# 0 - and 1-jettiness resummation for processes with coloured final states at the LHC 

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## Overview of the talk

- Motivation
- Geneva Monte Carlo GENEVA
http://geneva.physics.lbl.gov
- Zero-jettiness resummation for top-quark pair production at the LHC
- One-jettiness resummation for Z+jet production at the LHC
- Conclusions \& Outlook


## Motivation

- MC event generators are essential tools for particle physics phenomenology
- They provide realistic simulations: first principles QFT calculations are combined with parton showers and hadronization modelling
- They start from a perturbative description of the hard-interaction and predict the evolution of the event down to very small (nonperturbative) scales $\mathcal{O}(1) \mathrm{GeV}$
- State-of-the-art is the inclusion of partonic NNLO corrections. Several methods are available for colour-singlet processes (UNNLOPS, MiNNLOPS, GENEVA)


## N -Jettiness and Factorization

- N -jettiness resolution variables: given an M -particle phase space point with $M \geq N$

$$
\mathcal{T}_{N}\left(\Phi_{M}\right)=\sum_{k} \min \left\{\hat{q}_{a} \cdot p_{k}, \hat{q}_{b} \cdot p_{k}, \hat{q}_{1} \cdot p_{k}, \ldots, \hat{q}_{N} \cdot p_{k}\right\}
$$

- The limit $\mathcal{T}_{N} \rightarrow 0$ describes a N -jet event where the unresolved emissions can be either soft or collinear to the final state jets or initial state beams
- Color singlet final state, relevant variable is 0-jettiness aka "beam thrust" [Stewart, Tackmann,Waalewijn `09, `10]

$$
\mathcal{T}_{0}=\sum_{k}\left|\vec{p}_{k T}\right| e^{-\left|\eta_{k}-Y\right|}
$$



- When an extra jet is present the relevant jet resolution variable is 1-jettiness

$$
\mathcal{T}_{1}=\sum_{k} \min \left\{\frac{2 q_{a} \cdot p_{k}}{Q_{a}}, \frac{2 q_{b} \cdot p_{k}}{Q_{b}}, \frac{2 q_{J} \cdot p_{k}}{Q_{J}}\right\}
$$

- Class of geometric measures $Q_{i}=\rho_{i} 2 E_{i}$ ( $\rho_{i}$ dimensionless parameter), remove the dependence on the energies $E_{i}$ and only depends on the directions $\hat{q}_{i}$. Introduce frame dependence.
- Choice of the $\rho_{i}$ determines the frame in which the 1-jettiness is evaluated. We focus on 3 choices: Laboratory frame, Underlying Born (UB) frame ( $Y_{V j}=0$ ), Color Singlet (CS) frame ( $Y_{V}=0$ ).


## Monte Carlo implementation

- GENEVA [Alioli,Bauer,Berggren,Tackmann, Walsh `15], [Alioli,Bauer,Tackmann,Guns `16], [Alioli, Broggio,Lim, Kallweit,Rottoli `19],[Alioli,Broggio,Gavardi,Lim,Nagar,Napoletano,Kallweit,Rottoli `20-`21] combines 3 theoretical tools that are important for QCD predictions into a single framework
- fully differential fixed-order calculations, up to NNLO via 0-jettiness or $q_{T}$ subtraction
- up to NNLL` resummation for 0-jettiness in SCET or $\mathrm{N}^{3} \mathrm{LL}$ for $q_{T}$ via RadISH for colour singlet processes
- shower and hadronize events (PYTHIA8)
- IR-finite definition of events based on resolution parameters $\mathcal{T}_{0}^{\text {cut }}$ and $\mathcal{T}_{1}^{\text {cut }}$

| $\Phi_{0}$ events: | $\frac{\mathrm{d} \sigma_{0}^{\mathrm{MC}}}{\mathrm{d} \Phi_{0}}\left(\mathcal{T}_{0}^{\mathrm{cut}}\right)$, | $\Phi_{0}$ | $\Phi_{1}$ |  |
| :--- | :--- | :--- | :--- | :--- |
| $\Phi_{1}$ events: | $\frac{\mathrm{d} \sigma_{1}^{\mathrm{MC}}}{\mathrm{d} \Phi_{1}}\left(\mathcal{T}_{0}>\mathcal{T}_{0}^{\mathrm{cut}} ; \mathcal{T}_{1}^{\text {cut }}\right)$, |  |  |  |
| $\Phi_{2}$ events: | $\frac{\mathrm{d} \sigma_{\geq 2}^{\mathrm{MC}}}{\mathrm{d} \Phi_{2}}\left(\mathcal{T}_{0}>\mathcal{T}_{0}^{\mathrm{cut}}, \mathcal{T}_{1}>\mathcal{T}_{1}^{\mathrm{cut}}\right)$ | $\mathcal{T}_{0}<\mathcal{T}_{0}^{\text {cut }}$ |  | $\mathcal{T}_{0}>\mathcal{T}_{0}^{\text {cut }}$ |
|  |  |  | $\mathcal{T}_{0}^{\text {cut }}$ | $\mathcal{T}_{1}<\mathcal{T}_{1}^{\text {cut }}$ |
| $\mathcal{T}_{1}^{\text {cut }}$ | $\mathcal{T}_{1}>\mathcal{T}_{1}^{\text {cut }}$ |  |  |  |

- When we take $\mathcal{T}_{N}^{\text {cut }} \rightarrow 0$, large logarithms of $\mathcal{T}_{N}^{\text {cut }}, \mathcal{T}_{N}$ appear and need to be resummed
- Including the higher-order resummation will improve the accuracy of the predictions across the whole spectrum


# Zero-jettiness resummation for top-quark pair production at the LHC 

Based on arXiv:2111.03632, S. Alioli, AB, M.A. Lim

## 0 -jettiness resummation for $t \bar{t}$ production

- Top-quark properties are very interesting, interplay with the Higgs sector
- It is desirable to have a NNLO+PS calculation. Extrapolation from fiducial to inclusive phase space is done using NLO event generators [Behring, Czakon, Mitov, Papanastasiou, Poncelet `19]
- NNLO+PS for $t \bar{t}$ production available in MINNLOPS framework [Mazzitelli, Monni, Nason, Re, Wiesemann, Zanderighi `20, `21]
- Including higher-order resummation can improve the description of observables (this is the case of the GENEVA generator)


## 0 -jettiness resummation for $t \bar{t}$ production

- To reach NNLO+PS accuracy in GENEVA
- NLO calculations for $t \bar{t}$ and $t \bar{t}+$ jet
- Resummed calculation at NNLL` in the resolution variable $\mathcal{T}_{0}$
- $q_{T}$ resummation via SCET (NNLL in [1307.2464]) or direct QCD [1408.4564], [1806.01601] NNLL' ingredients (soft functions) in [s. Catani, s. Devoto, M. Grazzini, J. Mazzitelli 2301.11786], [Angeles-Martinez, Czakon, Sapeta 1809.01459]
- 0 -jettines resummation is used for colour-singlet in GENEVA, has to be extended for $t \bar{t}$ production
- Definition of 0-jettiness has to be adapted with top-quarks in the final state, we choose to treat them like EW particles and exclude them from the sum over radiation
- We first need to develop the resummation framework


## Factorization

We derived a factorization formula (see 2111.03632 Appendix A) using SCET+HQET in the region $\mathcal{T}_{0} \rightarrow 0$ when $M_{t \bar{t}} \sim m_{t} \sim \sqrt{\hat{s}}$ are all hard scales (in case of boosted regime $M_{t \bar{t}} \gg m_{t}$ situation similar to [Fleming, Hoang,Mantry,Stewart `07][Bachu,Hoang,Mateu,Pathak,Stewart `21])

Three different scales arise

$$
\mu_{H}=M_{t \bar{t}}, \quad \mu_{B}=\sqrt{\mathscr{T}_{0} M_{t \bar{t}}}, \quad \mu_{S}=\mathscr{T}_{0}
$$

Hard functions (color matrices) known to NLO

$$
\begin{aligned}
& \frac{\mathrm{d} \sigma}{\mathrm{~d} \Phi_{0} \mathrm{~d} \tau_{B}}=M \sum_{i j=\{q \bar{q}, \bar{q} q, g g\}} \int \mathrm{d} t_{a} \mathrm{~d} t_{b}\left(B_{i}\left(t_{a}, z_{a}, \mu\right) B_{j}\left(t_{b}, z_{b}, \mu\right) \operatorname{Tr}\left[\mathbf{H}_{i j}\left(\Phi_{0}, \mu\right) \mathbf{S}_{i j}\left(M \tau_{B}-\frac{t_{a}+t_{b}}{M}, \Phi_{0}, \mu\right)\right]\right. \\
& \text { Beam functions [Stewart, } \\
& \text { Tackmann, Waalewijn, [1002.2213], }
\end{aligned}
$$ the factorization formula is turn into a product of functions

$$
\mathscr{L}\left[\frac{\mathrm{d} \sigma}{\mathrm{~d} \Phi_{0} \mathrm{~d} \tau_{B}}\right]=M \sum_{i j=\{q \bar{q}, \bar{q} q, g g\}} \tilde{B}_{i}\left(\ln \frac{M \kappa}{\mu^{2}}, z_{a}\right) \tilde{B}_{j}\left(\ln \frac{M \kappa}{\mu^{2}}, z_{b}\right) \operatorname{Tr}\left[\mathbf{H}_{i j}\left(\ln \frac{M^{2}}{\mu^{2}}, \Phi_{0}\right) \tilde{\mathbf{S}}_{i j}\left(\ln \frac{\mu^{2}}{\kappa^{2}}, \Phi_{0}\right)\right]
$$

## Hard functions

The hard functions arise from matching the full theory onto the EFT, they can be extracted from colour decomposed loop amplitudes. At NLO it was first computed in [Ahrens, Ferroglia, Neubert, Pecjak, Yang, 1003.5827]. They satisfy the RG equations

$$
\frac{\mathrm{d}}{\mathrm{~d} \ln \mu} \mathbf{H}\left(M, \beta_{t}, \theta, \mu\right)=\boldsymbol{\Gamma}_{H}\left(M, \beta_{t}, \theta, \mu\right) \mathbf{H}\left(M, \beta_{t}, \theta, \mu\right)+\mathbf{H}\left(M, \beta_{t}, \theta, \mu\right) \boldsymbol{\Gamma}_{H}^{\dagger}\left(M, \beta_{t}, \theta, \mu\right)
$$

Solution:

$$
\begin{gathered}
\mathbf{H}\left(M, \beta_{t}, \theta, \mu\right)=\mathbf{U}\left(M, \beta_{t}, \theta, \mu_{h}, \mu\right) \mathbf{H}\left(M, \beta_{t}, \theta, \mu_{h}\right) \mathbf{U}^{\dagger}\left(M, \beta_{t}, \theta, \mu_{h}, \mu\right) \\
\mathbf{U}\left(M, \beta_{t}, \theta, \mu_{h}, \mu\right)=\exp \left[2 S\left(\mu_{h}, \mu\right)-a_{\Gamma}\left(\mu_{h}, \mu\right)\left(\ln \frac{M^{2}}{\mu_{h}^{2}}-i \pi\right)\right] \mathbf{u}\left(M, \beta_{t}, \theta, \mu_{h}, \mu\right)
\end{gathered}
$$

We have split the anomalous dimension into a cusp (diagonal in colour space) and non-cusp (not diagonal) part

$$
\begin{array}{r}
\boldsymbol{\Gamma}_{H}\left(M, \beta_{t}, \theta, \mu\right)=\Gamma_{\text {cusp }}\left(\alpha_{s}\right)\left(\ln \frac{M^{2}}{\mu^{2}}-i \pi\right)+\gamma^{h}\left(M, \beta_{t}, \theta, \alpha_{s}\right) \text { [Ferroglia, Neubert, Pecjak, Yang,09] } \\
\mathbf{u}\left(M, \beta_{t}, \theta, \mu_{h}, \mu\right)=\mathcal{P} \exp \int_{\alpha_{s}\left(\mu_{h}\right)}^{\alpha_{s}(\mu)} \frac{\mathrm{d} \alpha}{\beta(\alpha)} \gamma^{h}\left(M, \beta_{t}, \theta, \alpha\right) \quad \text { We evaluate the matrix exponential } \\
\mathbf{u} \text { as a series expansion in } \alpha_{s} \text { [1003.5827], } \\
\text { [Buchalla,Buras,Lautenbacher `96] }
\end{array}
$$

## Beam functions

The beam functions are given by convolutions of perturbative kernels with the standard PDFs $f_{i}(x, \mu)$

$$
B_{i}(t, z, \mu)=\sum_{j} \int_{z}^{1} \frac{d \xi}{\xi} I_{i j}(t, z / \xi, \mu) f_{j}(\xi, \mu) \quad I_{i j} \text { kernels are known up to } \mathrm{N}^{3} \mathrm{LO},
$$

RG equation in Laplace space is given by

$$
\frac{\mathrm{d}}{\mathrm{~d} \ln \mu} \tilde{B}_{i}\left(L_{c}, z, \mu\right)=\left[-2 \Gamma_{\text {cusp }}\left(\alpha_{s}\right) L_{c}+\gamma_{i}^{B}\left(\alpha_{s}\right)\right] \tilde{B}_{i}\left(L_{c}, z, \mu\right)
$$

with solution in momentum space

$$
B(t, z, \mu)=\exp \left[-4 S\left(\mu_{B}, \mu\right)-a_{\gamma^{B}}\left(\mu_{B}, \mu\right)\right] \tilde{B}\left(\partial_{\eta_{B}}, z, \mu_{B}\right) \frac{1}{t}\left(\frac{t}{\mu_{B}^{2}}\right)^{\eta_{B}} \frac{e^{-\gamma_{E} \eta_{B}}}{\Gamma\left(\eta_{B}\right)}
$$

where $\eta_{B} \equiv 2 a_{\Gamma}\left(\mu_{B}, \mu\right)$ and the collinear log is given by $L_{c}=\ln \left(M \kappa / \mu^{2}\right)$

## Soft functions

We computed the soft functions matrices at NLO which were unknown for this observable

$$
\begin{gathered}
\mathbf{S}_{\text {bare }, i j}^{(1)}\left(k_{a}^{+}, k_{b}^{+}, \beta_{t}, \theta, \epsilon, \mu\right)=\sum_{\alpha, \beta} \boldsymbol{w}_{i j}^{\alpha \beta} \hat{\mathcal{I}}_{\alpha \beta}\left(k_{a}^{+}, k_{b}^{+}, \beta_{t}, \theta, \epsilon, \mu\right) \\
\hat{\mathcal{I}}_{\alpha \beta}\left(k_{a}^{+}, k_{b}^{+}, \beta_{t}, \theta, \epsilon, \mu\right)=-\frac{2\left(\mu^{2} e^{\gamma_{E}}\right)^{\epsilon}}{\pi^{1-\epsilon}} \int \mathrm{d}^{d} k \frac{v_{\alpha} \cdot v_{\beta}}{v_{\alpha} \cdot k v_{\beta} \cdot k} \delta\left(k^{2}\right) \Theta\left(k^{0}\right) \\
\times\left[\delta\left(k_{a}^{+}-k \cdot n_{a}\right) \Theta\left(k \cdot n_{b}-k \cdot n_{a}\right) \delta\left(k_{b}^{+}\right)+\delta\left(k_{b}^{+}-k \cdot n_{b}\right) \Theta\left(k \cdot n_{a}-k \cdot n_{b}\right) \delta\left(k_{a}^{+}\right)\right]
\end{gathered}
$$

One can average over the two hemisphere momenta, the soft function satisfies the RG equation in Laplace space

$$
\frac{\mathrm{d}}{\mathrm{~d} \ln \mu} \tilde{\mathbf{S}}_{B}\left(L, \beta_{t}, \theta, \mu\right)=\left[\Gamma_{\text {cusp }} L-\gamma^{s^{\dagger}}\right] \tilde{\mathbf{S}}_{B}\left(L, \beta_{t}, \theta, \mu\right)+\tilde{\mathbf{S}}_{B}\left(L, \beta_{t}, \theta, \mu\right)\left[\Gamma_{\text {cusp }} L-\gamma^{s}\right]
$$

Solution in momentum space, where we used the consistency relation among anomalous dimensions $\gamma^{s}=\gamma^{h}+\gamma^{B} 1$

$$
\begin{aligned}
\mathbf{S}_{B}\left(l^{+}, \beta_{t}, \theta, \mu\right)= & \exp \left[4 S\left(\mu_{s}, \mu\right)+2 a_{\gamma^{B}}\left(\mu_{s}, \mu\right)\right] \\
& \times \mathbf{u}^{\dagger}\left(\beta_{t}, \theta, \mu, \mu_{s}\right) \tilde{\mathbf{S}}_{B}\left(\partial_{\eta_{s}}, \beta_{t}, \theta, \mu_{s}\right) \mathbf{u}\left(\beta_{t}, \theta, \mu, \mu_{s}\right) \frac{1}{l^{+}}\left(\frac{l^{+}}{\mu_{s}}\right)^{2 \eta_{s}} \frac{e^{-2 \gamma_{E} \eta_{s}}}{\Gamma\left(2 \eta_{s}\right)}
\end{aligned}
$$

## Resummed result for the cross section

- We have
- hard functions at NLO
- soft functions at NLO, by knowing the two-loop soft anomalous dimensions we can solve the RG equations order by order and obtain all the NNLO logarithmic contributions, we miss $\delta\left(\mathcal{T}_{0}\right)$ terms at NNLO
- beam functions at NNLO (both for $q \bar{q}$ and gg channels)
- two-loop anomalous dimensions
- We can resum to NNLL. We are missing $\delta\left(\mathcal{T}_{0}\right)$ terms (NNLO hard functions and NNLO soft). If we include everything we know we obtain a NNLL ${ }_{a}^{\prime}$ result
- We construct an approximate (N)NLO formula which reproduces the fixed-order behaviour of the spectrum (for $\mathcal{T}_{0}>0$ )


## Singular vs Fixed-order

Fixed-order comparisons, approximate NLO and approximate NNLO vs $\mathrm{LO}_{1}$ and $\mathrm{NLO}_{1}$



## Resummed results

NNLL' ${ }_{a}$ is our best prediction, it includes NNLO beam functions, all mixed NLO x NLO terms, NNLL evolution matrices, all NNLO soft logarithmic terms. Resummation is switched off via profile scales



## Matched results to fixed-order

$$
\frac{\mathrm{d} \sigma^{\text {match }}}{\mathrm{d} \mathcal{T}_{0}}=\frac{\mathrm{d} \sigma^{\text {resum }}}{\mathrm{d} \mathcal{T}_{0}}+\frac{\mathrm{d} \sigma^{\mathrm{FO}}}{\mathrm{~d} \mathcal{T}_{0}}-\left[\frac{\mathrm{d} \sigma^{\text {resum }}}{\mathrm{d} \mathcal{T}_{0}}\right]_{\mathrm{FO}}
$$



# One-jettiness resummation for Z+jet production at the LHC 

work in progress...

## 1-jettiness

- Start from expression for 1-jettiness in the Born frame, where $\rho_{i}=1$

$$
\hat{\mathcal{T}}_{1}=\sum_{k} \min \left\{\frac{\hat{q}_{a} \cdot \hat{p}_{k}}{\rho_{a}}, \frac{\hat{q}_{b} \cdot \hat{p}_{k}}{\rho_{b}}, \frac{\hat{q}_{J} \cdot \hat{p}_{J}}{\rho_{J}}\right\}
$$

- 1-jettiness in the color singlet frame by making a different choice of the $\rho_{i}$ 's (similar way to go to the laboratory frame)

$$
\begin{aligned}
& \rho_{a}=e^{\hat{Y}_{V}}, \\
& \rho_{b}=e^{-\hat{Y}_{V}}, \\
& \rho_{J}=\frac{e^{-\hat{Y}_{V}}\left(\hat{p}_{J}\right)_{+}+e^{\hat{Y}_{V}}\left(\hat{p}_{J}\right)_{-}}{2 \hat{E}_{J}}
\end{aligned}
$$

- We also employ a Fully-Recursive (FR) version of one-jettiness which is used in the fixed order calculations. Closest particles in the one-jettiness metric are merged together.
- Factorization formula in the region $\mathcal{T}_{1} \ll M_{l l} \sim \sqrt{s} \sim M_{T, l l}$ [Stewart, Tackmann,Waalewijn '09, '10]

$$
\begin{aligned}
\frac{\mathrm{d} \sigma}{\mathrm{~d} \Phi_{1} \mathrm{~d} \mathcal{T}_{1}}= & \sum_{\kappa=\{q \bar{q}, q g q, g g g\}} H_{\kappa}\left(\Phi_{1}\right) \int \mathrm{d} t_{a} \mathrm{~d} t_{b} \mathrm{~d} s_{J} B_{\kappa_{a}\left(t_{a}\right) B_{\kappa_{b}}\left(t_{b}\right) J_{\kappa_{J}}\left(s_{J}\right)} \\
& \times S_{\kappa}\left(n_{a, b} \cdot n_{J}, \mathcal{T}_{1}-\frac{t_{a}}{Q_{a}}-\frac{t_{b}}{Q_{b}}-\frac{s_{J}}{Q_{J}}\right)
\end{aligned}
$$

## Hard, Soft, Beam and Jet functions

- Hard functions: two-loop amplitudes for $q \bar{q} \rightarrow Z g$ known from [T. Gehrmann and L. Tancredi 1112.1531]. Recently available also the axial vector couplings [T. Gehrmann,T. Peraro,L. Tancredi 2211.13596] but not-included yet. IR-finite functions taken from [T. Becher, G. Bell, C. Lorentzen, S. Marti 1309.3245]. $\gamma^{*} / Z^{*} \rightarrow l^{+} l^{-}$added, squared amplitude complete analytic result. At NNLL` accuracy included the 1loop squared $g g \rightarrow Z g$.
- Beam and quark Jet functions known up to N³ LO [M. Ebert, B. Mistlberger, G. Vita 2006.03056] and [R. Bruser, Z.L. Liu, M. Stahlhofen 1804.09722], only needed up to NNLO here Beams [J.R. Gaunt, M. Stahlhofen, F. Tackmann 1401.5478, 1405.1044] and Jets [T. Becher and M. Neubert 0603140], [T. Becher and G. Bell 1104.4108].
- Soft function boundary terms at NLO implemented as on-the-fly integrals using results in [T.T. Jouttenus, I.W. Stewart, F. Tackmann, W. Waalewijn 1302.0846], kept full dependence on $\mathcal{T}_{1}$ frame dependence.
- Frame dependent NNLO soft function boundary contribution is provided by using the SoftSERVE [G. Bell, R. Rahn, J. Talbert 1812.08690, 2004.08396] method (thanks to Bahman Dehnadi, Guido Bell, Rudi Rahn) in the form of an interpolation grid over the parameters $\left\{\cos \theta_{J}, 1 / \rho_{a}, 1 / \rho_{J}\right\}$
- Validation against NLO result in different frames, at NNLO validated in UB frame against the interpolation in MCFM [J. Campbell, K. Ellis, R. Mondini, C. Williams, 1711.09984]. In CS and Lab frames new results.


## Resummation formula

Combine the solutions to the RG equations for the hard, soft, beam and jet functions to obtain
where we defined

$$
\begin{array}{cc}
L_{H}=\ln \left(\frac{Q^{2}}{\mu_{H}^{2}}\right) & L_{B}=\ln \left(\frac{Q_{a} Q}{\mu_{B}^{2}}\right), \quad L_{B}^{\prime}=\ln \left(\frac{Q_{b} Q}{\mu_{B}^{2}}\right) \\
L_{J}=\ln \left(\frac{Q_{J} Q}{\mu_{J}^{2}}\right) & L_{S}=\ln \left(\frac{Q^{2}}{\mu_{S}^{2}}\right)
\end{array}
$$

$$
K_{\gamma_{\text {tot }}}=-2 n_{g} K_{\gamma_{C}^{g}}\left(\mu_{S}, \mu_{H}\right)+2\left(n_{g}-3\right) K_{\gamma_{C}^{q}}\left(\mu_{S}, \mu_{H}\right)
$$

$$
-\left(n_{g}-n_{g}^{\kappa_{J}}\right) K_{\gamma_{J}^{g}}\left(\mu_{J}, \mu_{B}\right)-n_{g} K_{\gamma_{J}^{g}}\left(\mu_{S}, \mu_{J}\right)
$$

$$
+\left(n_{g}-2-n_{g}^{\kappa_{J}}\right) K_{\gamma_{J}^{q}}\left(\mu_{J}, \mu_{B}\right)+\left(n_{g}-3\right) K_{\gamma_{J}^{q}}\left(\mu_{S}, \mu_{J}\right)
$$

$$
\eta_{\text {tot }}=-2\left(C_{\kappa_{a}}+C_{\kappa_{b}}\right) \eta_{\Gamma_{\text {cusp }}}\left(\mu_{B}, \mu_{J}\right)+2\left(C_{\kappa_{a}}+C_{\kappa_{b}}+C_{\kappa_{J}}\right) \eta_{\Gamma_{\text {cusp }}}\left(\mu_{S}, \mu_{J}\right)
$$

$$
\begin{aligned}
& \frac{\mathrm{d} \sigma}{\mathrm{~d} \Phi_{1} \mathrm{~d} \mathcal{T}_{1}}=\sum_{\kappa} \exp \left\{4\left(C_{\kappa_{a}}+C_{\kappa_{b}}\right) K_{\Gamma_{\text {cusp }}}\left(\mu_{B}, \mu_{H}\right)+4 C_{\kappa_{J}} K_{\Gamma_{\text {cusp }}}\left(\mu_{J}, \mu_{H}\right)\right. \\
& -2\left(C_{\kappa_{a}}+C_{\kappa_{b}}+C_{\kappa_{J}}\right) K_{\Gamma_{\text {cusp }}}\left(\mu_{S}, \mu_{H}\right)-2 C_{\kappa_{J}} L_{J} \eta_{\Gamma_{\text {cusp }}}\left(\mu_{J}, \mu_{H}\right) \\
& -2\left(C_{\kappa_{a}} L_{B}+C_{\kappa_{b}} L_{B}^{\prime}\right) \eta_{\Gamma_{\text {cusp }}}\left(\mu_{B}, \mu_{H}\right)+\left[C_{\kappa_{a}} \ln \left(\frac{Q_{a}^{2} u}{s t}\right)+C_{\kappa_{b}} \ln \left(\frac{Q_{b}^{2} t}{s u}\right)\right. \\
& \left.\left.+C_{\kappa_{j}} \ln \left(\frac{Q_{J}^{2} s}{t u}\right)+\left(C_{\kappa_{a}}+C_{\kappa_{b}}+C_{\kappa_{J}}\right) L_{S}\right] \eta_{\Gamma_{\text {cusp }}}\left(\mu_{S}, \mu_{H}\right)+K_{\gamma_{\text {tot }}}\right\} \\
& \times \tilde{B}_{\kappa_{a}}\left(\partial_{\eta_{B}}+L_{B}, x_{a}, \mu_{B}\right) \tilde{B}_{\kappa_{b}}\left(\partial_{\eta_{B}^{\prime}}+L_{B}^{\prime}, x_{b}, \mu_{B}\right) \tilde{J}_{\kappa_{J}}\left(\partial_{\eta_{J}}+L_{J}, \mu_{J}\right) \\
& \times H_{\kappa}\left(\Phi_{1}, \mu_{H}\right) \tilde{S}_{\mathcal{T}_{1}}\left(\partial_{\eta_{S}}+L_{S}, \mu_{S}\right) \frac{Q^{-\eta_{\text {tot }}}}{\mathcal{T}_{1}^{1-\eta_{\text {tot }}}} \frac{\eta_{\text {tot }}}{\Gamma\left(1+\eta_{\text {tot }}\right)}+\mathcal{O}\left(\frac{\mathcal{T}_{1}}{Q}\right)
\end{aligned}
$$

## Singular vs Nonsingular

- Different frame choices for one-jettiness definition have different sizes of power corrections (fully-recursive results below, only fixed-order is different for $\mathscr{T}_{1}>0$ )
- CS frame as good as UB frame for different cuts, Lab. frame is worse




## Singular vs Nonsingular

- Reduced definition $\tau_{1}=2 \mathcal{T}_{1} / \sqrt{M_{l^{+} l^{-}}^{2}+q_{T}^{2}}$
- When we use as born defining cut the $Z$ boson transverse momentum $q_{T}$, differences in power corrections among definitions are reduced




## Resummed results

, We use profile scales to switch off resummation at $\mu_{H}=\sqrt{M_{l^{+} l^{-}}^{2}+q_{T}^{2}}$



## Matched results

$$
\frac{\mathrm{d} \sigma^{\text {match }}}{\mathrm{d} \mathcal{T}_{1}}=\frac{\mathrm{d} \sigma^{\text {resum }}}{\mathrm{d} \mathcal{T}_{1}}+\frac{\mathrm{d} \sigma^{\mathrm{FO}}}{\mathrm{~d} \mathcal{T}_{1}}-\left[\frac{\mathrm{d} \sigma^{\text {resum }}}{\mathrm{d} \mathcal{T}_{1}}\right]_{\mathrm{FO}}
$$



- $\mathcal{O}\left(\alpha_{s}^{3}\right)$ large corrections especially for small values of $\mathscr{T}_{0}^{\text {cut }}$
- We know that nonsingular in $\mathscr{T}_{1}$ is divergent for $\mathscr{T}_{0} \rightarrow 0$
- We sum in quadrature profile scales variations and fixed-order scale variations



## Outlook

- Calculate and extract all the missing ingredients to reach NNLL' accuracy for the topquark pair production process (hard and soft functions). Implement in GENEVA event generator
- Extend top-quark pair to study associated production of a top-pair and a heavy boson $t \bar{t} V$ ( $V=H, W^{ \pm}, Z$ ) [AB,Ferroglia,Pecjak,Signer, Yang `15], [AB,Ferroglia,Pecjak,Ossola `16], [AB,Ferroglia, Pecjak,Yang `16],[AB,Ferroglia,Pecjak,Ossola,Sameshima `17],[AB,Ferroglia,Frederix, Pagani,Pecjak,Tsinikos `19]
- Extend resummation to $N^{3}$ LL for $Z+j e t$ production. Implementation in Monte Carlo event generator


## Thank you!

## Backup slides

## N -Jettiness and Resummation

- At NNLO one needs a 0-jet and a 1-jet (for Z+j also 2-jet) resolution parameters
- Emissions below $\mathcal{T}_{N}^{\text {cut }}$ are unresolved (integrated over) and the kinematic considered is the one of the event before extra emissions
- Emissions above $\mathcal{T}_{N}^{\text {cut }}$ are kept and the full kinematics is considered
-When we take $\mathcal{T}_{N}^{\text {cut }} \rightarrow 0$, large logarithms of $\mathcal{T}_{N}^{\text {cut }}, \mathcal{T}_{N}$ appear and need to be resummed
- Including the higher-order resummation will improve the accuracy of the predictions across the whole spectrum



## Resummed result for the cross section

We can combine the solutions for the hard, soft and beam functions to obtain

$$
\begin{aligned}
\frac{\mathrm{d} \sigma}{\mathrm{~d} \Phi_{0} \mathrm{~d} \tau_{B}} & =U\left(\mu_{h}, \mu_{B}, \mu_{s}, L_{h}, L_{s}\right) \\
& \times \operatorname{Tr}\left\{\mathbf{u}\left(\beta_{t}, \theta, \mu_{h}, \mu_{s}\right)\left(\mathbf{H}\left(M, \beta_{t}, \theta, \mu_{h}\right)\right) \mathbf{u}^{\dagger}\left(\beta_{t}, \theta, \mu_{h}, \mu_{s}\right)\left(\tilde{\mathbf{S}}_{B}\left(\partial_{\eta_{s}}+L_{s}, \beta_{t}, \theta, \mu_{s}\right)\right\}\right. \\
& \times \underbrace{\tilde{B}_{a}\left(\partial_{\eta_{B}}+L_{B}, z_{a}, \mu_{B}\right) \tilde{B}_{b}\left(\partial_{\eta_{B}^{\prime}}+L_{B}, z_{b}, \mu_{B}\right)} \frac{1}{\gamma_{B}^{1-\eta_{\mathrm{tot}}}} \frac{e^{-\gamma_{E} \eta_{\mathrm{tot}}}}{\Gamma\left(\eta_{\mathrm{tot}}\right)}
\end{aligned}
$$

where
$U\left(\mu_{h}, \mu_{B}, \mu_{s}, L_{h}, L_{s}\right)=$

$$
\exp \left[4 S\left(\mu_{h}, \mu_{B}\right)+4 S\left(\mu_{s}, \mu_{B}\right)+2 a_{\gamma^{B}}\left(\mu_{s}, \mu_{B}\right)-2 a_{\Gamma}\left(\mu_{h}, \mu_{B}\right) L_{h}-2 a_{\Gamma}\left(\mu_{s}, \mu_{B}\right) L_{s}\right]
$$

and $L_{s}=\ln \left(M^{2} / \mu_{s}^{2}\right), L_{h}=\ln \left(M^{2} / \mu_{h}^{2}\right), L_{B}=\ln \left(M^{2} / \mu_{B}^{2}\right)$ and $\eta_{\text {tot }}=2 \eta_{S}+\eta_{B}+\eta_{B^{\prime}}$

## Singular vs Nonsingular contributions




## Resummed results

NNLL' is our best prediction, it includes NNLO beam functions, all mixed NLO x NLO terms, NNLL evolution matrices, all NNLO soft logarithmic terms. Resummation is switched off via profile scales



$$
\begin{aligned}
\mu_{H} & =\mu_{\mathrm{NS}} \\
\mu_{S}\left(\mathcal{T}_{0}\right) & =\mu_{\mathrm{NS}} f_{\text {run }}\left(\mathcal{T}_{0} / M\right) \\
\mu_{B}\left(\mathcal{T}_{0}\right) & =\mu_{\mathrm{NS}} \sqrt{f_{\text {run }}\left(\mathcal{T}_{0} / M\right)}
\end{aligned}
$$

$$
f_{\text {run }}(y)= \begin{cases}y_{0}\left[1+\left(y / y_{0}\right)^{2} / 4\right] & y \leq 2 y_{0}, \\ y & 2 y_{0} \leq y \leq y_{1}, \\ y+\frac{\left(2-y_{2}-y_{3}\right)\left(y-y_{1}\right)^{2}}{2\left(y_{2}-y_{1}\right)\left(y_{3}-y_{1}\right)} & y_{1} \leq y \leq y_{2}, \\ 1-\frac{\left(2-y_{1}-y_{2}\right)\left(y-y_{3}\right)^{2}}{2\left(y_{3}-y_{1}\right)\left(y_{3}-y_{2}\right)} & y_{2} \leq y \leq y_{3}, \\ 1 & y_{3} \leq y .\end{cases}
$$

$$
y_{0}=1.0 \mathrm{GeV} / M, \quad\left\{y_{1}, y_{2}, y_{3}\right\}=\{0.1,0.175,0.25\}
$$

## Resummed results

The evolution matrix $\mathbf{u}$ is evaluated in $\alpha_{s}$ expansion, we can choose to expand or not expand $U$, the difference is quite small



## Singular vs Nonsingular

- Result for exact one-jettiness in CS frame, very similar results to FR



