OPE-based sum rules for heavy hadrons

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Pushing the Limits of Theoretical Physics,
MITP, Mainz, May 9 , 2023

A bit of history:

- QCD sum rules in heavy-quark effective theory (HQET)

PHYSICAL REVIEW D VOLUME 45, NUMBER 7 1 APRIL 1992

Heavy-meson form factors from QCD sum rules

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(Received 16 December 1991)

A consistent framework is developed for studying hadronic form factors of heavy mesons using QCD sum rules in the heavy-quark effective theory, including the next-to-leading-order renormalization-group improvement. Sum rules are derived for the asymptotic value of the meson decay constant f_P and for the universal Isgur-Wise form factor. It is shown that renormalization-group effects considerably enhance the prediction for f_P and bring its asymptotic value in accordance with recent lattice results. Including finite-mass corrections, the dependence of the physical decay constant on the meson mass is investigated. We obtain $f_D \simeq 170 \pm 30$ MeV and $f_B \simeq 190 \pm 50$ MeV. The origin of the breakdown of the heavy-quark expansion for f_D is analyzed. In the case of heavy-meson transition form factors, both the QCD and $1/m_Q$ corrections are moderate and under control. A sum rule for the renormalized Isgur-Wise function is derived and evaluated. The theoretical result is compared to experimental data.

PACS number(s): 11.50.Li, 12.38.Cy, 12.38.Lg, 13.25.+m

- further discussed in the review:

M.Neubert, Heavy quark symmerty, Phys. Rept. 245 (1994) 259-396

□ Operator-Product Expansion

- originally formulated in K. G. Wilson, Phys. Rev. 179, 1499-1512 (1969).
- local OPE: $i\int d^4x\, e^{iqx}\, T\{j_A(x)j_B(0)\} = \sum_d C_d^{AB}(q^2)\hat{O}_d(0)$

  proved order by order in perturbative QCD at $q^2 \to -\infty$
- practical version of OPE including nonperturbative effects

  M.Shifman, A.Vainshtein and V.I.Zakharov, (1979)
- allowing one to calculate vacuum correlation functions

  $\Pi_A(q) = i\int d^4x\, e^{iqx}\langle 0|T\{j_A(x)j_A(0)\}|0\rangle$
- OPE generates vacuum averages of local operators

  $\Rightarrow$ condensate expansion of the correlation function:

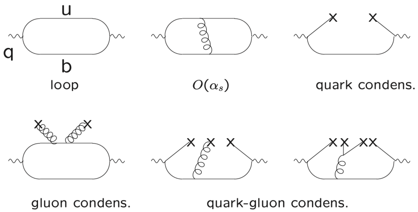
  $\Pi_A(q)^{OPE} = \sum_d C_d^{AB}(q^2)\langle 0|\hat{O}_d|0\rangle$ valid at $|q^2| \gg \Lambda^2_{QCD}$
- the heavy quark mass provides a large scale $m_Q >> \Lambda_{QCD}$ effectively replacing $\sqrt{|q^2|}$

□ QCD sum rule for the B-meson decay constant

- the sum rule for $f_B = \langle 0|j_5|B\rangle / m_B^2$, studied by Neubert in HQET limit
- correlation function

$$\Pi_5(q^2) = i \int d^4x\, e^{iqx} \langle 0|T\{j_5(x) j_5(0)\}|0\rangle, \quad j_5 = m_b \bar{b} i\gamma_5 u$$

- OPE diagrams for Wilson coefficients (known to $O(\alpha_s^2)$)



$$\Pi_5(q^2)^{OPE} = \sum_d C_{5d}(q^2, m_b^2) \langle 0|\hat{O}_d|0\rangle$$

valid at $q^2 \ll m_b^2$

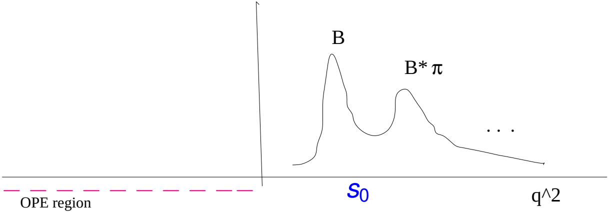
□ QCD sum rule for the *B*-meson decay constant

- use of unitarity $\oplus$ hadronic dispersion relation

$$\Pi_5(q^2) = \frac{f_B^2 m_B^4}{m_B^2 - q^2} + \int\limits_{s_h}^{\infty} ds \frac{\rho_5^h(s)}{s - q^2}\,, \qquad s_h = (m_B^* + m_\pi)^2$$

subtractions implied

- schematic view: there is a gap between B and $B^*\pi$



- quark-hadron duality: approximating the integral
  (a more general assumption than the local duality)

$$\int\limits_{s_h}^{\infty} ds \frac{\rho_5^h(s)}{s - q^2} = \frac{1}{\pi} \int\limits_{s_0}^{\infty} ds \frac{\mathrm{Im}\Pi_5^{OPE}(s)}{s - q^2}$$

□ f_B sum rule and others based on local OPE

- the (incomplete) list of papers on the f_B sum rule before 2013:

Phys. B147, 385 (1979).

[11] V. Novikov, M. A. Shifman, A. I. Vainshtein, M. B. Voloshin, and V. I. Zakharov, in Proceedings of Neutrinos 1978 (Purdue University, West Lafayette, 1978), pp. 278–288.

[12] T. M. Aliev and V. L. Eletsky, Yad. Fiz. **38**, 1537 (1983) [Sov. J. Nucl. Phys. **38**, 936 (1983)].

[13] C. A. Dominguez and N. Paver, Phys. Lett. B **197**, 423 (1987); **199**, 596(E) (1987).

[14] S. Narison, Z. Phys. C **14**, 263 (1982); Phys. Lett. B **198**, 104 (1987).

[15] E. V. Shuryak, Nucl. Phys. **B198**, 83 (1982).

[16] M. Neubert, Phys. Rev. D **45**, 2451 (1992).

[17] E. Bagan, P. Ball, V. M. Braun, and H. G. Dosch, Phys. Lett. B **278**, 457 (1992).

[18] D. J. Broadhurst and A. G. Grozin, Phys. Lett. B **274**, 421 (1992).

[19] M. Neubert, Phys. Rep. **245**, 259 (1994).

[20] D. J. Broadhurst, Phys. Lett. **B101**, 423 (1981).

[21] S. C. Generalis, J. Phys. G **16**, 785 (1990).

[22] C. A. Dominguez and N. Paver, Phys. Lett. B **246**, 493 (1990).

[23] K. G. Chetyrkin and M. Steinhauser, Eur. Phys. J. C **21**, 319 (2001).

[24] M. Jamin and B. O. Lange, Phys. Rev. D **65**, 056005 (2002).

[25] A. A. Penin and M. Steinhauser, Phys. Rev. D **65**, 054006 (2002).

- our analysis of the f_B sum rule

$$f_B = 201^{+17}_{-7}\ \text{MeV}$$

P.Gelhausen, AK, A.Pivoarov, D.Rosenthal (2013)

- comparison with the lattice QCD average:

$$f_B = 190.0 \pm 1.3\ \text{MeV}$$ [FLAG 2021 ($N_f = 4$)]

□ Advantages of the local-OPE QCD sum rules

- the number of universal inputs is minimal
- the spectral density is positive definite, with a gap,
  model-independent bounds possible
- if the hadronic spectral density is well measured
  $\Rightarrow$ extract QCD parameters,
  e.g. $m_{b,c}$ determination from a correlator of $\bar{Q}\gamma_\mu Q$ currents (Q=b,c)
  K. Chetyrkin, J. H. Kuhn, A. Maier, P. Maierhofer, P. Marquard, M. Steinhauser and C. Sturm, (2012)
- the choice of quark-hadron duality threshold s_0 is controlled by the B meson mass calculation

□ Heavy-light form factors from the sum rules based on local OPE

- in the past the local OPE was used to obtain $B \to h$ form factors
- need a three-point correlation function (cf lattice QCD)
- use of double dispersion relation
- quark-hadron duality in the two channels
- diagrams for the case of $b \to u$ exclusive transition

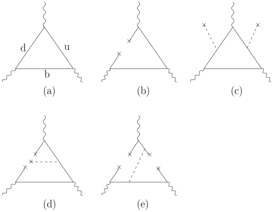


Figure 3: *Diagrams determining the Wilson coefficients of the OPE of the three-point correlation function (46). The symbols are as in Fig. 1.*

- however, there is a conceptual problem with local OPE:
  infinite towers of local operators contribute with the same power

discussed e.g., in the review by V. Braun, hep-ph/9801222.

□ OPE near the light-cone

- the best studied case: the $B \to \pi$ form factor
- a different type of correlation function

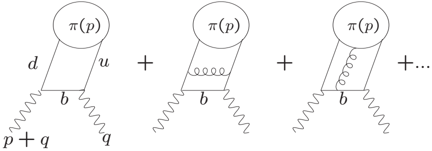
$$F_\mu(q,p) = i\int d^4x\, e^{iqx}\langle\pi^+(p)|T\{\bar{u}(x)\gamma_\mu b(x) m_b \bar{b}(0) i\gamma_5 d(0)\}|0\rangle$$

$$|q^2| \sim |(p+q)^2| \gg \Lambda^2_{QCD} \quad \Downarrow \quad x^2 \to 0$$

$$\boxed{\sum_t C_t(x^2, m_b^2)\langle\pi^+(p)|O_t(x,0)|0\rangle}$$

twist expansion

- OPE diagrams



- light-cone distribution amplitudes (DAs) of the pion emerge

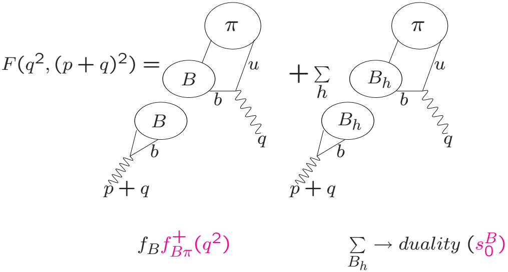
$\langle\pi(p)|O_t(x,0)|0\rangle$, $\quad O_{t=2} = \bar{u}(x)\gamma_\mu\gamma_5 d(0)$, ...

□ Light-cone sum rules (LCSR)

I.Balitsky,V.Braun, A.Kolesnichenko (1989); V.Chernyak, I.Zhitnisky (1989)

- hadronic dispersion relation in the B-meson channel

$$F_\mu^{OPE}(p,q) = \frac{\langle \pi^+(p)|\bar{u}\gamma_\mu b|B(p+q)\rangle\langle B(p+q)|m_b\bar{b}i\gamma_5 d|0\rangle}{m_B^2-(p+q)^2} + ...$$



- use of quark-hadron duality
- main input: the light-meson DAs (expansion, parameters)
- for each $B \to h$ transition we need a set of LCDAs of the hadron h (π, K, ρ, K^*...)

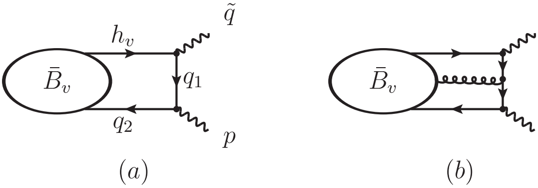
□ A different version of LCSRs for $B \to \pi$ form factors

- π is interpolated with a current $\oplus$ duality

$$F^{(B)}_{\mu\nu}(p,q) = i\int d^4x\, e^{ip\cdot x}\langle 0|T\left\{\bar{d}(x)\gamma_\mu\gamma_5 u(x), \bar{u}(0)\gamma_\nu b(0)\right\}|\bar{B}^0(p+q)\rangle\,.$$

- $q^2 \sim 0,\ p^2 < 0,\ |p^2| \gg \Lambda^2_{QCD}$, AK, Th.Mannel, N. Offen (2005)
- the diagrams: $(q_1 = u,\ q_2 = d)$, *u*-quark propagates near LC

.



- the nonlocal matrix element, defined in HQET leads to B-meson distribution amplitudes A.G.Grozin. M.Neubert (1997)

$$\langle 0|T\left\{\bar{d}_\alpha(x)[x,0]b_\beta(0)\right\}|\bar{B}^0(v)\rangle|_{x^2=0}$$

$$= -\frac{if_B m_B}{4}\left[(1+\not{v})\gamma_5\int\limits_0^\infty d\omega e^{-i\omega v\cdot x}\left\{\phi^B_+(\omega) + \frac{\phi^B_+(\omega)-\phi^B_-(\omega)}{2v\cdot x}\not{x}\right\}\gamma_5\right]_{\beta\alpha},$$

☐ *B*-meson distribution amplitudes in HQET

- the two key papers:
  - introducing the *B*-meson distribution amplitudes in HQET

PHYSICAL REVIEW D VOLUME 55, NUMBER 1 1 JANUARY 1997

Asymptotics of heavy-meson form factors

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M. Neubert
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(Received 16 July 1996)

  - understanding their renormalization

VOLUME 91, NUMBER 10 PHYSICAL REVIEW LETTERS week ending 5 SEPTEMBER 2003

Renormalization-Group Evolution of the *B*-Meson Light-Cone Distribution Amplitude

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□ New sum rule for $B_c \to J/\psi$ form factors

M.Bordone, AK, Th.Mannel, 2209.08851

- The $B_c \to J/\psi \ell \nu_\ell$ decay has created a lot of interest:
  - the measured ratio

$$R(J/\psi) = \frac{\mathcal{B}(B_c \to J/\psi \tau \bar{\nu}_\tau)}{\mathcal{B}(B_c \to J/\psi \mu \bar{\nu}_\mu)} = 0.71 \pm 0.17 \pm 0.18$$

R. Aaij et al. (LHCb Collab.), Phys. Rev. Lett. 120, 121801 (2018).

  - reveals a tension with the lattice QCD result:

$$R(J/\psi) = 0.2582 \pm 0.0038$$

J.Harrison, C.T.H. Davies, A.Lytle (HPQCD Collab.), Phys. Rev. Lett. 125, 222003 (2020)

- Lattice QCD provides the $B_c \to J/\psi$ form factors at all momentum transfers $0 < q^2 < (m_{B_c} - m_{J/\psi})^2$
- can we calculate these form factors with non-lattice methods at least at large recoil $q^2 \sim 0$?

□ $B_c \to J/\psi$- form factors

- definition:

$$\langle J/\psi(p,\varepsilon)|\left(\bar{c}\gamma_\nu b - \bar{c}\gamma_\nu\gamma_5 b\right)|\bar{B}_c(p+q)\rangle = \epsilon_{\nu\rho\alpha\beta}\varepsilon^{*\rho}q^\alpha p^\beta \frac{2V(q^2)}{M_{B_c}+M_{J/\psi}} + \text{axial FFs}$$

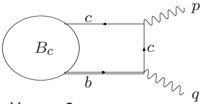
will concentrate on the vector-current form factor $V(q^2)$

- existing continuum QCD methods
  - NRQCD: direct calculation of the $B_c \to J/\psi$ matrix elements
  - three-point QCD sum rules: double dispersion relation only LO triangle loop +gluon condensate
- the task: an alternative sum rule approach, something similar to LCSR

□ Correlation function with on-shell B_c state

- correlator

$$
\begin{aligned}
F_{\mu\nu}(p,q) &= i\int d^4x\, e^{ipx}\langle 0|T\{\bar{c}(x)\gamma_\mu c(x)\bar{c}(0)(\gamma_\nu-\gamma_\nu\gamma_5)b(0)\}|\bar{B}_c(p+q)\rangle \\
&= \epsilon_{\mu\nu\alpha\beta}q^\alpha p^\beta F^V(p^2,q^2)+F^A_{\mu\nu}(p,q)
\end{aligned}
$$



- dispersion relation in p^2, e.g. for the V form factor

$$
F^V(p^2,q^2)=\frac{2f_{J/\psi}V(q^2)}{(m_{B_c}+m_{J/\psi})(m^2_{J/\psi}-p^2)}+\int\limits_{m^2_{\psi(2S)}}^{\infty} ds\,\frac{\rho^V(s,q^2)}{s-p^2}\,,
$$

- calculate $F^V(p^2,q^2)$ at $p^2,q^2\ll 4m_c^2$ from OPE in a dispersion form:

$$
F^{V(OPE)}(p^2,q^2)=\frac{1}{\pi}\int\limits_{s_{min}}^{\infty} ds\,\frac{\mathrm{Im}F^{V(OPE)}(s,q^2)}{s-p^2}
$$

- J/ψ duality interval expected from $s_{min}=4m_c^2$ to $s_0=(2m_c+\omega_0)^2$
- sum rule for the form factor in the Borel form (or power moments):

$$
\frac{2f_{J/\psi}V(q^2)}{(m_{B_c}+m_{J/\psi})}e^{-m^2_{J/\psi}/\mathcal{M}^2}=\frac{1}{\pi}\int\limits_{4m_c^2}^{s_0} ds\, e^{-s/\mathcal{M}^2}\mathrm{Im}F^{V(OPE)}(s,q^2)\,.
$$

□ Validity of local OPE

- heavy-quark limit, hierarchy of scales:

$$m_b \gg m_c \gg \bar{\Lambda} \sim \Lambda_{QCD}\,,$$
$$m_{B_c} \simeq m_b + m_c \equiv M, \quad p_{B_c} \simeq m_b v + m_c v = p + q$$

- external momenta such that $q^2 \ll M^2$, $p^2 \ll 4m_c^2$:
  the virtual c-quark in the correlation function is far off shell.
- LO diagram calculation

$$F^{(LO)}_{\mu\nu}(p,q) = i^2 \int \frac{d^4 f}{(2\pi)^4} \left[\gamma_\mu \frac{\not{f} + m_c}{f^2 - m_c^2} \Gamma_\nu\right]_{\alpha\beta} \int d^4x\, e^{i(px - fx)} \langle 0|\bar{c}_\alpha(x) b_\beta(0)|\bar{B}_c(p+q)\rangle\,,$$

- expanding the nonlocal matrix element in a series of local ones and integrating:

$$F^{(LO)}_{\mu\nu}(p,q) = -\frac{\epsilon_{\mu\nu\lambda\rho} v^\lambda p^\rho \sqrt{M}}{(p - m_c v)^2 - m_c^2} \hat{f}_{B_c} \left\{ 1 + \sum_{k=1}^{\infty} \frac{(i)^k}{k!} \bar{\Lambda}^k \left[\frac{2(p - m_c v)\cdot v}{m_c^2 - (p - m_c v)^2}\right]^k . \right\}$$

- all terms with $k \geq 1$ are suppressed by powers of $\bar{\Lambda}/m_c$

$$\frac{2(p - m_c v)\cdot v}{m_c^2 - (p - m_c v)^2} = \frac{1}{m_c} \left(\frac{1 - q^2/M^2 + p^2/M^2 - 2m_c/M}{1 - q^2/M^2 - p^2 m_b/(m_c M^2)} \right)$$

- replacing $m_c \to m_{u,d,s}$ we return to the situation with B meson DA

□ the LO spectral density

- The correlator in the local limit and at leading power:

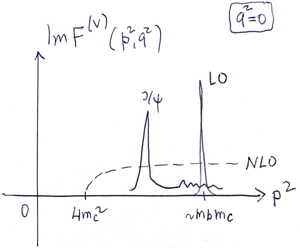
$$F^{(V)}(p^2,q^2) = \frac{f_{B_c} M}{m_b\left(m_c M^2/m_b - q^2 m_c/m_b - p^2\right)} + O(\alpha_s)\,,$$

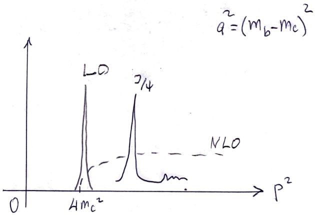
- the pole in p^2 is shifted from (naively expected) $4m_c^2$ to

  $p^2 \sim \frac{m_c}{m_b}\left((m_c+m_b)^2 + |q^2|\right)$

  - parametrically larger than $4m_c^2$
  - at $q^2_{max} = (m_b - m_c)^2$ (zero recoil) the pole at $p^2 = 4m_c^2$

- the LO spectral density is beyond the duality interval of J/ψ





□ Filling the gap with NLO spectral density

- In the sum rule $\mathrm{Im}F^{(OPE)}$ starts at NLO, $O(\alpha_S)$

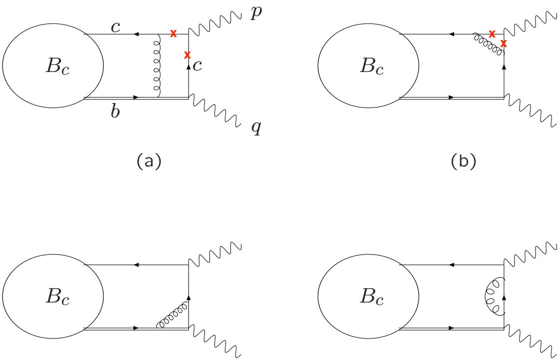
- $B_c \to J/\psi$ form factors at large recoil are dominated by hard gluon exchanges (no soft overlap mechanism)

- three-point QCD sum rules for $B_c \to J/\psi$ in LO – essentially incomplete

V.Kiselev, A.Likhoded, A.Onischenko (2000)

□ NLO diagrams

- $O(\alpha_s)$ diagrams



(a) (b) (c) (d)

- standard loop calculation is hard (too many scales)
- only two cut (x) diagrams with $\mathrm{Im}F(p^2) \sim \theta(p^2 - 4m_c^2)$.
- approximating the spectral density by NLO expression.

□ Results for the $B_c \to J/\psi$ form factors

- numerical results at $q^2 \leq 0$ and comparison with lattice QCD

Form factor	$q^2 = -20\,\text{GeV}^2$	$q^2 = -10\,\text{GeV}^2$	$q^2 = 0$	HPQCD at $q^2 = 0$ [4]
V	$0.044^{+0.016}_{-0.013}$	$0.112^{+0.043}_{-0.035}$	$0.705^{+0.364}_{-0.253}$	0.725 ± 0.055
A_1	$0.042^{+0.015}_{-0.012}$	$0.090^{+0.034}_{-0.027}$	$0.451^{+0.222}_{-0.158}$	0.457 ± 0.027
A_0	$0.028^{+0.010}_{-0.008}$	$0.071^{+0.026}_{-0.021}$	$0.443^{+0.219}_{-0.156}$	0.4770 ± 0.026
A_2	$0.033^{+0.012}_{-0.010}$	$0.081^{+0.031}_{-0.025}$	$0.466^{+0.228}_{-0.162}$	0.418 ± 0.086
A_{12}	$0.009^{+0.003}_{-0.002}$	$0.017^{+0.006}_{-0.005}$	$0.085^{+0.042}_{-0.030}$	0.091 ± 0.008

Table 3. Numerical results for the $B_c \to J/\psi$ form factors. The central values (asymmetric uncertainties) correspond to the medians of the distributions (the 68% confidence intervals).

- z-expansion to $q^2 > 0$ not available, no $R((J/\psi)$ prediction

Summary

- QCD sum rules for heavy hadrons and HQET,
  for both local and light-cone OPE.
- New sum rule for $B_c \to J/\psi$ form factors
  - the correlation function approximated with local OPE
  - f_{B_c} is the only nonperturbative parameter at leading power
  - at $q^2 \lesssim 0$ the duality region for J/ψ filled by NLO diags
  - no soft overlap for the large recoil $B_c \to J/\psi$ transition
- future perspectives: power corrections, $B_c \to \eta_c, \psi(2S), \ldots$

recollection of our MITP program in January 2020

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Light-Cone Distribution Amplitudes of Hadrons in QCD and their Applications