SCET at Next-to-leading Power:

Higgs form factor induced by light quarks

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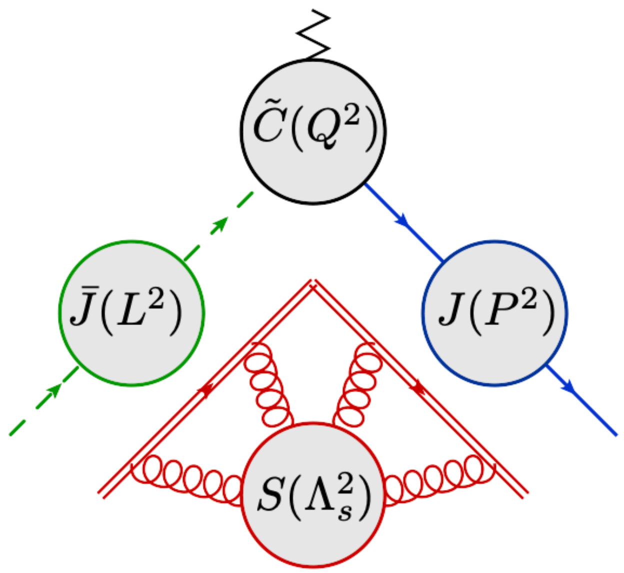
Based on works '21, '22 and in progress with Zelong Liu, Bianka Mecaj, Matthias Neubert, and Marvin Schnubel;

And in progress with Martin Beneke and Yao Ji

Leading-power (LP) Soft-Collinear Effective Theory



- One collinear field in each collinear sector $\longrightarrow$ jet functions
- Soft and collinear interaction is decoupled at Lagrangian level. Only soft Wilson lines capture soft radiations.

$$\hat{\sigma} \sim H \cdot \left[\prod_i J_j \right] \otimes S\big(\{n_i\}\big)$$

Fig from 1803.04310 by Thomas

Next-leading-power SCET

- Endpoint divergences
  - Unresolved particles in a given light-cone sector $\longrightarrow$ integrate over momentum fractions
  - NLP interactions (after decoupling) between soft and collinear modes $\longrightarrow$ integrate over interacting positions
- Renormalization: *not easy*
  - Large operator basis
  - Mixing
  - Interplay with endpoint divergences
- Resummation: *technical*

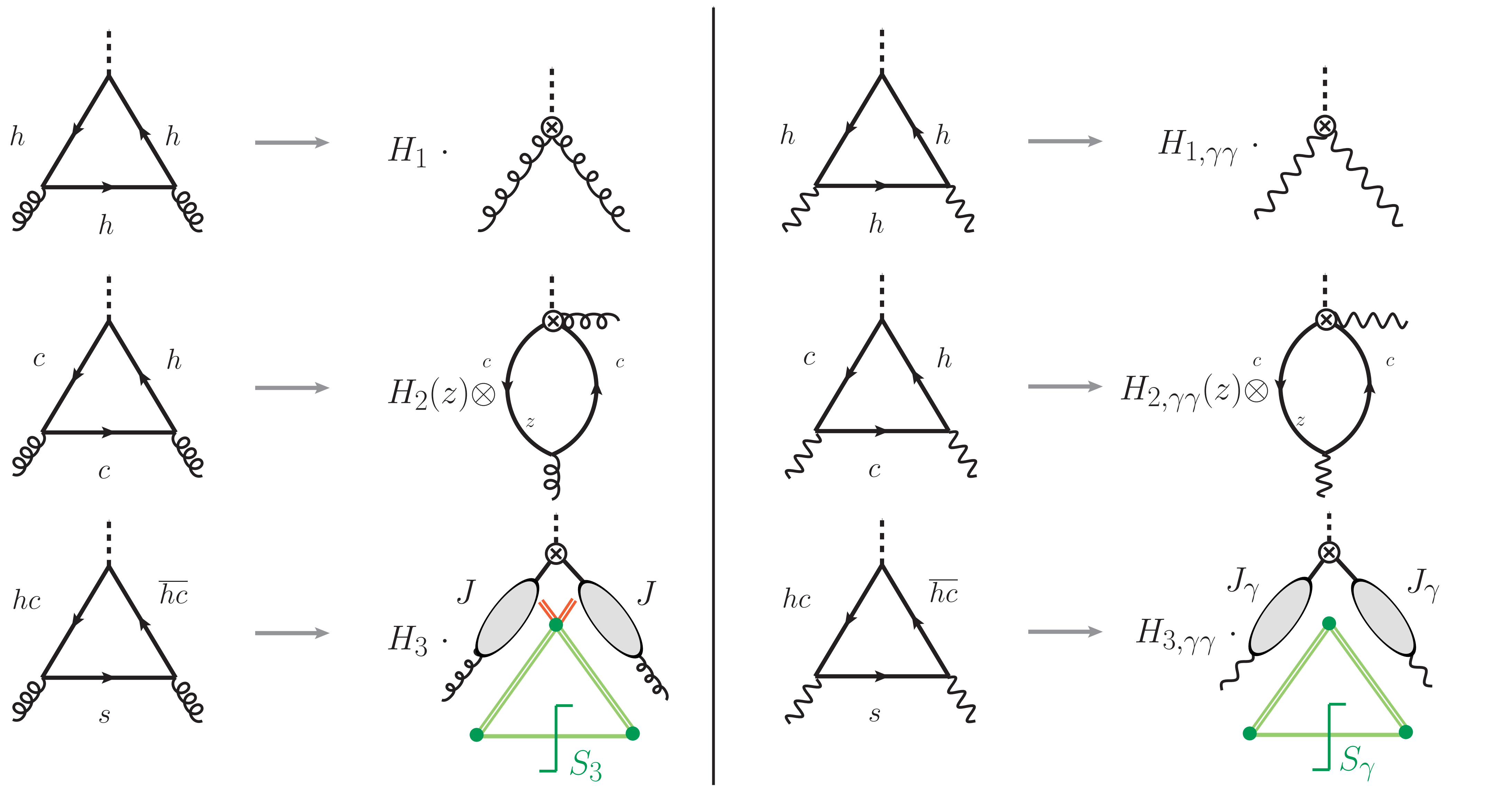
Next-leading-power: active frontier recently

- LHC physics
  - Threshold Drell-Yan: *Beneke et al, '18-'21*
  - **Higgs form factor:** *Liu, Neubert '19; Liu, Mecaj, Marvin, Neubert, XW '20 - '22*
  - Event shapes: *Ian et al, '18 - '19; Beneke et al, '22*
  - μe backscattering: *Bell, et al '22*
- Favour physics
  - inclusive $\bar{B} \to X_s \gamma$: *Robert, Tobias, '23*
  - QED corrections in leptonic B decays: *Feldmann et al, 22; Cornella et al '23*
- Gravity: *Patrick et al, '21 - '22*
- ....

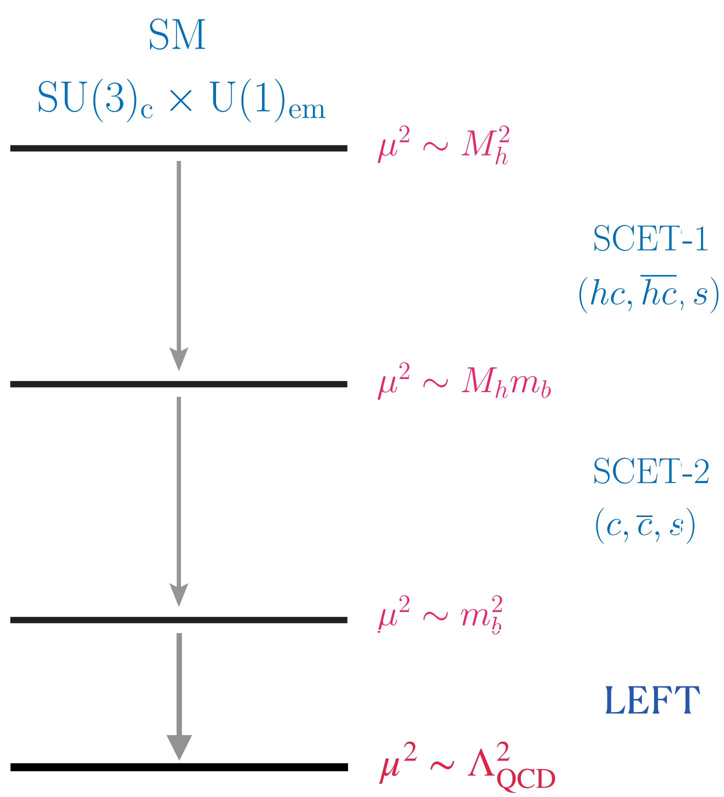
Next-leading-power: from Higgs FF's perspective

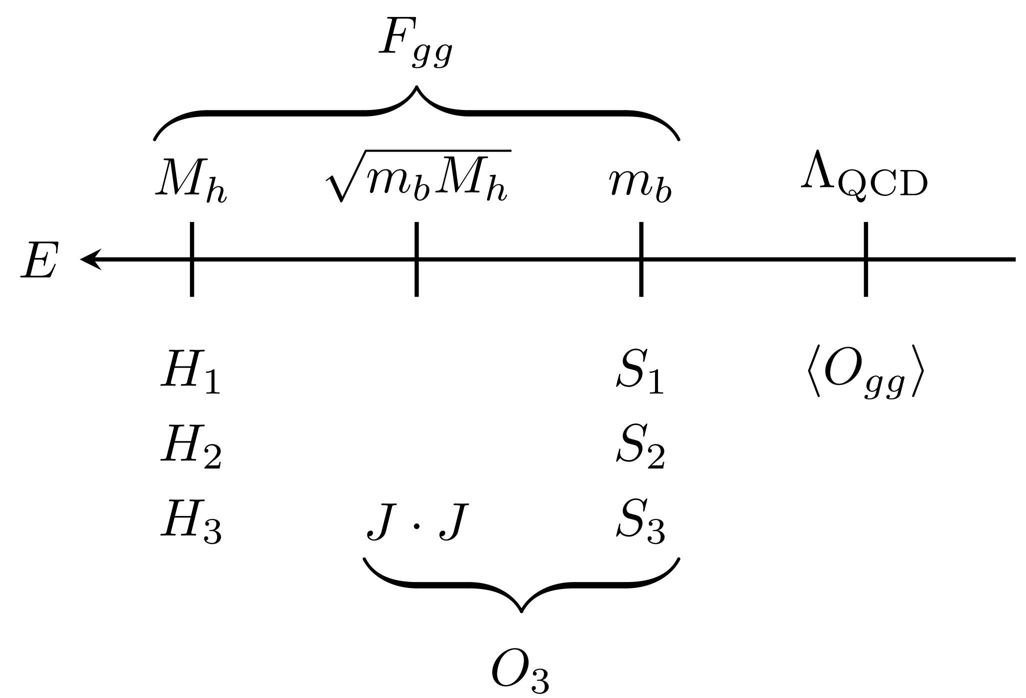
- Factorization
- Renormalization
- Resummation

Factorization



Factorization





$$F_{gg}^{(0)} = \overbrace{\left(H_1^{(0)} + \Delta H_1^{(0)}\right) S_1}^{T_1^{(0)}} + \overbrace{\int_0^1 \frac{\mathrm{d}z}{z} \left(\bar{H}_2^{(0)}(z) S_2^{(0)}(z) - [\![\bar{H}_2^{(0)}(z)]\!][\![S_2^{(0)}(z)]\!]\right)}^{T_2^{(0)}}$$

$$+ \underbrace{\lim_{\sigma \to -1} H_3^{(0)} \int_0^{M_h} \frac{\mathrm{d}\ell_-}{\ell_-} \int_0^{\sigma M_h} \frac{\mathrm{d}\ell_+}{\ell_+} J^{(0)}\left(-M_h \ell_-\right) J^{(0)}\left(M_h \ell_+\right) S_3^{(0)}\left(\ell_- \ell_+\right) \Big|_{LP}}_{T_3^{(0)}}$$

$$\langle pp\,|O_i|\,h\rangle = S_i \,\langle pp\,|O_{gg}|\,0\rangle\,; \quad i = 1, 2$$

$$\langle pp\,|O_3|\,h\rangle = J \otimes J \otimes S_3 \,\langle pp\,|O_{gg}|\,0\rangle$$

$$O_{gg} = \frac{1}{g_s^2} \mathcal{G}_{n_1}^{\perp a} \cdot \mathcal{G}_{n_2}^{\perp a}$$

Factorization: RBS scheme

$$F_{gg}^{(0)} = \overbrace{\left(H_1^{(0)} + \Delta H_1^{(0)}\right) S_1}^{T_1^{(0)}} + \overbrace{\int_0^1 \frac{\mathrm{d}z}{z} \left(\bar{H}_2^{(0)}(z) S_2^{(0)}(z) - [\![\bar{H}_2^{(0)}(z)]\!][\![S_2^{(0)}(z)]\!]\right)}^{T_2^{(0)}}$$

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subtraction

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subtraction

The add-backing subtraction has the same integrand
↪changing up-limits from ∞ to cutoffs
Ensure by re-factorization conditions

Factorization: RBS scheme

infinity-bin

subtraction

$$F_{gg}^{(0)} = \overbrace{\left(H_1^{(0)} + \overbrace{\Delta H_1^{(0)}}\right) S_1}^{T_1^{(0)}} + \overbrace{\int_0^1 \frac{\mathrm{d}z}{z}\left(\bar{H}_2^{(0)}(z) S_2^{(0)}(z) - [\![\bar{H}_2^{(0)}(z)]\!][\![S_2^{(0)}(z)]\!]\right)}^{T_2^{(0)}}$$
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Factorization: RBS scheme

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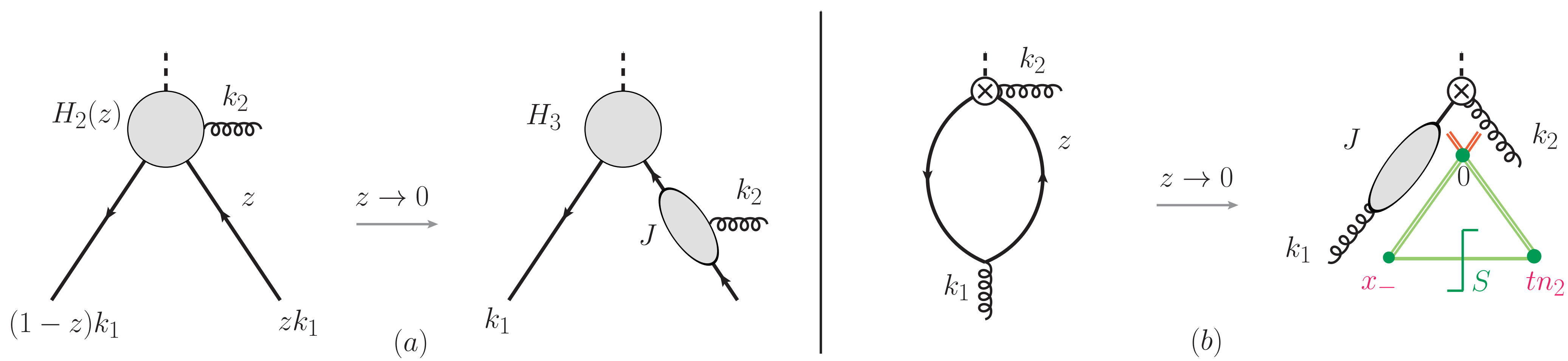
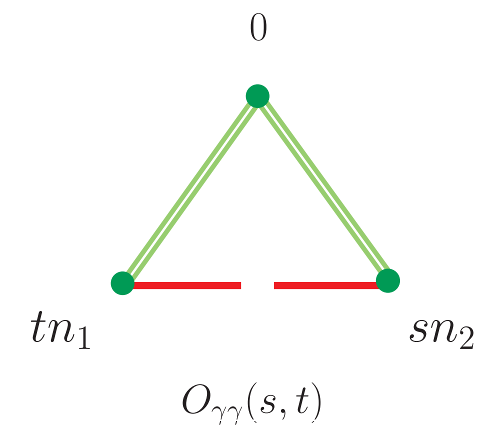


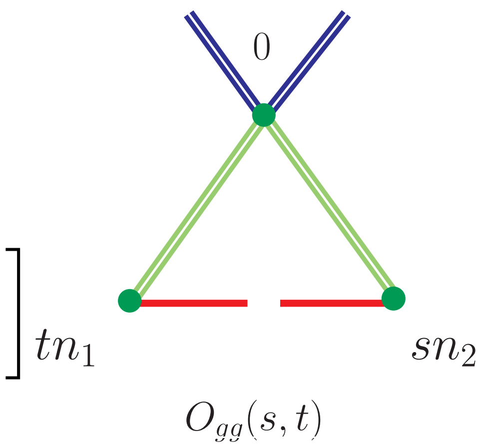
- Factorization
- **Renormalization**
- Resummation

Operator renormalization: soft function

$$O_{\gamma\gamma}(s,t) = \mathrm{Tr}_c \hat{\mathrm{T}} \left[\bar{q}(tn_1) Y_{n_1}(t) Y^\dagger_{n_1}(0) \frac{\not{n}_1 \not{n}_2}{4} Y_{n_2}(0) Y^\dagger_{n_2}(s) q(sn_2) \right]$$
$$= \mathrm{Tr}_c \hat{\mathrm{T}} \left[\bar{q}(tn_1) [tn_1, 0] \frac{\not{n}_1 \not{n}_2}{4} [0, sn_2] q(sn_2) \right]$$



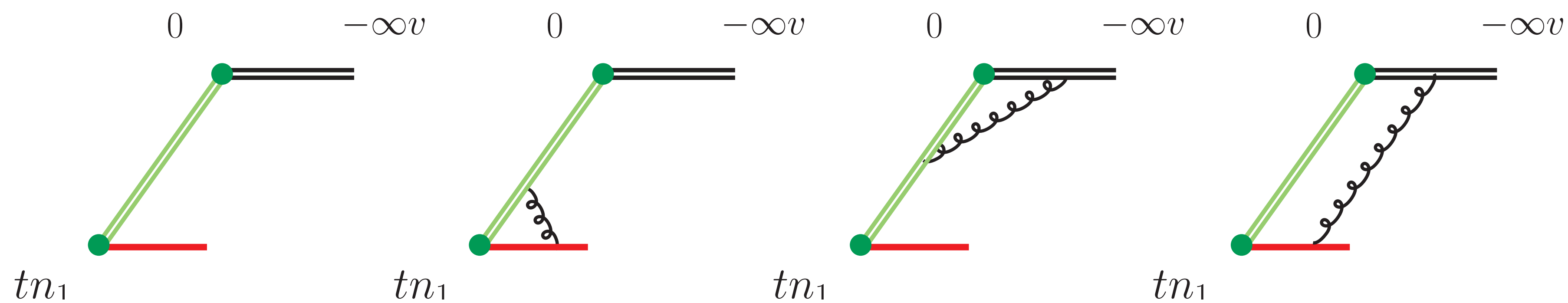
$$O_{gg}(s,t) = \mathrm{Tr}_c \hat{\mathrm{T}} \left[\bar{q}(tn_1) Y_{n_1}(t) T^a Y^\dagger_{n_1}(0) \frac{\not{n}_1 \not{n}_2}{4} Y_{n_2}(0) T^b Y^\dagger_{n_2}(s) q(sn_2) \right]$$
$$= \mathrm{Tr}_c \hat{\mathrm{T}} \left[\bar{q}(tn_1) [tn_1, 0] (\mathcal{Y}_{n_1}(0))^{ac} T^c \frac{\not{n}_1 \not{n}_2}{4} (\mathcal{Y}_{n_2}(0))^{bd} T^d [0, sn_2] q(sn_2) \right]$$



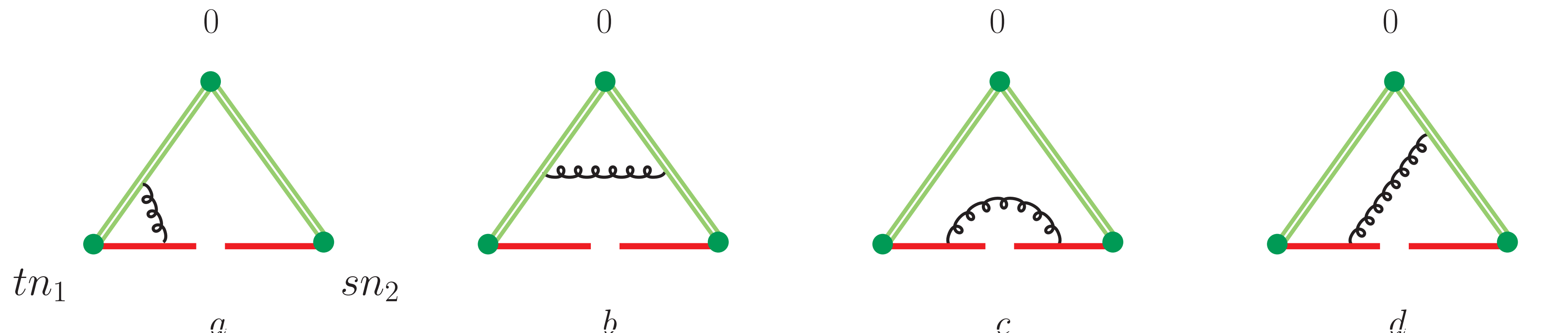
- We derived renormalization for these operators by RG consistency *['20, '21, '22]*;
- Then the one for $O_{\gamma\gamma}$ was calculated in momentum space by studying its analytic structure *[Bodwin et al, '21]*;

Operator renormalization: a new perspective (preliminary)

B-meson LCDA:



$S_{3,\gamma\gamma}$:



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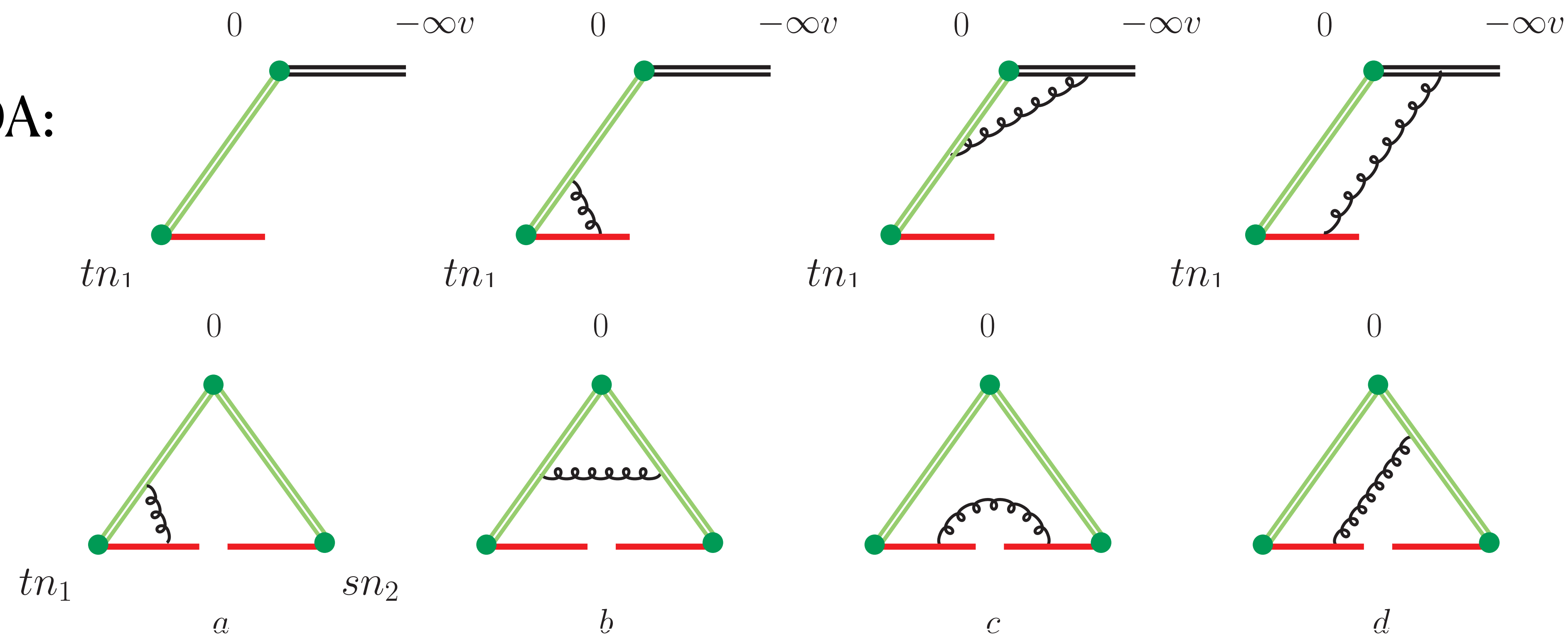

- The renormalization factor is calculated more efficiently and directly in position space *[In progress with Martin Beneke, and Yao Ji]*
  - ↪ other NLP operators?
  - No need to extensively analyse the analytical structure

Operator renormalization: a new perspective (preliminary)

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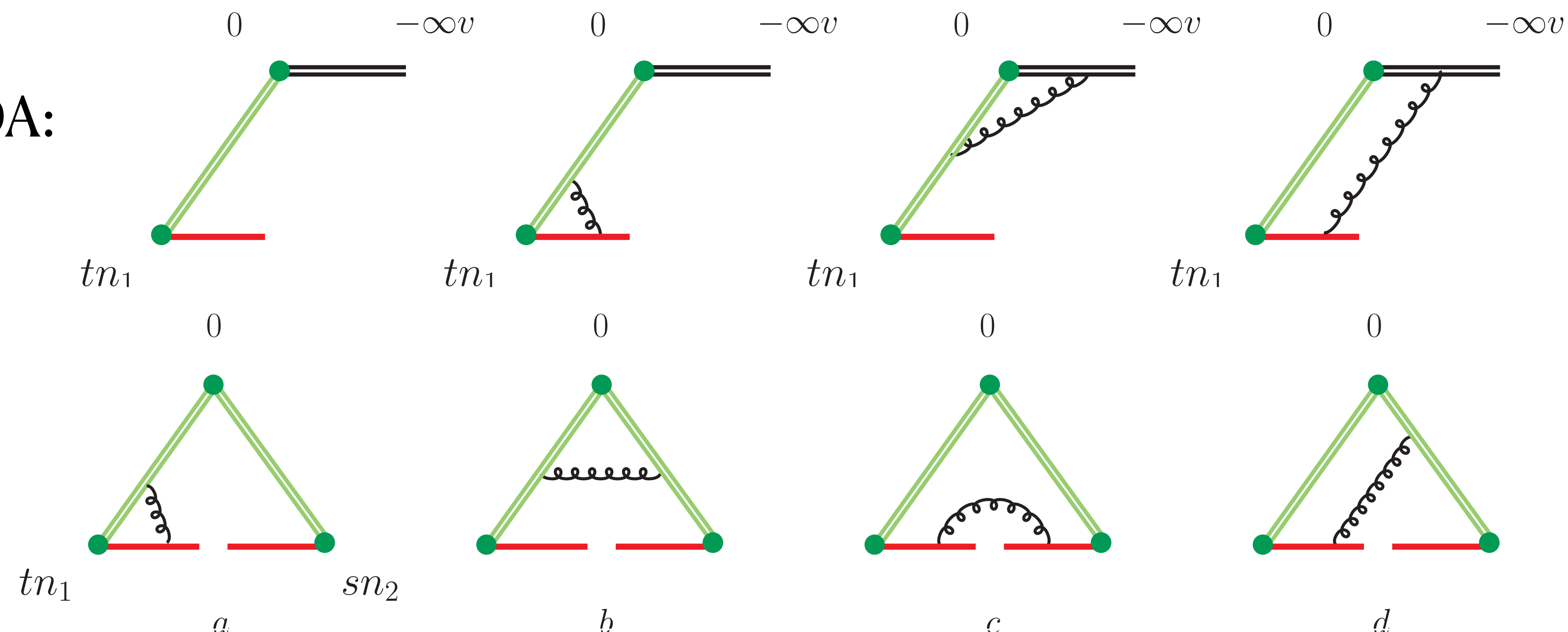

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$$S_3^{\gamma\gamma}(w;\mu) = \int_0^{+\infty} \mathrm{d}u \Bigg\{ \underbrace{\delta(1-u) + a_s C_F \left[\left(\frac{2}{\varepsilon^2} + \frac{2}{\varepsilon} \ln \frac{\mu^2}{w} - \frac{3}{\varepsilon} \right) \delta(1-u) - 4\Gamma(1,u) \right] + \mathcal{O}(a_s^2)}_{Z_S(w,uw)} \Bigg\} S_3^{\gamma\gamma}(uw)$$

Renormalized $H_1(\mu)$ in $h \to \gamma\gamma$

$$H_1(\mu) = H_1 Z_{11}^{-1} + 4\int_0^1 \frac{\mathrm{d}z}{z}\Big[\bar{H}_2(z,\mu)Z_{21}(z) - [\![\,\bar{H}_2(z,\mu)]\!][\![Z_{21}(z)]\!]\Big]Z_{11}^{-1}$$

$$+ \lim_{\sigma\to -1} H_3(\mu)\int_{M_h}^{\infty}\frac{\mathrm{d}\ell_-}{\ell_-}\int_{\sigma M_h}^{\infty}\frac{\mathrm{d}\ell_+}{\ell_+}J(M_h\ell_-,\mu)J(-M_h\ell_+,\mu)\frac{S_{3,\infty}(\ell_-\ell_+,\mu)}{m_b(\mu)}.$$

$$S_{3,\infty}(w,\mu) = \lim_{w\gg m_b^2} S_3(w,\mu)$$

Much more simpler than the soft function: power-like

Renormalized factorization formula

$$F_{gg}(\mu) = \overbrace{H_1(\mu)S_1(\mu)}^{T_1(\mu)} + \overbrace{\int_0^1 \frac{\mathrm{d}z}{z} \left(\bar{H}_2(z,\mu)S_2(z,\mu) - [\![\bar{H}_2(z,\mu)]\!][\![S_2(z,\mu)]\!]\right)}^{T_2(\mu)}$$

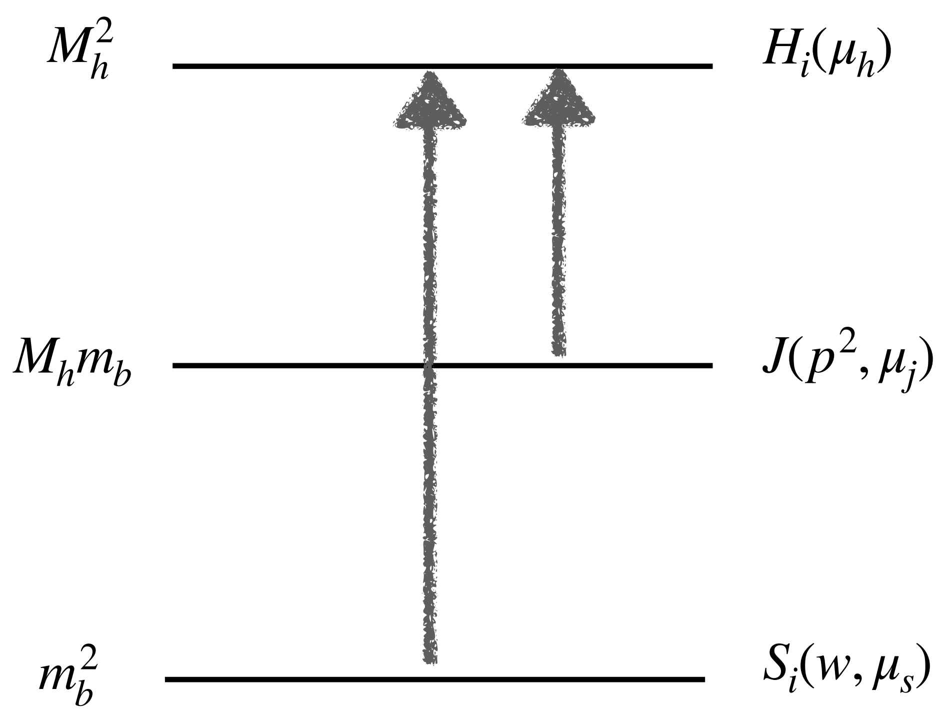
$$+ \underbrace{\lim_{\sigma\to -1} H_3(\mu) \int_0^{M_h} \frac{\mathrm{d}\ell_-}{\ell_-} \int_0^{\sigma M_h} \frac{\mathrm{d}\ell_+}{\ell_+} J\left(M_h\ell_-,\mu\right) J\left(-M_h\ell_+,\mu\right) S_3\left(\ell_-\ell_+,\mu\right)\Big|_{\mathrm{LP}}}_{T_3(\mu)}$$

- Free of endpoint divergences
- Cutoff arises from subtraction and rearrangement
- The price to pay: renormalization does not commute with cut-off convolution $\longrightarrow$ mismatch
    - For inclusive $\bar{B} \to X_s\gamma$: *[Robert, Tobias, '23]*, it does commute!

- Factorization
- Renormalization
- **Resummation**

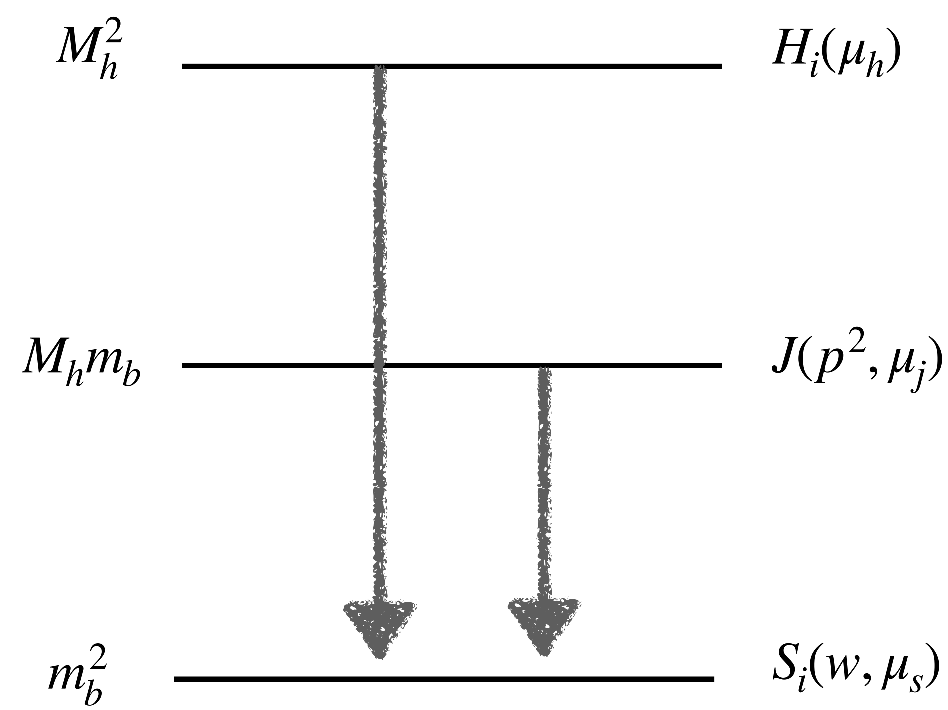
Evolve RGEs to resum logarithms

bottom-up



- No need to evolve H_1
- Resummed S_3 is complicated
- S_2 is the most complicated due to mixing and subtraction
- **Endpoint divergences cancel after evolution!**

top-down



- No need to evolve S_i
- Need to well understand RGE of H_1
- **Endpoint divergences cancel after evolution!**

In progress with Zelong, Marvin and Matthias

bottom-up: Logarithmic structure ($gg \to h$)

$$T_1(\mu_h) = T_F\,\delta_{ab}\frac{y_b(\mu_h)}{\sqrt{2}}\frac{\alpha_s(\mu_h)}{\pi}m_b(\mu_h)\left[-2 + \sum_{n\geq 1}\alpha_s(\mu_h)^n\,a_n\right],$$

$$T_2(\mu_h) = T_F\,\delta_{ab}\frac{y_b(\mu_h)}{\sqrt{2}}\frac{\alpha_s(\mu_h)}{\pi}m_b(\mu_h)\sum_{n\geq 1}\alpha_s(\mu_h)^n\sum_{i=0}^{n+1}b_{n,i}\,L^i\,,$$

$$L = \ln\frac{-M_h^2}{m_b^2}$$

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RG-impr. PT	Log. approx.	$\Gamma_{\text{cusp}}\,,\beta$	γ	$H_3\,,S_3\,,J$	$\alpha_s \cdot \alpha_s^n\,L^k$
–	LL	LO	–	LO	$k = 2n$
LO	NLL	NLO	LO	LO	$2n-1 \leq k \leq 2n$
–	NLL′	NLO	LO	NLO	$2n-2 \leq k \leq 2n$
NLO	NNLL	NNLO	NLO	NLO	$2n-3 \leq k \leq 2n$

bottom-up: RGE for S_3

$$\gamma_S^{gg}(w,w') = -\frac{\alpha_s}{\pi}\left\{\left[(C_F - C_A)L_w + \frac{3C_F-\beta_0}{2}\right]\delta(w-w') + 2\left(C_F - \frac{C_A}{2}\right)w\Gamma(w,w')\right\} + \mathcal{O}(\alpha_s^2)$$

$$S_{3,gg}^{\rm LO}(w,\mu) = U_S\,(w;\mu_s,\mu)\int_0^\infty \frac{{\rm d}w'}{w'} S_3^{\rm LO}\,(w',\mu_s)\, I_{2,2}^{1,1}\left(\begin{matrix}(-a_{\Delta\Gamma},1,2r_\Gamma) & ; & (1-a_{\Delta\Gamma},1,2r_\Gamma) \\ (1,1,2r_\Gamma) & ; & (0,1,2r_\Gamma)\end{matrix}\,\middle|\,\frac{w'}{w}\right)$$

$$S_{3,\gamma\gamma}^{\rm LO}(w,\mu) = U_S\,(w;\mu_s,\mu)\int_0^\infty \frac{{\rm d}w'}{w'} S_3^{\rm LO}\,(w',\mu_s)\, G_{4,4}^{2,2}\left(\begin{matrix}-a_\Gamma,-a_\Gamma,1-a_\Gamma,1-a_\Gamma \\ 1,1,0,0\end{matrix}\,\middle|\,\frac{w'}{w}\right)$$

$$= m_b U_S\,(w;\mu_s,\mu)\, G_{4,4}^{2,2}\left(\begin{matrix}-a_\Gamma,-a_\Gamma,1-a_\Gamma,1-a_\Gamma \\ 0,1,0,0\end{matrix}\,\middle|\,\frac{m_b^2}{w}\right)$$

$$S_{3,gg}^{\rm LO}\,(w,\mu_s)\,U_S\,(w;\mu_s,\mu)\left(\frac{\Gamma\left(1+a_{\Delta\Gamma}^{(0)}\,(\mu_s,\mu)\right)}{\Gamma\left(1-a_{\Delta\Gamma}^{(0)}\,(\mu_s,\mu)\right)}\right)^{2r_\Gamma} + \mathcal{O}\left(m_b^2/w\right)$$

$$r_\Gamma = \frac{C_F - C_A/2}{C_F - C_A}$$

bottom-up: NLL$'$ resummation ($gg \to h$)

$$
\begin{aligned}
T_3(\mu_h)|_{\mathrm{NLL}'} &= \mathcal{M}_0(\mu_h) L^2 \int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y \left[1 + \frac{\rho}{L}\frac{2\beta_0}{\Delta\Gamma_0}(x+y) + \frac{\rho^2}{L^2}\frac{4\beta_0^2}{(\Delta\Gamma_0)^2}(x+y)^2\right] \\
&\times \left\{1 + \frac{\rho}{L^2}\frac{2}{\Delta\Gamma_0}\left[C_F\left(8 - \frac{2\pi^2}{3}\right) + C_A\left(2 + \frac{\pi^2}{6}\right)\right]\right\} \\
&\times \exp\left[2S_{\Delta\Gamma}^{(0)}(\mu_s,\mu_h) - 2S_{\Delta\Gamma}^{(0)}(\mu_-,\mu_h) - 2S_{\Delta\Gamma}^{(0)}(\mu_+,\mu_h) + a_{\gamma_s}^{(0)}(\mu_s,\mu_h) + a_{\gamma_m}^{(0)}(\mu_s,\mu_h)\right]_{\mathrm{NLL}'} \\
&= \mathcal{M}_0(\mu_h)\frac{L^2}{2}\sum_{n=0}^{\infty}(-\rho)^n \frac{2\Gamma(n+1)}{\Gamma(2n+3)}\left\{1 + \frac{1}{L}\left[\rho\frac{-(\gamma_s^0+\gamma_m^0)+2\beta_0}{\Delta\Gamma_0}\frac{2n+2}{2n+3}\right.\right. \\
&\left. - \rho^2\frac{\beta_0}{\Delta\Gamma_0}\frac{(n+1)^2}{(2n+3)(2n+5)}\right] + \frac{1}{L^2}\left[\rho\frac{C_F\left(4-\frac{\pi^2}{3}\right)+C_A\left(1+\frac{\pi^2}{12}\right)}{C_F-C_A}\right. \\
&+ \rho^2\left(-\frac{\beta_0(\gamma_s^0+\gamma_m^0)}{(\Delta\Gamma_0)^2}\frac{n+1}{n+2} - \frac{\Delta\Gamma_1}{(\Delta\Gamma_0)^2}\frac{(n+1)^2}{(n+2)(2n+3)}\right. \\
&+ \left.\frac{(\gamma_s^0+\gamma_m^0)^2}{(\Delta\Gamma_0)^2}\frac{n+1}{2(n+2)} - \frac{\beta_0(\gamma_s^0+\gamma_m^0)}{(\Delta\Gamma_0)^2}\frac{2(n+1)}{n+2} + \frac{\beta_0^2}{(\Delta\Gamma_0)^2}\frac{4(n+1)}{n+2}\right) \\
&+ \rho^3\left(\frac{\beta_0(\gamma_s^0+\gamma_m^0)}{(\Delta\Gamma_0)^2}\frac{(n+1)^2}{2(n+3)(2n+3)} - \frac{\beta_0^2}{(\Delta\Gamma_0)^2}\frac{(n+1)^2(7n+18)}{6(n+3)(2n+3)(2n+5)}\right. \\
&\left.\left.\left. - \frac{\beta_0^2}{(\Delta\Gamma_0)^2}\frac{(n+1)^2}{(n+3)(2n+3)}\right) + \rho^4\frac{\beta_0^2}{(\Delta\Gamma_0)^2}\frac{(n+1)^2(n+2)}{8(n+4)(2n+3)(2n+5)}\right]\right\}
\end{aligned}
$$

$$
\rho = \frac{\alpha_s(\mu_h)}{4\pi}\frac{\Delta\Gamma_0}{2}L^2 = \frac{\alpha_s(\mu_h)}{2\pi}(C_F - C_A)L^2
$$

top-down: RGE for $H_1(h \to \gamma\gamma)$ (preliminary)

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} H_1(\mu) = H_1(\mu)\gamma_{11} + 4\int_0^1 \frac{\mathrm{d}z}{z}\left(\bar{H}_2(z,\mu)\gamma_{21}(z) - [\![\bar{H}_2(z,\mu)]\!][\![\gamma_{21}(z)]\!]\right) + D_{\text{cut}}$$

$$\text{with} \quad D_{\text{cut}} = -2\int_0^\infty \mathrm{d}x K(x) \int_{M_h}^{M_h/x} \frac{\mathrm{d}\ell_-}{\ell_-} [\![\bar{H}_2(x\ell_-/M_h,\mu)]\!] \underbrace{\left[\int_{\sigma M_h \ell_-}^\infty \frac{\mathrm{d}w}{w} \frac{S_{3,\infty}(w,\mu)}{S_1(\mu)} J(w/z,\mu) - 4[\![Z_{21}(z)]\!] Z_{11}^{-1}\right]}_{\Delta_{21}(z,\mu)}$$

$$\Delta_{21}(z,\mu) = 2\sum_{n\geq 0}\left(\frac{a_s\Gamma_0}{2}\right)^n \frac{(-L_z)^n}{(n+1)!}\left[B_0\left(L_h + \frac{L_z}{2}\right)^{n+1} - \frac{B_{n+1}}{(n+1)!} L_z^{n+1}\right] + \cdots$$

Logarithms in this anomalous dimension can be resummed to all orders!

[In progress with Zelong, Marvin and Marvin]

Hints NLP problems

- Find re-factorization conditions to establish re-factorization based subtraction to deal with endpoint divergences
  - Is subtraction the only way to kill endpoint divergences? Maybe not!
    If unfortunately yes, we'd better find a systematical way to construct it.
  - It works for endpoint divergences additive cancellation, non-additive cases? → *Guido's talk*.
- Renormalizing operators, which do ***not*** re-factorize, may be easier. To this extent, they are building blocks.
  - RG consistency
  - e.g., operators built with Wilson lines and soft-quark fields
    - Connection with QCD kernels? Symmetry tools?
- Mismatch or not? Rearrange cutoff contributions.
- Resummation is highly constrained by ignorance of solving RGEs
  - More tools to disentangle complicated functions therein.

Conclusion and Outlook

- So far, RBS scheme is the only way to cure endpoint divergences w/w.o. renormalization
- Factorization-renormalization-resummation workflow is now complete.
- Re-factorization conditions in Higgs form factors are relatively simple and useful. More insights for more complicated re-factorizations.
- Renormalize more NLP operators
- Interplay between "top-down" resummation and "bottom-up"

Backup

Operators

$$O_1 = \frac{m_b}{g_s^2}\, h\, \mathcal{G}_{n_1}^{\perp\mu,a}\, \mathcal{G}_{n_2\,\mu}^{\perp a}\,,$$

$$O_2(z) = h \left[\bar{\mathcal{X}}_{n_1} \gamma_\perp^\mu T^a \frac{\bar{\not{n}}_1}{2}\, \delta(z\,\bar{n}_1\cdot k_1 + i\bar{n}_1\cdot\partial)\, \mathcal{X}_{n_1}\right] \mathcal{G}_{n_2\,\mu}^{\perp,a}\,,$$

$$O_3 = T\left\{h\,\bar{\mathcal{X}}_{n_1}\mathcal{X}_{n_2}, i\int d^D x\, \mathcal{L}_{q\,\xi_{n_1}}^{(1/2)}(x), i\int d^D y\, \mathcal{L}_{\xi_{n_2}q}^{(1/2)}(y)\right\} + \text{h.c.}\,,$$

$$\langle pp|\, O_i\, |h\rangle = S_i\, \langle pp|\, O_{gg}\, |0\rangle\,; \quad i = 1, 2\,,$$

$$\langle pp|\, O_3\, |h\rangle = J \otimes J \otimes S_3\, \langle pp|\, O_{gg}\, |0\rangle$$

$$O_{gg} = \frac{1}{g_s^2}\mathcal{G}_{n_1}^{\perp a}\cdot\mathcal{G}_{n_2}^{\perp a}, \quad O_{\gamma\gamma} = \frac{1}{e_b^2}\mathcal{A}_{n_1}^{\perp}\cdot\mathcal{A}_{n_2}^{\perp}$$