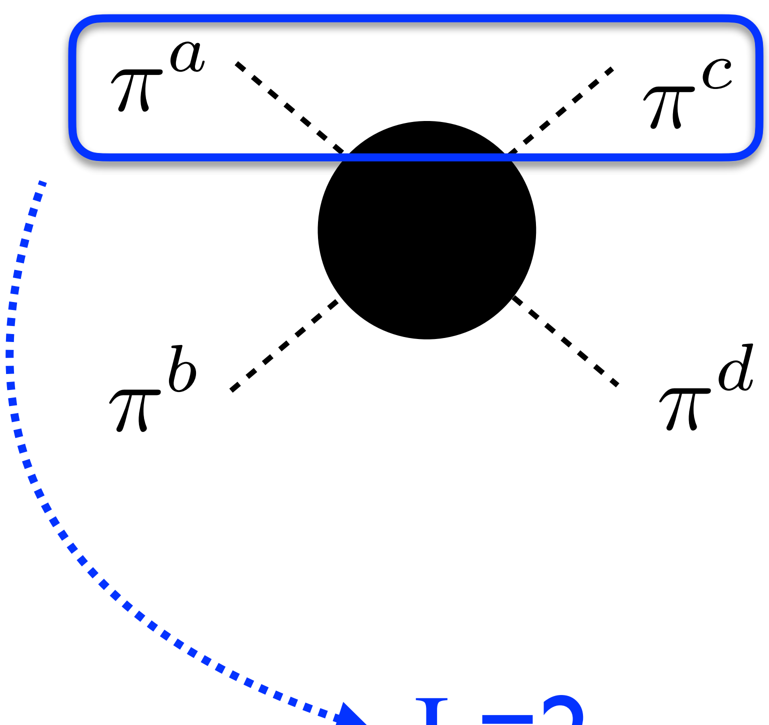
$\mathcal{M}_s^{I=2}$ cannot have poles in s

Extra condition from large-N$_c$:

Mesons = q $\bar{\text{q}}$

Isospin = I = 1/2⊗1/2 = 0, 1

☞ **no I =2 states**



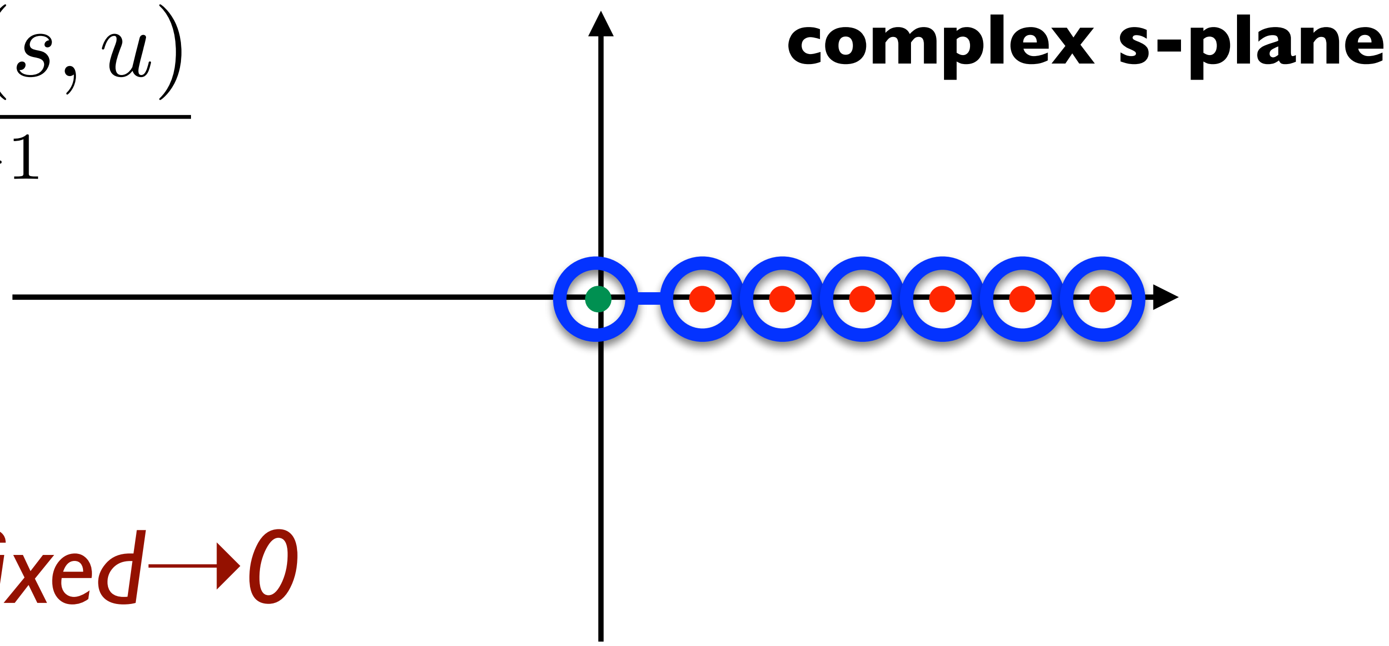
I =2

$\mathcal{M}_t^{I=2}$ cannot have poles in t

Working with $\mathcal{M}_t^{I=2}(s,u)$

(that cannot have poles in the t-channel)

$$\frac{\mathcal{M}_t^{I=2}(s,u)}{s^{k+1}}$$

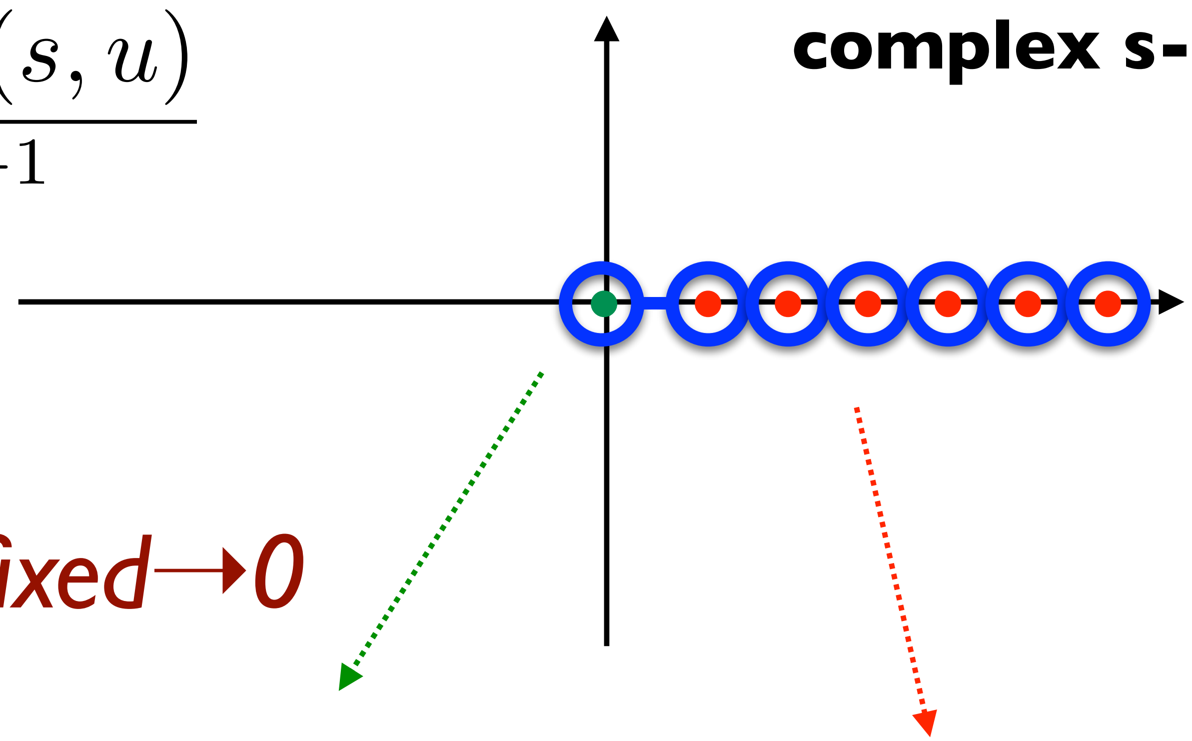


u fixed→0

Working with $\mathcal{M}_t^{I=2}(s,u)$

(that cannot have poles in the t-channel)

$$\frac{\mathcal{M}_t^{I=2}(s,u)}{s^{k+1}}$$



complex s-plane

u fixed→0

partial-wave expansion

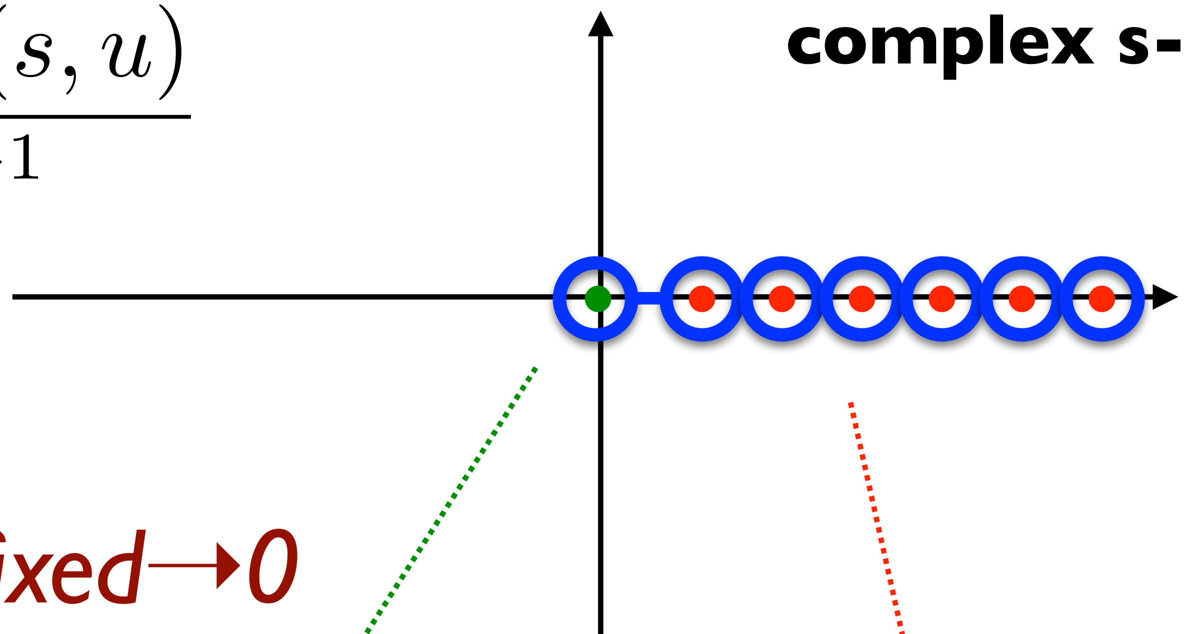
$$\text{Res}\,\frac{\mathcal{M}_t^{I=2}(s,u)}{s^{k+1}} = \sum_i \frac{|g_{\pi\pi\, i}|^2}{m_i^{2k}} P_{J_i}\left(1+\frac{2u}{m_i^2}\right)$$

$$\mathcal{M}_t^{I=2}(s,u) \xrightarrow{s,u\,\to\,0} g_{1,0}(s+u) + g_{2,0}(s^2+u^2) + g_{2,1}su + \cdots$$

Working with $\mathcal{M}_t^{I=2}(s,u)$

(that cannot have poles in the t-channel)

$$\frac{\mathcal{M}_t^{I=2}(s,u)}{s^{k+1}}$$



complex s-plane

u fixed→0

partial-wave expansion

$$\mathrm{Res}\,\frac{\mathcal{M}_t^{I=2}(s,u)}{s^{k+1}} = \sum_i \frac{|g_{\pi\pi\,i}|^2}{m_i^{2k}} P_{J_i}\left(1+\frac{2u}{m_i^2}\right)$$

crossing s↔u

$$\mathcal{M}_t^{I=2}(s,u) \xrightarrow{s,u\to 0} g_{1,0}(s+u) + g_{2,0}(s^2+u^2) + g_{2,1}su + \cdots$$

Legendre pol. and derivatives (all positive!)

small u expansion:

$$k=1: \qquad g_{1,0}+g_{2,1}u+g_{3,1}u^2+\ldots = \sum_i |g_{\pi\pi\,i}|^2\left(\frac{P_{J_i}(1)}{m_i^2}+2\frac{P'_{J_i}(1)}{m_i^4}u+2\frac{P''_{J_i}(1)}{m_i^6}u^2+\ldots\right),$$

$$k=2: \qquad g_{2,0}+g_{3,1}u+g_{4,2}u^2+\ldots = \sum_i |g_{\pi\pi\,i}|^2\left(\frac{P_{J_i}(1)}{m_i^4}+2\frac{P'_{J_i}(1)}{m_i^6}u+2\frac{P''_{J_i}(1)}{m_i^8}u^2+\ldots\right),$$

$$k=3: \qquad g_{3,0}+g_{4,1}u+g_{5,2}u^2+\ldots = \sum_i |g_{\pi\pi\,i}|^2\left(\frac{P_{J_i}(1)}{m_i^6}+2\frac{P'_{J_i}(1)}{m_i^8}u+2\frac{P''_{J_i}(1)}{m_i^{10}}u^2+\ldots\right),$$

$$\vdots$$

Legendre pol. and derivatives (all positive!)

small u expansion:

$$k=1: \qquad g_{1,0}+g_{2,1}u+g_{3,1}u^2+\ldots = \sum_i |g_{\pi\pi\,i}|^2\left(\frac{P_{J_i}(1)}{m_i^2}+2\frac{P'_{J_i}(1)}{m_i^4}u+2\frac{P''_{J_i}(1)}{m_i^6}u^2+\ldots\right),$$

$$k=2: \qquad g_{2,0}+g_{3,1}u+g_{4,2}u^2+\ldots = \sum_i |g_{\pi\pi\,i}|^2\left(\frac{P_{J_i}(1)}{m_i^4}+2\frac{P'_{J_i}(1)}{m_i^6}u+2\frac{P''_{J_i}(1)}{m_i^8}u^2+\ldots\right),$$

$$k=3: \qquad g_{3,0}+g_{4,1}u+g_{5,2}u^2+\ldots = \sum_i |g_{\pi\pi\,i}|^2\left(\frac{P_{J_i}(1)}{m_i^6}+2\frac{P'_{J_i}(1)}{m_i^8}u+2\frac{P''_{J_i}(1)}{m_i^{10}}u^2+\ldots\right),$$

$$\vdots$$

$$\left\{\begin{aligned} g_{n,0} &= \sum_i \frac{g_{i\pi\pi}^2}{m_i^{2n}} \\ g_{n+1,1} &= \sum_i \frac{g_{i\pi\pi}^2\, J_i(J_i+1)}{m_i^{2(n+1)}} \end{aligned}\right.$$

all mesons contribute positively!

small u expansion:

$$k=1: \qquad g_{1,0}+g_{2,1}u+\boxed{g_{3,1}}u^2+\ldots=\sum_i |g_{\pi\pi\, i}|^2\left(\frac{P_{J_i}(1)}{m_i^2}+2\frac{P'_{J_i}(1)}{m_i^4}u+2\frac{P''_{J_i}(1)}{m_i^6}u^2+\ldots\right),$$

$$k=2: \qquad g_{2,0}+\boxed{g_{3,1}}u+g_{4,2}u^2+\ldots=\sum_i |g_{\pi\pi\, i}|^2\left(\frac{P_{J_i}(1)}{m_i^4}+2\frac{P'_{J_i}(1)}{m_i^6}u+2\frac{P''_{J_i}(1)}{m_i^8}u^2+\ldots\right),$$

$$k=3: \qquad g_{3,0}+g_{4,1}u+g_{5,2}u^2+\ldots=\sum_i |g_{\pi\pi\, i}|^2\left(\frac{P_{J_i}(1)}{m_i^6}+2\frac{P'_{J_i}(1)}{m_i^8}u+2\frac{P''_{J_i}(1)}{m_i^{10}}u^2+\ldots\right),$$

$$\vdots$$

due to crossing, overconstrained system!

☞ **infinite constraints in the spectrum and couplings**

small u expansion:

$$k=1: \qquad g_{1,0}+g_{2,1}u+\boxed{g_{3,1}}u^2+\ldots = \sum_i |g_{\pi\pi\,i}|^2\left(\frac{P_{J_i}(1)}{m_i^2}+2\frac{P'_{J_i}(1)}{m_i^4}u+2\frac{P''_{J_i}(1)}{m_i^6}u^2+\ldots\right),$$

$$k=2: \qquad g_{2,0}+\boxed{g_{3,1}}u+g_{4,2}u^2+\ldots = \sum_i |g_{\pi\pi\,i}|^2\left(\frac{P_{J_i}(1)}{m_i^4}+2\frac{P'_{J_i}(1)}{m_i^6}u+2\frac{P''_{J_i}(1)}{m_i^8}u^2+\ldots\right),$$

$$k=3: \qquad g_{3,0}+g_{4,1}u+g_{5,2}u^2+\ldots = \sum_i |g_{\pi\pi\,i}|^2\left(\frac{P_{J_i}(1)}{m_i^6}+2\frac{P'_{J_i}(1)}{m_i^8}u+2\frac{P''_{J_i}(1)}{m_i^{10}}u^2+\ldots\right),$$

$$\vdots$$

due to crossing, overconstrained system!

☞ **infinite constraints in the spectrum and couplings**

e.g. $$\sum_i \frac{|g_{\pi\pi\,i}|^2}{m_i^6} J_i(J_i+1)(J_i-2)(J_i+3)=0$$

scalars do not enter

small u expansion:

$$k=1: \qquad g_{1,0}+g_{2,1}u+\boxed{g_{3,1}}u^2+\ldots=\sum_i |g_{\pi\pi\,i}|^2\left(\frac{P_{J_i}(1)}{m_i^2}+2\frac{P'_{J_i}(1)}{m_i^4}u+2\frac{P''_{J_i}(1)}{m_i^6}u^2+\ldots\right),$$

$$k=2: \qquad g_{2,0}+\boxed{g_{3,1}}u+g_{4,2}u^2+\ldots=\sum_i |g_{\pi\pi\,i}|^2\left(\frac{P_{J_i}(1)}{m_i^4}+2\frac{P'_{J_i}(1)}{m_i^6}u+2\frac{P''_{J_i}(1)}{m_i^8}u^2+\ldots\right),$$

$$k=3: \qquad g_{3,0}+g_{4,1}u+g_{5,2}u^2+\ldots=\sum_i |g_{\pi\pi\,i}|^2\left(\frac{P_{J_i}(1)}{m_i^6}+2\frac{P'_{J_i}(1)}{m_i^8}u+2\frac{P''_{J_i}(1)}{m_i^{10}}u^2+\ldots\right),$$

$$\vdots$$

due to crossing, overconstrained system!

☞ **infinite constraints in the spectrum and couplings**

e.g. $$\sum_i \frac{|g_{\pi\pi\,i}|^2}{m_i^6} J_i(J_i+1)(J_i-2)(J_i+3)=0$$

☞ **also from dispersion relations at fixed t**

Implications of Positivity bounds for Large N$_c$ QCD

Lets assume at $|s| \to \infty$ & either t or u fixed:

$$\frac{\mathcal{M}_t^{I=2}(s,u)}{s} \to 0$$

k=1

$$\sum_i \frac{|g_{\pi\pi\, i}|^2}{m_i^6} J_i(J_i+1)(J_i-2)(J_i+3) = 0$$

$$\frac{|g_{\pi\pi\, 1}|^2}{m_{J=1}^6} = 9\frac{|g_{\pi\pi\, 3}|^2}{m_{J=3}^6} + 35\frac{|g_{\pi\pi\, 4}|^2}{m_{J=4}^6} + \cdots$$

spin-1 must be in the spectrum with the largest coupling to pions

(apart from spin-0)

Vector Meson Dominance (VMD),

assumed in the past to explain QCD experimental data

$$\sum_i \frac{|g_{\pi\pi\, i}|^2}{m_i^6} J_i(J_i+1)(J_i-2)(J_i+3) = 0$$

$$\frac{|g_{\pi\pi\, 1}|^2}{m_{J=1}^6} = 9\frac{|g_{\pi\pi\, 3}|^2}{m_{J=3}^6} + 35\frac{|g_{\pi\pi\, 4}|^2}{m_{J=4}^6} + \cdots$$

spin-1 must be in the spectrum
with the largest coupling to pions

☞ from other constraints, one can shown that **J=2,3,...**
must also be in the spectrum
with couplings to pions that decreases with J

Upper bound on couplings

(normalized to m_i^2/F_π^2)

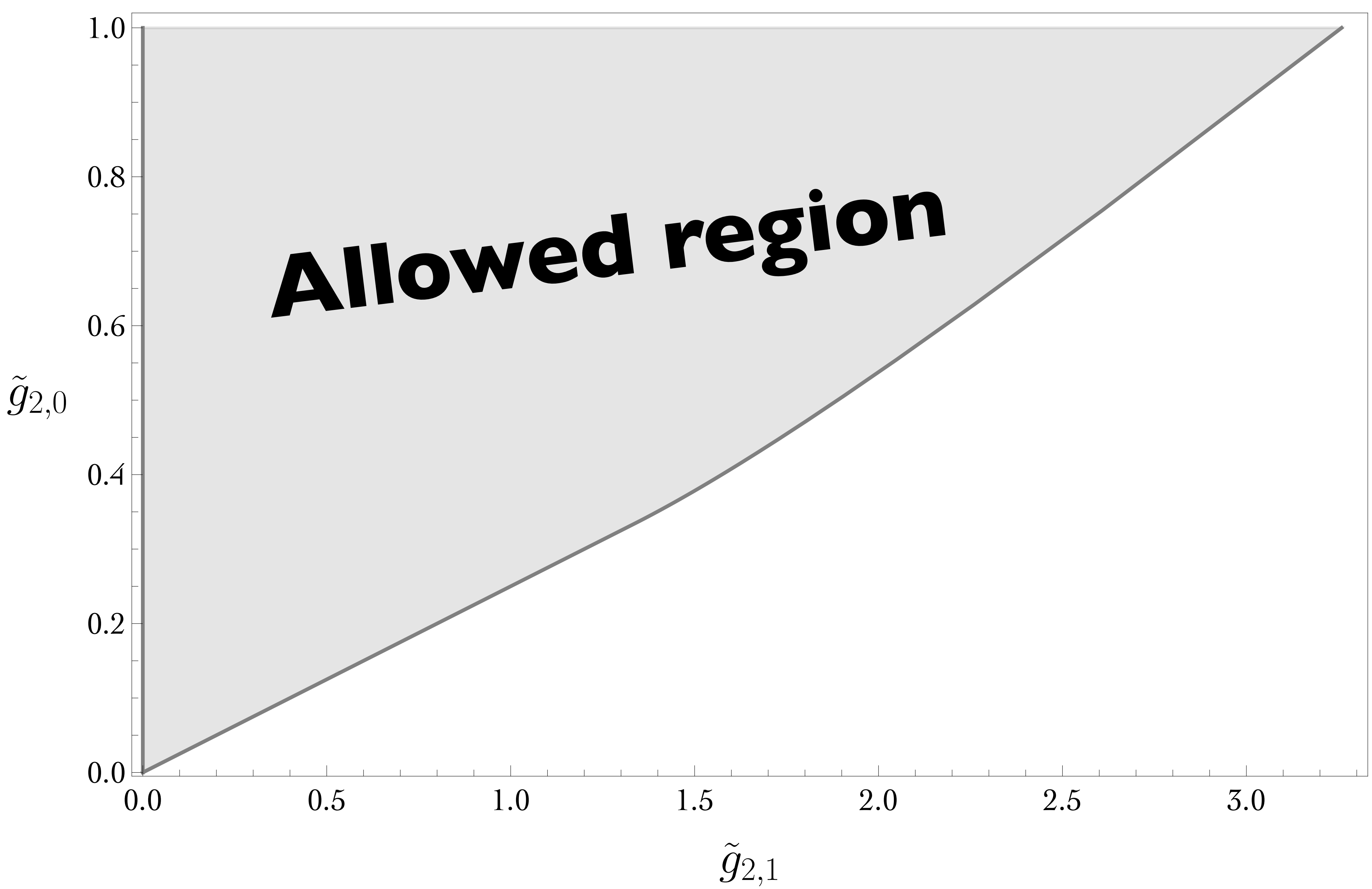
J	$\lvert g_{\pi\pi i}\rvert^2$	Exp. QCD
1	0.78	$\longrightarrow$ 0.5
2	0.18	$\longrightarrow$ 0.18
3	0.03	

Constraints on Wilson coefficients

$$\mathcal{L} = \frac{F_\pi^2}{4}\mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\right) + L_1 \mathrm{Tr}^2\left(\partial_\mu U^\dagger \partial^\mu U\right) + L_2 \mathrm{Tr}\left(\partial_\mu U^\dagger \partial_\nu U\right)\mathrm{Tr}\left(\partial^\mu U^\dagger \partial^\nu U\right) + L_3 \mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U \partial_\nu U^\dagger \partial^\nu U\right)$$

$$\tilde{g}_{2,0} = 4(2L_1 + 3L_2 + L_3)\frac{M^2}{F_\pi^2}\,, \qquad \tilde{g}_{2,1} = 16 L_2 \frac{M^2}{F_\pi^2}$$

mass of the 1st meson



"Polyhedronal" bounds

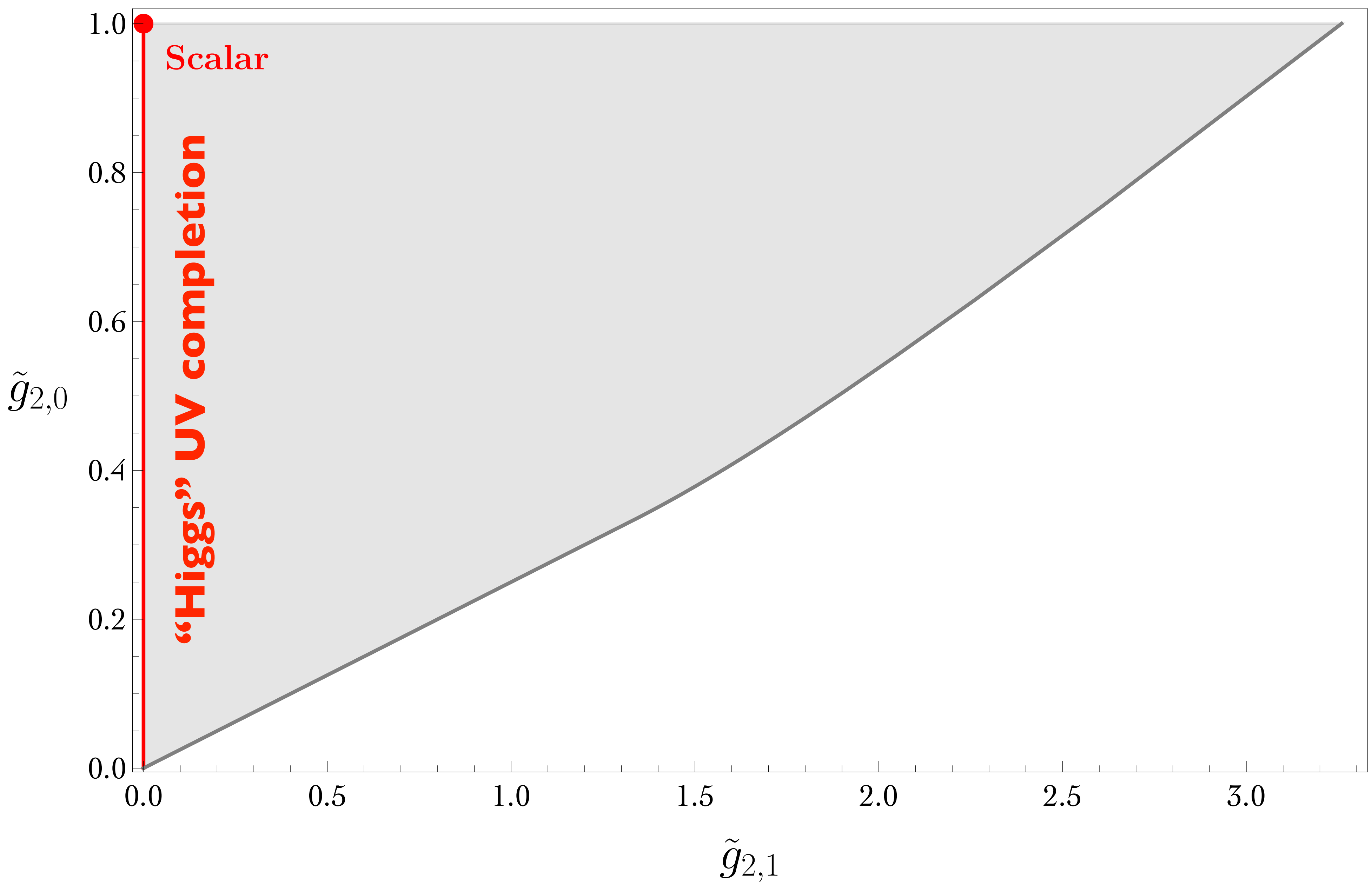
All EFT are "**EFT-hedron**"

Constraints on Wilson coefficients

$$\mathcal{L} = \frac{F_\pi^2}{4}\mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\right) + L_1 \mathrm{Tr}^2\left(\partial_\mu U^\dagger \partial^\mu U\right) + L_2 \mathrm{Tr}\left(\partial_\mu U^\dagger \partial_\nu U\right)\mathrm{Tr}\left(\partial^\mu U^\dagger \partial^\nu U\right) + L_3 \mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U \partial_\nu U^\dagger \partial^\nu U\right)$$

$$\tilde{g}_{2,0} = 4(2L_1 + 3L_2 + L_3)\frac{M^2}{F_\pi^2}\,, \qquad \tilde{g}_{2,1} = 16 L_2 \frac{M^2}{F_\pi^2}$$

mass of the 1st meson

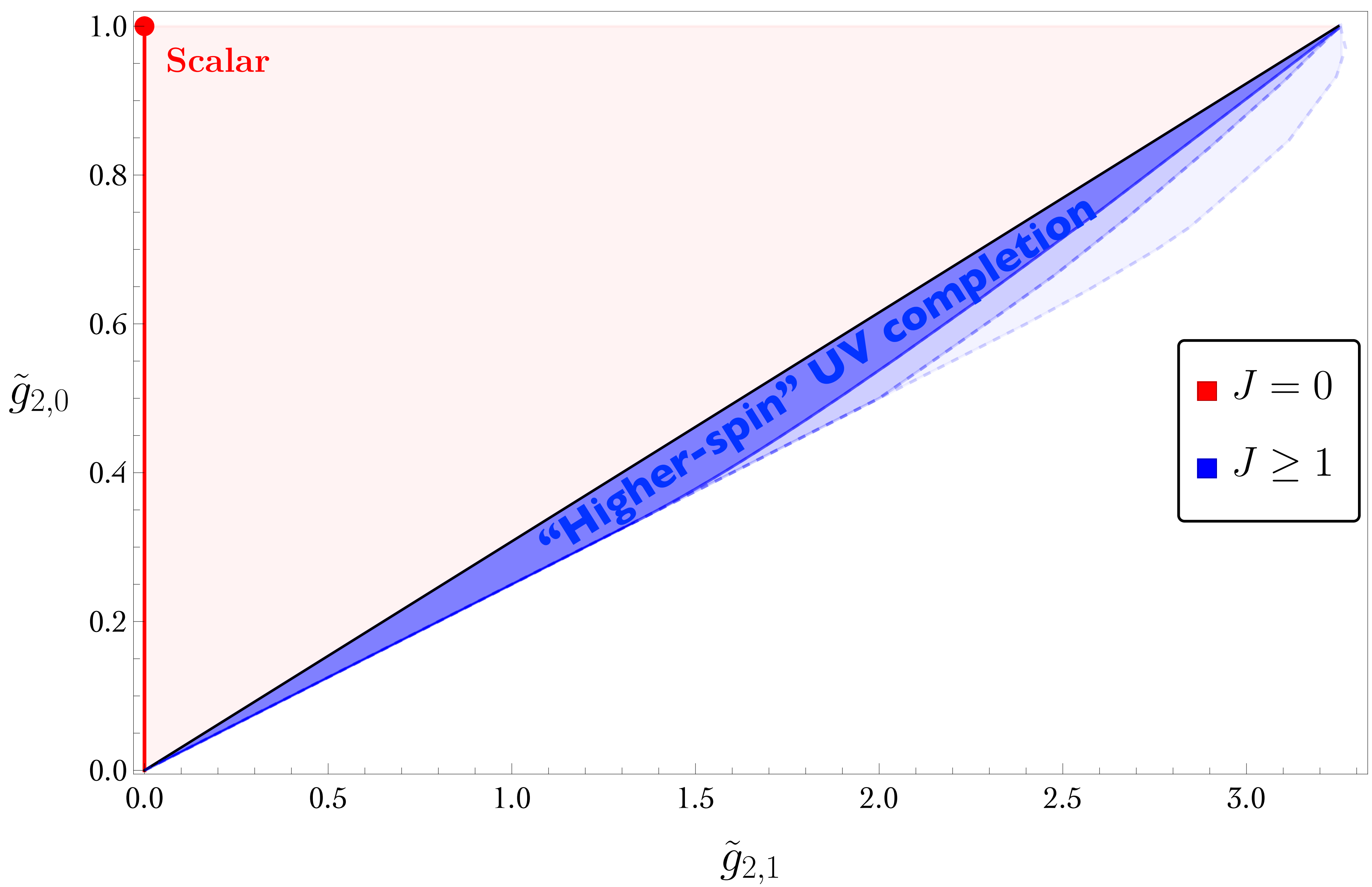


Constraints on Wilson coefficients

$$\mathcal{L} = \frac{F_\pi^2}{4}\mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\right) + L_1 \mathrm{Tr}^2\left(\partial_\mu U^\dagger \partial^\mu U\right) + L_2 \mathrm{Tr}\left(\partial_\mu U^\dagger \partial_\nu U\right)\mathrm{Tr}\left(\partial^\mu U^\dagger \partial^\nu U\right) + L_3 \mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U \partial_\nu U^\dagger \partial^\nu U\right)$$

$$\tilde{g}_{2,0} = 4(2L_1 + 3L_2 + L_3)\frac{M^2}{F_\pi^2}\,, \qquad \tilde{g}_{2,1} = 16 L_2 \frac{M^2}{F_\pi^2}$$

mass of the 1st meson

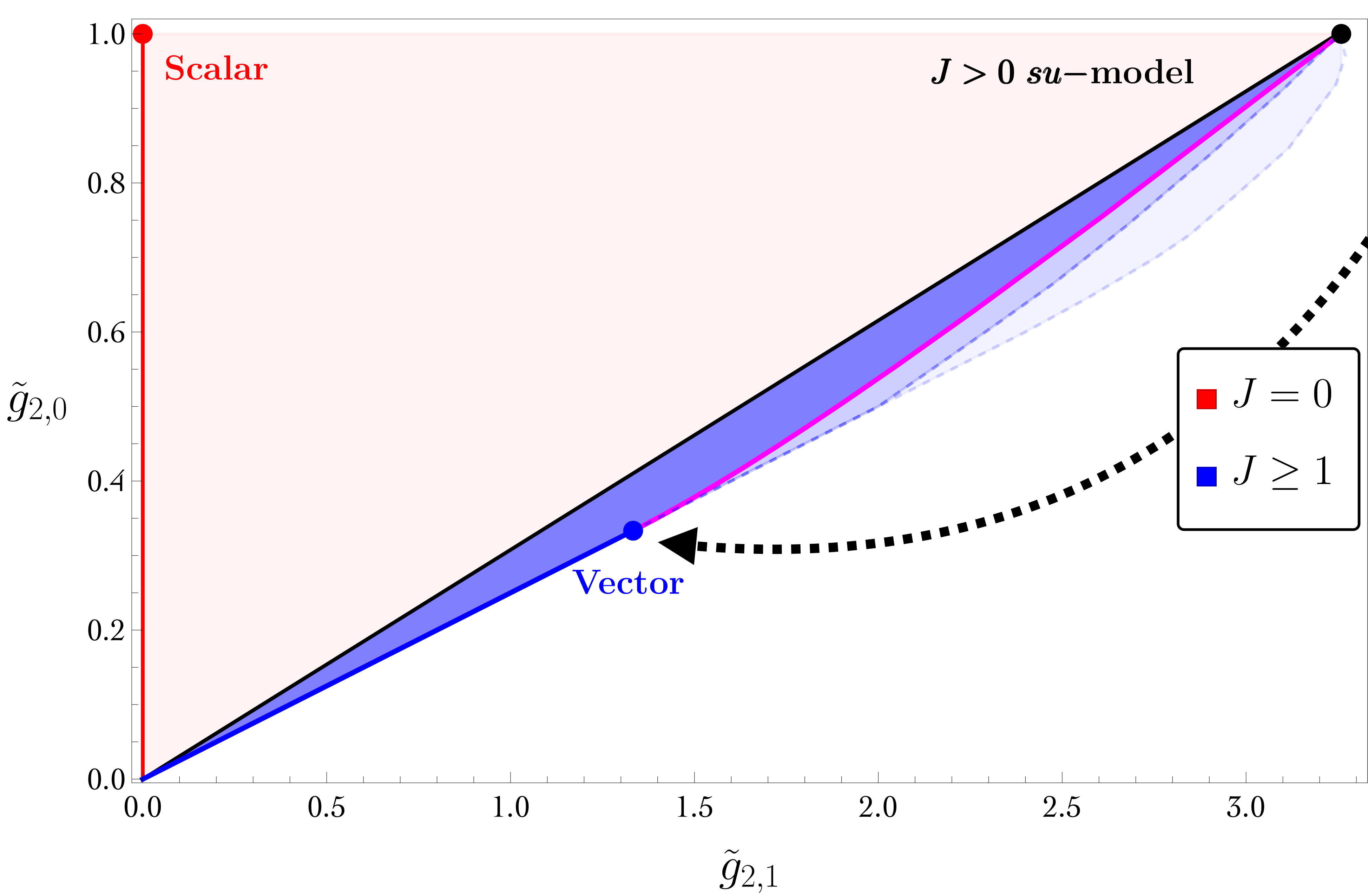


Constraints on Wilson coefficients

$$\mathcal{L} = \frac{F_\pi^2}{4}\mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\right) + L_1 \mathrm{Tr}^2\left(\partial_\mu U^\dagger \partial^\mu U\right) + L_2 \mathrm{Tr}\left(\partial_\mu U^\dagger \partial_\nu U\right)\mathrm{Tr}\left(\partial^\mu U^\dagger \partial^\nu U\right) + L_3 \mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U \partial_\nu U^\dagger \partial^\nu U\right)$$

$$\tilde{g}_{2,0} = 4(2L_1 + 3L_2 + L_3)\frac{M^2}{F_\pi^2}\,, \qquad \tilde{g}_{2,1} = 16 L_2 \frac{M^2}{F_\pi^2}$$

mass of the 1st meson



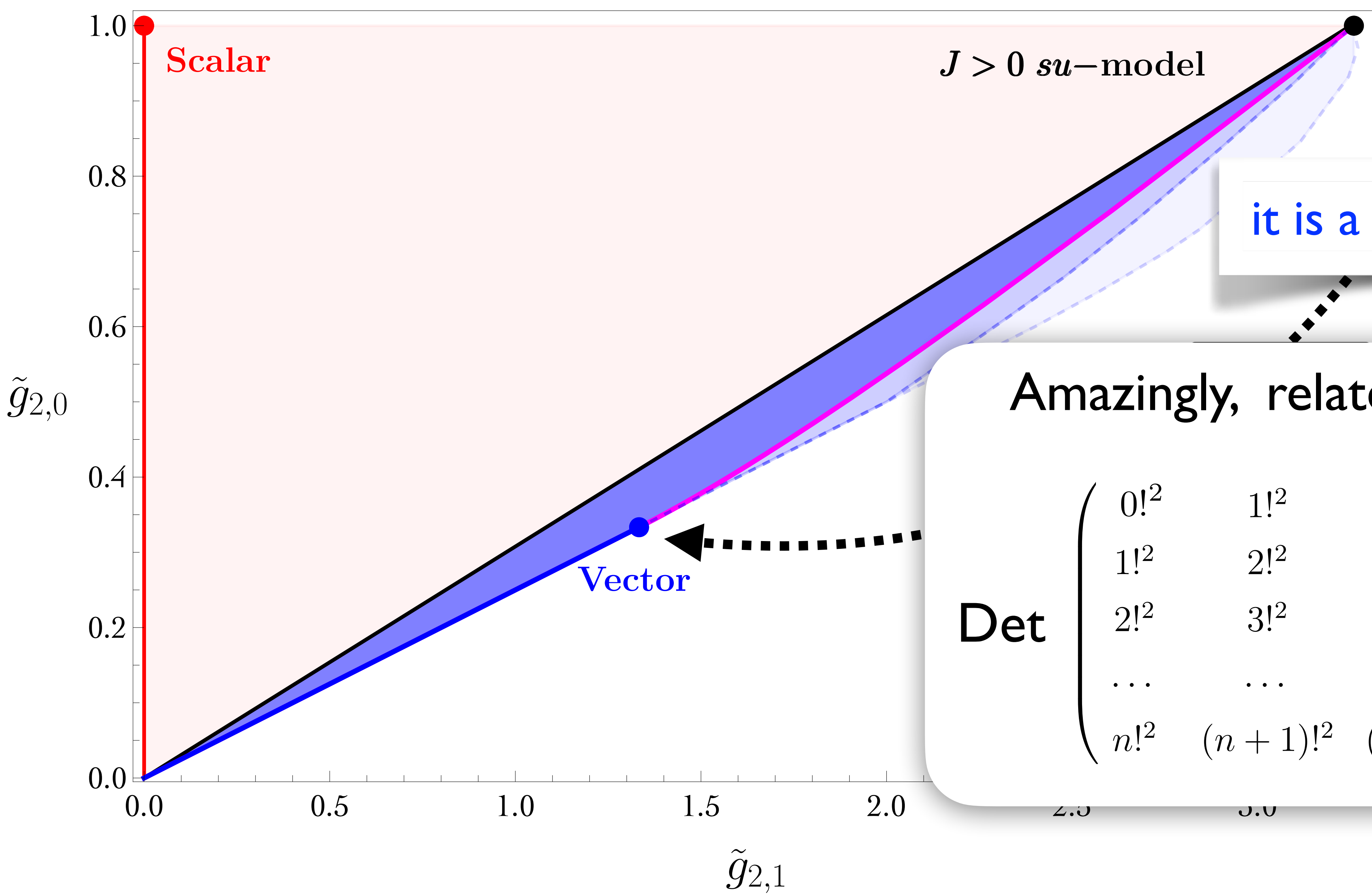
spin-1 model

Constraints on Wilson coefficients

$$\mathcal{L} = \frac{F_\pi^2}{4}\mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\right) + L_1 \mathrm{Tr}^2\left(\partial_\mu U^\dagger \partial^\mu U\right) + L_2 \mathrm{Tr}\left(\partial_\mu U^\dagger \partial_\nu U\right)\mathrm{Tr}\left(\partial^\mu U^\dagger \partial^\nu U\right) + L_3 \mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U \partial_\nu U^\dagger \partial^\nu U\right)$$

$$\tilde{g}_{2,0} = 4(2L_1 + 3L_2 + L_3)\frac{M^2}{F_\pi^2}\,, \qquad \tilde{g}_{2,1} = 16 L_2 \frac{M^2}{F_\pi^2}$$

mass of the 1st meson



spin-1 model

it is a kink of the boundary

Amazingly, related to the fact that

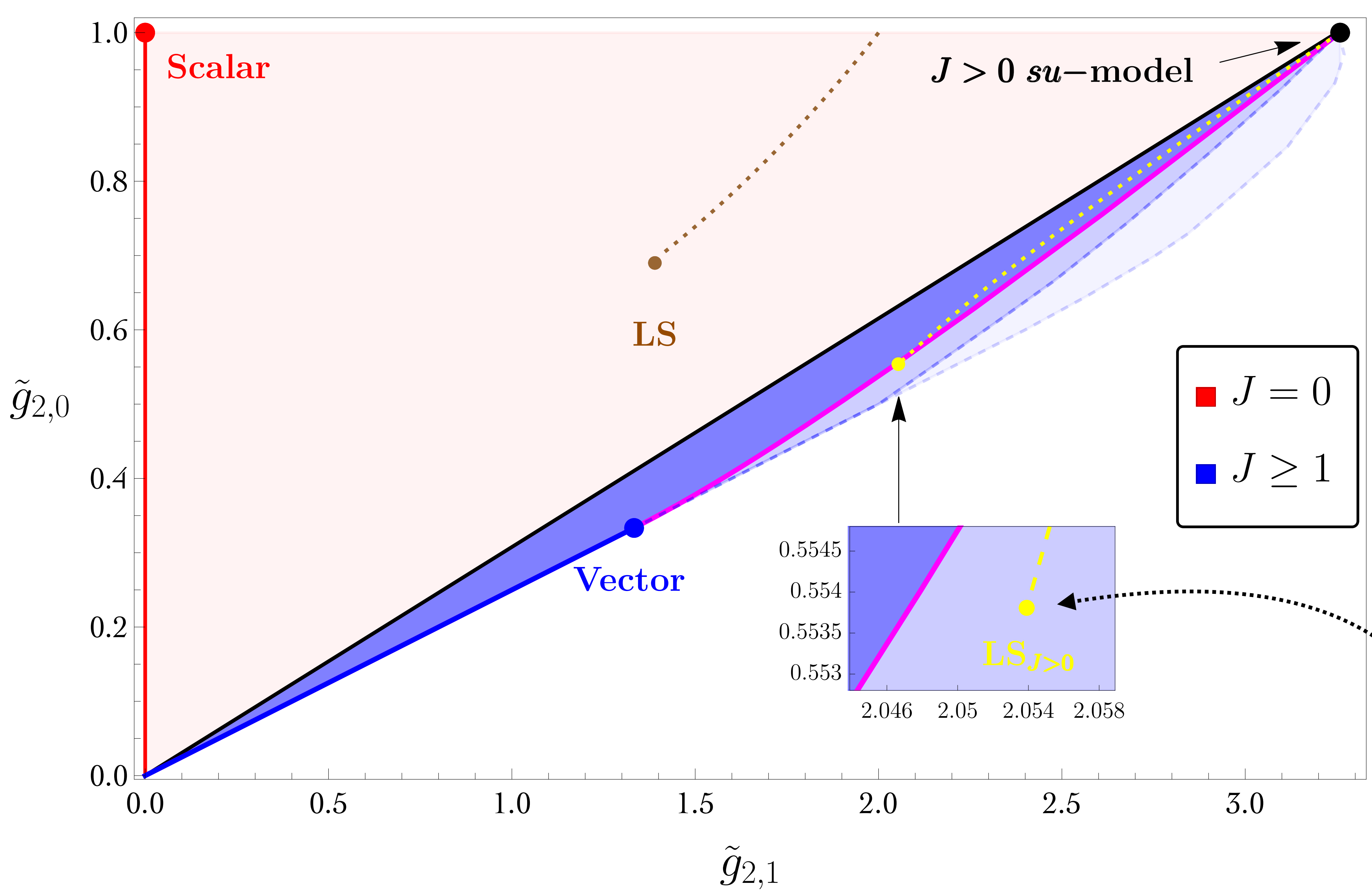
$$\mathrm{Det}\begin{pmatrix} 0!^2 & 1!^2 & 2!^2 & \cdots & n!^2 \\ 1!^2 & 2!^2 & 3!^2 & \cdots & (n+1)!^2 \\ 2!^2 & 3!^2 & 4!^2 & \cdots & (n+2)!^2 \\ \cdots & \cdots & \cdots & \ddots & \vdots \\ n!^2 & (n+1)!^2 & (n+2)!^2 & \cdots & (2n)!^2 \end{pmatrix} = 0$$

Constraints on Wilson coefficients

$$\mathcal{L} = \frac{F_\pi^2}{4}\mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\right) + L_1 \mathrm{Tr}^2\left(\partial_\mu U^\dagger \partial^\mu U\right) + L_2 \mathrm{Tr}\left(\partial_\mu U^\dagger \partial_\nu U\right)\mathrm{Tr}\left(\partial^\mu U^\dagger \partial^\nu U\right) + L_3 \mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U \partial_\nu U^\dagger \partial^\nu U\right)$$

$$\tilde{g}_{2,0} = 4(2L_1 + 3L_2 + L_3)\frac{M^2}{F_\pi^2}\ , \qquad \tilde{g}_{2,1} = 16 L_2 \frac{M^2}{F_\pi^2}$$

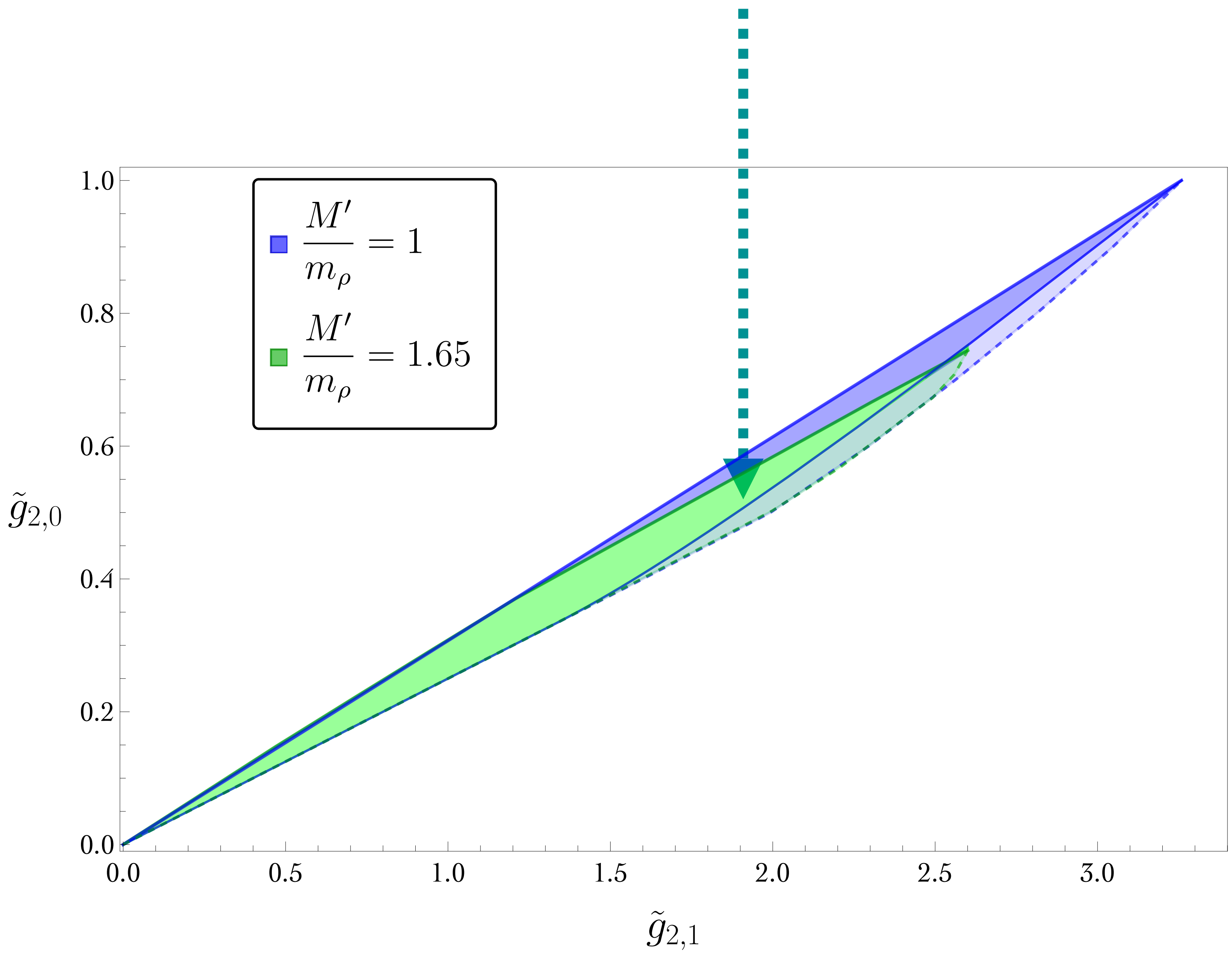
mass of the 1st meson



Lovelace-Shapiro:

$$\mathcal{M}^{(\mathrm{LS})}(s,u) = \frac{\Gamma\left(\frac{1}{2} - \frac{s}{2m_\rho^2}\right)\Gamma\left(\frac{1}{2} - \frac{u}{2m_\rho^2}\right)}{\Gamma\left(\frac{t}{2m_\rho^2}\right)}$$

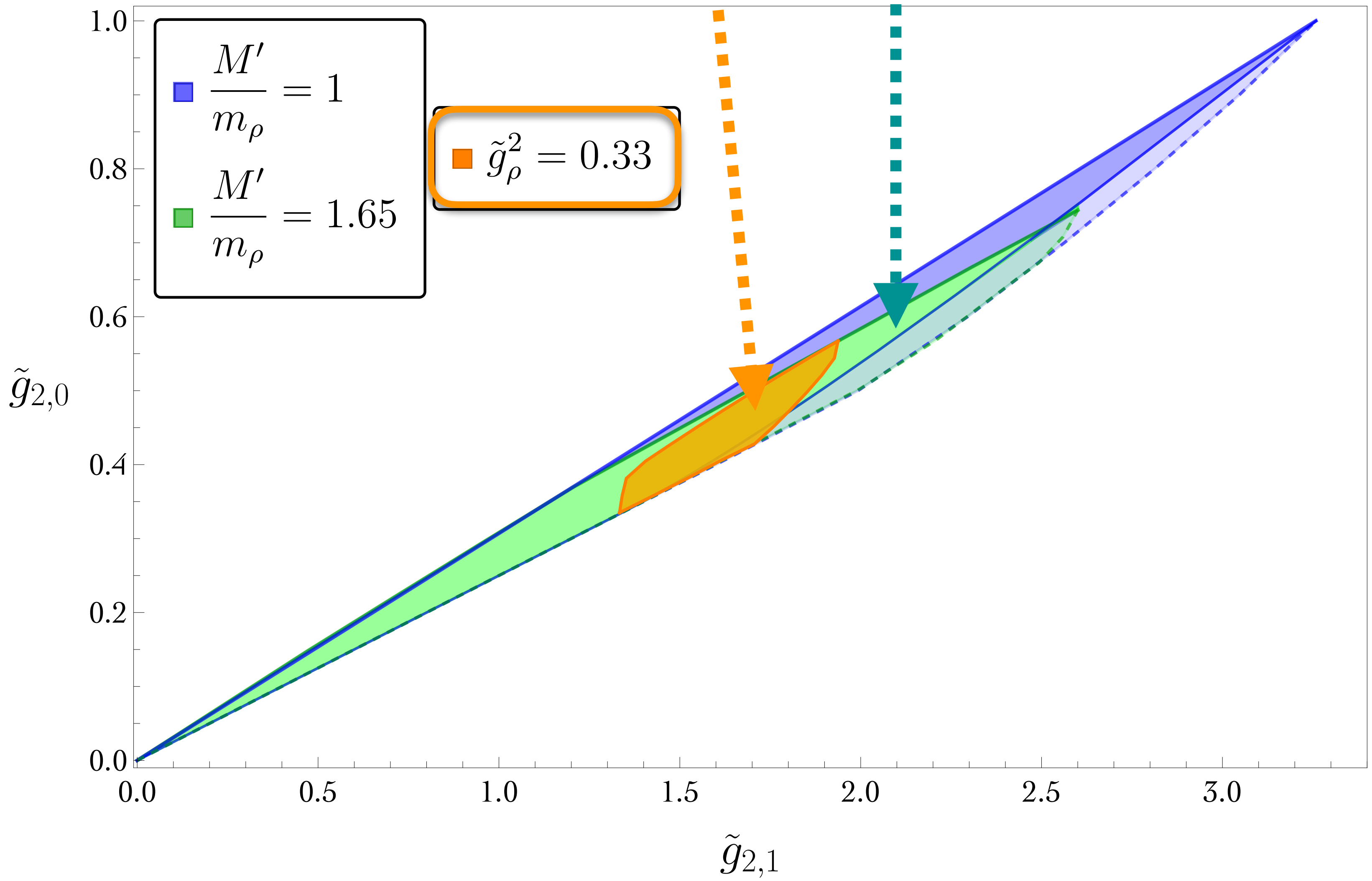
stronger bounds if we assume that, as in QCD, J>1 mesons are heavier



stronger bounds if we assume that, as in QCD, J>1 mesons are heavier

+ the spin-1 meson, the ρ, has a non-zero coupling to ππ

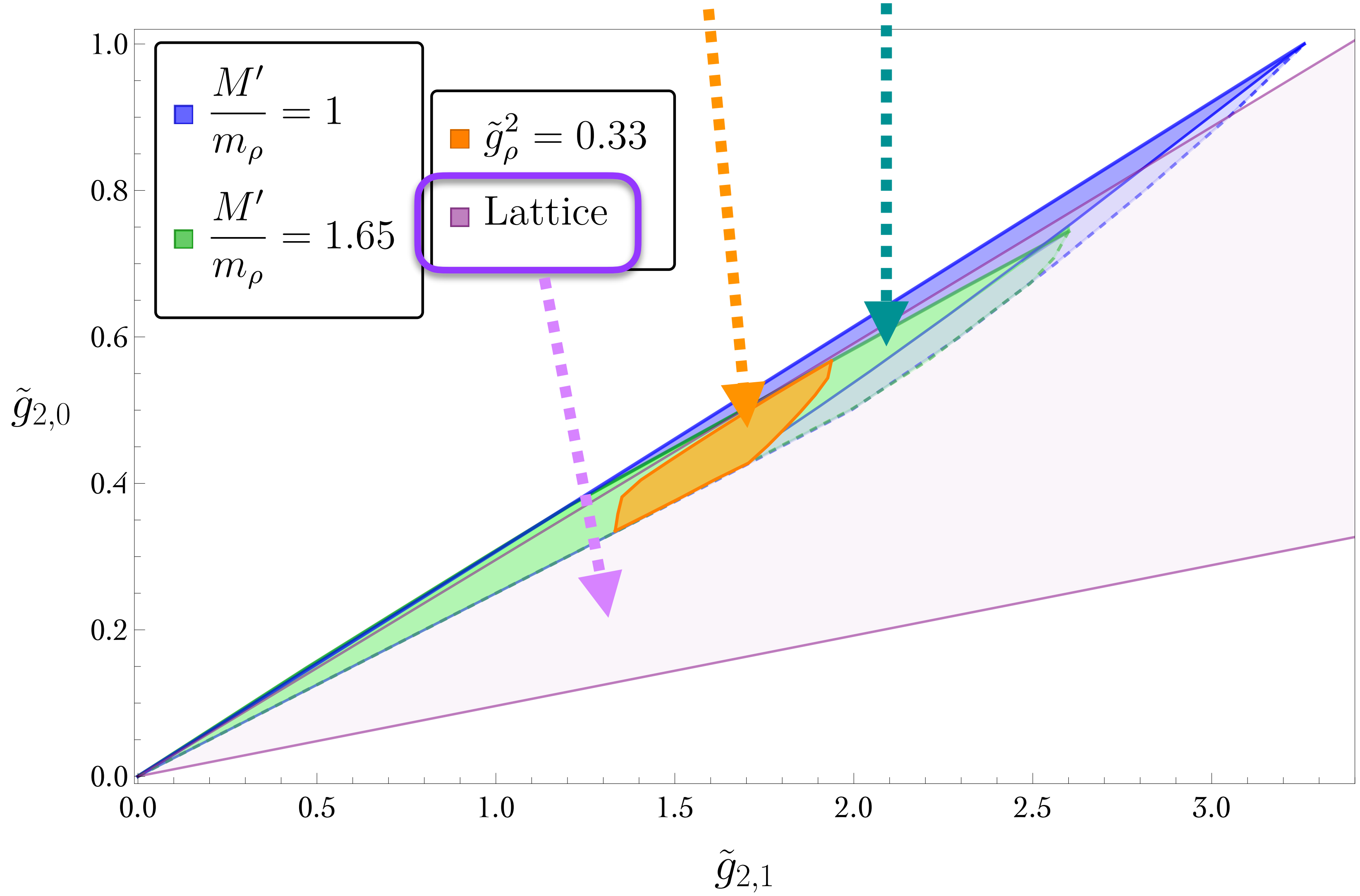
$$\tilde{g}_\rho^2 \equiv \frac{g_{\rho\pi\pi}^2}{g_{1,0} m_\rho^2}$$



stronger bounds if we assume that, as in QCD, J>1 mesons are heavier

+ the spin-1 meson, the ρ, has a non-zero coupling to ππ

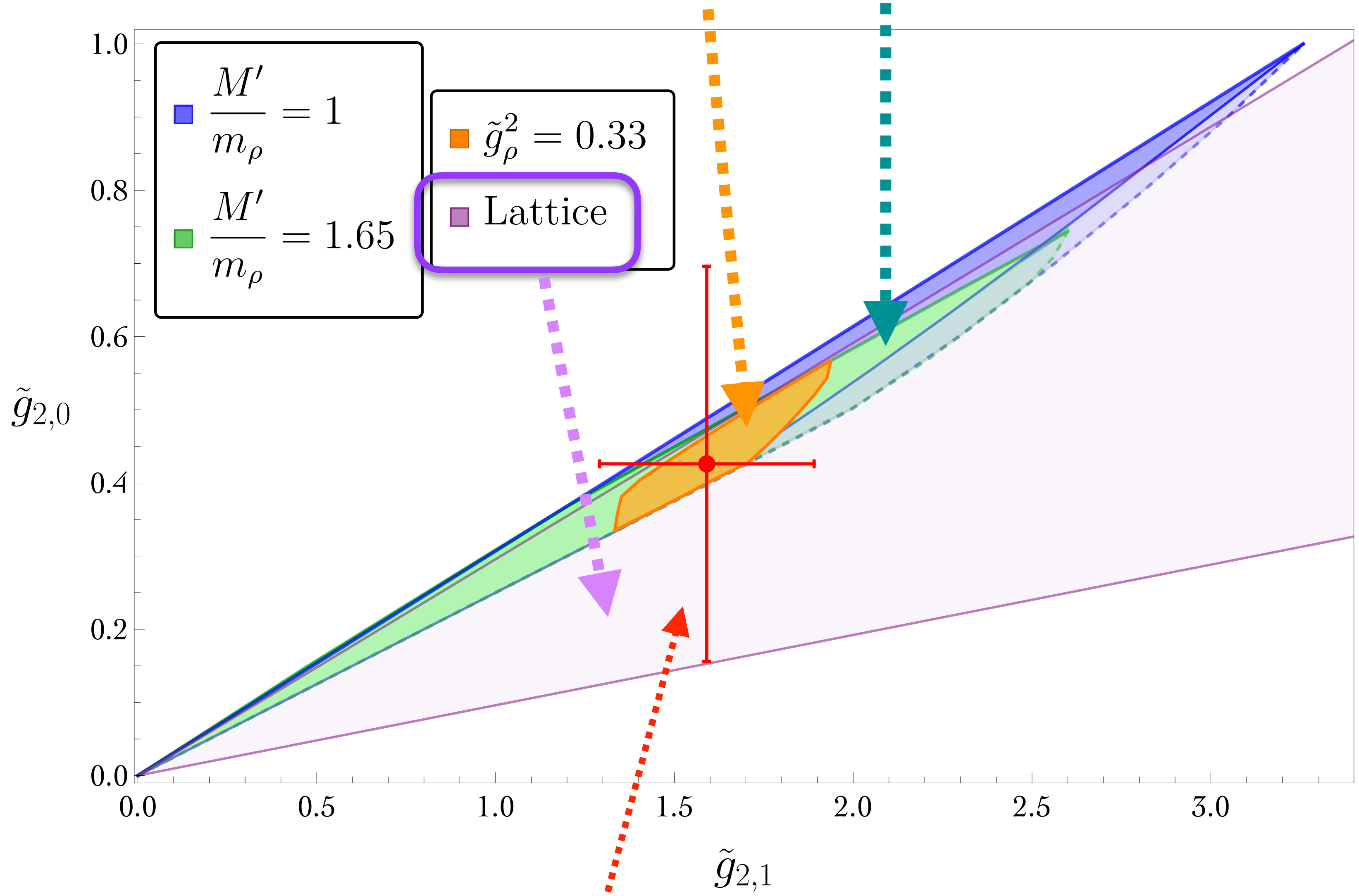
$$\tilde{g}_\rho^2 \equiv \frac{g_{\rho\pi\pi}^2}{g_{1,0}m_\rho^2}$$



stronger bounds if we assume that, as in QCD, J>1 mesons are heavier

+ the spin-1 meson, the ρ, has a non-zero coupling to ππ

$$\tilde{g}_\rho^2 \equiv \frac{g_{\rho\pi\pi}^2}{g_{1,0} m_\rho^2}$$



Experimental QCD data

Explaining the success of holography

AdS/QCD:

5D model for QCD mesons (spin=0,1):

$SU(2)_L \times SU(2)_R$ model:

Erlich+Katz+Son+Stephanov 05
Da Rold+Pomarol 05

$$\mathcal{L}_5 = \frac{M_5}{2} Tr\left[-L_{MN}L^{MN} - R_{MN}R^{MN} + |D_M\Phi|^2 + 3|\Phi|^2\right]$$

	Experiment	AdS_5	Deviation
m_ρ	775	824	+6%
m_{a_1}	1230	1347	+10%
m_ω	782	824	+5%
F_ρ	153	169	+11%
F_ω/F_ρ	0.88	0.94	+7%
F_π	87	88	+1%
$g_{\rho\pi\pi}$	6.0	5.4	−10%
L_9	$6.9\cdot 10^{-3}$	$6.2\cdot 10^{-3}$	−10%
L_{10}	$-5.2\cdot 10^{-3}$	$-6.2\cdot 10^{-3}$	−12%
$\Gamma(\omega\to\pi\gamma)$	0.75	0.81	+8%
$\Gamma(\omega\to 3\pi)$	7.5	6.7	−11%
$\Gamma(\rho\to\pi\gamma)$	0.068	0.077	+13%
$\Gamma(\omega\to\pi\mu\mu)$	$8.2\cdot 10^{-4}$	$7.3\cdot 10^{-4}$	−10%
$\Gamma(\omega\to\pi ee)$	$6.5\cdot 10^{-3}$	$7.3\cdot 10^{-3}$	+12%

Explaining the success of holography

AdS/QCD:

5D model for QCD mesons (spin=0,1):

$SU(2)_L \times SU(2)_R$ model:

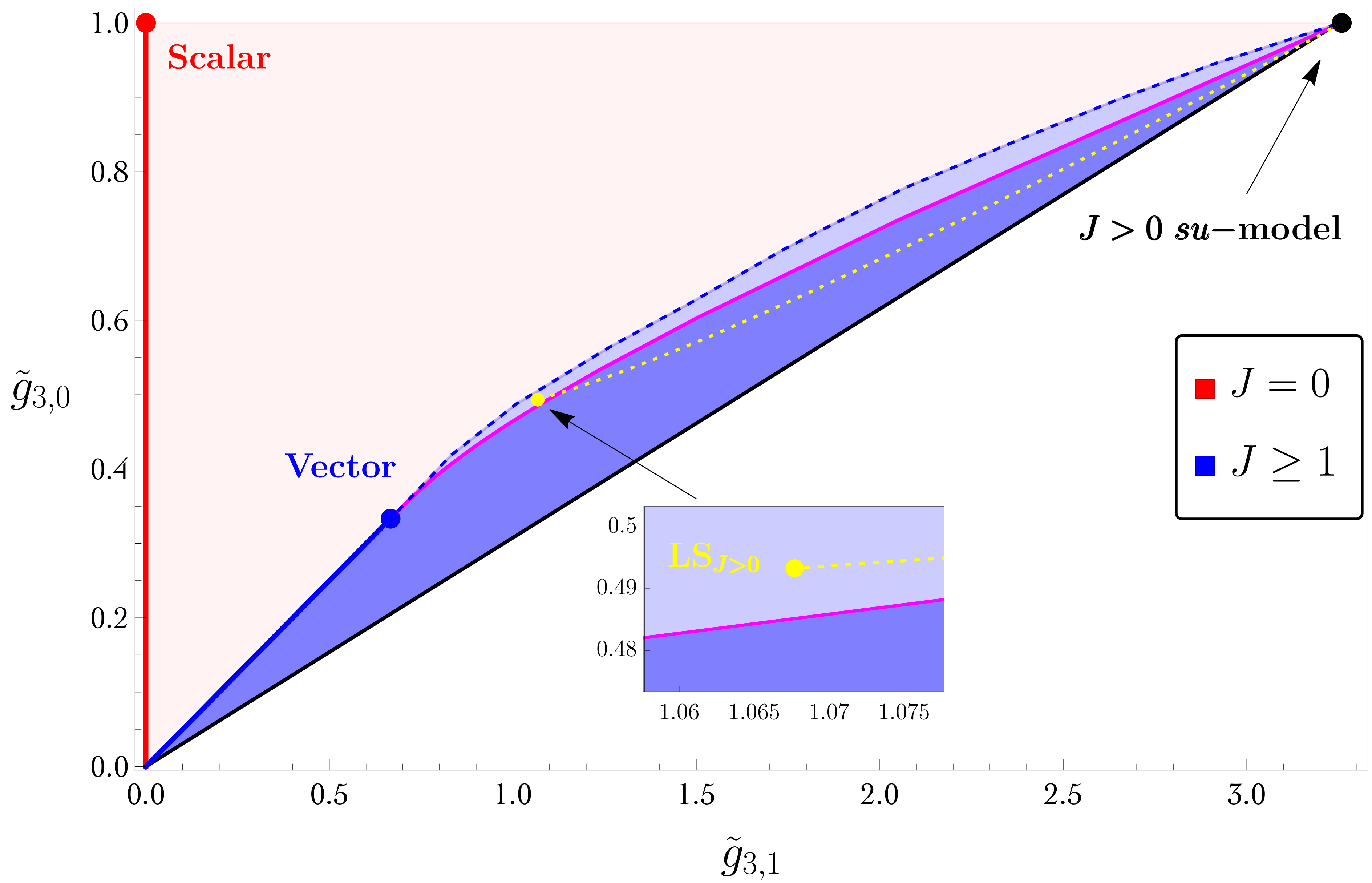
Erlich+Katz+Son+Stephanov 05
Da Rold+Pomarol 05

$$\mathcal{L}_5 = \frac{M_5}{2} Tr\left[-L_{MN}L^{MN} - R_{MN}R^{MN} + |D_M\Phi|^2 + 3|\Phi|^2\right]$$

	Experiment	AdS_5	Deviation
m_ρ	775	824	$+6\%$
m_{a_1}	1230	1347	$+10\%$
m_ω	782	824	$+5\%$
F_ρ	153	169	$+11\%$
F_ω/F_ρ	0.88	0.94	$+7\%$
F_π	87	88	$+1\%$
$g_{\rho\pi\pi}$	6.0	5.4	-10%
L_9	$6.9 \cdot 10^{-3}$	$6.2 \cdot 10^{-3}$	-10%
L_{10}	$-5.2 \cdot 10^{-3}$	$-6.2 \cdot 10^{-3}$	-12%
$\Gamma(\omega \to \pi\gamma)$	0.75	0.81	$+8\%$
$\Gamma(\omega \to 3\pi)$	7.5	6.7	-11%
$\Gamma(\rho \to \pi\gamma)$	0.068	0.077	$+13\%$
$\Gamma(\omega \to \pi\mu\mu)$	$8.2 \cdot 10^{-4}$	$7.3 \cdot 10^{-4}$	-10%
$\Gamma(\omega \to \pi ee)$	$6.5 \cdot 10^{-3}$	$7.3 \cdot 10^{-3}$	$+12\%$

Success can be understood from <u>positivity bounds</u> that restrict J>1 mesons to contribute little to low-energy observables

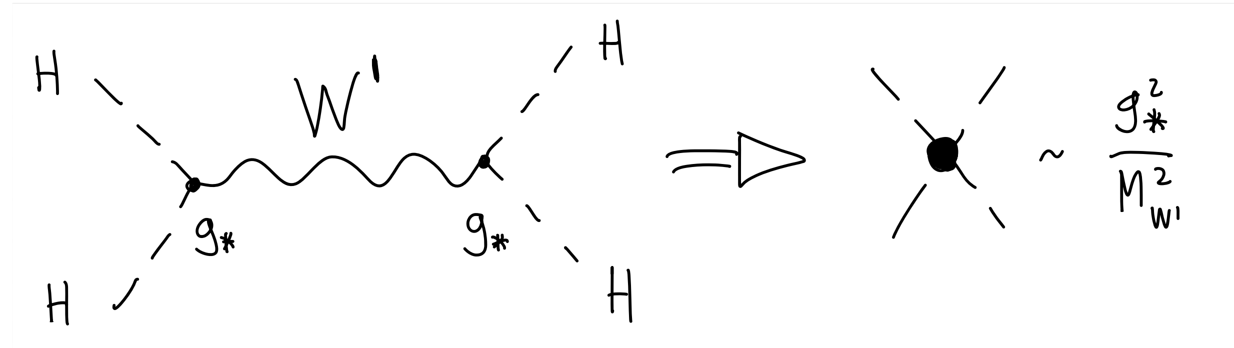
Similar structure for higher-order Wilson coeff.



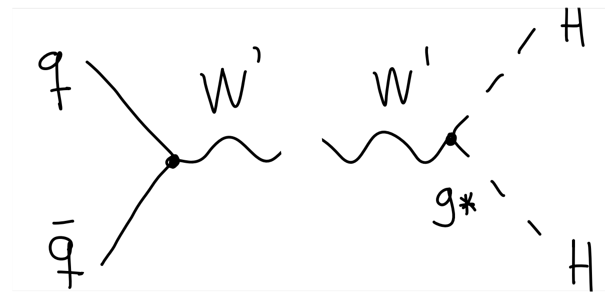
Impact on BSM searches at the LHC

Composite Higgs models:

Indirect probes:



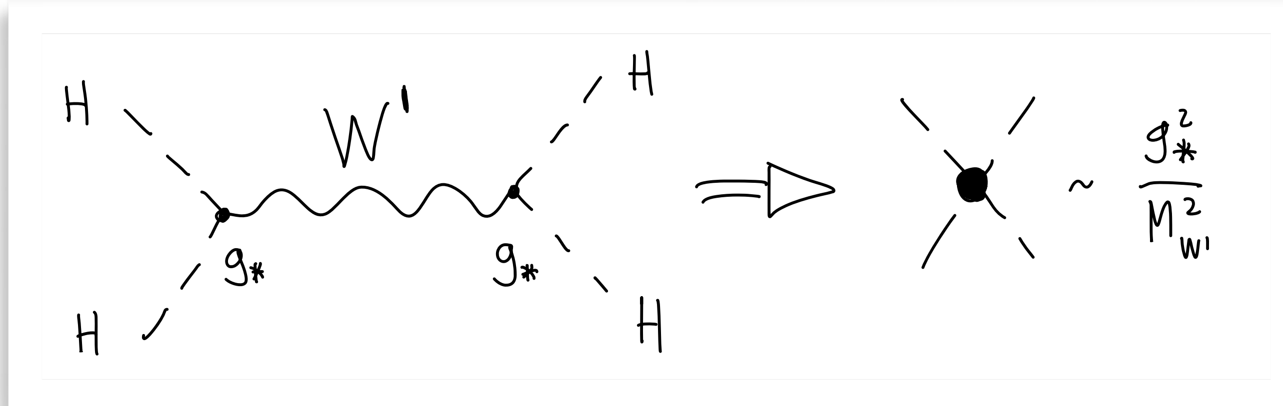
Direct probes:



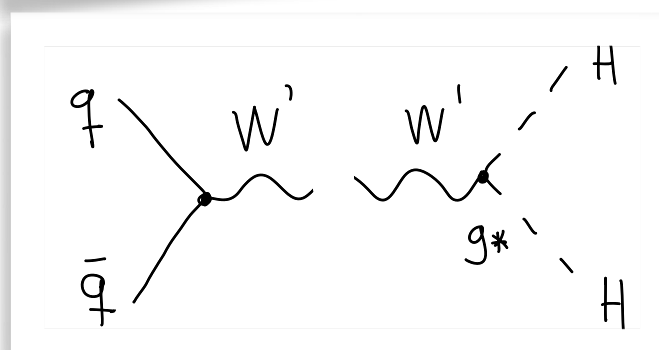
Impact on BSM searches at the LHC

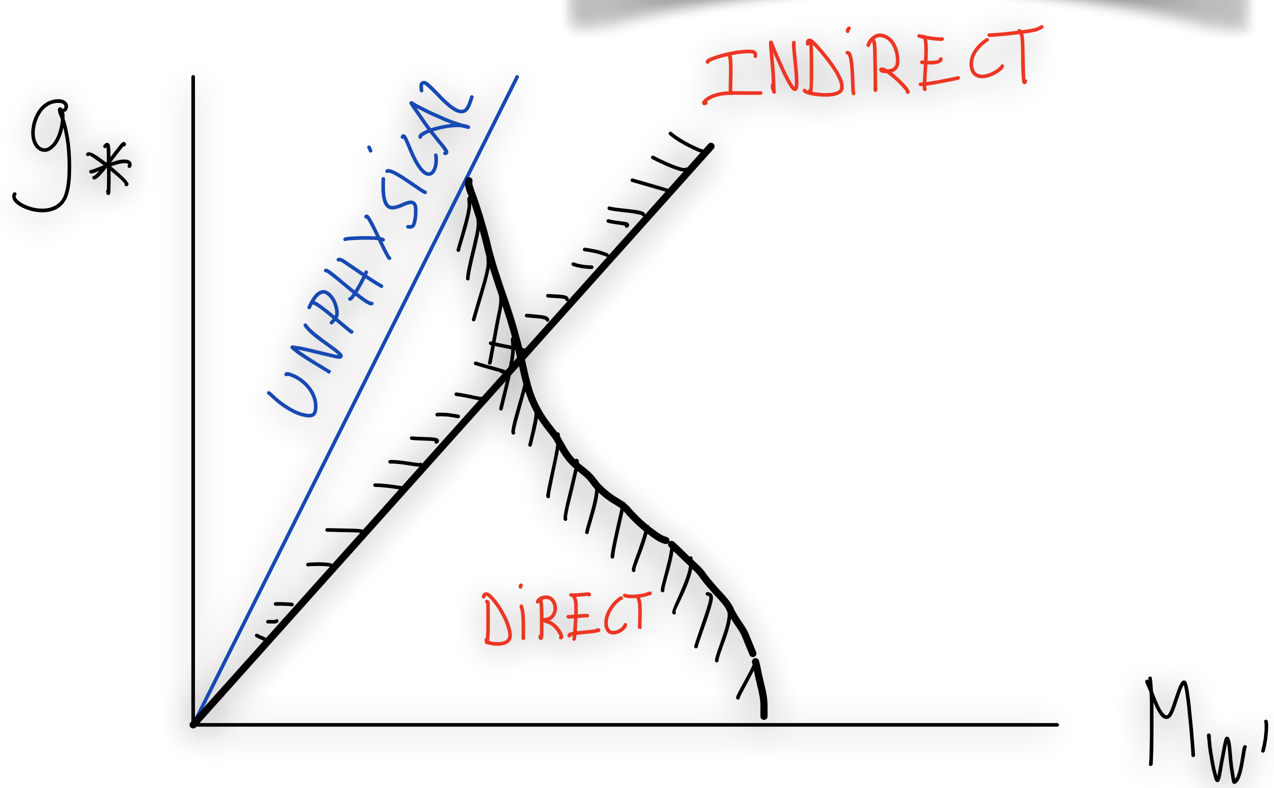
Composite Higgs models:

Indirect probes:



Direct probes:

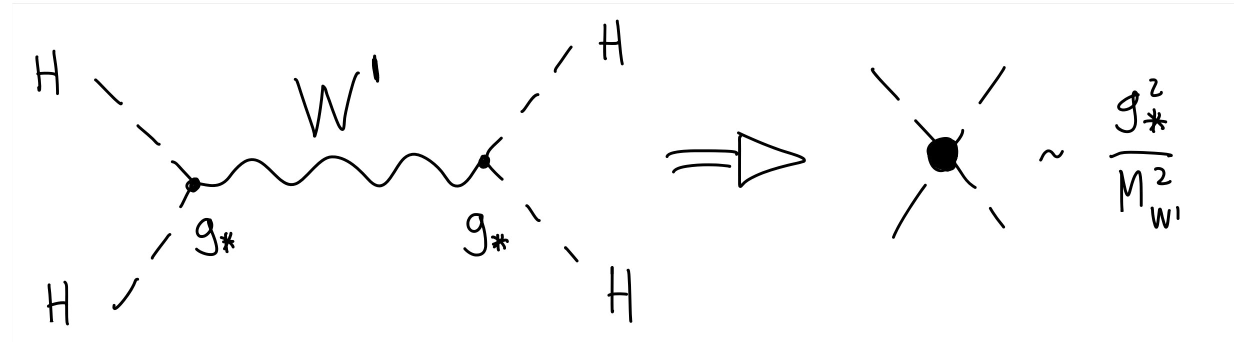




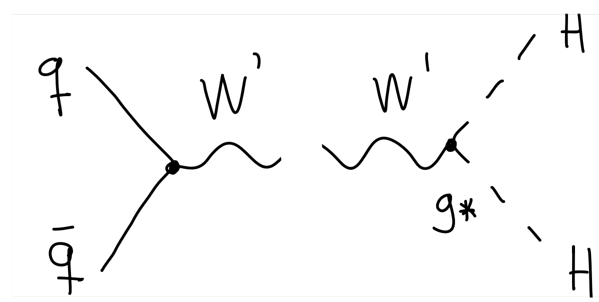
Impact on BSM searches at the LHC

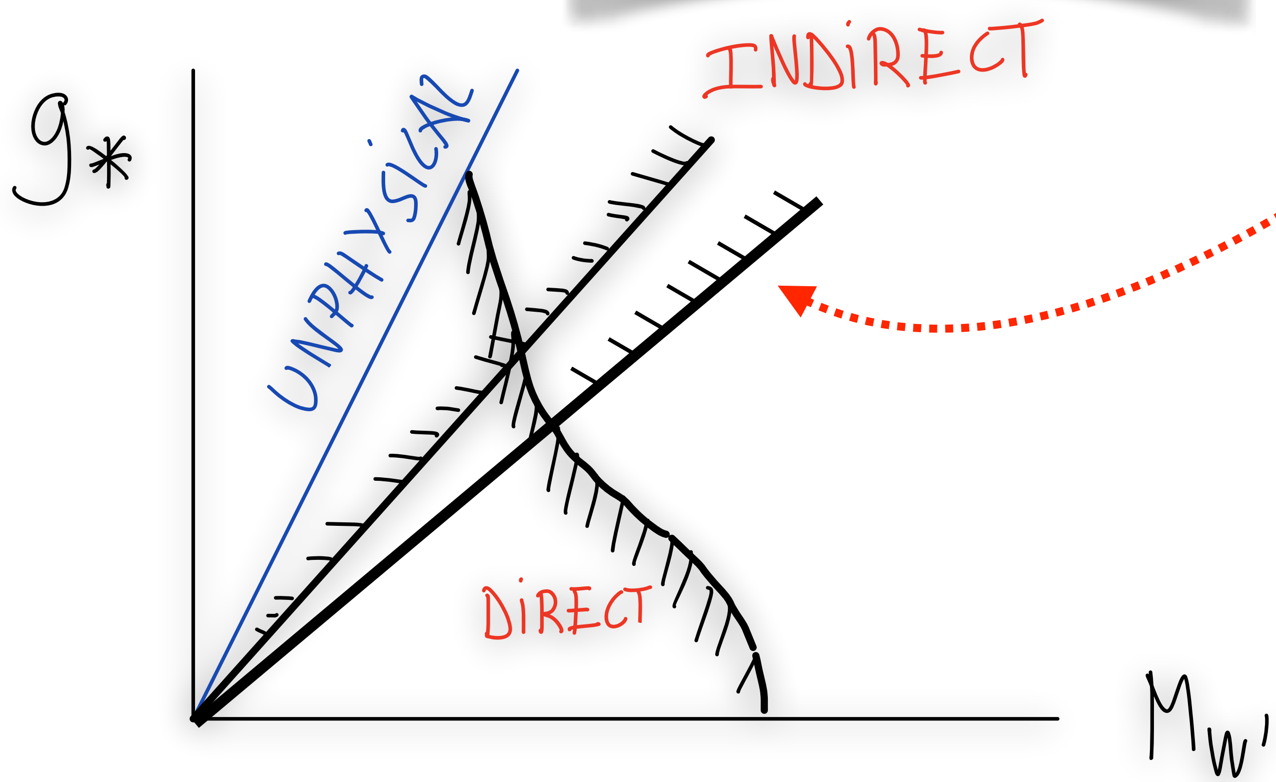
Composite Higgs models:

Indirect probes:



Direct probes:

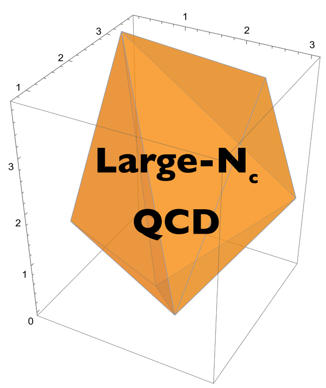




J>1 must **at least** contribute a **23%** to the Wilson coeff.

Conclusions

- Positivity bounds from ***Crossing* + *Analyticity* + *Unitarity*** shows the **"EFT-hedron"** structure of the Chiral Lagrangian at large-N_c



☞ ***Allows to get information on possible UV completions for a theory of Goldstones!***

e.g. mesons with all J needed

- Higher-spin (J>1) mesons are strongly constrained, giving a possible explanation for VMD & the success of holographic QCD

- Done for pion amplitudes, but "straightforward" generalizations to other amplitudes can allow to access to more information

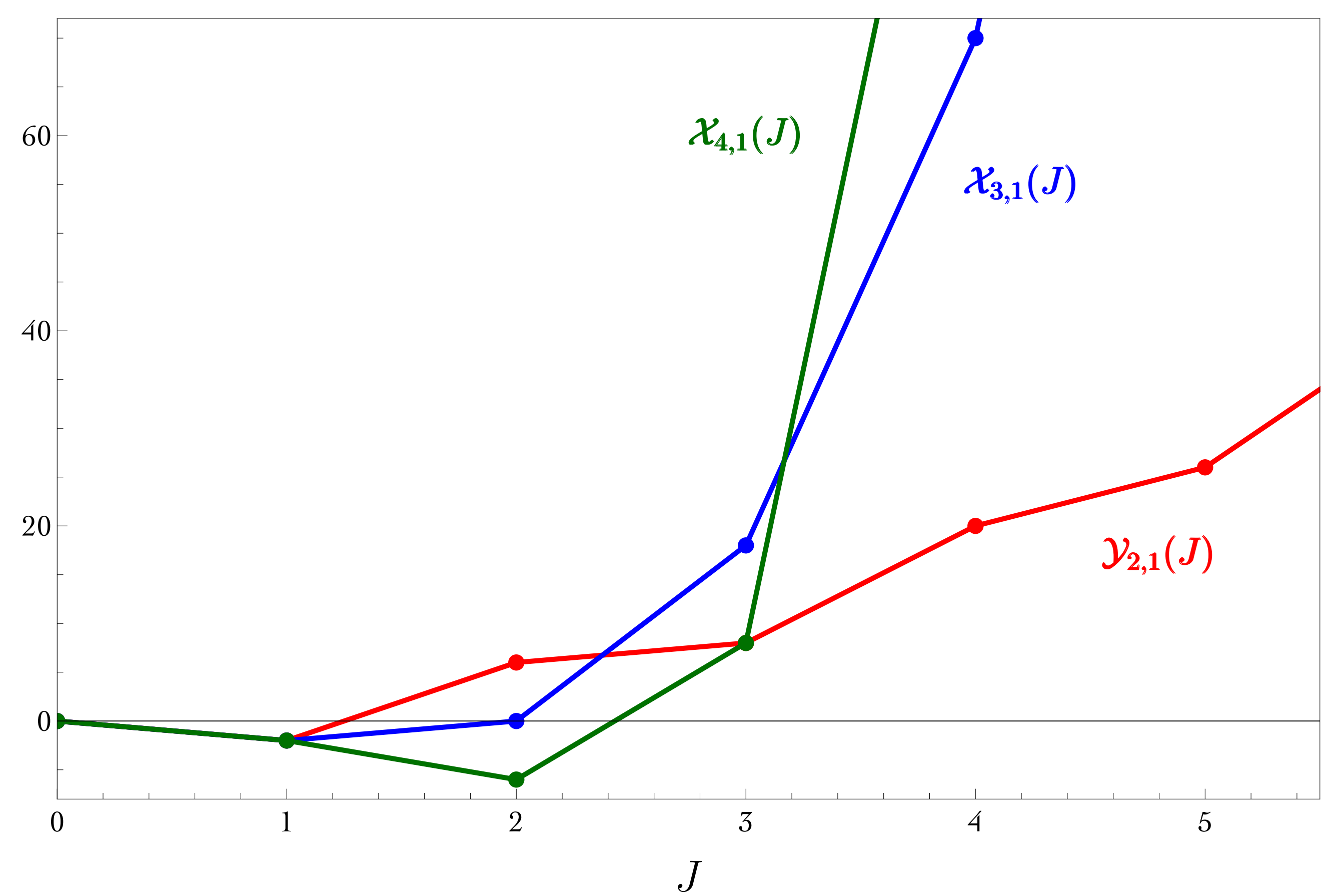
➥ potential interest to constrain SIMPs

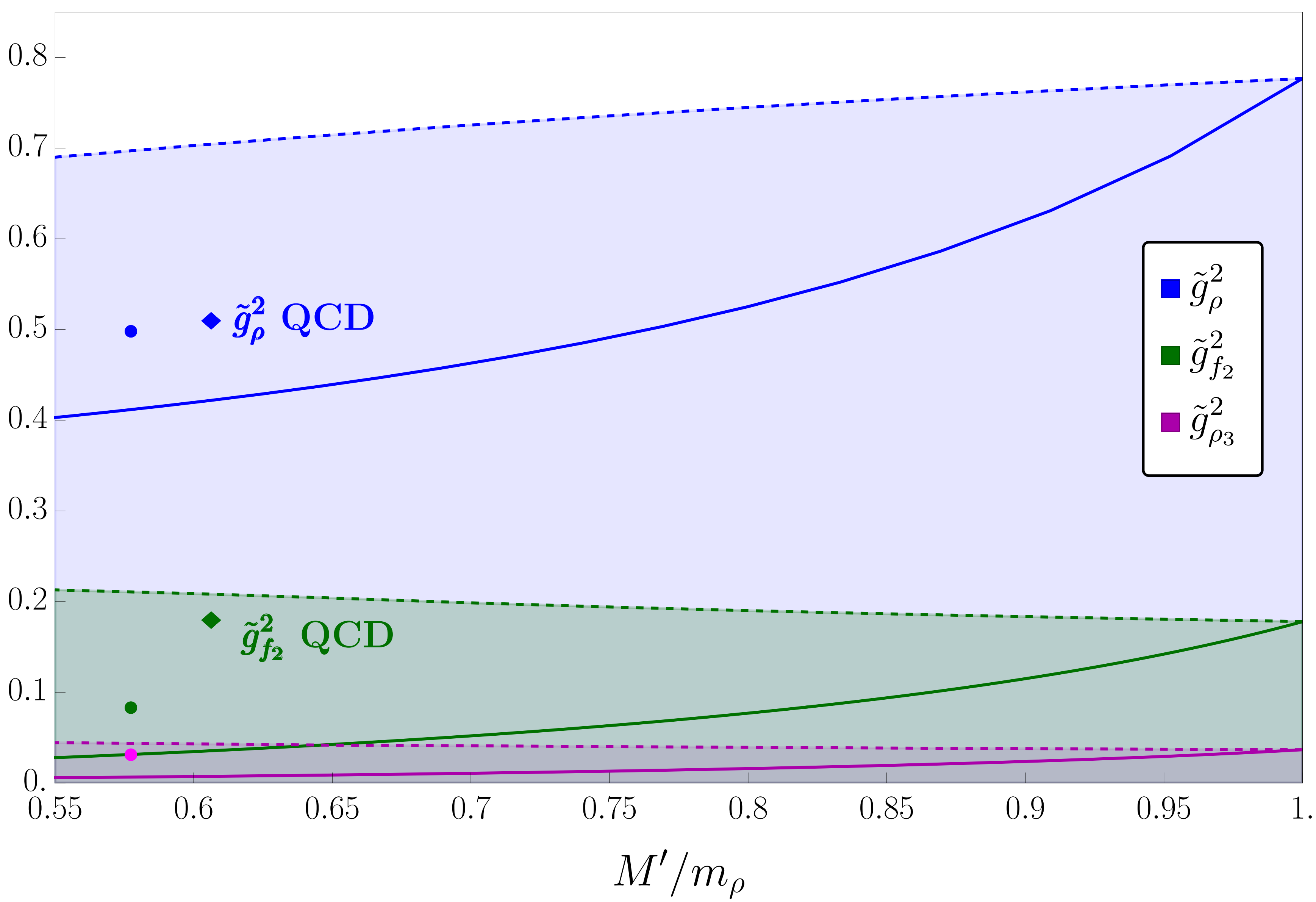

RESTRICTED AREA
MONITORED
BY VIDEO
CAMERA

$$0 = \sum_i \frac{|g_{\pi\pi\,i}|^2}{m_i^{2n}} \left(\underbrace{\frac{2^{n-1}}{(n-1)!^3} P_{J_i}^{(n-1)}(1) - \mathcal{J}_i^2}_{\mathcal{X}_{n,1}} \right)$$

n=2,3,4,...

$\mathcal{J}^2 \equiv J(J+1)$

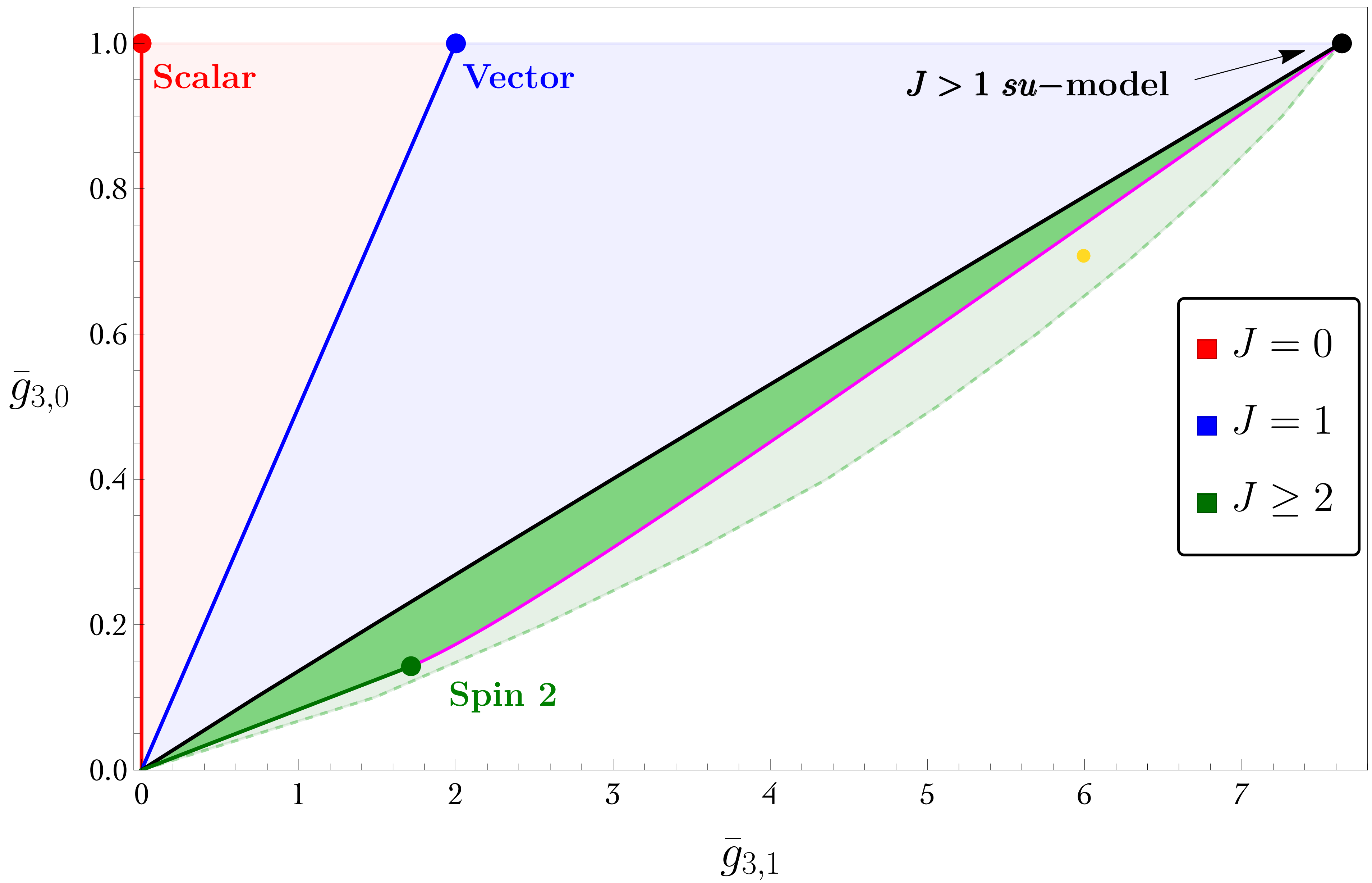




0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.
0.55
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95
1.
M'/m_ρ
$\tilde{g}_\rho^2$ QCD
$\tilde{g}_{f_2}^2$ QCD
$\tilde{g}_\rho^2$
$\tilde{g}_{f_2}^2$
$\tilde{g}_{\rho_3}^2$

Lets assume at $s \to \infty$ and either t or u fixed:

$$\frac{\mathcal{M}_t^{I=2}(s,u)}{s^2} \to 0$$



1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
5
6
7
$\bar{g}_{3,0}$
$\bar{g}_{3,1}$
Scalar
Vector
$J > 1$ $su-$model
Spin 2
$J = 0$
$J = 1$
$J \geq 2$

C The su-models

Let us consider the most general theory of a degenerate spectrum that contributes to the four-pion amplitude $\mathcal{M}(s,u)$ [7, 8]. This means that all states have equal mass m, and therefore the denominator of this amplitude is fixed to be $\mathcal{M}(s,u) \propto 1/((s-m^2)(u-m^2))$. If we further demand that Eq. (6a) and Eq. (6b) are satisfied for $k_{\min} = 1$, we are led to

$$\mathcal{M}(s,u) = \frac{a_1 m^4 + a_2 m^2 (s+u) + a_3 su}{(s-m^2)(u-m^2)}\,, \tag{91}$$

where a_i are constants. The Adler's zero condition fixes $a_1 = 0$. Then, aside from a global multiplicative factor, the amplitude has only one free parameter. We can write it as

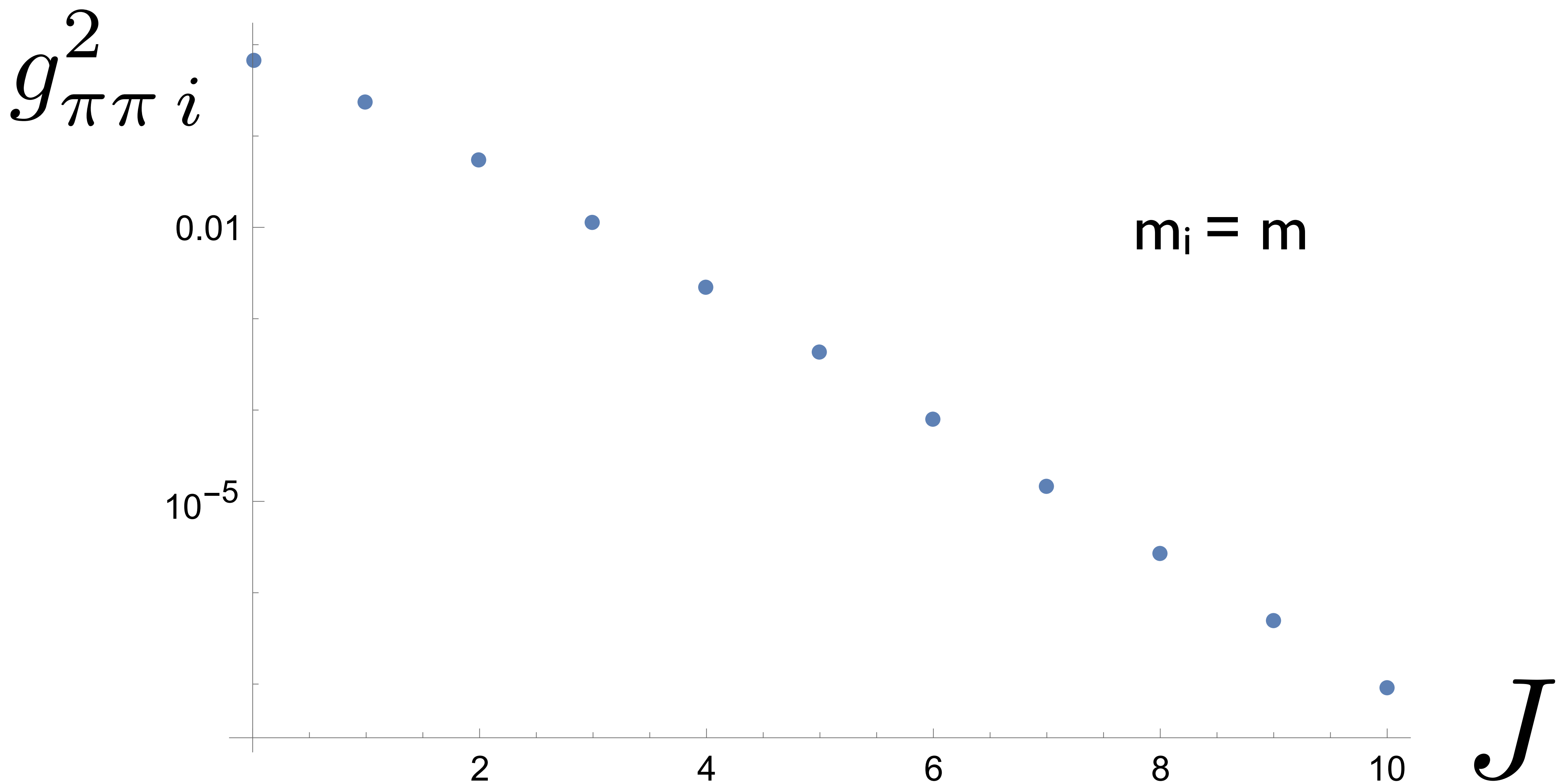
$$\mathcal{M}_1^{(su)}(s,u) = \frac{m^2(s+u) + \lambda su}{(s-m^2)(u-m^2)}\,, \tag{92}$$

where the possible values of λ are determined by unitarity. Indeed, imposing the positivity of the residues of Eq. (92), we obtain

$$-2 \leq \lambda \leq \frac{2\ln 2 - 1}{1 - \ln 2}\,. \tag{93}$$

In the limiting case $\lambda = -2$, the residues of all $J > 0$ states are zero, and we are left with the scalar amplitude Eq. (22). In the other limit,

$$\lambda = \frac{2\ln 2 - 1}{1 - \ln 2} \simeq 1.26\,, \tag{94}$$



$g^2_{\pi\pi\, i}$
0.01
10^{-5}
$m_i = m$
2
4
6
8
10
J

D The Lovelace-Shapiro amplitude

The Lovelace-Shapiro (LS) amplitude for the scattering of four pions is defined as [26, 27]

$$\mathcal{M}^{(\mathrm{LS})}(s,u)=\frac{\Gamma(1-\alpha(s))\Gamma(1-\alpha(u))}{\Gamma(1-\alpha(s)-\alpha(u))}\;, \tag{105}$$

where $\alpha(s)=\alpha_0+\alpha' s$ is referred as the Regge trajectory. We will fix the values of α_0 and α' by requiring that Eq. (106) satisfies the Adler zero condition, $\mathcal{M}^{(\mathrm{LS})}(s,u)\to 0$ for $s,u\to 0$, and that the first pole of Eq. (106) occurs for $s=m_\rho^2$. These two conditions lead to $\alpha_0=1/2$ and $\alpha'=1/(2m_\rho^2)$ [66] and then we can write

$$\mathcal{M}^{(\mathrm{LS})}(s,u)=\frac{\Gamma\left(\frac{1}{2}-\frac{s}{2m_\rho^2}\right)\Gamma\left(\frac{1}{2}-\frac{u}{2m_\rho^2}\right)}{\Gamma\left(\frac{t}{2m_\rho^2}\right)}\;. \tag{106}$$

By looking at the poles of Eq. (106), one can see that the LS amplitude corresponds to a theory of higher-spin states with masses

$$m_n^2=m_\rho^2(2n+1)\,,\quad n=0,1,2,\dots\,. \tag{107}$$

For a given n, there are at most $n+1$ states with spin $J=0,1,...,n+1$. Furthermore, Eq. (106) satisfies the condition Eq. (6a) and Eq. (6b) with $k_{\min}=1$.

E The Coon amplitude

The Lovelace-Shapiro amplitude presented in Appendix D can be generalized to a larger class of amplitudes depending on an additional parameter q. This is the so-called Coon amplitude, which was first proposed in [28][11]:

$$\mathcal{M}_q(s,u) = C(\sigma,\tau,q)\prod_{n=0}^{\infty}\frac{(1-q^{n+1})\,(\sigma\tau-q^{n+1})}{(\sigma-q^{n+1})\,(\tau-q^{n+1})}\ , \tag{118}$$

where $\sigma = 1+(q-1)(\alpha_0+\alpha' s)$ and $\tau = 1+(q-1)(\alpha_0+\alpha' u)$. As explained in Appendix D, we take $\alpha_0 = 1/2$ and $\alpha' = 1/(2m_\rho^2)$. The parameter q takes values between 0 and 1, and in the limit $q \to 1$ we recover the LS amplitude Eq. (106). There is some freedom in the choice of the prefactor C, as long as it satisfies $\lim_{q\to 1} C(\sigma,\tau,q) = 1$.

The Coon amplitude has an infinite number of simple poles at

$$s_n = m_\rho^2\frac{1+q-2q^{n+1}}{1-q}\ , \qquad n = 0,1,2,\ldots\ . \tag{119}$$