CP Violation in Effective Field Theory

How many BSM sources of CPV?
How large can they be?

Based on:
1. Q. Bonnefoy (DESY), E. Gendy (UHH), CG and J. Ruderman (NYU)
arXiv: 2112.03889 *"Beyond Jarlskog: 699 invariants for CP violation in SMEFT"*.
2. Q. Bonnefoy (DESY), CG, J. Kley (DESY)
arXiv: 2206.04182 *"The shift-invariant orders of an ALP"*.
3. Q. Bonnefoy (DESY), E. Gendy (UHH), CG and J. Ruderman (NYU)
arXiv: 2302.07288 *"Opportunistic CP violation"*.

Christophe Grojean DESY (Hamburg) & Humboldt University (Berlin) (christophe.grojean@desy.de)

Thank You

I’m glad that MITP exists

- I’m benefiting a lot from the various events organised.
- It gives me an example of a successful story when I’m arguing with the administration at DESY about the need and the benefits of having a strong Centre for Theoretical Physics.

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The problem is Matthias… because the directors keep repeating me

- Try to be as good as Matthias first.
- Then you can think of a Centre.

Outline

1) CP violation:

- The collective nature of **CPV**: real vs. imaginary interactions?
- The (flavour-)invariant measures of CPV
- Beyond Jarlskog: the 699 (minimal) CPV invariants of SMEFT_6
- Opportunistic CP violation: new interference with CKM phase.

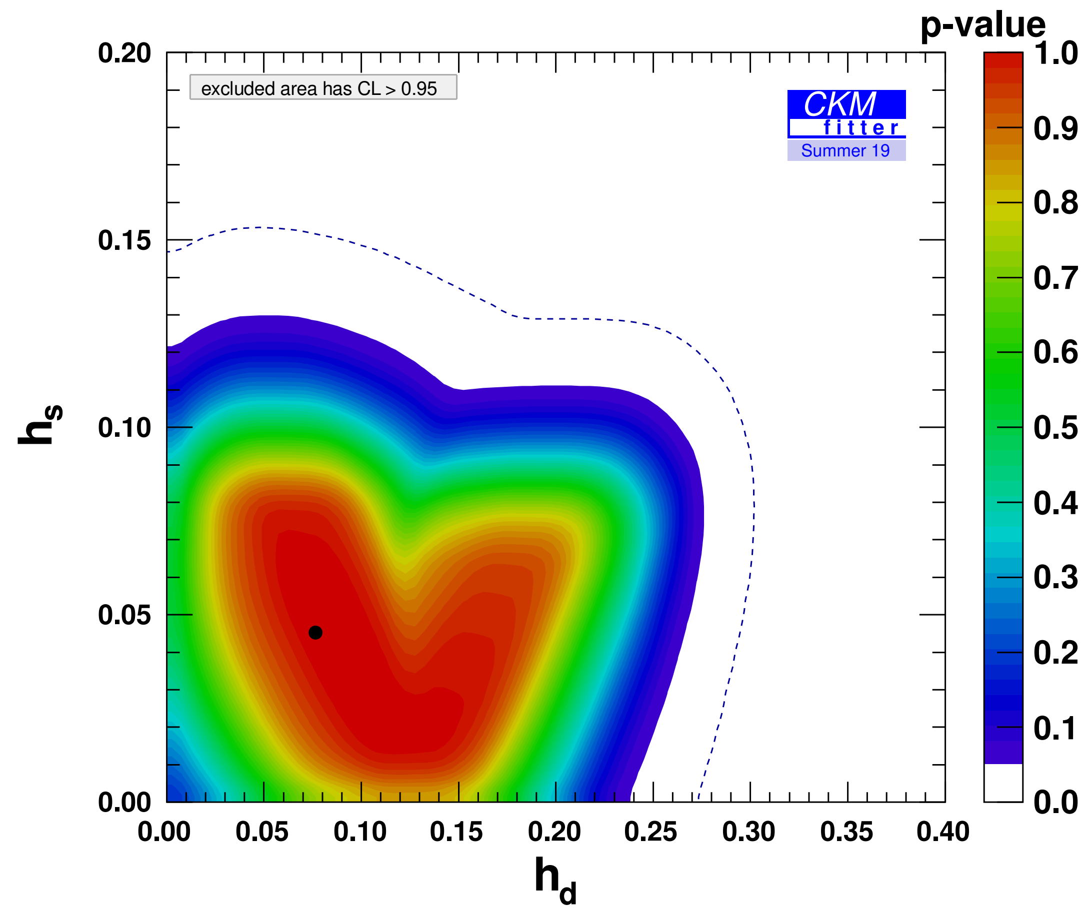
2) ALP shift symmetry

- Beyond Jarlskog: the 13 invariants of **ALP** shift-symmetry breaking
- The collective nature of shift-symmetry breaking
- RG invariance of the invariants

Note 1: I’ll consider only heavy/decoupling new physics
Note 2: I’ll assume that SU(2)xU(1) is linearly realised above the weak scale, i.e. SMEFT rather than HEFT. Our construction can be generalised but we haven’t gone through this exercise (yet). I’ll also assume that possible B and L violating effects are pushed to a high scale irrelevant for our discussion.

Part I. Does new physics break CP?

- Unlike B & L numbers, CP is not an accidental symmetry of SM4
- But its violation is “screened” by the CKM selection rules (see next slides). Not large enough for baryogenesis.
- BSM CPV effects can be O(1) in most loop-level FCNC processes



$h\,e^{2i\sigma} = A_{\rm NP}(B^0 \to \overline{B}^0)/A_{\rm SM}(B^0 \to \overline{B}^0)$

NP parameters

$$\frac{C_{ij}^2}{\Lambda^2}\left(\bar{q}_{i,L}\gamma_\mu q_{j,L}\right)^2 \quad \Rightarrow \quad h \simeq \frac{|C_{ij}|^2}{|V_{ti}^* V_{tj}|^2}\left(\frac{4.5\,\text{TeV}}{\Lambda}\right)^2$$

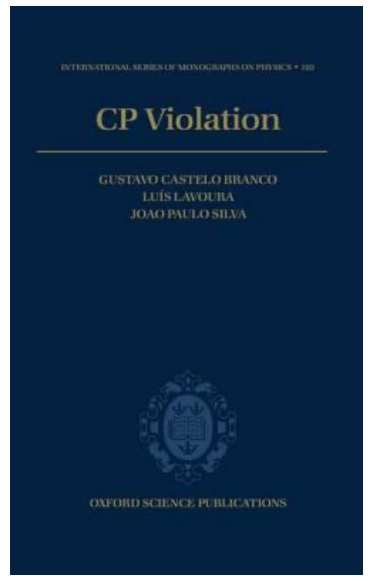
Couplings	NP loop order	Sensitivity for Summer 2019 [TeV]		Phase I Sensitivity [TeV]		Phase II Sensitivity [TeV]	
		B_d mixing	B_s mixing	B_d mixing	B_s mixing	B_d mixing	B_s mixing
$\|C_{ij}\| = \|V_{ti}V_{tj}^*\|$	tree level	9	13	17	18	20	21
(CKM-like)	one loop	0.7	1.0	1.3	1.4	1.6	1.7
$\|C_{ij}\| = 1$	tree level	1×10^3	3×10^2	2×10^3	4×10^2	2×10^3	5×10^2
(no hierarchy)	one loop	80	20	2×10^2	30	2×10^2	40

Charles et al. ‘20

- On the other hand, there are already strong (indirect) constraints, e.g., EDMs
- **We need a map/guide to explore CPV effects:**
  - What are the BSM sources of CPV?
  - What could be their sizes?
  - What should be the structure of CPV to allow new physics still accessible at colliders? MCPV?

CPV in SM_4

CPV comes from mixing among quarks and the resulting couplings to W



See for instance, G. Branco et al.

$$\mathcal{L}_{\rm mix} = \frac{e}{\sqrt{2}\sin\theta_w}\left[W_\mu^+ \bar{u} V \gamma^\mu \left(\frac{1-\gamma_5}{2}\right) d + W_\mu^- \bar{d} V^\dagger \gamma^\mu \left(\frac{1-\gamma_5}{2}\right) u\right]$$

$$\Big\downarrow \text{Proper CP}$$

$$\frac{e}{\sqrt{2}\sin\theta_w}\left[W_\mu^+ \bar{u}\,(V^\dagger)^T \gamma^\mu \left(\frac{1-\gamma_5}{2}\right) d + W_\mu^- \bar{d} V^T \gamma^\mu \left(\frac{1-\gamma_5}{2}\right) u\right]$$

Phases in CKM (can) break CP!

Are Phases a Sign of CPV?

Only after exhausting all flavour symmetries!

$$V_{\text{CKM}} = \begin{pmatrix} \frac{72-21i}{325} & \frac{4}{13} & -\frac{12i}{13} \\ -\frac{12}{13} & \frac{576+168i}{1625} & \frac{49-168i}{65} \\ -\frac{96-28i}{325} & -\frac{57}{65} & -\frac{24i}{65} \end{pmatrix}$$

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phases absorbed by redefining quark fields

no complex phase after appropriate phase shifts of quark fields

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if $m_u=m_c$,
enlarged U(2) flavour symmetry
that can be used to remove phase in CKM

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~~CPV ↔ ∃ phase in Lagrangian parameters~~

The SM$_4$ Collective CPV

The well-known KM counting

Kobayashi and Maskawa '73

	$SU(3)_Q$	$SU(3)_u$	$SU(3)_d$	$U(1)_u$	$U(1)_d$	$U(1)_B$
$Y_u\ (9R+9I)$	3	$\bar{3}$	1	1	0	0
$Y_d\ (9R+9I)$	3	1	$\bar{3}$	0	1	0
	$3R+5I$	$3R+5I$	$3R+5I$	$1I$	$1I$	$1I$

†

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$$\underbrace{3R+5I \quad 3R+5I \quad 3R+5I \quad 1I \quad 1I \quad \cancel{1I}}_{9R+17I}$$

physical
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$\underbrace{\qquad\qquad\qquad}_{9R+17I}$

$\boxed{\text{physical } 9R+1I}$

- The position of this physical phase is (flavour)-basis dependent, e.g.
  - Up-basis: Y_u=diag, Y_d= V_{CKM}.diag
  - Down-basis: Y_u=$V_{CKM}^\dagger$.diag, Y_d=diag
  - many other choices of flavour bases

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standard parametrisation
(particular choice of flavour basis)

$$V_{\rm CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13}e^{-i\delta_{\rm CKM}} \\ -c_{23}s_{12} - c_{12}s_{13}s_{23}e^{i\delta_{\rm CKM}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{\rm CKM}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\rm CKM}} & -c_{12}s_{23} - c_{23}s_{12}s_{13}e^{i\delta_{\rm CKM}} & c_{13}c_{23} \end{pmatrix}$$

Jarlskog Invariant

Jarlskog '85

see also Bernabeu, Branco, Gronau '86

The SM CPV order

- The lowest order flavour invariant sensitive to CPV

$$J_4 = \mathrm{ImTr}\left([Y_u Y_u^\dagger, Y_d Y_d^\dagger]^3\right)$$

- Explicitly

$$J_4 = \underbrace{6c_{12}s_{12}c_{13}^2 s_{13}c_{23}s_{23}}_{\mathcal{O}(\lambda^6)} \underbrace{\left(y_c^2 - y_u^2\right)\left(y_t^2 - y_u^2\right)\left(y_t^2 - y_c^2\right)\left(y_s^2 - y_d^2\right)\left(y_b^2 - y_d^2\right)\left(y_b^2 - y_s^2\right)}_{\mathcal{O}(\lambda^{30})} \underbrace{\sin\delta}_{\mathcal{O}(\lambda^0)}$$

Wolfenstein parametrisation

Wolfenstein '83

$$V_{\mathrm{CKM}} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4) \qquad \lambda \sim 0.22$$

- Even if δ~O(1), large suppression effects due to collective nature of CPV
- Important property: **CP is conserved iff $J_4=0$** (neglecting θ_{QCD} for now)

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exercise 2: check that for $N_F=2$, J_4 always vanishes

BSM CPV is also a Collective Effect

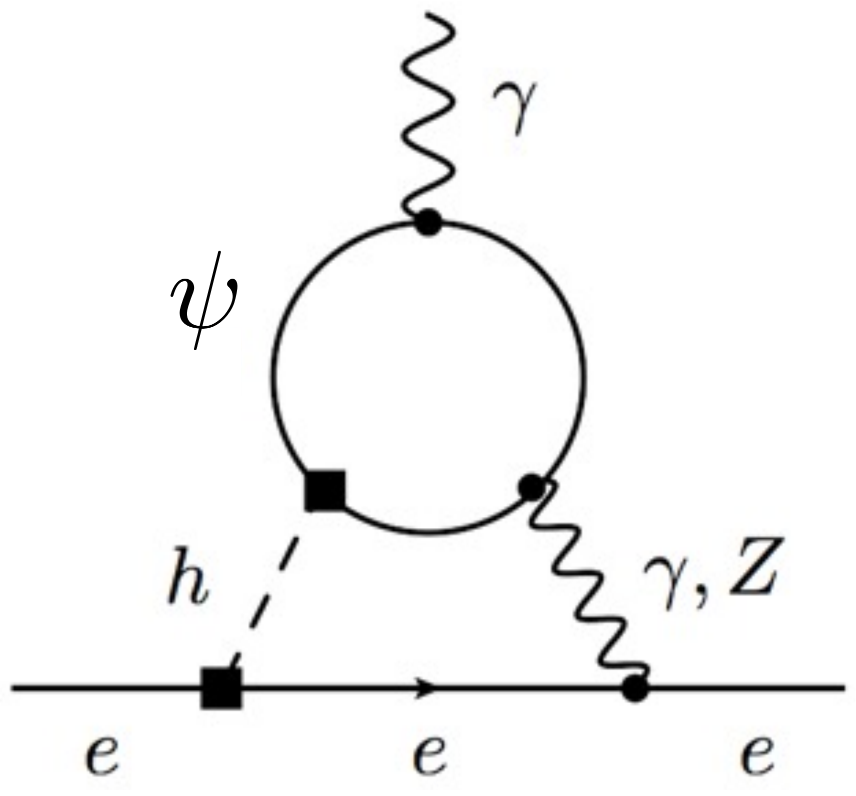
The example of electron EDM

- “Imaginary” Yukawa coupling gives rise to eEDM through Barr-Zee diagram

$$\mathcal{L} = Y_u \, \bar{Q}\tilde{H}U + C_{uH} \, |H|^2 \bar{Q}\tilde{H}U$$

$$\mathcal{L} = y \, h\bar{\psi}\psi$$

$$y_u = \frac{\sqrt{2}m_u}{v}\left(1 + C_{uH}v^2/\Lambda^2\right)$$



Brod, Haisch, Zupan ’13

$$\frac{d_e}{e} = -\frac{1}{48\pi^2}\frac{v m_e m_u}{m_h^2}\frac{\mathrm{Im}(C_{uH})}{\Lambda^2}F_1\left(\frac{m_u^2}{m_h^2}, 0\right)$$

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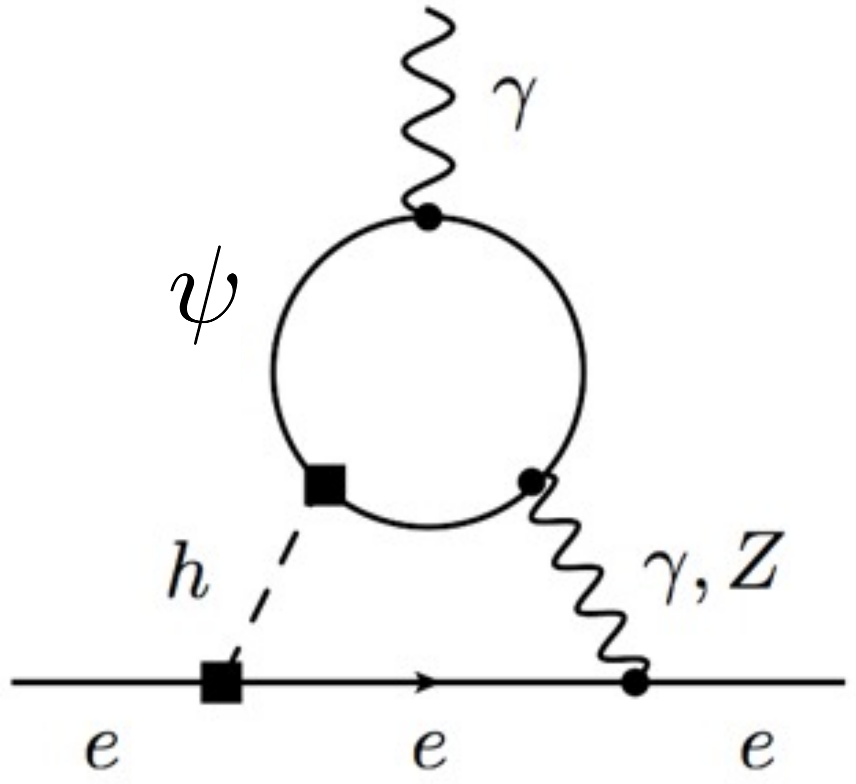
$$y_u = \frac{\sqrt{2}m_u}{v}\left(1 + C_{uH}v^2/\Lambda^2\right)$$



Brod, Haisch, Zupan ’13

$$\frac{d_e}{e} = -\frac{1}{48\pi^2}\frac{v m_e m_u}{m_h^2}\frac{\mathrm{Im}(C_{uH})}{\Lambda^2}F_1\left(\frac{m_u^2}{m_h^2}, 0\right)$$

$$\frac{d_e}{e} = -\frac{1}{48\pi^2}\frac{v m_e}{m_h^2}\frac{\mathrm{Im}(m_u^* C_{uH})}{\Lambda^2}F_1\left(\frac{|m_u|^2}{m_h^2}, 0\right)$$

- The Yukawa can be made real by chiral rotation: $\psi \to e^{i\theta\gamma^5}\psi$
- The “phase” will appear in the mass
- The CPV effect is captured by Im ($y^{\dagger}\cdot$m), which is invariant under chiral rotation

Trivial here, but can get complicated: flavour indices, links to UV parameters…

Dim-6 Yukawa's Contribution to EDMs

CP doesn't say Wilson coefficients are real

$$\mathcal{L} = \underbrace{Y_u}_{\substack{\text{3x3 complex} \\ \text{(9R+9I)}}} \bar{Q}\tilde{H}U + \underbrace{C_{uH}}_{\substack{\text{3x3 complex} \\ \text{(9R+9I)}}} |H|^2 \bar{Q}\tilde{H}U \quad \Rightarrow \quad \underbrace{g^{ij}_{huu}}_{Y_u^{ij} + 3v^2 C_{uH}^{ij}} h\bar{u}_i u_j$$

One can choose $U(3)_Q \times U(3)_U$ transformations to make C_{uH} (or g_{huu}) *real*

CPV effects $\overset{!}{\leftrightarrow}$ Im C_{uH}

Phases can be moved to mass matrices — even in mass basis, ∃ residual U(1)'s to move phase around

(flavour basis fully specified by the location of the phase in the CKM matrix)

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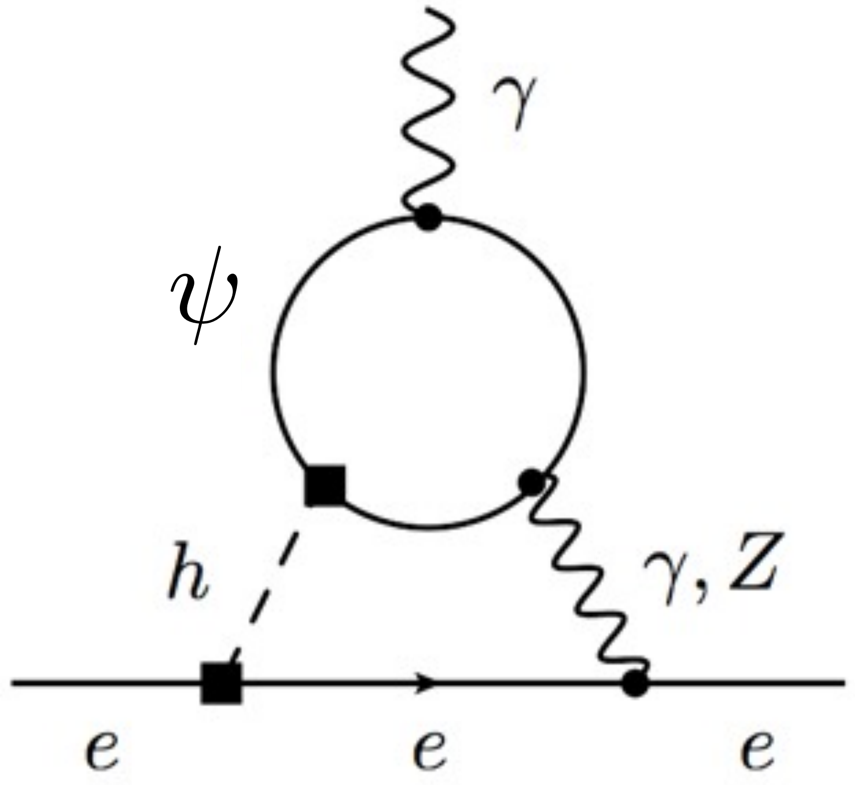
At two loops and $1/\Lambda^2$ order, **Barr-Zee** diagrams depends only on three phases captured by **three invariants**
(only diagonal phases can contribute at 2-loops because no FCNC in SM)



$$\frac{d_e}{e} \propto \frac{\alpha y_e}{16\pi^3} (a\, I_1 + b\, I_2 + c\, I_3) \qquad \text{with} \qquad I_n = \text{Im Tr}\left(Y_u^\dagger \left(Y_u Y_u^\dagger\right)^n C_{uH}\right)$$

a, b, c functions of Y_u only

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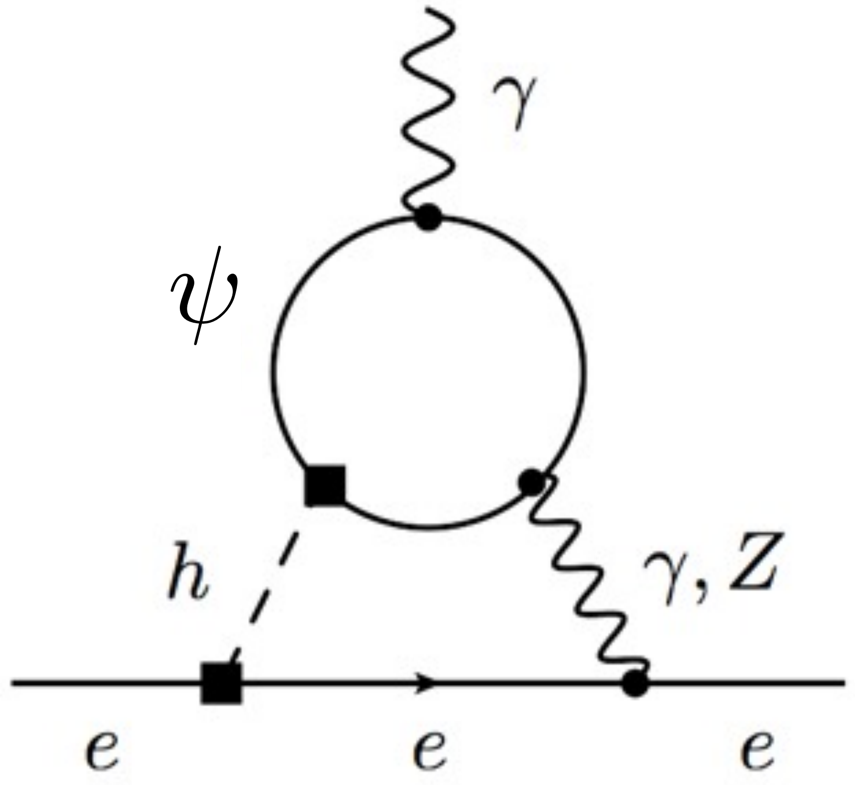

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- How many?
- How many constraints should we impose to ensure CP is conserved?

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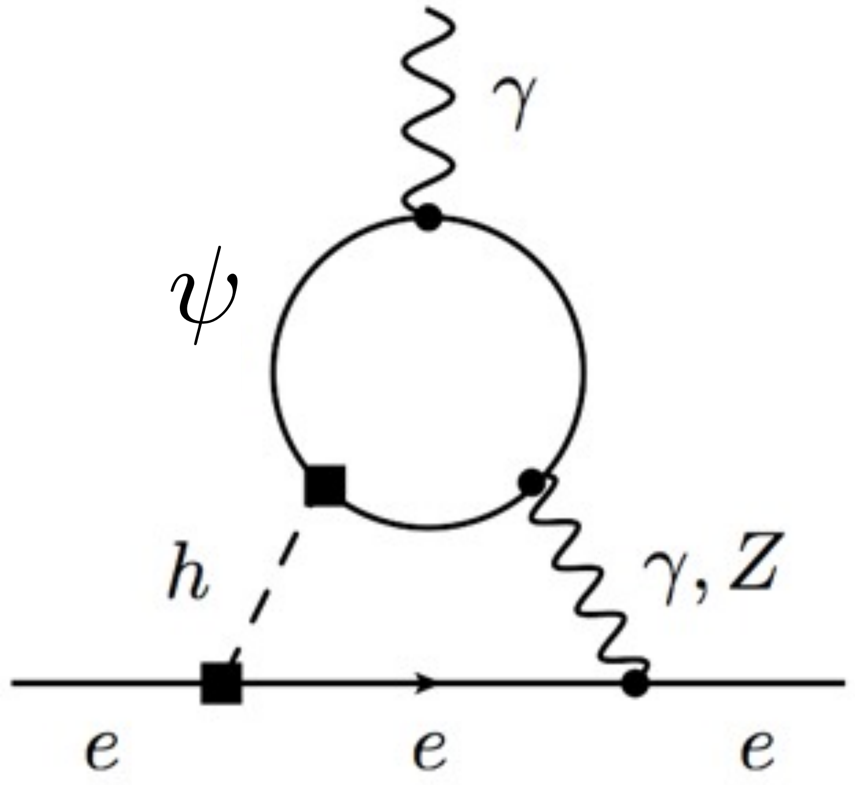

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~~CP ↔ C_{uH} real matrix~~

Beyond Jarlskog

Necessary and sufficient conditions for CPV?

$$\mathcal{A} = \mathcal{A}^{(4)} + \mathcal{A}^{(6)} + \ldots \Rightarrow |\mathcal{A}^{(4)}|^2 + 2\mathrm{Re}\left(\mathcal{A}^{(4)}\mathcal{A}^{(6)*}\right)$$

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CP iff J_4=0 & ???

How many conditions?

Any relation with the number of phases that can appear in L_{SM6}?

Beyond Jarlskog: Building SM_6 invariants

Examples of invariants from with bilinear operators

- For each operators, e.g. the dim-6 Yukawa operators, we can build a series of CP-odd invariants:

$$I_{u_1\ldots d_k} = \mathrm{Im}\ \mathrm{Tr}\left(Y_u^\dagger \left(Y_u Y_u^\dagger\right)^{u_1} \left(Y_d Y_d^\dagger\right)^{d_1} \ldots \left(Y_u Y_u^\dagger\right)^{u_k} \left(Y_d Y_d^\dagger\right)^{d_k} C_{uH}\right)$$

- Of course, they are not all independent:

e.g., for 3 families, $I_3 = \mathrm{Tr}\left(Y_u Y_u^\dagger\right) I_2 + \frac{1}{2}\left(\mathrm{Tr}\left(\left(Y_u\ Y_u^\dagger\right)^2\right) - \mathrm{Tr}^2\left(Y_u\ Y_u^\dagger\right)\right) I_1$

- Only need to consider only a finite set of invariants:

Cayley-Hamilton: $A^3 = A^2\,\mathrm{Tr}(A) - \frac{1}{2}A\left[\mathrm{Tr}(A)^2 - \mathrm{Tr}\left(A^2\right)\right] + \frac{1}{6}\left[\mathrm{Tr}(A)^3 - 3\,\mathrm{Tr}\left(A^2\right)\mathrm{Tr}(A) + 2\,\mathrm{Tr}\left(A^3\right)\right]\mathbb{I}_{3\times 3}$

→ enough to consider $\mathrm{Tr}\left(X_u^a X_d^b X_u^c X_d^d C\right)$ a,b,c,d=0,1,2, a≠b,c≠d $\qquad X_{u/d} = Y_{u/d}\ Y_{u/d}^\dagger$

Can find a basis of invariants linearly independent from each others (see backup)

Opportunistic CP violation

Opportunistic CPV = interference with CKM phase

- If J_4=0, we can find 699 independent invariants ⇒ **minimal** basis of invariants.

"CP is conserved iff J_4 and the invariants of the minimal basis are all vanishing"

- If $J_4 \neq 0$, we can actually build more invariants! Not surprising, because CP-even BSM can interfere with CP-odd SM. But what was maybe unexpected is that many of these interfering invariants can be much larger than J_4 → **maximal** basis of invariants.

dim (maximal basis) = number of physical (real and imaginary) parameters
that can interfere with SM
and thus can show up in observables at leading $O(1/\Lambda^2)$

Opportunistic CPV relies on interference with SM phase but it doesn't
have to suffer from the same collective suppression!

How many independent invariants at a given order in Cabibbo expansion?

Taylor Rank

$$\boxed{\text{Taylor Rank}_{|\epsilon^n}(M) = \underset{N=M+\mathcal{O}(\epsilon^{n+1})}{\text{Min}} \text{Rank}\,(N)}$$

$$M = \begin{pmatrix} 1 & \epsilon \\ \epsilon & \epsilon^2 \end{pmatrix}$$

$$\text{Taylor Rank}_{|\epsilon^0} = 1 = \text{Rank}\left(M_{|\epsilon^0}\right)$$

$$\text{Taylor Rank}_{|\epsilon^1} = 1 \neq \text{Rank}\left(M_{|\epsilon^1}\right) = 2$$

Scaling of Collective CPV BSM Effects

The new invariants don't suffer from the same suppression factors

- The invariants can be evaluated in e.g. the up-flavour basis:

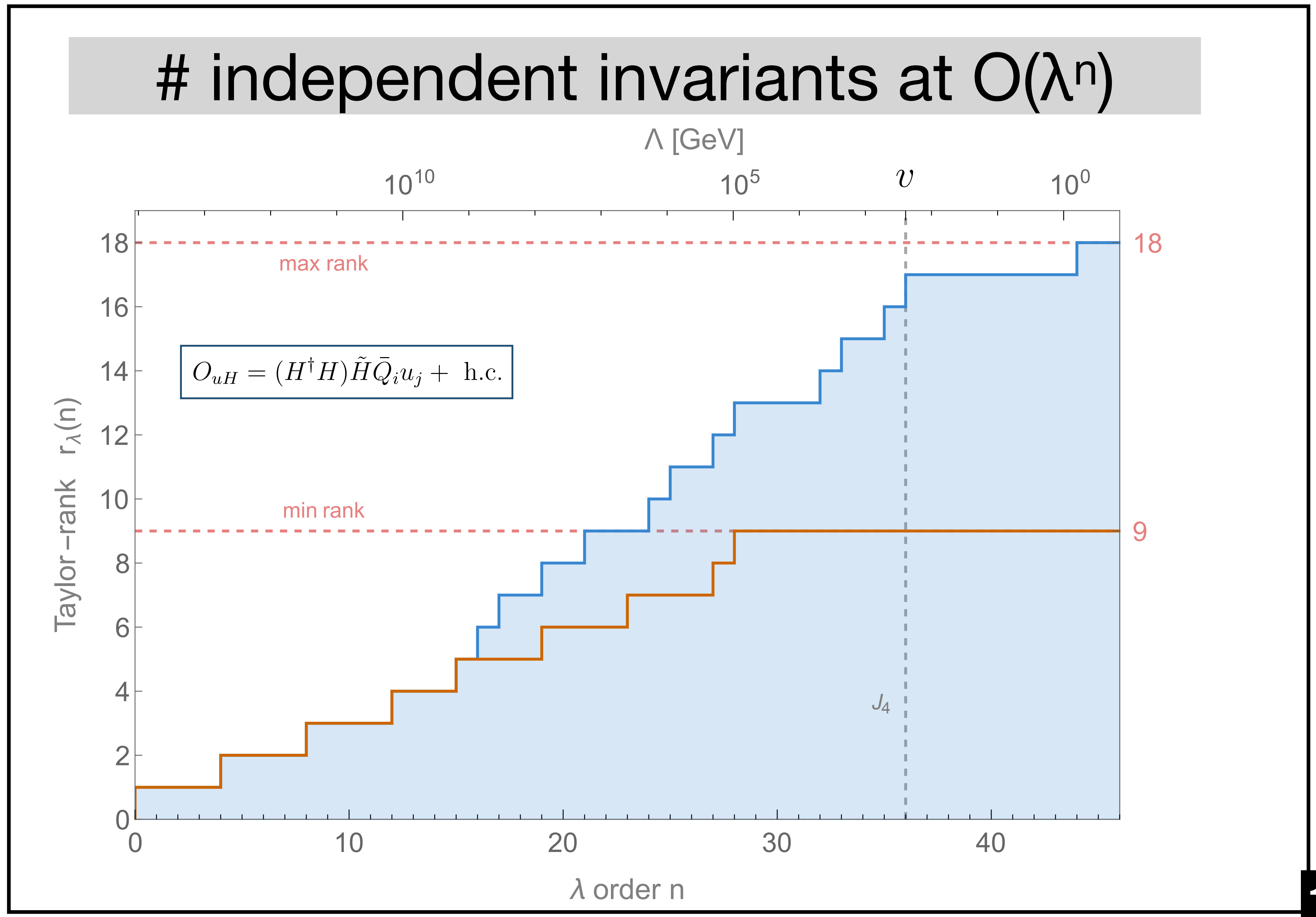
$\boxed{\text{dim.6 up-Yukawa operator}}$

$$I_n = \underbrace{y_u^{2n+1}}_{\mathcal{O}\left(\lambda^{16n+8}\right)}\eta_u + \underbrace{y_c^{2n+1}}_{\mathcal{O}\left(\lambda^{8n+4}\right)}\eta_c + \underbrace{y_t^{2n+1}}_{\mathcal{O}\left(\lambda^{0}\right)}\eta_t \qquad I_n = \mathrm{Im}\ \mathrm{Tr}\left(Y_u^\dagger\left(Y_uY_u^\dagger\right)^n C_{uH}\right)$$

$$I_{1,1} = \underbrace{c_{13}c_{23}s_{13}s_\delta}_{\mathcal{O}\left(\lambda^{3}\right)}\underbrace{\left(y_b^2 - c_{12}^2y_d^2 - s_{12}^2y_s^2\right)y_t}_{\mathcal{O}\left(\lambda^{6}\right)}\rho_{ut} + \ldots \qquad I_{1,1} = \mathrm{Im}\ \mathrm{Tr}\left(Y_u^\dagger\left(Y_uY_u^\dagger\right)\left(Y_dY_d^\dagger\right)C_{uH}\right)$$

- The BSM invariants are suppressed by scale of new physics
- but not necessarily by small Yukawa/mixing angles as J_4

$$I \sim \lambda^n \frac{v^2}{\Lambda^2}$$



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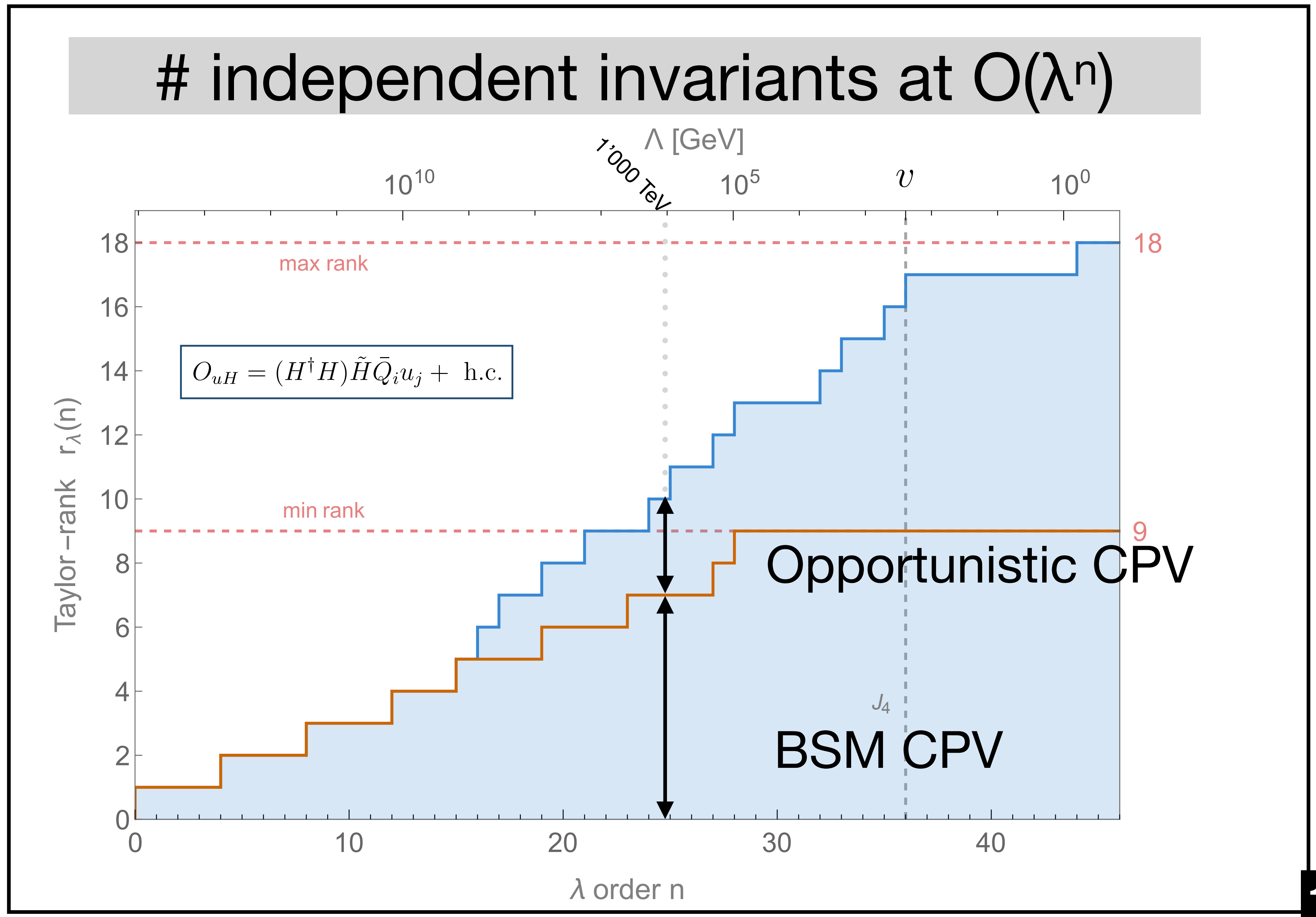
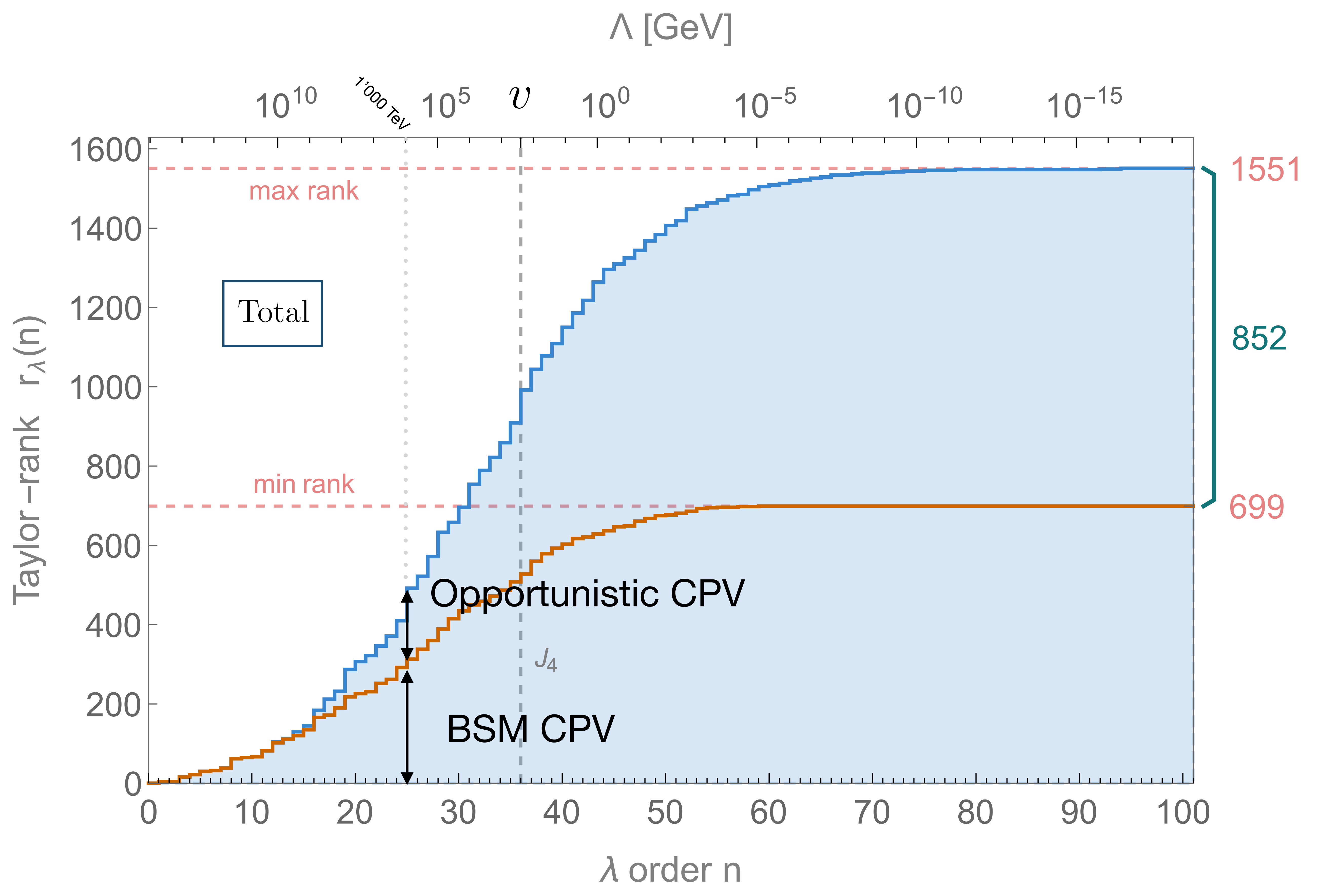
$$I \sim \lambda^n\frac{v^2}{\Lambda^2} > J_4 \sim \lambda^{36} \leftrightarrow \Lambda < \lambda^{n/2-18}\,v$$

Λ~1'000 TeV → 7 BSM and 3 Opportunistic invariants larger than J_4



Scaling of Collective CPV BSM Effects

independent invariants at $O(\lambda^n)$ for dim-6 operators



maximal basis
(interference with CKM)

minimal basis
("new phases")

Λ~1'000 TeV → ~250 BSM and ~250 Opportunistic invariants larger than J_4

Models of Flavours

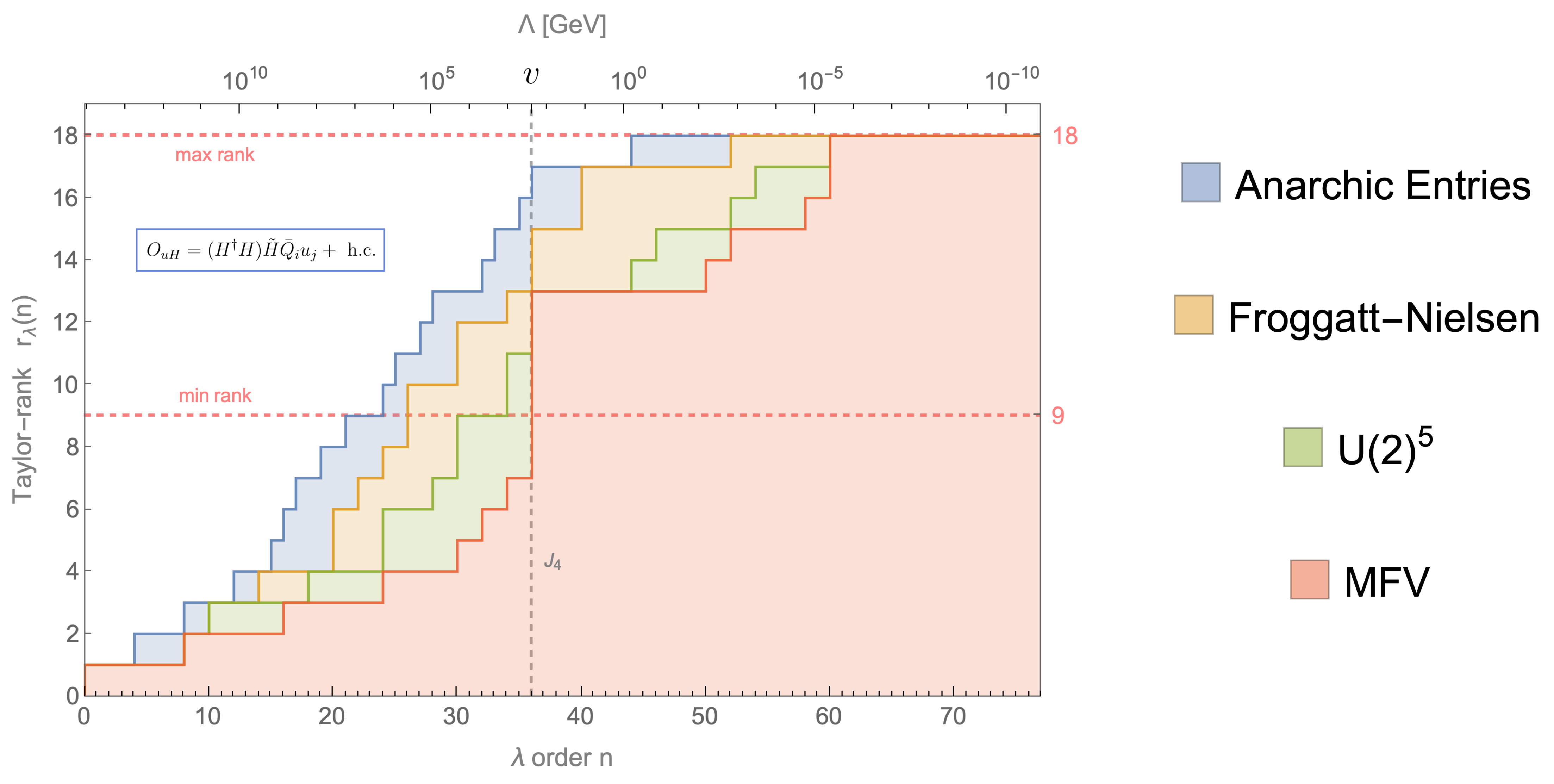
Beyond generic flavour model: MFV

- Other constraints from CP-even observables: totally flavour generic/anarchic dim-6 operators are severely constrained. How do additional flavour structure affect the orders of CPV computed above in the generic case?

- Let's first stick to the canonical flavour "model": Minimal Flavour Violation

$$c_{uH} = aY_u + b\left(Y_u Y_u^\dagger\right) Y_u + c\left(Y_d Y_d^\dagger\right) Y_u + \dots$$

Generic Flavour		MFV
$\text{Rank } 1 \to \mathcal{O}\left(\lambda^0\right)$		$\text{Rank } 1 \to \mathcal{O}\left(\lambda^0\right)$
$\text{Rank } 2 \to \mathcal{O}\left(\lambda^4\right)$	➡	$\text{Rank } 2 \to \mathcal{O}\left(\lambda^8\right)$
$\text{Rank } 3 \to \mathcal{O}\left(\lambda^8\right)$		$\text{Rank } 3 \to \mathcal{O}\left(\lambda^{16}\right)$
$\vdots$		$\vdots$

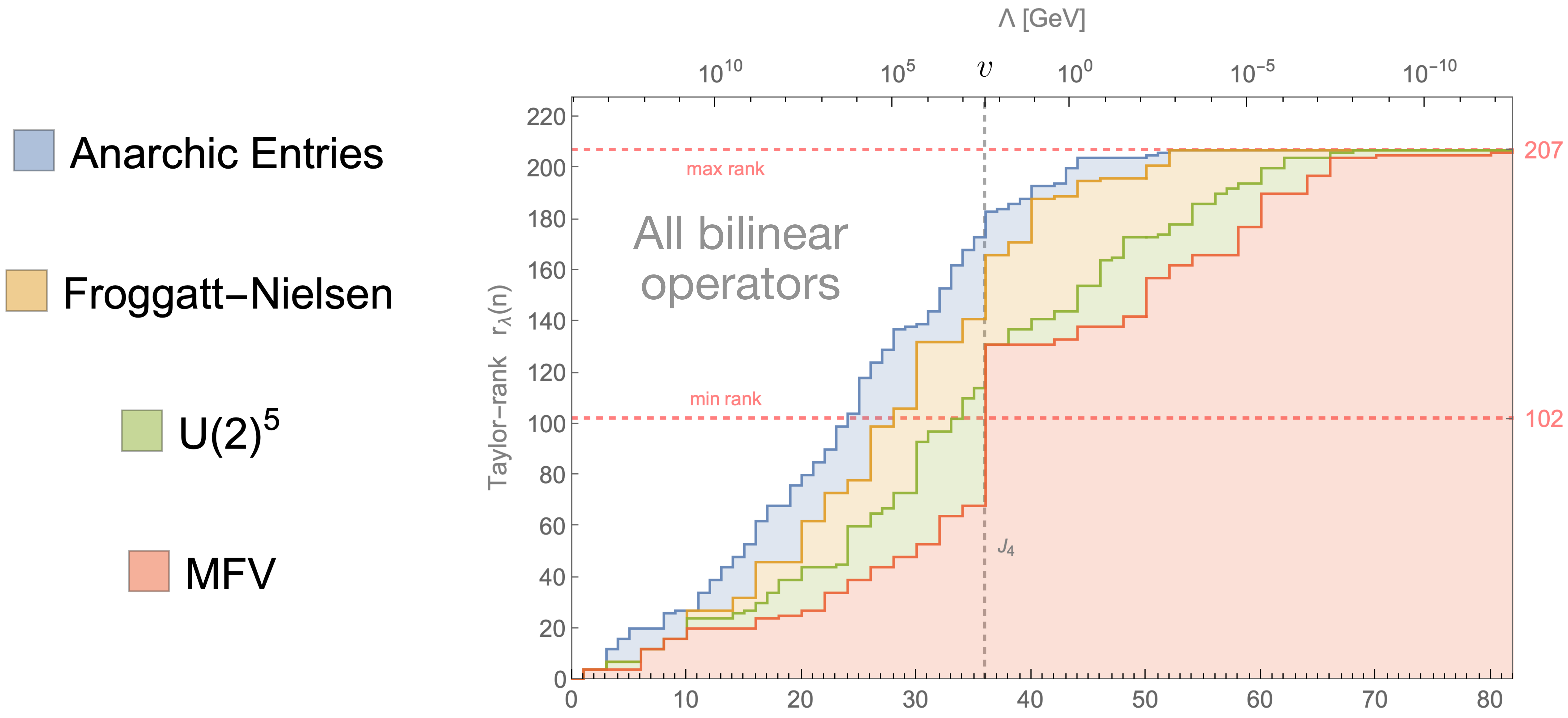


CPV Orders in Alignment Models

Froggatt-Nielsen-type & U(2)³ Flavour Structure

- Another popular flavour structure is alignment inherited e.g. from $U(1)_{FN}$ symmetry
- The U(1) charges of the quarks will imprint a particular scaling of the dim.6 WC:

$$Y_u = \begin{pmatrix} \lambda^8 & \lambda^5 & \lambda^3 \\ \lambda^7 & \lambda^4 & \lambda^2 \\ \lambda^5 & \lambda^2 & 1 \end{pmatrix} \qquad Y_d = \begin{pmatrix} \lambda^7 & \lambda^6 & \lambda^6 \\ \lambda^6 & \lambda^5 & \lambda^5 \\ \lambda^4 & \lambda^3 & \lambda^3 \end{pmatrix} \qquad C_{uH} = \text{generic} = \begin{pmatrix} \lambda^8 & \lambda^5 & \lambda^3 \\ \lambda^7 & \lambda^4 & \lambda^2 \\ \lambda^5 & \lambda^2 & 1 \end{pmatrix}$$



Generic Flavour Structure
Λ >1'000 TeV ⇒ ~120 sources of CPV larger than SM

MFV Flavour Structure
Λ > 5-10 TeV ⇒ ~50 sources of CPV larger than SM

We couldn't explore effects of Flavour assumptions on 4 Fermi operators (too computational intensive)

Part II. CPV in Axion couplings to fermions?

Axion/ALP=Goldstone boson → shift-symmetry

$$a \to a + \epsilon f$$

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{2}\left(\partial_\mu a\right)\left(\partial^\mu a\right) + \frac{\partial_\mu a}{f}\sum_{\psi \in \rm SM} \bar{\psi}\boxed{c_\psi}\gamma^\mu\psi + \mathcal{O}\left(\frac{1}{f^2}\right)$$

hermitian matrices
(26 CP-even and 13 CP-odd couplings)

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But shift-symmetry cannot be exact (PQ as approximate symmetry)

What are the allowed couplings of an ALP after (soft) breaking of shift-symmetry?

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What are the allowed couplings of an ALP after (soft) breaking of shift-symmetry?

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{a}{f}\left(\bar Q \boxed{\tilde Y_u} \tilde H u + \bar Q \boxed{\tilde Y_d} H d + \bar L \boxed{\tilde Y_e} H e + {\rm h.c.}\right)$$

generic matrices
(27 CP-even and 25 CP-odd couplings)

What is the power counting of these new couplings?

What are the conditions to recover a shift-symmetry?

ALP shift invariance and CP

1. Conditions to enforce ALP shift-symmetry

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) + \frac{\partial_\mu a}{f}\sum_{\psi\in{\rm SM}}\bar{\psi}\boxed{c_\psi}\gamma^\mu\psi + \mathcal{O}\left(\frac{1}{f^2}\right) \xrightarrow{\psi \to e^{-ic_\psi a/f}\psi} \mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{a}{f}\left(\bar{Q}\boxed{\tilde{Y}_u}\tilde{H}u + \bar{Q}\boxed{\tilde{Y}_d}Hd + \bar{L}\boxed{\tilde{Y}_e}He + \text{h.c.}\right)$$

$$\tilde{Y}_{u,d} = i(Y_{u,d}c_{u,d} - c_Q Y_{u,d})\ , \quad \tilde{Y}_e = i(Y_e c_e - c_L Y_e)$$

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Numbers of physical parameters

hermitian matrices (6 angles and 3 phases) — generic matrices (9 angles and 9 phases)

	Shift-symmetric Wilson coefficients $c_{Q,u,d,L,e}$		Generic Wilson coefficients $\tilde{Y}_{u,d,e}$		Number of constraints	
	CP-even	CP-odd	CP-even	CP-odd	CP-even	CP-odd
Quark sector	17	9	18	18	1	9
Lepton sector	9	4	9	7	0	3

U(1)$_B$ and U(1)$_{L_i}$ conserved currents

$\partial_\mu a J^\mu$ added to Lagrangian

3*6-1=17

2*6-3=9

#L_i remove 2 phases

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13 conditions on $\tilde Y$ to recover a shift symmetry (1 CP-even and 12 CP-odd)

ALP shift invariance

The conditions for shift-symmetry can be written in an invariant way

$X_x = Y_x Y_x^\dagger$

- **Lepton sector**

$$\mathrm{Re\,Tr}\Big(X_e^{0,1,2} \tilde{Y}_e Y_e^\dagger \Big) = 0$$

3 invariants

- **Quark sector**

$$
\begin{aligned}
I_u^{(1)} &= \mathrm{Re\,Tr}\left(\tilde{Y}_u Y_u^\dagger\right), \quad I_u^{(2)} = \mathrm{Re\,Tr}\left(X_u \tilde{Y}_u Y_u^\dagger\right), \quad I_u^{(3)} = \mathrm{Re\,Tr}\left(X_u^2 \tilde{Y}_u Y_u^\dagger\right),\\
I_d^{(1)} &= \mathrm{Re\,Tr}\left(\tilde{Y}_d Y_d^\dagger\right), \quad I_d^{(2)} = \mathrm{Re\,Tr}\left(X_d \tilde{Y}_d Y_d^\dagger\right), \quad I_d^{(3)} = \mathrm{Re\,Tr}\left(X_d^2 \tilde{Y}_d Y_d^\dagger\right),\\
I_{ud}^{(1)} &= \mathrm{Re\,Tr}\left(X_d \tilde{Y}_u Y_u^\dagger + X_u \tilde{Y}_d Y_d^\dagger\right),\\
I_{ud,u}^{(2)} &= \mathrm{Re\,Tr}\left(X_u^2 \tilde{Y}_d Y_d^\dagger + \{X_u, X_d\} \tilde{Y}_u Y_u^\dagger\right),\\
I_{ud,d}^{(2)} &= \mathrm{Re\,Tr}\left(X_d^2 \tilde{Y}_u Y_u^\dagger + \{X_u, X_d\} \tilde{Y}_d Y_d^\dagger\right),\\
I_{ud}^{(3)} &= \mathrm{Re\,Tr}\left(X_d X_u X_d \tilde{Y}_u Y_u^\dagger + X_u X_d X_u \tilde{Y}_d Y_d^\dagger\right)\\
I_{ud}^{(4)} &= \mathrm{Im\,Tr}\left(\left[X_u, X_d\right]^2 \left(\left[X_d, \tilde{Y}_u Y_u^\dagger\right] - \left[X_u, \tilde{Y}_d Y_d^\dagger\right]\right)\right)
\end{aligned}
$$

$I_i = 0$

4 entangled conditions

between up and down sectors

⇒ collective nature

one algebraic relation ⇒ only **10 independent invariants**

13 flavour invariants all linear in $\tilde{Y}$ (CP ensure that all but $I_{ud}^{(4)}$ vanish)

RG invariance

The set of invariants is closed under RG

$$\dot{I}_e^{(1)} = 2\gamma_e I_e^{(1)} + 6I_e^{(2)} + 2\,\mathrm{Tr}(X_e)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right),$$
$$\dot{I}_e^{(2)} = 4\gamma_e I_e^{(2)} + 9I_e^{(3)} + 2\,\mathrm{Tr}\left(X_e^2\right)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right),$$
$$\dot{I}_e^{(3)} = 6\gamma_e I_e^{(3)} + 12I_e^{(4)} + 2\,\mathrm{Tr}\left(X_e^3\right)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right)$$
$$\dot{I}_u^{(1)} = 2\gamma_u I_u^{(1)} + 6I_u^{(2)} - 3I_{ud}^{(1)} - 2\,\mathrm{Tr}(X_u)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right),$$
$$\dot{I}_u^{(2)} = 4\gamma_u I_u^{(2)} + 9I_u^{(3)} - 3I_{ud,u}^{(2)} - 2\,\mathrm{Tr}\left(X_u^2\right)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right),$$
$$\dot{I}_u^{(3)} = 6\gamma_u I_u^{(3)} + 12I_u^{(4)} - 3I_u' - 2\,\mathrm{Tr}\left(X_u^3\right)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right),$$
$$\dot{I}_d^{(1)} = 2\gamma_d I_d^{(1)} + 6I_d^{(2)} - 3_{ud}^{(1)} + 2\,\mathrm{Tr}(X_d)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right),$$
$$\dot{I}_d^{(2)} = 4\gamma_d I_d^{(2)} + 9I_d^{(3)} - 3I_{ud,d}^{(2)} + 2\,\mathrm{Tr}\left(X_d^2\right)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right),$$
$$\dot{I}_d^{(3)} = 6\gamma_d I_d^{(3)} + 12I_d^{(4)} - 3I_d' + 2\,\mathrm{Tr}\left(X_d^3\right)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right),$$
$$\dot{I}_{ud}^{(1)} = 2(\gamma_u + \gamma_d)I_{ud}^{(1)},$$
$$\dot{I}_{ud,u}^{(2)} = (4\gamma_u + 2\gamma_d)I_{ud,u}^{(2)} + 3I_u' - 6I_{ud}^{(3)} - 2\,\mathrm{Tr}(X_u X_d X_u)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right),$$
$$\dot{I}_{ud,d}^{(2)} = (4\gamma_d + 2\gamma_u)I_{ud,d}^{(2)} + 3I_d' - 6I_{ud}^{(3)} + 2\,\mathrm{Tr}(X_d X_u X_d)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right),$$
$$\dot{I}_{ud}^{(3)} = 4(\gamma_u + \gamma_d)I_{ud}^{(3)},$$
$$\dot{I}_{ud}^{(4)} = 6\left(\gamma_u + \gamma_d + \frac{1}{2}\,\mathrm{Tr}(X_u + X_d)\right)I_{ud}^{(4)} - \mathrm{Im}\,\mathrm{Tr}\left([X_u, X_d]^3\right)(I_u^{(1)} + I_d^{(1)}).$$

$$\gamma_e = -\tfrac{15}{4}g_1^2 - \tfrac{9}{4}g_2^2 + \mathrm{Tr}\left(X_e + 3(X_u + X_d)\right)$$

$$\gamma_u \equiv -\tfrac{17}{12}g_1^2 - \tfrac{9}{4}g_2^2 - 8g_3^2 + \mathrm{Tr}\left(X_e + 3(X_u + X_d)\right)$$

$$\gamma_d \equiv -\tfrac{5}{12}g_1^2 - \tfrac{9}{4}g_2^2 - 8g_3^2 + \mathrm{Tr}\left(X_e + 3(X_u + X_d)\right)$$

RG invariance

The set of invariants is closed under RG

$$\dot{I}_e^{(1)} = 2\gamma_e I_e^{(1)} + 6I_e^{(2)} + 2\,\mathrm{Tr}(X_e)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right),$$

$$\dot{I}_e^{(2)} = 4\gamma_e I_e^{(2)} + 9I_e^{(3)} + 2\,\mathrm{Tr}\left(X_e^2\right)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right),$$

$$\dot{I}_e^{(3)} = 6\gamma_e I_e^{(3)} + 12I_e^{(4)} + 2\,\mathrm{Tr}\left(X_e^3\right)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right)$$

$$\dot{I}_u^{(1)} = 2\gamma_u I_u^{(1)} + 6I_u^{(2)} - 3I_{ud}^{(1)} - 2\,\mathrm{Tr}(X_u)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right),$$

$$\dot{I}_u^{(2)} = 4\gamma_u I_u^{(2)} + 9I_u^{(3)} - 3I_{ud,u}^{(2)} - 2\,\mathrm{Tr}\left(X_u^2\right)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right),$$

$$\dot{I}_u^{(3)} = 6\gamma_u I_u^{(3)} + 12I_u^{(4)} - 3I_u' - 2\,\mathrm{Tr}\left(X_u^3\right)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right),$$

$$\dot{I}_d^{(1)} = 2\gamma_d I_d^{(1)} + 6I_d^{(2)} - 3_{ud}^{(1)} + 2\,\mathrm{Tr}(X_d)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right),$$

$$\dot{I}_d^{(2)} = 4\gamma_d I_d^{(2)} + 9I_d^{(3)} - 3I_{ud,d}^{(2)} + 2\,\mathrm{Tr}\left(X_d^2\right)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right),$$

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$$\dot{I}_{ud}^{(1)} = 2(\gamma_u + \gamma_d)I_{ud}^{(1)},$$

$$\dot{I}_{ud,u}^{(2)} = (4\gamma_u + 2\gamma_d)I_{ud,u}^{(2)} + 3I_u' - 6I_{ud}^{(3)} - 2\,\mathrm{Tr}(X_u X_d X_u)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right),$$

$$\dot{I}_{ud,d}^{(2)} = (4\gamma_d + 2\gamma_u)I_{ud,d}^{(2)} + 3I_d' - 6I_{ud}^{(3)} + 2\,\mathrm{Tr}(X_d X_u X_d)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right),$$

$$\dot{I}_{ud}^{(3)} = 4(\gamma_u + \gamma_d)I_{ud}^{(3)},$$

$$\dot{I}_{ud}^{(4)} = 6\left(\gamma_u + \gamma_d + \frac{1}{2}\,\mathrm{Tr}(X_u + X_d)\right)I_{ud}^{(4)} - \mathrm{Im}\,\mathrm{Tr}\left([X_u, X_d]^3\right)(I_u^{(1)} + I_d^{(1)}).$$

$$\gamma_e = -\tfrac{15}{4}g_1^2 - \tfrac{9}{4}g_2^2 + \mathrm{Tr}\left(X_e + 3(X_u + X_d)\right)$$

$$\gamma_u \equiv -\tfrac{17}{12}g_1^2 - \tfrac{9}{4}g_2^2 - 8g_3^2 + \mathrm{Tr}\left(X_e + 3(X_u + X_d)\right)$$

$$\gamma_d \equiv -\tfrac{5}{12}g_1^2 - \tfrac{9}{4}g_2^2 - 8g_3^2 + \mathrm{Tr}\left(X_e + 3(X_u + X_d)\right)$$

closed set except for:

$$I_e^{(4)} = \mathrm{Re}\,\mathrm{Tr}\left(X_e^3 \tilde{Y}_e Y_e^\dagger\right)$$

$$I_u' = \mathrm{Re}\,\mathrm{Tr}\left((X_u X_d X_u + \{X_d, X_u^2\})\tilde{Y}_u Y_u^\dagger + X_u^3 \tilde{Y}_d Y_d^\dagger\right)$$

$$I_d' = I_u'(u \leftrightarrow d)$$

RG invariance

The set of invariants is closed under RG

$$\dot{I}_e^{(1)} = 2\gamma_e I_e^{(1)} + 6I_e^{(2)} + 2\,\mathrm{Tr}(X_e)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right),$$
$$\dot{I}_e^{(2)} = 4\gamma_e I_e^{(2)} + 9I_e^{(3)} + 2\,\mathrm{Tr}(X_e^2)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right),$$
$$\dot{I}_e^{(3)} = 6\gamma_e I_e^{(3)} + 12\boxed{I_e^{(4)}} + 2\,\mathrm{Tr}(X_e^3)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right)$$
$$\dot{I}_u^{(1)} = 2\gamma_u I_u^{(1)} + 6I_u^{(2)} - 3I_{ud}^{(1)} - 2\,\mathrm{Tr}(X_u)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right),$$
$$\dot{I}_u^{(2)} = 4\gamma_u I_u^{(2)} + 9I_u^{(3)} - 3I_{ud,u}^{(2)} - 2\,\mathrm{Tr}(X_u^2)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right),$$
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$$\dot{I}_d^{(1)} = 2\gamma_d I_d^{(1)} + 6I_d^{(2)} - 3_{ud}^{(1)} + 2\,\mathrm{Tr}(X_d)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right),$$
$$\dot{I}_d^{(2)} = 4\gamma_d I_d^{(2)} + 9I_d^{(3)} - 3I_{ud,d}^{(2)} + 2\,\mathrm{Tr}(X_d^2)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right),$$
$$\dot{I}_d^{(3)} = 6\gamma_d I_d^{(3)} + 12I_d^{(4)} - 3\boxed{I_d'} + 2\,\mathrm{Tr}(X_d^3)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right),$$
$$\dot{I}_{ud}^{(1)} = 2(\gamma_u + \gamma_d)I_{ud}^{(1)},$$
$$\dot{I}_{ud,u}^{(2)} = (4\gamma_u + 2\gamma_d)I_{ud,u}^{(2)} + 3\boxed{I_u'} - 6I_{ud}^{(3)} - 2\,\mathrm{Tr}(X_u X_d X_u)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right),$$
$$\dot{I}_{ud,d}^{(2)} = (4\gamma_d + 2\gamma_u)I_{ud,d}^{(2)} + 3\boxed{I_d'} - 6I_{ud}^{(3)} + 2\,\mathrm{Tr}(X_d X_u X_d)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right),$$
$$\dot{I}_{ud}^{(3)} = 4(\gamma_u + \gamma_d)I_{ud}^{(3)},$$
$$\dot{I}_{ud}^{(4)} = 6\left(\gamma_u + \gamma_d + \frac{1}{2}\mathrm{Tr}(X_u + X_d)\right)I_{ud}^{(4)} - \mathrm{Im}\,\mathrm{Tr}\left([X_u, X_d]^3\right)(I_u^{(1)} + I_d^{(1)}).$$

$$\gamma_e = -\tfrac{15}{4}g_1^2 - \tfrac{9}{4}g_2^2 + \mathrm{Tr}\left(X_e + 3(X_u + X_d)\right)$$
$$\gamma_u \equiv -\tfrac{17}{12}g_1^2 - \tfrac{9}{4}g_2^2 - 8g_3^2 + \mathrm{Tr}\left(X_e + 3(X_u + X_d)\right)$$
$$\gamma_d \equiv -\tfrac{5}{12}g_1^2 - \tfrac{9}{4}g_2^2 - 8g_3^2 + \mathrm{Tr}\left(X_e + 3(X_u + X_d)\right)$$

closed set except for:

$$\boxed{I_e^{(4)}} = \mathrm{Re}\,\mathrm{Tr}\left(X_e^3 \tilde{Y}_e Y_e^\dagger\right)$$
$$\boxed{I_u'} = \mathrm{Re}\,\mathrm{Tr}\left((X_u X_d X_u + \{X_d, X_u^2\})\tilde{Y}_u Y_u^\dagger + X_u^3 \tilde{Y}_d Y_d^\dagger\right)$$
$$\boxed{I_d'} = I_u'(u \leftrightarrow d)$$

but Cayley-Hamilton eq. tells us that these 3 invariants are actually linear combinations of our original set

$\Downarrow$

shift-invariance conditions are closed under RG

Conclusions

EDM constraints don't exclude all BSM sources of CPV!

- CPV is a **collective effect**.
- CP is not an accidental symmetry but **CPV is accidentally small** in SM_4.
- Many new possible (large) sources of CPV at dim-6 level.
- **Shift-symmetry of an ALP** reduces to Jarlskog-like invariant conditions
- ALP shift-symmetry is surprisingly closely connected to CP-symmetry

BONUS

SM$_6$

Basis of dim-6 operators, aka Warsaw basis

$1: X^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
$Q_{\widetilde{G}}$	$f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
Q_W	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$
$Q_{\widetilde{W}}$	$\epsilon^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$

$2: H^6$	
Q_H	$(H^\dagger H)^3$

$3: H^4 D^2$	
$Q_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
Q_{HD}	$\left(H^\dagger D_\mu H\right)^* \left(H^\dagger D_\mu H\right)$

$5: \psi^2 H^3 + \text{h.c.}$	
Q_{eH}	$(H^\dagger H)(\bar{l}_p e_r H)$
Q_{uH}	$(H^\dagger H)(\bar{q}_p u_r \widetilde{H})$
Q_{dH}	$(H^\dagger H)(\bar{q}_p d_r H)$

$4: X^2 H^2$	
Q_{HG}	$H^\dagger H\, G_{\mu\nu}^A G^{A\mu\nu}$
$Q_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$
Q_{HW}	$H^\dagger H\, W_{\mu\nu}^I W^{I\mu\nu}$
$Q_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$
Q_{HB}	$H^\dagger H\, B_{\mu\nu} B^{\mu\nu}$
$Q_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu} B^{\mu\nu}$
Q_{HWB}	$H^\dagger \tau^I H\, W_{\mu\nu}^I B^{\mu\nu}$
$Q_{H\widetilde{W}B}$	$H^\dagger \tau^I H\, \widetilde{W}_{\mu\nu}^I B^{\mu\nu}$

$6: \psi^2 X H + \text{h.c.}$	
Q_{eW}	$(\bar{l}_p \sigma^{\mu\nu} e_r) \tau^I H W_{\mu\nu}^I$
Q_{eB}	$(\bar{l}_p \sigma^{\mu\nu} e_r) H B_{\mu\nu}$
Q_{uG}	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r) \widetilde{H}\, G_{\mu\nu}^A$
Q_{uW}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tau^I \widetilde{H}\, W_{\mu\nu}^I$
Q_{uB}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \widetilde{H}\, B_{\mu\nu}$
Q_{dG}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) H\, G_{\mu\nu}^A$
Q_{dW}	$(\bar{q}_p \sigma^{\mu\nu} d_r) \tau^I H\, W_{\mu\nu}^I$
Q_{dB}	$(\bar{q}_p \sigma^{\mu\nu} d_r) H\, B_{\mu\nu}$

$7: \psi^2 H^2 D$	
$Q_{Hl}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{l}_p \gamma^\mu l_r)$
$Q_{Hl}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{l}_p \tau^I \gamma^\mu l_r)$
Q_{He}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{e}_p \gamma^\mu e_r)$
$Q_{Hq}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{q}_p \gamma^\mu q_r)$
$Q_{Hq}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{q}_p \tau^I \gamma^\mu q_r)$
Q_{Hu}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{u}_p \gamma^\mu u_r)$
Q_{Hd}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{d}_p \gamma^\mu d_r)$
Q_{Hud} + h.c.	$i(\widetilde{H}^\dagger D_\mu H)(\bar{u}_p \gamma^\mu d_r)$

$8: (\bar{L}L)(\bar{L}L)$	
Q_{ll}	$(\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{q}_s \gamma^\mu q_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p \gamma_\mu \tau^I q_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{q}_s \gamma^\mu q_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p \gamma_\mu \tau^I l_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$

$8: (\bar{R}R)(\bar{R}R)$	
Q_{ee}	$(\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t)$
Q_{uu}	$(\bar{u}_p \gamma_\mu u_r)(\bar{u}_s \gamma^\mu u_t)$
Q_{dd}	$(\bar{d}_p \gamma_\mu d_r)(\bar{d}_s \gamma^\mu d_t)$
Q_{eu}	$(\bar{e}_p \gamma_\mu e_r)(\bar{u}_s \gamma^\mu u_t)$
Q_{ed}	$(\bar{e}_p \gamma_\mu e_r)(\bar{d}_s \gamma^\mu d_t)$
$Q_{ud}^{(1)}$	$(\bar{u}_p \gamma_\mu u_r)(\bar{d}_s \gamma^\mu d_t)$
$Q_{ud}^{(8)}$	$(\bar{u}_p \gamma_\mu T^A u_r)(\bar{d}_s \gamma^\mu T^A d_t)$

$8: (\bar{L}L)(\bar{R}R)$	
Q_{le}	$(\bar{l}_p \gamma_\mu l_r)(\bar{e}_s \gamma^\mu e_t)$
Q_{lu}	$(\bar{l}_p \gamma_\mu l_r)(\bar{u}_s \gamma^\mu u_t)$
Q_{ld}	$(\bar{l}_p \gamma_\mu l_r)(\bar{d}_s \gamma^\mu d_t)$
Q_{qe}	$(\bar{q}_p \gamma_\mu q_r)(\bar{e}_s \gamma^\mu e_t)$
$Q_{qu}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{u}_s \gamma^\mu u_t)$
$Q_{qu}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{u}_s \gamma^\mu T^A u_t)$
$Q_{qd}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{d}_s \gamma^\mu d_t)$
$Q_{qd}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{d}_s \gamma^\mu T^A d_t)$

$8: (\bar{L}R)(\bar{R}L) + \text{h.c.}$	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_{tj})$

$8: (\bar{L}R)(\bar{L}R) + \text{h.c.}$	
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\epsilon_{jk}(\bar{q}_s^k d_t)$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\epsilon_{jk}(\bar{q}_s^k T^A d_t)$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\epsilon_{jk}(\bar{q}_s^k u_t)$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r)\epsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM}^{(4)} + \sum_{n\geq 5} \frac{c_n}{\Lambda^{n-4}} \mathcal{O}^{(n)}$$

59 types of operators.
2499 independent Wilson coefficients
(1350 real and 1149 imaginary).

SM$_6$

Basis of dim-6 operators, aka Warsaw basis

$1: X^3$		$2: H^6$		$3: H^4 D^2$		$5: \psi^2 H^3 + \text{h.c.}$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_H	$(H^\dagger H)^3$	$Q_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$	Q_{eH}	$(H^\dagger H)(\bar l_p e_r H)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$			Q_{HD}	$\left(H^\dagger D_\mu H\right)^* \left(H^\dagger D_\mu H\right)$	Q_{uH}	$(H^\dagger H)(\bar q_p u_r \widetilde H)$
Q_W	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$					Q_{dH}	$(H^\dagger H)(\bar q_p d_r H)$
$Q_{\widetilde W}$	$\epsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$						

$4: X^2 H^2$		$6: \psi^2 X H + \text{h.c.}$		$7: \psi^2 H^2 D$	
Q_{HG}	$H^\dagger H\, G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I H W_{\mu\nu}^I$	$Q_{Hl}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar l_p \gamma^\mu l_r)$
$Q_{H\widetilde G}$	$H^\dagger H\, \widetilde G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r) H B_{\mu\nu}$	$Q_{Hl}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}_\mu^I H)(\bar l_p \tau^I \gamma^\mu l_r)$
Q_{HW}	$H^\dagger H\, W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\widetilde H\, G_{\mu\nu}^A$	Q_{He}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar e_p \gamma^\mu e_r)$
$Q_{H\widetilde W}$	$H^\dagger H\, \widetilde W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r)\tau^I \widetilde H\, W_{\mu\nu}^I$	$Q_{Hq}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar q_p \gamma^\mu q_r)$
Q_{HB}	$H^\dagger H\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r)\widetilde H\, B_{\mu\nu}$	$Q_{Hq}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}_\mu^I H)(\bar q_p \tau^I \gamma^\mu q_r)$
$Q_{H\widetilde B}$	$H^\dagger H\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r) H\, G_{\mu\nu}^A$	Q_{Hu}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar u_p \gamma^\mu u_r)$
Q_{HWB}	$H^\dagger \tau^I H\, W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r)\tau^I H\, W_{\mu\nu}^I$	Q_{Hd}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar d_p \gamma^\mu d_r)$
$Q_{H\widetilde W B}$	$H^\dagger \tau^I H\, \widetilde W_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r) H\, B_{\mu\nu}$	Q_{Hud} + h.c.	$i(\widetilde H^\dagger D_\mu H)(\bar u_p \gamma^\mu d_r)$

$8: (\bar L L)(\bar L L)$		$8: (\bar R R)(\bar R R)$		$8: (\bar L L)(\bar R R)$	
Q_{ll}	$(\bar l_p \gamma_\mu l_r)(\bar l_s \gamma^\mu l_t)$	Q_{ee}	$(\bar e_p \gamma_\mu e_r)(\bar e_s \gamma^\mu e_t)$	Q_{le}	$(\bar l_p \gamma_\mu l_r)(\bar e_s \gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar q_s \gamma^\mu q_t)$	Q_{uu}	$(\bar u_p \gamma_\mu u_r)(\bar u_s \gamma^\mu u_t)$	Q_{lu}	$(\bar l_p \gamma_\mu l_r)(\bar u_s \gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar q_p \gamma_\mu \tau^I q_r)(\bar q_s \gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar d_p \gamma_\mu d_r)(\bar d_s \gamma^\mu d_t)$	Q_{ld}	$(\bar l_p \gamma_\mu l_r)(\bar d_s \gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar l_p \gamma_\mu l_r)(\bar q_s \gamma^\mu q_t)$	Q_{eu}	$(\bar e_p \gamma_\mu e_r)(\bar u_s \gamma^\mu u_t)$	Q_{qe}	$(\bar q_p \gamma_\mu q_r)(\bar e_s \gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar l_p \gamma_\mu \tau^I l_r)(\bar q_s \gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar e_p \gamma_\mu e_r)(\bar d_s \gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar u_s \gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar u_p \gamma_\mu u_r)(\bar d_s \gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar q_p \gamma_\mu T^A q_r)(\bar u_s \gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar u_p \gamma_\mu T^A u_r)(\bar d_s \gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar d_s \gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar q_p \gamma_\mu T^A q_r)(\bar d_s \gamma^\mu T^A d_t)$

$8: (\bar L R)(\bar R L) + \text{h.c.}$		$8: (\bar L R)(\bar L R) + \text{h.c.}$	
Q_{ledq}	$(\bar l_p^j e_r)(\bar d_s q_{tj})$	$Q_{quqd}^{(1)}$	$(\bar q_p^j u_r)\epsilon_{jk}(\bar q_s^k d_t)$
		$Q_{quqd}^{(8)}$	$(\bar q_p^j T^A u_r)\epsilon_{jk}(\bar q_s^k T^A d_t)$
		$Q_{lequ}^{(1)}$	$(\bar l_p^j e_r)\epsilon_{jk}(\bar q_s^k u_t)$
		$Q_{lequ}^{(3)}$	$(\bar l_p^j \sigma_{\mu\nu} e_r)\epsilon_{jk}(\bar q_s^k \sigma^{\mu\nu} u_t)$

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM}^{(4)} + \sum_{n\geq 5} \frac{c_n}{\Lambda^{n-4}} \mathcal{O}^{(n)}$$

59 types of operators.
2499 independent Wilson coefficients
(1350 real and 1149 imaginary).

1. How many new sources of CPV?
2. Which ones can appear at BSM leading order ($1/\Lambda^2$)?

— Not because a parameter is $O(1/\Lambda^2)$ that it can contribute at leading order in any physical observable!
We'll see indeed that there are general non-interference theorems —

3. What are the collective breaking patterns associated to these new sources of CPV?
4. Where should we look for CPV?

Beyond Jarlskog: Building SM_6 invariants

Playing with new fermion bilinear interactions first

- In the Warsaw basis, Manohar et al. counted 7 Hermitian (6R+3I) and 12 generic bilinear (9R+9I) operators for a total of 129 phases (and 150 real parameters)

	$5:\psi^2H^3+\text{h.c.}$		$6:\psi^2XH+\text{h.c.}$
generic matrices	Q_{eH}	$(H^\dagger H)(\bar{l}_p e_r H)$	Q_{eW}, Q_{eB}
	Q_{uH}	$(H^\dagger H)(\bar{q}_p u_r \widetilde{H})$	Q_{uG} , Q_{uW}, Q_{uB}
	Q_{dH}	$(H^\dagger H)(\bar{q}_p d_r H)$	Q_{dG}, Q_{dW}, Q_{dB}

	$7:\psi^2H^2D$	
Hermitian matrices	$Q_{Hl}^{(1)}$, $Q_{Hl}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{l}_p\gamma^\mu l_r)$, $(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{l}_p\tau^I\gamma^\mu l_r)$
	Q_{He}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{e}_p\gamma^\mu e_r)$
	$Q_{Hq}^{(1)}$, $Q_{Hq}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{q}_p\gamma^\mu q_r)$, $(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{q}_p\tau^I\gamma^\mu q_r)$
	Q_{Hu}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}_p\gamma^\mu u_r)$
	Q_{Hd}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{d}_p\gamma^\mu d_r)$
generic	Q_{Hud}	$i(\widetilde{H}^\dagger D_\mu H)(\bar{u}_p\gamma^\mu d_r)$

$SU(3)_Q$	$SU(3)_u$	$SU(3)_d$	$SU(3)_L$	$SU(3)_e$
1	1	1	3	$\bar{3}$
3	$\bar{3}$	1	1	1
3	1	$\bar{3}$	1	1
1	1	1	$8+1$	1
1	1	1	1	$8+1$
$8+1$	1	1	1	1
1	$8+1$	1	1	1
1	1	$8+1$	1	1
1	3	$\bar{3}$	1	1

- In the limit m_ν=0, lepton numbers in each family are conserved. The WC not invariant under these U(1)'s can never show up at linear order in any amplitude: 129 → 102 phases (and 150 → 123 real parameters) — see later for more details

Beyond Jarlskog: Minimal Basis

Transfer matrix of maximal rank

$$\begin{pmatrix} I_1 \\ I_2 \\ \dots \\ I_n \end{pmatrix} = \left(T^R \; T^I\right) \begin{pmatrix} \mathrm{Re}C_1 \\ \mathrm{Re}C_2 \\ \dots \\ \mathrm{Re}C_p \\ \mathrm{Im}C_1 \\ \dots \\ \mathrm{Im}C_q \end{pmatrix}$$

transfer matrix that depends
only on Y_u and Y_d

Beyond Jarlskog: Minimal Basis

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transfer matrix that depends
only on Y_u and Y_d

The problem boils down to find what is the maximal rank of the transfer matrix
in general and also when $J_4=0$

Beyond Jarlskog: Minimal Basis

Transfer matrix of maximal rank

Seems a simple exercise to compute the rank!
But the invariants are real monsters when computed explicitly in a particular flavour basis
(up to $9^7 \approx 5\text{x}10^6$ of terms for some of the invariants)
Hopeless to analytically compute ranks.
Numerically tricky too → compute ranks for rational matrices

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Numerically tricky too → compute ranks for rational matrices

	Type of op.	# of ops	# real	# im.	# CP-odd invariants
bilinears	Yukawa	3	27	27	21
bilinears	Dipoles	8	72	72	60
bilinears	current-current	8	51	30	21
	all bilinears	19	150	129	102

Beyond Jarlskog: Minimal Basis

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	Dipoles	8	72	72	60
	current-current	8	51	30	21
	all bilinears	19	150	129	102

Note that there are fewer CP-odd invariants than phases

Not all the phases can appear in observables — not interference theorems

Non-Interference

Conservation of individual family lepton numbers

Let us see it in a fixed basis, e.g.

$$Y_u = \mathrm{diag}(y_u, y_c, y_t) \qquad Y_d = V_{\mathrm{CKM}}\mathrm{diag}(y_d, y_s, y_b) \qquad Y_e = \mathrm{diag}(y_e, y_\mu, y_\tau)$$

In the lepton sector, this choice breaks the $U(3)_L \times U(3)_e$ of the free Lagrangian down to the $U(1)^3$ described by the transformation

$$(L, e) \to \mathrm{diag}(e^{i\delta_1}, e^{i\delta_2}, e^{i\delta_3})(L, e)$$

At dimension 6, operators containing leptons are charged under this symmetry, e.g.

$$\mathcal{O}_{He} = \frac{1}{\Lambda^2} C_{He,mn} \, (H^\dagger i \overleftrightarrow{D}_\mu H) \, \bar{e}_m \gamma^\mu e_n \qquad C_{He,mn} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{12}^* & c_{22} & c_{23} \\ c_{13}^* & c_{23}^* & c_{33} \end{pmatrix} \xrightarrow{U(1)^3} \begin{pmatrix} c_{11} & c_{12} e^{i(\delta_2-\delta_1)} & c_{13} e^{i(\delta_3-\delta_1)} \\ c_{12}^* e^{-i(\delta_2-\delta_1)} & c_{22} & c_{23} e^{i(\delta_3-\delta_2)} \\ c_{13}^* e^{-i(\delta_3-\delta_1)} & c_{23}^* e^{-i(\delta_3-\delta_2)} & c_{33} \end{pmatrix}$$

The off-diagonal elements cannot enter into observables at linear order!

Non-Interference

Conservation of individual family lepton numbers

	Type of op.	# of ops	# real	# im.	inv. under $U(1)_{L_i} - U(1)_{L_j}$ # real	# im.	# CP-odd invariants
bilinears	Yukawa	3	27	27	21	21	21
	Dipoles	8	72	72	60	60	60
	current-current	8	51	30	42	21	21
	all bilinears	19	150	129	123	102	102

Minimal sets can be built explicitly
— not a unique choice —

Minimal Sets for Fermion Bilinear Operators

Wilson coefficient	Number of phases	Minimal set
$C_e \equiv \begin{cases} C_{eH} \\ C_{eW} \\ C_{eB} \end{cases}$	3	$\left\{ L_0\left(C_e Y_e^\dagger\right) L_1\left(C_e Y_e^\dagger\right) L_2\left(C_e Y_e^\dagger\right) \right\}$
$C_u \equiv \begin{cases} C_{uH} \\ C_{uG} \\ C_{uW} \\ C_{uB} \end{cases}$	9	$\left\{ \begin{array}{lll} L_{0000}\left(C_u Y_u^\dagger\right) & L_{1000}\left(C_u Y_u^\dagger\right) & L_{0100}\left(C_u Y_u^\dagger\right) \\ L_{1100}\left(C_u Y_u^\dagger\right) & L_{0110}\left(C_u Y_u^\dagger\right) & L_{2200}\left(C_u Y_u^\dagger\right) \\ L_{0220}\left(C_u Y_u^\dagger\right) & L_{1220}\left(C_u Y_u^\dagger\right) & L_{0122}\left(C_u Y_u^\dagger\right) \end{array} \right\}$
$C_d \equiv \begin{cases} C_{dH} \\ C_{dG} \\ C_{dW} \\ C_{dB} \end{cases}$		Same with $C_u Y_u^\dagger \to C_d Y_d^\dagger$
C_{Hud}		Same with $C_u Y_u^\dagger \to Y_u C_{Hud} Y_d^\dagger$
$C_{HL}^{(1,3)}, C_{He}$	0	$\varnothing$
$C_{HQ}^{(1,3)}$	3	$\left\{ L_{1100}\left(C_{HQ}^{(1,3)}\right) L_{2200}\left(C_{HQ}^{(1,3)}\right) L_{1122}\left(C_{HQ}^{(1,3)}\right) \right\}$
C_{Hu}		Same with $C_{HQ}^{(1,3)} \to Y_u C_{Hu} Y_u^\dagger$
C_{Hd}		Same with $C_{HQ}^{(1,3)} \to Y_d C_{Hd} Y_d^\dagger$
	102	

One explicit basis of invariants

$$L_{abcd}(\tilde{C}) \equiv \operatorname{Im}\operatorname{Tr}\left(X_u^a X_d^b X_u^c X_d^d \tilde{C}\right)$$

4-Fermi Operators

4F invariants from bilinear invariants

- In the Warsaw basis, Manohar et al. also counted the free-parameters in 4F operators: 1014 phases. As before, not all these phases can show up at leading order when the neutrino masses are taken to vanish: only 597 survive (adding to the 102 bilinear ones and J_4 for a total of 700 phases)

e.g.

$$C_{QuQd}\,\bar{Q}u\bar{Q}d \qquad \begin{array}{ccc} SU(3)_Q & SU(3)_u & SU(3)_d \\ \hline 1+3+6 & \bar{3} & \bar{3} \end{array}$$

- One can build two types of 4F-invariants out of the bilinear invariants:

A-type

$$\mathrm{Im}\left(\underbrace{M^{uH}_{ij}}\,\underbrace{M^{dH}_{kl}}\,C^{QuQd}_{ijkl}\right)$$

B-type

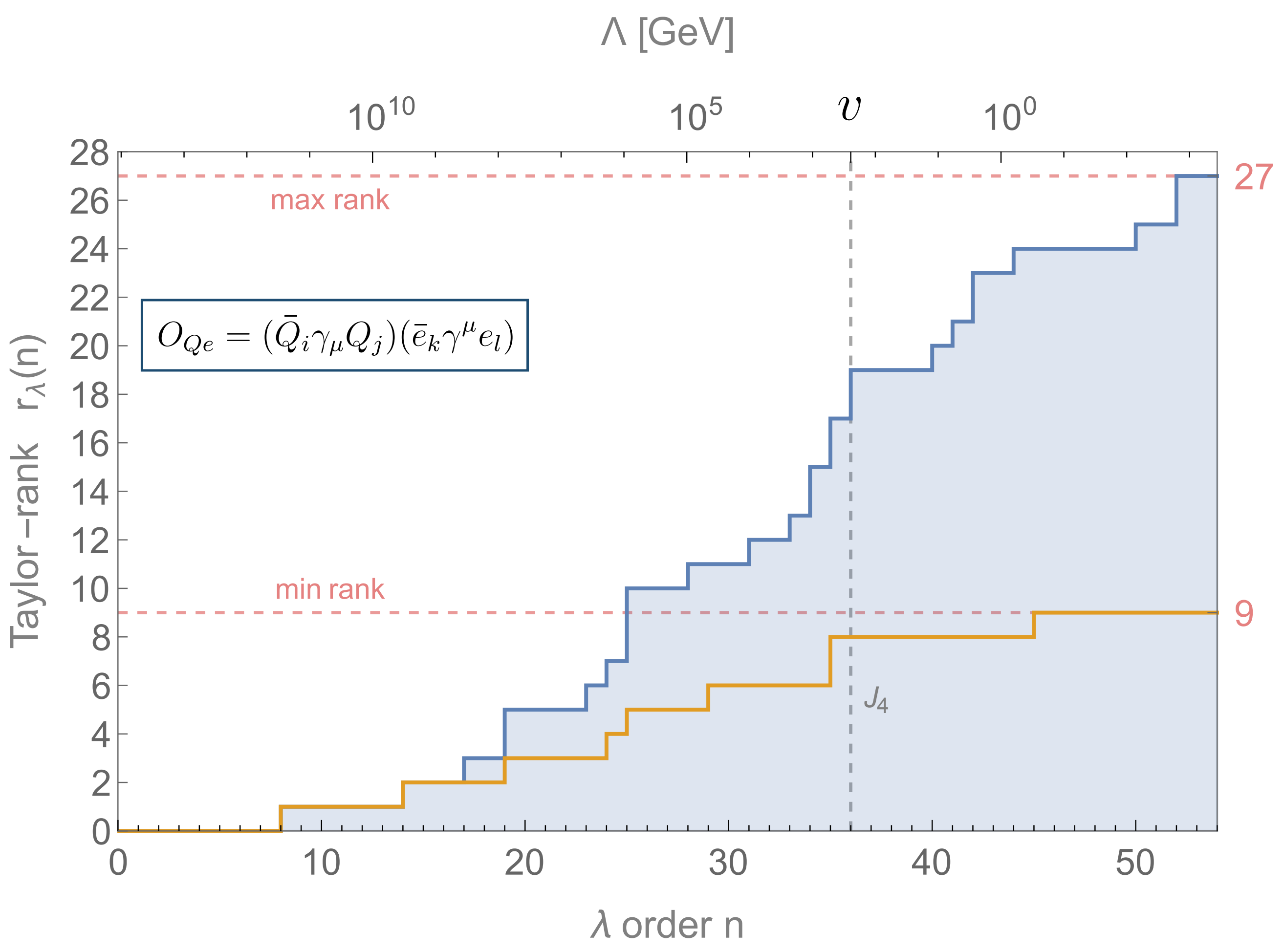
$$\mathrm{Im}\left(\underbrace{M^{dH}_{il}}\,\underbrace{M^{uH\dagger}_{jk}}\,C^{QuQd}_{ijkl}\right)$$

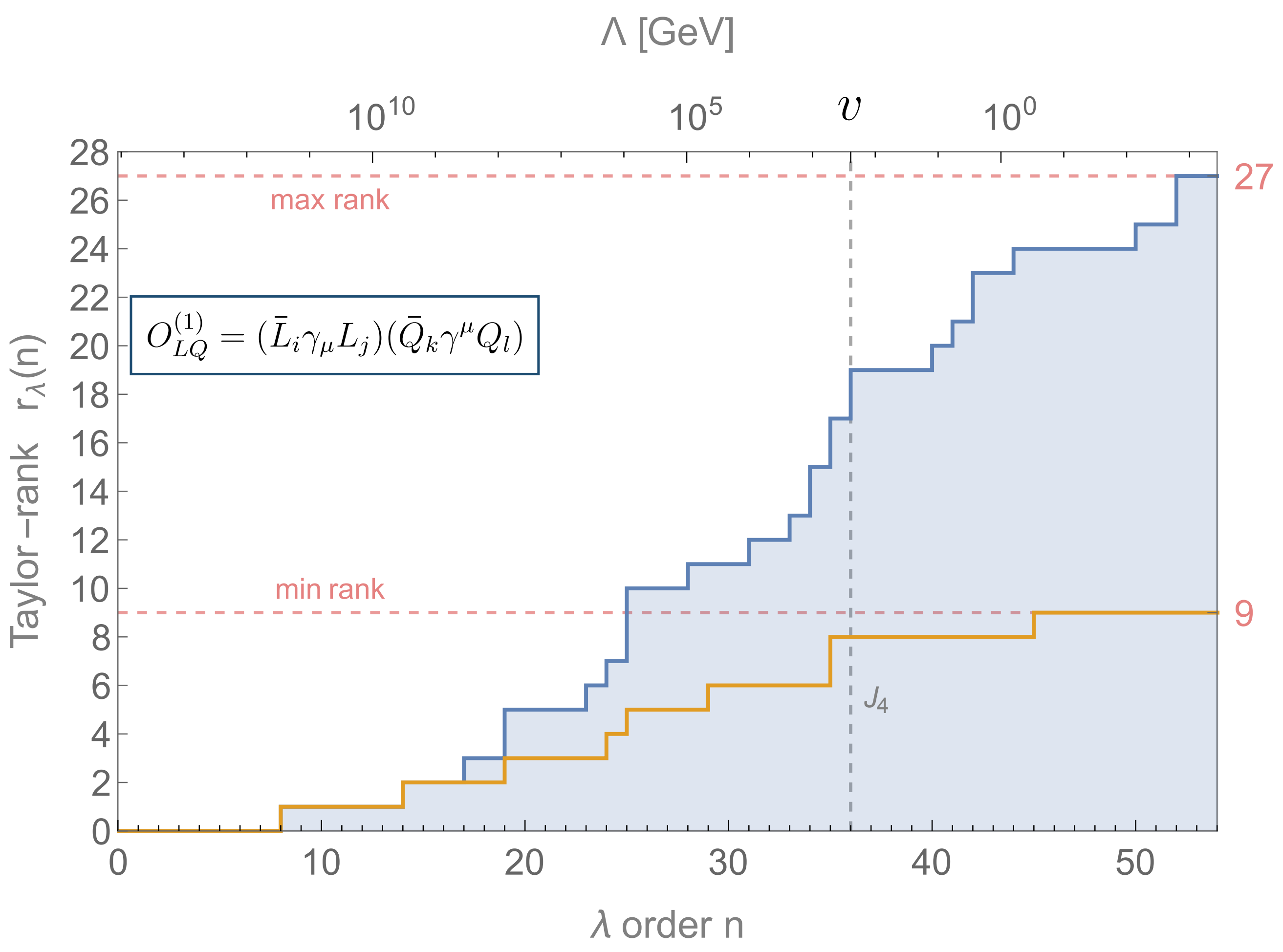
matrices built out of Yu and Yd that to form bilinear invariants, e.g., $\mathrm{Im}\ \mathrm{Tr}\left(M^{uH}C_{uH}\right)$

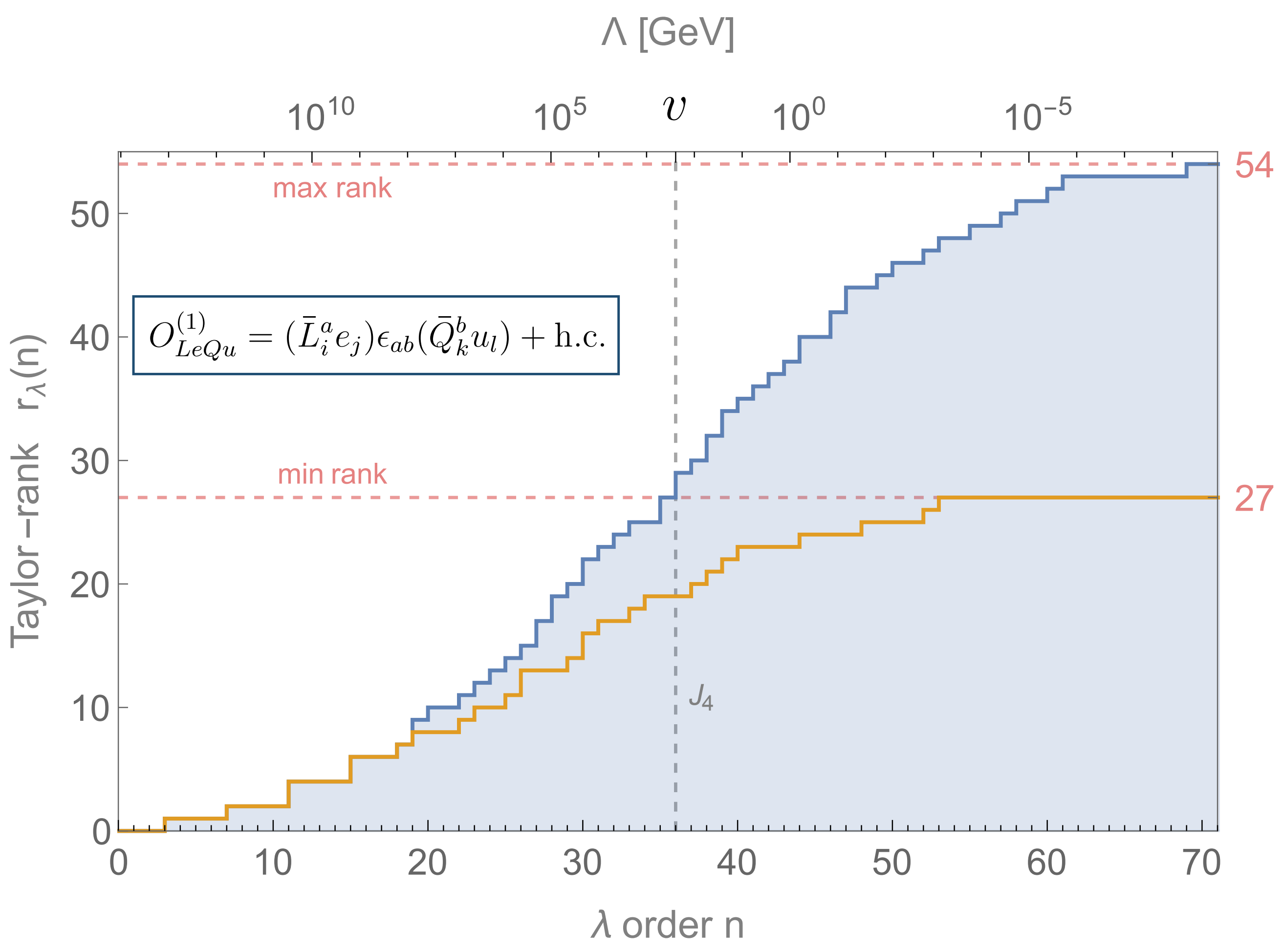
An explicit basis of 597 invariants for the 4F operators can be built (see bonus slides)

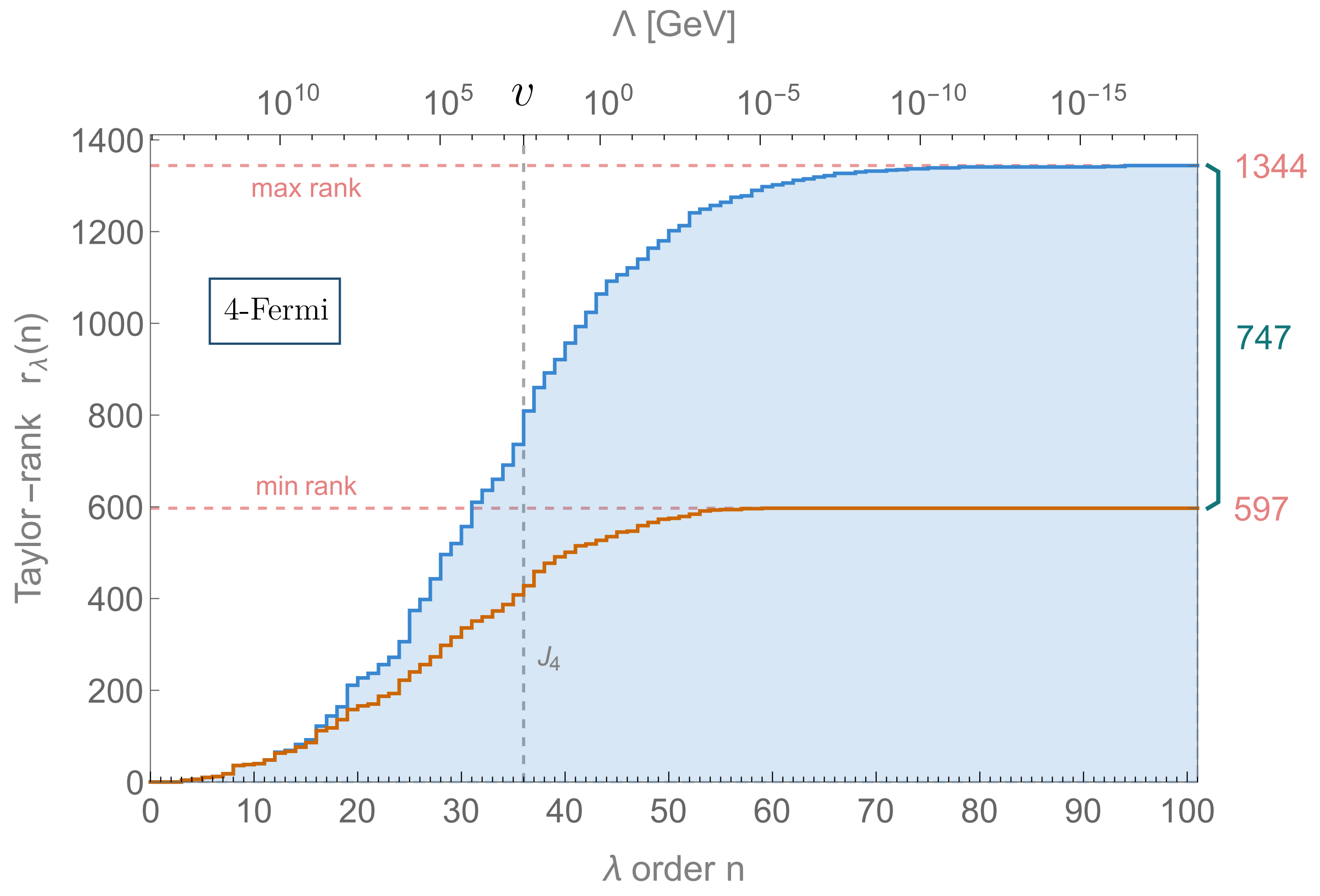
4-Fermi Operators

independent invariants at O(λⁿ) for some 4F operators









Minimal Set

parameters for the different types of operators

	Type of op.	# of ops	# real	# im.	inv. under $U(1)_{L_i} - U(1)_{L_j}$ # real	inv. under $U(1)_{L_i} - U(1)_{L_j}$ # im.
bilinears	Yukawa	3	27	27	21	21
bilinears	Dipoles	8	72	72	60	60
bilinears	current-current	8	51	30	42	21
	all bilinears	19	150	129	123	102
4-Fermi	LLLL	5	171	126	99	54
4-Fermi	RRRR	7	255	195	186	126
4-Fermi	LLRR	8	360	288	246	174
4-Fermi	LRRL	1	81	81	27	27
4-Fermi	LRLR	4	324	324	216	216
	all 4-Fermi	25	1191	1014	774	597
	all		1341	1143	897	699

primary sources of CPV

	Bilinears				4-Fermi						
Flavour symmetries of the quark sector of the SM	C_{eH} C_{eW} C_{eB}	C_{uH} C_{uG} C_{uW} C_{uB} C_{dH} C_{dG} C_{dW} C_{dB} C_{Hud}	$C_{HL}^{1,3}$ C_{He}	$C_{HQ}^{1,3}$ C_{Hu} C_{Hd}	C_{LL} C_{ee}	C_{Le}	$C_{QQ}^{1,3}$ C_{uu} C_{dd}	$C_{LQ}^{1,3}$ C_{Qe} C_{Lu} C_{eu} C_{Ld} C_{ed}	$C_{ud}^{1,8}$ $C_{Qu}^{1,8}$ $C_{Qd}^{1,8}$	C_{LedQ} $C_{LeQu}^{1,3}$	$C_{QuQd}^{1,8}$
$U(1)_B$	3	9	0	3	0	3	18	9	36	27	81
$U(1)^2$	3	5	0	1	0	3	5	3	12	15	33
$U(1)^3$	3	3	0	0	0	3	0	0	3	9	15
$U(2) \times U(1)$	3	2	0	0	0	3	0	0	1	6	7
$U(3)$	3	1	0	0	0	3	0	0	0	3	2
Two degenerate electron-type leptons	$\times\frac{2}{3}$	$\times 1$		$\times 1$		$\times\frac{2}{3}$	$\times 1$	$\times\frac{2}{3}$	$\times 1$	$\times\frac{2}{3}$	$\times 1$
All electron-type leptons degenerate	$\times\frac{1}{3}$	$\times 1$		$\times 1$		$\times\frac{1}{3}$	$\times 1$	$\times\frac{1}{3}$	$\times 1$	$\times\frac{1}{3}$	$\times 1$

CPV for Degenerate Spectrum

- As noticed already in SM$_4$, degenerate spectra (equal mass, zero or maximal mixing angle) have different CPV counting than generic case

Parameter values		Flavor symmetries of the SM$_4$ Lagrangian
$m_u \neq m_c \neq m_t$, $m_d \neq m_s \neq m_b$	Generic $V_{\rm CKM}$	$U(1)_B$
	$\lvert V_{{\rm CKM},i_0 j_0}\rvert = 1\,,\ V_{{\rm CKM},i j_0} = V_{{\rm CKM},i_0 j} = 0$, $i \neq i_0,\ j \neq j_0$	$U(1)^2$
	$\lvert V_{{\rm CKM},i_1 j_1}\rvert = \lvert V_{{\rm CKM},i_2 j_2}\rvert = \lvert V_{{\rm CKM},i_3 j_3}\rvert = 1$ for $i_1 \neq i_2 \neq i_3$, $j_1 \neq j_2 \neq j_3$; $V_{{\rm CKM},ij} = 0$ elsewhere	$U(1)^3$
$m_u \neq m_c = m_t$, $m_d \neq m_s \neq m_b$	Generic $V_{\rm CKM}$ (see Eq. (4.16))	$U(1)_B$
	$\lvert V_{{\rm CKM},i_0 j_0}\rvert = 1\,,\ V_{{\rm CKM},i j_0} = V_{{\rm CKM},i_0 j} = 0$, $i \neq i_0,\ j \neq j_0$	$U(1)^2$
	$\lvert V_{{\rm CKM},i_1 j_1}\rvert = \lvert V_{{\rm CKM},i_2 j_2}\rvert = \lvert V_{{\rm CKM},i_3 j_3}\rvert = 1$ for $i_1 \neq i_2 \neq i_3$, $j_1 \neq j_2 \neq j_3$; $V_{{\rm CKM},ij} = 0$ elsewhere	$U(1)^3$
$m_u \neq m_c \neq m_t$, $m_d = m_s \neq m_b$	Same as the previous case with $V_{\rm CKM} \leftrightarrow V^\dagger_{\rm CKM}$	
$m_u \neq m_c = m_t$, $m_d = m_s \neq m_b$	Generic $V_{\rm CKM}$	$U(1)^2$
	$\lvert V_{{\rm CKM},11}\rvert = \lvert V_{{\rm CKM},22}\rvert = \lvert V_{{\rm CKM},33}\rvert = 1$, $V_{{\rm CKM},ij} = 0$ elsewhere	$U(1)^3$
	$\lvert V_{{\rm CKM},13}\rvert = \lvert V_{{\rm CKM},22}\rvert = \lvert V_{{\rm CKM},31}\rvert = 1$, $V_{{\rm CKM},ij} = 0$ elsewhere	$U(2)\times U(1)$
$m_u = m_c = m_t$	$m_d \neq m_s \neq m_b$	$U(1)^3$
	$m_d = m_s \neq m_b$	$U(2)\times U(1)$
	$m_d = m_s = m_b$	$U(3)$
$m_d = m_s = m_b$	$m_u \neq m_c \neq m_t$	$U(1)^3$
	$m_u \neq m_c = m_t$	$U(2)\times U(1)$
	$m_u = m_c = m_t$	$U(3)$

	Bilinears			
Flavour symmetries of the quark sector of the SM	C_{eH} C_{eW} C_{eB}	C_{uH} C_{uG} C_{uW} C_{uB} C_{dH} C_{dG} C_{dW} C_{dB} C_{Hud}	$C^{1,3}_{HL}$ C_{He}	$C^{1,3}_{HQ}$ C_{Hu} C_{Hd}
$U(1)_B$	3	9	0	3
$U(1)^2$	3	5	0	1
$U(1)^3$	3	3	0	0
$U(2)\times U(1)$	3	2	0	0
$U(3)$	3	1	0	0
Two degenerate electron-type leptons	$\times\frac{2}{3}$	$\times 1$		$\times 1$
All electron-type leptons degenerate	$\times\frac{1}{3}$	$\times 1$		$\times 1$

maximal rank of transfer matrix
for different flavour symmetries of the Yukawa matrices

Minimal Sets for 4-Fermi Operators

Wilson coefficient	Number of phases	Minimal set
C_{LL}, C_{ee}	0	$\varnothing$
C_{Le}	3	$\left\{ B_0^0\left(C_{LL\tilde{e}\tilde{e}}\right)\ B_0^1\left(C_{LL\tilde{e}\tilde{e}}\right)\ B_0^2\left(C_{LL\tilde{e}\tilde{e}}\right) \right\}$
C_{Qe}		$\left\{ \begin{matrix} A_0^{1100}(C_{QQee}) & A_1^{1100}(C_{QQee}) & A_2^{1100}(C_{QQee}) \\ A_0^{2200}(C_{QQee}) & A_1^{2200}(C_{QQee}) & A_2^{2200}(C_{QQee}) \\ A_0^{1122}(C_{QQee}) & A_1^{1122}(C_{QQee}) & A_2^{1122}(C_{QQee}) \end{matrix} \right\}$
C_{ed}		Same with $C_{QQee} \to C_{ee\tilde{d}\tilde{d}}$ (exchanging upper with lower indices and with $Y_e \leftrightarrow Y_e^\dagger$)
C_{eu}	9	Same with $C_{QQee} \to C_{ee\tilde{u}\tilde{u}}$ (exchanging upper with lower indices and with $Y_e \leftrightarrow Y_e^\dagger$)
$C_{LQ}^{(1,3)}$		$\left\{ \begin{matrix} A_{1100}^0\left(C_{LQ}^{(1,3)}\right) & A_{1100}^1\left(C_{LQ}^{(1,3)}\right) & A_{1100}^2\left(C_{LQ}^{(1,3)}\right) \\ A_{2200}^0\left(C_{LQ}^{(1,3)}\right) & A_{2200}^1\left(C_{LQ}^{(1,3)}\right) & A_{2200}^2\left(C_{LQ}^{(1,3)}\right) \\ A_{1122}^0\left(C_{LQ}^{(1,3)}\right) & A_{1122}^1\left(C_{LQ}^{(1,3)}\right) & A_{1122}^2\left(C_{LQ}^{(1,3)}\right) \end{matrix} \right\}$
C_{Ld}		Same with $C_{LQ}^{(1,3)} \to C_{LL\tilde{d}\tilde{d}}$
C_{Lu}		Same with $C_{LQ}^{(1,3)} \to C_{LL\tilde{u}\tilde{u}}$
$C_{LeQu}^{(1,3)}$	27	$\left\{ \begin{matrix} A_{0000}^0(C_{L\tilde{e}Q\tilde{u}}) & A_{0000}^1(C_{L\tilde{e}Q\tilde{u}}) & A_{0000}^2(C_{L\tilde{e}Q\tilde{u}}) \\ A_{1000}^0(C_{L\tilde{e}Q\tilde{u}}) & A_{1000}^1(C_{L\tilde{e}Q\tilde{u}}) & A_{1000}^2(C_{L\tilde{e}Q\tilde{u}}) \\ A_{0100}^0(C_{L\tilde{e}Q\tilde{u}}) & A_{0100}^1(C_{L\tilde{e}Q\tilde{u}}) & A_{0100}^2(C_{L\tilde{e}Q\tilde{u}}) \\ A_{1100}^0(C_{L\tilde{e}Q\tilde{u}}) & A_{1100}^1(C_{L\tilde{e}Q\tilde{u}}) & A_{1100}^2(C_{L\tilde{e}Q\tilde{u}}) \\ A_{0110}^0(C_{L\tilde{e}Q\tilde{u}}) & A_{0110}^1(C_{L\tilde{e}Q\tilde{u}}) & A_{0110}^2(C_{L\tilde{e}Q\tilde{u}}) \\ A_{2200}^0(C_{L\tilde{e}Q\tilde{u}}) & A_{2200}^1(C_{L\tilde{e}Q\tilde{u}}) & A_{2200}^2(C_{L\tilde{e}Q\tilde{u}}) \\ A_{0220}^0(C_{L\tilde{e}Q\tilde{u}}) & A_{0220}^1(C_{L\tilde{e}Q\tilde{u}}) & A_{0220}^2(C_{L\tilde{e}Q\tilde{u}}) \\ A_{1220}^0(C_{L\tilde{e}Q\tilde{u}}) & A_{1220}^1(C_{L\tilde{e}Q\tilde{u}}) & A_{1220}^2(C_{L\tilde{e}Q\tilde{u}}) \\ A_{0122}^0(C_{L\tilde{e}Q\tilde{u}}) & A_{0122}^1(C_{L\tilde{e}Q\tilde{u}}) & A_{0122}^2(C_{L\tilde{e}Q\tilde{u}}) \end{matrix} \right\}$
C_{LedQ}		Same with $C_{L\tilde{e}Q\tilde{u}} \to C_{L\tilde{e}\tilde{d}Q}$ and $A^a_{bcde} \to A^a_{edcb}$

Wilson coefficient	Number of phases	Minimal set
$C_{QQ}^{(1,3)}$	18	$\left\{ \begin{matrix} A_{1100}^{0000}(C_{QQQQ}) & A_{1100}^{1000}(C_{QQQQ}) & A_{1100}^{0100}(C_{QQQQ}) \\ A_{2200}^{0000}(C_{QQQQ}) & A_{1100}^{1100}(C_{QQQQ}) & A_{2200}^{1000}(C_{QQQQ}) \\ A_{2200}^{0100}(C_{QQQQ}) & A_{1122}^{0000}(C_{QQQQ}) & A_{2200}^{1100}(C_{QQQQ}) \\ A_{2100}^{1200}(C_{QQQQ}) & A_{1122}^{1000}(C_{QQQQ}) & A_{1122}^{0100}(C_{QQQQ}) \\ A_{1122}^{1100}(C_{QQQQ}) & A_{2200}^{2200}(C_{QQQQ}) & B_{1100}^{0000}(C_{QQQQ}) \\ B_{2200}^{0000}(C_{QQQQ}) & B_{1122}^{0000}(C_{QQQQ}) & A_{1122}^{2200}(C_{QQQQ}) \end{matrix} \right\}$
C_{uu}	18	$\left\{ \begin{matrix} A_{1100}^{0000}(C_{uu\tilde{u}\tilde{u}}) & A_{1100}^{1000}(C_{\tilde{u}\tilde{u}\tilde{u}\tilde{u}}) & A_{1100}^{0100}(C_{\tilde{u}\tilde{u}\tilde{u}\tilde{u}}) \\ A_{2200}^{0000}(C_{uu\tilde{u}\tilde{u}}) & A_{1100}^{1100}(C_{\tilde{u}\tilde{u}\tilde{u}\tilde{u}}) & A_{1100}^{0200}(C_{\tilde{u}\tilde{u}\tilde{u}\tilde{u}}) \\ A_{2200}^{0100}(C_{\tilde{u}\tilde{u}\tilde{u}\tilde{u}}) & A_{1122}^{0000}(C_{uu\tilde{u}\tilde{u}}) & A_{2200}^{1100}(C_{\tilde{u}\tilde{u}\tilde{u}\tilde{u}}) \\ A_{1122}^{1000}(C_{\tilde{u}\tilde{u}\tilde{u}\tilde{u}}) & A_{1122}^{0100}(C_{\tilde{u}\tilde{u}\tilde{u}\tilde{u}}) & A_{0122}^{1100}(C_{\tilde{u}\tilde{u}\tilde{u}\tilde{u}}) \\ A_{2200}^{1200}(C_{\tilde{u}\tilde{u}\tilde{u}\tilde{u}}) & B_{1100}^{0000}(C_{u\tilde{u}\tilde{u}u}) & B_{1100}^{0100}(C_{\tilde{u}\tilde{u}\tilde{u}\tilde{u}}) \\ B_{2100}^{0200}(C_{\tilde{u}\tilde{u}\tilde{u}\tilde{u}}) & A_{1122}^{1200}(C_{\tilde{u}\tilde{u}\tilde{u}\tilde{u}}) & B_{1200}^{1000}(C_{\tilde{u}\tilde{u}\tilde{u}\tilde{u}}) \end{matrix} \right\}$
C_{dd}	18	$\left\{ \begin{matrix} A_{1100}^{0000}(C_{dd\tilde{d}\tilde{d}}) & A_{1100}^{1000}(C_{\tilde{d}\tilde{d}\tilde{d}\tilde{d}}) & A_{2200}^{0000}(C_{dd\tilde{d}\tilde{d}}) \\ A_{2000}^{1100}(C_{\tilde{d}\tilde{d}\tilde{d}\tilde{d}}) & A_{1100}^{0100}(C_{\tilde{d}\tilde{d}\tilde{d}\tilde{d}}) & A_{1100}^{1100}(C_{\tilde{d}\tilde{d}\tilde{d}\tilde{d}}) \\ A_{2200}^{1000}(C_{\tilde{d}\tilde{d}\tilde{d}\tilde{d}}) & A_{1122}^{0000}(C_{dd\tilde{d}\tilde{d}}) & A_{2200}^{1100}(C_{\tilde{d}\tilde{d}\tilde{d}\tilde{d}}) \\ A_{1122}^{1000}(C_{\tilde{d}\tilde{d}\tilde{d}\tilde{d}}) & A_{1220}^{1100}(C_{\tilde{d}\tilde{d}\tilde{d}\tilde{d}}) & A_{2110}^{1200}(C_{\tilde{d}\tilde{d}\tilde{d}\tilde{d}}) \\ A_{0122}^{2100}(C_{\tilde{d}\tilde{d}\tilde{d}\tilde{d}}) & A_{1220}^{2200}(C_{\tilde{d}\tilde{d}\tilde{d}\tilde{d}}) & B_{1100}^{0000}(C_{d\tilde{d}\tilde{d}d}) \\ B_{2100}^{0100}(C_{\tilde{d}\tilde{d}\tilde{d}\tilde{d}}) & B_{1100}^{1000}(C_{\tilde{d}\tilde{d}\tilde{d}\tilde{d}}) & B_{2000}^{1200}(C_{\tilde{d}\tilde{d}\tilde{d}\tilde{d}}) \end{matrix} \right\}$
$C_{Qu}^{(1,8)}$	36	$\left\{ \begin{matrix} A_{0000}^{1100}(C_{QQuu}) & A_{1100}^{0000}(C_{QQ\tilde{u}\tilde{u}}) & A_{1100}^{1000}(C_{QQ\tilde{u}\tilde{u}}) \\ A_{0100}^{1100}(C_{QQ\tilde{u}\tilde{u}}) & A_{1100}^{1100}(C_{QQ\tilde{u}\tilde{u}}) & A_{0110}^{1100}(C_{QQ\tilde{u}\tilde{u}}) \\ A_{1000}^{1200}(C_{QQ\tilde{u}\tilde{u}}) & A_{0000}^{2200}(C_{QQuu}) & A_{2200}^{1100}(C_{QQ\tilde{u}\tilde{u}}) \\ A_{0220}^{1100}(C_{QQ\tilde{u}\tilde{u}}) & A_{0110}^{2200}(C_{QQ\tilde{u}\tilde{u}}) & A_{1122}^{1100}(C_{QQ\tilde{u}\tilde{u}}) \\ A_{1220}^{1200}(C_{QQ\tilde{u}\tilde{u}}) & A_{1122}^{2200}(C_{QQ\tilde{u}\tilde{u}}) & B_{0100}^{0000}(C_{QQ\tilde{u}\tilde{u}}) \\ B_{1000}^{0000}(C_{QQ\tilde{u}\tilde{u}}) & B_{0110}^{0000}(C_{QQ\tilde{u}\tilde{u}}) & B_{0220}^{0000}(C_{QQ\tilde{u}\tilde{u}}) \\ B_{1100}^{0000}(C_{QQ\tilde{u}\tilde{u}}) & B_{0221}^{0000}(C_{QQ\tilde{u}\tilde{u}}) & B_{1000}^{0100}(C_{QQ\tilde{u}\tilde{u}}) \\ B_{1100}^{0100}(C_{QQ\tilde{u}\tilde{u}}) & B_{2200}^{0100}(C_{QQ\tilde{u}\tilde{u}}) & B_{2110}^{0100}(C_{QQ\tilde{u}\tilde{u}}) \\ B_{2000}^{0200}(C_{QQ\tilde{u}\tilde{u}}) & B_{2100}^{0200}(C_{QQ\tilde{u}\tilde{u}}) & B_{2110}^{0200}(C_{QQ\tilde{u}\tilde{u}}) \\ B_{0110}^{1000}(C_{QQ\tilde{u}\tilde{u}}) & B_{0220}^{1000}(C_{QQ\tilde{u}\tilde{u}}) & B_{0221}^{1000}(C_{QQ\tilde{u}\tilde{u}}) \\ B_{1100}^{1100}(C_{QQ\tilde{u}\tilde{u}}) & B_{2200}^{1100}(C_{QQ\tilde{u}\tilde{u}}) & B_{2100}^{1200}(C_{QQ\tilde{u}\tilde{u}}) \\ B_{2210}^{1200}(C_{QQ\tilde{u}\tilde{u}}) & B_{1200}^{2100}(C_{QQ\tilde{u}\tilde{u}}) & B_{0221}^{0110}(C_{QQ\tilde{u}\tilde{u}}) \end{matrix} \right\}$
$C_{Qd}^{(1,8)}$	36	$\left\{ \begin{matrix} A_{0000}^{1100}(C_{QQdd}) & A_{1100}^{0000}(C_{QQ\tilde{d}\tilde{d}}) & A_{1100}^{1000}(C_{QQ\tilde{d}\tilde{d}}) \\ A_{1000}^{1100}(C_{QQ\tilde{d}\tilde{d}}) & A_{0000}^{2200}(C_{QQdd}) & A_{1100}^{0100}(C_{QQ\tilde{d}\tilde{d}}) \\ A_{2200}^{0000}(C_{QQ\tilde{d}\tilde{d}}) & A_{1100}^{1100}(C_{QQ\tilde{d}\tilde{d}}) & A_{2100}^{1100}(C_{QQ\tilde{d}\tilde{d}}) \\ A_{0000}^{1122}(C_{QQdd}) & A_{1122}^{0000}(C_{QQ\tilde{d}\tilde{d}}) & A_{2200}^{1100}(C_{QQ\tilde{d}\tilde{d}}) \\ A_{0220}^{1100}(C_{QQ\tilde{d}\tilde{d}}) & A_{1122}^{1000}(C_{QQ\tilde{d}\tilde{d}}) & A_{1122}^{1100}(C_{QQ\tilde{d}\tilde{d}}) \\ A_{0122}^{2100}(C_{QQ\tilde{d}\tilde{d}}) & B_{0100}^{0000}(C_{QQ\tilde{d}\tilde{d}}) & B_{1000}^{0000}(C_{QQ\tilde{d}\tilde{d}}) \\ B_{0110}^{0000}(C_{QQ\tilde{d}\tilde{d}}) & B_{0220}^{0000}(C_{QQ\tilde{d}\tilde{d}}) & B_{1100}^{0000}(C_{QQ\tilde{d}\tilde{d}}) \\ B_{0221}^{0000}(C_{QQ\tilde{d}\tilde{d}}) & B_{2200}^{0000}(C_{QQ\tilde{d}\tilde{d}}) & B_{2210}^{0000}(C_{QQ\tilde{d}\tilde{d}}) \\ B_{1000}^{0100}(C_{QQ\tilde{d}\tilde{d}}) & B_{0120}^{0100}(C_{QQ\tilde{d}\tilde{d}}) & B_{1100}^{0100}(C_{QQ\tilde{d}\tilde{d}}) \\ B_{2210}^{0100}(C_{QQ\tilde{d}\tilde{d}}) & B_{0110}^{1000}(C_{QQ\tilde{d}\tilde{d}}) & B_{0220}^{1000}(C_{QQ\tilde{d}\tilde{d}}) \\ B_{0221}^{1000}(C_{QQ\tilde{d}\tilde{d}}) & B_{1200}^{1000}(C_{QQ\tilde{d}\tilde{d}}) & B_{2200}^{1100}(C_{QQ\tilde{d}\tilde{d}}) \\ B_{2210}^{1100}(C_{QQ\tilde{d}\tilde{d}}) & B_{2100}^{1200}(C_{QQ\tilde{d}\tilde{d}}) & B_{2211}^{2100}(C_{QQ\tilde{d}\tilde{d}}) \end{matrix} \right\}$

Wilson coefficient	Number of phases	Minimal set
$C_{ud}^{(1,8)}$	36	$\left\{ \begin{matrix} A_{0000}^{1100}(C_{\tilde{u}\tilde{u}dd}) & A_{1100}^{0000}(C_{uu\tilde{d}\tilde{d}}) & A_{1100}^{1000}(C_{\tilde{u}\tilde{u}\tilde{d}\tilde{d}}) \\ A_{1000}^{1100}(C_{\tilde{u}\tilde{u}\tilde{d}\tilde{d}}) & A_{0000}^{2200}(C_{\tilde{u}\tilde{u}dd}) & A_{1100}^{0100}(C_{\tilde{u}\tilde{u}\tilde{d}\tilde{d}}) \\ A_{2200}^{0000}(C_{uu\tilde{d}\tilde{d}}) & A_{1100}^{1100}(C_{\tilde{u}\tilde{u}\tilde{d}\tilde{d}}) & A_{0110}^{1100}(C_{\tilde{u}\tilde{u}\tilde{d}\tilde{d}}) \\ A_{2200}^{1000}(C_{\tilde{u}\tilde{u}\tilde{d}\tilde{d}}) & A_{2100}^{1100}(C_{\tilde{u}\tilde{u}\tilde{d}\tilde{d}}) & A_{0000}^{1122}(C_{\tilde{u}\tilde{u}dd}) \\ A_{2200}^{0100}(C_{\tilde{u}\tilde{u}\tilde{d}\tilde{d}}) & A_{1122}^{0000}(C_{uu\tilde{d}\tilde{d}}) & A_{2200}^{1100}(C_{\tilde{u}\tilde{u}\tilde{d}\tilde{d}}) \\ A_{1122}^{1000}(C_{\tilde{u}\tilde{u}\tilde{d}\tilde{d}}) & A_{1122}^{0100}(C_{\tilde{u}\tilde{u}\tilde{d}\tilde{d}}) & A_{1122}^{1100}(C_{\tilde{u}\tilde{u}\tilde{d}\tilde{d}}) \\ B_{0100}^{0000}(C_{\tilde{u}\tilde{u}\tilde{d}\tilde{d}}) & B_{1000}^{0000}(C_{\tilde{u}\tilde{u}\tilde{d}\tilde{d}}) & B_{0110}^{0000}(C_{\tilde{u}\tilde{u}\tilde{d}\tilde{d}}) \\ B_{1100}^{0000}(C_{\tilde{u}\tilde{u}\tilde{d}\tilde{d}}) & B_{0221}^{0000}(C_{\tilde{u}\tilde{u}\tilde{d}\tilde{d}}) & B_{2200}^{0000}(C_{\tilde{u}\tilde{u}\tilde{d}\tilde{d}}) \\ B_{1000}^{0100}(C_{\tilde{u}\tilde{u}\tilde{d}\tilde{d}}) & B_{0110}^{0100}(C_{\tilde{u}\tilde{u}\tilde{d}\tilde{d}}) & B_{2110}^{0100}(C_{\tilde{u}\tilde{u}\tilde{d}\tilde{d}}) \\ B_{2000}^{0200}(C_{\tilde{u}\tilde{u}\tilde{d}\tilde{d}}) & B_{2110}^{0200}(C_{\tilde{u}\tilde{u}\tilde{d}\tilde{d}}) & B_{0110}^{1000}(C_{\tilde{u}\tilde{u}\tilde{d}\tilde{d}}) \\ B_{0221}^{1000}(C_{\tilde{u}\tilde{u}\tilde{d}\tilde{d}}) & B_{1200}^{1000}(C_{\tilde{u}\tilde{u}\tilde{d}\tilde{d}}) & B_{2200}^{1100}(C_{\tilde{u}\tilde{u}\tilde{d}\tilde{d}}) \\ B_{2211}^{1100}(C_{\tilde{u}\tilde{u}\tilde{d}\tilde{d}}) & B_{2100}^{1200}(C_{\tilde{u}\tilde{u}\tilde{d}\tilde{d}}) & B_{1200}^{2100}(C_{\tilde{u}\tilde{u}\tilde{d}\tilde{d}}) \end{matrix} \right\}$
$C_{QuQd}^{(1,8)}$	81	$\left\{ \begin{matrix} A_{0000}^{0000}(C_{Q\tilde{u}Q\tilde{d}}) & A_{1000}^{0000}(C_{Q\tilde{u}Q\tilde{d}}) & A_{0000}^{1000}(C_{Q\tilde{u}Q\tilde{d}}) \\ A_{1000}^{1000}(C_{Q\tilde{u}Q\tilde{d}}) & A_{0100}^{0000}(C_{Q\tilde{u}Q\tilde{d}}) & A_{0000}^{0100}(C_{Q\tilde{u}Q\tilde{d}}) \\ A_{1100}^{0000}(C_{Q\tilde{u}Q\tilde{d}}) & A_{0110}^{0000}(C_{Q\tilde{u}Q\tilde{d}}) & A_{1000}^{0100}(C_{Q\tilde{u}Q\tilde{d}}) \\ A_{0100}^{1000}(C_{Q\tilde{u}Q\tilde{d}}) & A_{0000}^{1100}(C_{Q\tilde{u}Q\tilde{d}}) & A_{0000}^{0110}(C_{Q\tilde{u}Q\tilde{d}}) \\ A_{1100}^{1000}(C_{Q\tilde{u}Q\tilde{d}}) & A_{0110}^{1000}(C_{Q\tilde{u}Q\tilde{d}}) & A_{1000}^{1100}(C_{Q\tilde{u}Q\tilde{d}}) \\ A_{0100}^{0100}(C_{Q\tilde{u}Q\tilde{d}}) & A_{1100}^{0100}(C_{Q\tilde{u}Q\tilde{d}}) & A_{0110}^{0100}(C_{Q\tilde{u}Q\tilde{d}}) \\ A_{0100}^{0110}(C_{Q\tilde{u}Q\tilde{d}}) & A_{2200}^{0000}(C_{Q\tilde{u}Q\tilde{d}}) & A_{0220}^{0000}(C_{Q\tilde{u}Q\tilde{d}}) \\ A_{2000}^{0200}(C_{Q\tilde{u}Q\tilde{d}}) & A_{1100}^{1100}(C_{Q\tilde{u}Q\tilde{d}}) & A_{0110}^{1100}(C_{Q\tilde{u}Q\tilde{d}}) \\ A_{0200}^{2000}(C_{Q\tilde{u}Q\tilde{d}}) & A_{0100}^{2100}(C_{Q\tilde{u}Q\tilde{d}}) & A_{1100}^{0110}(C_{Q\tilde{u}Q\tilde{d}}) \\ A_{0110}^{0110}(C_{Q\tilde{u}Q\tilde{d}}) & A_{1000}^{0210}(C_{Q\tilde{u}Q\tilde{d}}) & A_{1220}^{0000}(C_{Q\tilde{u}Q\tilde{d}}) \\ A_{2000}^{1200}(C_{Q\tilde{u}Q\tilde{d}}) & A_{0122}^{0000}(C_{Q\tilde{u}Q\tilde{d}}) & A_{1220}^{0100}(C_{Q\tilde{u}Q\tilde{d}}) \\ A_{0122}^{1000}(C_{Q\tilde{u}Q\tilde{d}}) & A_{2200}^{1100}(C_{Q\tilde{u}Q\tilde{d}}) & A_{0220}^{1100}(C_{Q\tilde{u}Q\tilde{d}}) \\ A_{2100}^{1200}(C_{Q\tilde{u}Q\tilde{d}}) & A_{1200}^{2100}(C_{Q\tilde{u}Q\tilde{d}}) & A_{0210}^{2100}(C_{Q\tilde{u}Q\tilde{d}}) \\ A_{0110}^{2200}(C_{Q\tilde{u}Q\tilde{d}}) & A_{2200}^{0110}(C_{Q\tilde{u}Q\tilde{d}}) & A_{0220}^{0110}(C_{Q\tilde{u}Q\tilde{d}}) \\ A_{2000}^{0112}(C_{Q\tilde{u}Q\tilde{d}}) & A_{1220}^{1100}(C_{Q\tilde{u}Q\tilde{d}}) & A_{0112}^{2100}(C_{Q\tilde{u}Q\tilde{d}}) \\ A_{1220}^{1200}(C_{Q\tilde{u}Q\tilde{d}}) & A_{2200}^{2200}(C_{Q\tilde{u}Q\tilde{d}}) & A_{1122}^{0110}(C_{Q\tilde{u}Q\tilde{d}}) \\ A_{2100}^{0122}(C_{Q\tilde{u}Q\tilde{d}}) & A_{0220}^{0220}(C_{Q\tilde{u}Q\tilde{d}}) & B_{0000}^{0000}(C_{Q\tilde{u}Q\tilde{d}}) \\ B_{0100}^{0000}(C_{Q\tilde{u}Q\tilde{d}}) & B_{1000}^{0000}(C_{Q\tilde{u}Q\tilde{d}}) & B_{1100}^{0000}(C_{Q\tilde{u}Q\tilde{d}}) \\ B_{2200}^{0000}(C_{Q\tilde{u}Q\tilde{d}}) & B_{0110}^{0000}(C_{Q\tilde{u}Q\tilde{d}}) & B_{0122}^{0000}(C_{Q\tilde{u}Q\tilde{d}}) \\ B_{0220}^{0000}(C_{Q\tilde{u}Q\tilde{d}}) & B_{0000}^{0100}(C_{Q\tilde{u}Q\tilde{d}}) & B_{1000}^{0100}(C_{Q\tilde{u}Q\tilde{d}}) \\ B_{1100}^{0100}(C_{Q\tilde{u}Q\tilde{d}}) & B_{2100}^{0100}(C_{Q\tilde{u}Q\tilde{d}}) & B_{0120}^{0100}(C_{Q\tilde{u}Q\tilde{d}}) \\ B_{1220}^{0100}(C_{Q\tilde{u}Q\tilde{d}}) & B_{1120}^{0200}(C_{Q\tilde{u}Q\tilde{d}}) & B_{0000}^{1000}(C_{Q\tilde{u}Q\tilde{d}}) \\ B_{0100}^{1000}(C_{Q\tilde{u}Q\tilde{d}}) & B_{1200}^{1000}(C_{Q\tilde{u}Q\tilde{d}}) & B_{0110}^{1000}(C_{Q\tilde{u}Q\tilde{d}}) \\ B_{0122}^{1000}(C_{Q\tilde{u}Q\tilde{d}}) & B_{0210}^{1000}(C_{Q\tilde{u}Q\tilde{d}}) & B_{0000}^{1100}(C_{Q\tilde{u}Q\tilde{d}}) \\ B_{1100}^{1100}(C_{Q\tilde{u}Q\tilde{d}}) & B_{2200}^{1100}(C_{Q\tilde{u}Q\tilde{d}}) & B_{0110}^{1100}(C_{Q\tilde{u}Q\tilde{d}}) \\ B_{0220}^{1100}(C_{Q\tilde{u}Q\tilde{d}}) & B_{1122}^{1100}(C_{Q\tilde{u}Q\tilde{d}}) & B_{2100}^{1200}(C_{Q\tilde{u}Q\tilde{d}}) \\ B_{0122}^{2100}(C_{Q\tilde{u}Q\tilde{d}}) & B_{0000}^{2200}(C_{Q\tilde{u}Q\tilde{d}}) & A_{1122}^{2200}(C_{Q\tilde{u}Q\tilde{d}}) \end{matrix} \right\}$

the 597 invariants associated to the 4F operators

4-Fermi Operators

Minimal and maximal bases

- As for the bilinears, one can construct a minimal basis of invariants:

"CP is conserved iff J_4 and the invariants of a minimal basis are all vanishing"

- The dimension of the **minimal** basis is always equal to the number of physical phases associated to an operator: QQQQ → 18, QuQd → 81, LLuu → 36/9 (w/wo neutrino masses) …

- But the real coefficients also contribute to CPV: the dimension of the **maximal** basis is equal to the total number of parameters associated to an operator: QQQQ → 45, QuQd → 162, LLuu → 81/27 (w/wo neutrino masses) …

SMEFT CPV Invariants with Theta QCD

Can we build new invariants using ϴ$_{QCD}$?

	$SU(3)_{Q_L}$	$U(1)_{Q_L}$	$SU(3)_{u_R}$	$U(1)_{u_R}$	$SU(3)_{d_R}$	$U(1)_{d_R}$
Q_L	$\mathbf{3}$	1	$\mathbf{1}$	0	$\mathbf{1}$	0
u_R	$\mathbf{1}$	0	$\mathbf{3}$	1	$\mathbf{1}$	0
d_R	$\mathbf{1}$	0	$\mathbf{1}$	0	$\mathbf{3}$	1
Y_u	$\mathbf{3}$	1	$\bar{\mathbf{3}}$	-1	$\mathbf{1}$	0
Y_d	$\mathbf{3}$	1	$\mathbf{1}$	0	$\bar{\mathbf{3}}$	-1
$e^{i\theta_{QCD}}$	$\mathbf{1}$	6	$\mathbf{1}$	-3	$\mathbf{1}$	-3

- Given that $\bar{\theta} = \theta - \arg\det\left(Y_u Y_d\right)$ is a flavour invariant, no new SM$_4$ invariant can be constructed
- In SM$_6$, in principle, new structure can emerge

$$\mathrm{Im}\left(e^{-i\theta_{QCD}}\epsilon^{ABC}\epsilon^{abc}Y_{u,Aa}Y_{u,Bb}C_{uH,Cc}\det Y_d\right)$$

- Probably highly suppressed in the perturbative regime of QCD ($e^{-8\pi^2/g_s^2} \sim \lambda^{37}$)
- Relevant at low scale?

Shift-invariant axion: non-pertubative condition

θ$_{QCD}$ again

$$-\frac{C_g g_3^2}{16\pi^2}\frac{a}{f}\,\mathrm{Tr}\big(G_{\mu\nu}\tilde{G}^{\mu\nu}\big)$$

breaks shift-invariance non-perturbatively (instanton effects)

(in the operator basis where fermion couplings are derivative)

Shift-invariant axion: non-pertubative condition

θ$_{QCD}$ again

$$-\frac{C_g g_3^2}{16\pi^2}\frac{a}{f}\,\mathrm{Tr}\Big(G_{\mu\nu}\tilde{G}^{\mu\nu}\Big)$$

breaks shift-invariance non-perturbatively (instanton effects)

(in the operator basis where fermion couplings are derivative)

$$I_g \equiv C_g + \mathrm{Im}\,\mathrm{Tr}\left(Y_u^{-1}\tilde{Y}_u + Y_d^{-1}\tilde{Y}_d\right) = 0$$

is the basis independent condition for the shift-invariance to be maintained at the non-perturbative level

Shift-invariant axion: non-pertubative condition

θ_{QCD} again

$$-\frac{C_g g_3^2}{16\pi^2}\frac{a}{f}\,\mathrm{Tr}\big(G_{\mu\nu}\tilde{G}^{\mu\nu}\big)$$

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(in the operator basis where fermion couplings are derivative)

$$I_g \equiv C_g + \mathrm{Im}\,\mathrm{Tr}\left(Y_u^{-1}\tilde{Y}_u + Y_d^{-1}\tilde{Y}_d\right) = 0$$

is the basis independent condition for the shift-invariance to be maintained at the non-perturbative level

It can be shown again that this condition is **RG invariant**

$$\mu\frac{dI_g}{d\mu} = 0$$

whenever shift-symmetry holds ($I_g = I_i = 0$ for i=1…13)