## 0- and 1-jettiness resummation for processes with coloured final states at the LHC

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#### **Overview of the talk**

Motivation

Geneva Monte Carlo



http://geneva.physics.lbl.gov

Zero-jettiness resummation for top-quark pair production at the LHC

One-jettiness resummation for Z+jet production at the LHC

Conclusions & Outlook



## Motivation

- MC event generators are essential tools for particle physics phenomenology
- They provide realistic simulations: first principles QFT calculations are combined with parton showers and hadronization modelling
- They start from a perturbative description of the hard-interaction and predict the evolution of the event down to very small (nonperturbative) scales  $\mathcal{O}(1)$  GeV
- State-of-the-art is the inclusion of partonic NNLO corrections. Several methods are available for colour-singlet processes (UNNLOPS, MiNNLOPS, GENEVA)



## **N-Jettiness and Factorization**

N-jettiness resolution variables: given an M-particle phase space point with  $M \ge N$ 



When an extra jet is present the relevant jet resolution variable is 1-jettiness

$$\mathcal{T}_1 = \sum_k \min\{\frac{2q_a \cdot p_k}{Q_a}, \frac{2q_b \cdot p_k}{Q_b}, \frac{2q_J \cdot p_k}{Q_J}\}$$

- Class of geometric measures  $Q_i = \rho_i 2 E_i$  ( $\rho_i$  dimensionless parameter), remove the dependence on the energies  $E_i$  and only depends on the directions  $\hat{q}_i$ . Introduce frame dependence.
- Choice of the  $\rho_i$  determines the frame in which the 1-jettiness is evaluated. We focus on 3 choices: Laboratory frame, Underlying Born (UB) frame ( $Y_{Vi} = 0$ ), Color Singlet (CS) frame ( $Y_V = 0$ ).

## **Monte Carlo implementation**

- GENEVA [Alioli,Bauer,Berggren,Tackmann, Walsh `15], [Alioli,Bauer,Tackmann,Guns `16], [Alioli,Broggio,Lim, Kallweit,Rottoli `19],[Alioli,Broggio,Gavardi,Lim,Nagar,Napoletano,Kallweit,Rottoli `20-`21] combines 3 theoretical tools that are important for QCD predictions into a single framework
  - fully differential fixed-order calculations, up to NNLO via 0-jettiness or  $q_T$  subtraction
  - up to NNLL` resummation for 0-jettiness in SCET or N<sup>3</sup>LL for  $q_T$  via RadISH for colour singlet processes
  - shower and hadronize events (PYTHIA8)
- IR-finite definition of events based on resolution parameters  $\mathcal{T}_0^{ ext{cut}}$  and  $\mathcal{T}_1^{ ext{cut}}$



- When we take  $\mathcal{T}_N^{ ext{cut}} o 0$ , large logarithms of  $\mathcal{T}_N^{ ext{cut}}$ ,  $\mathcal{T}_N$  appear and need to be resummed
- Including the higher-order resummation will improve the accuracy of the predictions across the whole spectrum



# Zero-jettiness resummation for top-quark pair production at the LHC

Based on arXiv:2111.03632, S. Alioli, AB, M.A. Lim



## **0**-jettiness resummation for $t\overline{t}$ production

- ▶ Top-quark properties are very interesting, interplay with the Higgs sector
- It is desirable to have a NNLO+PS calculation. Extrapolation from fiducial to inclusive phase space is done using NLO event generators [Behring, Czakon, Mitov, Papanastasiou, Poncelet `19]
- NNLO+PS for tt production available in MINNLOPS framework [Mazzitelli, Monni, Nason, Re, Wiesemann, Zanderighi `20, `21]
- Including higher-order resummation can improve the description of observables (this is the case of the GENEVA generator)



## **0**-jettiness resummation for $t\overline{t}$ production

- To reach NNLO+PS accuracy in GENEVA
  - NLO calculations for  $t\overline{t}$  and  $t\overline{t}$ +jet
  - Resummed calculation at NNLL` in the resolution variable  $\mathcal{T}_0$
  - *q<sub>T</sub>* resummation via SCET (NNLL in [1307.2464]) or direct QCD [1408.4564], [1806.01601]
     NNLL' ingredients (soft functions) in [S. Catani, S. Devoto, M. Grazzini, J. Mazzitelli 2301.11786], [Angeles-Martinez, Czakon, Sapeta 1809.01459]

  - Definition of 0-jettiness has to be adapted with *top-quarks* in the final state, we choose to *treat them like EW particles* and exclude them from the sum over radiation
  - We first need to develop the resummation framework



#### Factorization

We derived a factorization formula (see 2111.03632 Appendix A) using SCET+HQET in the region  $\mathcal{T}_0 \to 0$  when  $M_{t\bar{t}} \sim m_t \sim \sqrt{\hat{s}}$  are all hard scales (in case of boosted regime  $M_{t\bar{t}} \gg m_t$  situation similar to [Fleming, Hoang, Mantry, Stewart `07] [Bachu, Hoang, Mateu, Pathak, Stewart `21])

> $\mu_H = M_{t\bar{t}}, \quad \mu_B = \sqrt{\mathcal{T}_0 M_{t\bar{t}}}, \quad \mu_S = \mathcal{T}_0$ Three different scales arise



known up to N<sup>3</sup>LO

known to NLO

It is convenient to transform the soft and beam functions in Laplace space to solve the RG equations, the factorization formula is turn into a product of functions

$$\mathscr{L}\left[\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_{0}\mathrm{d}\tau_{B}}\right] = M \sum_{ij=\{q\bar{q},\bar{q}q,gg\}} \tilde{B}_{i}\left(\ln\frac{M\kappa}{\mu^{2}},z_{a}\right) \tilde{B}_{j}\left(\ln\frac{M\kappa}{\mu^{2}},z_{b}\right) \mathrm{Tr}\left[\mathbf{H}_{ij}\left(\ln\frac{M^{2}}{\mu^{2}},\Phi_{0}\right) \tilde{\mathbf{S}}_{ij}\left(\ln\frac{\mu^{2}}{\kappa^{2}},\Phi_{0}\right)\right]$$

versitat

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## Hard functions

The hard functions arise from matching the full theory onto the EFT, they can be extracted from colour decomposed loop amplitudes. At NLO it was first computed in [Ahrens, Ferroglia, Neubert, Pecjak, Yang, 1003.5827]. They satisfy the RG equations

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\mathbf{H}(M,\beta_t,\theta,\mu) = \mathbf{\Gamma}_H(M,\beta_t,\theta,\mu)\mathbf{H}(M,\beta_t,\theta,\mu) + \mathbf{H}(M,\beta_t,\theta,\mu)\mathbf{\Gamma}_H^{\dagger}(M,\beta_t,\theta,\mu)$$

Solution:

$$\mathbf{H}(M,\beta_t,\theta,\mu) = \mathbf{U}(M,\beta_t,\theta,\mu_h,\mu)\mathbf{H}(M,\beta_t,\theta,\mu_h)\mathbf{U}^{\dagger}(M,\beta_t,\theta,\mu_h,\mu)$$

$$\mathbf{U}(M,\beta_t,\theta,\mu_h,\mu) = \exp\left[2S(\mu_h,\mu) - a_{\Gamma}(\mu_h,\mu)\left(\ln\frac{M^2}{\mu_h^2} - i\pi\right)\right]\mathbf{u}(M,\beta_t,\theta,\mu_h,\mu)$$

We have split the anomalous dimension into a cusp (diagonal in colour space) and non-cusp (not diagonal) part

$$\Gamma_H(M,\beta_t,\theta,\mu) = \Gamma_{\rm cusp}(\alpha_s) \left( \ln \frac{M^2}{\mu^2} - i\pi \right) + \gamma^h(M,\beta_t,\theta,\alpha_s) \quad \text{[Ferroglia, Neubert, Pecjak, Yang,`09]}$$

$$\mathbf{u}(M,\beta_t,\theta,\mu_h,\mu) = \mathcal{P}\exp\int_{\alpha_s(\mu_h)}^{\alpha_s(\mu)} \frac{\mathrm{d}\alpha}{\beta(\alpha)} \boldsymbol{\gamma}^h(M,\beta_t,\theta,\alpha)$$

We evaluate the matrix exponential **u** as a series expansion in  $\alpha_s$  [1003.5827], [Buchalla,Buras,Lautenbacher `96]



#### **Beam functions**

The beam functions are given by convolutions of perturbative kernels with the standard PDFs  $f_i(x, \mu)$ 

$$B_i(t,z,\mu) = \sum_j \int_z^1 \frac{d\xi}{\xi} I_{ij}(t,z/\xi,\mu) f_j(\xi,\mu)$$

 $I_{ij}$  kernels are known up to N<sup>3</sup>LO, process independent

RG equation in Laplace space is given by

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\tilde{B}_i(L_c,z,\mu) = \left[-2\Gamma_{\mathrm{cusp}}(\alpha_s)L_c + \gamma_i^B(\alpha_s)\right]\tilde{B}_i(L_c,z,\mu)$$

with solution in momentum space

$$B(t,z,\mu) = \exp\left[-4S(\mu_B,\mu) - a_{\gamma^B}(\mu_B,\mu)\right] \tilde{B}(\partial_{\eta_B},z,\mu_B) \frac{1}{t} \left(\frac{t}{\mu_B^2}\right)^{\eta_B} \frac{e^{-\gamma_E \eta_B}}{\Gamma(\eta_B)}$$

where  $\eta_B \equiv 2a_{\Gamma}(\mu_B, \mu)$  and the collinear log is given by  $L_c = \ln(M\kappa/\mu^2)$ 



## Soft functions

We computed the soft functions matrices at NLO which were unknown for this observable

$$\begin{aligned} \mathbf{S}_{\text{bare},\,ij}^{(1)}(k_a^+,k_b^+,\beta_t,\theta,\epsilon,\mu) &= \sum_{\alpha,\beta} \boldsymbol{w}_{ij}^{\alpha\beta} \hat{\mathcal{I}}_{\alpha\beta}(k_a^+,k_b^+,\beta_t,\theta,\epsilon,\mu) \\ \hat{\mathcal{I}}_{\alpha\beta}(k_a^+,k_b^+,\beta_t,\theta,\epsilon,\mu) &= -\frac{2(\mu^2 e^{\gamma_E})^\epsilon}{\pi^{1-\epsilon}} \int \mathrm{d}^d k \frac{v_\alpha \cdot v_\beta}{v_\alpha \cdot k \, v_\beta \cdot k} \,\delta(k^2) \Theta(k^0) \\ &\times \left[ \delta(k_a^+-k \cdot n_a) \Theta(k \cdot n_b - k \cdot n_a) \,\delta(k_b^+) + \delta(k_b^+-k \cdot n_b) \Theta(k \cdot n_a - k \cdot n_b) \,\delta(k_a^+) \right] \end{aligned}$$

One can average over the two hemisphere momenta, the soft function satisfies the RG equation in Laplace space

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\tilde{\mathbf{S}}_B(L,\beta_t,\theta,\mu) = \left[\Gamma_{\mathrm{cusp}}L - \boldsymbol{\gamma}^{s^{\dagger}}\right]\tilde{\mathbf{S}}_B(L,\beta_t,\theta,\mu) + \tilde{\mathbf{S}}_B(L,\beta_t,\theta,\mu) \left[\Gamma_{\mathrm{cusp}}L - \boldsymbol{\gamma}^s\right]$$

Solution in momentum space, where we used the consistency relation among anomalous dimensions  $\gamma^s = \gamma^h + \gamma^B \, {f 1}$ 

$$\mathbf{S}_{B}(l^{+},\beta_{t},\theta,\mu) = \exp\left[4S(\mu_{s},\mu) + 2a_{\gamma^{B}}(\mu_{s},\mu)\right] \\ \times \mathbf{u}^{\dagger}(\beta_{t},\theta,\mu,\mu_{s}) \,\tilde{\mathbf{S}}_{B}(\partial_{\eta_{s}},\beta_{t},\theta,\mu_{s}) \,\mathbf{u}(\beta_{t},\theta,\mu,\mu_{s}) \,\frac{1}{l^{+}} \left(\frac{l^{+}}{\mu_{s}}\right)^{2\eta_{s}} \frac{e^{-2\gamma_{E}\eta_{s}}}{\Gamma(2\eta_{s})} \\ \xrightarrow{\text{universität}} \eta_{s} \equiv -2a_{\Gamma}(\mu_{s},\mu) \quad 12$$

#### • We have

- hard functions at NLO
- soft functions at NLO, by knowing the two-loop soft anomalous dimensions we can solve the RG equations order by order and obtain all the NNLO logarithmic contributions, we miss  $\delta(T_0)$  terms at NNLO
- beam functions at NNLO (both for  $q\bar{q}$  and gg channels)
- two-loop anomalous dimensions
- We can resum to NNLL. We are missing  $\delta(\mathcal{T}_0)$  terms (NNLO hard functions and NNLO soft). If we include everything we know we obtain a NNLL'<sub>a</sub> result
- We construct an approximate (N)NLO formula which reproduces the fixed-order behaviour of the spectrum (for  $T_0 > 0$ )



## **Singular vs Fixed-order**

Fixed-order comparisons, approximate NLO and approximate NNLO vs LO<sub>1</sub> and NLO<sub>1</sub>



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#### **Resummed results**

 $NNLL'_{a}$  is our best prediction, it includes NNLO beam functions, all mixed NLO x NLO terms, NNLL evolution matrices, all NNLO soft logarithmic terms. Resummation is switched off via profile scales





#### Matched results to fixed-order





## One-jettiness resummation for Z+jet production at the LHC

work in progress...



## 1-jettiness

Start from expression for 1-jettiness in the Born frame, where  $\rho_i = 1$ 

$$\hat{\mathcal{T}}_1 = \sum_k \min\{\frac{\hat{q}_a \cdot \hat{p}_k}{\rho_a}, \frac{\hat{q}_b \cdot \hat{p}_k}{\rho_b}, \frac{\hat{q}_J \cdot \hat{p}_J}{\rho_J}\}$$

• 1-jettiness in the color singlet frame by making a different choice of the  $\rho_i$ 's (similar way to go to the laboratory frame)

$$\rho_{a} = e^{\hat{Y}_{V}},$$

$$\rho_{b} = e^{-\hat{Y}_{V}},$$

$$\rho_{J} = \frac{e^{-\hat{Y}_{V}}(\hat{p}_{J})_{+} + e^{\hat{Y}_{V}}(\hat{p}_{J})_{-}}{2\hat{E}_{J}}$$

- ▶ We also employ a Fully-Recursive (FR) version of one-jettiness which is used in the fixed order calculations. Closest particles in the one-jettiness metric are merged together.
- Factorization formula in the region  $T_1 \ll M_{ll} \sim \sqrt{s} \sim M_{T,ll}$  [Stewart, Tackmann, Waalewijn `09, `10]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_{1}\mathrm{d}\mathcal{T}_{1}} = \sum_{\kappa = \{q\bar{q}g,qgq,ggg\}} H_{\kappa}(\Phi_{1}) \int \mathrm{d}t_{a} \mathrm{d}t_{b} \mathrm{d}s_{J} B_{\kappa_{a}}(t_{a}) B_{\kappa_{b}}(t_{b}) J_{\kappa_{J}}(s_{J})$$

$$\times S_{\kappa} \left( n_{a,b} \cdot n_{J}, \mathcal{T}_{1} - \frac{t_{a}}{Q_{a}} - \frac{t_{b}}{Q_{b}} - \frac{s_{J}}{Q_{J}} \right)$$
Dependence on the frame



## Hard, Soft, Beam and Jet functions

- ► Hard functions: two-loop amplitudes for  $q\bar{q} \rightarrow Zg$  known from [T. Gehrmann and L. Tancredi 1112.1531]. Recently available also the axial vector couplings [T. Gehrmann,T. Peraro,L. Tancredi 2211.13596] but not-included yet. IR-finite functions taken from [T. Becher, G. Bell, C. Lorentzen, S. Marti 1309.3245].  $\gamma^*/Z^* \rightarrow l^+l^-$  added, squared amplitude complete analytic result. At NNLL` accuracy included the 1loop squared  $gg \rightarrow Zg$ .
- Beam and quark Jet functions known up to N<sup>3</sup>LO [M. Ebert, B. Mistlberger, G. Vita 2006.03056] and [R. Bruser, Z.L. Liu, M. Stahlhofen 1804.09722], only needed up to NNLO here Beams [J.R. Gaunt, M. Stahlhofen, F. Tackmann 1401.5478, 1405.1044] and Jets [T. Becher and M. Neubert 0603140], [T. Becher and G. Bell 1104.4108].
- Soft function boundary terms at NLO implemented as on-the-fly integrals using results in [T.T. Jouttenus, I.W. Stewart, F. Tackmann, W. Waalewijn 1302.0846], kept full dependence on  $\mathcal{T}_1$  frame dependence.
- Frame dependent NNLO soft function boundary contribution is provided by using the SoftSERVE [G. Bell, R. Rahn, J. Talbert 1812.08690, 2004.08396] method (thanks to Bahman Dehnadi, Guido Bell, Rudi Rahn) in the form of an interpolation grid over the parameters { $\cos \theta_J$ ,  $1/\rho_a$ ,  $1/\rho_J$ }
- Validation against NLO result in different frames, at NNLO validated in UB frame against the interpolation in MCFM [J. Campbell, K. Ellis, R. Mondini, C. Williams, 1711.09984]. In CS and Lab frames new results.



#### **Resummation formula**

Combine the solutions to the RG equations for the hard, soft, beam and jet functions to obtain

$$\begin{aligned} \frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_{1}\mathrm{d}\mathcal{T}_{1}} &= \sum_{\kappa} \exp\left\{4(C_{\kappa_{a}}+C_{\kappa_{b}})K_{\Gamma_{\mathrm{cusp}}}(\mu_{B},\mu_{H}) + 4C_{\kappa_{J}}K_{\Gamma_{\mathrm{cusp}}}(\mu_{J},\mu_{H}) \\ &\quad -2(C_{\kappa_{a}}+C_{\kappa_{b}}+C_{\kappa_{J}})K_{\Gamma_{\mathrm{cusp}}}(\mu_{S},\mu_{H}) - 2C_{\kappa_{J}}L_{J} \eta_{\Gamma_{\mathrm{cusp}}}(\mu_{J},\mu_{H}) \\ &\quad -2(C_{\kappa_{a}}L_{B}+C_{\kappa_{b}}L'_{B})\eta_{\Gamma_{\mathrm{cusp}}}(\mu_{B},\mu_{H}) + \left[C_{\kappa_{a}}\ln\left(\frac{Q_{a}^{2}u}{st}\right) + C_{\kappa_{b}}\ln\left(\frac{Q_{b}^{2}t}{su}\right) \right. \\ &\quad + C_{\kappa_{j}}\ln\left(\frac{Q_{J}^{2}s}{tu}\right) + (C_{\kappa_{a}}+C_{\kappa_{b}}+C_{\kappa_{J}})L_{S}\right]\eta_{\Gamma_{\mathrm{cusp}}}(\mu_{S},\mu_{H}) + K_{\gamma_{\mathrm{tot}}}\right\} \\ &\quad \times \tilde{B}_{\kappa_{a}}(\partial_{\eta_{B}}+L_{B},x_{a},\mu_{B})\tilde{B}_{\kappa_{b}}(\partial_{\eta'_{B}}+L'_{B},x_{b},\mu_{B})\tilde{J}_{\kappa_{J}}(\partial_{\eta_{J}}+L_{J},\mu_{J}) \\ &\quad \times H_{\kappa}(\Phi_{1},\mu_{H})\tilde{S}_{\mathcal{T}_{1}}^{\kappa}\left(\partial_{\eta_{S}}+L_{S},\mu_{S}\right)\frac{Q^{-\eta_{\mathrm{tot}}}}{\mathcal{T}_{1}^{1-\eta_{\mathrm{tot}}}}\frac{\eta_{\mathrm{tot}}e^{-\gamma_{E}\eta_{\mathrm{tot}}}}{\Gamma(1+\eta_{\mathrm{tot}})} + \mathcal{O}\left(\frac{\mathcal{T}_{1}}{Q}\right) \end{aligned}$$

where we defined

$$L_{H} = \ln\left(\frac{Q^{2}}{\mu_{H}^{2}}\right) \qquad L_{B} = \ln\left(\frac{Q_{a}Q}{\mu_{B}^{2}}\right), \qquad L'_{B} = \ln\left(\frac{Q_{b}Q}{\mu_{B}^{2}}\right) \qquad K_{\gamma_{\text{tot}}} = -2n_{g}K_{\gamma_{C}^{g}}(\mu_{S},\mu_{H}) + 2(n_{g}-3)K_{\gamma_{C}^{g}}(\mu_{S},\mu_{H}) \\ -(n_{g}-n_{g}^{\kappa_{J}})K_{\gamma_{J}^{g}}(\mu_{J},\mu_{B}) - n_{g}K_{\gamma_{J}^{g}}(\mu_{S},\mu_{J}) \\ +(n_{g}-2-n_{g}^{\kappa_{J}})K_{\gamma_{J}^{g}}(\mu_{J},\mu_{B}) + (n_{g}-3)K_{\gamma_{J}^{g}}(\mu_{S},\mu_{J})$$

 $\eta_{\text{tot}} = -2(C_{\kappa_a} + C_{\kappa_b})\eta_{\Gamma_{\text{cusp}}}(\mu_B, \mu_J) + 2(C_{\kappa_a} + C_{\kappa_b} + C_{\kappa_J})\eta_{\Gamma_{\text{cusp}}}(\mu_S, \mu_J)$ 



## Singular vs Nonsingular

- Different frame choices for one-jettiness definition have different sizes of power corrections (fully-recursive results below, only fixed-order is different for  $\mathcal{T}_1 > 0$ )
- ▶ CS frame as good as UB frame for different cuts, Lab. frame is worse





 $10^{2}$ 

## Singular vs Nonsingular

- Reduced definition  $\tau_1 = 2 T_1 / \sqrt{M_{l^+l^-}^2 + q_T^2}$
- When we use as born defining cut the Z boson transverse momentum  $q_T$ , differences in power corrections among definitions are reduced





#### **Resummed results**

• We use profile scales to switch off resummation at  $\mu_H = \sqrt{M_{l^+l^-}^2 + q_T^2}$ 



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## **Matched results**



- $\mathcal{O}(\alpha_s^3)$  large corrections especially for small values of  $\mathcal{T}_0^{\text{cut}}$
- We know that nonsingular in  $\mathcal{T}_1$  is divergent for  $\mathcal{T}_0 \to 0$
- We sum in quadrature profile scales variations and fixed-order scale variations universität wien

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 $10^{1}$  $\mathcal{T}_1$  [GeV]

 $pp \rightarrow \ell^+ \ell^- + j + X$ 

 $50 < M_{\ell^+\ell^-}/{
m GeV} < 150$  $\sqrt{S} = 13 \text{ TeV}; \ \mathcal{T}_0 > 1 \text{ GeV}$ 

 $10^{0}$ 1.0

- 0.5 0.0 o - utio - 0.5

 $^{-1.0}$ 

 $10^{2}$ 

 $10^{2}$ 

## Outlook

- Calculate and extract all the missing ingredients to reach NNLL' accuracy for the topquark pair production process (hard and soft functions). Implement in GENEVA event generator
- Extend top-quark pair to study associated production of a top-pair and a heavy boson tt V (V = H, W<sup>±</sup>, Z) [AB,Ferroglia,Pecjak,Signer, Yang `15], [AB,Ferroglia,Pecjak,Ossola `16], [AB,Ferroglia,Pecjak,Yang `16],[AB,Ferroglia,Pecjak,Ossola,Sameshima `17],[AB,Ferroglia,Frederix, Pagani,Pecjak,Tsinikos `19]
- Extend resummation to N<sup>3</sup>LL for Z+jet production. Implementation in Monte Carlo event generator

## Thank you!



## **Backup slides**



## **N-Jettiness and Resummation**

- At NNLO one needs a 0-jet and a 1-jet (for Z+j also 2-jet) resolution parameters
- Emissions below  $\mathcal{T}_N^{\text{cut}}$  are unresolved (integrated over) and the kinematic considered is the one of the event before extra emissions
- Emissions above  $\mathcal{T}_N^{\text{cut}}$  are kept and the full kinematics is considered
- When we take  $\mathcal{T}_N^{\text{cut}} \to 0$ , large logarithms of  $\mathcal{T}_N^{\text{cut}}$ ,  $\mathcal{T}_N$  appear and need to be resummed
- Including the higher-order resummation will improve the accuracy of the predictions across the whole spectrum





#### **Resummed result for the cross section**

We can combine the solutions for the hard, soft and beam functions to obtain

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_{0}\mathrm{d}\tau_{B}} = U(\mu_{h}, \mu_{B}, \mu_{s}, L_{h}, L_{s}) \times \mathrm{Tr}\left\{\mathbf{u}(\beta_{t}, \theta, \mu_{h}, \mu_{s}) \mathbf{H}(M, \beta_{t}, \theta, \mu_{h}) \mathbf{u}^{\dagger}(\beta_{t}, \theta, \mu_{h}, \mu_{s}) \mathbf{\tilde{S}}_{B}(\partial_{\eta_{s}} + L_{s}, \beta_{t}, \theta, \mu_{s}) \right\} \times \left[\tilde{B}_{a}(\partial_{\eta_{B}} + L_{B}, z_{a}, \mu_{B}) \tilde{B}_{b}(\partial_{\eta_{B}'} + L_{B}, z_{b}, \mu_{B}) \frac{1}{\tau_{B}^{1-\eta_{\mathrm{tot}}}} \frac{e^{-\gamma_{E}\eta_{\mathrm{tot}}}}{\Gamma(\eta_{\mathrm{tot}})} \right]$$

where

$$U(\mu_h, \mu_B, \mu_s, L_h, L_s) = \exp\left[4S(\mu_h, \mu_B) + 4S(\mu_s, \mu_B) + 2a_{\gamma^B}(\mu_s, \mu_B) - 2a_{\Gamma}(\mu_h, \mu_B)L_h - 2a_{\Gamma}(\mu_s, \mu_B)L_s\right]$$

and 
$$L_s = \ln(M^2/\mu_s^2)$$
,  $L_h = \ln(M^2/\mu_h^2)$ ,  $L_B = \ln(M^2/\mu_B^2)$  and  $\eta_{\text{tot}} = 2\eta_S + \eta_B + \eta_{B'}$ 



#### **Singular vs Nonsingular contributions**





#### **Resummed results**

NNLL' is our best prediction, it includes NNLO beam functions, all mixed NLO x NLO terms, NNLL evolution matrices, all NNLO soft logarithmic terms. Resummation is switched off via profile scales





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#### **Resummed results**

The evolution matrix **u** is evaluated in  $\alpha_s$  expansion, we can choose to expand or not expand U, the difference is quite small



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#### **Singular vs Nonsingular**

▶ Result for *exact* one-jettiness in CS frame, very similar results to FR



