MUON-ELECTRON BACKWARD SCATTERING AND ENDPOINT DIVERGENCES IN SCET

[GUIDO BELL]

based on: GB, P. Böer, T. Feldmann, JHEP 09 (2022), 183 [2205.06021]

A personal remark

Matthias' work has been a source of inspiration throughout my career . . .

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Physics Reports 245 (1994) 259–395

PHYSICS REPORTS

Heavy-quark symmetry

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Received February 1994; editor: R. Slansky

Contents:

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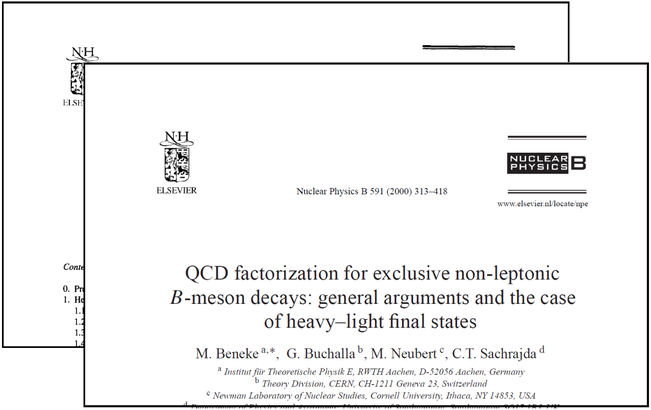
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Nuclear Physics B 591 (2000) 313–418
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QCD factorization for exclusive non-leptonic B-meson decays: general arguments and the case of heavy–light final states

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THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Drell–Yan production at small q_T, transverse parton distributions and the collinear anomaly

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Abstract Using methods from effective field theory, an exact all-order expression for the Drell–Yan cross section at small transverse momentum is derived directly in q_T space, in which all large logarithms are resummed. The anomalous dimensions and matching coefficients necessary for resummation at NNLL order are given explicitly. The precise relation between our result and the Collins–Soper–Sterman

and Sterman (CSS) [4], and explicit results for the ingredients necessary for resummation at next-to-next-to-leading logarithmic (NNLL) order were derived in [5–7]. The region of small q_T is of phenomenological importance, since it has the largest cross section and is used to extract the W-boson mass and width. In fact, the measurement of the charged-lepton q_T spectrum now gives the most precise determina-

A personal remark

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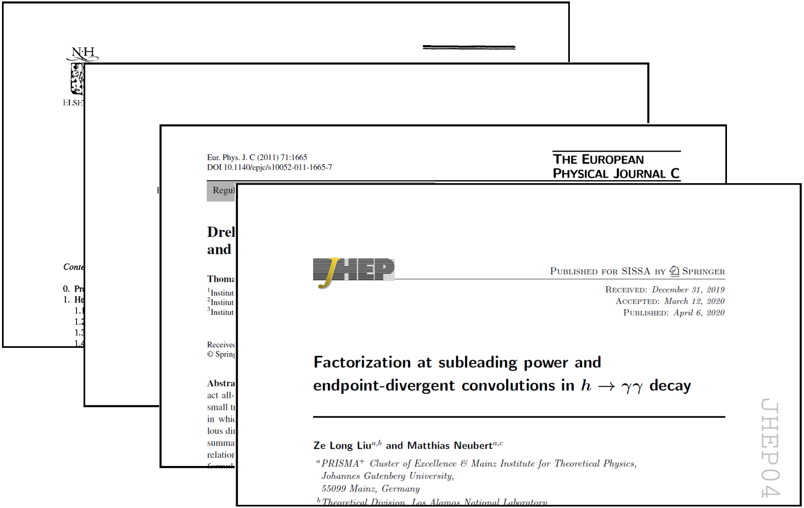

A personal remark

. . . but despite the common interests I only wrote one paper with him so far

Physics Letters B 704 (2011) 276–283

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Factorization and resummation for jet broadening

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ABSTRACT

Jet broadening is an event-shape variable probing the transverse momenta of particles inside jets. It has been measured precisely in e^+e^- annihilations and is used to extract the strong coupling constant. The factorization of the associated cross section at small values of the broadening is afflicted by a collinear anomaly. Based on an analysis of this anomaly, we present an all-order factorization theorem for jet-broadening distributions, which is free of large perturbative logarithms in the two-jet limit. Our formula reproduces known results at next-to-leading logarithmic order but also extends to higher orders.

A personal remark

. . . but despite the common interests I only wrote one paper with him so far

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Back to physics

At leading power soft-collinear factorisation is well understood

$$\frac{d\sigma}{d\tau} = H(Q,\mu) \int d\tau_n \, d\tau_{\bar{n}} \, d\tau_s \, J(\sqrt{\tau_n}Q,\mu) \, J(\sqrt{\tau_{\bar{n}}}Q,\mu) \, S(\tau_s Q,\mu) \, \delta(\tau - \tau_n - \tau_{\bar{n}} - \tau_s)$$

Back to physics

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At subleading power one often finds that the convolutions diverge at the endpoints

$$\int_0^1 \mathrm{d}z \; h(z) \, j(z) = \int_0^1 \mathrm{d}z \;\; z^{-\varepsilon} \; z^{-1-\varepsilon} \neq \int_0^1 \mathrm{d}z \left[1 - \varepsilon \ln z + \ldots\right] \left[-\frac{1}{\varepsilon}\delta(z) + \frac{1}{z_+} + \ldots\right]$$

$\Rightarrow$ convolution and renormalisation of EFT operators do not commute

$\Rightarrow$ prevents the use of RG techniques at subleading power

Back to physics

At leading power soft-collinear factorisation is well understood

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$\Rightarrow$ convolution and renormalisation of EFT operators do not commute

$\Rightarrow$ prevents the use of RG techniques at subleading power

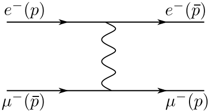
Generic problem in SCET at subleading power

- event shapes, DIS, $H \to \gamma\gamma$, $B \to X_s\gamma$, $B \to \pi\ell\nu$, $B \to \pi\pi$, $\ldots$

$\Rightarrow$ for some problems this has been solved at next-to-leading power

[Liu, Neubert 19; Beneke et al 22; Feldmann et al 22; Cornella, König, Neubert 22; Hurth, Szafron 23; ...]

Muon-electron scattering



backward scattering at high energies

$$s \sim -t \gg m_\mu^2 \sim m_e^2 \gg u$$

- equal masses $m_\mu = m_e \equiv m$
  exact backward limit $\theta = \pi$
  $\Bigg\}$ expansion parameter $\lambda = \frac{m}{\sqrt{s}} \ll 1$

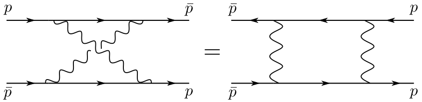
- $$\mathcal{M}(e^-\mu^- \to e^-\mu^-) = F_1(\lambda)\,\underbrace{\mathcal{M}^{(0)}}_{\text{helicity-conserving}} + F_2(\lambda)\,\underbrace{\widetilde{\mathcal{M}}}_{\text{helicity-flipping}} + \mathcal{O}(\lambda)$$

- two form factors

$$F_1(\lambda) \simeq 1 + \frac{\alpha_{\rm em}}{2\pi}\,\frac{1}{2}\ln^2\lambda^2 + \ldots$$ $\Leftarrow$ focus on double logarithms $\left(\frac{\alpha_{\rm em}}{2\pi}\right)^n \ln^{2n}\lambda^2$

$$F_2(\lambda) \simeq \frac{\alpha_{\rm em}}{2\pi}\,\ln\lambda^2 + \ldots$$

NLO analysis



$$p^\mu = \frac{\sqrt{s}}{2}\, n_-^\mu + \frac{m^2}{2\sqrt{s}}\, n_+^\mu$$

$$\bar{p}^\mu = \frac{\sqrt{s}}{2}\, n_+^\mu + \frac{m^2}{2\sqrt{s}}\, n_-^\mu$$

Double logarithms arise from configurations in which the fermions are soft $k^\mu \sim m$

$$F_1^{(1)}(\lambda) \sim \int \frac{d^d k}{(2\pi)^d} \frac{k_\perp^2}{(k^2-m^2)^2 (k-p)^2 (k-\bar{p})^2} \sim \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2-m^2)(k-p)^2(k-\bar{p})^2}$$

$\Rightarrow$ photon propagators become eikonal

$\Rightarrow$ integration over $k_\perp$ yields discontinuity of fermion propagator

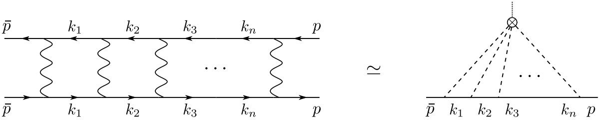
$\Rightarrow$ in the traditional approach one regularises the remaining integrations with hard cutoffs

$$F_1^{(1)}(\lambda) \sim \int_0^1 \frac{dx}{x} \int_0^1 \frac{dy}{y}\, \theta\big(xy - \lambda^2\big) = \frac{1}{2} \ln^2 \lambda^2$$

$$n_+ k = x\sqrt{s}$$

$$n_- k = y\sqrt{s}$$

All-order structure



Each subloop generates the double logarithms in a similar way

$\Rightarrow$ all fermion propagators go on-shell, but their long. momenta are strongly ordered

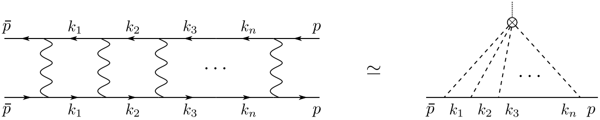
$$\frac{m^2}{\sqrt{s}} \approx n_+\bar{p} \ll n_+k_1 \ll \ldots \ll n_+k_n \ll n_+p \approx \sqrt{s}$$

$$\frac{m^2}{\sqrt{s}} \approx n_-p \ll n_-k_n \ll \ldots \ll n_-k_1 \ll n_-\bar{p} \approx \sqrt{s}$$

$\Rightarrow$ all photon propagators become eikonal

$$F_1^{(n)}(\lambda) \sim \int_{\lambda^2}^1 \frac{dx_1}{x_1} \int_{x_1}^1 \frac{dx_2}{x_2} \ldots \int_{x_{n-1}}^1 \frac{dx_n}{x_n} \int_{\lambda^2/x_1}^{1} \frac{dy_1}{y_1} \int_{\lambda^2/x_2}^{y_1} \frac{dy_2}{y_2} \ldots \int_{\lambda^2/x_n}^{y_{n-1}} \frac{dy_n}{y_n} = \frac{\ln^{2n}\lambda^2}{n!\,(n+1)!}$$

All-order structure



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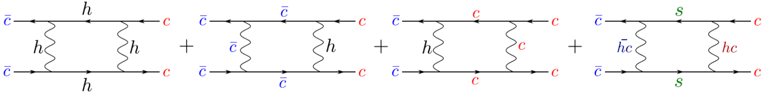
$$\frac{m^2}{\sqrt{s}} \approx n_-p \ll n_-k_n \ll \ldots \ll n_-k_1 \ll n_-\bar{p} \approx \sqrt{s}$$

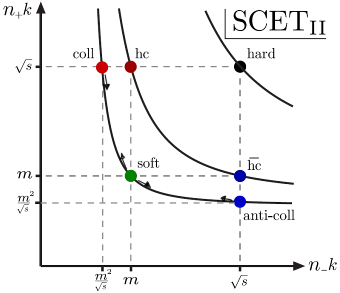
Nested integrals resum to a modified Bessel function [Gorshkov, Gribov, Lipatov, Frolov 66]

$$F_1(\lambda) = \sum_{n=0}^{\infty} \left(\frac{\alpha_{\rm em}}{2\pi}\right)^n F_1^{(n)}(\lambda) \simeq \frac{I_1\left(2\sqrt{z}\right)}{\sqrt{z}} \qquad\qquad z = \frac{\alpha_{\rm em}}{2\pi} \ln^2 \lambda^2$$

SCET formulation

At fixed order one can reproduce the double logarithms with the method of regions





Loop integrals require rapidity regulator

- $\left(\dfrac{\nu}{n_+k + n_-k - i0}\right)^{\alpha}$

$\Rightarrow$ symmetry between collinear/anti-collinear

- $\left(\dfrac{\nu}{n_+k - i0}\right)^{\alpha}$

$\Rightarrow$ soft integrals are scaleless and vanish

Factorisation theorem

$$F_1(\lambda) = \int \frac{dx}{x} \int \frac{dy}{y}\, f_c(x)\, H(xy)\, f_{\bar{c}}(y)$$

$$+ \int \frac{dx}{x} \int \frac{dy}{y} \int \frac{d\rho}{\rho} \int \frac{d\omega}{\omega}\, f_c(x)\, J_{hc}(x\rho)\, S(\rho\omega)\, J_{\bar{hc}}(\omega y)\, f_{\bar{c}}(y)$$

Factorisation theorem

$$F_1(\lambda) = \int \frac{dx}{x} \int \frac{dy}{y} \; f_c(x)\; H(xy)\; f_{\bar{c}}(y)$$

$$+ \int \frac{dx}{x} \int \frac{dy}{y} \int \frac{d\rho}{\rho} \int \frac{d\omega}{\omega} \; f_c(x)\; J_{hc}(x\rho)\; S(\rho\omega)\; J_{\bar{hc}}(\omega y)\; f_{\bar{c}}(y)$$

Standard hard-scattering picture:

- hard function from QED $\rightarrow$ SCET-1 matching
- flavour off-diagonal parton distribution functions

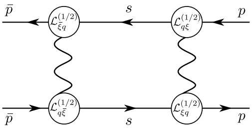
$$\langle \mu^-(p)| \underbrace{\bar{\chi}_c^{(\mu)}(\tau n_+) \frac{\not{n}_+}{2} P_R\, \chi_c^{(e)}(0)}_{\text{leading twist}} |e^-(p)\rangle = \int dx\, e^{ix\tau n_+ p} \Big\{ f_c(x)\, \underbrace{[\bar{u}_\xi^{(\mu)} \frac{\not{n}_+}{2} P_R u_\xi^{(e)}]}_{\text{helicity-conserving}} + \widetilde{f}_c(x)\, \underbrace{[\bar{u}_\xi^{(\mu)} \frac{\not{n}_+}{2} P_L u_\xi^{(e)}]}_{\text{helicity-flipping}} \Big\}$$

Factorisation theorem

$$F_1(\lambda) = \int \frac{dx}{x} \int \frac{dy}{y} \; f_c(x)\; H(xy)\; f_{\bar{c}}(y)$$

$$+ \int \frac{dx}{x} \int \frac{dy}{y} \int \frac{d\rho}{\rho} \int \frac{d\omega}{\omega} \; f_c(x)\; J_{hc}(x\rho)\; S(\rho\omega)\; J_{\bar{hc}}(\omega y)\; f_{\bar{c}}(y)$$

Soft-enhancement mechanism:



suppression from subleading Lagrangian insertions is compensated by soft propagators

- jet functions from SCET-1 $\to$ SCET-2 matching
- $S(\rho\omega) \sim \int d(n_- x_1) \int d(n_+ x_2)\; e^{\frac{i}{2}\rho(n_+ x_2)}\; e^{-\frac{i}{2}\omega(n_- x_1)} \int d^d x_3$

$$\times \langle 0|\, T\,[\bar{\psi}_s^{(\mu)} \bar{S}_{n_+}](x_{1+}) \frac{\not{n}_+}{2} P_R\,[S^\dagger_{n_+} \psi_s^{(e)}](0)\,[\bar{\psi}_s^{(e)} \bar{S}_{n_-}](x_3) \frac{\not{n}_-}{2} P_R\,[S^\dagger_{n_-} \psi_s^{(\mu)}](x_{2-} + x_3)|0\rangle$$

Factorisation theorem

$$F_1(\lambda) = \int_0^1 \frac{dx}{x} \int_0^1 \frac{dy}{y}\, f_c(x)\, H(xy)\, f_{\bar{c}}(y)$$

$$+ \int_0^1 \frac{dx}{x} \int_0^1 \frac{dy}{y} \int_0^\infty \frac{d\rho}{\rho} \int_0^\infty \frac{d\omega}{\omega}\, f_c(x)\, J_{hc}(x\rho)\, S(\rho\omega)\, J_{\bar{hc}}(\omega y)\, f_{\bar{c}}(y)$$

Bare factorisation theorem suffers from endpoint divergences

- rapidity divergences cancel only in the sum of the two terms

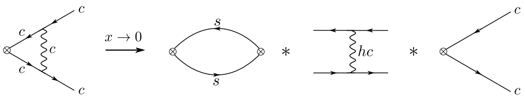
$\Rightarrow$ both terms must describe the same physics in the endpoint region

Factorisation theorem

$$F_1(\lambda) = \int_0^1 \frac{dx}{x} \int_0^1 \frac{dy}{y}\, f_c(x)\, H(xy)\, f_{\bar{c}}(y)$$

$$+ \int_0^1 \frac{dx}{x} \int_0^1 \frac{dy}{y} \int_0^\infty \frac{d\rho}{\rho} \int_0^\infty \frac{d\omega}{\omega}\, f_c(x)\, J_{hc}(x\rho)\, S(\rho\omega)\, J_{\overline{hc}}(\omega y)\, f_{\bar{c}}(y)$$

Refactorisation of collinear matrix elements [Böer 18; Liu,Neubert 19]



$$f_c(x \to 0) \simeq \int \frac{dx'}{x'} \int \frac{d\rho}{\rho}\, f_c(x')\, J_{hc}(x'\rho)\, S(\rho x)$$

$\Rightarrow$ brings the hard-scattering term into the same form as the soft contribution!

note: the problem of endpoint divergences repeats itself in the refactorisation condition

Resummation

Double logarithms can be resummed using consistency relations

- poles in $1/\alpha$ and $1/\varepsilon$ must cancel in $F_1(\lambda)$
- each pole comes with characteristic logarithm
- non-trivial constraint from refactorisation condition

Derivation uses asymmetric rapidity regulator

$$F_1(\lambda) \simeq \int_0^1 \frac{dx}{x}\, f_c\Big(x; \frac{\mu}{m}, \frac{\nu}{\sqrt{s}}\Big) \int_0^1 \frac{dy}{y}\, f_{\bar{c}}\Big(y; \frac{\mu}{m}, \frac{\nu\sqrt{s}}{m^2}\Big)\, H\Big(\frac{\mu^2}{xys}\Big)$$

$$\simeq \sum_{n=0}^{\infty} z_h^n\, h^{(n)}\, \langle x^{-1-n\varepsilon}\rangle_{f_c}\Big(\frac{\mu}{m}, \frac{\nu}{\sqrt{s}}\Big)\, \langle y^{-1-n\varepsilon}\rangle_{f_{\bar{c}}}\Big(\frac{\mu}{m}, \frac{\nu\sqrt{s}}{m^2}\Big) \qquad z_h = \frac{\alpha_{\rm em}}{2\pi}\,\frac{1}{\varepsilon^2}\Big(\frac{\mu^2}{s}\Big)^{\varepsilon}$$

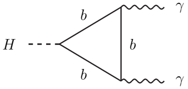
$$\simeq \sum_{n=0}^{\infty} z_h^n\, h^{(n)}\, r_n(\mu/m) \times \Big(\frac{m^2}{s}\Big)^{\mathcal{F}_n(\mu/m)}$$

$\Rightarrow$ reproduce modified Bessel function order-by-order in perturbation theory

(up to a single coefficient that can be extracted from the one-loop expressions)

$H \to \gamma\gamma$

Bottom-quark contribution



- scale hierachy $m_b \ll M_H$
- subleading power due to helicity suppression

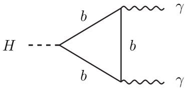
Bare factorisation theorem

[Liu, Neubert 19]

$$\begin{aligned}
\mathcal{M}_b(H \to \gamma\gamma) \sim\; & H_1 \, \langle O_1 \rangle \\
& + 2 \int_0^1 \frac{dz}{z} \, \bar{H}_2(z) \, \langle O_2(z) \rangle \\
& + H_3 \int_0^\infty \frac{d\ell_-}{\ell_-} \int_0^\infty \frac{d\ell_+}{\ell_+} \; J(M_H \ell_+) \, J(M_H \ell_-) \, S(\ell_+ \ell_-)
\end{aligned}$$

$H \to \gamma\gamma$

Bottom-quark contribution



- scale hierachy $m_b \ll M_H$
- subleading power due to helicity suppression

Bare factorisation theorem is spoilt by endpoint divergences [Liu, Neubert 19]

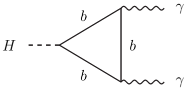
$$\mathcal{M}_b(H \to \gamma\gamma) \sim H_1 \, \langle O_1 \rangle + 2 \int_0^1 \frac{dz}{z} \, \bar{H}_2(z) \, \langle O_2(z) \rangle + H_3 \int_0^\infty \frac{d\ell_-}{\ell_-} \int_0^\infty \frac{d\ell_+}{\ell_+} \, J(M_H \ell_+) \, J(M_H \ell_-) \, S(\ell_+ \ell_-)$$

- in the endpoint regions the two terms describe the same physics

$\Rightarrow$ is there a way to combine these contributions?

$H \to \gamma\gamma$

Bottom-quark contribution



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- subleading power due to helicity suppression

Bare factorisation theorem is free from endpoint divergences [Liu, Neubert 19]

$$\mathcal{M}_b(H \to \gamma\gamma) \sim (H_1 + \Delta H_1) \langle O_1 \rangle$$

$$+ 2 \int_0^1 \frac{dz}{z} \left\{ \bar{H}_2(z) \langle O_2(z) \rangle - \left[\!\left[\bar{H}_2(z) \langle O_2(z) \rangle \right]\!\right]_0 - \left[\!\left[\bar{H}_2(z) \langle O_2(z) \rangle \right]\!\right]_1 \right\}$$

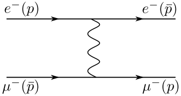
$$+ H_3 \int_0^{M_H} \frac{d\ell_-}{\ell_-} \int_0^{M_H} \frac{d\ell_+}{\ell_+} \, J(M_H \ell_+) \, J(M_H \ell_-) \, S(\ell_+ \ell_-)$$

- rearrangement based on refactorisation conditions

$$\left[\!\left[\bar{H}_2(z) \langle O_2(z) \rangle \right]\!\right]_0 = \frac{H_3}{2} \int_0^\infty \frac{d\ell_+}{\ell_+} \, J(M_H \ell_+) \, J(z \, M_H^2) \, S(\ell_+ z \, M_H)$$

Muon-electron scattering

High-energy backward scattering



- scale hierachy $m \ll \sqrt{s}$
- leading power QED process

Bare factorisation theorem

[GB, Böer, Feldmann 22]

$$F_1(\lambda) = \int_0^1 \frac{dx}{x} \int_0^1 \frac{dy}{y} \, f_c(x) \, H(xy) \, f_{\bar{c}}(y)$$

$$+ \int_0^1 \frac{dx}{x} \int_0^1 \frac{dy}{y} \int_0^\infty \frac{d\rho}{\rho} \int_0^\infty \frac{d\omega}{\omega} \, f_c(x) \, J(x\rho) \, S(\rho\omega) \, J(\omega y) \, f_{\bar{c}}(y)$$

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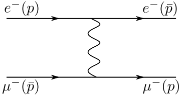

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Bare factorisation theorem is spoilt by endpoint divergences [GB, Böer, Feldmann 22]

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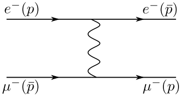

- scale hierachy $m \ll \sqrt{s}$
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Bare factorisation theorem is spoilt by endpoint divergences [GB, Böer, Feldmann 22]

$$F_1(\lambda) = \int_0^1 \frac{dx}{x} \int_0^1 \frac{dy}{y}\; f_c(x)\; H(xy)\; \Big\{ f_{\bar{c}}(y) - \big[\!\big[f_{\bar{c}}(y) \big]\!\big]_0 \Big\}$$

$$+ \int_0^1 \frac{dx}{x} \int_0^1 \frac{dy}{y} \int_0^{\sqrt{s}} \frac{d\rho}{\rho} \int_0^{\infty} \frac{d\omega}{\omega}\; f_c(x)\; J(x\rho)\; S(\rho\omega)\; J(\omega y)\; f_{\bar{c}}(y)$$

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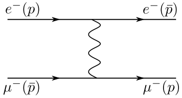

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$$+ f_c \otimes J \otimes S \otimes J \otimes S \otimes J \otimes f_{\bar{c}} + \ldots$$

$\Rightarrow$ system does not close under rearrangements

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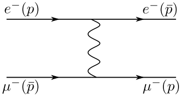


- scale hierachy $m \ll \sqrt{s}$
- leading power QED process

Double logarithms can be derived from self-consistency relation [GB, Böer, Feldmann 22]

$$F_1(\lambda) = \mathcal{F}_1(z) = 1 + z \int_0^1 d\xi \int_0^1 d\eta\, \mathcal{F}_1(\xi^2 z)\, \theta(1-\xi-\eta)\, \mathcal{F}_1(\eta^2 z) = \frac{I_1(2\sqrt{z})}{\sqrt{z}}$$

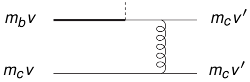
in terms of logarithmic variables $z = \frac{\alpha}{2\pi} \ln^2 \lambda^2 , \quad \xi = \frac{\ln x}{\ln \lambda^2} , \quad \eta = \frac{\ln y}{\ln \lambda^2}$

Compare to $H \to \gamma\gamma$

$$\mathcal{F}_b(z) = 2 \int_0^1 d\xi \int_0^1 d\eta\, \theta(1-\xi-\eta)\, e^{-2\xi\eta z} = {}_2F_2\big(1,1;3/2,2;-z/2\big)$$

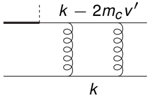
Exclusive B decays

$B_c \to \eta_c \ell \nu$ in non-relativistic approximation



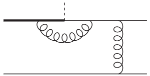
- $m_b \gg m_c \gg \Lambda_{QCD}$
- large recoil $q^2 \ll m_b^2$

Pattern of endpoint divergences similar to muon-electron backward scattering



- no soft-enhancement mechanism
- recover same modified Bessel function (in QED)

Additional source of double logarithms



- Sudakov logarithms from heavy-light vertex
- non-trivial interplay of cusp and endpoint logs

Conclusions

Progress in understanding endpoint divergences in SCET

- partial solutions based on refactorisation conditions and rearrangement formulae
- more complicated for partonic $2 \to 2$ processes and beyond

Muon-electron backward scattering

- iterative pattern of endpoint-divergent convolution integrals
- endpoint logarithms sum up to modified Bessel function
- relation to exclusive B decays

Looking forward to the next MITP workshop . . . and further interactions with Matthias!